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Quasi-Infinite Horizon Model Predictive Control with Fixed-Time Disturbance Observer for Underactuated Surface Vessel Path Following

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Abstract: As a flexible, autonomous and intelligent motion platform, underactuated surface vessels (USVs) are expected to be an ideal means of transport in dangerous and complex marine environments. The success and efficiency of maritime missions performed by USVs depend on their ability to accurately follow paths and remain robust against wind and wave disturbances. To this end, this paper focuses on accurate and robust path following control for USVs under wave disturbances. Model predictive control with a quasi-infinite horizon is proposed which converts the objective function from an infinite horizon to an approximate finite horizon, providing the convergence performance in long prediction horizons and reducing the computation load explicitly. To enhance robustness against disturbances, a fixed-time disturbance observer is applied to estimate the time-varying and bounded disturbances. The estimated value is provided to the controller input to form a robust control framework with disturbance feedforward compensation and predictive control feedback correction, which is substantially different from existing works. The convergence and optimality of the proposed algorithm are presented mathematically. Finally, we demonstrate the advantages of the algorithm in both theory and simulation.

Keywords: underactuated surface vessels; path following; model predictive control; quasi-infinite horizon; fixed-time disturbance observer



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1. Introduction

Maritime incidents have been a significant concern within the global maritime community since the inception of shipping. As indicated in [1], the total number of maritime accidents over the past 40 years is more than 63991 and there has been a reduction in the number of maritime incidents from 3152 in 1979 to 959 in 2001. Autonomous ships have the potential to significantly reduce the high rate of maritime accidents caused by human error, improving safety at sea [2]. Autonomous ships are most effective in complex heat, cold and deep-water environments. Path following control technology is a crucial aspect of autonomous underactuated surface vessels (USVs) performing maritime missions, as it determines the accuracy and autonomy of USVs navigating according to predetermined or self-planned paths [3].

A substantial amount of research has been conducted on the problem of USV path following with various control methods, including [4–10]. In particular, the line-of-sight (LOS) guidance law, initially developed by Fossen [4] for USV motion control, is a widely accepted guidance algorithm used in path following a USV. Reference [5] highlights the efficacy of the LOS guidance algorithm, which is a straightforward, intuitive and effective method that enables vessels to converge to the desired path in an efficient manner. The

authors of [5] state that traditional autopilot can readily incorporate LOS, with the LOS angle being calculated through the minimization of the cross-track error. As long as the vessel is able to adhere to the LOS angle, its position will ultimately converge to the desired path [6]. Furthermore, the path tracking control algorithm is based on the traditional PID algorithm [7]. Qiu et al. [8] proposed an adaptive sliding mode control scheme for USVs based on a neural network, an auxiliary dynamic system, sliding mode control and an inverse stepping technique, which effectively addresses the issues of model uncertainty and input saturation. However, path following vehicles are affected by external ocean disturbances in practical applications, which reduce their accuracy [9,10]. The previous state-of-the-art studies have only emphasized path tracking accuracy, ignoring the effects of waves and wind. This leads to the generation of dynamic models that are not applicable to the actual ocean environment. Furthermore, there is an upper limit on the force output due to the input characteristics and physical properties of the USV thruster.

The vessel dynamic model incorporates both internal and external disturbances, which presents a significant challenge for controller design. To address this, different observers have been designed and applied. Gao et al. propose a finite-time trajectory tracking control approach for USVs with unknown dead-zones and unknown disturbances [11]. Han et al. introduce an active disturbance rejection controller that employs an extended state observer (ESO) to estimate states and disturbances [12]. Naghdi and Sadrnia in [13] propose a simple fuzzy system to modify the gain of linear ESO (LESO) to improve its performance. Li et al. in [14] investigate the path following problem for a USV in the presence of ocean currents, model uncertainties and input saturation. In [15], Seongik et al. propose a predefined-time and prescribed-performance control method for an unmanned planning hull system with input delay and unknown disturbance. In our previous work [16], we designed a path following controller for USVs and a disturbance observer to compensate for time-varying disturbances such as wind, waves and current. Furthermore, the authors of [17] conducted extensive research into model predictive control algorithms and robust high-speed multihull control. A robust anti-pitching control method with Kalman filter measurement is proposed in [17], which effectively solves the problem of excessive vertical movement of high-speed multihulls, as well as the difficulty of directly measuring surge and pitch angle speed. Despite the inherent complexity of designing a controller for this model given the time-varying disturbance of wind, waves, current and the physical model of the ship itself, the proposed disturbance observer combined with a robust controller provides a reliable solution.

To address this problem, Fu et al. [18] focus on the path following control problem for USVs in the presence of unknown time-varying environmental disturbances, using an enhanced adaptive integral line-of-sight guidance law that is suitable for any parametric paths and is capable of handling time-varying ocean currents. A disturbance observer is proposed in [19] as a means of estimating and compensating for disturbances in the designed controller for a highly nonlinear and strongly coupled ship's path following model. The implementation of a model predictive control (MPC) law with a disturbance observer enables the effective mitigation of the effects of environmental disturbances and the provision of smooth control commands [20]. Furthermore, the proposed MPC approach in [20] is capable of achieving path following for underactuated surface ships, even in the presence of input saturation and parameter uncertainties. In standard finite horizon MPC, the prediction and control horizon lengths are often selected through tuning [21]. However, it is important to note that considering an infinite horizon, as demonstrated in [22], leads to more accurate results. In contrast to finite horizon model predictive control (FH-MPC), which is susceptible to a decrease in system performance with an increase in the horizon length, infinite horizon model predictive control (iH-MPC) exhibits a consistent level of performance, despite its increased complexity.

This paper presents a robust controller for USV path following with a disturbance observer. To this end, this paper proposes a quasi-infinite horizon model predictive control

(QiH-MPC) scheme with a fixed-time disturbance observer for USV robust path following. The main contributions of this paper are summarized as follows:

- (1) In this paper, a state-of-the-art state space control model considering time-varying wave disturbances is established for USV path following, in which the states of tracking error, heading angle, heading angular velocity and disturbance are taken into account.
- (2) A QiH-MPC is proposed that provides convergence performance in long prediction horizons, and it explicitly reduces the computational burden due to the fact that it converts the objective function from an infinite horizon to an approximate finite horizon. Moreover, the convergence of the proposed algorithm is given mathematically.
- (3) Through the application of the fixed-time disturbance observer, the disturbance estimation is compensated in the controller input to form a robust control framework with disturbance feedforward compensation and predictive control feedback correction, which differs significantly from previous state-of-the-art works. This is of great value for the determination of optimal control actions for robust USV path following.

This paper is structured as follows. In Section 2, we present a description of the system model, including a USV path following description, a state space model of USV path following and QiH-MPC with a disturbance compensation framework. Section 3 presents the proposed controller for USV path following. Section 4 gives the fixed-time observer used for disturbance estimation. Section 5 gives the convergence analysis and Section 6 details the numerical simulation. This paper concludes in Section 7 and outlines future work.

2. System Modeling

2.1. Path following Description of USV

In this paper, we study the USV path following problem in Serret–Frenet (SF) coordinates (SFCs) [23], as shown in Figure 1. The origin of the coordinate system {SF} is located at the point on the given path which is closest to the origin of the body-fixed coordinate system {B}. Ω is the given path, T is the tangent direction of the given path, M is the USV direction, ψ is the heading angle formed by rotating the USV direction counterclockwise to the X axis, ψ_{SF} is the desired heading angle formed by rotating the tangent direction of the path counterclockwise to the X axis, e is the tracking error and X_d is the desired point.

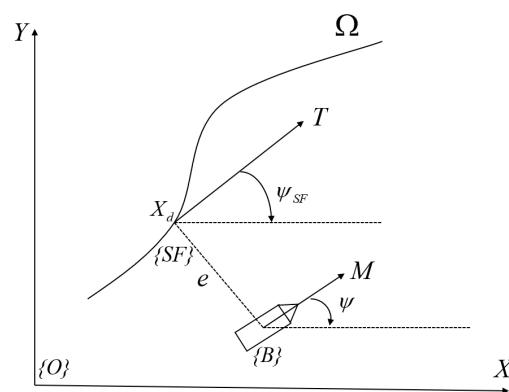


Figure 1. Illustration of the coordinates in the earth frame (inertial frame) {O}, the surface vessel body-fixed frame {B} and the Serret–Frenet {SF} frame.

In Figure 1, the error dynamics based on the SFCs are given by

$$\begin{cases} \dot{e} = U \sin(\bar{\psi}) + V \cos(\bar{\psi}) \\ \dot{\bar{\psi}} = \frac{\rho}{1-e\rho} (U \sin(\bar{\psi}) - V \cos(\bar{\psi})) + r \\ \bar{\psi} = \psi - \psi_d \end{cases} \quad (1)$$

where U , V and r are the surge, sway and yaw velocities; ρ is the curvature of the desired path. In general, the control objective of USV path following is to drive tracking error e and heading error $\bar{\psi}$ to zero, thus accurately tracking the desired heading angle and the desired path.

