Quantification of Expert Knowledge in Describing COLREGs Linguistic Variables

Miho Kristič 1,*, and Srdan Žuškin 2,*

1 Maritime Department, University of Dubrovnik, Ćira Carića 4, 20000 Dubrovnik, Croatia
2 Faculty of Maritime Studies, University of Rijeka, Studentska ulica 2, 51000 Rijeka, Croatia
* Correspondence: miho.kristic@unidu.hr (M.K.); srdan.zuskin@pfri.uniri.hr (S.Ž.); Tel.: +385-20445-700 (M.K.); +385-51338-411 (S.Ž.)

Abstract: The International Regulations for Preventing Collisions at Sea 1972 (COLREGs) have been the cornerstone of maritime navigation since their introduction. Knowledge and implementation of these rules are paramount in collision avoidance at sea. However, terms found in these rules are sometimes imprecise or fuzzy, as they are written by humans for humans, giving them some freedom in interpretation. The term Very Large Ship used in Rule 7 of the COLREGs is, by its nature, fuzzy. While human navigators understand this term’s meaning, it could be challenging for machines or autonomous ships to understand such an unprecise expression. Fuzzy sets could easily describe unprecise terms used in maritime navigation. A fuzzy set consists of elements with degrees of membership in a set, making them perfect for interpreting some terms where boundaries are unclear. This research was conducted among 220 navigational experts to describe linguistic variables used in maritime regulations. This research consists of an internationally distributed questionnaire. Membership data were collected with the adapted horizontal method, and the results were statistically analyzed, followed by regression analyses to describe the range and shape of membership functions. A conceptual model of the implementation of linguistic variables is presented. The novelty of this study derives from the data collecting, modeling, and quantification of the important but neglected linguistic term Very Large Ship based on a large number of navigational experts. The same quantification method could be easily used for other COLREGs linguistic variables, which could easily lift barriers to advances in intelligent solutions based on fuzzy sets. The obtained quantified fuzzy sets can be used in decision support or control systems used by conventional or autonomous ships in the future.

Keywords: fuzzy set; collision avoidance; COLREGs; safety of navigation

1. Introduction

Although among the most important regulations for maritime navigators, the COLREGs contain numerous imprecise terms and definitions. Nevertheless, the implementation of these rules is a daily routine for navigational ranks. The obligation set by these rules to maintain proper lookout at all times means that the navigator has to follow the COLREGs from taking the watch until being relieved. During their education periods, professional navigators will extensively study collision regulations. However, misunderstanding rules is a serious problem [1,2]. Research by Mohovič et al. showed gaps in understanding some parts of the COLREGs by navigators and students due to wrong interpretations of these rules [3]. Furthermore, the number of collisions is gradually increasing, and many of them result in catastrophic consequences [4]. The COLREGs Convention (1972) was created to update and replace the Collision Regulations of 1960 [5]. New regulations in 1972 implemented traffic separation schemes, which made traffic in congested areas safer.

These rules were written and designed for the human operation of ships, so they are based on human judgments of navigational situations and the common sense of navigators [6–9]. In the meantime, different maritime intelligent technologies have become more
accepted and proposed [10–15]. While some terms used in the COLREGs are easily understandable to human navigators, explaining some of them mathematically can be difficult. Despite progress made in autonomous shipping, the International Maritime Organization (IMO) still plans to keep these rules as a reference point and keep them in their current form [16]. Rødseth et al. studied interactions between conventional and autonomous ships, and as a result, advocated changes in the COLREGs to improve safety at sea [17].

An analytical review of relevant literature by Chang et al. indicates that the present COLREGs represent a significant linguistic barrier to advances in autonomous navigation [18]. Additionally, the development of artificial intelligence solutions for collision avoidance is facing a lack of quantitative data regarding the COLREGs [19], so quantifying these rules is becoming imperative [20,21]. Research by Kim and Park recognized the COLREGs represent a significant linguistic barrier to advances in autonomous navigation. Furthermore, it was proven that these rules is becoming imperative [20,21]. Research by Kim and Park recognized the COLREGs represent a significant linguistic barrier to advances in autonomous navigation [17].

Another study confirmed that there is a significant difference between a navigator’s understanding of rules and collision-avoidance algorithms. Additionally, it was proven that navigators exhibit differing interpretations of collision regulation [23]. A study by Zhou et al. indicates that the COLREGs need to eliminate the uncertainty of interpretation as a barrier to the use of autonomous ships [24]. Research by Kim aimed to quantify COLREGs terms that were not previously quantified, such as “narrow channel” and “restricted visibility” [25], while Li suggested a model of autonomous collision avoidance under the constraint of the COLREGs [26]. Hagen et al. proposed a practical tool for the development of a collision avoidance system that is COLREGs-compliant [27].

