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Application of a Statistical Regression Technique for Dynamic Analysis of Submarine Pipelines

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Abstract: This study employs a statistical regression technique to investigate the maximum displacement, stress, and natural vibration frequencies of a submarine pipeline subjected to hydrodynamic wave forces. Eighteen pipeline models are designed, varying in wall thickness from 10 mm to 30 mm and diameter from 500 mm to 1000 mm. The hydrodynamic drag and inertia forces are performed by using the Morison equation. Computer-aided Finite Element Analysis is employed to simulate the complex interactions between the fluid and structure in 18 pipelines. Multiple Regression technique is used to evaluate the reliability metrics, considering uncertainties in geometrical properties affecting pipeline performance. Full Quadratic models are developed for expressing more effective and concise mathematical equations. Analysis of Variance (ANOVA) is performed to determine the adequacy of the model in representing the observed data. The Coefficient of Determination ($R^2$), Mean Square Error (MSE), and Mean Absolute Error (MAE) are calculated to assess the equation’s predictive accuracy and reliability. The results confirm the suitability of the suggested regression technique for analyzing the relationships between predictor variables and the response variable.

Keywords: full quadratic model; hydrodynamic forces; multiple regression technique; pipeline

1. Introduction

The design of submarine pipelines involves a comprehensive approach that integrates knowledge from multiple disciplines, aiming to ensure the long-term integrity and functionality of critical components in the oil and gas industry’s transportation infrastructure. Environmental loads, including wave forces, are carefully analyzed, with particular attention to the physical properties of the water and the characteristics of the pipeline itself, such as diameter ($D$), thickness ($t$), and material composition. Additionally, factors like wave height ($H$), wave period ($T$), and water depth ($d$) play a significant role in influencing hydrodynamic processes and the dynamic behaviors of pipelines. Researchers have proposed different methodologies, such as numerical modeling, experimental testing, field measurements, and fault detection techniques in subsea environments, to analyze and mitigate stability concerns [1–7]. Repeated and complex analyses are performed under variable conditions to determine the structural design.

In recent years, where the importance of achieving results in a shorter time and with fewer resources has increased, the use of statistical prediction methods in the analysis of submarine pipelines has also increased. Youssef et al. employ a statistical method to create a response surface model that predicts the maximum horizontal displacement of a pipeline during storm conditions. The Monte Carlo simulation technique is applied alongside the established response surface model to determine extreme response statistics [8]. Xu and Sinha introduce a thorough analytical structure for statistically analyzing field performance data for water pipelines. The paper provides a detailed explanation of the methodology’s implementation steps, along with preliminary analyses conducted on datasets from two different water utility systems [9]. The study by Zhang and Weng presents a
Bayesian network model for analyzing buried gas pipeline failures caused by corrosion and external interference. The model offers a probabilistic framework for assessing the risk of pipeline failure, taking into account various contributing factors and their uncertainties [10]. Zhang et al. develop an optimal statistical regression model capable of predicting the wave-induced equilibrium scour depth beneath pipelines in sandy and silty seabeds [11]. Most of the studies conducted on offshore structures are based on ground movements occurring on the seabed [12,13]. In this study, a suspended pipeline model is examined to avoid the impact of seabed movement.

This study aims to provide a more comprehensive understanding of the factors influencing pipeline dynamic behavior and to enhance the accuracy of stability assessments by using statistical methods. This study focuses on the structural model rather than seabed movement and material properties. This approach allows for a comprehensive assessment of pipeline stability under hydrodynamic wave forces, enabling informed decision-making in design processes.

The multiple regression model, which is most commonly used for complex decision problems in many fields [14], is employed to demonstrate the general tendencies of the relationship between the pipeline design parameters and stability criteria. The hydrodynamic wave forces are obtained by using Airy Wave Theory. Le Méhauté’s diagram is used to determine the applicability theories of water waves according to the values of normalized wave height $H$ and the water depth $d$ [15]. The hydrodynamic drag force ($F_D$) and inertia force ($F_M$) are performed by using the Morison equation. Since the ratio of the gap between the pipe and the seabed to the pipe diameter is considered to be greater than 1.0, the hydrodynamic lift force ($F_L$) has been neglected [16]. The vibration and stability of a simply supported steel pipeline are analyzed when lateral hydrodynamic forces ($F_H$) act on it. For this, the dynamic behavior of the pipeline is modeled by computer-aided Time History Analyses based on Newmark $\beta$ [17]. The pipe wall thickness ranges from 10 to 30 mm, and the pipe outer diameter ranges from 500 to 1000 mm. The initial three natural frequencies, maximum values of stress, and displacement are used to create a dataset to explore relationships between variables for 18 pipe models. The multiple regression method is employed to assess the reliability metrics while accounting for uncertainties in the geometric properties that influence pipeline performance. Full quadratic models are constructed to present more efficient and concise mathematical formulations. An Analysis of Variance is conducted to ascertain the model’s adequacy in accurately representing the collected data. The performance of the regression methods is evaluated based on different metrics: Mean Squared Error (MSE), Mean Absolute Error (MAE), and Coefficient of Determination ($R^2$). The results highlighted that the proposed regression methods can be an alternative practical way that provides the correct solution in a shorter time in the preliminary design of submarine pipelines. The relationship between the variables can be implemented in any computational calculation software.

