Article

Nonparametric Modeling and Control of Ship Steering Motion Based on Local Gaussian Process Regression

Zi-Lu Ouyang, Zao-Jian Zou and Lu Zou

Abstract: This paper aims to study the nonparametric modeling and control of ship steering motion. Firstly, the black box response model is derived based on the Nomoto model. Then, the establishment of a nonparametric response model and prediction of ship steering motion are realized by applying the local Gaussian process regression (LGPR) algorithm. To assess the performance of LGPR, two cases are studied, including a Mariner class vessel by using simulation data and a KVLCC2 tanker model by using experimental data. The results reveal that the response model identified by LGPR presents good prediction accuracy and low computational burden. Finally, the identified response model is used as the basis for developing the ship heading controller, and the results demonstrate that the proposed controller is able to achieve good dynamic performance.

Keywords: ship steering response model; nonparametric modeling; system identification; local gaussian process regression; ship heading control

1. Introduction

In the study of ship maneuverability, an accurate mathematical model of ship maneuvering motion plays a significant role. On the one hand, the ship dynamic model is the basis of developing simulators for the prediction of ship maneuvering behaviors. On the other hand, the design and validation of motion control methods always need a reliable ship dynamic model.

The classic mathematical models of ship maneuvering motion include the whole-ship model [1], the modular model [2] and the response model [3]. Among them, the whole-ship model and the modular model are nonlinear models, which can accurately predict large amplitude ship maneuvering motion. However, acquiring the unknown coefficients of the nonlinear models is always laborious and costly. On the other hand, the response model is usually used to design a controller of ship steering motion with small amplitude. It can be derived from the linearized whole-ship model, and its parameters (the maneuverability indices) consist of some linear hydrodynamic derivatives in the whole-ship model. With the response model, the three degrees of freedom (3-DoF) model of ship maneuvering motion on the horizontal plane is simplified to the mathematical relationship between the yaw rate and the rudder angle which has the advantages of a simple structure and the ability to capture the response characteristics of ship steering motion to the control input and has been extensively used to predict ship steering motion as well as to design ship autopilots [4–8].

As artificial intelligence technology develops, the unknown maneuverability indices in the response model can be identified by system identification methods in combination with machine learning algorithms in an easy to operate way. Since the mathematical expression of the response model is explicit, the parameter identification of the maneuverability indices is essentially a linear regression problem. Zhu et al. [9] proposed an improved least-squares support vector regression (LSSVR) method, which was optimized by using an artificial bee
colony algorithm for the purposes of identifying the parameters of the response model. Xie et al. [10] developed a least-squares algorithm based on the multi-innovation extended Kalman filtering method to improve the identification accuracy. Wang et al. [11] applied the nonlinear Gaussian filter (NGF) algorithm to identify the parameters of the response model in real time. Zhang et al. [12] employed the target-oriented crow search algorithm for identifying the response model, and the effectiveness of the identified model was validated on the ship heading control.

Although the parametric identification of the response model has achieved satisfactory results, the inherent problems of parametric identification, i.e., parameter drift and sensitivity to the noise in the training dataset caused by the measurement uncertainties and environmental interferences, are always hard to deal with. To solve these problems, non-parametric modeling methods have been used to model ship maneuvering motion. Among them, artificial neural network (ANN) methods have shown satisfactory performance due to their strong approximation capability [13–15]. However, the training of ANNs usually requires a large amount of sample data. Recently, kernel-based methods represented by Gaussian process regression (GPR) [16–18], support vector machines (SVM) [19], and locally weighted learning (LWL) [20,21] have also been successfully applied. With the aid of kernel-based methods, the training dataset can be mapped onto a high-dimensional Hilbert feature space, where the regression performance can be improved effectively compared with those in low-dimensional space. However, the calculation of the kernel matrix is time-consuming due to its large dimensions.

