Article

Probabilistic Prediction of Floating Offshore Wind Turbine Platform Motions via Uncertainty Quantification and Information Integration

Na Li 1, Guang Zou 1,*, Yu Feng 2,3 and Liaqat Ali 1

1 Department of Ocean Science and Engineering, Southern University of Science and Technology, Shenzhen 518055, China; nl940927@outlook.com (N.L.); liaqata@sustech.edu.cn (L.A.)
2 School of Civil Engineering, Sun Yat-sen University, Zhuhai 519082, China; fengy253@mail.sysu.edu.cn
3 Department of Offshore Energy, Norwegian Geotechnical Institute, 0484 Oslo, Norway; yu.feng@ngi.no
* Correspondence: zoug@sustech.edu.cn

Abstract: The accurate prediction of short-term platform motions in a real environment is crucial for the safe design, operation, and maintenance of floating offshore wind turbines (FOWTs). Numerical simulations of motions are typically associated with high uncertainties due to abstracted theoretical models, empirical parameters, initial environment parameters, etc. Therefore, it is necessary to integrate other sources of information associated with less uncertainty, e.g., monitoring data, for accurate predictions. In this paper, we propose a probabilistic prediction based on the Bayesian approach that logically integrates motion monitoring data with simulated motion predictions of FOWTs, considering uncertainties in the environment model, structural properties, motion prediction method, monitoring data, etc. The approach consists of constructing a prior probability density function (PDF) of a random variable (which characterizes the largest value of the initial motion response) via numerical simulations and a likelihood function based on platform motion monitoring data and deriving a posterior PDF of the random variable by Bayesian updating. Then, posterior distributions of short-term extreme motion responses are derived using the posterior PDF of the random variable, representing lower uncertainty and improved accuracy. A Metropolis–Hastings algorithm is adopted to obtain PDFs of complex probability distributions. The effectiveness of the approach is demonstrated on a real FOWT platform in Scotland. The proposed probabilistic prediction approach results in posterior distributions of short-term extreme platform motions associated with less uncertainty and higher accuracy, which is attributed to integrating prior knowledge with monitoring data.

Keywords: floating offshore wind turbines; monitoring; Bayesian updating; Metropolis–Hastings sampling; extreme value distribution

1. Introduction

In recent years, renewable energy has attracted significant attention due to the rising energy demand and growing concerns about environmental degradation. Among renewable energy sources, offshore wind energy holds substantial promise, as evidenced by its rapid growth in the market. Despite this expansion, numerous technical challenges exist within the offshore wind energy industry [1]. For example, considerable efforts have been devoted to understanding their structural response and motion behavior. However, existing studies predominantly focus on analyses under simplified conditions, and in-depth studies on dynamic responses under complex marine conditions (e.g., combining waves, wind, and ocean currents) and real environments, especially extreme environments, are lacking [2]. China is one of the world’s high-incidence countries for typhoons, with several typhoons along the coast each summer, causing significant impacts and losses to wind farms. A review published in Science in 2019 also highlighted that typhoon-induced structural damage...
to wind turbines poses a significant threat to the operation of wind farms [3]. Therefore, it is crucial to deeply understand and predict the complex dynamic responses of floating offshore wind turbines (FOWTs) under real environments accurately. Reducing uncertainties in simulating dynamic responses of FOWTs is key to accurate predictions.

Guedes Soares [4] classified uncertainty into intrinsic and epistemic types. Epistemic uncertainty arises from assumptions, simplifications, or incomplete information concerning a system or process, which can be reduced by collecting more information about physical quantities and improving measurement methods. Several studies have been conducted to assess the uncertainty in simulation models (e.g., abstracted theoretical models, empirical parameters) of FOWTs and revealed that a significant portion of the discrepancies between simulation results and measured data can be attributed to the uncertainty in simulation models [5–9]. Consequently, some research efforts have been made to optimize simulation models to achieve more accurate results for detecting structural risk probability [10–13].

On the other hand, structural health monitoring (SHM) serves as another powerful technique to collect real-world data about the platform’s motion responses and monitor structural risks. A variety of classical inference methods based on SHM have been developed to identify and predict structure responses [14–18]. However, structural health monitoring in complex ocean environments is costly and subject to observational noise, particularly in the early stages when monitoring data are scarce or unavailable [19,20]. Frangopol [21] proposed that incorporating prior information about a structural system can help augment limited monitoring data, leading to more reliable structural assessment. Classical statistical methods do not allow for effectively combining the prior knowledge, reflecting experiential judgments, with the likelihood of quantifying the chance of observations, whereas Bayesian methods are better suited for this task [22–25]. In Bayesian inference, a prior model is a probability distribution that represents the existing state of knowledge about a parameter or property before obtaining additional data, and a posterior model (obtained via Bayesian updating) denotes the probability distribution after obtaining data.