Most problems pertaining to USV path following can be solved by tracking a straight line or a waypoint path, which can be segmented into several piecewise straight lines with low curvature. Furthermore, this solution can be derived with minimal control input, resulting in $\sin(\bar{\psi}) \rightarrow \bar{\psi}$. Assuming that the ship is moving near the equilibrium point and disregarding the drift angle, we can deduce that surge velocity $U \gg 0$ and sway velocity $V \rightarrow 0$. This analysis enables the error dynamics (1) to be expressed in a compact form as

$$\begin{cases} \dot{e} = U \cdot \bar{\psi} \\ \dot{\bar{\psi}} = r \\ \bar{\psi} = \psi - \psi_d \end{cases} \quad (2)$$

where e is the tracking error, $\bar{\psi}$ is the tracking heading error, U is the USV speed and r is the heading angle velocity.

2.2. State Space Model of USV Path Following

This paper investigates a ship featuring typical underactuated characteristics with a control rudder angle and the speed of the propeller thruster considered to be constant. This represents a significant and hitherto unrecognized threat to the stability of the ship, the reliability of its equipment and the safety of navigation. In extreme cases, the vessel can be capsized if the stability cannot be maintained.

Consequently, it is imperative to model and analyze the actuator and attitude kinematics. The Nomoto model [24] was employed to analyze ship maneuverability since it can model ship motion problems through easily obtained operational parameters. The Nomoto model is presented in detail below:

$$\eta \ddot{\psi} + \dot{\psi} - \zeta \delta = 0 \quad (3)$$

where ψ is the heading angle, δ represents the constrained rudder angle and serves as the control input and ζ and η are the operational parameters, which can be obtained by the designated rotary and Z-type experiments. Equation (3) can be rewritten into a concise form as follows:

$$\ddot{\psi} = -\frac{1}{\eta} \dot{\psi} + \frac{\zeta}{\eta} \delta \quad (4)$$

The heading deviation is defined as $\bar{\psi} = \psi - \psi_d$. If the desired path is a straight line or can be decomposed into multiple straight lines, we can derive that $\dot{\bar{\psi}} = \dot{\psi}$ because $\dot{\psi}_d = 0$ for a straight line. Therefore, Formula (4) evolves into

$$\ddot{\bar{\psi}} = -\frac{1}{\eta} \dot{\bar{\psi}} + \frac{\zeta}{\eta} \delta \quad (5)$$

Combined with the error dynamics of USV path following (2) and the sum of external bounded disturbance, it follows that

$$\begin{cases} \dot{r} = -\frac{1}{\eta} r + \frac{\zeta}{\eta} \delta + d \\ \dot{e} = U \cdot \bar{\psi} \\ \dot{\bar{\psi}} = r \\ \bar{\psi} = \psi - \psi_d \end{cases} \quad (6)$$

We can rewrite (6) into the form of a state space equation as

$$\begin{bmatrix} \dot{r} \\ \dot{\psi} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} -1/\eta & 0 & 0 \\ 1 & 0 & 0 \\ 0 & U & 0 \end{bmatrix} \begin{bmatrix} r \\ \psi \\ e \end{bmatrix} + \begin{bmatrix} \zeta/\eta \\ 0 \\ 0 \end{bmatrix} \delta + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} d \quad (7)$$

Remark 1. The path following state space model in (7) describes the dynamics of USV path following with a control rudder angle as the input, where it is assumed that the speed of the propeller thruster is constant, featuring typical underactuated characteristics. By applying the following QiH-MPC to adjust the rudder angle of the controller, the positional and directional discrepancies between the current path and the target one are gradually reduced and ultimately eliminated.

2.3. QiH-MPC with Disturbance Compensation Framework

In conventional MPC, the time domain parameters of both prediction and control are adjustable, which directly affects the actual control performance [25]. The results will be better if an infinite horizon is considered, especially when the approximation of the desired minimization cost is more accurate. However, iH-MPC raises two concerns: (1) as the prediction and control time domain increase, the algorithm's complexity grows, reducing real-time performance; and (2) although feedback correction enhances robustness, iH-MPC lacks the capability to effectively mitigate time-varying wave disturbances [26].

This paper presents QiH-MPC with a fixed-time disturbance observer for USV path following, which achieves optimal performance while also ensuring robustness. In detail, the objective function of infinite horizon prediction is constructed, and the terminal cost term is based on the linear quadratic regulator optimization theory. This maintains the performance of predictive control and reduces the amount of online optimization computation, which is substantially different from the existing works. The system control framework is shown in Figure 2, which includes the following:

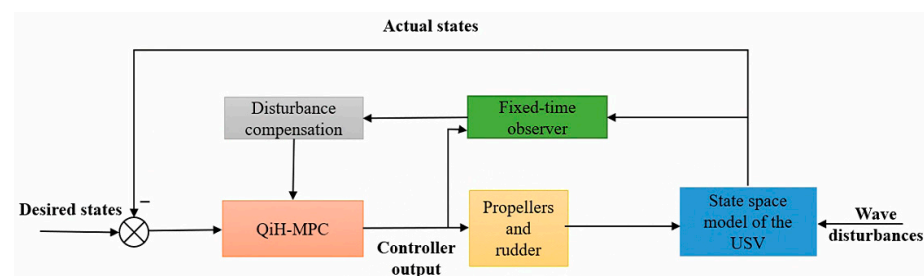


Figure 2. USV path following with disturbance compensation framework based on QiH-MPC and fixed-time observer.

- (1) The feedback control strategy with QiH-MPC aims to obtain the optimal control law for USV path following. Specifically, the terminal cost term of finite time domain optimization in conventional MPC is re-constructed based on the linear quadratic regulator optimization. This approximation maintains the predictive performance of conventional MPC and reduces the amount of online computation significantly.
- (2) To improve the accuracy of the prediction model and the ability to suppress the wave disturbance, a fixed-time observer is proposed to estimate aggregate disturbances efficiently. The prediction model incorporates the estimation of aggregate disturbances, encompassing time-varying wave and nonlinear convolution terms of heading angle velocity, to improve the system's resilience within a fixed time frame.

3. QiH-MPC for USV Path Following

MPC is an efficient computer control algorithm that utilizes model prediction, roll optimization and feedback correction to achieve excellent control performance and robust-

ness. This paper focuses on the path following control of surface unmanned vehicles using iH-MPC. To implement MPC, the USV path following control model can be discretized as

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}_d \mathbf{x}(k) + \mathbf{B}_d \mathbf{u}(k) + \hat{\mathbf{v}}_{dk} \\ \mathbf{y}(k) = \mathbf{C} \mathbf{x}(k) \end{cases} \quad (8)$$

where $\mathbf{x}(k) = [r(k) \ \bar{\psi}(k) \ e(k)]^T$ and $\mathbf{u}(k) = \delta(k)$ are the discretized state and control variables; $\mathbf{A}_d$, $\mathbf{B}_d$ and $\mathbf{C}$ are system coefficient matrices; and $\hat{\mathbf{v}}_{dk}$ is the discretized form of the estimated disturbance. $\mathbf{y}(k)$ is the output of the system. The discrete predictive control objective function within the finite-time horizon is defined as

$$J = \frac{1}{2} \left(\sum_{i=k+1}^{k+N} (\mathbf{y}_{i|k}^T \mathbf{Q} \mathbf{y}_{i|k}) + \sum_{i=k}^{k+N} (\mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) \right) \quad (9)$$

where N denotes the predictive horizon and control horizon; $\mathbf{y}_{i|k}$, $\mathbf{u}_{i|k}$ denote the corresponding predictive output and predictive input at the future sample instant i based on the current sample instant k . $\mathbf{Q} > 0$, $\mathbf{R} \geq 0$ denote weighing matrices of the state and the control. It is clear that increasing the prediction horizon improves the prediction accuracy of the prediction model, thereby improving the steady-state and dynamic performance of the controller and the stability of the control system. However, this also increases the dimension of the optimization variable and computation load, leading to a deterioration in the real-time performance of the control system. As a result, a QiH-MPC scheme is designed in this section to accurately approximate the infinite horizon model predictive control with less computation, which can improve the robustness of the system while ensuring real-time performance.