Rule 7, covering the risk of collision, is among the most essential rules. The rule states that the risk of collision may exist even when the bearing of the approaching vessel changes if the approaching ship is very large. The term Very Large Ship used in this rule is challenging to define. When is the ship large enough to be considered very large? What is the limit or border between a Not Very Large Ship and a Very Large Ship crisp?

Hypothetically, let us consider that a Very Large Ship is anything above 200 m; in that case, a ship with a length of 199.99 m would not be very large, and a ship 200.01 m long would be very large (Figure 1a). Of course, this makes no sense by human reasoning, but it is the only possible solution based on the rules of classical binary logic. The solution to this problem is implementing fuzzy sets, in which elements can have a degree of membership (Figure 1b).

![Figure 1](image_url)

**Figure 1.** Membership example: (a) crisp set; (b) fuzzy set.

According to fuzzy set theory, the same element can partially belong to a given set and several fuzzy sets simultaneously. In our case, the ship can be large or very large, with a partial degree of membership in both sets. This is very close to human reasoning, so fuzzy theory and fuzzy logic are valuable tools for modeling variables humans use to describe surroundings. Fuzzy logic is a widely used method by authors researching maritime collision risk and avoidance [28–36], maritime fuzzy controllers [37], Fuzzy Fault Tree Analysis [38], and others. The essential part of classical fuzzy systems and the
Fuzzy Analytic Hierarchy Process (Fuzzy AHP) is the membership function, but there are difficulties related to defining parameters that define the shape of these functions [39]. In several previous studies, heuristic methods of assigning membership functions were used, but some approaches try to use probability distribution to describe membership functions [40]. Several studies have already described the linguistic variable Ship’s size as part of creating fuzzy systems. However, the risk of collision for the purpose of intelligent solutions is mainly modeled by the use of only two variables, namely, Distance to the Closest Point of Approach (DCPA) and Time to the Closest Point of Approach (TCPA) [41], while not including linguistic variable Very Large Ship. That omission means that the rule, as stated in the COLREGs, is not adequately modeled. From the perspective of humans, avoiding an ultra-large tanker is not the same as avoiding a small tug boat. It is not only size that matters but maneuvering characteristics are also completely different. Sheng-Long Kao et al. (2007) defined the linguistic variable Size of the Ship in a range from 130 m to 250 m to create a fuzzy collision risk system [28], but the term Very Large Ship was not modeled. Linguistic variables are based on the Nippon Kaiji Kyokai classification society’s (Class NK) ship data, and that paper does not show how the authors created membership functions. The authors created default triangular and partial trapezoidal membership functions to describe the terms Small Ship, Medium Ship, and Large Ship. As suggested by the authors, membership functions do not include ships below 130 m or above 250 m; thus, this linguistic variable does not include a full universe of discourse. Ren et al. [42] used the variable Ship’s Length to modify the input variable DCPA. As a result, this linguistic variable was created using triangular and partially trapezoidal functions. Nevertheless, the variable Very Large Ship, as defined in the COLREGs, has not been quantified by previous research or used in modeling the risk of collision.

The quantified variable Very Large Ship could be further used in classical fuzzy systems or Fuzzy AHP to improve existing models or create new models that define Rule 7 adequately. Fuzzy AHP systems have been used in several recent studies as an alternative to classical fuzzy logic systems to describe the risk of collision [43–45]. Fuzzy AHP is used by Luo et al. to describe collision risk using basic triangular membership functions, and the risk of collision is described by the variables DCPA and TCPA. Gaussian membership functions have been used in research to overcome problems with basic triangular and trapezoidal membership functions [46,47]. However, research [48] has found that, on observed data, fuzzy logic system results are more conservative than those of Fuzzy AHP and the equal weights method. This could suggest that the classical fuzzy system is more appropriate for risk assessment. Another study’s [49] verification results indicated that the classical fuzzy model performed better than AHP on a given data set. Therefore, because the uncertainty of the interpretation of ship size needs to be eliminated to facilitate collision avoidance for autonomous ships, this paper deals with defining the range of the linguistic variable Ship’s Size in the COLREGs. As defined in COLREGs Rule 7, the linguistic variable Very Large Ship might completely change the estimation of the risk of collision and, therefore, needs to be defined for an autonomous ship’s collision avoidance algorithm to function adequately. According to that, the novelty of this study is the definition of the important COLREGs linguistic variable Very Large Ship through the quantification of expert knowledge, which was not adequately defined in previous studies, and in that way, this study opens up opportunities for further research and improvements of collision avoidance algorithms for autonomous ships. Once defined, linguistic variables could be further used in fuzzy systems, such as fuzzy decision support systems.