The Bayesian model updating has demonstrated significant computational capabilities in reducing the uncertainty of engineering models and provides valuable insights into model predictions based on observational data [26]. Garbatov and Guedes Soares [27] adopted a Bayesian approach to update parameters of probability distributions (which govern the reliability assessment of maintained floating structures) using inspection data, achieving improved reliability estimation. Okasha et al. [28] proposed a Bayesian updating approach to combine prior design code-based knowledge with SHM data for improved life-cycle performance assessment of ship structures under uncertainty. This approach utilized advanced modeling tools and techniques for lifetime reliability computations, including an optimization-based method for the ultimate failure moment, a progressive collapse method for the first failure moment, and hybrid Latin hypercube sampling. SHM data obtained from a scaled model testing of a joint high-speed sealift ship were used to update the structural life-cycle performance in the Bayesian framework. Okasha and Frangopol [29] extended the approach mentioned earlier into a life-cycle bridge management framework, where prior knowledge and SHM data about an existing bridge located in Wisconsin were integrated for better performance assessment. Zhu and Frangopol [30] further applied this approach to update wave-induced load effects and evaluate the structural performance of a ship. Their study focused on two parameters (location parameter and scale parameter) in the Type I extreme value distribution. It explored three general cases associated with parameter updating (i.e., one parameter, two parameters separately, and two correlated parameters simultaneously). Decò and Frangopol [31] proposed a risk-informed approach for ship hulls, which integrated SHM data via Bayesian updating and presented an approach for real-time optimal routing of ships. In the last five years, there has been growing attention to the use of Bayesian updating methods to reduce the uncertainties of characteristics of structures via response measurements. For example, Fang et al. [32] used displacement influence line data obtained from GPS measurements to update the finite element characteristics of the Tsing
Ma Bridge, a long-span cable-stayed bridge in Hong Kong. The results showed that the updated finite element characteristics based on the measured dynamic properties are consistent with the structural characteristics of the bridge. Additionally, Bayesian updating methods have been used to probabilistically update the prior distribution of resistance model uncertainty related to specific combinations of modeling hypotheses in reinforced concrete systems, thereby improving the final model’s accuracy [33]. Kamariotis et al. [34] applied the classical Bayesian updating method to study the deteriorating bridge systems, ultimately achieving the purpose of quantifying the benefit of the availability of long-term SHM vibration data. In order to address the critical issue of premature failure of bridge expansion joints, Ni et al. [35] employed Bayesian updating to update multiple parameters in a linear regression model between bridge temperature and expansion joint displacement, thus considering the uncertainty in the monitoring data and quantifying the model error and the prediction uncertainty.

In the context of wind turbines, Xu and Guedes Soares [36] highlighted Bayesian inference as a robust method for studying extreme mooring tension. They demonstrated the advantages of the mixture gamma-generalized Pareto distribution model in estimating extreme mooring tension for all investigated cases. Ding et al. [37] integrated wind turbine gear physical models and available health condition data via a Bayesian method, predicting wind turbine gearbox fatigue life under instantaneously varying load conditions. In order to accurately predict the performance of unsteady aeroelastic forces in a blade accounting for the uncertainty in aeroelastic characteristics, Sarma et al. [38] proposed a Bayesian method in which experimental data were used to update the structural and rotational parameters of a system of downwind wind turbine blades. Valikhani et al. [39] presented a Bayesian method to update the parameters of a physics-based wind turbine drivetrain model and input loads using measurement data. Moynihan et al. [40] updated the effective stiffness of soil springs for a 6 MW offshore monopile wind turbine and compared the deterministic and probabilistic (Bayesian) methods. Bayesian model updating results successfully estimate the posterior distribution of updating model parameters with an increasing degree of certainty as more data are used. Wang et al. [41] extracted the natural frequencies and mode shapes of offshore jacket platforms from the structural dynamic response of a finite element model and the sensory measurements in the physical system to construct the likelihood function for the Bayesian updating method. The results indicated that the proposed model updating method maintained high effectiveness even with high-level measurement noise and model uncertainty.

The literature review reveals that the Bayesian updating method has shown great potential in reducing structural model uncertainty and could be applied to dynamically and probabilistically predict the extreme value of motion response of FOWTs under real environments. To the best of the authors’ knowledge, there is no such study to date, and a research gap exists. Also, it would be meaningful if we could maximize the value of limited available monitoring data to improve the accuracy of FOWT platform motion predictions. Platform motion monitoring is costly since FOWTs are mostly situated in remote, hash, and complex oceans.

The objective of this study is to bridge the research gap by developing a Bayesian updating approach that integrates prior knowledge (represented by numerical simulation results) with real monitoring data of FOWT motion responses. By probabilistically accounting for uncertainty in both sources of information, this approach leads to more accurate motion response predictions and an improved safety level of FOWTs. The flowchart (Figure 1) of the proposed approach involves constructing prior probability density functions (PDFs) and likelihood functions of model parameters, respectively, from simulation and monitoring data of platform motion responses and obtaining posterior distributions via Bayesian updating, based on which posterior predictions of short-term extreme platform motion responses are obtained and improved in terms of lower uncertainty. Specifically, numerical results of motion responses (performed with the effective knowledge and information of FOWTs) are used to construct the prior PDF of a random variable ($\mu_a$),
which characterizes the largest value of motion response. The likelihood function is established by analyzing platform motion monitoring data, and the posterior probability distribution is derived via Bayesian updating. The posterior prediction results of short-term extreme platform motion responses are obtained based on the posterior probability distribution of $\mu_\text{in}$. A Markov chain Monte Carlo (MCMC) method coded with a Metropolis–Hastings algorithm is adopted to obtain PDFs of complex probability distributions. Compared to existing studies, the prior distribution model in this paper is more informative, since the distribution is obtained by making full use of all available knowledge and information about FOWTs while performing hydrodynamic simulations. The more informative prior distribution helps to improve the accuracy of posterior distribution with fewer monitoring data. The proposed approach is thus especially useful when monitoring data are limited. A case study on the Hywind wind farm in Scotland demonstrates the effectiveness of the proposed method.

