New Adaptive Super-Twisting Extended-State Observer-Based Sliding Mode Scheme with Application to FOWT Pitch Control

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Abstract: This paper details the transformation of the velocity or position-tracking problem of a class of uncertain systems using finite time stability control for first-order uncertain systems. A new composite extended-state observer sliding mode (ESOSM) scheme is proposed, which includes an adaptive super-twisting-like ESO and an adaptive super-twisting controller. The adaptive super-twisting controller is implemented through a barrier function-based second-order sliding mode algorithm. To further reduce control chattering and improve control performance, the adaptive super-twisting-like ESO, which employs high-order terms in the super-twisting algorithm to accelerate convergence, is designed to observe the lumped uncertainty in real time. The advantages of the proposed scheme are verified by a numerical example and application with regard to floating offshore wind turbine (FOWT) pitch control. Compared with proportional integral (PI) and adaptive super-twisting sliding mode (ASTSM) schemes, better results are obtained in velocity tracking and fatigue load suppression. For the FOWT pitch control application, the platform roll, pitch, and yaw are decreased by 3%, 2%, and 4%, respectively, compared to the PI scheme at an average turbulent wind speed of 17 m/s and turbulence intensity of 17.27%.

Keywords: extended-state observer; super-twisting sliding mode; adaptive gain; floating offshore wind turbine

1. Introduction

Wind power generation based on a floating platform is a development trend in deepsea wind energy utilization [1]. However, the floating platform motion exacerbates output power fluctuations and fatigue loads [2]. Repeated platform motion not only leads to fatigue damage to the platform structure but also causes fluctuations in power output. Therefore, developing an appropriate pitch control strategy is crucial to mitigate platform motion, minimize fatigue loads, and ensure the stability of power production. Yet, floating offshore wind turbines (FOWTs) are often intricate dynamic systems, characterized by significant nonlinearity and interconnection and subject to disturbances such as ocean waves and wind. Traditional algorithms used for onshore wind turbine control can no longer maintain high control performance when the operating point is altered.

The sliding mode control (SMC) method exhibits robust performance in the presence of uncertainties, including parameter perturbations, unmodeled dynamics, and external disturbances. It has been extensively utilized in the context of uncertain nonlinear systems [3]. However, traditional SMC still has two major drawbacks: the chattering problem and relative order limitation [4]. As a form of the SMC algorithm, the high-order SMC algorithm can overcome the relative order restriction and chattering phenomenon of traditional first-order SMC, while retaining strong robustness and higher control accuracy.
Second-order SMC belongs to the category of higher-order SMC techniques, with its algorithmic approaches predominantly encompassing the twisting algorithm, suboptimal algorithm, quasi-continuous method, and super-twisting algorithm [9]. The super-twisting algorithm is a special second-order SMC algorithm that requires only the knowledge of the sliding mode variables and does not necessitate the acquisition of their derivatives [10]. It can produce continuous control and reduce sliding mode chattering for first-order uncertain systems. At present, it is widely used in electromechanical systems and has also undergone hardware verification [11,12].

Although the control effect of the super-twisting algorithm is continuous when applied to systems with a relative degree of one, this is actually achieved by hiding discontinuous terms in the integral action. The chattering phenomenon is not thoroughly eliminated but is relatively suppressed [13,14]. Meanwhile, the standard super-twisting SMC algorithm requires the upper bound of the uncertainty derivative, which is difficult to determine in many practical systems [15]. If the upper bound is large, control gain parameters will be larger to guarantee system stability and facilitate establishment of the sliding mode. This will undoubtedly increase the chattering amplitude. In the event that the upper bound is relatively small and the control gain is smaller, it is impossible to establish a sliding mode, the system stability cannot be guaranteed, and the robustness is significantly affected [16].

In order to overcome the challenge of determining an upper bound on the uncertainty derivative in the context of the super-twisting algorithm, scholars have explored adaptive super-twisting algorithms [17]. According to the development trend in recent years, adaptive super-twisting algorithms can be roughly classified into four types. The first type involves the control gain constantly increasing until the sliding surface is attained and remains unchanged, and also constantly increasing as the upper bound of the uncertainty derivative becomes large [18,19]. With this type of algorithm, an ideal second-order sliding mode with monotonically increasing control gain is established. As the subsequent uncertainty effect decreases, the control gain remains constant. This indicates that this type of control is not truly adaptive. The second type involves using equivalent control as a disturbance estimate. After the sliding mode is established, the high-frequency control input is filtered through a low-pass filter and treated as a disturbance variable in the control gain [20]. However, the equivalent control signal method has the disadvantage that its filter constant must be much smaller than the reciprocal of the upper bound of the disturbance derivative. Therefore, the use of low-pass filters in control design implicitly requires information about this upper bound. The third type ensures that the sliding variable converges to the origin while decreasing the overestimate of adaptive gain [21]. Compared with an equivalent control-based strategy, the advantage of this method is that there is no need to know the upper bound of disturbance derivative. However, this method still suffers from the disadvantage that the sliding variables are not accurately constrained within a predetermined neighborhood of the origin. As a result, the control accuracy of this type is not as good as that of equivalent control-based methods and cannot be controlled according to the preset accuracy. The fourth type relies on the barrier function for adjusting the control gain up or down. It guarantees control accuracy and does not require knowledge of the upper bound of the disturbance derivative [22,23]. This method can further reduce the overestimate of the adaptive gain by strictly restricting the sliding variables to a predefined neighborhood of the origin.