2. Analyzing Procedure

In this paper, a 2D structural model is performed to predict the dynamic behavior of submarine pipelines under wave forces according to displacements, maximum stresses, and natural vibration frequencies. Airy Wave Theory is used to obtain wave forces, which are adopted as environmental loads. Numerical simulations are also applied using the Time History Tool of SAP 2000. The natural vibration frequencies, maximum values of stress, and deflection are derived to create a dataset for the regression process. The multiple regression model is used to learn relationships between four categories of input and five categories of output from example data and to predict novel inputs. Moreover, 144 input values are used for estimating the relationships between dependent and independent variables for each case. The performance metrics of full quadratic equations have demonstrated that regression analyses can be used for forecasting and predicting submarine pipeline reactions based on geometrical properties and effective hydrodynamic forces.
2.1. Structural Model Application

In application, the predominant configuration of suspended submarine pipelines consists of multiple spans (refer to Figure 1), with support provided at intermediate locations through various support mechanisms. In this study, the pipe is selected as similar to the one used in Gücüyen [18], having $L_p = 10$ m span length and consisting of uniform and homogeneous material.

![Figure 1. Pipeline model.](image)

The structural behavior of the 18 pipe models is achieved under regular waves. The outside diameters of the pipe range from $\phi 500$ to $\phi 1000$, and the wall thicknesses are chosen as 10 mm, 20 mm, and 30 mm to verify the effect of cross-section on frequency, stress, and displacement values. The properties of the pipe models are given in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (mm)</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>600</td>
<td>600</td>
<td>600</td>
<td>700</td>
<td>700</td>
<td>700</td>
</tr>
<tr>
<td>$t_w$ (mm)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

The material properties of the pinned–pinned pipe are chosen to represent the steel, with Young’s modulus of $21 \times 10^7$ kN/m$^2$, Poisson’s ratio of 0.3, and density of 78.50 kN/m$^3$. Various combinations of pipe geometrical properties are performed using Time History analyses. The outer diameter of pipe models influences the value of hydrodynamic forces [19,20]. However, the wall thickness changes the behavior of the pipe in response to the force.

2.2. Environmental Conditions and Load Assessment

The submarine pipeline is designed by considering a combination of dead loads and wave loads. The environmental conditions are modeled in the numerical analysis based on Airy Wave Theory, referring to Le Méhauté’s diagram, which indicates the validity domain of each theory-type of water waves [21].

Water depth, wave period, and height are three major parameters in the design of marine facilities [22]. The employed parameters $d = 10.00$ m, $T = 8.00$ s, and $H = 0.31$ m are considered for the applicability criteria of Airy Wave Theory, which is widely used to model a marine environment [23]. The wave loads are calculated with water particle velocities ($u$) and accelerations ($\ddot{u}$) by two different approaches presented as follows:
where the z-axis is directed vertically upward from the still water level, which is positive. The x-axis is along the direction of the propagation of waves. When the value of the relative water depth (water depth/wavelength, $d/L$) is between 0.005 and 0.05, it is classified as an intermediate depth wave, as mentioned in this study [24]. The free surface elevation $\eta$ during the wave generation process is obtained based on the potential flow approach and is given in Figure 2.

Figure 2. Surface elevations based on Airy Wave Theory.

The seawater characteristics are assigned as fluid properties, with a salty water density ($\rho$) of 1025 kg/m$^3$ and a dynamic viscosity ($\mu$) of 0.0015 Ns/m$^2$. The wavelength ($L$) is determined as $\approx 70.90$ m by considering wave parameters.

Description of Hydrodynamic Loads Acting on Pipeline

Drag, lift, and inertia forces are hydrodynamic loads caused by the motion of the surrounding water in a pipe. Drag and inertia are defined as the components that are parallel to the flow direction, and lift is the component of the hydrodynamic load that is perpendicular to the flow direction. In this study, the magnitude of the gap ratio between the pipe and the seabed is assumed to be $e/D \approx 1.2$. It is observed that when $e/D > 1$, the importance of the lift force vanishes compared to the in-line forces. This means that the importance of the cross-flow vibration becomes negligible, and in-line vibrations dominate the behavior of the pipe [25]. Hydrodynamic stability is established by using the Morison equation, which relates hydraulic inertial ($F_I$), drag ($F_D$) forces to local values of particle velocity and acceleration. The total force ($F_H$) can be derived by integrating Equation (3) along the z-axis as follows:

$$F_H = F_D + F_I = \int_{-d}^{d} \left[ \frac{1}{2} \rho C_D D u(z,t) u_z(z,t) \right] dz + \int_{-d}^{d} \rho C_M \frac{\pi D^2}{4} u_z(z,t) dz\text{ (3)}$$

where $D$ is represented as the diameter of the pipe. As seen in Equation (3), the force components are required to determine the drag force coefficient $C_D$ and the inertia force coef-
C. In this paper, the unknown coefficients are assumed to be 2.0 and 0.7, respectively [26]. The dynamic behavior related to changes in hydrodynamic forces over a wave period is analyzed by collecting data at 1 s intervals.