Let $N = \infty$ and the objective function (9) evolve into

$$\begin{aligned} J &= \frac{1}{2} \left(\sum_{i=k+1}^{\infty} (\mathbf{y}_{i|k}^T \mathbf{Q} \mathbf{y}_{i|k}) + \sum_{i=k}^{\infty} (\mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) \right) \\ &= \frac{1}{2} \left(\sum_{i=k+1}^{k+N-1} (\mathbf{y}_{i|k}^T \mathbf{Q} \mathbf{y}_{i|k}) + \sum_{i=k}^{k+N-1} (\mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) \right) + \frac{1}{2} \left(\sum_{i=k+N}^{\infty} (\mathbf{y}_{i|k}^T \mathbf{Q} \mathbf{y}_{i|k}) + \sum_{i=k+N}^{\infty} (\mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) \right) \\ &= J_{finite} + J_{infinite} \end{aligned} \quad (10)$$

Moreover, we denote that

$$J_{infinite} = \frac{1}{2} \sum_{i=k+N}^{\infty} \left((\mathbf{y}_{i|k}^T \mathbf{Q} \mathbf{y}_{i|k}) + (\mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) \right) = \frac{1}{2} \sum_{i=k+N}^{\infty} \left((\mathbf{x}_{i|k}^T \tilde{\mathbf{Q}} \mathbf{x}_{i|k}) + (\mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) \right) \quad (11)$$

where $\tilde{\mathbf{Q}} = \mathbf{C}^T \mathbf{Q} \mathbf{C}$ is symmetric and positive semi-definite. To make (11) approximate a finite terminal cost term, consider a discrete Riccati equation (DRE) following LQR theory as

$$\mathbf{A}_d^T \mathbf{P} \mathbf{A}_d - \mathbf{P} - \mathbf{A}_d^T \mathbf{P} \mathbf{B}_d [\mathbf{R} + \mathbf{B}_d^T \mathbf{P} \mathbf{B}_d]^{-1} \mathbf{B}_d^T \mathbf{P} \mathbf{A}_d + \tilde{\mathbf{Q}} = 0 \quad (12)$$

where $\tilde{\mathbf{Q}} = \mathbf{C}^T \mathbf{Q} \mathbf{C}$ is symmetric and positive semi-definite, $(\mathbf{A}_d, \mathbf{B}_d)$ is controllable and $(\mathbf{A}_d, \mathbf{C})$ is observable. $\mathbf{P}$ is the solution of (12). From [27], one can derive that there exists a unique solution for the matrix $\mathbf{P}$ that minimizes $J_{infinite}$. Thus, the obtained control law can be expressed as

$$\mathbf{u}_{i|k} = -\tilde{\mathbf{K}} \mathbf{x}_{i|k} \quad (13)$$

where $\tilde{\mathbf{K}} = [\mathbf{R} + \mathbf{B}_d^T \mathbf{P} \mathbf{B}_d]^{-1} \mathbf{B}_d^T \mathbf{P} \mathbf{A}_d$. Therefore, the dynamics of the closed-loop system can be written as

$$\mathbf{x}_{i+1|k} = (\mathbf{A}_d - \mathbf{B}_d \tilde{\mathbf{K}}) \mathbf{x}_{i|k} \quad (14)$$

We proceed by constructing a Lyapunov function as $\mathbf{x}^T \mathbf{P} \mathbf{x}$, satisfying the following condition:

$$\mathbf{P} - (\mathbf{A}_d - \mathbf{B}_d \tilde{\mathbf{K}})^T \mathbf{P} (\mathbf{A}_d - \mathbf{B}_d \tilde{\mathbf{K}}) = \tilde{\mathbf{Q}} + \tilde{\mathbf{K}}^T \mathbf{R} \tilde{\mathbf{K}} \quad (15)$$

When we multiply both sides of (15) by $\mathbf{x}_{j|k}^T$ on the left and $\mathbf{x}_{j|k}$ on the right simultaneously, we derive that

$$\mathbf{x}_{j|k}^T \mathbf{P} \mathbf{x}_{j|k} - \mathbf{x}_{j|k}^T (\mathbf{A}_d - \mathbf{B}_d \tilde{\mathbf{K}})^T \mathbf{P} (\mathbf{A}_d - \mathbf{B}_d \tilde{\mathbf{K}}) \mathbf{x}_{j|k} = \mathbf{x}_{j|k}^T \tilde{\mathbf{Q}} \mathbf{x}_{j|k} + \mathbf{x}_{j|k}^T \tilde{\mathbf{K}}^T \mathbf{R} \tilde{\mathbf{K}} \mathbf{x}_{j|k} \quad (16)$$

which can be written as

$$\mathbf{x}_{j|k}^T \mathbf{P} \mathbf{x}_{j|k} - \mathbf{x}_{j+1|k}^T \mathbf{P} \mathbf{x}_{j+1|k} = \mathbf{x}_{j|k}^T \tilde{\mathbf{Q}} \mathbf{x}_{j|k} + \mathbf{u}_{j|k}^T \mathbf{R} \mathbf{u}_{j|k} \quad (17)$$

Taking an accumulation from $j = k + N$ to $j = \infty$ for Equation (17), one follows

$$\mathbf{x}_{k+N|k}^T \mathbf{P} \mathbf{x}_{k+N|k} - \lim_{a \rightarrow \infty} (\mathbf{x}_{a|k}^T \mathbf{P} \mathbf{x}_{a|k}) = \sum_{i=k+N}^{\infty} (\mathbf{x}_{i|k}^T \tilde{\mathbf{Q}} \mathbf{x}_{i|k} + \mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) \quad (18)$$

According to optimal control theory [28], the linear quadratic regulator optimization controller in the infinite horizon is closed-loop stable. Then, $\lim_{a \rightarrow \infty} (\mathbf{x}_{a|k}^T \mathbf{P} \mathbf{x}_{a|k}) = 0$ is established and Equation (18) can be expressed as

$$\mathbf{x}_{k+N|k}^T \mathbf{P} \mathbf{x}_{k+N|k} = \sum_{i=k+N}^{\infty} (\mathbf{y}_{i|k}^T \mathbf{Q} \mathbf{y}_{i|k}) + \sum_{i=k+N}^{\infty} (\mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) = 2J_{\text{infinite}} \quad (19)$$

Based on (19), the discrete predictive control objective function (10) becomes a finite sum as

$$J = \frac{1}{2} \left(\sum_{i=k+1}^{k+N-1} (\mathbf{y}_{i|k}^T \mathbf{Q} \mathbf{y}_{i|k}) + \sum_{i=k}^{k+N-1} (\mathbf{u}_{i|k}^T \mathbf{R} \mathbf{u}_{i|k}) + \mathbf{x}_{k+N|k}^T \mathbf{P} \mathbf{x}_{k+N|k} \right) \quad (20)$$

It is shown that J is an expression of a finite sum that can be represented as

$$J = \frac{1}{2} (\boldsymbol{\alpha}_{\text{inf}}^T \mathbf{Q}_{\text{inf}} \boldsymbol{\alpha}_{\text{inf}} + \mathbf{u}_{k|k,N}^T \mathbf{R}_{\text{inf}} \mathbf{u}_{k|k,N}) \quad (21)$$

$$\text{where } \mathbf{Q}_{\text{inf}} = \begin{bmatrix} Q & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & Q & 0 \\ 0 & \cdots & 0 & P \end{bmatrix}, \quad \mathbf{R}_{\text{inf}} = \begin{bmatrix} R & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & R & 0 \\ 0 & \cdots & 0 & R \end{bmatrix}, \quad \boldsymbol{\alpha}_{\text{inf}} = \begin{bmatrix} \mathbf{y}_{k+1|k} \\ \mathbf{y}_{k+2|k} \\ \vdots \\ \mathbf{y}_{k+N-1|k} \\ \mathbf{x}_{k+N|k} \end{bmatrix},$$

$$\mathbf{u}_{k|k,N} = \begin{bmatrix} \mathbf{u}_{k|k} \\ \mathbf{u}_{k+1|k} \\ \vdots \\ \mathbf{u}_{k+N-2|k} \\ \mathbf{u}_{k+N-1|k} \end{bmatrix}.$$

Based on the above mathematical analysis, the problem of USV path following can be expressed in the following form:

$$\begin{aligned} & \min_{\mathbf{u}(t)^*} J(\mathbf{x}(t)) \\ & \text{s.t. } \mathbf{x}_{i|k} = \mathbf{A}_d \mathbf{x}_{i-1|k} + \mathbf{B}_d \mathbf{u}_{i-1|k} + \hat{\mathbf{v}}_{dk} i = k+1, k+2, \dots, k+N \end{aligned} \quad (22)$$