While retaining the advantages of adaptive SMC, disturbance observers can be combined to further improve tracking performance and robustness and reduce control chattering [24,25]. In order to observe the lumped system disturbance caused by matching uncertainty and non-matching uncertainty, extended state observer (ESO) can be employed [26]. To make ESO robust, sliding-mode ESO may be used. The trajectory
tracking control problem of photoelectric tracking systems with friction and other nonlinear disturbances was studied based on super-twisting ESO and fractional terminal SMC in [27]. In [28], DC motor position control was realized based on super-twisting control, and super-twisting ESO was used to observe the matching and non-matching uncertainties. In [29], to address the problem of fast estimation and compensation of the temporal disturbance torque acting on the front wheel of an automobile steer-by-wire system under different working conditions, a control scheme combining the super-twisting algorithm and the fast super-twisting disturbance observer is proposed. In these studies, the super-twisting ESO observers are not adaptive, and the upper bound of the observation error derivative is presumed to be known in advance. An adaptive super-twisting ESO was designed in [30], but the gain was monotonically increased, and a first-order sliding mode was used in the SMC part, which affected the overall control performance.

Thus, this paper transforms the tracking problem into a first-order uncertain system SMC problem with state feedback control consideration. A new composite extended state observer sliding mode (ESOSM) scheme is proposed, which is suitable for FOWT pitch control and can solve the fatigue structural load and power fluctuation problems induced by sea waves and wind on the floating support platform. FAST (Fatigue, Aerodynamics, Structure, and Turbulence) is employed to model FOWT, and MATLAB/Simulink is used to execute combination schemes. These two simulation tools are used jointly to perform simulation experiments. The mechanical stress of the tower and blade is analyzed by MLife, and the described force and moment are quantified by the fatigue damage equivalent load (DEL).

The main innovations include the following: a new adaptive SMC and observation composite scheme is presented; a new adaptive super-twisting-like ESO is developed to observe the lumped uncertainty, further suppress the chattering, and enhance the control quality; and the proposed composite ESOSM scheme is applied to FOWT pitch control, and the fatigue load suppression is improved.

This paper is organized as follows. Section 2 details the wind turbine modeling and problem formulation. The ESOSM scheme design and the convergence proof are outlined in Section 3. Section 4 includes two application simulations: comparison using a numerical example and FOWT pitch control as analyzed using the proposed scheme. The last section includes a concluding discussion.

2. Wind Turbine Modeling and Problem Formulation

Figure 1 shows the barge type FOWT model. Wind turbine and platform parameters are as specified in [31].
The rotor angular velocity dynamic of the FOWT is determined as follows:

\[ \dot{\omega}_w = - \frac{\mu_w}{J_w} \omega_w + \frac{T_w}{J_w} - \frac{N_w}{J_w} \dot{T}_g \]  

(1)

where \( \omega_w \) is the rotor angular velocity, \( \mu_w \) indicates the viscous friction coefficient at the low-speed shaft, \( T_g \) represents the generator torque, \( N_w \) is the gear transmission ratio, and \( J_w \) is the inertia coefficient. \( \mu_w \) and \( J_w \) can be represented as \( \mu_w = \pi_w + \Delta \mu_w \) and \( J_w = \bar{T}_w + \Delta J_w \) with \( |\Delta \mu_w| \leq d_{\mu_w} \), \( |\Delta J_w| \leq d_{\mu_w} \), and \( \bar{T}_w, \bar{J}_w \) are nominal values. \( \Delta \mu_w \) and \( \Delta J_w \) are uncertain items. \( d_{\mu_w} \) and \( d_{\mu_w} \) are unknown upper bounds.

The mechanical torque of the FOWT is

\[ T_w = \frac{k_r r_w C_p(\lambda_w, \beta_f) \omega_w^3}{\lambda_w} \]  

(2)

where \( k_r = \frac{\pi}{2} \rho_w r_w^2 \), \( r_w \) and \( \lambda_w \) represent the rotor radius and tip speed ratio, \( C_p(\lambda_w, \beta_f) \) indicates the power conversion efficiency, \( v_w \) is the inflow wind speed, \( \beta_f \) is the blade pitch angle, and \( \rho_w \) is air density.

The power conversion efficiency of the FOWT is expressed as

\[ C_p(\lambda_w, \beta_f) = p_1(p_1 y_w + p_2 \beta_f + p_3)\exp(p_4 v_w) \]  

(3)

\[ y_w = \frac{1}{\lambda_w + 0.08\beta_f} - \frac{0.035}{\beta_f^3 + 1} \]  

(4)

