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Wideband Vibro-Acoustic Coupling Investigation in Three Dimensions Using Order-Reduced Isogeometric Finite Element/Boundary Element Method

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Abstract: This study introduces an innovative model-order reduction (MOR) technique that integrates boundary element and finite element methodologies, streamlining the analysis of wideband vibro-acoustic interactions within aquatic and aerial environments. The external acoustic phenomena are efficiently simulated via the boundary element method (BEM), while the finite element method (FEM) adeptly captures the dynamics of vibrating thin-walled structures. Furthermore, the integration of isogeometric analysis within the finite element/boundary element framework ensures geometric integrity and maintains high-order continuity for Kirchhoff–Love shell models, all without the intermediary step of meshing. Foundational to our reduced-order model is the application of the second-order Arnoldi method coupled with Taylor expansions, effectively eliminating the frequency dependence of system matrices. The proposed technique significantly enhances the computational efficiency of wideband vibro-acoustic coupling analyses, as demonstrated through numerical simulations.

Keywords: wideband; FEM/BEM; fluid–structure interaction; vibro-acoustic; isogeometric; model-order reduction



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1. Introduction

In a multitude of engineering domains, the interaction between the vibrations of shell-like structures and acoustic fields is a common occurrence. To accurately evaluate these interactions, a robust and credible numerical simulation methodology is paramount. The synergy of the finite element method (FEM) [1–4] and the boundary element method (BEM) emerged as an efficacious strategy for tackling vibro-acoustic interaction challenges [5–11]. While FEM boasts a sophisticated treatment of structural dynamics, BEM distinguishes itself by inherently fulfilling the Sommerfeld radiation condition at the far field, necessitating only the discretization of the boundary, as defined by the structure. This attribute renders BEM particularly well suited for addressing unbounded problems, such as those prevalent in the realm of external acoustic research [12,13]. At the interface where structural and acoustic realms converge, the continuity of pressure applied to the structures and normal velocity are maintained as critical surface boundary conditions.

In real-world acoustic scenarios, the stimulating sound wave can encompass a broad spectrum of frequencies. To mitigate extensive computational demands, model-order reduction (MOR) methodologies are employed to construct a low-order surrogate of the actual system, and they are specifically tailored for wide-band frequency analyses. The second-order Arnoldi algorithm (SOAR) [14] is a sophisticated and effective MOR technique for linear systems. This algorithm facilitates the development of a reduced-order model (ROM) by generating orthogonal bases from the full-order model (FOM) through second-order Arnoldi processes, subsequently projecting the model into a second-order Krylov subspace. The merits of SOAR include rapid convergence and the retention of the original model's matrix properties and second-order structural integrity.

While SOAR is directly applicable to FEM subsystems, its utility is limited when it comes to BEM subsystems due to the frequency-dependent nature of Green's functions, which require frequent recalculations of BEM matrix entries. However, by decoupling the frequency-varying components from Green's functions and employing series expansions to approximate integrals, we can navigate around this limitation [15–17]. Thereafter, SOAR can be adeptly utilized to formulate orthonormal bases that are liberated from frequency constraints, thereby enabling reduced-order simulations that are both efficient and accurate.

The adoption of isogeometric analysis, specifically through the isogeometric boundary element method (IGABEM) [18–22] and the isogeometric finite element method (IGAFEM) [23–26], has been instrumental in enhancing the precision and efficacy of our analytical procedures. This approach seamlessly integrates computer-aided design (CAD) with computer-aided engineering (CAE), thereby offering substantial advantages over conventional FEM and BEM techniques that rely on Lagrangian polynomial basis functions [27,28]. A key distinction lies in IGAFEM/BEM's adherence to the C^1 continuity criterion, which is essential for the thin-walled Kirchhoff theory [29,30], ensuring a high degree of geometric fidelity [31,32]. This methodology eliminates the need for a cumbersome meshing process, allowing for simulations to be executed directly from CAD models. This feature is particularly beneficial in scenarios involving fluid–structure interaction analyses, where the data exchange between CAD and CAE platforms is paramount [33–36].

In this study, we integrate MOR into the IGAFEM/BEM framework for thin-shell structures engaged in acoustic wave interactions. By employing BEM, we partition the boundary integral equation (BIE), while FEM is utilized to discretize the weak form associated with structural vibrations. The reduced bases essential for MOR are crafted through the SOAR method, and frequency dependence is mitigated via Taylor series expansions.

Here are the main novelties of this paper: 1. We innovatively introduced a formulation of MOR that leverages frequency decoupling and the SOAR algorithm, and it is specifically tailored for vibro-acoustic analysis within the FEM/BEM framework. This represents a significant advancement in the capability to analyze complex vibro-acoustic interactions. 2. Our research innovatively applied series expansion for the frequency decoupling of vibro-acoustic-coupled systems, integrated with IGAFEM/BEM and Loop subdivision surfaces. This approach not only ensures high-fidelity modeling, but also significantly boosts computational efficiency.

The subsequent sections of this paper are meticulously organized as follows: Section 2 delineates the IGAFEM/BEM formulation for fluid–structure vibro-acoustic systems, establishing a solid foundation for our analysis. Section 3 delves into the formulation of MOR, leveraging SOAR and Taylor expansions to enhance computational efficiency. Section 4 showcases a comprehensive numerical analysis of the MOR's application to underwater vibro-acoustic coupling, demonstrating its efficacy. Finally, Section 5 encapsulates our findings and concludes this study.

2. IGAFEM/BEM Formulation for Vibro-Acoustic Analysis

Figure 1 illustrates a common scenario in acoustic–structure interactions: an elastic thin-shell structure immersed in an infinite fluid domain. This setup is prevalent in various engineering applications and poses significant challenges in terms of vibration and noise control. The system can be conceptualized as consisting of two distinct regions: the acoustic domain Ω_f , which encompasses the fluid; and the structural domain Ω_s . The interface between these two domains is denoted by Γ_{sf} . The shell is filled with air, Ω_a , which, due to its significantly lower density compared to the structure, can be neglected in the analysis.

On the Boundaries Γ_s and Γ_f , the unit normal vectors are represented by n_s and n_f , respectively. Notably, at the Coupling Interface Γ_{sf} , these vectors are opposite in direction, i.e., $n_s = -n_f$. In the realm of vibro-acoustic coupling, the acoustic wave propagation within the fluid is governed by the Helmholtz equation, while the mechanical behavior of the thin-shell structure is dictated by the Kirchhoff–Love shell theory. The structure is

excited by an incoming plane wave f_p with angular frequency ω and an external mechanical load f_s .

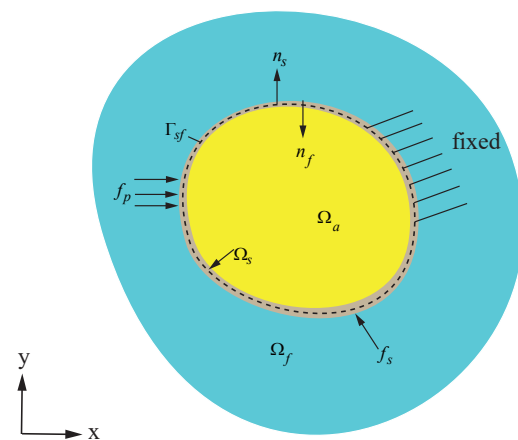


Figure 1. System of vibro-acoustic interaction. The dashed line represents the shell structure's surface. The sound pressure of incoming plane sound wave is f_p , and the external mechanical load is f_s .

2.1. Geometric Modeling Using Subdivision Surfaces

In this study, we selected subdivision surfaces, with a particular focus on Loop subdivision surfaces, as our CAD modeling approach. This choice was motivated by their ability to generate intricate, water-tight geometries that can accommodate any topology. By employing the quadrissection method on the components and aligning the control mesh with spline basis functions, a smooth surface can be effortlessly crafted at any given point.