The general form of the prediction output is obtained iteratively as

$$\underbrace{\begin{bmatrix} \mathbf{y}_{k+1|k} \\ \mathbf{y}_{k+2|k} \\ \vdots \\ \mathbf{y}_{k+N-1|k} \\ \mathbf{x}_{k+N|k} \end{bmatrix}}_{\mathbf{a}_{inf}} = \underbrace{\begin{bmatrix} \mathbf{CA}_d \mathbf{x}_{k|k} \\ \mathbf{CA}_d^2 \mathbf{x}_{k|k} \\ \vdots \\ \mathbf{CA}_d^{N-1} \mathbf{x}_{k|k} \\ \mathbf{A}_d^N \mathbf{x}_{k|k} \end{bmatrix}}_{\mathbf{M}} + \underbrace{\begin{bmatrix} \mathbf{C} & 0 & \cdots & 0 & 0 \\ \mathbf{CA}_d & \mathbf{C} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{CA}_d^{N-2} & \mathbf{CA}_d^{N-3} & \cdots & \mathbf{C} & 0 \\ \mathbf{A}_d^{N-1} & \mathbf{A}_d^{N-3} & \cdots & \mathbf{A}_d & \mathbf{I} \end{bmatrix}}_{\mathbf{D}} \underbrace{\begin{bmatrix} \hat{\mathbf{v}}_{dk} \\ \hat{\mathbf{v}}_{dk} \\ \vdots \\ \hat{\mathbf{v}}_{dk} \\ \hat{\mathbf{v}}_{dk} \end{bmatrix}}_{\mathbf{v}_{dk}} + \underbrace{\begin{bmatrix} \mathbf{CB}_d & 0 & \cdots & 0 & 0 \\ \mathbf{CA}_d \mathbf{B}_d & \mathbf{CB}_d & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{CA}_d^{N-2} \mathbf{B}_d & \mathbf{CA}_d^{N-3} \mathbf{B}_d & \cdots & \mathbf{CB}_d & 0 \\ \mathbf{A}_d^{N-1} \mathbf{B}_d & \mathbf{A}_d^{N-3} \mathbf{B}_d & \cdots & \mathbf{A}_d \mathbf{B}_d & \mathbf{B}_d \end{bmatrix}}_{\mathbf{L}} \mathbf{u}_{k|k,N} \quad (23)$$

Based on the definition of $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{L}$ in (23), the discrete predictive control objective function J evolves into

$$J = \frac{1}{2} \left[\left(\mathbf{M} + \mathbf{D} + \mathbf{L} \mathbf{u}_{k|k,N} \right)^T \mathbf{Q}_{inf} \left(\mathbf{M} + \mathbf{D} + \mathbf{L} \mathbf{u}_{k|k,N} \right) + \mathbf{u}_{k|k,N}^T \mathbf{R}_{inf} \mathbf{u}_{k|k,N} \right] \quad (24)$$

By differentiating for the objective function J with respect to $\mathbf{u}_{k|k,N}$, the following equation holds according to the necessary conditions for generalized optimality:

$$\frac{dJ}{d\mathbf{u}_{k|k,N}} = (\mathbf{L}^T \mathbf{Q}_{inf} \mathbf{L} + \mathbf{R}_{inf}) \mathbf{u}_{k|k,N} + \mathbf{L}^T \mathbf{Q}_{inf} (\mathbf{M} + \mathbf{D}) = 0 \quad (25)$$

Then, the optimal control sequence is

$$\mathbf{u}_{k|k,N}^{opt} = - \left(\mathbf{L}^T \mathbf{Q}_{inf} \mathbf{L} + \mathbf{R}_{inf} \right)^{-1} \left[\mathbf{L}^T \mathbf{Q}_{inf} (\mathbf{M} + \mathbf{D}) \right] \quad (26)$$

The model predictive control only selects the current instant as

$$\mathbf{u}(t) = \mathbf{I}_{1*N} \mathbf{u}_{k|k,N}^{opt} \quad (27)$$

where $\mathbf{I}_{1*N} = [1, 0, \dots, 0]$ and $\mathbf{u}_{k|k,N}^{opt}$ is derived from Equation (26).

Remark 2. It can be seen that controller form (26) performs the compensation for the wave disturbances by adding the integrated disturbance compensation matrix $\mathbf{D}$ in the finite horizon. This significantly reduces the inaccuracy of the model prediction state and improves the robustness of the model predictive control. Furthermore, utilizing predicted states in the finite horizon allows for a precise estimation of infinite horizon predictive control, resulting in a substantial reduction in computation and facilitating practical applications.

4. Fixed-Time Observer (FTO) for Disturbance Estimation

The disturbance observer is capable of estimating both external and internal disturbances in the system, regardless of accurate modeling, thereby improving robustness. In contrast to the finite-time observer, the fixed-time disturbance observer for aggregate disturbance estimation ensures that the estimation error converges to zero within a fixed time. Furthermore, the upper bound of the convergence time is independent of the system's initial state. As a result, this paper employs fixed-time observers for online estimation of aggregate disturbances for USV path following control, which can be incorporated into the

QiH-MPC, forming a feed-forward and feedback integrated framework to improve its path following accuracy and robustness.

The fixed-time disturbance observer in this work was specifically designed for USV path following state space model (7). Incorporating the wave disturbance force and moment into the heading angular velocity is a deliberate choice, as the observer is more responsive to changes in wind and wave currents. We denote $x_1 = r$, $x_2 = \bar{\psi}$ and $x_3 = e$. Then, the USV path following model, with the added wave disturbance force and moment, can be expressed as

$$\begin{cases} \dot{x}_1 = f(x_1, x_2, x_3, \delta) + d(F_{\text{wave}}, M_{\text{wave}}) \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = Ux_2 \end{cases} \quad (28)$$

where $d(F_{\text{wave}}, M_{\text{wave}})$ denotes the aggregate bounded disturbance imposed by the disturbance force and moment. We define the tracking error as

$$e_1 = x_1 - x_{1d} \quad (29)$$

where e_1 denotes the tracking error of the heading angle velocity; x_{1d} denotes the desired heading angle velocity. Combined with (28) and (29), one derives that

$$\dot{e}_1 = f(x_1, x_2, x_3, \delta) + d(F_{\text{wave}}, M_{\text{wave}}) - \dot{x}_{1d} \quad (30)$$

Following [29], we define a fixed-time disturbance observer as

$$\dot{w}_1 = f(x_1, x_2, x_3, \delta) + v_1 - \dot{x}_{1d} \quad (31)$$

where v_1 is the estimate of the disturbance. With the introduction of sliding mode auxiliary variables, s_1 is

$$\begin{cases} s_1 = e_1 - w_1 \\ \dot{s}_1 = d_1 - v_1 \end{cases} \quad (32)$$

Based on Equation (32), when s_1 and $\dot{s}_1$ converge to zero, one derives $v_1 = d_1$ accordingly. An auxiliary sliding mode differential equation is constructed based on the continuous fixed-time second-order sliding mode control algorithm, proposed as

$$\begin{cases} \dot{s}_1 = -\lambda_1 |s_1|^{\frac{1}{2}} \text{sign}(s_1) - \lambda_2 |s_1|^p \text{sign}(s_1) + \eta \\ \dot{\eta} = -\alpha \text{sign}(s_1) + \dot{d}_1, \eta(t_0) = 0 \end{cases} \quad (33)$$

where η is an auxiliary variable, $|\dot{d}_1| \leq \vartheta$ is the aggregate bounded disturbance and $\lambda_1, \lambda_2, \alpha, \vartheta, p$ are positive constants, satisfying $\lambda_1, \lambda_2 > 0, \alpha > \vartheta > 0, p > 1$. t_0 denotes the initial instant of the system. Following [29], one derives that given a constant $\sigma > 0$, when $|s_1(t_0)| > \sigma$,

$$\frac{d|s_1(t)|}{dt} \leq -\lambda_2 |s_1(t)|^p \quad (34)$$

We rewrite (34) in the following form:

$$|s_1(t)|_{-p} d|s_1(t)| \leq -\lambda_2 dt \quad (35)$$

By integrating both sides of Equation (35), it follows that

$$\int_{t_0}^t |s_1(t)|_{-p} d|s_1(t)| \leq \int_{t_0}^t -\lambda_2 dt \quad (36)$$

that is,

$$\frac{|s_1(t)|^{1-p}}{1-p} - \frac{|s_1(t_0)|^{1-p}}{1-p} \leq -\lambda_2(t - t_0) \quad (37)$$

and, noting that $p > 1$,

$$\frac{|s_1(t)|^{1-p}}{1-p} \leq -\lambda_2(t-t_0) + \frac{|s_1(t_0)|^{1-p}}{1-p} \leq -\lambda_2(t-t_0) \quad (38)$$