Although considerable progress has been achieved in the nonparametric modeling of 3-DoF ship maneuvering motion, until now there have been few studies of the nonparametric modeling of ship steering motion. In this study, a novel algorithm proposed in the previous study [22], local Gaussian process regression (LGPR), is applied to the modeling and prediction of ship steering motion. From the research results presented in [22], it is noted that LGPR does not need a large amount of data to establish the nonparametric model, while is more efficient and has acceptable prediction accuracy compared with classic GPR. Moreover, on the basis of the identified response model, an identification-based ship heading controller is developed in this paper.

The rest of this paper is structured as follows: Section 2 provides the derivation of the black box response model used in LGPR; Section 3 describes the mathematical derivation of the LGPR algorithm; in Section 4, the nonparametric response models of a Mariner class vessel and a KVLCC2 tanker model are established by LGPR; and in Section 5, the identified nonparametric response model is utilized to develop an identification-based ship heading controller, and the simulation test is performed for the purpose of evaluating the performance of the developed controller. Section 6 presents conclusions and prospects.

2. Response Model of Ship Steering Motion

The ship response model was derived by Nomoto [3] from the aspect of control engineering. It regards ship steering motion as the response to the input of the rudder angle. The second-order Nomoto model is mathematically expressed as follows:

\[ T_1 T_2 \ddot{r} + (T_1 + T_2) \dot{r} + r = K_\delta (T_3 \delta + \delta) \]

where \( r \) is the yaw rate, \( \delta \) is the rudder angle, \( T_1, T_2 \) and \( T_3 \) are the time coefficients of the model, and \( K_\delta \) is the gain coefficient.

Based on Equation (1), the transfer function between the rudder angle and the ship yaw motion can be derived by Laplace transform:

\[ G(s) = \frac{K_\delta (T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)} \]
Considering that the ship maneuvering motion is of low-frequency, Equation (2) can be simplified by neglecting the second-order variables as follows:

\[ Y(s) = \frac{K_\delta}{1 + Ts} \] (3)

Based on Equation (3), the first-order Nomoto model can be obtained by applying inverse Laplace transform:

\[ T\dot{r} + r = K_\delta \delta \] (4)

where \( T = T_1 + T_2 - T_3 \).

As can be seen from Equations (1) and (4), both the second-order and the first-order Nomoto models depict the relationship between \( r \) and \( \delta \). The mathematical expression of the black box model is given as Equation (5) on the basis of Equations (1) and (4):

\[ \dot{r} = g(r, \delta) \] (5)

where \( g \) is the nonlinear function which needs to be identified by the machine learning algorithm.

Figure 1 shows the flowchart of the modeling and prediction of ship steering motion on the basis of the identified nonparametric response model, where \( \dot{r} \) represents \( \dot{r} \); \( r(k) \), \( \dot{r}(k) \) and \( \delta(k) \) represent the motion variables and control variable of the \( k \)-th sample in the training dataset; \( r(t) \) and \( \delta(t) \) represent the inputted motion variable and control variable; and \( \dot{r}(t) \) represents the yaw acceleration in the predicted ship steering motion at time \( t \).

Figure 1. Flowchart of the modeling and prediction of ship steering motion based on the identified nonparametric response model.

As indicated in Figure 1, the nonparametric response model is identified based on the black box model given as Equation (5) and the training dataset, and the ship steering motion is predicted by using the identified response model.
3. Local Gaussian Process Regression Algorithm

3.1. Classic Gaussian Process Regression

As a nonparametric modeling method based on Bayesian theory, Gaussian process regression (GPR) can be mathematically derived on the basis of the definition of a Gaussian process (GP). A GP can be depicted by its mean and variance:

\[ f(x) \sim \text{GP}(\mu(x), k(x, x')) \]  

(6)

where \( f(x) \) is a GP, \( \mu(x) \) is the mean function, \( k(x, x') \) is the covariance function, and their mathematical expressions are as below:

\[ \mu(x) = E[f(x)] \]  

(7)

\[ k(x, x') = E[(f(x) - \mu(x))(f(x') - \mu(x'))] \]  

(8)