2. Method of Eliciting Expert Knowledge

Research data were collected using an internationally distributed online questionnaire. The questionnaire was prepared according to the observed problem, targeting randomly selected navigational experts from different shipping companies. It was prepared using the Lime Survey software (LimeSurvey Cloud Version 5.6.62, https://www.limesurvey.org/, accessed on 17 May 2024), an online open-source survey tool [50], and distributed among
active sailing navigational experts. All questions were designed as closed-type questions due to a large sample size and to allow for easier data analysis.

However, eliciting expert knowledge is sometimes problematic, as the definition of an expert is quite broad. The literature uses the term expert very broadly and often describes anyone with any knowledge about a matter [51]. This makes sense with certain types of research and data collection, but in this paper, the questionnaire aims to consider participants with knowledge, skills, and navigation practice. This was achieved by disseminating a questionnaire among masters and navigational officers only, not including cadets, petty officers, or ratings. Additionally, it was intended to reach experts sailing on all ship types and sizes. The questionnaire was prepared by authors who are navigational experts, trying to resolve the meaning of some linguistic variables by examining a large number of navigational experts.

Research that aims to interpret COLREGs linguistic variables consists of two parts:

- Personal profile questions—PPQs.
- Fuzzy variable range questions—RFQs.

The first part of the questionnaire contains personal profile questions:

- PPQ1: Rank on board.
- PPQ2: Sea experience in years.
- PPQ3: Type of your last ship.
- PPQ4: Length overall in meters of your last ship.

The second part of the questionnaire consists of five questions:

- RFQ1: Please describe the linguistic variable Very Small Ship by recording minimum and maximum values in meters, in a range from 0 to 400 m.
- RFQ2: Please describe the linguistic variable Small Ship by recording minimum and maximum values in meters, in a range from 0 to 400 m.
- RFQ3: Please describe the linguistic variable Medium Ship by recording minimum and maximum values in meters, in a range from 0 to 400 m.
- RFQ4: Please describe the linguistic variable Large Ship by recording minimum and maximum values in meters, in a range from 0 to 400 m.
- RFQ5: Please describe the linguistic variable Very Large Ship by recording minimum and maximum values in meters, in a range from 0 to 400 m.

The standard membership estimation techniques are the horizontal and vertical methods, but both methods could have a lack of continuity as a drawback [52]. An adapted horizontal method was used by authors, where the drawback of continuity was handled by questioning experts about a full range of elements in a specific set. In the classical horizontal method, experts provide opinions if predefined elements belong to a specified set. Such a method was not practical for this research, as a number of predefined elements would be excessive for an effective questionnaire. The authors decided to question experts on the lower and upper limits of elements that belong to the set. Elements in the set are based on the obtained limits as follows:

\[ X = \{ x_{ll}, x_{ll} + 1, x_{ll} + 2, \ldots, x_{ul} \}, \]

where \( x_{ll} \) is the lower bound of the set, and \( x_{ul} \) is the upper bound of the set. Given “\( n \)” number of experts, the ratio between the number of repeating elements and the total number of replies is the membership degree. When all experts agree, membership is equal to one. The Lime Survey software was chosen because it has enough flexibility to allow for some custom-made question types. This was especially important for questions that were supposed to define a specific fuzzy set with all its elements. To achieve this, a custom-made double slider available through the open-source website GitHub [53] was used (Figure 2). This tool assists in determining the lower and upper limits of some linguistic variables. Furthermore, this approach avoided leading experts toward a predefined range for any
linguistic value. All linguistic values, from Very Small Ship to Very Large Ship, were examined on the same scale, from zero to four hundred meters.

![Custom-made double-sided slider used in the survey.](image)

Figure 2. Custom-made double-sided slider used in the survey.

Respondents could move both sliders to a precision of 1 m, so the answers represented all values within the defined range with a step of 1 m. Descriptive statistics of the research data were created using MATLAB R2018a Update 6(9.4.0.949201) ([https://www.mathworks.com/products/matlab.html](https://www.mathworks.com/products/matlab.html), accessed on 17 May 2024), which is widely used to analyze data [54]. Outliers in the data were removed using the quartile method, removing elements that were more than 1.5 interquartile ranges above the upper quartile or below the lower quartile [55,56]. The obtained data were further used to form sets of linguistic values. The data were normalized for use from zero to one, as fuzzy membership functions are defined in the same range. Finally, appropriate curves were fitted to analyze data using regression analysis.