Figure 2b illustrates the resulting smooth surface, which is derived from a regular triangular element configuration, as depicted in Figure 2a. This configuration is anchored by twelve controlling vertices and is generated through the application of Equation (1):

$$\mathbf{x}_e(\theta_1, \theta_2) = \sum_{i=1}^{12} B_i(\theta_1, \theta_2) \mathbf{C}_i, \quad (1)$$

where $\mathbf{x}_e$ represents the Cartesian coordinates of a point within the Element Γ_e . The parametric coordinates, denoted by θ_1 and θ_2 , are constrained within the unit square, i.e., $(\theta_1, \theta_2) \in [0, 1]^2$. The position of the i -th vertex is represented by the Cartesian coordinates $\mathbf{C}_i$. Furthermore, the basis functions for the quadratic box-splines, which are instrumental in our modeling approach, are denoted by B_i [27].

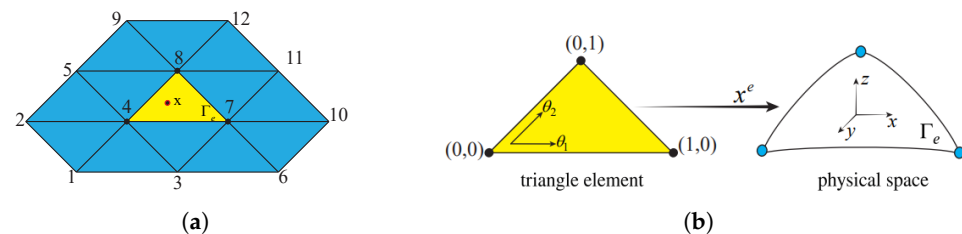


Figure 2. A Loop-subdivision patch for a regular triangle element. (a) A regular triangle element with 12 control vertices and (b) the correspondence between parametric and physical space.

The surface of an irregular triangle element within the Cartesian coordinate system presents a challenge for the fitting process previously outlined, as the basis functions are not explicitly defined. This issue can be circumvented by further subdividing the irregular pieces. Stam has contributed a significant solution [37], which leverages the eigendecomposition of the subdivision matrix for triangles with irregularly positioned

vertices. The process necessitates multiple refining stages, ensuring that the point of interest is eventually embedded within a patch composed of regular elements.

2.2. IGAFEM for Structural Vibrating Analysis

In our research, we employed Loop subdivision to construct geometrical representations that are integral to our analysis. The structural domain was meticulously discretized using IGAFEM, which leverages the same spline basis functions utilized in CAD, as detailed in [27]. This harmonization between the discretization and CAD processes ensures a seamless transition and consistency in our modeling approach. Consequently, by employing the Loop subdivision basis functions, we are able to estimate the displacement within the structural domain, as articulated in Equation (2):

$$\mathbf{u}_e = \sum_{i=1}^{12} B_i(\theta_1, \theta_2) \mathbf{u}_i^e, \quad (2)$$

where the nodal displacement parameters $\mathbf{u}_i^e$ represent the displacements associated with the i -th control vertex of the e -th element. This parameterization is crucial for capturing the mechanical behavior of the structure.

After discretizing the physical domain and assembling the individual element matrices, we can derive the resulting system of linear equations. This process is encapsulated in Equation (3), which forms the foundation of our FEM dynamic analysis.

$$(\mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M})\mathbf{u} = \mathbf{f} = \mathbf{f}_s + \mathbf{f}_p, \quad (3)$$

where the Structural Stiffness Matrix $\mathbf{K}$, Damping Matrix $\mathbf{C}$, and Mass Matrix $\mathbf{M}$ are all frequency-independent, which simplifies the analysis by reducing the need for recalculating these matrices across different frequencies. The external Mechanical Load Vector $\mathbf{f}_s$ and the Sound Pressure Load Vector $\mathbf{f}_p$ collectively constitute the Load Vector $\mathbf{f}$. It is important to note that the structural vibration also contributes to the Sound Pressure Load Vector $\mathbf{f}_p$. This interdependence means that Equation (3) cannot be solved directly due to the coupling between the structural response and the acoustic loads.

2.3. IGABEM for External Acoustic Analysis

In this subsection, we introduce the IGABEM formulation, which was specifically designed to evaluate the radiating and scattering of sound waves resulting from incident waves or structural vibrations. To achieve high precision in the assessment of acoustic pressure, we employed the boundary integral equation formulated based on the Burton–Miller method [38,39], as illustrated in Equation (4). This approach allows us to accurately capture the acoustic effects, providing a robust framework for our analysis.

$$\begin{aligned} & C(\mathbf{x})p(\mathbf{x}) + \alpha C(\mathbf{x})q(\mathbf{x}) + \int_{\Gamma_{sf}} [F(\mathbf{x}, \mathbf{y}) + \alpha H(\mathbf{x}, \mathbf{y})]p(\mathbf{y})d\Gamma_{sf}(\mathbf{y}) \\ &= \int_{\Gamma_{sf}} [G(\mathbf{x}, \mathbf{y}) + \alpha K(\mathbf{x}, \mathbf{y})]q(\mathbf{y})d\Gamma_{sf}(\mathbf{y}) + p_{\text{inc}}(\mathbf{x}) + \alpha \frac{\partial p_{\text{inc}}(\mathbf{x})}{\partial n(\mathbf{x})}, \end{aligned} \quad (4)$$

where the interaction coefficient, denoted by α , is defined differently depending on the Wave Number k . Specifically, α equals i for $k < 1$, and, for $k \geq 1$, it is given by $-i/k$. The constant $C(x)$ is set to $1/2$ when the Field Point $\mathbf{x}$ is located on the smoothed Boundary Γ_{sf} . The sound pressure at the Source Point $\mathbf{y}$ is denoted by $p(\mathbf{y})$, and its normal derivative by $q(\mathbf{y})$. The acoustic pressure of the incident plane wave is represented by $p_{\text{inc}}(\mathbf{x})$.

The fundamental functions G , F , K , and H , which play a crucial role in our formulation, are detailed in Equation (5). In this equation, the Distance r between the Source Point $\mathbf{y}$ and the Field Point $\mathbf{x}$ is expressed as the Euclidean distance $r = |\mathbf{x} - \mathbf{y}|$.

$$\begin{aligned} G(\mathbf{x}, \mathbf{y}) &= \frac{e^{ikr}}{4\pi r}, \\ F(\mathbf{x}, \mathbf{y}) &= -\frac{e^{ikr}}{4\pi r^2} (1 - ikr) \frac{\partial r}{\partial n(\mathbf{y})}, \\ K(\mathbf{x}, \mathbf{y}) &= -\frac{e^{ikr}}{4\pi r^2} (1 - ikr) \frac{\partial r}{\partial n(\mathbf{x})}, \\ H(\mathbf{x}, \mathbf{y}) &= \frac{e^{ikr}}{4\pi r^3} \left[(3 - 3ikr - k^2 r^2) \frac{\partial r}{\partial n(\mathbf{y})} \frac{\partial r}{\partial n(\mathbf{x})} + (1 - ikr) n_i(\mathbf{x}) n_i(\mathbf{y}) \right]. \end{aligned} \quad (5)$$

Through employing the Loop subdivision basis functions, we discretize the Sound Pressure p_e and its associated Flux q_e at a point (θ_1, θ_2) within the Element Γ_e . This discretization process is elegantly captured in Equation (6), which serves as a pivotal step in our numerical scheme. By leveraging these basis functions, we were able to achieve a high level of accuracy in representing the acoustic field across the element, thereby facilitating a robust analysis of the sound wave behavior.

$$\begin{aligned} p_e &= \sum_{i=1}^{12} B_i(\theta_1, \theta_2) p_i^e, \\ q_e &= \sum_{i=1}^{12} B_i(\theta_1, \theta_2) q_i^e, \end{aligned} \quad (6)$$

where the nodal Sound Pressure p_i^e and its normal Derivative q_i^e are defined at the i -th control vertex of the Element Γ_e . These quantities represent the acoustic pressure and the flux normal to the boundary at the respective nodal points, which are integral to our analysis.