The remainder of this paper is structured as follows: Section 2 presents the prior information and analysis of extreme platform motion responses. Sections 3 and 4 describe the statistical analysis of platform motion monitoring data and the MCMC-based Bayesian updating approach. A case study of the Hywind wind farm in Scotland is presented in Section 5. Finally, conclusions are drawn in Section 6.

2. Prior Motion Response Analysis

To assess the safety of FOWTs, it is vital to accurately predict the platform motion responses in six degrees of freedom (DOFs). The six DOFs of a rigid floating structure consist of three translation DOFs (surge, sway, and heave) and three rotational DOFs (roll,
pitch, and yaw). Physical monitoring and numerical simulation are two common methods for obtaining FOWT motion responses. Platform motion monitoring data are collected with sensors installed on the platform and the tower base. Numerical simulation is often used without monitoring data. A wide range of numerical techniques and software packages (e.g., FAST, HAWC2, AQWA, SESAM, etc.) are used for FOWT motion simulation [42]. Typically, a numerical model of a new design is developed and then validated by model-scale laboratory tests to assess the uncertainties associated with numerical models. In this study, the software package Ansys AQWA 2021 R1 with FAST v7 is utilized to simulate the motion responses of an FOWT subjected to wind, wave, and current loads [43,44]. Considering interactions between the FOWT and the mooring lines, the time-domain equation of motion of FOWTs can be written as follows [45]:

\[(M + A_w)\ddot{X} + \int_0^t k(t - \tau)\dot{X}d\tau + C\dot{X} = F_{WA} + F_{CU} + F_{MR} + F_{WI}\]  

where \(M\) is the inertia matrix; \(A_w\) is the added mass matrix at the infinite wave frequency; \(k\) is the delay function matrix; \(C\) is hydrostatic resilience; and \(\dot{X}, \ddot{X}\) and \(X\) represent the acceleration vector, velocity vector, and displacement vector, respectively. \(F_{WA}\) is the wave force, \(F_{WI}\) and \(F_{CU}\) are the wind load and current load, and \(F_{MR}\) is the force passed from the mooring line to the FOWT.

An extreme value distribution of the platform motion responses is adopted as the choice for the prior distribution. Obtaining an accurate extreme value distribution model is a challenging task. One of three asymptotic extreme value distributions is selected to address this challenge based on the tail behavior of the probability distribution of the platform motion responses. Previous studies have indicated that the motion responses of floating structures typically follow the Rayleigh distribution and tend to the Gaussian distribution under severe sea conditions [46]. It is worth noting that exponential tails characterize both the Rayleigh distribution and the Gaussian distribution. Specifically, when the probability distribution of the platform motion responses exhibits an exponentially decaying tail, the Type I extreme value distribution, i.e., the Gumbel distribution, is assigned [47]. Therefore, the Gumbel distribution is usually chosen as an appropriate extreme value distribution for motion response given as follows:

\[f_X(x) = \alpha_n e^{-\alpha_n(x-\mu_n)} \exp[-e^{-\alpha_n(x-\mu_n)}]\]  

where the variable of simulated motion responses is defined as the initial variable \(X\). \(\mu_n\) is the characteristic largest value of the initial variable \(X\), and \(\alpha_n\) is an inverse measure of the dispersion of the largest value of \(X\). Therefore, the initial values of \(\mu_n\) and \(\alpha_n\) are related to the mean \(\mu_X\) and standard deviation \(\delta_X\) of \(X\) as follows:

\[\alpha_n = \frac{\pi}{\sqrt{6}\delta_X}\]  

\[\mu_{n0} = \mu_X - \frac{\gamma}{\alpha_n}\]  

where \(\gamma = 0.577216\) is the Euler number. In comparison with the parameter \(\alpha_n\), the parameter \(\mu_n\) has a larger effect on the extreme value. In this paper, \(\alpha_n\) is deterministic, and the initial value of \(\mu_{n0}\) is estimated based on prior knowledge and then updated with platform motion monitoring data in the proposed Bayesian framework. Based on previous studies, \(\mu_n\) follows a Lognormal distribution, which is given by Ang and Tang [47].

\[\pi(\mu_n) = \frac{1}{\zeta\lambda\mu_n\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln\mu_n - \lambda}{\zeta}\right)^2}\]  

where \(\zeta\) and \(\lambda\) are the hyperparameters of the variable \(\mu_n\) [48] and are related to the mean and standard deviation of the variable \(\mu_n\) according to Equations (6) and (7).
\[
\zeta = \frac{\ln(1 + \delta^2)}{\mu_n^2}
\tag{6}
\]
\[
\lambda = \ln{\mu_n} - \frac{1}{2}I^2
\tag{7}
\]

where the mean \(\mu_n\) is obtained from prior knowledge, and \(\delta\) is set to be the coefficient of variation (COV) of 10\% [28].

3. Statistical Analysis of Platform Motion Monitoring Data

In this study, we use monitoring datasets of a real FOWT to construct the likelihood function. We define positive directions as north, east, and above sea level. To obtain the datasets of the extreme value of the six-DOF motion responses, an extreme value extraction algorithm is developed and applied to the signals of the six-DOF motion responses. The signals of six DOFs are represented by \(X_1, X_2, X_3, X_4, X_5,\) and \(X_6\). The algorithm is as follows:

1. Let \(X_k = [x_{k1}, x_{k2}, \ldots, x_{kn}]\) be the recorded platform motion monitoring signals of one DOF without duplicate data and calculate the mean value motion of this DOF. (\(n\) is the number of non-duplicate platform motion monitoring data, \(k = 1, 2, 3, 4, 5, 6\)).
2. Let the initial \(i = 1\), starting from \(x_{k1}\) to \(x_{kn}\); find the \(x_{k(i+1)}\) that satisfies \((x_{k(i+1)} - x_{ki}) \times (x_{k(i+2)} - x_{k(i+1)}) < 0\).
3. If \(x_{k(i+1)} > x_{ki}\), plug the absolute value of \(x_{k(i+1)}\) into the \(P_k = \max(x_{k(i+1)}, x_{k(k+2)}, \ldots, x_{k(n-1)})\) (\(P\) means positive direction) array.
4. If \(x_{k(i+1)} < x_{ki}\), plug the absolute value of \(x_{k(i+1)}\) into the \(N_k = \max(x_{k(i+1)}, x_{k(k+2)}, \ldots, x_{k(n-1)})\) (\(N\) means negative direction) array.
5. Calculate the mean value of the cell in the \(P_k\) array and calculate the mean value of the \(N_k\) array.
6. Compare the absolute value of these data. The array that has a larger mean value would be implemented in the Bayesian updating.

4. MCMC-Based Bayesian Updating

4.1. Bayesian Framework

The Bayesian theory is described by Gelman et al. [49] and Kruschke [50], which is given as follows:

\[
\pi(\theta \mid x) = \frac{h(x, \theta)}{m(x)} = \frac{p(\theta \mid x)\pi(\theta)}{m(x)}
\tag{8}
\]

where \(\theta\) is the vector of unknown parameters; \(x\) are the measured data; \(\pi(\theta)\) is the prior distribution of the model parameters, and \(\pi(\theta \mid x)\) is the posterior PDF; \(p(x \mid \theta)\) is the likelihood function.

Referring to the above equation, a distinctive feature of the Bayesian theory is the capability of combining prior knowledge with the likelihood based on SHM data to obtain the posterior probability distribution of \(\theta\). In this paper, the posterior PDF of \(\mu_n\) in the Gumbel distribution of extreme motion responses are updated. The likelihood function is constructed based on the discrete peak samples extracted from platform motion monitoring data, which is given as follows:

\[
p(x \mid \theta) = \prod_{i=1}^{N} f_X(x \mid \theta) = \prod_{i=1}^{W} \alpha_n e^{-\alpha_n(x_i - \mu_n)} \exp[-e^{-\alpha_n(x_i - \mu_n)}]
\tag{9}
\]

where \(f_X(x \mid \theta)\) is the PDF of extreme platform motion responses; \(X\) is evaluated based on the platform motion monitoring data value \(x_i\), given that the parameter of the PDF is \(\theta\). \(m(x)\) is a normalizing constant required to make \(\pi(\theta \mid x)\) a proper PDF and is calculated independently of \(\theta\) using Equation (10).
\[ m(x) = \int_\theta p(x | \theta) \pi(\theta) d\theta \]  

(10)

The new PDF of \( X \) is obtained based on the original PDF of \( X \) and the posterior PDF of its parameter \( \theta \). The updated PDF of \( X \) could be described with Equation (11) using the total probability theorem.

\[ f'_x(x) = \int_\theta f_x(x | \theta) \pi(\theta | x) d\theta \]  

(11)

Closed-form solutions for Equation (11) are not always possible. The MCMC method is used, and the algorithm is as follows:

1. Use the MCMC method to sample from the posterior distribution \( \pi(\theta | x) \), obtaining a set of parameter samples \( \theta \).
2. For each parameter sample \( \theta_i \), calculate \( f_x(x | \theta_i) \).
3. Take the average of \( f_x(x | \theta_i) \) to approximate \( f'_x(x) \).

4.2. Metropolis–Hasting Algorithm

In Bayesian statistical analysis, the evaluation of the PDFs of complex posterior distributions can be challenging. To address this, the MCMC method is utilized. In this method, a Markov chain is constructed with a stationary distribution that is equal to the target sampling distribution, using the chain’s states to generate random samples after an initial burn-in period in which the state distribution converges to the target. Developments on Bayes estimation using MCMC are summarized in detail by Li [51]. Among the multiple available MCMC algorithms, here, we employ the Metropolis–Hasting algorithm. The Metropolis–Hasting algorithm was first introduced by Metropolis et al. [52] and improved by Hastings [53]. It has been extensively employed in Bayesian data analysis. The Metropolis–Hasting algorithm generates samples from a target distribution by simulating an easy Markov chain that has the target density as its stationary distribution [54]. The MATLAB R2022b software package was used for implementation in this study and the Metropolis–Hasting algorithm is as follows:

1. Assume an initial value \( X(t) \).
2. Draw a sample \( y(t) \) from a proposal distribution \( q(y | X(t)) \).
3. Accept \( y(t) \) as the next sample \( X(t + 1) \) with probability \( r(X(t), y(t)) \) and keep \( X(t) \) as the next sample \( X(t + 1) \) with probability \( 1 - r(X(t), y(t)) \), where \( r(x, y) = \min \left\{ \frac{f(y) q(x | y)}{f(x) q(y | x)}, 1 \right\} \).

\( f(x) \) is the target distribution, also known as the probability density function that we are trying to sample from.
4. Increment \( t \to t + 1 \) and repeat steps 2 and 3 until the desired number of samples is met.