3. Results of Survey

The research results are presented as descriptive statistics for the first part of the questionnaire, which contains the personal profiles of the navigational experts. The second part consists of descriptive statistics on the linguistic variable and a curve-fitting method for analyzing data.

3.1. Personal Profile Questions

An online questionnaire was distributed internationally to active navigational ranks, and 220 active maritime experts participated in this research (Figure 3). This research contains answers from 113 Masters, 65 Chief Officers, 28 Second Officers, and 14 Third Officers. The largest group of respondents is Masters, with a share of 51%, followed by Chief Officers (30%). The total experience of the respondents closely corresponds to rank distribution; the most experienced participants with more than 15 years of experience occupy a share of 58.2%; those with less than 15 years but more than 10 years have a share of 16.3%; those with 5 to 10 years take a share of 15.5%; those with 3 to 5 years take up 4.5%; and finally, those with 1 to 3 years take up a share of 5.5%.

![Participants in the questionnaire (a) by navigational rank and (b) by their total experience at sea.](image)

Figure 3. Participants in the questionnaire (a) by navigational rank and (b) by their total experience at sea.

The remaining personal profile answers reveal the ship’s type and the ship’s length overall on which participants sail (Figure 4). The largest group of participants sailed on liquified natural gas (LNG) carriers (91), followed by cruisers (59), dry bulk carriers (27), container ships (22), tankers (13), and others (8). Participants, based on the length of the
ship on which they serve, can be categorized into six groups: (i) 4.1% of respondents serve on ships from 100 to 150 m; (ii) 12.3% of respondents sail on ships from 150 m to 200 m; (iii) 17.3% of participants serve on ships from 200 m to 250 m; (iv) 46% of respondents represent the largest group, and they serve on ships from 250 m to 300 m; (v) 17.3% serve on ships from 300 m to 350 m; and (vi) last, the smallest group, 6%, are the navigators, who sail on the largest vessels, larger than 350 m.

![Figure 4](image)

**Figure 4.** Participants in the questionnaire (a) by type of ship and (b) by the length of their last ship.

### 3.2. Fuzzy Range Questions

The results of questions in the second part of the survey are summarized in Table 1, presenting respondents’ answers on the lower and upper limits of the range of linguistic variables expressed in meters. Common expressions used in fuzzy theory for describing linguistic values were used: the linguistic value *Very Small Ship* is defined by the fuzzy value name *Very Low* (VL); *Small Ship* by the fuzzy value name *Low* (L); *Medium Ship* by the fuzzy value name *Medium* (M); *Large Ship by the fuzzy value name High* (H); and finally, *Very Large Ship* by the fuzzy value name *Very High* (VH). The abbreviations are as follows: standard deviation (SD), minimum value (Min), arithmetic mean (Mean), maximum value (Max), coefficient of Kurtosis (Ku), coefficient of skewness (Sk), interquartile range (IQR), and median (C).

<table>
<thead>
<tr>
<th>Ship’s Size</th>
<th>Range Limits</th>
<th>SD</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Ku</th>
<th>Sk</th>
<th>IQR</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>VL</td>
<td>lower</td>
<td>10.3983</td>
<td>0</td>
<td>5.9659</td>
<td>41</td>
<td>4.8606</td>
<td>1.6843</td>
<td>10.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>upper</td>
<td>20.2872</td>
<td>10</td>
<td>43.3450</td>
<td>101</td>
<td>3.2339</td>
<td>0.6615</td>
<td>24.25</td>
<td>45</td>
</tr>
<tr>
<td>L</td>
<td>lower</td>
<td>17.5196</td>
<td>0</td>
<td>39.0455</td>
<td>83</td>
<td>2.8815</td>
<td>−0.0821</td>
<td>25</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>upper</td>
<td>28.0226</td>
<td>30</td>
<td>95.1724</td>
<td>180</td>
<td>3.1751</td>
<td>0.2994</td>
<td>24</td>
<td>100</td>
</tr>
<tr>
<td>M</td>
<td>lower</td>
<td>28.2761</td>
<td>30</td>
<td>95.1667</td>
<td>167</td>
<td>2.8211</td>
<td>0.1738</td>
<td>29</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>upper</td>
<td>35.7669</td>
<td>100</td>
<td>185.0506</td>
<td>272</td>
<td>3.1328</td>
<td>−0.0768</td>
<td>45</td>
<td>187.5</td>
</tr>
<tr>
<td>H</td>
<td>lower</td>
<td>37.2397</td>
<td>80</td>
<td>187.2880</td>
<td>274</td>
<td>3.3676</td>
<td>−0.1965</td>
<td>41</td>
<td>193</td>
</tr>
<tr>
<td></td>
<td>upper</td>
<td>36.8226</td>
<td>200</td>
<td>284.0000</td>
<td>350</td>
<td>3.8887</td>
<td>−0.2651</td>
<td>50</td>
<td>299</td>
</tr>
<tr>
<td>VH</td>
<td>lower</td>
<td>34.2945</td>
<td>197</td>
<td>279.2707</td>
<td>359</td>
<td>2.9865</td>
<td>−0.3982</td>
<td>50</td>
<td>297</td>
</tr>
<tr>
<td></td>
<td>upper</td>
<td>0</td>
<td>400</td>
<td>400.0000</td>
<td>400</td>
<td>Na</td>
<td>Na</td>
<td>0</td>
<td>400</td>
</tr>
</tbody>
</table>