The examinations of total hydrodynamic forces are made on the basis of the definition of Airy Wave Theory, and the results are given in Figure 3.

![Figure 3. Time-varying hydrodynamic forces.](image)

The hydrodynamic forces are assigned as time-varying external loads on the pipeline in the finite element analysis.

2.3. Time History Analyses

Time history analysis is a sophisticated technique used in structural engineering to assess the dynamic response of structures under various time-dependent loads [27]. This method involves inputting a detailed time series of loading conditions into a computer model of the structure, which then calculates the resulting stresses and displacements at each time step.

The horizontal hydrodynamic forces ($F_H$) resulting from $F_D$ and $F_M$ are performed in computer-aided Time History analyses based on Newmark $\beta$ [17]. Time History analyses provide for both linear and nonlinear structural dynamic responses and sinusoidal and non-sinusoidal loading. The time step method, which is supported by computer-aided Time History analyses, determines the solution of an equation at a succession of values of $t$, $t + \Delta t$, $t + 2\Delta t$, etc. For finite element formulation, it was decided to use the well-known commercial software SAP2000, which is based on the implicit time step method. Various implicit methods are available. The widely used Newmark $\beta$ method is used in the method given below. Starting from a Taylor series [28] as follows:

$$
Y_{t+\Delta t} = \left[ \frac{m}{\Delta t^2} + \frac{c}{2\Delta t} + \beta k \right]^{-1} \left( \beta F_{t+\Delta t} + (1 - 2\beta) F_t + \beta F_{t-\Delta t} \right) - 2 \left[ \frac{1}{2} - \beta \right] k \left[ -\frac{m}{\Delta t^2} + \frac{c}{2\Delta t} - \beta k \right] Y_t
$$

(4)

Here, $m$ is mass, $k$ is stiffness, $c$ is damping, and $F$ is the loading term. The accuracy of the solution depends on the length of the step interval ($\Delta t$). It must be short enough for the load time history, the response time history, and, in many cases, the shortest natural periods to be well defined [29]. The analysis is executed over a duration of 30 s, employing a step interval of 0.01 s and a $\beta$ value of 0.5. Numerical analysis uses the time-varying
external loads as time history functions presented in Figure 3. Various combinations of pipe geometrical properties and hydrodynamic loading conditions are performed using Time History analyses, which provide a time-dependent function for obtaining dynamic structural response under wave forces. The natural frequencies for the initial three modes, the maximum lateral displacement of the pipeline, and the maximum values of stresses are used to generate a set of 144 input–output data pairs.

2.4. Regression Techniques Implemented

Regression, as denoted in the statistical literature, contains the prediction or learning of numeric features [30]. Regression holds significance across numerous engineering applications, as a multitude of real-world issues can be effectively addressed through the modeling of regression problems. Extensive research has been conducted in the field of maritime transportation [31–33].

Multiple regression (MR) is commonly used to investigate the causal relationship between multiple independent variables and a dependent variable simultaneously based on mathematical equations [34]. Each independent variable has its own coefficient, representing its unique contribution to the prediction of the dependent variable, while holding other variables constant. The algorithm calculates coefficients for the equation in a way that minimizes the sum of squared errors between the predicted values and the actual values [35]. The general mathematical form of multiple regression with “n” predictors is expressed as follows [36]:

$$y = \beta_0 + \sum_{j=1}^{m} (\beta_j x_j) + \varepsilon$$  \hspace{1cm} (5)

Here, $y$ is the dependent variable, $x_j$ is an independent variable, $\beta_j$ represents the regression coefficient associated with each regressor, $\beta_0$ is the constant term, and $\varepsilon$ denotes the random error term.

The prediction accuracy of the regression model is investigated by performance metrics containing Coefficient of Determination ($R^2$), Mean Square Error (MSE), and Mean Absolute Error (MAE).

The performance of all regression models is also investigated based on the Coefficient of Determination ($R^2$) as given by the following expression:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (\text{predicted}_i - \text{actual}_i)^2}{\sum_{i=1}^{n} (\text{actual}_i - \text{mean}(\text{actual}))^2}$$  \hspace{1cm} (6)

$R^2$ measures the effectiveness of a model in fitting its data and presents variances in the measured variable. Its scale spans from 0 (indicating no explanatory power) to 1 (representing a perfect fit).

$MSE$ determines the variations between predicted and actual values by averaging their squared disparities. Smaller values of $MSE$ signify higher algorithm performance; however, it is susceptible to the influence of outliers [33].

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (\text{predicted}_i - \text{actual}_i)^2$$  \hspace{1cm} (7)
MAE evaluates the mean variation between predicted and actual values, treating all errors uniformly. It presents lower sensitivity to outlier data, rendering it advantageous in scenarios where substantial errors are undesirable [37].

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |predicted_i - actual_i|
\]  

(8)

where the number of observations is represented by \( n \). Within this manuscript, six main stages are used in the system architecture, as presented in Figure 4. These steps are assigned distinct and crucial responsibilities within the system.

