

Article

Inverse Beamforming Algorithm for Strong Interference Suppression Based on Parameter-Adaptive Optimization

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Abstract

To improve the interference suppression performance of the inverse beamforming algorithm, an improved inverse beamforming algorithm based on the least squares criterion is proposed in this paper. Firstly, the array manifold matrix is constructed, and its generalized inverse matrix is multiplied by the beam-domain data of interference to obtain the element-domain interference data. Subsequently, the constructed element-domain interference data is subtracted from the original element-domain data to realize interference cancellation. This paper theoretically analyzes why the traditional inverse beamforming algorithm forms nulls towards interference directions yet retains prominent residual power in the regions immediately adjacent to the nulls. The proposed algorithm adopts grid search and Bayesian optimization to realize adaptive parameter selection. The theoretical analysis proves that the new algorithm can completely eliminate residual power, adjust the width of interference nulls, and further improve the local signal-to-noise ratio (SNR). Simulation results show that compared with the conventional inverse beamforming algorithm, the proposed algorithm increases the local SNR by 2–3 dB, suppresses the in-mainlobe power by no less than 30 dB, and enables adjustable width of interference nulls. Sea trial data further verifies the effectiveness and superior performance of the proposed algorithm.

Keywords: inverse beamforming; interference suppression; least squares; residual power; grid search optimization; Bayesian optimization

1. Introduction

In modern underwater acoustic information detection systems, the interference suppression capability in complex environments is the core metric determining the performance of underwater target detection [1,2]. With the increasing normalization of marine resource development operations and the continuous growth in the number of navigable vessels on the sea surface, radiated noise from various ships has gradually become the main interference source for underwater detection by passive sonar. In particular, the interference noise generated by large tugboats exhibits characteristics of wide bandwidth, high energy, and time-varying non-stationarity, with its power in the observation frequency band being 20–40 dB higher than that of long-range weak targets. This enormous power difference easily submerges the radiated signals of weak targets [3]. If the strong interference is spatially close enough to the weak target to be detected, the mainlobe and sidelobes of the interference formed by conventional beamforming will further obscure the target spectral peak, making it impossible for the weak target to be effectively resolved in the spatial



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spectrum. This ultimately significantly degrades the detection performance of weak targets and restricts the operational effectiveness and battlefield survivability of underwater acoustic detection equipment. Therefore, achieving strong interference suppression while preserving the effective signals of weak targets is a critical problem that urgently needs to be solved to improve the detection capability of passive sonar.

Current various adaptive anti-interference algorithms for strong tugboat interference scenarios have different technical focuses but generally suffer from inherent application limitations [4]. The performance of the Minimum Variance Distortionless Response (MVDR) algorithm is highly dependent on the estimation accuracy of the co-variance matrix from training samples and the consistency of statistical characteristics between training samples and test samples. The number of snapshots, array scale, and signal-to-interference-plus-noise ratio (SINR) all constrain the minimum number of snapshots required by the algorithm [5,6]. When tugboat maneuvering induces array shape distortion and non-stationary noise changes, sample statistical mismatch will directly cause covariance estimation distortion and adaptive weighting failure. Adaptive side-lobe cancellers are prone to self-cancellation of useful desired signals, and array element deformation and non-stationary interference characteristics further reduce their interference suppression effect [7]. Although the post-beamforming interference cancellation technology has been implemented in active sonar equipment, it can only achieve narrowband interference suppression and cannot adapt to tugboat broadband interference scenarios [8]. Empirical Mode Decomposition (EMD) separates interference components by decomposing intrinsic mode functions, but its operation relies on prior parameters of the tugboat noise spectrum and requires manual selection of mode components, resulting in low automation and limited engineering practicality [9]. Blind Source Separation (BSS) algorithms complete signal-noise separation based on the constraints of non-Gaussianity and statistical independence of signals, while complex marine hydrological environments cannot meet these strict statistical assumptions, leading to limited applicable scenarios [10]. Acoustic shielding algorithms need to pre-identify the interference transfer function, and the target signal gain improvement after shielding processing is limited, making it difficult to balance computational efficiency and interference suppression performance [11].