The proposed distribution is crucial for producing high-quality samples effectively using this algorithm. There is currently no consensus in academia for the selection of a burn-in period for Markov chains. In this study, the burn-in period was selected as the initial 10% of samples based on the convergence of the Markov chain shown in the time series plot. To assess convergence, the Markov chain time series plot of three chains was examined, with 200,000 samples generated per chain. Convergence was determined by observing small fluctuations above and below a horizontal line without any trend or period. Additionally, the mean and variance of the samples for each chain were compared. Meanwhile, a sequential updating approach combined with the Metropolis–Hasting algorithm was adopted in this study. In this approach, a posterior distribution is obtained by multiple sequential updates of another update, and 50 SHM samples as one part are used for updating [49].
5. Case Study: Hywind Scotland

5.1. Brief Introduction of the Hywind Wind Farm

The world’s first floating wind farm, the 30 MW Hywind Scotland pilot park, has been in operation since 2017, demonstrating the feasibility of floating wind farms that could be much larger. The platform motion monitoring data of FOWT HS4 were employed in this study. Figure 2 shows the location of the FOWT HS4.

![Figure 2. The location of the floating offshore wind turbine (FOWT) HS4.](image)

5.2. The Dimensions of Hywind and Some Considerations

Each Hywind unit consists of a turbine SWT-6.0-154 provided by Siemens Gamesa, a tower, and a floater. Referring to Chen et al. [55] and Equinor [56], the main dimensions of the Hywind Spar floater and tower, are shown in Figure 3. In addition, Figure 4 shows the mooring system. The properties of the mooring system are given in Table 1. The mooring system in the initial configuration should provide a vertical load of about 2660 KN. The tension at the upper end of the delta line (bridle) is about 590 KN, while the tension at the upper end of the main line is about 760 KN. Table 2 summarizes the properties of the FOWT.

The turbine of the Hywind is Siemens Gamesa SWT-6.0-154, which is similar to the 5 MW Hywind turbine and tower in OC3 [57]. Relevant information to specify the structural parameters is not available. The parameters of the 5 MW Hywind turbine and tower in OC3 are used to replace the wind turbine of HS4 in this study [57]. The mooring system is also a vital structure that connects a floating foundation to the seabed. The floating foundation is positioned by the restoring force and moment provided by the mooring lines. To enhance the stability of the Hywind floating structure, a delta-connection mooring chain design proposed by Koo et al. [58] was used, comprising three long anchor chains (120 degrees to each other) and six short anchor chains. This design effectively restricts the motion of a floater in the yaw degree of freedom. In this study, the delta-connection of mooring lines was considered to be directly linked to the hull with additional yaw stiffness [55]. These simplifications and alternatives also reflect some real situations in engineering.

Table 1. The properties of the mooring system.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth</td>
<td>m</td>
<td>100</td>
</tr>
<tr>
<td>The radius of the anchors</td>
<td>m</td>
<td>640</td>
</tr>
<tr>
<td>Draft of mooring points</td>
<td>m</td>
<td>20.6</td>
</tr>
<tr>
<td>Number of lines</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>Connection point outside wall</td>
<td>m</td>
<td>0.8</td>
</tr>
<tr>
<td>Segment name (from top to bottom)</td>
<td>-</td>
<td>Bridle</td>
</tr>
<tr>
<td>Property</td>
<td>Unit</td>
<td>Value</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>-------</td>
<td>--------</td>
</tr>
<tr>
<td>Segment length</td>
<td>m</td>
<td>50</td>
</tr>
<tr>
<td>Nominal diameter</td>
<td>mm</td>
<td>132</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>Mpa</td>
<td>53,941</td>
</tr>
<tr>
<td>Segment dry mass per meter</td>
<td>kg/m</td>
<td>348.5</td>
</tr>
<tr>
<td>Weight in water per meter</td>
<td>KN/m</td>
<td>3.403</td>
</tr>
</tbody>
</table>

Figure 3. The main dimensions of the Hywind Spar floater and tower.

Table 2. A summary of the properties of this FOWT.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draft</td>
<td>m</td>
<td>77.6</td>
</tr>
<tr>
<td>Displacement</td>
<td>tons</td>
<td>11,754</td>
</tr>
<tr>
<td>Dry mass</td>
<td>tons</td>
<td>11,483</td>
</tr>
</tbody>
</table>

Figure 4. The mooring system.
Mooring tension  tons  270.9
Center of gravity  m  X  Y  Z
Center of buoyancy  m  0  0  −50.03

5.3. Source of Platform Motion Monitoring Data

The platform motion monitoring data of 11 cases of Hywind in Scotland were provided by Equinor and ORE Catapult [56]. The operation condition (Case 7) and idle condition (Case 10) of the FOWT were chosen to be the representative cases for the FOWT. Wind direction is defined as the direction from which the wind is coming from. Thus, a “northerly wind” is coming from the north, and the wind direction is defined as 0 degrees, while an “easterly wind” has a wind direction of 90 degrees. Wave direction is similarly defined as the direction from which the waves are coming from. Thus, “northerly waves” are coming from the north, and the wave direction is defined as 0 degrees, etc. In contrast, the current direction is defined as the direction toward which the current is going. Thus, a “northerly current” is coming from the south, and the current direction is defined as 0 degrees, while an “easterly current” is coming from the west toward the east, and the current direction is defined as 90 degrees. The wind speed is the undisturbed wind speed measured with the turbine’s anemometer located at elevation +99 relative to sea level [56]. Details of the environmental conditions of Cases 7 and 10 are given in Table 3 and Figure 5.