The discretization process of the BIE in Equation (4) is executed through Equation (6), which effectively partitions the BIE into manageable components. The result of this operation is presented in Equation (7), providing a discretized form of the BIE that is well suited for numerical computation and analysis.

$$\begin{aligned} C(\mathbf{x})p(\mathbf{x}) + \alpha C(\mathbf{x})q(\mathbf{x}) + \sum_{e=1}^{N_e} \sum_{i=1}^{12} p_i^e \int_{\Gamma_e} B_i(\theta_1, \theta_2) [F(\mathbf{x}, \mathbf{y}) + \alpha H(\mathbf{x}, \mathbf{y})] d\Gamma_e \\ = \sum_{e=1}^{N_e} \sum_{i=1}^{12} q_i^e \int_{\Gamma_e} B_i(\theta_1, \theta_2) [G(\mathbf{x}, \mathbf{y}) + \alpha K(\mathbf{x}, \mathbf{y})] d\Gamma_e + p_{\text{inc}}(\mathbf{x}) + \alpha \frac{\partial p_{\text{inc}}(\mathbf{x})}{\partial n(\mathbf{x})}. \end{aligned} \quad (7)$$

Equation (7), which encapsulates the discretized boundary integral equations, was further refined into a matrix-vector format, as shown in Equation (8). This transformation is crucial for our numerical solution strategy. It involves ensuring that the discretized equations accurately represent the physical behavior at the chosen collocation sites. By carefully assembling these equations into a comprehensive matrix-vector equation, we create a structured framework that facilitates the computational process.

This approach not only simplifies the implementation of numerical methods, but also enhances the stability and efficiency of the solution. The matrix-vector formulation allows for a direct application of iterative solvers, which is particularly advantageous when dealing with large-scale problems in solid mechanics and engineering mechanics.

$$\mathbf{H}\mathbf{p} - \mathbf{G}\mathbf{q} = \mathbf{p}_{\text{inc}}, \quad (8)$$

where the Coefficient Matrices $\mathbf{H}$ and $\mathbf{G}$, which are of size $\mathbb{C}^{N \times N}$, exhibit frequency dependency, asymmetry, and full rank. These matrices play a pivotal role in the matrix-vector equation that we have derived. At the collocation points, the acoustic pressure and its corresponding normal derivative are represented by the Column Vectors $\mathbf{p}$ and $\mathbf{q}$, respectively. Additionally, the incoming wave is denoted by $\mathbf{p}_{\text{inc}}$.

It is important to note that Equation (8) cannot be solved directly at this stage, as the values of $\mathbf{p}$ and $\mathbf{q}$ remain unknown. This highlights the complexity of the problem and the necessity for an iterative approach or additional boundary conditions to obtain a solvable system.

2.4. IGAFEM/BEM Interaction Analysis

Equations (3) and (8) are not standalone-solvable due to the indeterminate nature of the variables $\mathbf{u}$, $\mathbf{f}_p$, $\mathbf{p}$, and $\mathbf{q}$. These equations necessitate a unified approach, leveraging boundary conditions to reframe them into a solvable system. With a focus on Equation (3), we can reconfigure the expression for the Acoustic-Loading Force $\mathbf{f}_p$, as depicted in Equation (9).

$$\begin{aligned}\mathbf{f}_p &= \mathbf{C}_{sf}\mathbf{p}, \\ \mathbf{C}_{sf} &= \int_{\Gamma_{sf}} \mathbf{B}_s^T \mathbf{n} \mathbf{B}_f d\Gamma_{sf},\end{aligned}\quad (9)$$

where $\mathbf{B}_s$ and $\mathbf{B}_f$ represent the interpolation functions specific to the structural and fluid domains, respectively. The Matrix $\mathbf{n}$ encapsulates the unit normal vectors $\mathbf{n}$ along the fluid–structure interface Γ_{sf} . Utilizing these definitions, we can adeptly reformulate Equation (3), presenting it anew as Equation (10).

$$(\mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M})\mathbf{u} = \mathbf{f}_s + \mathbf{C}_{sf}\mathbf{p}. \quad (10)$$

Through a process of manipulation and transformation, Equation (10) was modified to yield Equation (11).

$$\begin{aligned}\mathbf{u} &= \mathbf{A}^{-1}(\mathbf{f}_s + \mathbf{C}_{sf}\mathbf{p}), \\ \mathbf{A} &= \mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M}.\end{aligned}\quad (11)$$

At the pivotal junction where the structural and fluid domains intersect, we imposed the essential continuity condition [40]. This condition is captured in Equation (12), which serves as a bridge between the disparate physical domains, ensuring seamless integration of their respective behaviors.

$$\begin{aligned}\mathbf{q} &= \frac{\partial \mathbf{p}}{\partial \mathbf{n}} = i\omega\rho_f\mathbf{v}_f, \\ \mathbf{v}_f &= -i\omega\mathbf{C}_{fs}\mathbf{u}, \\ \mathbf{C}_{fs} &= \mathbf{C}_{sf}^T,\end{aligned}\quad (12)$$

where $\mathbf{v}_f$ symbolizes the fluid’s normal velocity vector, while ρ_f denotes its density. By seamlessly integrating Equation (12), which embodies the continuity condition, into the IGABEM framework, as articulated in Equation (8), we derived the comprehensive coupling equation, which is presented as Equation (13).

$$\mathbf{H}\mathbf{p} - G\omega^2\rho_f\mathbf{C}_{fs}\mathbf{u} = \mathbf{p}_{inc}. \quad (13)$$

By combining Equations (11) and (13), we meticulously derived the coupling system equation, which encapsulates the intricate interplay between the structural and fluid dynamics. This unified formulation is meticulously presented in Equation (14).

$$\begin{aligned}(\mathbf{H} - G\mathbf{W}\mathbf{C}_{sf})\mathbf{p} &= G\mathbf{W}\mathbf{f}_s + \mathbf{p}_{inc}, \\ \mathbf{W} &= \omega^2\rho_f\mathbf{C}_{fs}\mathbf{A}^{-1}.\end{aligned}\quad (14)$$

In our current research, we employed the generalized minimal residual method (GMRES) to solve the reduced system equations. GMRES is a powerful iterative technique

known for its efficiency in handling large systems, which is particularly advantageous in our context. Once the Pressure Vector $\mathbf{p}$ was obtained, we could similarly evaluate the Displacement Vector $\mathbf{u}$.

While the techniques discussed can effectively model vibro-acoustic coupling at a specific frequency, the broad excitation frequency spectrum encountered in engineering applications necessitates solving problems on a large scale, which can be quite labor-intensive. To address this challenge, MOR will be applied in the subsequent sections to accelerate the analysis across a wide frequency range.

3. Model-Order Reduction for Wideband Vibro-Acoustic Coupling Analysis

In the field of computational mathematics, the wideband, reduced-order calculation for FEM using SOAR has gained recognition. The essence of SOAR lies in constructing a second-order reduced system within the framework of a projection method tailored for linear systems. The concept of projection can be visualized as employing an alternate state vector confined to a subspace to approximate the state vector of the original system [41].

This study proposes the application of the IGAFEM/BEM model-order reduction to the wideband vibro-acoustic interaction system. Our findings indicate that this approach yields positive results, significantly enhancing the efficiency of wideband calculations.