By multiplying both sides of Equation (38) with $p-1$, it follows that

$$-|s_1(t)|^{1-p} \leq -\lambda_2(t-t_0)(p-1) \quad (39)$$

that is,

$$|s_1(t)|^{p-1} \leq \frac{1}{\lambda_2(p-1)(t-t_0)} \quad (40)$$

Thus, one derives that $|s_1(t)|$ is decreasing. Due to the fact that $\text{sign}(s_1(t))$ and $\text{sign}(\eta(t))$ have opposite signs within the interval $[t_0, T_1]$, then $|\eta(t)|$ exhibits an increasing trend over the interval $[t_0, T_1]$. When $|s_1(t)| \leq \sigma$, it follows from (33) that

$$\frac{d|s_1(t)|}{dt} \leq -\lambda_1|s_1(t)|^{\frac{1}{2}} \quad (41)$$

and

$$|s_1(t)|^{-\frac{1}{2}} d|s_1(t)| \leq -\lambda_1 dt \quad (42)$$

By integrating both sides of Equation (42), it follows that

$$\int_{T_1}^t |s_1(t)|^{-\frac{1}{2}} d|s_1(t)| \leq \int_{T_1}^t -\lambda_1 dt \quad (43)$$

and

$$2|s_1(t)|^{-1/2} - 2|s_1(T_1)|^{-1/2} \leq -\lambda_1(t-T_1) \quad (44)$$

Therefore, $|s_3(t)|$ can reach zero at $t = T_2 \leq 2\sigma^{1/2}/\lambda_1$. Similarly, because $\text{sign}(s_1(t))$ and $\text{sign}(\eta(t))$ have opposite signs within the interval $[t_0, T_2]$, it follows that $|\eta(t)|$ exhibits an increasing trend over the interval $[t_0, T_1]$. Then, $|\eta(t)|$ is bounded as

$$|\eta(T_2)| < M(T_2 - t_0) = M \left(\frac{1}{\lambda_2(p-1)\sigma^{p-1}} + \frac{2\sigma^{1/2}}{\lambda_1} \right) \quad (45)$$

where $M = \alpha + L$, L is a constant to bound the disturbance as $\|d(F_{\text{wave}}, M_{\text{wave}})\| \leq L(t-t_0)$ and t_0 is the initial time moment of the system. Based on (45), it follows that compared with the start point of $(0, \eta')$ for sliding system (33), the point of $(0, \eta(T_2))$ performs a faster convergence to zero, where

$$\eta' = M \left(\frac{1}{\lambda_2(p-1)\sigma^{p-1}} + \frac{2\sigma^{1/2}}{\lambda_1} \right) \quad (46)$$

Reference [29] presented an estimation of trajectories in a finite time as

$$T_{STW} \leq \sum_i \dot{s}_1(t_i)/m < \eta'/(1-q)m, i = 1, 2, \dots \quad (47)$$

where $m = \alpha - L$, $q = |\dot{s}(t_2)|/|\dot{s}(t_1)| < Mh(\lambda_1)/\lambda_1$ and t_2 and t_2 are the subsequent time moments. $h(\lambda_1) = 1/\lambda_1 + (2e/m\lambda_1)$ and e is the symbol of the base of the natural logarithm. Then, the convergence time can be obtained:

$$T_{STW} < M \left(\frac{1}{\lambda_2(p-1)\sigma^{p-1}} + \frac{2\sigma^{1/2}}{\lambda_1} \right) / (1 - Mh(\lambda_1)/\lambda_1)m \quad (48)$$

Therefore, s_1 and $\dot{s}_1$ must converge to zero within T_f as

$$T_f \leq T_1 + T_2 + T_{STW} = \left(\frac{1}{\lambda_2(p-1)\sigma^{p-1}} + \frac{2\sigma^{\frac{1}{2}}}{\lambda_1} \right) \left(1 + \frac{1}{m((1/M) - (h(\lambda_1)/\lambda_1))} \right) \quad (49)$$

Then, the aggregate disturbance estimation can be obtained as

$$\begin{cases} v_1 = \lambda_1 |s_1|^{\frac{1}{2}} \text{sign}(s_1) + \lambda_2 |s_1|^p \text{sign}(s_1) + z_1 \\ \dot{z}_1 = \alpha \text{sign}(s_1), z_1(0) = 0 \end{cases} \quad (50)$$

Remark 3. By adding the disturbance estimation v_1 of wave forces and moments, it follows that the USV path following state space model becomes (7) exactly. The aggregate disturbances, including time-varying wave and nonlinear convolution terms of the heading angle velocity, are then introduced into the prediction model. To improve the accuracy of the prediction model and the ability to suppress the wave disturbance, the fixed-time observer above was employed to estimate aggregate disturbances efficiently, significantly enhancing the system's robustness for external disturbances.

5. Convergence Analysis

Although the proposed QiH-MPC with a fixed-time disturbance observer can be briefly explained mathematically, it is essential to present a convergence and stability analysis. For the discretized form of USV path following control system (8), it is recommended to express it as a general form with

$$x(k+1) = f(x(k), u(k), \hat{v}(k)) \quad (51)$$

Since the control action in each period is obtained by solving optimization problem (22), we rewrite the objective function as

$$V(k) = \min_{x,u} \sum_{i=k+1}^{k+N-1} l(x(k+i|k), u(k+i-1|k), \hat{v}(k+i|k)) \quad (52)$$

where $l(x, u, \hat{v}) \geq 0$ and $l(x, u, \hat{v}) = 0$ if and only if $x = 0$, $u = 0$ and $\hat{v}(k)$ is bounded satisfying $|\dot{\hat{v}}| \leq \vartheta$. Then, we present the following two assumptions that are easily satisfied based on the proposed QiH-MPC with a fixed-time disturbance observer:

Assumption 1. $x = 0$, $u = 0$ and $|\dot{\hat{v}}| \leq \vartheta$ is an equilibrium condition of USV path following control system (51).

Assumption 2. Optimization problem (22) for each period has a feasible solution and the globally optimal solution can be obtained. Moreover, only the first optimal controller action $u^*(k)$ is imposed on the system for the optimal control sequence $\{u^*(k+i|k) | i = 0, \dots, N-1\}$.

Then, we present Theorem 1 about the convergence of the proposed Qi-MPC with a fixed-time disturbance observer as follows:

Theorem 1. If Assumption 1 and Assumption 2 hold, then system (51) is stable at $x = 0$, $u = 0$ and $|\dot{\hat{v}}| \leq \vartheta$.

Proof of Theorem 1. Based on the Lyapunov stability theory, it is necessary to determine a Lyapunov function of the system that is positive definite, and that has a negative definite time-derivative (i.e., the Lyapunov function is decreasing). We define the Lyapunov function as $V(k)$. It is obvious that $V(k)$ is positive definite due to the fact that the form of $V(k)$ is quadratic. Then,

$$\begin{aligned}
 V(k+1) &= \min_{x,u} \sum_{i=k+1}^{k+N-1} l(x(k+i+1), u(k+i), \hat{\vartheta}(k+i+1)) \\
 &= \min_{x,u} \left(\sum_{i=k+1}^{k+N-1} l(x(k+i), u(k+i-1), \hat{\vartheta}(k+i)) - l(x(k+1), u(k), \hat{\vartheta}(k+1)) \right) \\
 &\quad + l(x(k+1+N), u(k+N), \hat{\vartheta}(k+1+N)) \\
 &= -l(x(k+1), u(k), \hat{\vartheta}(k+1)) + \min_{x,u} \left(\sum_{i=k+1}^{k+N-1} l(x(k+i), u(k+i-1), \hat{\vartheta}(k+i)) \right. \\
 &\quad \left. + l(x(k+1+N), u(k+N), \hat{\vartheta}(k+1+N)) \right) \\
 &\leq -l(x(k+1), u^*(k), \vartheta) + V(k) + \min_{x,u} \{l(x(k+1+N), u(k+N), \hat{\vartheta}(k+1+N))\}
 \end{aligned} \tag{53}$$

For the end of optimization, we have the endpoint state constraint that $x(k+N|k) = 0$. Then, one notes that $\min_{x,u} \{l(x(k+1+N), u(k+N), \hat{\vartheta}(k+1+N))\} = 0$. At the same time, we have $l(x(k+1), u^*(k), \vartheta) \geq 0$. Then, it follows that

$$V(k+1) \leq V(k) \tag{54}$$

Based on the Lyapunov stability theory, USV path following control system (51) is stable at $x = 0$, $u = 0$ and $|\dot{\vartheta}| \leq \vartheta$. $\square$

6. Simulations

The proposed QiH-MPC for USV path following was verified in a MATLAB environment (v.2023b) on the author's laptop with a 13th Gen Intel Core i9-13900HX processor (Intel, Santa Clara, CA, USA) and 16 GB DDR3 memory. The USV parameters are listed in Table 1. To elucidate the device characteristics under investigation, we present a block diagram of the control system of the USV SL-20Y in Figure 3, which can contribute to comprehending the system components and controller principles in detail.

In this section, a simulation of the performance of QiH-MPC for USV path following is given first. Then, we make detailed comparisons between the proposed algorithm and conventional MPC. Finally, we present the performance of QiH-MPC with and without the fixed-time observer (FTO) to further show the effectiveness of the QiH-MPC with FTO.