The values of coefficient \( p_i (i = 1, 2, \cdots, 5) \) are determined by the blade and aerodynamic performance. The nominal values of NREL 5-MW FOWT are \( \bar{\rho}_1 = 7.02 \), \( \bar{\rho}_2 = -0.0418 \), \( \bar{\rho}_3 = -0.386 \), \( \bar{\rho}_4 = -14.52 \), and \( \bar{\rho}_5 = 6.909 \). \( \exp(p_4 v_w) \) can be approximately represented by a series of nominal linear regressions. \( \bar{\rho}_{i_1} \) and \( \bar{\rho}_{i_2} \) are fixed values. \( \frac{0.08\beta_f}{\lambda_w} \leq 1 \) and \( \frac{1}{\lambda_w + 0.08\beta_f} \gg \frac{0.035}{\beta_f^3 + 1} \) are assumed to hold. Then, \( y_w \) is approximately represented as \( \frac{1}{\lambda_w (1 - \frac{0.08\beta_f}{\lambda_w})} \). The power conversion efficiency of the FOWT can be simplified as

\[ \tilde{C}_p(\lambda_w, \beta_f) = \bar{\rho}_s(\bar{\rho}_{i_1} \bar{\lambda}_w + \bar{\rho}_{i_2} h_{i_1}(\lambda_w) - \beta_f \cdot h_{i_2}(\lambda_w)) + d_{i_1}(\lambda_w, \beta_f) \]  

(5)

where \( h_{i_1}(\lambda_w) = \frac{p_1}{\lambda_w + \rho_3} \), \( h_{i_2}(\lambda_w) = 0.08 \rho_4 / \lambda_w^3 - \rho_2 \), and \( d_{i_1}(\lambda_w, \beta_f) \) are the simplified errors.

Combining Formulas (1), (2), and (5), and considering the influence of wind and waves, the FOWT model is represented as

\[ \dot{\omega}_w = \frac{k_r r_w \omega_w^3}{J_w \lambda_w} \bar{\rho}_s(\bar{\rho}_{i_1} \bar{\lambda}_w + \bar{\rho}_{i_2} h_{i_1}(\lambda_w) - \beta_f \cdot h_{i_2}(\lambda_w)) + d_{i_1}(\lambda_w, \beta_f, \omega_w, v_w, w_w) \]

\[ d(\lambda_w, \beta_f, \omega_w, v_w, w_w) = \frac{k_r r_w \omega_w^3}{J_w \lambda_w} \bar{\rho}_s(\bar{\rho}_{i_1} \bar{\lambda}_w + \bar{\rho}_{i_2} h_{i_1}(\lambda_w) - \bar{\rho}_s \omega_w) 
- \frac{N_w \dot{\bar{P}}_w}{\omega_w} + \frac{k_r r_w \omega_w^3}{J_w \lambda_w} d_{i_1}(\lambda_w, \beta_f, \omega_w, v_w, w_w) \]  

(6)
where \( d_{w2}(\lambda_w, \beta_j, v_w, w_w) \) is aroused by internal uncertainty as well as wind and wave disturbance, and it has an upper bound. Considering physical constraints of the FOWT, the derivative of \( d(\cdot) \) is bounded. The FOWT operates above the rated wind speed. The generator power is \( P_w = N_w T_g \cdot \omega_w \), and \( T_g \) is the generator rated torque.

The FOWT model cannot be simply represented by a linear state equation due to system parameter perturbations as well as wind and wave disturbances. It is a complex uncertain nonlinear system.

Considering that the proposed algorithm is suitable for a class of system and has wide application, this section details the study of a class of uncertain affine nonlinear systems:

\[
\dot{x} = f(x, t) + g(x, t)u
\]  

(7)

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R} \) is the control input, \( f(x, t) \) and \( g(x, t) \) are smooth uncertain functions, \( f(x, t) \) contains an unmeasurable disturbance term, and \( g(x, t) \neq 0 \).

The sliding mode variable is defined as \( \sigma = \sigma(x, t) \). It is assumed that the sliding mode variable is either a position tracking error, a velocity tracking error, or some linear combination of state variables. The relative degree of (7) for \( \sigma \) is one.

\[
\dot{\sigma} = \psi(x, t) + \varphi(x, t)u + \Delta f(t)
\]  

(8)

where \( \psi(x, t) = \varphi + \Delta \psi \) and \( \varphi(x, t) = \varphi + \Delta \varphi \) are smooth uncertain functions, \( \varphi \) and \( \varphi \) are the known nominal parts of the corresponding function, and \( \Delta \psi \) and \( \Delta \varphi \) are uncertain terms. \( \Delta f(t) = \Delta f \) is the external disturbance (e.g., wind and wave disturbance).

Then,

\[
\dot{\sigma} = \varphi + \Delta \psi + (\varphi + \Delta \varphi)u + \Delta f
\]  

\[
= \varphi + \Delta \psi u + \Delta \varphi u + \Delta f
\]  

\[
= \varphi + \Delta \psi u + d
\]  

(9)

where \( d \) represents lumped uncertainty, \( |d| \leq D_d \) is satisfied, and \( D_d \) is the unknown upper bound.

For Formula (9), feedback control law \( u \) is constructed as

\[
u = \frac{1}{\varphi}(v - \varphi)
\]  

(10)

Substituting Formula (10) into Formula (9) yields

\[
\dot{\sigma} = v + d
\]  

(11)

where \( v \) is the auxiliary control law.