In our approach, we employ Taylor expansions to eliminate the frequency dependency inherent in the IGABEM coefficient matrix. This technique allows us to express the Green's function in a more manageable form. Specifically, Equation (15) utilizes this expansion to represent the term e^{ikr} , where k is the wave number and r represents the distance. By doing so, we transform the frequency-dependent component of the Green's function into a series that can be more easily handled in our computations.

$$e^{ikr} = e^{ik_0 r} \sum_{m=0}^{\infty} \frac{(ir)^m (k - k_0)^m}{m!}, \quad (15)$$

where we establish a fixed-frequency expansion point, represented by the symbol k_0 . Should the incoming wave advance parallel to the j -axis, Equation (16) delineates the expanded form of the latter components featured within Equation (7) on its right-hand side.

$$p_{\text{inc}}(\mathbf{x}) + \alpha \frac{\partial p_{\text{inc}}(\mathbf{x})}{\partial n(\mathbf{x})} = P_{\text{inc}}(\mathbf{x}) = e^{ikx_j} \left(1 + \alpha ik \frac{\partial x_j}{\partial n(\mathbf{x})} \right), \quad (16)$$

where x_j is designated as the coordinate along the j -axis, with j taking values of 1, 2, or 3 corresponding to the three principal axes. By substituting Equations (15) and (16) into our foundational equation, Equation (7), we arrive at an alternative expression, which is presented as Equation (17).

$$\begin{aligned} C(\mathbf{x})p(\mathbf{x}) + \alpha C(\mathbf{x})q(\mathbf{x}) + \sum_{m=0}^{\infty} \frac{(k - k_0)^m}{m!} \left[\left(I_{f1}^m - I_s^m - I_{k1}^m + I_{h1}^m \right) + k \left(I_{f2}^m - I_{k2}^m + I_{h2}^m \right) + k^2 I_{h3}^m \right] \\ = \sum_{m=0}^{\infty} \frac{(k - k_0)^m}{m!} [P_1 + \alpha ik P_2], \end{aligned} \quad (17)$$

in which

$$\begin{aligned} I_s^m &= \int_{\Gamma} \frac{e^{ik_0 r} (ir)^m}{4\pi r} q(\mathbf{y}) \, d\Gamma(\mathbf{y}), \\ I_{f1}^m &= \int_{\Gamma} \frac{-e^{ik_0 r} (ir)^m}{4\pi r^2} \frac{\partial r}{\partial n(\mathbf{y})} p(\mathbf{y}) \, d\Gamma(\mathbf{y}), \\ I_{f2}^m &= \int_{\Gamma} \frac{e^{ik_0 r} (ir)^{m+1}}{4\pi r^2} \frac{\partial r}{\partial n(\mathbf{y})} p(\mathbf{y}) \, d\Gamma(\mathbf{y}), \end{aligned}$$

$$\begin{aligned}
I_{k1}^m &= \int_{\Gamma} \frac{-\alpha e^{ik_0 r} (ir)^m}{4\pi r^2} \frac{\partial r}{\partial n(\mathbf{x})} q(\mathbf{y}) d\Gamma(\mathbf{y}), \\
I_{k2}^m &= \int_{\Gamma} \frac{\alpha e^{ik_0 r} (ir)^{m+1}}{4\pi r^2} \frac{\partial r}{\partial n(\mathbf{x})} q(\mathbf{y}) d\Gamma(\mathbf{y}), \\
I_{h1}^m &= \int_{\Gamma} \frac{\alpha e^{ik_0 r} (ir)^m}{4\pi r^3} \left[3 \frac{\partial r}{\partial n(\mathbf{x})} \frac{\partial r}{\partial n(\mathbf{y})} + n_i(\mathbf{x}) n_i(\mathbf{y}) \right] p(\mathbf{y}) d\Gamma(\mathbf{y}), \\
I_{h2}^m &= \int_{\Gamma} \frac{\alpha e^{ik_0 r} (ir)^{m+1}}{4\pi r^3} \left[-3 \frac{\partial r}{\partial n(\mathbf{x})} \frac{\partial r}{\partial n(\mathbf{y})} - n_i(\mathbf{x}) n_i(\mathbf{y}) \right] p(\mathbf{y}) d\Gamma(\mathbf{y}), \\
I_{h3}^m &= \int_{\Gamma} \frac{-\alpha e^{ik_0 r} (ir)^m}{4\pi r} \frac{\partial r}{\partial n(\mathbf{x})} \frac{\partial r}{\partial n(\mathbf{y})} p(\mathbf{y}) d\Gamma(\mathbf{y}), \\
P_1 &= e^{ik_0 x_j} (ix_j)^m, \\
P_2 &= e^{ik_0 x_j} (ix_j)^m \frac{\partial x_j}{\partial n(\mathbf{x})}.
\end{aligned}$$

Through employing the Loop subdivision basis functions for the discretization process, we transformed Equation (17) into its matrix representation, yielding Equation (18). This transition to the matrix form is pivotal for the numerical solution of the problem at hand.

$$\sum_{m=0}^{\infty} \frac{(k - k_0)^m}{m!} \left\{ \left[\mathbf{I}_{p1}^m + k \mathbf{I}_{p2}^m + k^2 \mathbf{I}_{p3}^m \right] \mathbf{p} - \left[\mathbf{I}_{q1}^m + k \mathbf{I}_{q2}^m \right] \mathbf{q} \right\} = \sum_{m=0}^{\infty} \frac{(k - k_0)^m}{m!} [\mathbf{P}_1 + \alpha i k \mathbf{P}_2], \quad (18)$$

in which the coefficient matrices, denoted as $\mathbf{I}_{p1}^m$, $\mathbf{I}_{p2}^m$, $\mathbf{I}_{p3}^m$, $\mathbf{I}_{q1}^m$, and $\mathbf{I}_{q2}^m$, are elements of the complex plane $\mathbb{C}^{N \times N}$ and are independent of frequency. This property implies that, for wideband-frequency problems, these matrices require computation only once, thereby simplifying the process. However, in large-scale scenarios, the direct application of GMRES to solve Equation (18) presents significant challenges. The primary obstacle is the substantial memory requirement, which scales with the order of $O(5(M+1)N^2)$, where M represents the number of truncation components and the matrices involved are asymmetric and fully dense.

Through utilizing SOAR, we generated a set of orthonormal bases, denoted as $\mathbf{Q}_n^{\text{FE}}$, which reside within the complex domain of Dimension $\mathbb{C}^{N \times n}$. These bases span the second-order Krylov subspace, defined as $\mathcal{G}_n(\tilde{\mathbf{A}}, \tilde{\mathbf{B}}; \mathbf{r}_0)$, where n is significantly smaller than N and is specifically tailored for the finite element (FE) system. This construction is detailed in Equation (19).

$$\mathcal{G}_n(\tilde{\mathbf{A}}^{\text{FE}}, \tilde{\mathbf{B}}^{\text{FE}}; \mathbf{r}_0^{\text{FE}}) = \text{span}\{\mathbf{Q}_n^{\text{FE}}\} = \text{span}\{\mathbf{r}_0^{\text{FE}}, \mathbf{r}_1^{\text{FE}}, \mathbf{r}_2^{\text{FE}}, \dots, \mathbf{r}_{n-1}^{\text{FE}}\}, \quad (19)$$

in which

$$\begin{aligned}
\tilde{\mathbf{A}}^{\text{FE}} &= -\tilde{\mathbf{K}}^{-1} \tilde{\mathbf{C}}, \\
\tilde{\mathbf{B}}^{\text{FE}} &= -\tilde{\mathbf{K}}^{-1} \tilde{\mathbf{M}}, \\
\mathbf{r}_0^{\text{FE}} &= \tilde{\mathbf{K}}^{-1} \mathbf{f}_s, \\
\mathbf{r}_1^{\text{FE}} &= \tilde{\mathbf{A}}^{\text{FE}} \mathbf{r}_0, \\
\mathbf{r}_j^{\text{FE}} &= \tilde{\mathbf{A}}^{\text{FE}} \mathbf{r}_{j-1} + \tilde{\mathbf{B}}^{\text{FE}} \mathbf{r}_{j-2}, \quad \text{for } j \geq 2, \\
\tilde{\mathbf{K}} &= \mathbf{K} + s_0 \mathbf{C} + s_0^2 \mathbf{M}, \\
\tilde{\mathbf{C}} &= \mathbf{C} + 2s_0 \mathbf{M}, \\
\tilde{\mathbf{M}} &= \mathbf{M}.
\end{aligned}$$