Since the answers determined the range limits, creating a set containing all elements of the set between limits was necessary, and we did so by discretizing continuous values to be discrete with an interval of 1 m. An interval of 1 m was also an interval used in the online questionnaire. After the discrete sets were created, the sets were analyzed with the results shown in Table 2. Elements of the descriptive statistics are as follows: standard deviation (SD), arithmetic mean (Mean), and median (C).
Table 2. Descriptive statistics for linguistic value data set.

<table>
<thead>
<tr>
<th>Ship’s Size</th>
<th>SD</th>
<th>Mean</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>VL</td>
<td>18.3734</td>
<td>26.729</td>
<td>24</td>
</tr>
<tr>
<td>L</td>
<td>27.1491</td>
<td>68.5861</td>
<td>67</td>
</tr>
<tr>
<td>M</td>
<td>39.8209</td>
<td>141.8283</td>
<td>140</td>
</tr>
<tr>
<td>H</td>
<td>45.3227</td>
<td>235.6199</td>
<td>236</td>
</tr>
<tr>
<td>VH</td>
<td>41.5732</td>
<td>336.1744</td>
<td>340</td>
</tr>
</tbody>
</table>

The analysis results show no significant difference between the median and arithmetic mean, indicating the data’s lack of significant outliers and skewness. The distribution of data for the linguistic variable can be visually represented by histograms, as in Figure 5a–e.

Figure 5. Distribution of data for linguistic values: (a) Very Low; (b) Low; (c) Medium; (d) High; (e) Very High.

It is visible on the obtained graphs that there is some fuzziness in the linguistic values, which was expected and needed to be further described by using the appropriate membership function. To use the results and define membership functions, the data obtained had to be normalized to a range between zero and one. The Min–Max method was used to convert values, as it is generally used as a normalization method for transferring values to a defined scale [57]. It can be expressed as

\[ x' = \frac{x - \min(x)}{\max(x) - \min(x)} \]  

where \( x' \) is the normalized value, \( x \) is the original value, \( \min(x) \) is the minimum value in the set, and \( \max(x) \) is the maximum value in a given set. As in the analyzed set, \( \min(x) \) is equal to zero, and the formula can be reduced to

\[ x' = \frac{x}{\max(x)} \]
By using a new normalized value, a graph showing rough fuzzy linguistic values could be represented as in Figure 6.

![Graph of linguistic values in a full range of linguistic variables.](image)

**Figure 6.** Graph of linguistic values in a full range of linguistic variables.

A so-defined linguistic value would not be practical for use in fuzzy systems. Therefore, normalized data will be further used to define membership functions by regression analysis and by assigning the best fit.

3.3. Regression Analysis to Obtain Membership Functions

The normalized data were further used to determine the range and shape of membership functions. Membership functions were modeled using the MATLAB curve-fitting tool, which allows for regression analysis using available linear and non-linear models [58]. Regression modeling uses statistical processing and analysis to mathematically describe dependencies between variables [59]. The results of the regression are presented and analyzed graphically, followed by an examination of goodness-of-fit figures. The MATLAB toolbox provides the sum of squares due to error, R-square, adjusted R-square, and root mean squared error as goodness-of-fit statistics.

Gaussian curves were used to obtain the best fit for the linguistic variables *Low*, *Medium*, and *High* (Figure 7a–c).