Table 3. The details of environmental conditions of Cases 7 and 10.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Wave Height (m)</th>
<th>Wave Period (s)</th>
<th>Wave Direction (deg)</th>
<th>Wind Speed (m/s)</th>
<th>Wind Direction (deg)</th>
<th>Current Speed (m/s)</th>
<th>Current Direction (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4.4</td>
<td>10.9</td>
<td>17</td>
<td>13.7</td>
<td>11</td>
<td>0.21</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>3.9</td>
<td>8.3</td>
<td>174</td>
<td>30</td>
<td>213</td>
<td>0.27</td>
<td>27</td>
</tr>
</tbody>
</table>

Figure 5. The direction of wave, wind, and current of Cases 7 and 10.

The displacement of surge and sway was measured by GPS located at elevation +15.3 relative to sea level. The north motion is the positive direction for sway, and the east motion is defined as the positive motion for the surge. It is worth noting that since the GPS is located at the platform level, some of the translational motions may be due to rotations (roll/pitch motions) [56]. The platform motion angles at roll and pitch DOFs were evaluated based on the angles of the tower’s motion measured with sensors. The coordinate system of the tower motion measurements is presented in Figure 6; its origin is set at +16.9 m relative to sea level. This paper concerns the measured data at four DOFs (i.e., surge, sway, roll, and pitch). In this study, any noise treatments to the monitoring data were not applied before updating, because the quality of the monitoring data used was considered acceptable by using fast Fourier transform analysis. Also, filtering techniques for
mitigating noise probably introduce subjective judgment biases and increase uncertainty. The Bayesian updating method has an inherent advantage in handling uncertainties, making it reducible and common to use monitoring signals with noise. The posterior distribution can quantify the relative plausibility of different values of the model parameters based on the available incomplete information, such as signals with noise [59].

![Figure 6. The coordinate system of the tower motions.](image)

5.4. Prior Analysis by Numerical Simulation

An aero-hydro-servo-elastic coupling framework for dynamic analysis of FOWTs based on AQWA and FAST v7 (F2A), developed by Yang [44], was utilized to simulate the motion responses of the FOWT subjected to wind, wave, and current loads in an operation condition (Case 7) and an idle condition (Case 10). In the idle condition, the blades were locked at a 90° pitch angle and aligned with the wind direction, while the nacelle yaw was maintained at 0 degrees to mitigate wind loads and motions.

The Blade element momentum (BEM) theory and the potential theory were used for calculating aerodynamics and hydrodynamics, respectively. The Kaimal wind spectrum was adopted in FAST v7. The JONSWAP wave spectrum and current load parameters were defined in the Hydrodynamic Response module of the AQWA Workbench 2021 R1. The full low-frequency quadratic transfer function (QTF) was applied due to the importance of the low-frequency wave loads for the spar buoy. Equation (1) is used to calculate the motion responses of FOWT. Each simulation lasted 3600 s with a time step of 0.005 s. Ten different time-domain realizations were obtained for each condition (of significant wave height, peak-spectral period, wind speed, etc.) by changing random numbers of wave and wind between simulations.

5.4.1. Natural Periods

It is crucial to verify the natural periods of the FOWT and tune damping by free decay tests. In this study, we found that the simulation results and the measurement results from Equinor [56] are in good agreement, with a discrepancy of approximately 5% or less, as shown in Table 4.

<table>
<thead>
<tr>
<th></th>
<th>Simulated Data</th>
<th>Monitoring Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Natural Period</td>
<td>Damping Ratio</td>
</tr>
<tr>
<td>Surge</td>
<td>91.63</td>
<td>0.0434</td>
</tr>
<tr>
<td>Sway</td>
<td>91.6</td>
<td>0.0427</td>
</tr>
<tr>
<td>Heave</td>
<td>26.28</td>
<td>0.0272</td>
</tr>
</tbody>
</table>

Table 4. Comparison of the results from numerical simulations and monitoring data.
5.4.2. Motion Response Results

Figure 7 presents the time series and spectral analysis of surge and roll motion responses for both simulation data and monitoring data of Case 7. Figure 8 presents the time series and spectral analysis of surge and roll motion responses for both simulation data and monitoring data of Case 10. Figures 7b,d and 8b,d show dominant frequencies of motion responses, but low-frequency motion responses are underestimated even with full QTF. Some differences in the mean value of motion response can be observed between simulation data and monitoring data in Figures 7a,c and 8a,c. The reasons can be attributed to three main aspects. Firstly, alternative parameters of the wind turbine and tower in OC3 were used. Secondly, it is challenging to obtain accurate values of drag coefficients. Thirdly, the real ocean conditions may not always follow the wind and wave spectrums we used. The existing differences highlight the value of our study, which aims to reduce the differences and improve predictions by integrating limited monitoring information with numerical simulations via Bayesian updating. At present, the simulated motion responses represent the best of current knowledge about FOWTs and should be utilized to develop a prior model. Table 5 shows the mean and standard deviation values of extreme motion responses for surge and roll in Cases 7 and 10. Table 6 provides the parameters of the corresponding Gumbel and Lognormal distributions for the surge and roll motion responses in Cases 7 and 10. Figures 9 and 10 show the statistical histogram of extreme motion response and fitted Gumbel PDF in Cases 7 and 10.

<table>
<thead>
<tr>
<th></th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>34.36</td>
<td>0.0245</td>
<td>33.7</td>
</tr>
<tr>
<td></td>
<td>34.59</td>
<td>0.0239</td>
<td>33.7</td>
</tr>
<tr>
<td></td>
<td>12.89</td>
<td>0.0414</td>
<td>13</td>
</tr>
</tbody>
</table>
Figure 7. Comparison of surge and roll motion responses predicted by AQWA+FAST and full-scale monitoring in Case 7: (a,c) time series of motion responses; (b,d) spectral analysis results.