For a predictive sample \( Z \), in order to predict \( f_s = f(Z) \) over the training dataset \( D = \{x_i, y_i\}_{i=1}^{n} \), GPR first establishes a prior function and then transforms the prior function into a posterior distribution on the basis of Bayesian theory. The vector \( y \), which is composed of \( y_i(i = 1, 2, \ldots, n) \), has a joint Gaussian distribution with \( f_s \):

\[ \begin{bmatrix} y \\ f_s \end{bmatrix} \sim N(0, \begin{bmatrix} K(X, X) + \sigma_n^2 I_n & K(Z, X)^T \\ K(Z, X) & K(Z, Z) \end{bmatrix}) \]  

(9)

where \( X = [x_1, x_2, \ldots, x_n]^T \), \( K \) is the covariance matrix, whose elements can be calculated as \( K_{ij} = k_c(x_i, x_j) \), and \( k_c \) is the covariance function. \( I_n \) is a unit matrix of \( n \)-dimensions. \( \sigma_n^2 \) is the variance of the noise contained in the target value \( y \). Based on Equation (9), the distribution of \( f_s \) can be expressed as

\[ p(f_s|D, Z) = N(\mu, \hat{S}) \]  

(10)

\[ \mu = K(Z, X)Ay \]  

(11)

\[ \hat{S} = K(Z, Z) - K(Z, X)AK(Z, X)^T \]  

(12)

where \( \mu \) is the mean of \( f_s \), \( \hat{S} \) is the variance of \( f_s \), and \( A \) refers to the equation

\[ A = [K(X, X) + \sigma_n^2 I_n]^{-1} \]  

Equations (10)–(12) are the key regression equations of GPR, and the detailed mathematical derivation can be found in [23].

3.2. Local Gaussian Process Regression

A novel algorithm proposed in the previous study [22], local Gaussian process regression (LGPR) is used to identify the response model and predict the ship steering motion with high computational efficiency. According to the results in the previous study [22], LGPR is more efficient than other methods and the prediction accuracy is acceptable.

In LGPR, according to similarity criterion among the samples, the whole training dataset is segmented into \( k \) clusters automatically by the clustering analysis algorithm. The \( n \) samples in \( X = [x_1, x_2, \ldots, x_n]^T \) can be divided into \( k \) clusters \( X_1, X_2, \ldots, X_f, \ldots, X_k \):

\[ \text{L}(X_1) + \text{L}(X_2) + \ldots + \text{L}(X_f) + \ldots + \text{L}(X_k) = n \]  

(13)

where \( \text{L}(X_i)(i = 1, 2, \ldots, k) \) is the length of cluster \( X_i \). A single cluster contains samples which are similar to each other to the greatest extent, while samples in different clusters are different from each other to the greatest extent. In LGPR, the prediction result of the predictive sample \( Z \) is obtained for the cluster \( X_f = [x_{f1}, x_{f2}, \ldots, x_{fh}]^T (h < n) \) of which the center is closest to \( Z \), rather than for the whole training dataset \( X = [x_1, x_2, \ldots, x_n]^T \),
considering that the prediction result of $Z$ is mainly determined by the samples which have high similarities with $Z$ in the GPR method [24]. Then, the key regression equations become

$$
\hat{\mu} = K(Z, X_f)B y_f
$$

(14)

$$
\hat{\sigma}^2 = K(Z, Z) - K(Z, X_f)BK(Z, X_f)^T
$$

(15)

where $B = [K(X_f, X_f) + \sigma^2 I_h]^{-1}$, and $y_f = [y_{f1}, y_{f2}, \ldots, y_{fh}]^T (h < n)$ is the target value vector of $X_f = [x_{f1}, x_{f2}, \ldots, x_{fh}]^T$. According to Equations (14) and (15), the computational complexity of calculating the predictive mean and variance of $f_s$ are $O(h)$ and $O(h^2)$, respectively.

In this study, the squared exponential (SE) function $k_{SE}$ is used, whose mathematical expression is given as

$$
k_{SE} = \exp\left[-\frac{1}{2\sigma^2}(x_i - Z)^T(x_i - Z)\right]
$$

(16)

where $\sigma^2$ is the length scale of the SE covariance function. In this study, in order to guarantee the generalization ability of the identified model, a genetic algorithm (GA) is employed to optimize the hyperparameters in the GPR for the whole training dataset. The detailed implementation process of the hyperparameters’ optimization by a GA can be found in [18].