Gaussian curves for the linguistic variables *Low*, *Medium*, and *High* can be expressed by the expression

\[ f(x) = a * e^{\left(-\frac{(x-b)^2}{c}\right)} \]  

(3)

where *a* is amplitude, *b* is the centroid (location), and *c* is related to the peak width. Coefficients for the above variables are provided in Table 3.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><em>a</em></td>
<td>66.65</td>
<td>139.7</td>
<td>237</td>
</tr>
<tr>
<td><em>b</em></td>
<td>36.2</td>
<td>57.5</td>
<td>66.89</td>
</tr>
</tbody>
</table>

Table 3. Coefficients for linguistic values *Low*, *Medium*, and *High*.

After analyzing the data for the variables *Very Low* and *Very High*, a sigmoidal logistic curve achieved the best fit. A graphic representation of the fit reveals a good fit for these two values (Figure 8a,b).
Figure 7. Fitting of Gaussian curve to linguistic values (a) Low, (b) Medium, and (c) High.
Table 3. Coefficients for linguistic values Low, Medium, and High.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>66.65</td>
<td>139.7</td>
<td>237</td>
</tr>
<tr>
<td>c</td>
<td>36.2</td>
<td>57.5</td>
<td>66.89</td>
</tr>
</tbody>
</table>

After analyzing the data for the variables Very Low and Very High, a sigmoidal logistic curve achieved the best fit. A graphic representation of the fit reveals a good fit for these two values (Figure 8a,b).

Figure 8. Fitting of sigmoidal logistic curve to linguistic values (a) Very Low and (b) Very High.

The sigmoidal logistic curve for linguistic variables Very Low and Very High can be expressed by the expression

\[
f(x) = \frac{a}{1 + e^{-b(x-c)}}
\]

where \( a \) is a parameter for the horizontal asymptote, \( b \) is a growth rate parameter, and \( c \) is the midpoint between the horizontal asymptotes and the inflection point. Coefficients for the above variables are provided in Table 4.

Table 4. Coefficients for the linguistic values Very Low and Very High.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Very Low</th>
<th>Very High</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>-0.0931</td>
<td>0.0535</td>
</tr>
<tr>
<td>c</td>
<td>44.587</td>
<td>282.1991</td>
</tr>
</tbody>
</table>

The goodness of fit for all the linguistic variables, after fitting the curves, is provided in Table 5, with SSE (sum of squares due to error), R-square (coefficient of determination), adjusted R-square, and RMSE (root-mean-square error).
Table 5. The goodness of fit for the linguistic values.

<table>
<thead>
<tr>
<th>Linguistic Variables</th>
<th>SSE</th>
<th>R-Square</th>
<th>Adjusted R-Square</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low</td>
<td>0.3699</td>
<td>0.9716</td>
<td>0.9713</td>
<td>0.0621</td>
</tr>
<tr>
<td>Low</td>
<td>0.6204</td>
<td>0.9646</td>
<td>0.9644</td>
<td>0.0625</td>
</tr>
<tr>
<td>Medium</td>
<td>0.7338</td>
<td>0.9733</td>
<td>0.9732</td>
<td>0.0564</td>
</tr>
<tr>
<td>High</td>
<td>1.5820</td>
<td>0.9549</td>
<td>0.9547</td>
<td>0.0771</td>
</tr>
<tr>
<td>Very high</td>
<td>0.8787</td>
<td>0.9719</td>
<td>0.9717</td>
<td>0.0664</td>
</tr>
</tbody>
</table>

Among the linguistic values described by the Gaussian curve, the lowest SSE can be observed for the variable Low (Small Ship). Other statistics for the same variable indicate a good fit. For the other two variables described by the Gaussian curve, it can be seen that the appropriate SSE and other statistics also indicate a good fit. The same is visible when the graphical representation of fit is examined. For linguistic values described by the sigmoidal logistic curve, the linguistic value Very Low has a better fit with the lower SSE value than Very High, but not significantly. Altogether, the proposed curves fit the values properly, with the goodness-of-fit statistics showing excellent fitting.

3.4. Review of Obtained Results with Conceptual Model

After membership functions are defined, it is possible to review and evaluate membership in fuzzy sets for any element in a continuous range from 0 to 400 m. The degree of membership for some sizes of the ships in the fuzzy sets (Very Low, Low, Medium, High, and Very High) can be found in Table 6.

Table 6. Membership degree of some elements in the fuzzy sets.