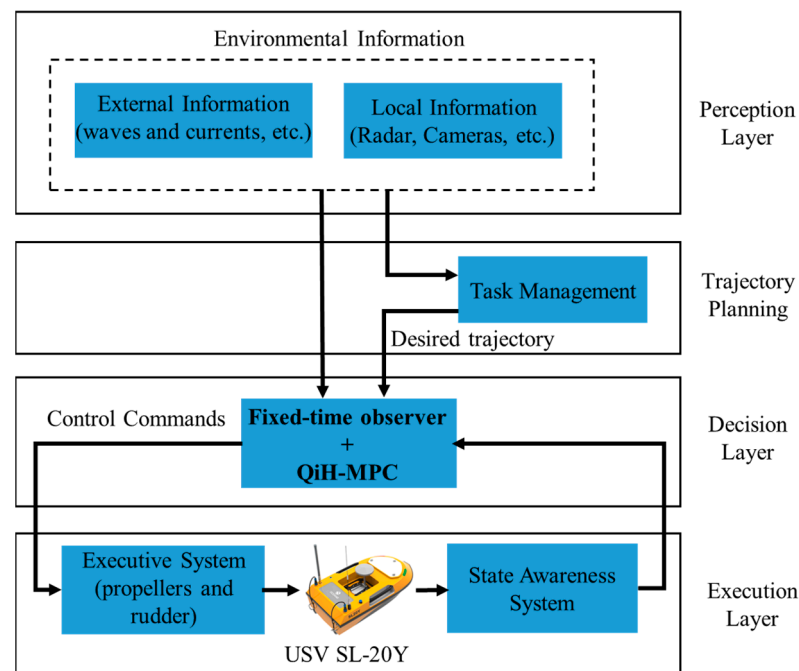


Figure 3. USV SL-20Y path following block diagram based on QiH-MPC with fixed-time observer.

Table 1. Main details of the USV SL-20Y.

Index	Parameter
Size	1.05 m (Length) $\times$ 0.55 m (Width)
Weight	15 kg
Load Capacity	10 kg
Maximum Speed	5 m/s
Communication Distance	Remote control: 1 km Base station: 2 km
Turning Radius	227 mm
Mode of advancement	Jet Pump
Wave Resistance	Force 3 winds Waves 0.5 m
Maneuverability parameter	$\zeta = 0.6463$
Followability parameter	$\eta = 1.0674$

6.1. Performance of QiH-MPC for USV Path Following

In this simulation, we give the performance of QiH-MPC for USV path following in detail. The maneuverability parameter $\zeta = 0.6463$ and followability parameter $\eta = 1.0674$ are noted in Table 1. The speed of the USV is set as $U = 2.29$ m/s. The multi-segment linear tracking scenario is set up here, in which the initial values of the state are set as $[0 \ -90 \ 50]^T$ (deg/s, deg, m) for the first target path and $[0 \ 45^\circ \ -20]^T$ (deg/s, deg, m) for the second target path. The sampling time is set as $T_s = 0.05$ s. The controller input is limited to $[-30, 30]$ (deg, deg) and the rate of change of the controller input is constrained to $[-2, 2]$ (deg/s, deg/s). To verify the QiH-MPC controller performance, we set different parameters of the output error matrix Q and the input control matrix R for this simulation. The results are shown in Figures 4 and 5.

The variation in the performance of the QiH-MPC path following controller is shown in Figures 4 and 5. By setting two desired paths with different slopes, the QiH-MPC path following controller designed in this paper can be seen to be able to achieve accurate path following. Moreover, as the output error matrix increases and the input control matrix decreases, the tracking error converges more rapidly (see the tracking error state curve in Figures 4b and 5b), such that the USV more aggressively follows the desired path (see the path following results in Figures 4a and 5a). However, this is at the cost of a subsequent increase in the heading angular velocity, the heading angular error and the magnitude of rudder angle oscillations, which will make the USV less stable. Therefore, the parameter values of the output error matrix and the input control matrix must be chosen as a compromise between the efficiency and stability of path following.

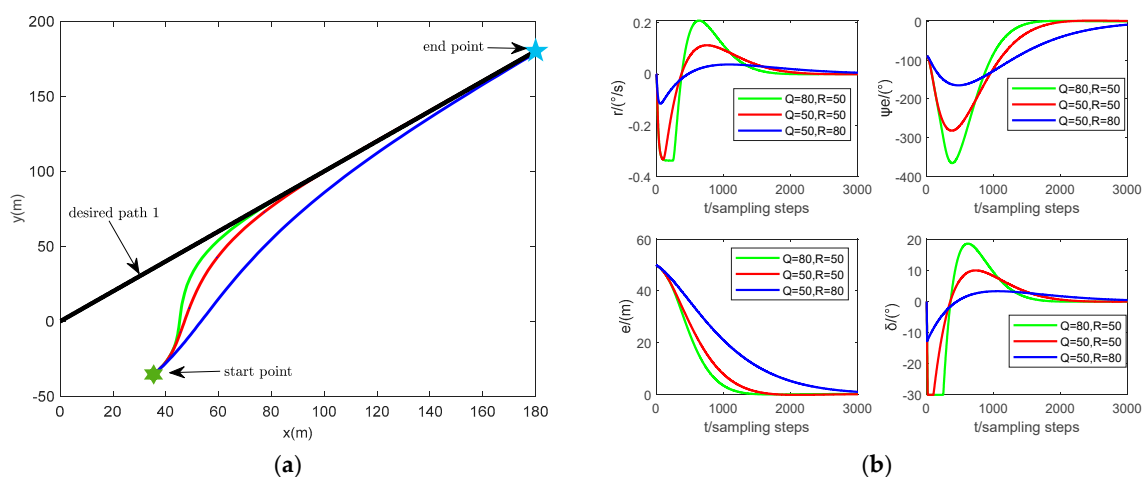


Figure 4. (a) Simulation results for USV desired path 1 following; (b) the states and controller output for desired path 1 following.

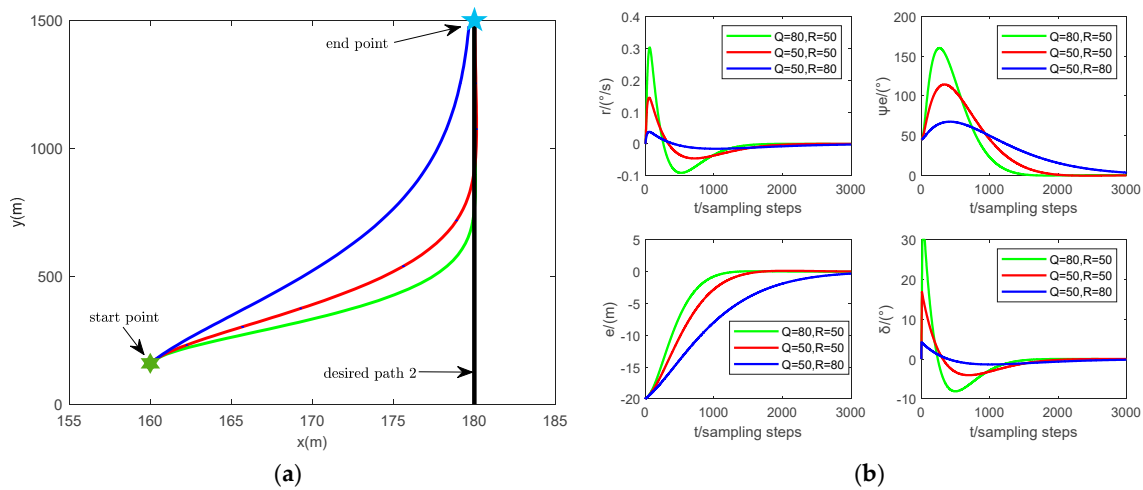


Figure 5. (a) Simulation results for USV desired path 2 following; (b) the states and controller output for desired path 2 following.

6.2. Comparisons between Proposed Algorithm and Conventional MPC

In this simulation, we give the performance comparisons between the proposed algorithm and conventional MPC. A multi-segment linear tracking scenario is set up here, in which the initial values of the state are set as $[0 \ -90 \ 30]^T$ (deg/s, deg, m) for the first target path and $[0 \ 45^\circ \ -20]^T$ (deg/s, deg, m) for the second target path. The controller input is limited to $[-60, 60]$ (deg, deg). To eliminate any uncertainty, the other parameters of QiH-MPC and conventional MPC are both set the same as in Section 6.1. The output error matrix $Q = 80$ and the input control matrix $R = 50$ for this simulation. The results are shown in Figures 6 and 7. Moreover, to further quantify the improvement in algorithm performance, the state and control quantities are further compared in terms of the maximum difference (absolute value), standard deviation and regulation time for the proposed QiH-MPC and conventional MPC.