For Formula (11), the standard super-twisting algorithm can be applied to establish the second-order sliding mode. Convergence to the equilibrium point is achieved in finite time. The standard super-twisting algorithm is

\[
\begin{align*}
\dot{v} &= -\alpha_1 \sigma |\sigma|^{1/2} \text{sign}(\sigma) + v_1 \\
\dot{v}_1 &= -\alpha_2 \text{sign}(\sigma)
\end{align*}
\]  

(12)

where \( \alpha_1 \) and \( \alpha_2 \) are control gains. If \( \alpha_1 = 1.5\sqrt{D_d} \) and \( \alpha_2 = 1.1D_d \) are satisfied, \( \sigma \) converges to the equilibrium point in finite time. However, in an actual system, \( d \) includes matching uncertainty, unmatched uncertainty, and the control term, such that \( D_d \) is very difficult to determine. When \( \alpha_1 \) and \( \alpha_2 \) are large, it can easily cause a large
chattering amplitude value. When \( \alpha_1 \) and \( \alpha_2 \) are small, the sliding mode \( \sigma \) cannot be established, and the robustness is affected.

Thus, to handle the problem, we propose the new ESOSM scheme to obtain a real second-order sliding mode, reduce control chattering, and obviate the requirement for an unspecified upper limit of the uncertainty derivative. Three schemes were compared and validated in order to assess the efficacy of the newly devised strategy.

3. Adaptive Super-Twisting ESO-Based Sliding Mode Scheme

3.1. The Composite Scheme Design

In practical applications, it is desirable for the controlled object to meet the required control accuracy. Indeed, considering the system uncertainty and discrete computing, it is also impractical to set up an ideal second-order sliding mode. Thus, it is expected that \( \alpha_1 \) and \( \alpha_2 \) can adaptively adjust according to the change of disturbance \( d \), and the control effect can continuously change, such that \( \sigma = \sigma(x, t) \) converges to a predetermined domain range in finite time and establishes a real second-order sliding mode with respect to \( \sigma \).

The adaptive super-twisting algorithm, which can adapt the control gain according to the derivative change of the uncertain upper bound, is more suitable for practical applications. In recent years, some promising studies have been conducted in this field [22, 32, 33]. For Formula (11), an adaptive super-twisting algorithm is presented, as follows [22]:

\[
\begin{align*}
| \sigma_t & = -k(t, \sigma_0) | \sigma |^2 \text{sign}(\sigma) + v_1 \\
\dot{v}_1 & = -k^2(t, \sigma_0) \text{sign}(\sigma) \\
k(t, \sigma_0) & = \begin{cases} 
  k_1 + k_0 & 0 \leq t < t_f(\sigma_0) \\
  k_2(\sigma) & t \geq t_f(\sigma_0)
\end{cases} \\
k_2(\sigma) & = \frac{\sqrt{\mu \rho}}{(\mu-|\sigma|)^{\frac{3}{2}}}
\end{align*}
\]

where \( t_f(\sigma_0) \) is the first moment that \( |\sigma_0| = |\sigma(0)| \leq \mu/2 \) is satisfied. \( k(t, \sigma_0) \) is the variable gain. \( k_0 \) and \( k_1 \) are arbitrary positive constants. \( k_2(\sigma) \) is a barrier function, the property of which is as specified as [23]. \( \mu \) is a constant that specifies the converging neighborhood of \( \sigma \). \( \rho \) is a positive constant. Control gains are designed as (13) for (11); then, for any \( \sigma_0 \in \mathbb{R} \), the existence of \( t_f(\sigma_0) \) makes \( 1 \sigma \leq \mu/2 \) hold. Then, for all \( t \geq t_f(\sigma_0), 1 \sigma < \mu \) is satisfied.

If the lumped uncertainty can be observed via a disturbance observer, then super-twisting control gains will be significantly reduced, thereby further reducing control chattering. Formula (9) is rewritten as

\[
\dot{\sigma} = \bar{v} + \bar{v}_u + \hat{d} + \tilde{d}
\]

where \( \hat{d} \) is the observed value and \( \tilde{d} \) is the observation error; \( \hat{d} = d - \tilde{d} \).

Feedback control \( u \) is constructed as

\[
u = \frac{1}{\bar{v}}(v - \bar{v} - \hat{d})
\]

If we substitute Formula (15) into Formula (16), then

\[
\dot{\sigma} = v + \tilde{d}
\]
where \( v \) is the auxiliary control. For Formula (16), the adaptive super-twisting algorithm (13) may be employed for \( v \) to achieve the control objectives.

\( \hat{d} \) is unknown in Formula (15). Next, the adaptive super-twisting-like ESO is constructed in detail to observe disturbance \( \hat{d} \).