![Diagram](image_url)

**Figure 4.** Main steps of the system architecture.

3. Numerical Results and Discussions

3.1. Natural Vibration Frequencies, Displacements, and Stresses of Pipeline Models

The analysis is conducted for a pipeline with a single opening and simple supported ends using the Euler–Bernoulli beam theory. The basis of the Euler–Bernoulli beam theory relies on three assumptions of continuum mechanics: first, that straight lines initially normal to the beam’s axis retain their straightness after deformation; second, that these lines are inextensible; and third, that they rotate as rigid lines perpendicular to the bent axis of the beam [38]. The axis \( x \) of the pipe is defined as a longer dimension, and the transverse vibration of the pipe that occurred on this axis is calculated.

The pipeline system’s dynamic performance can be described by identifying one or more natural vibration frequencies [39]. The high-frequency modes exhibit a more rapid decay in amplitude compared to the slower decay observed in the low-frequency modes. The dominant behavior is caused by the lower frequency modes [40]. Hence, the natural vibration frequencies of the pipe have been calculated for the first three modes in this study. Figure 5 shows the specific displacements required for the first three modes, initiating harmonic vibrations in the pipe.
The comparative results in Table 2 present the obtained natural vibration frequency values from the analysis conducted for a single opening with simply supported ends of the pipeline.

Table 2. Comparison of natural vibration frequency values.

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>t_w (mm)</th>
<th>$\omega_1$ (Hz)</th>
<th>$\omega_2$ (Hz)</th>
<th>$\omega_3$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>10</td>
<td>7.903</td>
<td>21.538</td>
<td>41.597</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>7.750</td>
<td>21.128</td>
<td>40.833</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>7.599</td>
<td>20.730</td>
<td>40.080</td>
</tr>
<tr>
<td>600</td>
<td>10</td>
<td>9.474</td>
<td>25.694</td>
<td>49.334</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>9.322</td>
<td>25.297</td>
<td>48.591</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>9.173</td>
<td>24.900</td>
<td>47.870</td>
</tr>
<tr>
<td>700</td>
<td>10</td>
<td>11.024</td>
<td>29.727</td>
<td>56.689</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>10.906</td>
<td>29.343</td>
<td>55.991</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>10.726</td>
<td>28.935</td>
<td>55.279</td>
</tr>
<tr>
<td>800</td>
<td>10</td>
<td>12.550</td>
<td>33.625</td>
<td>63.654</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>12.402</td>
<td>33.256</td>
<td>62.972</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>12.255</td>
<td>32.884</td>
<td>62.344</td>
</tr>
<tr>
<td>900</td>
<td>10</td>
<td>14.047</td>
<td>37.383</td>
<td>70.175</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>13.902</td>
<td>37.023</td>
<td>69.589</td>
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<tr>
<td></td>
<td>30</td>
<td>13.757</td>
<td>36.657</td>
<td>68.966</td>
</tr>
<tr>
<td>1000</td>
<td>10</td>
<td>15.516</td>
<td>40.984</td>
<td>86.993</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>15.373</td>
<td>40.634</td>
<td>75.758</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>15.230</td>
<td>40.290</td>
<td>75.131</td>
</tr>
</tbody>
</table>

In assessments where the internal flow in the pipe is neglected, it is observed that the natural vibration frequency values increase with an increase in the pipe diameter. Additionally, for each diameter, three different values of wall thickness also affect the frequency. It is evident that as the wall thickness increases, the results obtained for the same diameter decrease. The highest natural vibration frequency occurs when the wall diameter is 1000 mm and the wall thickness is 10 mm.
The maximum values of stress and displacement are used to create a dataset to explore relationships between variables for 18 pipe models. The maximum stress values contain the most significant strains occurring at the section where the maximum displacement is observed. The hydrodynamic forces vary depending on the velocity of fluid particles changing direction throughout the wave period. This leads to stresses calculated at the cross-section of the pipeline having different effects, with positive (+) and negative (−) pressures, as given in Figure 6. “BS EN 1993-1-1:2005+A1:2014 Eurocode 3. Design of steel structures General rules and rules for buildings” [41] and “BS EN 10025-2:2019 Hot rolled products of structural steels Technical delivery conditions for non-alloy structural steels” [42] are selected as steel standards in this study. The stress levels for all models remain within permissible limits.

![Figure 6. Comparison of maximum values of stress.](image)

The blue, red, and black colors represent the results for wall thicknesses of 10 mm, 20 mm, and 30 mm, respectively. As observed in the figure, results obtained at $t = T/2$, $T/4$, $3T/4$, and $T$ are compared. The stress values in the pipeline are highest at 2 s and 6 s, where wave crests and troughs occur, representing the peak hydrodynamic forces. An increase in wall thickness enhances strength, leading to a decrease in stress. However, the decrease in stress is not significant due to the increase in hydrodynamic force according to the Morison equation resulting from the increase in diameter. The maximum displacement that can occur in the cross-section of the pipeline is as crucial as stress. Particularly in systems where the gap between the pipeline and the seabed is small, erroneous calculations can lead to the deformation of the pipeline and significant financial losses [43]. Comparative results of maximum values are presented in Figure 7, considering the time intervals as $t = T/2$, $T/4$, $3T/4$, and $T$.