For the construction of a localized orthonormal basis, we can strategically leverage the pre-computed inverse, $\tilde{\mathbf{K}}^{-1}$, across each subsequent iteration, thereby necessitating only a solitary evaluation. Prior to the iterative computation of the Orthonormal Basis $\mathbf{Q}_n^{\text{FE}}$, this

matrix can be efficiently decomposed through methods such as LU decomposition. The selection of the expansion point is pivotal and is set at $s_0 = -i\omega_0$, with ω_0 denoting the angular frequency's expansion point. As delineated in Equation (20), the initially computed Displacement Vector $\mathbf{u}$ is subsequently projected onto the subspace $\mathcal{G}_n(\tilde{\mathbf{A}}, \tilde{\mathbf{B}}; \mathbf{r}_0)$, which is spanned by the orthonormal basis $\mathbf{Q}_n^{\text{FE}}$.

$$\mathbf{u} \approx \mathbf{Q}_n^{\text{FE}} \mathbf{u}_n, \quad (20)$$

where we identify a vector of Dimension n , represented as $\mathbf{u}_n$, within the confines of the subspace $\mathcal{G}_n(\tilde{\mathbf{A}}, \tilde{\mathbf{B}}; \mathbf{r}_0)$. The derivation of the second-order ROM from the FEM equation of Equation (3) is captured in Equation (21). This is achieved by the multiplication of the FEM equation by the transpose of the orthonormal basis matrix, $(\mathbf{Q}_n^{\text{FE}})^T$, subsequent to the incorporation of Equation (20) into the original Equation (3).

$$\mathbf{Z}_n \mathbf{u}_n = \mathbf{f}_{s,n} + \mathbf{f}_{p,n}, \quad (21)$$

in which

$$\begin{aligned} \mathbf{Z}_n &= \mathbf{K}_n + i\omega \mathbf{C}_n - \omega^2 \mathbf{M}_n, \\ \mathbf{K}_n &= (\mathbf{Q}_n^{\text{FE}})^T \mathbf{K} \mathbf{Q}_n^{\text{FE}}, \\ \mathbf{C}_n &= (\mathbf{Q}_n^{\text{FE}})^T \mathbf{C} \mathbf{Q}_n^{\text{FE}}, \\ \mathbf{M}_n &= (\mathbf{Q}_n^{\text{FE}})^T \mathbf{M} \mathbf{Q}_n^{\text{FE}}, \\ \mathbf{f}_{s,n} &= (\mathbf{Q}_n^{\text{FE}})^T \mathbf{f}_s, \\ \mathbf{f}_{p,n} &= (\mathbf{Q}_n^{\text{FE}})^T \mathbf{f}_p. \end{aligned}$$

The order of the orthonormal basis and the number of frequency expansion points are pivotal parameters that significantly influence both the precision and computational efficiency of our analysis. In this investigation, we employed an adaptive computational technique, which intelligently determines the requisite order of the orthonormal basis and the corresponding number of expansion points, thereby streamlining the process.

In this work, we forged a link between the nodal values of acoustic pressure and the associated flux, thereby integrating the FEM equation that encapsulates structural vibrations with the BEM equation that are pertinent to acoustic analysis. Through a transformative modification from Equation (9) to its reduced counterpart, Equation (21), we derived Equation (22), which integrates the dynamics of structural vibrations with acoustic analysis.

$$\begin{aligned} \mathbf{u}_n &= \mathbf{Z}_n^{-1} \mathbf{f}_{s,n} + \mathbf{Z}_n^{-1} \mathbf{f}_{p,n} \\ &= \mathbf{Z}_n^{-1} (\mathbf{Q}_n^{\text{FE}})^T \mathbf{f}_s + \mathbf{Z}_n^{-1} (\mathbf{Q}_n^{\text{FE}})^T \mathbf{C}_{sf} \mathbf{p}, \quad \mathbf{C}_{sf} \in \mathbb{R}^{N \times N}, \quad \mathbf{p} \in \mathbb{C}^N. \end{aligned} \quad (22)$$

By substituting the reduced-order formulation encapsulated in Equation (22) into our displacement approximation equation, Equation (20), we derived an alternative expression for the displacement vector. This refined formulation is meticulously detailed in Equation (23), offering a more nuanced perspective on the system's dynamic response.

$$\begin{aligned} \mathbf{u} &\approx \mathbf{Q}_n^{\text{FE}} \mathbf{Z}_n^{-1} \mathbf{f}_{s,n} + \mathbf{Q}_n^{\text{FE}} \mathbf{Z}_n^{-1} \mathbf{C}_{sf,n} \mathbf{p}, \\ \mathbf{C}_{sf,n} &= (\mathbf{Q}_n^{\text{FE}})^T \mathbf{C}_{sf}, \quad \mathbf{C}_{sf,n} \in \mathbb{C}^{n \times N}. \end{aligned} \quad (23)$$

Upon incorporating the impedance boundary condition articulated in Equation (24)

$$\mathbf{q} = \omega^2 \rho_f \mathbf{C}_{fs} \mathbf{u}, \quad (24)$$

we proceeded to derive an alternative formulation by integrating the displacement vector expression from Equation (23). This integration leads to the development of Equation (25), which offers a refined insight into boundary dynamics.

$$\begin{aligned} \mathbf{q} &= \mathbf{E} \mathbf{f}_{s,n} + \mathbf{E} \mathbf{C}_{sf,n} \mathbf{p}, \\ \mathbf{E} &= \omega^2 \rho_f \mathbf{C}_{fs} \mathbf{Q}_n^{\text{FE}} \mathbf{Z}_n^{-1}. \end{aligned} \quad (25)$$

Equation (26) encapsulates the essence of the IGAFEM/BEM coupling, which is achieved by the substitution of Equation (25) into the foundational Equation (18). This formulation is particularly noteworthy for its coefficient matrices, which remain steadfastly independent of frequency variations.

$$\begin{aligned} & \sum_{m=0}^{\infty} \frac{(k - k_0)^m}{m!} \left\{ \left[\mathbf{I}_{p1}^m + k \mathbf{I}_{p2}^m + k^2 \mathbf{I}_{p3}^m - \mathbf{I}_{q1}^m \mathbf{E} \mathbf{C}_{sf,n} - k \mathbf{I}_{q2}^m \mathbf{E} \mathbf{C}_{sf,n} \right] \mathbf{p} \right\} \\ &= \sum_{m=0}^{\infty} \frac{(k - k_0)^m}{m!} \left\{ [\mathbf{P}_1 + \alpha i k \mathbf{P}_2] + [\mathbf{I}_{q1}^m + k \mathbf{I}_{q2}^m] \mathbf{E} \mathbf{f}_{s,n} \right\}. \end{aligned} \quad (26)$$

In the context of the interaction described by Equation (26), we constructed a sequence of orthonormal bases, denoted as $\mathbf{Q}_n^{\text{FE/BE}}$, within the complex Vector Space $\mathbb{C}^{N \times n}$. These bases span the Krylov subspace $\mathcal{G}_n(\tilde{\mathbf{A}}, \tilde{\mathbf{B}}; \mathbf{r}_0)$, where n is a small fraction of N . This construction is a critical step in the development of our reduced-order models, as depicted in Equation (27).

$$\mathcal{G}_n(\tilde{\mathbf{A}}^{\text{FE/BE}}, \tilde{\mathbf{B}}^{\text{FE/BE}}; \mathbf{r}_0^{\text{FE/BE}}) = \text{span}\{\mathbf{Q}_n^{\text{FE/BE}}\} = \text{span}\{\mathbf{r}_0^{\text{FE/BE}}, \mathbf{r}_1^{\text{FE/BE}}, \mathbf{r}_2^{\text{FE/BE}}, \dots, \mathbf{r}_{n-1}^{\text{FE/BE}}\}, \quad (27)$$

in which

$$\begin{aligned} \tilde{\mathbf{A}}^{\text{FE/BE}} &= -(\mathbf{I}_{p1}^0)^{-1} \mathbf{I}_{p2}^0, \\ \tilde{\mathbf{B}}^{\text{FE/BE}} &= -(\mathbf{I}_{p1}^0)^{-1} \mathbf{I}_{p3}^0, \\ \mathbf{r}_1^{\text{FE/BE}} &= \tilde{\mathbf{A}}^{\text{FE/BE}} \mathbf{r}_0^{\text{FE/BE}}, \\ \mathbf{r}_j^{\text{FE/BE}} &= \tilde{\mathbf{A}}^{\text{FE/BE}} \mathbf{r}_{j-1}^{\text{FE/BE}} + \tilde{\mathbf{B}}^{\text{FE/BE}} \mathbf{r}_{j-2}^{\text{FE/BE}}, \quad \text{for } j \geq 2. \end{aligned}$$