When \( d \) is extended to a new state variable, \( v \), it is deduced as

\[
\begin{align*}
\dot{\sigma} &= \overline{\psi} + \overline{q} u + v \\
\dot{v} &= \xi
\end{align*}
\]

(17)

where \( \xi \) is the derivative of \( v \), and the observed value \( \hat{\sigma} \) of \( \sigma \) and the observed value \( \hat{\xi} \) of \( \xi \) are \( Z_1 \) and \( Z_2 \), respectively. On the basis of analysis of the barrier function sliding mode controller [22,34,35], an adaptive super-twisting-like ESO is constructed as

\[
\begin{align*}
e_1 &= Z_1 - \sigma \\
Z_1 &= \overline{\psi} + \overline{q} u + Z_2 - \gamma_1 \sigma e^{1/2} \text{sign}(e_1) - \gamma_0 \sigma e^{1/2} \text{sign}(e_1) \\
Z_2 &= -\frac{\gamma_2}{2} \sigma e^{1/2} \text{sign}(e_1)
\end{align*}
\]

(18)

where \( \gamma_1 \) and \( \gamma_2 \) are the adaptive observer gains to be designed. \( \gamma_0 \) is positive constant. If we define the observer error as \( e_1 = Z_1 - \sigma \), \( e_2 = Z_2 - \sigma \), then

\[
\begin{align*}
\dot{e}_1 &= -\gamma_1 \sigma e^{1/2} \text{sign}(e_1) - \gamma_0 \sigma e^{1/2} \text{sign}(e_1) + e_2 \\
\dot{e}_2 &= -\frac{\gamma_2}{2} \sigma e^{1/2} \text{sign}(e_1) + \eta_i
\end{align*}
\]

(19)

where \( \eta_i = -\xi \). It is observed that (19) is converted to a standard super-twisting algorithm when \( \gamma_0 \) is chosen as 0. The term \( \gamma_0 \sigma e^{1/2} \text{sign}(e_1) \) can speed up convergence.

Adaptive observer gains \( \gamma_1 \) and \( \gamma_2 \) are constructed as

\[
\gamma_1 = \begin{cases} 
  c_1 t + c_0 & 0 \leq t < t_i \\
  M t^{1/3} & t \geq t_i
\end{cases}
\]

(20)

\[
\gamma_2 = 2l_1 \gamma_1 + 2(l_2 + 4l_1^2) \gamma_1 \gamma_1
\]

where \( c_1 \) and \( c_0 \) are arbitrary positive constants, \( l_1, l_2, l_3 \) are positive constants to be set, and \( t_i \) is the time taken for \( \| e_1 \| \) to arrive at \( J/2 \) for the first time.

3.2. Disturbance Observer Error Dynamic Stability Analysis

**Theorem 1.** Considering perturbed augmented systems (17), the observer errors \( e_1 \) and \( e_2 \) can converge in finite time and \( \| e_1 \| < J/2, \| e_2 \| < e \) hold, when the adaptive gains are constructed as (20) for the proposed disturbance observer (18).

First, to prove the existence of finite time \( t_i \), such that \( \| e_1 \| < J/2 \) holds under the action of adaptive gain (20), without loss of generality, \( \| e_i(0) \| < J/2 \) can be supposed to hold immediately. It is observed from (20) that control gain \( \gamma_1 \) is \( c_1 t + c_0 \) as long as \( \| e_i(t) \| < J/2 \) holds. The time interval is \([0, T_0]\), which satisfies \( \| e_i(0) \| > J/2 \). We need to prove that \( T_0 \) is finite, which means that \( T_0 \) is the desired time \( t_i \).
Proof by contradiction is employed. The inequality \( \| e_i(t) \| J \geq J/2 \) is assumed to be satisfied for interval \([0, T_0]\), and \( e_i \) is positive without losing generality. From the second equation of (19), it is deduced as

\[
-\frac{\gamma_2}{2} - D_2 \leq \dot{e}_2 \leq -\frac{\gamma_2}{2} + D_2
\]  

(21)

It is easy to determine that the time interval for (21) to hold is \([0, \infty)\). In particular, \( e_2 \) becomes and remains negative. From the first part of (19), \( \dot{e}_1 \leq -\gamma_1 \| e_1 \|^2 - \gamma_0 \| e_1 \|^2 \) is satisfied. Thus, \( e_1 \) converges to 0 in finite time, which contradicts \( \| e_i(t) \| J \geq J/2 \) being satisfied in the time interval \([0, \infty)\). Then, the proof of \( \| e_i \| J < J \) under adaptive gains (20) is completed.

Then, let us prove that \( \| e_i \| < J \) and \( \| e_2 \| < e \) are always satisfied for all \( t > t_1 \). To define a new state vector, \( \tau = [\tau_1, \tau_2]^T \),

\[
\begin{cases}
\tau_1 = \| e_1 \| \text{ sign}(e_1) \\
\tau_2 = e_2
\end{cases}
\]

(22)

To define \( \dot{\tau} = \eta_1 = \frac{c_1 \tau_1}{2} \), \( c_3 \) is an uncertain coefficient and satisfies

\[
\| c_3 \| \leq 2D_2
\]

(23)

For Formula (19), the time derivative of \( \tau \) can be represented as

\[
\dot{\tau} = \frac{1}{2 \| \tau_1 \|} \left( A \tau + \begin{bmatrix} 0 \\ c_3 \end{bmatrix} \right)
\]

(24)

where \( A = \begin{bmatrix} \gamma_1 - \gamma_0 \| e_1 \|^2 & 1 \\ -\gamma_2 & 0 \end{bmatrix} \).

Take note that \( \text{sign}(e_1) = \text{sign}(\tau_1) \), and \( e_1, e_2 \) will converge to 0 in finite and fixed time if \( \tau_1 \) and \( \tau_2 \) converge to 0 in finite and fixed time \( T \).