![Figure 7. Comparison of maximum values of displacement.](image)
Maximum displacements are observed again at the wave crest and trough positions. However, unlike stress values, it is evident that both the diameter and wall thickness values contribute to the decrease in displacement. Evaluating the scenario where displacement takes small values aims to ensure safety within the limits under the internal flow conditions, which will be considered in subsequent studies. The largest displacement, calculated as 0.113 mm, occurs at \( t = 6 \) s with \( D = 500 \) mm and \( t_w = 10 \) mm. The smallest value, 0.003 mm, is determined at \( t = 8 \) s when the surface profile returns to a calm sea surface, with \( D = 1000 \) mm and \( t = 30 \) mm. The relationship between datasets consisting of maximum stress and displacement values, independent of time, is presented in Figure 8.

![Figure 8](image)

Figure 8. The maximum stress value versus the maximum displacement.

It is observed that the relationship between maximum stress and displacement values is significant based on the Coefficient of Determination. Deviation in the values occurs when the hydrodynamic forces change from positive to negative. In this wave position, while stress increases, displacement values numerically decrease due to the change in direction.

### 3.2. Comparative Results of Regression Models

In this section of this study, ANOVA (Analysis of Variance) results are generated. The effect magnitude of input parameters (\( D, t_w, F_H, \) and \( t \)) on the outcomes (\( w_{\text{max}}, \sigma_{\text{max}}, \omega_1, \omega_2, \) and \( \omega_3 \)) is determined. Thus, the impact ratios of pipe diameter \( D \), pipe wall thickness \( t_w \), hydrodynamic force \( F_H \), and time \( t \) are evaluated. In analyses where time is considered as input, the \( t/T \) approach is used to reach a general correlation. By using the dimensionless time term, the mathematical expression obtained ensures validity for wave models with different wave periods. Full quadratic models that include not only linear terms but also quadratic terms are developed for expressing more effective and concise mathematical equations [44]. The squares of the independent variables appear as part of the quadratic term. These quadratic terms are included to capture nonlinear relationships between the predictor variables. It is expressed mathematically as follows:

\[
Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \ldots + \beta_{i-1} X_{i-1} + \beta_i X_i + \beta_{11} X_1 X_2 + \beta_{22} X_2 X_2 + \beta_{33} X_3 X_3 + \ldots + \beta_{kk} X_k X_k + \varepsilon
\]

where \( Y \) is the dependent variable, and \( X_1, X_2, \ldots, X_k \) are the predictor variables. \( \beta_0, \beta_i, \beta_{11}, \beta_{22}, \ldots, \beta_{kk} \) are the coefficients for the linear terms; \( \beta_{11}, \beta_{22}, \beta_{33}, \ldots, \beta_{kk} \) are the coefficients for the quadratic terms; \( \beta_{ij}, \beta_{i3}, \ldots, \beta_{i-1,i-1} \) are the coefficients for the interaction terms; and \( \varepsilon \) represents the error term.

Additionally, the effects of input parameters on the outcome parameters are demonstrated with 3D graphics.
3.2.1. The Effect of Input Variables on Displacement Values

Displacement is an important parameter in the strength of the pipe. Knowing the extent to which each input affects displacement is of great importance. New mathematical models have been created using statistical methods to solve this problem in pipeline transportation systems. The results of the ANOVA analysis are provided in Table 3.

Table 3. Analysis of Variance for $w_{\text{max}}$ values.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F-Value</th>
<th>p-Value</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00</td>
<td>0.969</td>
<td>0.00</td>
</tr>
<tr>
<td>$t_w$</td>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00</td>
<td>0.968</td>
<td>0.00</td>
</tr>
<tr>
<td>$F_H$</td>
<td>1</td>
<td>0.07225</td>
<td>0.07225</td>
<td>438.81</td>
<td>0.000</td>
<td>29.476</td>
</tr>
<tr>
<td>$t_w \times F_H$</td>
<td>1</td>
<td>0.01893</td>
<td>0.01893</td>
<td>115.03</td>
<td>0.000</td>
<td>7.727</td>
</tr>
<tr>
<td>$F_H$</td>
<td>1</td>
<td>0.00203</td>
<td>0.00203</td>
<td>12.38</td>
<td>0.001</td>
<td>0.832</td>
</tr>
<tr>
<td>$t_w \times t$</td>
<td>1</td>
<td>0.00615</td>
<td>0.00615</td>
<td>37.38</td>
<td>0.000</td>
<td>2.511</td>
</tr>
<tr>
<td>Error</td>
<td>135</td>
<td>0.02222</td>
<td>0.00016</td>
<td>9.068</td>
<td>0.000</td>
<td>9.068</td>
</tr>
<tr>
<td>Total</td>
<td>143</td>
<td>0.24511</td>
<td></td>
<td>100.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As seen in Table 3, within a 95% confidence interval, $F_H$, $D \times F_H$, and $t_w \times F_H$ are the most influential parameters on the maximum displacement occurring throughout the independent period. At the same time frame, both the diameter and wall thickness of the pipe have a significant impact on displacement values. However, when a general regression model independent of time is constructed, hydrodynamic force and the conditions under which this parameter is multiplied become important.