Figure 8. Comparison of sway and pitch motion responses predicted by AQWA+FAST and full-scale monitoring in Case 10: (a,c) time series of motion responses; (b,d) spectral analysis results.
**Table 5.** The mean and standard deviation of extreme motion responses (surge and roll) in Cases 7 and 10.

<table>
<thead>
<tr>
<th>Monitoring Data</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge (Case 7)</td>
<td>7.5799</td>
<td>1.1507</td>
</tr>
<tr>
<td>Roll (Case 7)</td>
<td>1.2324</td>
<td>0.3547</td>
</tr>
<tr>
<td>Surge (Case 10)</td>
<td>2.0699</td>
<td>0.8446</td>
</tr>
<tr>
<td>Roll (Case 10)</td>
<td>−0.3601</td>
<td>0.5436</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulated Data</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge (Case 7)</td>
<td>8.1088</td>
<td>0.6490</td>
</tr>
<tr>
<td>Roll (Case 7)</td>
<td>0.8840</td>
<td>0.5725</td>
</tr>
<tr>
<td>Surge (Case 10)</td>
<td>2.2042</td>
<td>0.6504</td>
</tr>
<tr>
<td>Roll (Case 10)</td>
<td>−2.2183</td>
<td>1.0627</td>
</tr>
</tbody>
</table>

**Table 6.** The parameters of the Gumbel and Lognormal distributions (surge and roll) in Cases 7 and 10.

<table>
<thead>
<tr>
<th>Likelihood Function</th>
<th>$\alpha_n$</th>
<th>Distribution Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge (Case 7)</td>
<td>1.9763</td>
<td>Gumbel</td>
</tr>
<tr>
<td>Roll (Case 7)</td>
<td>2.2402</td>
<td>Gumbel</td>
</tr>
<tr>
<td>Surge (Case 10)</td>
<td>1.9719</td>
<td>Gumbel</td>
</tr>
<tr>
<td>Roll (Case 10)</td>
<td>1.2374</td>
<td>Gumbel</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prior Function</th>
<th>$\zeta$</th>
<th>$\lambda$</th>
<th>Distribution Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge (Case 7)</td>
<td>0.0998</td>
<td>2.0513</td>
<td>Lognormal</td>
</tr>
<tr>
<td>Roll (Case 7)</td>
<td>0.0998</td>
<td>−0.4728</td>
<td>Lognormal</td>
</tr>
<tr>
<td>Surge (Case 10)</td>
<td>0.0998</td>
<td>0.6429</td>
<td>Lognormal</td>
</tr>
<tr>
<td>Roll (Case 10)</td>
<td>0.0998</td>
<td>0.5628</td>
<td>Lognormal</td>
</tr>
</tbody>
</table>

**Figure 9.** Statistical histograms of extreme motion responses and fitted Gumbel probability density functions (PDFs) for (a) surge and (b) roll in Case 7.
5.5. Platform Motion Monitoring and Bayesian Updating

The prior distribution model about the extreme motion responses was obtained using AQWA 2021 R1 and FAST v7. For the likelihood function, the parameter \( \alpha_n \) of the Gumbel distribution was derived by calculating the standard deviation \( \delta_{\chi} \) of motion responses at each DOF and using Equation (3), while the parameter \( \mu_n \) (as a significant uncertain parameter) was updated by Bayesian updating. Using the mean \( \mu_{n0} \) from the prior analysis, with a COV of 10\%, the hyperparameters of the prior PDF of the parameter \( \mu_n \) was calculated using Equations (6) and (7). The platform motion monitoring data were used to update the prior PDF of the parameter \( \mu_n \) via the Metropolis–Hasting algorithm. Approximately 250 platform motion monitoring samples were applied sequentially in an increment of 50 samples in the Bayesian updating based on the rationale discussed in Section 4.2. The updated PDF of the parameter \( \mu_n \) in the Gumbel distribution at surge and roll DOFs in Cases 7 and 10 are shown in Figures 11 and 12, while Figures 13 and 14 provide a comparison before and after the update of \( \mu_n \) for four DOFs in Cases 7 and 10. The COV of the Gumbel distribution parameter \( \mu_n \) diminishes in both Cases 7 and 10, which means the uncertainties associated with \( \mu_n \) are reduced by incorporating the platform motion monitoring data. As monitoring and updating continue, it is reasonable that uncertainties decrease until they become acceptable, at which point the updating process could be terminated. The decision to stop the updating process was made based on convergence analysis (of the posterior distribution), according to the desired level of reduction in the uncertainty of a specific engineering problem. Meanwhile, Laskey [60] reported that the posterior distribution would be concentrated around the maximum likelihood estimate and relatively insensitive to the prior distribution when the sample size is very large. Finally, although the prior and posterior PDFs of the Gumbel distribution parameter \( \mu_n \) only reflect the reduction in the uncertainty of \( \mu_n \) by Bayesian updating based on the platform motion monitoring data, it is clear that the updated posterior PDFs of the parameter \( \mu_n \) have a favorable impact on the underlying Gumbel distribution of platform motion responses.
Figure 11. Posterior histograms of the Gumbel distribution parameter $\mu_n$ of (a) surge and (b) roll motions in Case 7.

Figure 12. Posterior histograms of the Gumbel distribution parameter $\mu_n$ of (a) surge and (b) roll motions in Case 10.

Figure 13. Bayesian updating results of the Gumbel distribution parameter $\mu_n$ for (a) surge and (b) roll motions in Case 7.
Figure 14. Bayesian updating results of the Gumbel distribution parameter $\mu_\alpha$ for (a) surge and (b) roll motions in Case 10.