Upon the specification of k as k_0 within Equation (26), Equation (28) is consequently derived, highlighting $\mathbf{E}_0$ as the impedance Matrix $\mathbf{E}$ precisely at this critical frequency.

$$\mathbf{r}_0^{\text{FE/BE}} = \left[\mathbf{I}_{p1}^0 + k_0 \mathbf{I}_{p2}^0 + k_0^2 \mathbf{I}_{p3}^0 - (\mathbf{I}_{q1}^0 + k_0 \mathbf{I}_{q2}^0) \mathbf{E}_0 \mathbf{C}_{sf,n} \right]^{-1} \left[\mathbf{P}_1 + \alpha i k_0 \mathbf{P}_2 + (\mathbf{I}_{q1}^0 + k_0 \mathbf{I}_{q2}^0) \mathbf{E}_0 \mathbf{f}_{s,n} \right], \quad (28)$$

Advancing further, by employing the approximation technique that utilizes the Vector $\mathbf{p}_n$, a dimension- n element within the Krylov subspace $\mathcal{G}_n(\tilde{\mathbf{A}}, \tilde{\mathbf{B}}; \mathbf{r}_0)$, which is spanned by the orthonormal basis $\mathbf{Q}_n^{\text{FE/BE}}$, we arrive at the refined formula presented in Equation (29).

$$\mathbf{p} \approx \mathbf{Q}_n^{\text{FE/BE}} \mathbf{p}_n, \quad (29)$$

The formulation of the reduced-order system, as articulated in Equation (30), emerged through an adept manipulation of Equation (26). This involves the insertion of Equation (29) into the original Equation (26), which is followed by the multiplication by the transpose of the orthonormal basis matrix $(\mathbf{Q}_n^{\text{FE/BE}})^T$. This nuanced approach yields a system that encapsulates the reduced complexity while retaining the core dynamics.

$$\left\{ \sum_{m=0}^{\infty} \frac{(k - k_0)^m}{m!} \left[\mathbf{I}_{p1}^{m,n} + k \mathbf{I}_{p2}^{m,n} + k^2 \mathbf{I}_{p3}^{m,n} - \mathbf{I}_{q1}^{m,n} \mathbf{E} \mathbf{C}_{sf,nm} - k \mathbf{I}_{q2}^{m,n} \mathbf{E} \mathbf{C}_{sf,nm} \right] \right\} \mathbf{p}_n$$

$$= \sum_{m=0}^{\infty} \frac{(k - k_0)^m}{m!} \left[(\mathbf{P}_{1,n} + \alpha i k \mathbf{P}_{2,n}) + (\mathbf{I}_{q1}^{m,n} + k \mathbf{I}_{q2}^{m,n}) \mathbf{E} \mathbf{f}_{s,n} \right], \quad (30)$$

in which

$$\begin{aligned} \mathbf{I}_{p1}^{m,n} &= [\mathbf{Q}_n^{\text{FE/BE}}]^T \mathbf{I}_{p1}^m \mathbf{Q}_n^{\text{FE/BE}}, \\ \mathbf{I}_{p2}^{m,n} &= [\mathbf{Q}_n^{\text{FE/BE}}]^T \mathbf{I}_{p2}^m \mathbf{Q}_n^{\text{FE/BE}}, \\ \mathbf{I}_{p3}^{m,n} &= [\mathbf{Q}_n^{\text{FE/BE}}]^T \mathbf{I}_{p3}^m \mathbf{Q}_n^{\text{FE/BE}}, \\ \mathbf{I}_{q1}^{m,n} &= [\mathbf{Q}_n^{\text{FE/BE}}]^T \mathbf{I}_{q1}^m, \\ \mathbf{I}_{q2}^{m,n} &= [\mathbf{Q}_n^{\text{FE/BE}}]^T \mathbf{I}_{q2}^m, \\ \mathbf{P}_{1,n} &= [\mathbf{Q}_n^{\text{FE/BE}}]^T \mathbf{P}_1, \\ \mathbf{P}_{2,n} &= [\mathbf{Q}_n^{\text{FE/BE}}]^T \mathbf{P}_2, \\ \mathbf{C}_{sf,nm} &= \mathbf{C}_{sf,n} \mathbf{Q}_n^{\text{FE/BE}}. \end{aligned}$$

Equation (30) presents the reduced-order model derived from the IGAFEM/BEM full-order model, as delineated in Equation (14). Within this ROM, the coefficient matrices $\mathbf{I}_{p1}^{m,n}$, $\mathbf{I}_{p2}^{m,n}$, and $\mathbf{I}_{p3}^{m,n}$ are defined within the Complex Space $\mathbb{C}^{n \times n}$, with the dimension n being significantly smaller than N ($n \ll N$). This compact representation maintains the essential characteristics of the original IGAFEM/BEM system. By solving Equation (30), we accurately determine the Sound Pressure Vector $\mathbf{p}$. Subsequently, the Displacement Vector $\mathbf{u}$ is effortlessly computed utilizing the relationship established in Equation (23).

4. Numerical Examples

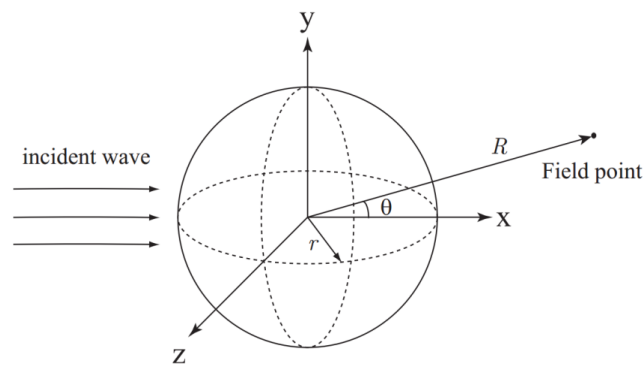
In this section, we present a pair of illustrative examples designed to reinforce the vibro-acoustic analysis conducted via the IGAFEM/BEM framework, and it is enhanced by the MOR technique. The structural material is characterized by a density of 7860 kg/m^3 , a Young's modulus of $2.1 \times 10^{11} \text{ N/m}^2$, and a Poisson's ratio of 0.3, showcasing that its mechanical properties are suitable for the analysis. For the acoustic medium, we selected water with a density of 1000 kg/m^3 and a wave propagation velocity of 1482 m/s , which are standard parameters for aquatic environments. Our computations were performed utilizing our proprietary Fortran code, ensuring both precision and efficiency. Adhering to the international system of units (SI), the numerical examples provided were grounded in a consistent and universally recognized system of measurement.

4.1. Elastic Spherical Shell under Incoming Plane Waves

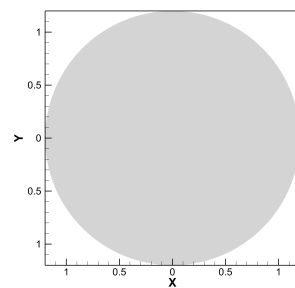
Building upon established analytical insights [42], this study presents a simulation of an elastic spherical shell submerged in water, with a radius of $r = 1.2 \text{ m}$ and a thickness of $t = 0.012 \text{ m}$. As depicted in Figure 3, the shell is subjected to the incidence of a plane wave, propagating in the x -direction with a unit amplitude. The geometric center of the shell coincides with the coordinate origin $(0, 0, 0)$, from which the sound pressure at various points is meticulously examined.

The distribution of sound pressure across the shell surface, examined at distinct frequencies of 200, 300, 400, and 500 Hz, is portrayed in Figure 4. A discernible axisymmetry in the acoustic pressure was observed from the contour plot, which is indicative of the shell's symmetric response to the incident wave. Furthermore, the contour patterns exhibited distinct phase variations with frequency, which is consistent with the expectation that the

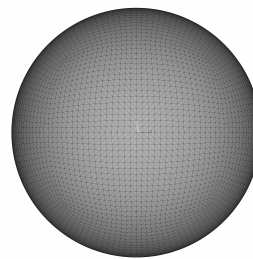
plane wave propagates along the x -axis. These observations underscore the frequency-dependent nature of the acoustic field's interaction with the shell structure.