The full quadratic model equation obtained using these parameters is given in Equation (10).

$$w_{\text{max}} = -0.0745 + 0.000061D + 0.001261t_w + 0.003184F_H + 0.1330t - 0.000002D \times F_H - 0.000109D \times t - 0.000022t_w \times F_H - 0.002251t_w \times T$$ (10)

$F_H$ is the most influential parameter on maximum displacement, accounting for 29.476% of the effect. The effect ratio for $D \times F_H$ is 7.727%, while the effect ratio for $t_w \times F_H$ is 2.511%. The Pareto chart related to effective parameters is given in Figure 9.

![Figure 9. Pareto chart of the standardized effects for $w_{\text{max}}$.](image-url)
The regression model achieves a high $R^2$ of $8.952 \times 10^{-1}$, which is close to the 1.0 perfect fit. Additionally, the model’s MSE value is $1.182 \times 10^{-2}$ and its MAE value is $8.081 \times 10^{-2}$. The results show that the model can accurately predict the maximum displacements of the pipeline. The relationship between maximum displacement values and input data is presented in Figure 10.

![Figure 10. Maximum displacement versus independent variables.](image)

It is observed that hydrodynamic forces reach their maximum values under the crest and trough of the wave surface profile. When the hydrodynamic force is at its maximum, displacement values also increase. The increase in displacement values is more pronounced when the wall thickness is taken as 10 mm. Displacement values decrease as the pipe diameter increases. For the smallest selected diameter of 500 mm, the pipe experiences more oscillation due to the hydrodynamic force, resulting in a displacement change figure resembling the wave surface profile.

For the largest selected diameter of 1000 mm, the displacement values are maximum at the (-) and (+) maximum points of the hydrodynamic force, but the change curve reaches a more linear form. This indicates that increasing the pipe diameter and thickness reduces the destabilizing effect on stability in the transition regions of the hydrodynamic force causing tension and compression. The results obtained also highlight the importance of independent variable effect coefficients determined by ANOVA analysis in the design process.

3.2.2. The Effect of Input Variables on Stress Values

Stress values serve as key indicators of the structural performance of submarine pipelines, and their proper evaluation and management are essential for ensuring the safety, reliability, and sustainability of these critical infrastructure systems [45]. The statistical analysis results, aimed at succinctly and practically illustrating the variation of this important parameter with respect to the geometric properties of the submarine pipeline, are presented in Table 4.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F-Value</th>
<th>p-Value</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00</td>
<td>0.998</td>
<td>0.00</td>
</tr>
<tr>
<td>$t_w$</td>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00</td>
<td>0.998</td>
<td>0.00</td>
</tr>
<tr>
<td>$F_H$</td>
<td>1</td>
<td>2.31716</td>
<td>2.31716</td>
<td>955.12</td>
<td>0.000</td>
<td>29.49</td>
</tr>
<tr>
<td>$t$</td>
<td>1</td>
<td>0.00206</td>
<td>0.00206</td>
<td>0.85</td>
<td>0.359</td>
<td>0.026</td>
</tr>
<tr>
<td>$D \times F_H$</td>
<td>1</td>
<td>0.35671</td>
<td>0.35671</td>
<td>147.04</td>
<td>0.000</td>
<td>4.540</td>
</tr>
<tr>
<td>$D \times t$</td>
<td>1</td>
<td>0.01926</td>
<td>0.01926</td>
<td>7.94</td>
<td>0.006</td>
<td>0.245</td>
</tr>
<tr>
<td>$t_w \times F_H$</td>
<td>1</td>
<td>0.33188</td>
<td>0.33188</td>
<td>136.80</td>
<td>0.000</td>
<td>4.224</td>
</tr>
<tr>
<td>$t_w \times t$</td>
<td>1</td>
<td>0.03215</td>
<td>0.03215</td>
<td>13.25</td>
<td>0.000</td>
<td>0.409</td>
</tr>
<tr>
<td>Error</td>
<td>135</td>
<td>0.32751</td>
<td>0.00243</td>
<td></td>
<td></td>
<td>4.168</td>
</tr>
<tr>
<td>Total</td>
<td>143</td>
<td>7.85733</td>
<td></td>
<td></td>
<td></td>
<td>100.000</td>
</tr>
</tbody>
</table>
As seen in Table 4, \( F_H \) is the most influential parameter on maximum stress values, accounting for 29.49\% of the effect. The effect ratio for \( D \times F_H \) is 4.54\%, while the effect ratio for \( t_w \times F_H \) is 4.22\%. In the mathematical model, the properties of the pipeline are represented by 2-Way Interaction terms: \( D \times F_H \) and \( t_w \times F_H \). The full quadratic model equation is presented in Equation (11).

\[
\sigma_{\text{max}} = 0.2530 - 0.000188D - 0.00503t_w - 0.016386F_H - 0.450t + 0.000010D \times F_H + 0.000334D \times t_w + 0.000164t_w \times F_H + 0.00894t \times t
\] (11)

The Pareto chart related to effective parameters is given in Figure 11.