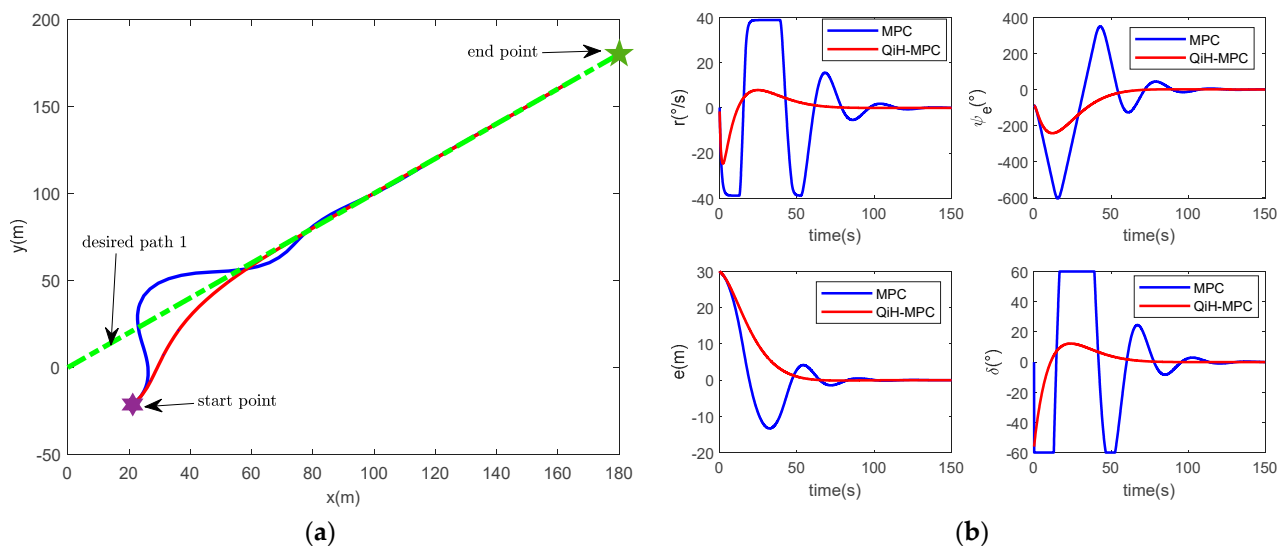


Figure 6. Comparisons between QiH-MPC and conventional MPC for desired path 1 following: (a) USV path 1 following result; (b) the states and controller output simulation result for target path 1.

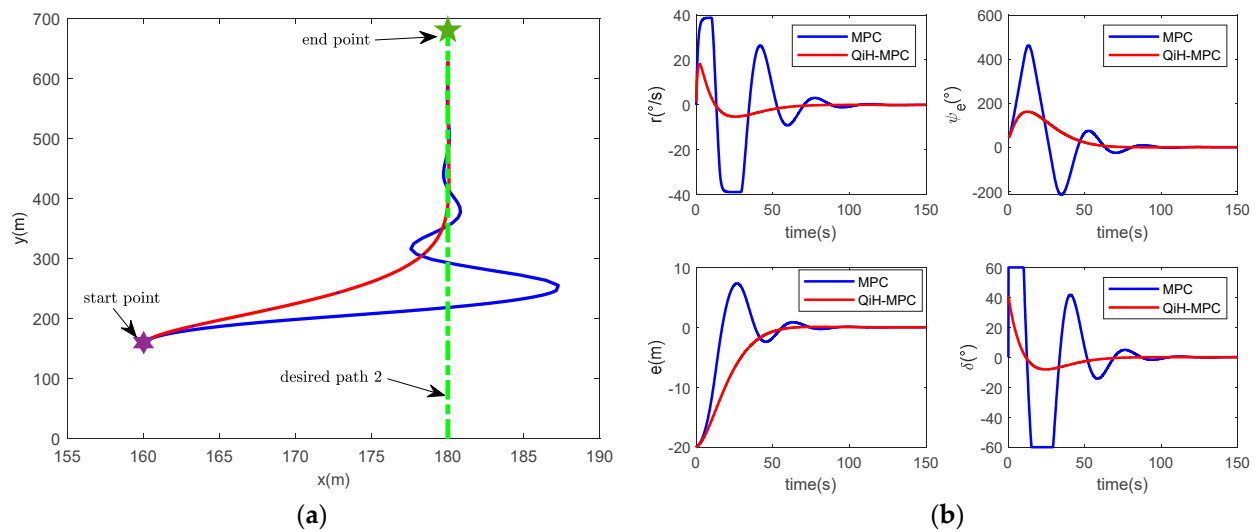


Figure 7. Comparisons between QiH-MPC and conventional MPC for desired path 2 following: (a) USV path 2 following result; (b) the states and controller output simulation result for target path 2.

The results from QiH-MPC and conventional MPC for USV path following are shown in Figures 6 and 7 by setting two desired paths with different slopes as well. Compared to conventional MPC, the QiH-MPC path following controller designed in this paper converges faster for each state variable and the controller input shows fewer oscillations; therefore, QiH-MPC achieves faster and more stable path following results.

Moreover, we present the regulation time, standard deviation and maximum difference comparison for the QiH-MPC and conventional MPC. For the regulation time, the tracking error of QiH-MPC is reduced by 44.1% for target path 1 and by 32.05% for target path 2, so QiH-MPC can track the target paths faster. For the standard deviation, the heading angle error of QiH-MPC is reduced by 59.17% for target path 1 and by 60.14% for target path 2; thus, QiH-MPC can track the target paths more stably. For the control quantity, the maximum difference is reduced by 43.13% for target 1 and by 59.21% for target path 2, so that the actuator of USV can track the desired paths more easily with the proposed QiH-MPC. Other detailed quantitative performance comparisons are shown in Tables 2 and 3. The downward arrow denotes the percentage decrease of the variable corresponding to QiH-MPC in comparison to MPC.

6.3. Performance of QiH-MPC with and without FTO for USV Path Following

The proposed QiH-MPC implements the path following problem with and without the fixed-time observer under time-varying wave disturbances. The QiH-MPC parameters are set up following the same procedure as in Section 6.2. For the fixed-time observer, the parameters are set as $\alpha = 0.019$, $\lambda_1 = \lambda_2 = 1.1$ and $p = 1.3$. Waves are described by the two-parameter spectrum that is recommended by ITTC in [30] as

$$S(\omega_e) = \frac{172.75h_{1/3}^2}{T_a^4\omega_e^5} \exp\left(-\frac{691}{T_a^4\omega_e^4}\right) \quad (55)$$

where $S(m^2 \cdot s)$ denotes the power density, ω_e refers to the angular frequency of the wave, T_a is the average period of the wave and $h_{1/3}$ is the significant wave height.

Table 2. Numerical comparisons between QiH-MPC and conventional MPC for target path 1 following.

Variables		$r(^{\circ}/s)$		$\overline{\psi}(^{\circ})$		$e(m)$		$\delta(^{\circ})$				
Algorithms	MPC	QiH-MPC	performance	MPC	QiH-MPC	performance	MPC	QiH-MPC	performance	MPC	QiH-MPC	performance
Maximum difference (absolute value)	77.5557	32.5149	58.08%↓	953.5068	243.5468	74.45%↓	43.3451	30.0922	30.57%↓	120	68.242	43.13%↓
Standard deviation	21.8243	5.1822	76.25%↓	192.5823	78.6287	59.17%↓	8.5027	7.9198	6.85%↓	34.5431	8.6243	75.03%↓
Regulation time (sec)	142.2	77.8	45.21%↓	140.7	77.5	44.91%↓	106.9	59.75	44.1%↓	144.1	79.7	44.69%↓

Table 3. Numerical comparisons between QiH-MPC and conventional MPC for target path 2 following.

Variables		$r(^{\circ}/s)$		$\overline{\psi}(^{\circ})$		$e(m)$		$\delta(^{\circ})$				
Algorithms	MPC	QiH-MPC	performance	MPC	QiH-MPC	performance	MPC	QiH-MPC	performance	MPC	QiH-MPC	performance
Maximum difference (absolute value)	77.5525	23.3469	69.89%↓	677.6342	161.8532	76.11%↓	27.3171	20.0642	26.55%↓	120	48.9403	59.21%↓
Standard deviation	17.4187	3.7445	78.5%↓	130.879	52.168	60.14%↓	5.0674	4.3694	13.77%↓	27.6849	6.2359	77.47%↓
Regulation time (sec)	119	75.2	36.8%↓	132.6	100.4	24.28%↓	85.8	58.3	32.05%↓	119.5	77.5	35.14%↓

In this simulation, we give performance comparisons between the proposed algorithm QiH-MPC with and without FTO for USV path following. A multi-segment linear tracking scenario is set up here, in which the initial values of the state are set as $[0 \ -30 \ 50]^T$ (deg/s, deg, m) for the first target path and $[0 \ 135^\circ \ -40]^T$ (deg/s, deg, m) for the second target path. Additionally, we positioned it 5 m away from the end of the current desired path, prompting the controller to follow to the next waypoint. To verify the constraint effect of the controller on different input control quantity, the controller input is limited to $[-50, 50]$ (deg, deg). To eliminate any uncertainty, the other parameters of QiH-MPC and conventional MPC are both set the same as in Section 6.1. The output error matrix $Q = 50$ and the input control matrix $R = 50$ for this simulation. The results are shown in Figures 8 and 9. Moreover, to further quantify the improvement in algorithm performance, the state and control quantities are further compared in terms of the maximum difference (absolute value) and standard deviation in Tables 4 and 5. The downward arrow denotes the percentage decrease of the variable corresponding to QiH-MPC with FTO in comparison to QiH-MPC without FTO.