<table>
<thead>
<tr>
<th>Length Overall (m)</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
<th>350</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low</td>
<td>0.91</td>
<td>0.38</td>
<td>0.04</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Low</td>
<td>0.19</td>
<td>0.81</td>
<td>0.87</td>
<td>0.24</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Medium</td>
<td>0.01</td>
<td>0.09</td>
<td>0.34</td>
<td>0.77</td>
<td>1</td>
<td>0.76</td>
<td>0.33</td>
<td>0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>High</td>
<td>0</td>
<td>0</td>
<td>0.03</td>
<td>0.12</td>
<td>0.37</td>
<td>0.74</td>
<td>0.99</td>
<td>0.89</td>
<td>0.53</td>
</tr>
<tr>
<td>Very High</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.06</td>
<td>0.23</td>
<td>0.6</td>
<td>0.88</td>
</tr>
</tbody>
</table>

As previously mentioned, fuzzy set theory allows some elements to have only partial membership in a given set and to be simultaneously present in two or more sets. Ships with a length overall (LOA) of 260 m are, for instance, members of three sets. Membership in Medium Ship is just 0.01, but membership in the set of Large Ships is 0.89. At the same time, this ship size is also a member of the Very Large Ship set with a membership of 0.23. Of course, due to the continuous nature of the linguistic variables, it is not possible to present each element in the table, but the membership of any ship by length overall can be calculated by the above-presented formulas and coefficients.

The implementation of the quantified linguistic value is defined by the COLREGs. The rules state that if during a determination of the risk of collision, it appears to the navigator that the risk does not exist, risk still has to be confirmed if the approaching ship is very large. This definition in the COLREGs indicates the importance of a ship’s size in evaluating risk of collision.

The quantified linguistic value Very Large Ship could be easily used to improve existing collision avoidance systems or to create new collision avoidance fuzzy or Fuzzy AHP systems. Furthermore, this variable can be easily implemented as a new variable in a hierarchical fuzzy system. A hierarchical fuzzy system consists of two or more fuzzy systems that are arranged in a hierarchical tree structure (Figure 9).
At the same time, disagreement was modeled by partial membership in a fuzzy set. The fuzziness in the experts’ knowledge was modeled with Gaussian and sigmoidal logistic membership function like a triangular function, as the result of this research confirmed. Very Large Ship

The ship’s length, the membership value in the fuzzy set comes with time spent on board. The results show that most respondents were Masters, human expert knowledge to describe the linguistic variables. This was achieved through the application of fuzzy theory based on the responses of 220 navigational experts. As the COLREGs are written by humans for humans, this research focused on modeling human expert knowledge to describe the linguistic variables.

The questionnaire targeted experienced seafarers only, considering that experience comes with time spent on board. The results show that most respondents were Masters, followed by Chief Officers, making up about 80% of the total respondents. Considering total experience at sea, respondents with more than ten years of experience at sea constituted more than 70%. This would indicate that the respondents’ expertise is appropriate for creating a model based on expert knowledge. The respondents’ profiles regarding the type and size of the ship reveal that groups and ship size do not reflect the actual distribution of the world fleet, with most respondents sailing on LNG carriers and on ships from 250 to 300 m. The mentioned characteristic of the sample could induce some skewness in the obtained results, so in future research, the authors will try to study different samples and seek differences.

The results of the second part of the questionnaire revealed significant facts. Firstly, the sigmoidal linguistic curve was the best fit for extreme values of the linguistic variables (Very Low and Very High). This was logically expected and confirmed as, by incrementing the ship’s length, the membership value in the fuzzy set Very High can only increase or remain the same. It goes vice versa for the value Very Low. Furthermore, the Gaussian curve fitted the other three values as the best option. Also, as previous studies were mainly concentrated on creating a fuzzy system, the definition of a fuzzy membership function was not detailed enough. Expert knowledge cannot be modeled with the most straightforward membership function like a triangular function, as the result of this research confirmed. Fuzziness in the experts’ knowledge was modeled with Gaussian and sigmoidal logistic functions, where the agreement of experts was expressed with a full membership value. At the same time, disagreement was modeled by partial membership in a fuzzy set. The
membership value for various lengths of ships in the fuzzy sets was presented. For example, it was shown that a ship with a length of 260 m is a member of the Very Large Ship set, with a partial membership of just 0.23, while simultaneously, it would also be a member of the Large Ship fuzzy set at 0.89. This example presents the benefits of fuzzy sets compared with classical sets as a mathematically based representation of vagueness and uncertainty. Additionally, fuzzy sets and related fuzzy systems are easily understandable to humans, making them ideal solutions for resolving ambiguities in maritime terminology. Fuzzy systems are examples of White Box models, which are easily interpretable and, as such, suitable for applications that require transparency in prediction [62]. Since the COLREGs rules are written vaguely, relying on the navigator’s knowledge, skills, and experience, modeling rules has to be achieved by using expert opinions. Fuzzy theory was developed to model such vague concepts, as it relies on human language, which has been successfully used for centuries.