![Pareto chart](image)

**Figure 11.** Pareto chart of the standardized effects for \( \sigma_{\text{max}} \).

The regression model exhibits a strong \( R^2 \) value of \( 9.518 \times 10^{-1} \), indicating that it explains a significant portion of the variance in the data. Moreover, MSE is low at \( 2.691 \times 10^{-3} \) and MAE is \( 3.903 \times 10^{-2} \). The results of quality metrics showed that the model’s predictions are generally close to the actual values. The relationship between the independent variables and maximum stress is given in Figure 12.

![Graphs](image)

**Figure 12.** Maximum stress versus independent variables.

The sudden changes in maximum stress values occurring under the wave crest and trough decrease with increasing diameter values. As the cross-sectional area of the pipeline increases, stability is enhanced, resulting in smaller oscillations in frequency. While the increase in pipe diameter leads to a higher hydrodynamic inertia force at \( t = 0 \), the resulting reductions in maximum stress values are relatively lower. However, over time, a significant decrease in stress becomes apparent.

The time-dependent maximum stress variation figure is obtained to be consistent with the water surface profile. The maximum stress values decrease with increasing wall thickness. The greatest difference between positive and negative stress values is obtained...
under the conditions of $D = 500$ and $t_w = 10$ mm. Significant variations in maximum tensile and compressive stresses can lead to fatigue of the pipeline over time and the occurrence of additional stresses.

3.2.3. The Effect of Input Variables on Natural Vibration Frequencies

The natural vibration frequencies of submarine pipelines are essential considerations in ensuring their structural integrity and resilience against dynamic forces in underwater environments [46]. Hence, in this study, a variation analysis of the natural vibration frequencies for the initial three modes is conducted. The results of the Analysis of Variance (ANOVA) are presented in Table 5.

### Table 5. Analysis of Variance for $\omega_1$, $\omega_2$, and $\omega_3$ values.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F-Value</th>
<th>p-Value</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>1</td>
<td>976.437</td>
<td>976.437</td>
<td>795364.29</td>
<td>0.000</td>
<td>99.768</td>
</tr>
<tr>
<td>$t_w$</td>
<td>1</td>
<td>2.098</td>
<td>2.098</td>
<td>1708.81</td>
<td>0.000</td>
<td>0.214</td>
</tr>
<tr>
<td>$D \times t_w$</td>
<td>1</td>
<td>0.001</td>
<td>0.001</td>
<td>0.72</td>
<td>0.397</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>140</td>
<td>0.172</td>
<td>0.001</td>
<td></td>
<td></td>
<td>0.018</td>
</tr>
<tr>
<td>Total</td>
<td>143</td>
<td>978.708</td>
<td></td>
<td></td>
<td></td>
<td>100.000</td>
</tr>
</tbody>
</table>

| $D$    | 1  | 6398.67   | 6398.67| 17918183.400| 0.000   | 99.718|
| $t_w$  | 1  | 13.82     | 13.82  | 38706.740  | 0.000   | 0.215|
| $D \times D$ | 1  | 4.21      | 4.21   | 11784.430 | 0.000   | 0.066|
| Error  | 140| 0.05      | 0.000  |         |         | 0.001|
| Total  | 143| 6416.75   |        |         |         | 100.000|

| $D$    | 1  | 26667.7   | 26667.70| 2471.800  | 0.000   | 90.280|
| $t_w$  | 1  | 551.9     | 551.900 | 51.160   | 0.000   | 1.868|
| $D \times D$ | 1  | 154.3     | 154.300| 14.300   | 0.000   | 0.522|
| $t \times t$ | 1  | 94.7      | 94.700  | 8.780    | 0.004   | 0.321|
| $D \times t$ | 1  | 581.3     | 581.300| 53.880   | 0.000   | 1.968|
| Error  | 138| 1488.9    | 10.800  |         |         | 5.040|
| Total  | 143| 29538.8   |        |         |         | 100.000|

Typically, the lowest frequency mode, termed the first mode, is characterized by a uniform bending motion along the length of the pipeline. As seen in Table 5, the effect of $D$ in the regression model is calculated as 99.768%, while the effect of $t_w$ is computed as 0.214%. In contrast, the second mode presents a single-loop oscillation, resembling a half-wave pattern. Therefore, in addition to the effects of $D$ at 99.718% and $t_w$ at 0.215%, the term $D \times D$ also exhibits an effect of 0.066%. The third mode introduces an additional loop, leading to a more intricate vibration pattern. This also increases the number of terms required to define the mathematical model. In addition to $D$ being the most effective parameter at 90.280% and $t_w$ at 1.868%, the square ($D \times D$, $t_w \times t_w$) and 2-Way Interaction ($D \times t_w$) parameters also gain importance.