The flowchart of LGPR is shown in Figure 2.

![Flowchart of LGPR](image)

**Figure 2.** Flowchart of LGPR.
4. Nonparametric Modeling and Validation

4.1. Mariner Class Vessel with Simulation Data

Firstly, the Mariner class vessel is taken as the research object and the effectiveness of LGPR is evaluated by using the simulation data. The simulation tests are performed in MATLAB R2020a. The main parameters of the ship are given in [25].

The datasets are generated by carrying out simulations of maneuvers with a nonlinear whole-ship model, where the hydrodynamic derivatives are taken from Chislett and Tejsen [25]. The training datasets are collected from simulated 5°/5°, 10°/10° and 35°/5° zigzag maneuvers at a sampling time interval of 0.5 s, and the sample size of each maneuver is 400. The parameters in LGPR are configured as follows: 0.9 for the crossover possibility, 0.1 for the mutation possibility, 30 for the evolutional generation, and 3 for the number of clusters \( k \). Meanwhile, a classic GPR without clustering analysis process and a neural network (NN) based on a back-propagation algorithm are also used to model and predict the ship steering motion for comparison purposes. NNs are a widely used nonparametric modeling method due to its universal approximation ability. In this study, the NN consists of three layers: an input layer, a hidden layer with 10 hidden nodes, and an output layer. The Levenberg–Marquardt algorithm is used as the optimization method for tuning the weights and the bias values in the NN, the mean-square error is utilized as the loss function, the hyperbolic tangent function is adopted as the activation function for the input layer and the hidden layer, and the linear transfer function is adopted as the activation function for the output layer. The maximum training iterations and the learning rate are set as 1000 and 0.1, respectively.

In LGPR, the k-means clustering analysis algorithm is applied to divide the training dataset automatically [26], and the min–max normalization is carried out on the training dataset before performing the clustering analysis. In respect of the clustering analysis, the results are presented in Figure 3 and Table 1. The yellow dotted line in Figure 3 represents the borders of the clusters.

![Figure 3. Clustering results (Mariner).](image)

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>725</td>
</tr>
<tr>
<td>B</td>
<td>166</td>
</tr>
<tr>
<td>C</td>
<td>309</td>
</tr>
<tr>
<td>Sum</td>
<td>1200</td>
</tr>
</tbody>
</table>

Table 1. Dimensions of clusters (Mariner).

The hyperparameters tuned and optimized by GA are presented in Table 2, where \( \sigma_r \) and \( \sigma_d \) are the length scales of the covariance function of the input variables, i.e., yaw rate
and rudder angle, respectively; \( \sigma_f \) is the change magnitude of the output of the covariance function; and \( \sigma_n \) is the standard deviation of the noise contained in the target value \( y \).

Table 2. Hyperparameters tuned and optimized by GA (Mariner).

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>( \sigma_r )</th>
<th>( \sigma_d )</th>
<th>( \sigma_f )</th>
<th>( \sigma_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.410</td>
<td>0.339</td>
<td>0.0225</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

The generalization ability of the identified response model is validated by predicting the maneuvers which are excluded in the identification procedure, including 10°/5° and 15°/15° zigzag maneuvers, and random maneuvers. For the predicted maneuvers, the results are presented in Figures 4–6, where ‘RHM’ denotes the results simulated by the hydrodynamic model (here the whole-ship model) for ease of exposition. Figure 7 presents the results of the comparison among GPR, LGPR and the NN in respect of the time consumed for prediction, while the results of the comparison in respect of the root-mean-square error (RMSE) values of the yaw rate are given in Table 3, where \( R_r (\text{deg/s}) \) is calculated as

\[
R_r = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (r_i - \hat{r}_i)^2}
\]

(17)

where \( r_i \) is the RHM value and \( \hat{r}_i \) is the value predicted by the identified response model.

Figure 4. Predictions of 10°/5° zigzag maneuver (Mariner): (a) yaw rate, and (b) heading angle.
Figure 4. Predictions of 10°/5° zigzag maneuver (Mariner): (a) yaw rate, and (b) heading angle.