Consider the Lyapunov function:

\[
V(\tau) = \tau^T \Gamma \tau
\]

(25)

where \( \Gamma = \begin{bmatrix} l_2 + 4l_1^2 & -2l_1 \\ -2l_1 & 1 \end{bmatrix} \), \( l_2 > 0; l_1 > 0 \).

From (19), we can derive the following:

\[
\dot{V}(\tau) = \dot{\tau}^T \dot{\tau} + \dot{\tau}^T \Gamma \dot{\tau} = \frac{1}{2 \| \tau_1 \|} \left( \tau^T \left( A^T \Gamma + \Gamma A \right) \tau + 2\tau^T \Gamma \begin{bmatrix} 0 \\ c_3 \end{bmatrix} \right) = -\frac{1}{2 \| \tau_1 \|} \tau^T \Theta(\tau) \tau
\]

(26)

where \( \Theta(\tau) = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix} \), \( \Theta_{11} = 4l_1(2l_1 \gamma_1 - \gamma_2) + 4(l_1 + 4l_1^2) \gamma_0 \| \tau_1 \|^2 + 4l_1 c_3 + 2l_2 \gamma_1 \gamma_3 \), and \( \Theta_{12} = \gamma_2 - 2l_1 \gamma_1 - 2(l_1 + 4l_1^2) \gamma_0 \| \tau_1 \|^2 - l_2 - 4l_1^2 - c_3 \), \( \Theta_{22} = 4l_1 \).

In order to guarantee the positive definiteness of \( \Theta(\tau) \), \( \gamma_2 \) has to satisfy

\[
\gamma_2 = 2l_1 \gamma_1 + 2(l_1 + 4l_1^2) \gamma_0 \gamma_3
\]

(27)
If $\gamma_1 > \gamma$, holds, $\Theta(\tau)$ will be a positive definite, with a minimum eigenvalue of $\lambda_{\min}(\Theta(\tau)) > 2l_1$.

$$\gamma_1 = \frac{l_1(4D_d + 1) + 2D_d + l_2 + 4l_3}{l_1(1-c_q)}$$

where $0 < c_q < 1$. Then,

$$\dot{V}(\tau) \leq \frac{l_1}{2|\tau|} \|e\|^2$$

Because Formulas (30) and (31) hold,

$$\lambda_{\max}(\Gamma) \|\tau\|^2 \leq V(\tau) \leq \lambda_{\min}(\Gamma) \|\tau\|^2$$

$$l_\tau \leq \sqrt{l_1^2 + l_2^2} = \|\tau\| \leq \frac{V^1/2(\tau)}{\sqrt{\lambda_{\min}(\Gamma)}}$$

Then,

$$\dot{V}(\tau) \leq -\rho V^{1/2}(\tau)$$

where $\rho = \frac{l_1}{2} \sqrt{\frac{\lambda_{\min}(\Gamma)}{\lambda_{\max}(\Gamma)}}$.

Considering the inequality $|e_i(t)| < J$ and where $\gamma_1 = \frac{M J^{1/5}}{J^{1/5} - |e_i|^{1/5}}$ is a monotone increasing function with regard to $|e_i|$ when $|e_i| \in [0, J]$, then $e_{11} < J$:

$$e_{11} = \begin{cases} J \left(1 - \frac{M}{\gamma_1^{1/5}}\right)^w & M < \gamma_1^{1/5} \\ 0 & M \geq \gamma_1^{1/5} \end{cases}$$

Thus, $\gamma_1 = \frac{M J^{1/5}}{J^{1/5} - |e_i|^{1/5}} > \gamma_1^{1/5}$ and Formula (32) hold if $|e_i| > \gamma_1^{1/5}$ is satisfied; then, $V(\tau)$ is a negative definite for $e_{11} \leq |e_i| < J$. Therefore, $|e_i| \leq e_{11} < J$ holds for $t > t_1$. $|e_i|$ will be increased to $e_{11}$ from $J/2$ after time interval $T$ if $J/2 < e_{11}$ holds. Therefore, the bound of $e_2$ may be assessed as $\varepsilon = \int_{t_1}^{t_f} \left(\frac{\gamma_2(t)}{2} + D_d\right) dt$, where $t_f$ is the moment when $\tau_1$ converges to 0.

To sum up, the composite scheme designed in this section is shown in Figure 2. The lumped uncertainty is effectively addressed through the implementation of the adaptive super-twisting-like ESO, while the feedback control is seamlessly integrated with the adaptive super-twisting algorithm to successfully attain the desired control objective.
The efficacy of the suggested ESOSM scheme is evaluated through a comparison with the adaptive super-twisting sliding mode (ASTSM) control scheme outlined in [22], as depicted in (13). The parameters in (9) are chosen as $\varphi = 3$ and $\varphi = 2$. The disturbances are:

$$d = \begin{cases} 1.5\sin(5t), & \text{if } t \leq 3\pi s \\ 2.75\sin(3t), & \text{if } 3\pi s < t \leq 5\pi s \\ 5.5\sin(3t), & \text{if } t > 3\pi s \end{cases}$$

(34)

Then, Formula (9) can be represented as

$$\dot{\sigma} = 3 + 2u + d$$

(35)

Table 1. Parameter values.