The equations for the full quadratic model, obtained to describe the relationships between variables, are presented for the initial three natural vibration frequencies in Equations (12)–(14).

$$\omega_1 = 0.4928 + 0.015212D - 0.01612t + 0.000002D \times t$$  \hspace{1cm} (12)$$

The model given by Equation (12) exhibits a strong $R^2$ value of $9.998 \times 10^{-1}$, indicating that it explains a significant portion of the variance in the data. Moreover, MSE is low at $1.207 \times 10^{-3}$ and MAE is $2.849 \times 10^{-2}$.

$$\omega_2 = -1.0575 + 0.049312D - 0.037945t - 0.000007D \times D$$  \hspace{1cm} (13)$$
The second vibration frequency model accounts for 100% of the variance in the dependent variable. It means that the model explains all the variability of the dependent variable around its mean using the independent variables. Additionally, both the MSE and MAE are low, recorded at $9.283 \times 10^{-3}$ and $8.667 \times 10^{-2}$, respectively.

$$\omega_3 = 11.39 + 0.0463D + 0.153t + 0.000041D \times D + 0.01720 D \times t - 0.001441 D \times t$$  \hspace{1cm} (14)$$

The model obtained using the third natural vibration frequency value exhibits a strong $R^2$ value of $9.437 \times 10^{-1}$, indicating that it explains a significant portion of the variance in the data. Even though defining the equation with more variables may decrease the actual prediction percentage, the results are within acceptable limits. The MSE value is determined to be 5.461, accompanied by a MAE value of 1.758.

The pareto charts, which are used to identify the most significant parameters for natural vibration frequencies, and 3D graphics are presented in Figure 13.
Figure 13. Pareto charts of the standardized effects for natural vibration frequencies and 3D graphics. (a) $\omega_1$, (b) $\omega_2$, and (c) $\omega_3$.

As seen in Figure 13, the geometry of the pipeline, including diameter and wall thickness, affects its natural vibration frequencies. Thicker pipelines tend to have lower frequency values. The effect of pipe thickness on natural vibration frequency increases with larger pipe diameters. Developing explicit expressions that define the relationship between the inputs and the natural vibration frequencies of the pipe would be particularly advantageous in terms of time and computational load, especially in the presence of internal flow.

Table 6 summarizes the previously obtained results, where the models are sorted in descending order, in terms of performance criteria, depending on the train data set used.

<table>
<thead>
<tr>
<th>Performance Criteria</th>
<th>Prediction Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>$\omega_2 \rightarrow \omega_1 \rightarrow \sigma_{max} \rightarrow \omega_3 \rightarrow W_{max}$</td>
</tr>
<tr>
<td>MSE</td>
<td>$\omega_1 \rightarrow \sigma_{max} \rightarrow \omega_2 \rightarrow W_{max} \rightarrow \omega_3$</td>
</tr>
<tr>
<td>MAE</td>
<td>$\omega_1 \rightarrow \sigma_{max} \rightarrow W_{max} \rightarrow \omega_2 \rightarrow \omega_3$</td>
</tr>
</tbody>
</table>

As seen in Table 6, the suggested equations for the full quadratic model exhibit the best performance for the initial two natural vibration frequencies. However, the relationship for displacement estimation, while remaining within the validity bounds, demonstrates weaker performance in terms of the $R^2$ value. The natural vibration frequency values for the third mode yield numerically larger results compared to other outputs. Therefore, MSE and MAE values are correspondingly larger.

4. Conclusions

The primary objective of this investigation is to establish the correlation among the factors influencing submarine pipelines without an extensive and complex analysis process. In this study, the widely used Airy Wave Theory is employed. Hydrodynamic wave forces acting on submarine pipeline models, each characterized by 18 different geometric properties, are calculated using the Morison Equation.

Dynamic behaviors are investigated through Finite Element-based computer-aided Time History analyses, with consideration given to constant clearance and environmental factors for each pipeline. A dataset including maximum displacement, stress, and the initial three natural vibration frequencies is generated. Multiple Regression analysis is performed using 144 data pairs with 4 inputs and 5 outputs. The relationship between inputs...
and outputs is defined by mathematical equations using a Full Quadratic model. Generally, it is observed that variations in diameter had a greater effect on displacement values than on stress values. The largest variation between predicted and actual values for maximum displacement and stress values occurs under conditions where the diameter is 1000 mm at T/4 and 3T/4. This finding supports the notion that hydrodynamic forces are the predominant variables in stress and displacement prediction equations. Examination of natural vibration frequency values reveals that increasing the number of variables required to describe the relationship between inputs and outputs reduced prediction performance by 5.630%. Therefore, this study aims to obtain the simplest mathematical equation possible, yielding the best results. Considering the influence of submarine pipeline geometry on hydrodynamic force variations, this study is significant in elucidating and interpreting the relationship between maximum stress, displacement, and natural vibration frequency. Furthermore, performance criteria are determined for each equation. These results show that the proposed models provide a highly accurate depiction of the data and give a great deal of confidence in the accuracy and reliability of the model. The results are observed in the case of neglecting the internal flow of the pipeline to simplify the analysis. When the internal flow of the pipeline is taken into account, correlations that include the effects on the outputs, especially on the natural vibration frequency, will become more significant. Future research will incorporate the effects of internal and external fluid flow. From this perspective, this study provides valuable insights into the underlying dynamics of submarine pipeline systems and analyzes inflection points in the relationships between variables.

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**References**


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