Finally, a conceptual model of the implementation of a new linguistic variable was presented. Due to its specific nature, hierarchical fuzzy systems can easily adopt this new variable into their systems and allow for refined modeling of collision risk in cases in which another vessel is very large. Other ship variables, including LOA, are easily available through the Automatic Identification System (AIS), a system used to exchange navigation information between ships [63]. A definition of linguistic variables in the full universe of discourse, from Very Small Ship to Very Large Ship, is required as an important step in developing a new fuzzy decision support system. Additionally, there is a very fine division in the five linguistic values: Very Small Ship, Small Ship, Medium Ship, Large Ship, and Very Large Ship help in achieving a more detailed and precise fuzzy system. Classical fuzzy systems and Fuzzy AHP systems depend on input variables, so the detailed definition of linguistic variables used in the COLREGs is essential in developing successful intelligent decision support systems for collision risk assessment. Decision support systems evolve gradually as additions to existing integrated navigational systems on ships and aim to improve the safety of navigation. Furthermore, the integration of decision support intelligent systems within integrated navigational systems is advancing toward autonomous navigation. The assessment of the risk of collision on humanless ships represents a specific problem, especially considering that the COLREGs will continue to have a primary role in collision avoidance. A fuzzy decision support system for collision risk assessment complying with the COLREGs requires the definition of other fuzzy variables influencing the risk of collision. As such, it could be a decision support system solution on ships with human crews and humanless ships. Future research will concentrate on the definition of other COLREGs linguistic variables by using fuzzy theory and expert opinion.

5. Conclusions

Fuzziness in the maritime world, particularly in rules, is often seen as a problematic issue, especially in intelligent solution advances. However, the theory of fuzzy sets is a great tool for modeling fuzzy or unprecise terms used in the human world. While the fuzzy theory approach has been used in significant fields of research and maritime study, modeling the membership functions of linguistic variables has not been the focus of previous studies. The shape and range of linguistic variables greatly influence the performance of fuzzy systems, and to express human reasoning, they have to be modeled on a navigational expert’s opinion. In this research, the data used to define membership functions were collected using double-sided sliders. This approach to data collection is proposed when the sample size and the range of linguistic variables are large, followed by using an adapted horizontal method. This research aimed to elicit expert knowledge, which was achieved through the participation of 220 international navigational experts with significant experience in navigation. In future research, the authors plan to include ANOVA testing to study if there are statistically important differences between groups of respondents based on personal profiles or sailing areas. Fuzziness in responses between
different groups is expected but normally handled by the inherent ability of fuzzy numbers. The responses were analyzed and statistically modeled to determine the appropriate shape of the linguistic variables. Gaussian and sigmoidal logistic curves proved to be the best fit for the linguistic values, confirming that simple basic curves like triangular curves are not the best option when defining expert opinions. Most importantly, the linguistic value Very High was mathematically defined, describing the unprecise term Very Large Ship used in the COLREGs, which has been previously neglected in creating collision avoidance systems. Additionally, all linguistic values within the linguistic variables were defined, resolving an interpretation of the linguistic variable Ship’s Size. Using the same method, other linguistic variables abundantly present in the COLREGs could be modeled. Further research into quantifying methods for COLREGs terms is essential for developing intelligent solutions in maritime navigation. The obtained curves could be further used in various fuzzy systems, from control systems to decision support systems on conventional ships with human crews on board or autonomous ships. Finally, an implementation of quantified value within hierarchical fuzzy systems was presented as a simple and efficient method of including new variables in collision avoidance fuzzy systems.

Author Contributions: Conceptualization, M.K. and S.Ž.; methodology, M.K. and S.Ž.; validation, M.K. and S.Ž.; formal analysis, M.K. and S.Ž.; writing—original draft preparation, M.K.; writing—review and editing, S.Ž.; visualization, M.K. and S.Ž.; supervision, S.Ž.; project administration, S.Ž. All authors have read and agreed to the published version of the manuscript.

Funding: This research was partially funded by European Union’s Horizon Europe under the call HORIZON-CL5-2022-D6-01 (Safe, Resilient Transport and Smart Mobility services for passengers and goods), grant number 101077026, project name SafeNav. The views and opinions expressed are, however, those of the author(s) only and do not necessarily reflect those of the European Union or Executive Agency (CINEA). Neither the European Union nor the granting authority can be held responsible for them. Also, this study was partially funded by the University of Rijeka under the Faculty of Maritime Studies project ECDIS CoDe (UNIRI-ZIP-2103-10-22).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

References