(a) Spherical shell model with incoming wave.

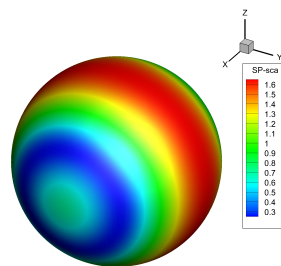


(b) Spherical shell coordinates.

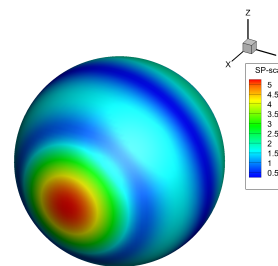


(c) Spherical shell meshing.

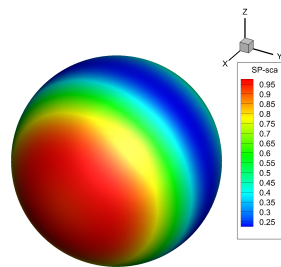
Figure 3. The spherical shell model.



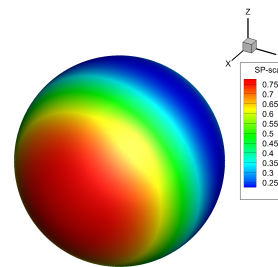
(a) At 200 Hz.



(b) At 300 Hz.



(c) At 400 Hz.



(d) At 500 Hz.

Figure 4. Sound pressure on the spherical shell surface at different frequencies. The shell Thickness $t = 0.012$ m and Radius $r = 1.2$ m.

In our comprehensive fluid–structure, vibro-acoustic analysis, we scrutinized the sound pressure at multiple discrete points. A comparison of the numerical result using IGAFEM/BEM and the analytical result, as depicted in Figure 5, revealed a remarkable concordance between the two, substantiating the precision and validity of IGAFEM/BEM. Moreover, the figure eloquently demonstrates a pronounced escalation in sound pressure at the frequencies of peak resonance. This observation underscores the system’s inherent frequency dependency, a critical aspect when conducting frequency-domain analyses of fluid–structure, vibro-acoustic interactions.

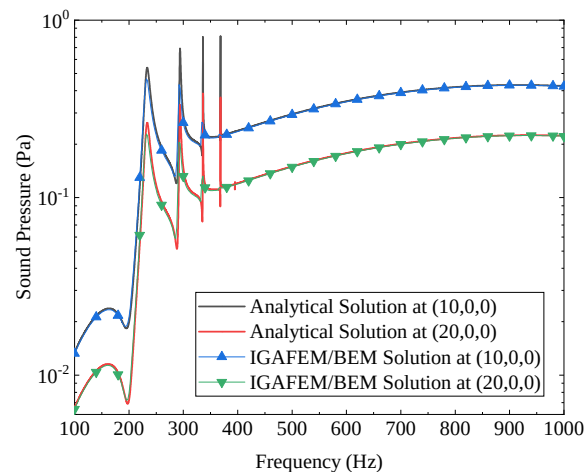


Figure 5. Sound pressure for the spherical shell model at points (10, 0, 0) and (20, 0, 0): a comparison of the analytical and IGAFEM/BEM results. The shell Thickness $t = 0.012$ m and Radius, $r = 1.2$ m.

Let us focus on the sound pressure at the specific coordinates (4, 0, 0). A comparison between the numerical outcomes yielded by the IGAFEM/BEM framework, which was enhanced by model-order reduction, and the corresponding analytical solutions is presented in Figure 6. The striking congruence between the numerical and analytical data validates the efficacy of our MOR-integrated approach. Moreover, the computational time for the IGAFEM/BEM simulation, when employing MOR, is a mere 32 min, in contrast to the 95 min required by the full-order model-based approach. This stark disparity underscores the potential of MOR in significantly bolstering computational efficiency without compromising the accuracy of the results.

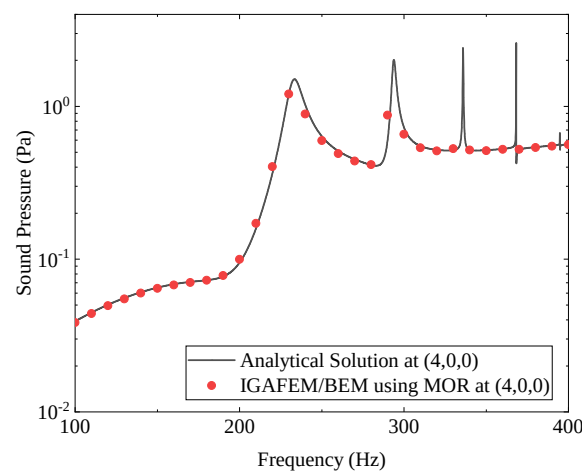


Figure 6. Sound pressure for the spherical shell model at point (4, 0, 0): a comparison of the analytical result and the result acquired with IGAFEM/BEM using MOR. The shell Thickness $t = 0.012$ m and Radius $r = 1.2$ m.

4.2. Elastic Submarine Model under Incoming Plane Wave

In this illustrative case, we examined the behavior of an underwater elastic submarine shell model subjected to the incidence of a plane wave, as depicted in Figure 7. This wave propagates in the x -direction and is characterized by a unit amplitude. The geometric confines of the model are defined by a specific coordinate range (as illustrated in Figure 7a): x spans from -0.888 m to 1.112 m, y extends from -0.255 m to 0.255 m, and z ranges between -0.127 m and 0.387 m. The shell is meticulously crafted with a uniform Thickness $t = 0.003$ m. Within the scope of this analysis, we delve into the sound pressure at various discrete points along the shell's surface.

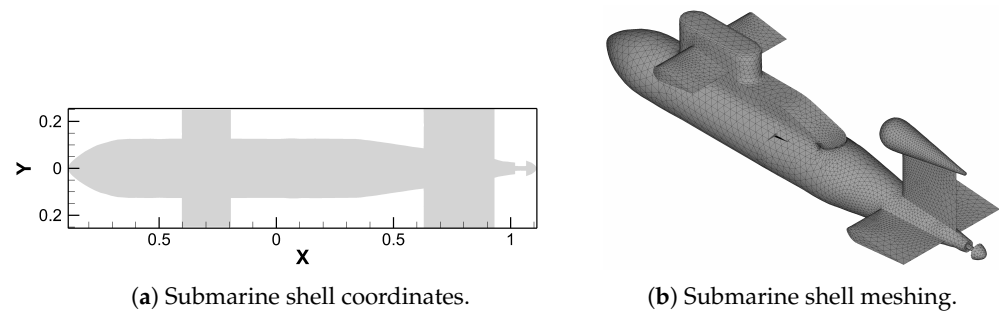
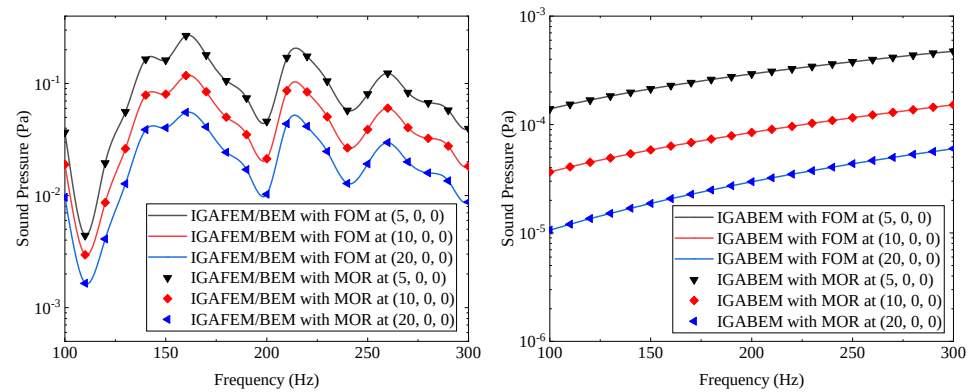


Figure 7. The submarine shell model.