Figure 5. Predictions of 15°/15° zigzag maneuver (Mariner): (a) yaw rate, and (b) heading angle.

Figure 6. Cont.
Figure 5. Predictions of 15°/15° zigzag maneuver (Mariner): (a) yaw rate, and (b) heading angle.

Figure 6. Rudder angle and predictions of random maneuver (Mariner): (a) rudder angle during random maneuver, (b) predictions of short time (0–5 s) random maneuver, (c) predictions of medium time (0–100 s) random maneuver, and (d) predictions of long time (0–600 s) random maneuver.
Figure 6. Rudder angle and predictions of random maneuver (Mariner): (a) rudder angle during random maneuver, (b) predictions of short time (0–5 s) random maneuver, (c) predictions of medium time (0–100 s) random maneuver, and (d) predictions of long time (0–600 s) random maneuver.

Figure 7. Comparison among GPR, LGPR and the NN of time consumed for prediction (Mariner).

Table 3. Comparison among GPR, LGPR and the NN of RMSE value of yaw rate (Mariner).

<table>
<thead>
<tr>
<th>Maneuver Type</th>
<th>GPR</th>
<th>LGPR</th>
<th>NN</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°/5° Zigzag</td>
<td>3.35 × 10⁻²</td>
<td>3.57 × 10⁻²</td>
<td>3.87 × 10⁻²</td>
</tr>
<tr>
<td>15°/15° Zigzag</td>
<td>9.43 × 10⁻²</td>
<td>9.81 × 10⁻²</td>
<td>9.64 × 10⁻²</td>
</tr>
<tr>
<td>Random</td>
<td>4.90 × 10⁻²</td>
<td>5.00 × 10⁻²</td>
<td>5.65 × 10⁻²</td>
</tr>
</tbody>
</table>

Figure 4a shows that in the predicted 10°/5° zigzag maneuver, the predicted yaw rates by GPR, LGPR and the NN are smaller than the RHM in the early stage. After 100 s, the predicted yaw rates by the three methods begin to fit well with the RHM. It can be seen from Figure 4b that the accumulative prediction errors of the heading angle by the three methods are rather large, and those of the NN are the most noticeable. Figure 5 shows that in the prediction of 15°/15° zigzag maneuver, the results of the yaw rate predicted by the three methods are smaller than the RHM during 0–50 s, while after 50 s the prediction results for the yaw rate are larger than the RHM, leading to the accumulative prediction errors for the heading angle. The heading angles predicted by the three methods are larger than the RHM later in the prediction, and those predicted by LGPR and the NN exhibit more noticeable deviations.

The response model is usually applied to develop the controller of ship steering motion. Generally, the rudder angle during the motion control operation is not as regular as that in the standard zigzag maneuvers, but rather random. Therefore, it is necessary to assess the prediction accuracy of the identified response model for the simulation of random rudder angle, as shown in Figure 6a. In this study, the predictions of the random maneuver for short time (0–5 s), medium time (0–100 s) and long time (0–600 s) are performed, and the results are shown in Figure 6b–d.

As can be seen from Figure 6b, in the prediction of the short time random maneuver, the heading angles predicted by GPR and the NN show acceptable accuracy, while those predicted by LGPR exhibit rather distinct deviations during 3–5 s. Figure 6c shows that in the prediction of the medium time random maneuver, the results of the three methods fit well with the RHM in the first 30 s. After 30 s the prediction results begin to show errors. The performances of the three methods are similar and GPR gives slightly better results after 70 s. One can find from Figure 6d that in the prediction of the long time random maneuver, the prediction results of GPR and LGPR are in accord with the RHM overall, while those of the NN exhibit quite large deviations. These results indicate that the performances of the three methods are similar in the predictions of short time and medium time random maneuvers, while in the long time prediction the performance of the NN is not satisfactory, indicating that the nonparametric response model identified by GPR has better generalization ability.
As shown in Table 3, the values of \( R_r \) of the NN are larger than those of GPR and LGPR in the predictions of \( 10^\circ /5^\circ \) zigzag and random maneuvers, while in the prediction of \( 15^\circ /15^\circ \) zigzag maneuver, the \( R_r \) values of the three methods are close. Figure 7 demonstrates that LGPR is the least time-consuming of the three methods. These results reveal that LGPR is efficient, while the loss of prediction accuracy is not distinct. The improvement in computational efficiency and the guarantee of prediction accuracy can be attributed to the introduction of the clustering analysis method. The prediction result of the predictive sample is obtained on the cluster for which the center is closest to that of the predictive sample, rather than on the whole training dataset, thereby the computational burden is significantly reduced.