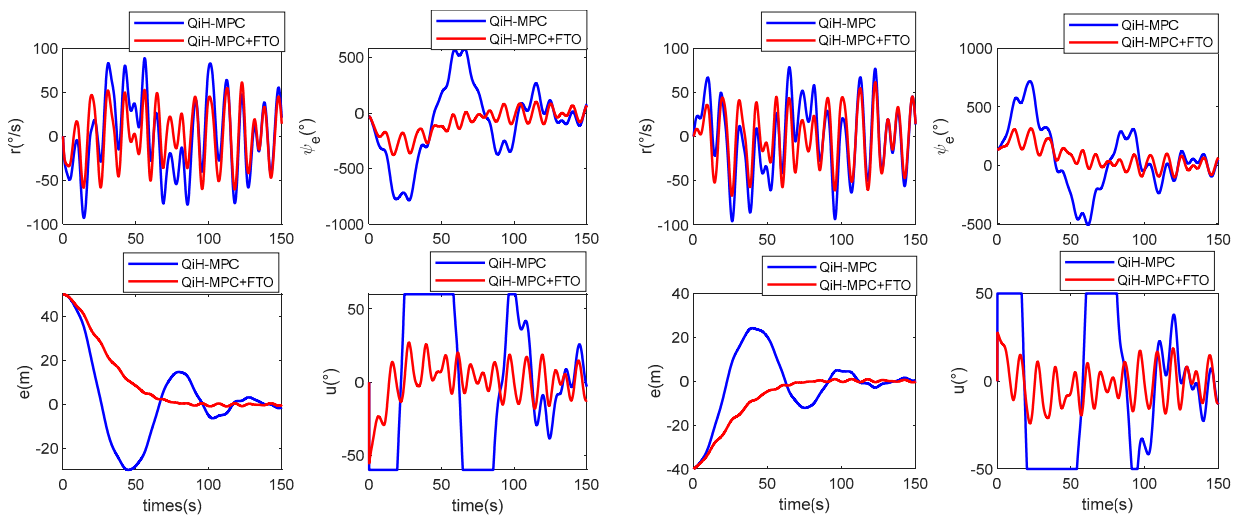


Figure 8. (a) Performance of QiH-MPC with and without FTO for USV path 1 following; (b) performance of QiH-MPC with and without FTO for USV path 2 following.

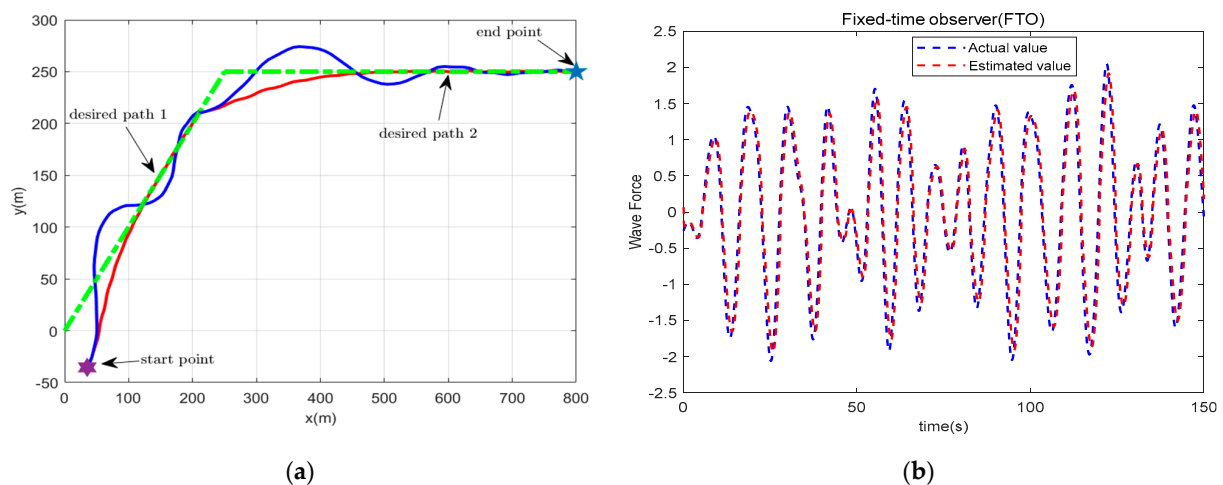


Figure 9. (a) USV path following results of QiH-MPC with and without FTO; (b) the actual wave disturbances and their estimation based on FTO.

Table 4. Performance details of QiH-MPC with and without FTO for USV path 1 following.

Variables		$r(^{\circ}/s)$			$\bar{\psi}(^{\circ})$			$e(m)$			$\delta(^{\circ})$	
Algorithms	QiH-MPC	QiH-MPC +FTO	performance	QiH-MPC	QiH-MPC +FTO	performance	QiH-MPC	QiH-MPC +FTO	performance	QiH-MPC	QiH-MPC +FTO	performance
Maximum difference (absolute value)	183.0536	121.8055	33.45%↓	1415.872	478.0977	66.23%↓	80.95	51.016	36.97%↓	100	76.9652	23.03%↓
Standard deviation	41.6379	31.5932	24.12%↓	334.748	112.2367	66.47%↓	20.6361	15.5204	24.79%↓	42.7789	13.9504	67.39%↓

Table 5. Performance details of QiH-MPC with and without FTO for USV path 2 following.

Variables		$r(^{\circ}/s)$			$\bar{\psi}(^{\circ})$			$e(m)$			$\delta(^{\circ})$	
Algorithms	QiH-MPC	QiH-MPC +FTO	Performance	QiH-MPC	QiH-MPC +FTO	Performance	QiH-MPC	QiH-MPC +FTO	Performance	QiH-MPC	QiH-MPC +FTO	Performance
Maximum difference (absolute value)	174.386	128.5276	25.99%↓	1227.289	418.103	65.93%↓	64.1968	40.9477	36.21%↓	100	51.5565	48.44%↓
Standard deviation	39.9568	31.1563	22.02%↓	281.7829	99.0569	64.84%↓	14.5054	11.4749	20.89%↓	38.9469	11.1136	71.46%↓

The results of QiH-MPC with and without FTO for USV path following are shown in Figures 8 and 9. Clearly, the compensation of the state and control variables for wave disturbances is more effective using QiH-MPC with FTO designed in this paper, resulting in a significant reduction in the oscillations. In Figure 9a, one can verify that QiH-MPC with FTO performs with superior efficiency and stability in following the desired paths compared to QiH-MPC without FTO. And the wave disturbance can be estimated accurately by the FTO (see Figure 9b).

In Tables 4 and 5, we present the maximum difference and standard deviation comparison for the QiH-MPC with and without FTO. For USV path 1 following, the maximum difference is reduced by 36.97% for the track error and by 23.03% for the controller input rudder angle. This allows the USV to follow the desired path more easily and efficiently with the proposed QiH-MPC with FTO. The standard deviation of USV path 2 following is reduced by 64.84% for the heading angle error and by 22.02% for the heading angle velocity with the proposed QiH-MPC with FTO. This contributes to the stability of the target paths. Further quantitative performance data can be found in Tables 4 and 5.

7. Conclusions

This paper presents a QiH-MPC with a fixed-time disturbance observer for USV path following, taking into account the accuracy and robustness of the USV under wind and wave disturbances. The QiH-MPC provides convergence performance under long prediction horizons and reduces the computing load. The advantages of the algorithm are demonstrated from both theoretical and simulation aspects. Specifically, the proposed algorithm results in a 36% reduction in the maximum track error and a more than 23% reduction in the maximum difference of the controller input rudder angle. Furthermore, there is a reduction of over 64% in the standard deviation of the course angle error and of more than 22% in the standard deviation of the course angular velocity. These improvements enable the USV to track the desired path with greater efficiency and accuracy.

The work presented in this paper, however, is subject to certain limitations. Firstly, the model introduced here is not suitable for tracking paths with significant curvature due to its linearization approximation. Furthermore, the proposed disturbance observer proves ineffective in compensating for colored noise within wave disturbances. Therefore, future steps will consider real marine environments encompassing complex desired paths, diverse wave disturbances and multiple obstacle scenarios.

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