Figures 15 and 16 show the original and updated PDFs of the extreme motion responses of surge and roll DOFs in Cases 7 and 10, along with frequency histograms of platform motion monitoring data. It can be observed that the updated distributions of extreme motion responses satisfy the Lognormal distribution and shift toward the platform motion monitoring data. The reduction in the standard deviation in Table 7 is a conclusion to show the reduction in uncertainty. It is clearly observed that the uncertainty of $\mu_\alpha$ is significantly reduced in most cases after Bayesian updating, with a maximum reduction of up to 91%. Although the uncertainty of variable $\mu_\alpha$ increases to some extent for roll motion in Case 7, the updated posterior mean value converges toward the monitoring data, outperforming the prior simulation results. The uncertainty of the extreme motion responses $X$ is also slightly reduced due to the favorable impact of $\mu_\alpha$. As more data were used, the Bayesian updating method could successfully estimate the posterior distribution of variables with an increasing degree of certainty. This paper presents the framework with two representative load cases, as the primary objective was to develop a Bayesian updating and inference framework leading to improved FOWT motion response probabilistic predictions. The framework can be applied to other load cases or sea states as well since load cases or sea states are just part of the inputs in this framework.

Figure 15. Original and updated PDFs of the extreme motion responses for (a) surge and (b) roll motions in Case 7.
Figure 16. Original and updated PDFs of the extreme motion responses for (a) surge and (b) roll motions in Case 10.

Table 7. Reduction in uncertainty using Bayesian updating in Cases 7 and 10.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Wave Height</th>
<th>Motion</th>
<th>Prior Mean</th>
<th>Prior Standard Deviation</th>
<th>Posterior Mean</th>
<th>Posterior Standard Deviation</th>
<th>Reduction in Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Variable $\mu_n$</td>
<td>Surge 7.821</td>
<td>0.781</td>
<td>6.420</td>
<td>0.070</td>
<td>91.04%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Roll 0.626</td>
<td>0.062</td>
<td>0.981</td>
<td>0.070</td>
<td>-12.9%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Extreme motion</td>
<td>Surge 8.109</td>
<td>0.649</td>
<td>6.712</td>
<td>0.640</td>
<td>1.39%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Roll 0.884</td>
<td>0.573</td>
<td>1.240</td>
<td>0.570</td>
<td>0.52%</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Variable $\mu_n$</td>
<td>Surge 1.991</td>
<td>0.191</td>
<td>1.990</td>
<td>0.069</td>
<td>63.87%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Roll 1.764</td>
<td>0.184</td>
<td>0.955</td>
<td>0.055</td>
<td>70.11%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Extreme motion</td>
<td>Surge 2.204</td>
<td>0.650</td>
<td>2.283</td>
<td>0.650</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Roll -2.218</td>
<td>1.063</td>
<td>-1.426</td>
<td>1.034</td>
<td>2.73%</td>
<td></td>
</tr>
</tbody>
</table>

6. Conclusions

In this paper, we introduced an effective Bayesian updating method to dynamically and probabilistically predict the extreme value of motion response of FOWTs by using monitoring data under real sea conditions for the first time. The method provides a theoretical framework for rational and logical integration of monitoring data with prior motion predictions, enhancing the precision and reliability of these predictions under both normal and extreme conditions. The numerical results of motion responses (performed with the effective knowledge and information of FOWTs) were used to construct the prior PDF of a random variable ($\mu_n$), which characterized the largest value of motion response. The likelihood function was established by analyzing platform motion monitoring data, and the posterior probability distribution was derived via Bayesian updating. The posterior prediction results of short-term extreme platform motion responses were obtained based on the posterior probability distribution of $\mu_n$. Finally, accurate posterior predictions play an important role in platform structural design, risk management, and saving costs.

Our proposed approach was demonstrated to be effective in reducing uncertainty in the extreme response prediction of FOWT platforms using the Hywind wind farm in Scotland as a case study. The platform motion response data obtained from Equinor were used. It was clearly observed that the uncertainty of the variables $\mu_n$ was significantly reduced in most cases after Bayesian updating, with a maximum reduction of up to 91%. Although the uncertainty of variable $\mu_n$ increased to some extent for roll motion in Case 7, the updated posterior mean value converged toward the monitoring data, outperforming the prior simulation results. The uncertainty of the extreme motion responses X was also slightly reduced due to the favorable impact of $\mu_n$. As more data were used, the
Bayesian updating method could successfully estimate the posterior distribution of variables with an increasing degree of certainty.

**Author Contributions:** Conceptualization, G.Z. and N.L.; methodology, G.Z. and N.L.; software, N.L.; validation, N.L. and Y.F.; investigation, N.L., G.Z., and Y.F.; resources, G.Z.; data curation, N.L.; writing—original draft preparation, N.L.; writing—review and editing, G.Z., Y.F., L.A., and N.L.; visualization, N.L.; supervision, G.Z. and Y.F.; funding acquisition, G.Z. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the Southern University of Science and Technology, grant number Y01316134.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** The data utilized in this study were obtained from publicly available sources and previously published datasets, which are appropriately cited in the references. No new data were created for this research. Due to privacy or ethical restrictions, certain data used in this study cannot be shared.

**Acknowledgments:** Thanks to Equinor for sharing operational data of Hywind Scotland through ORE Catapult.

**Conflicts of Interest:** The authors declare no conflicts of interest.

**References**


Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.