4.2. KVLCC2 Tanker Model with Experimental Data

To further verify the robustness of LGPR, the nonparametric modeling of the ship response model is conducted by using the experimental data of the KVLCC2 tanker model. The detailed information about this ship and the experimental data are given in SIMMAN 2008 Workshop [27].

The training datasets are collected from the \( 10^\circ /5^\circ, 10^\circ /10^\circ, 30^\circ /5^\circ \) and \( 35^\circ /5^\circ \) zigzag maneuvers at a sampling time interval of 0.05 s, and the whole training datasets include 1288 samples. The parameters set in LGPR are the same as those set in the previous subsection. In respect of the clustering analysis, the results are presented in Figure 8 and Table 4. The yellow dotted line in Figure 8 represents the borders of the clusters. The hyperparameters tuned and optimized by GA are presented in Table 5.

![Figure 8. Clustering results (KVLCC2).](image)

<table>
<thead>
<tr>
<th>Table 4. Dimensions of clusters (KVLCC2).</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cluster</strong></td>
</tr>
<tr>
<td>Dimension</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5. Hyperparameters tuned and optimized by GA (KVLCC2).</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hyperparameter</strong></td>
</tr>
<tr>
<td>Value</td>
</tr>
</tbody>
</table>

Similar to the previous subsection, the maneuvers which are excluded in the identification procedure are predicted by the identified models, including \( 20^\circ /10^\circ \) (S), \( 20^\circ /10^\circ \) (P) and \( 20^\circ /5^\circ \) zigzag maneuvers, where ‘(S)’ and ‘(P)’ stand for the zigzag maneuvers started...
with the rudder turning to the starboard and port sides, respectively. GPR and the NN are used for comparison purposes and the structure of the NN is the same as that used in the previous subsection. The results of the predicted maneuvers are shown in Figures 9–11; the results of the comparison among GPR, LGPR and the NN in respect of time consumed for prediction are presented in Figure 12; and the results of the comparison of RMSE values of yaw rate are given in Table 6.

Figure 9. Predictions of 20°/10° (S) zigzag maneuver (KVLCC2): (a) yaw rate, and (b) heading angle.

Figure 10. Cont.
Figure 10. Predictions of $20^\circ$/10° (P) zigzag maneuver (KVLCC2): (a) yaw rate, and (b) heading angle.

Figure 11. Predictions of $20^\circ$/5° zigzag maneuver (KVLCC2): (a) yaw rate, and (b) heading angle.
According to Figures 9a and 11a, in the predictions of 20°/10° (S) and 20°/5° zigzag maneuvers, the yaw rate predicted by GPR, LGPR and the NN are smaller than the experimental data in the early stage of prediction. As the prediction progresses, the results of the predicted yaw rate become larger than the experimental data, leading to the deviations in the predictions of the heading angle. Figures 9b and 11b show that the heading angle predicted by LGPR exhibits rather large deviations in the later stage of prediction, and the heading angle predicted by GPR shows lower accuracy than LGPR and the NN. Figure 10b shows that in the prediction of 20°/10° (P) zigzag maneuver, the heading angle predicted by the NN exhibits distinct errors for the valley points. The prediction deviations of the nonparametric model identified by the NN may be attributed to the limited training samples. In general, a larger sample size is required by the NN to identify the mapping relationship between the input and output data [28], while GPR is able to establish a rather robust model with a small sample size [23].