In this analysis, we selected three distinct locations— $(5, 0, 0)$, $(10, 0, 0)$, and $(20, 0, 0)$ —to evaluate the sound pressure, as illustrated in Figure 8. Utilizing these points, we compared the outcomes derived from both the full-order model and the model-order reduction approach. Incident plane waves induce vibrations in the structure by exciting its scattering acoustic field, thereby radiating noise omnilaterally. The fluid–structure vibro-acoustic analysis, as depicted in Figure 8a, was performed using the IGAFEM/BEM method, which accounts for the interaction between the fluid and the structure. In contrast, the rigid acoustic analysis, as shown in Figure 8b, employed the IGABEM method, disregarding such interactions. The expansion frequencies considered were 125 Hz, 175 Hz, 225 Hz, and 275 Hz.

A discernible concordance between the MOR and FOM results is evident in Figure 8, validating the effectiveness of the MOR technique. A juxtaposition of Figure 8a,b revealed significant disparities between simulations that incorporate fluid–structure, vibro-acoustic effects in elastic structures and those that treat structures as being rigid without considering interactive effects. The elastic model displayed multiple resonant peaks, while the rigid model exhibited a uniform increase in response with frequency. This underscores the necessity of incorporating hydrodynamic excitation effects in the analysis of underwater thin-shell structures. Moreover, the computational time for the IGAFEM/BEM simulation was substantially reduced when employing MOR, taking only 5.1 h as opposed to the 22.67 h required by the FOM-based approach. This comparison reaffirms that MOR not only enhances computational efficiency, but also retains the accuracy of the computations.

Figure 9 illustrates the distribution of sound pressure across the submarine shell surface, as determined by the fluid–structure, vibro-acoustic IGAFEM/BEM analysis. Similar to the findings in Figure 4, the contour plot revealed an axisymmetric pattern of sound pressure with respect to the x - z plane—a consequence of the plane wave's unidirectional propagation along the x -axis. Moreover, the observed variations in pressure at distinct frequencies underscored the frequency-dependent nature of the acoustic response. This observation was pivotal, suggesting that any forthcoming investigation into sensitivity, optimization, and sound field analysis must encompass a thorough frequency-band analysis to capture the dynamic range of the shell's acoustic behavior.



(a) Elastic analysis using IGAFEM/BEM.

(b) Rigid analysis using IGABEM.

Figure 8. Sound pressure for the submarine shell model at calculating locations (5, 0, 0), (10, 0, 0), and (20, 0, 0). Comparison of the result using a full-order model (FOM) and the result using model-order reduction (MOR). The shell Thickness $t = 0.003$ m.

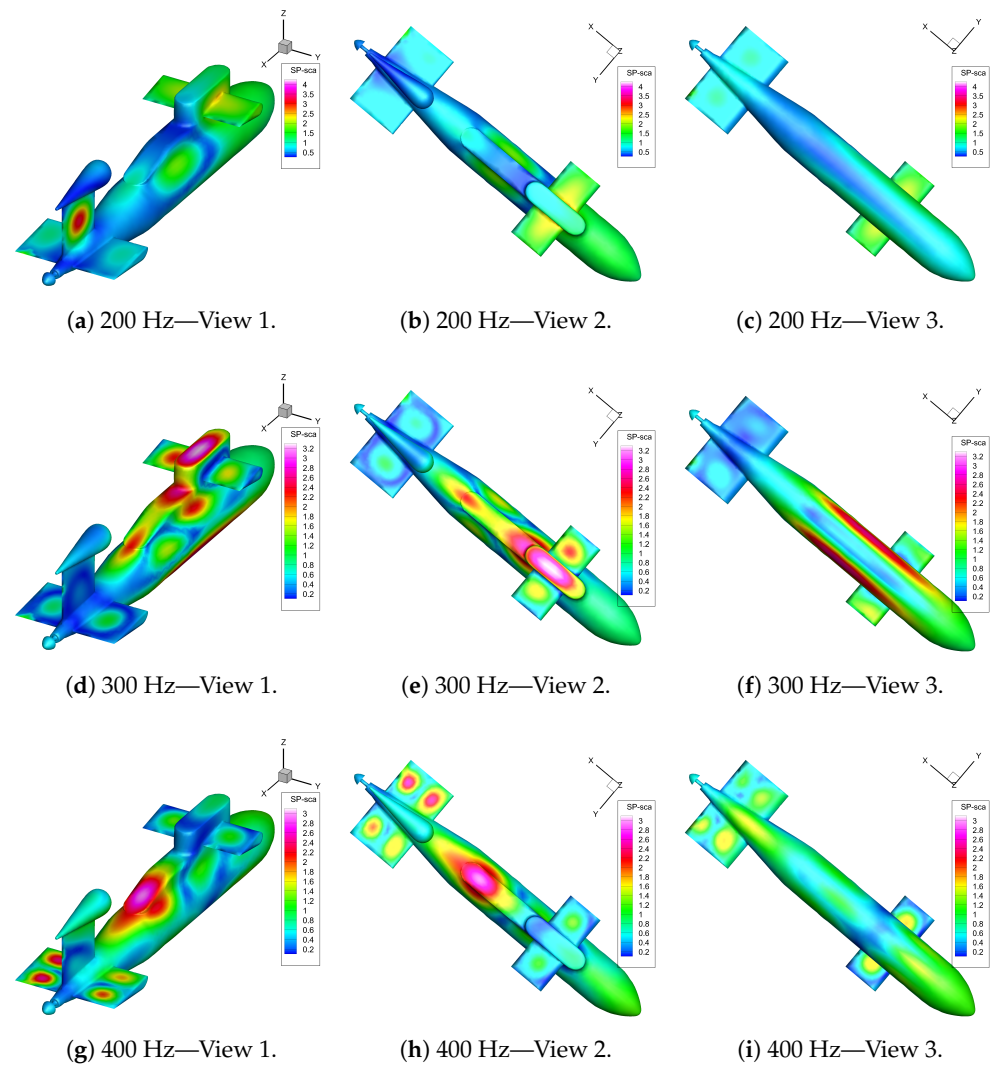


Figure 9. Sound pressure on the submarine shell surface at different frequencies. The shell Thickness $t = 0.003$ m.

5. Conclusions

This research successfully integrated the MOR technique with IGAFEM/BEM to address the complexities of wideband vibro-acoustic interactions within fluid–structure systems. This study capitalized on the individual strengths of BEM for external acoustic solutions and FEM for the analysis of shell vibrations, culminating in a synergistic approach that is particularly adept at handling unbounded domain problems.

Through the adoption of subdivision surfaces for geometric modeling, this study facilitated the construction of water-tight geometries with a controlled curvature, thereby enhancing the fidelity of the model. The integration of isogeometric analysis not only ensured high-order continuity for Kirchhoff–Love shell elements, but it also significantly reduced the meshing burden, streamlining the workflow from CAD to CAE.

The innovative application of SOAR, in conjunction with Taylor expansions, effectively decoupled the frequency dependence of system matrices, leading to a ROM that retains the essential characteristics of the original system. The proposed technique demonstrated a marked improvement in computational efficiency without compromising accuracy, as evidenced by the numerical simulations presented.

The results of this study underscore the potential of the MOR-integrated IGAFEM/BEM approach in providing rapid and accurate analyses for a wide range of frequencies, which is of paramount importance in engineering applications. Future work will explore the incorporation of coupling sensitivity analysis and shape/topology optimization, taking into account frequency uncertainty to further enrich the applicability of the method.

In conclusion, the integration of MOR with IGAFEM/BEM, as presented in this study, offers a robust and efficient framework for the analysis of fluid–structure, vibro-acoustic interactions. The findings pave the way for advancements in the design and optimization of structures subjected to acoustic environments, promising significant contributions to the fields of naval architecture, aerospace, and mechanical engineering.

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