<table>
<thead>
<tr>
<th>Control Scheme</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESOSM</td>
<td>$k_1 = 1$, $k_0 = 0.1$, $\mu = 0.01$, $\varphi = 0.01$, $c_1 = 1$, $c_0 = 0.1$, $\gamma_0 = 1$, $l_1 = 1$, $l_2 = 1$, $l_3 = 1$</td>
</tr>
<tr>
<td>ASTSM</td>
<td>$k_1 = 2$, $k_0 = 0.1$, $\mu = 0.01$</td>
</tr>
</tbody>
</table>

The observed and actual lumped disturbances are shown in Figure 3a, which shows good observation performance. The error between the observed disturbances and the actual disturbances is shown in Figure 3b. The sliding variable under the two schemes is shown in Figure 4a, which indicates faster convergence and smaller overshoot under the proposed scheme. Figure 4b shows the control inputs under the two schemes. As can be observed, the control inputs of the proposed scheme have less clattering. Overall, the observation of external disturbance, the establishment of second-order sliding mode dynamics, the improvement of dynamic performance, and the reduction of control input chattering are achieved under the proposed composite scheme.
Figure 3. (a) The actual value (blue) and observed value (red) of the lumped disturbances. (b) The observation error of the lumped disturbances.

Figure 4. (a) The sliding mode surface $\sigma$ and (b) the control input $u$.

4.2. Blade Pitch Control of the FOWT

In this section, the proposed scheme is applied to FOWT pitch control for stabilizing power output and suppressing platform motion and fatigue load simultaneously. NREL 5MW FOWT is employed.

For the FOWT model, to choose the integral sliding surface,
\begin{equation}
\sigma_w = \omega_w - \omega_w^* + c_w \int_0^t (\omega_w - \omega_w^*) d\tau
\end{equation}

(36)

where \( \omega_w^* \) is the rated rotor angular velocity, taking into account platform displacement, and \( c_w \) is a positive constant. \( \omega_w^* \) is depicted as

\begin{equation}
\omega_w^* = \omega_{wr} - k_w \dot{\theta}
\end{equation}

(37)

where \( \omega_{wr} \) is the rated rotor speed, \( \dot{\theta} \) is the platform pitch speed, and \( k_w \) is a positive parameter to be defined.

When the FOWT is tilted into the wind, higher energy can be extracted from the wind by raising the rated rotor speed \( \omega_w^* \); then, platform motion suppression is achieved. By substituting Formula (6) into (36), it can be deduced as

\[
\dot{\sigma}_w = \dot{\omega}_w - \dot{\omega}_w^* + c_w (\omega_w - \omega_w^*) = \frac{k_r v^2}{J_w \lambda_w} \left( \frac{\beta_2}{\beta_1} \right) \frac{\lambda_w}{\lambda_w(\lambda_w + \lambda_w(1 + 1 \gamma)} \left[ h_{w2}(\lambda_w) - \beta_2 h_{w2}(\lambda_w) + d(\lambda_w, \beta_2, v_w, \omega_w) - \dot{\omega}_w^* + c_w (\omega_w - \omega_w^*) \right]
\]

(38)

where \( \dot{\sigma}_w \) represents the nominal value, \( d_w \) is the lumped system uncertainty, and \( D_{aw} \) is the unknown upper bound of the lumped uncertainty derivative.

Considering Formulas (13), (18), (20), and (38), the control block diagram is also shown, as in Figure 2, to achieve the multiple control objectives of stabilizing power output and suppressing platform motion and tower loads. The parameters under the ESOSM scheme are set as \( L_1 = 0.1, \ L_0 = 0.01, \ \epsilon_1 = 20, \ \epsilon_2 = 1, \ c_1 = 1, \ c_0 = 0.1, \ \gamma_0 = 1, \ l_1 = 1, \ l_2 = 1, \) and \( l_3 = 1. \) The control parameters of the comparative ASTSM scheme are chosen as \( L_1 = 0.1, \ L_0 = 0.01, \ \epsilon_1 = 20, \ K_f = 0.1, \) and \( K_i = 0.02 \) and arranged for the PI scheme.

The co-simulation is performed via FAST and MATLAB/Simulink 2023b. The simulation running time is set to 600 s. The wind data are produced by Turbsim v2.0.07a software, and the FOWT parameters are detailed in Table 2.

Table 2. Main parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Blades</td>
<td>3</td>
</tr>
<tr>
<td>Rated Power</td>
<td>5 MW</td>
</tr>
<tr>
<td>Rotor, Hub Diameter</td>
<td>126 m, 3 m</td>
</tr>
<tr>
<td>Hub Height</td>
<td>90 m</td>
</tr>
<tr>
<td>Rated Rotor Speed</td>
<td>12.1 rpm</td>
</tr>
<tr>
<td>Cut-In, Rated, Cut-Out Wind Speed</td>
<td>3 m/s, 11.4 m/s, 25 m/s</td>
</tr>
<tr>
<td>Rotor Mass</td>
<td>110,000 kg</td>
</tr>
<tr>
<td>Nacelle Mass</td>
<td>240,000 kg</td>
</tr>
<tr>
<td>Tower Mass</td>
<td>347,460 kg</td>
</tr>
<tr>
<td>Rated Generator Torque</td>
<td>43,093.55 N m</td>
</tr>
</tbody>
</table>

Figure 5 presents the height profile of wind speed and wave. The average speed of the turbulent wind is 17 m/s, and the turbulence intensity is 17.27%. The effective height
of the wave is 3 m, and the cross-zero period is 9 s. Taking into account practical wind turbine applications, the blade pitch range is specified as $[0^\circ, 90^\circ]$, and the variable pitch rate is restricted to 8°/s. In this study, the fatigue loads on the blades and tower were evaluated using the DEL of MLife, with a frequency of 1 Hz and a calculated service life of 20 years. To minimize the impact of the initial conditions, all performance indexes were calculated between 100 s and 600 s.