From Figure 12 and Table 6, it can be seen that LGPR is able to give high computational efficiency and acceptable prediction accuracy when compared with GPR and the NN. Moreover, $R_r$ of the predicted maneuvers given in Table 6 are of the order of $10^{-1}$, which are larger than those given in Table 3. This indicates that modeling based on the experimental data is more difficult than that based on the clean simulation data, since the experimental data contain noise caused by environmental interferences and measurement uncertainties.

In the above study of the two cases, the results predicted by the nonparametric response model exhibit rather distinct accumulative errors in the later stage of prediction. On the one hand, this may be attributed to the modeling errors of the identification algorithm. On the other hand, it may be attributed to the mathematical structure of the black box model (Equation (5)). In this study, the yaw acceleration is determined by yaw rate and rudder angle, which are derived from the Nomoto model. However, based on the mathematical model of 3-DoF ship maneuvering motion, the yaw acceleration is considered to be determined by surge speed, sway speed, yaw rate, and rudder angle [15,16,18,19,22]. Equation (5) can be regarded as a simplified black box model in comparison with that used in the modeling of 3-DoF ship maneuvering motion. Therefore, the identification algorithm cannot capture the complete dynamic characteristics of ship steering motion from the
training dataset constructed based on Equation (5). Nevertheless, the prediction accuracy of the nonparametric response model is acceptable, and the computational efficiency is expected to be higher than that of the nonparametric model of 3-DoF ship maneuvering motion because of neglecting the motion variables surge speed and sway speed.

5. Ship Heading Control Based on Identified Nonparametric Response Model

5.1. Identification-Based Ship Heading Controller

According to the results in the previous section, LGPR is able to give acceptable prediction results of ship steering motion with low computational costs. In this section, the identified nonparametric response model and the efficient prediction method of ship steering motion based on LGPR are applied to develop a ship heading controller. Proportion–integration–differentiation (PID) control law is utilized as the basis for the design of the identification-based heading controller in this study:

\[
\delta = k_p\Delta\psi(t) + k_i\int_0^t \Delta\psi(\tau)d\tau + k_d\frac{d\Delta\psi(t)}{dt}
\]

\[(18)\]

\[
\Delta\psi(t) = \psi_d(t) - \psi(t)
\]

\[(19)\]

where \(\Delta\psi(t)\) is the control error; \(\psi_d(t)\) and \(\psi(t)\) are the desired heading angle and current heading angle, respectively; and \(k_p, k_i\) and \(k_d\) are the proportional, integral and differential control parameters, respectively.

PID control is a classic and widely used method characterized by a simple mathematical structure and robust performance in the field of ship motion control. However, the tuning of the control parameters \(k_p, k_i\) and \(k_d\) is an open problem. Traditional tuning methods include the cut-and-trial method and the expert experience method, with which it is always hard to obtain satisfactory solutions in view of the high nonlinearity of ship maneuvering motion. LGPR can establish the nonparametric model and predict ship steering motion efficiently, providing an approach for tuning the control parameters in a model-based way. The detailed implementation process is shown in Figure 13.

![Flowchart of identification-based ship heading controller based on LGPR.](image)

The optimization algorithm used to tune the control parameters in PID controllers is the GA, and the fitness function \(F\) in the GA is defined as

\[
F = \frac{1}{\int_0^T (\omega_1|\psi(t) - \psi_d(t)| + \omega_2\delta^2(t))dt}
\]

\[(20)\]

where \(T\) is the duration of the computer simulations of ship heading control tests, performed by LGPR, and \(\omega_1\) and \(\omega_2\) are the weights. The term \(\omega_1|\psi(t) - \psi_d(t)|\) stands for the
accumulative control error, which is used to improve the accuracy of control. The term $\omega_2\delta^2(t)$ is used to make the rudder angle moderate and not change too frequently, to avoid unnecessary steering and excessive mechanical wear of the steering gear.

Based on Equation (20), the control parameters are optimized in the evolution procedure by carrying out computer simulations of heading control tests by using the identified response model and the efficient prediction method of ship steering motion, and the control parameters whose fitness function values are highest are obtained at the end of evolution.