![Wind Speed and Waves](image.png)

**Figure 5.** Wind speed and waves.

To compare the performance of ASTSM, PI, and the proposed ESOSM scheme, the root mean square (RMS) of some physical quantities was calculated. Table 3 shows the RMS of the rotor speed and output power, which indicates that they are closer to the rated rotor speed of 12.1 rpm and the rated output power of 5297.75 KW under the ESOSM scheme. The RMS normalized values of rotor speed, output power, and output power error portrayed in Figure 6a demonstrate the enhanced efficacy of the ESOSM scheme in regulating output power in contrast to both the PI scheme and the ASTSM scheme. Figure 6b shows the RMS normalized values of platform motion. Platform roll, pitch, and yaw exhibit a reduction of 3%, 2%, and 4%, respectively, under the ESOSM scheme in comparison to the performance achieved under the PI scheme. In summary, the advanced ESOSM scheme exhibits superior capabilities in mitigating platform motion when compared to the PI and ESOSM schemes. Figure 7 illustrates the time series response depicting the generator output power and platform motion angular velocity.

**Table 3.** RMS values.

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Rotor Speed (rpm)</th>
<th>Generator Power (GP) (KW)</th>
<th>Generator Power Error (KW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>12.25</td>
<td>5362.77</td>
<td>65.02</td>
</tr>
<tr>
<td>ASTSM</td>
<td>12.15</td>
<td>5319.80</td>
<td>22.25</td>
</tr>
<tr>
<td>ESOSM</td>
<td>12.1</td>
<td>5299.52</td>
<td>1.77</td>
</tr>
</tbody>
</table>
Figure 6. Normalized RMS values under the three schemes.

Figure 7. Power and platform motion response. (a) Generator power (b) Platform roll (c) Platform pitch (d) Platform yaw.

The comparison of the normalized DEL values of the tower base moment and tower top bending moment is presented in Figure 8. The analysis reveals a notable decrease in the total normalized DEL values under the ESOSM scheme when juxtaposed with the other two schemes. This outcome signifies the superior efficacy of the ESOSM scheme in mitigating tower load, as evidenced by the results. The normalized DEL values of the blade root moment under three control schemes are shown in Figure 9. Under the ESOSM scheme, the normalized DEL values of inside and outside plane moment at the blade root are lower than those under PI control. However, the normalized DEL values of the blade root edgewise moment and the blade root moment flap wise are increased. Figure 10 shows that the blade pitch angle fluctuates more under the ASTSM and ESOSM control schemes than under the PI control scheme. However, the simulation takes into account the blade pitch saturation and change rate limit, resulting in no significant increase in the blade root moment DEL value. Thus, the impact of blade actuator fluctuation on blade fatigue life remains within an acceptable range.
The new ESOSM scheme demonstrates enhanced performance in stabilizing generator output power, mitigating floating platform motion, and reducing tower loads. The scheme offers a notable benefit by enabling the observation and compensation of disturbances. As depicted in Figure 11, the observer gains are dynamically fine-tuned according to the lumped uncertainty. This is beneficial in reducing the parameter adjustment effort and improving the observer robustness. This can be clearly observed from the sliding surface error shown in Figure 12, which converges to near zero, further confirming the effective observing capability.
Figure 11. Adaptive control gains \( \gamma_1 \) and \( \gamma_2 \) under ESOSM scheme.

Figure 12. Observation error under ESOSM scheme.

5. Conclusions

A new adaptive ESOSM scheme for tracking the position or velocity of uncertain systems is studied and applied to the FOWT pitch control in this paper. A super-twisting-like ESO is constructed to achieve the lumped uncertainty observation, and the observer gains are deployed as an adaptive type. A barrier function is employed for the adaptive SMC gain design. The effectiveness of the proposed algorithm is verified using a numerical example. The proposed scheme demonstrates advantages over the ASTSM scheme in control chattering suppression, disturbance observation, and dynamic tracking performance. The scheme also indicates effectiveness when utilized in the context of FOWT pitch control. The experimental results show that the proposed ESOSM control scheme achieves better output power tracking and lower overall DEL normalized values for tower torque and tower top torque. A normalized RMS value of the generator power error of 0.03 is achieved. At the same time, the platform roll, pitch, and yaw are reduced by 3%, 2%, and 4%, respectively, compared to the PI scheme at an average turbulent wind speed of 17 m/s and turbulence intensity of 17.27%. The experimental verification of the proposed scheme in the FOWT is a challenging research direction.

Author Contributions: Conceptualization, R.M. and F.L.S.; methodology, R.M.; software, W.Y.; validation, R.M.; formal analysis, W.Y.; investigation, T.H.G.T.; writing—original draft preparation,
References

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