5.2. Simulation Test of Ship Heading Control

The Mariner class vessel is used as the control object, and the control parameters in Equation (18) are tuned and optimized based on the nonparametric response model established in Section 4.1. The initial heading angle is $0^\circ$, and the desired heading angle is set as $10^\circ$. The parameters in the GA for tuning the control parameters are set as follows: $\omega_1$ is 0.7, $\omega_2$ is 0.02, the crossover probability is 0.9, the mutation probability is 0.1, the population size is 30, and the number of evolution iterations is 30. The proposed identification-based ship heading controller is denoted as the LGPR-GA-PID.

To evaluate the effectiveness of the LGPR-GA-PID, the whole-ship model of Mariner class vessel is also used for the GA to optimize the control parameters, and the corresponding controller is denoted as the whole-GA-PID. Obviously, with the control parameters tuned by the LGPR-GA-PID closer to those tuned by the whole-GA-PID, more reliable performance of the LGPR-GA-PID is obtained. The comparison of the results of tuning the control parameters using the whole-ship model and the nonparametric response model identified in Section 4.1 is given in Table 7. It is noticed that the results of tuning the control parameters $k_p$, $k_i$ and $k_d$ on the basis of the whole-ship model and the nonparametric response model identified by LGPR are very similar, indicating that the identified nonparametric response model and the proposed prediction method of ship steering motion are reliable.

Table 7. Comparison of control parameters tuned based on whole-ship model and nonparametric response model identified by LGPR.

<table>
<thead>
<tr>
<th>Control Parameter</th>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$k_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole-GA-PID</td>
<td>1.5863</td>
<td>$\approx0$</td>
<td>36.6454</td>
</tr>
<tr>
<td>LGPR-GA-PID</td>
<td>1.6859</td>
<td>$\approx0$</td>
<td>37.3302</td>
</tr>
</tbody>
</table>

Figure 14 shows the comparison between the proposed identification-based heading controller and a standard PID controller used in the Marine System Simulator (MSS) [29]. It reveals that the control performance of the LGPR-GA-PID controller is satisfactory, no obvious overshoot is observed, and the steady-state error is within $1^\circ$, while the overshoot of the standard PID controller is rather distinct and the steady-state error is larger than that of LGPR-GA-PID. Table 8 gives the comparison results of the dynamic performance indexes. It is noted that the rise time of the LGPR-GA-PID is larger than that of the PID controller used in the MSS, while the settling time and the overshoot of the PID controller in the MSS are larger than those of the LGPR-GA-PID, indicating that the LGPR-GA-PID has better dynamic performance. These results all show that the proposed ship heading control method based on the identified nonparametric response model is reliable and can achieve satisfactory control performance.

Table 8. Dynamic performance indexes of the LGPR-GA-PID and the PID used in the MSS [29].

<table>
<thead>
<tr>
<th></th>
<th>Rise Time (s)</th>
<th>Settling Time (s)</th>
<th>Overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>44</td>
<td>172</td>
<td>23.30%</td>
</tr>
<tr>
<td>LGPR-GA-PID</td>
<td>60</td>
<td>102</td>
<td>9.41%</td>
</tr>
</tbody>
</table>
control method based on the identified nonparametric response model is reliable and can achieve satisfactory control performance.

Figure 14. Comparison of the control performances between the LGPR-GA-PID and the PID used in the MSS [29]: (a) heading angle, and (b) rudder angle.

6. Conclusions

In this paper, the nonparametric modeling and prediction of ship steering motion is studied. A local Gaussian process regression (LGPR) algorithm is employed to establish the nonparametric response model and predict ship steering motion with low computational burden and acceptable prediction accuracy. The modeling and prediction results demonstrate the high modeling accuracy and the low computational burden of the identified nonparametric response model.

Based on the identified nonparametric response model, an identification-based ship heading controller is developed. The heading control simulation results indicate the satisfactory control performance of the proposed controller compared with that of a standard PID controller because of taking the ship steering characteristics into consideration.

In this study, the proposed controller is evaluated by carrying out simulation tests. As the proposed method is easy to operate, an attempt will be made to apply the proposed method to real physical experiments in a future study so as to further verify its engineering application value.

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