In summary, existing interference suppression schemes generally have defects such as high dependence on prior information, strict application constraints, and insufficient signal gain improvement, which cannot adapt to complex marine measured environments. First, the uneven spatiotemporal distribution of seawater temperature and salinity causes fluctuations in propagation loss, coupled with the dynamic changes in tugboat sailing speed, resulting in real-time non-stationarity of the tugboat radiated noise spectrum. This makes it difficult to obtain accurate and timely estimates of the interference prior spectrum and transfer function, and traditional prior-driven algorithms are highly prone to environmental mismatch, leading to problems such as interference suppression failure and weak target false cancellation. Second, complex working conditions, such as multi-tugboat superimposed interference and carrier maneuvering array shape distortion, further amplify the shortcomings of traditional algorithms. Methods such as BSS and acoustic shielding cannot balance environmental adaptability and real-time computing requirements. In addition, the signal-to-noise ratio (SNR) of underwater weak targets against strong interference backgrounds is extremely low, and traditional algorithms cannot avoid target signal attenuation while suppressing interference, making it difficult to meet the detection gain requirements for long-range small-scale weak targets. Accordingly, the development of a new interference suppression algorithm that is independent of prior information, environmentally adaptive, and balances computational efficiency with weak target gain

improvement has become a key research direction in the field of array underwater acoustic signal processing.

Inverse Beamforming (IBF), with its core architecture of interference Direction of Arrival (DOA) estimation, interference signal reconstruction, and element-domain interference cancellation, can be deeply integrated with Conventional Beamforming (CBF), providing a feasible idea to solve the above technical pain points [12,13]. The working logic of this algorithm is as follows: it quickly calculates characteristic parameters such as interference DOA and radiated power based on the spatial spectrum output by CBF, then reconstructs element-domain interference data using interference characteristics, and directly subtracts the reconstructed interference from the original array received data. This approach removes broadband non-stationary interference such as tugboat noise from the element dimension. Different from adaptive beamforming systems such as MVDR that optimize weighting coefficients from the power domain, IBF gets rid of the rigid constraint of accurate covariance matrix estimation and has stronger robustness to sample snapshots, array deformation, and non-stationary noise changes.

Some studies have used CBF spatial spectrum to locate interference DOA and combine the coherent interference step-by-step cancellation idea to achieve element-domain noise reduction. Relevant works have successively proposed derivative, such as sector IBF, time-domain broadband IBF, narrowband robust IBF, and frequency-domain broadband IBF, and have completed algorithm robustness verification on vector hydrophone arrays [14–17]. Existing studies have uniformly standardized IBF into three execution steps: CBF interference parameter estimation, interference signal reconstruction, and element-domain interference cancellation. Among these, frequency-domain broadband IBF is the most widely adopted enhanced approach. After element-domain interference cancellation followed by CBF beam output, the power at the frequency points directly facing the interference can be effectively suppressed, but obvious residual interference power still remains at the position offset by the half-power beamwidth from the interference DOA. The root cause is that the reconstructed interference and the real interference only achieve energy matching within the mainlobe -3 dB bandwidth, and they do not follow the same random distribution outside the bandwidth, resulting in incomplete interference cancellation.

Aiming at the defect of excessive residual power in interference sidelobes of frequency-domain broadband IBF, this paper constructs an modified IBF algorithm based on the least squares optimization approach. It solves the least squares analytical solution of element-domain interference at each frequency point by combining the array manifold matrix and CBF beam-domain data, and flexibly controls the interference cancellation interval by adjusting the dimension of the array manifold matrix. Although the modified scheme optimizes the completeness of interference cancellation, the key parameter of interference notch width in the algorithm still relies on manual debugging and selection, and cannot be adaptively matched according to interference distribution and target DOA.

To solve the problem of manual parameter tuning, this paper introduces an intelligent optimization strategy. As an efficient global optimization algorithm, Bayesian optimization is suitable for black-box and high-cost evaluation optimization problems such as beam optimization and array layout, and its comprehensive performance is superior to traditional grid traversal and heuristic search [18,19]. This paper builds a parameter-adaptive optimization framework based on grid search combined with Bayesian optimization to realize automatic optimization of the notch width parameter of the modified IBF. It can autonomously obtain the optimal parameters under different interference–target spatial layout conditions, simultaneously achieve residual interference power suppression and weak target signal protection, and reduce manual parameter tuning costs, which is the core research focus of this paper.

2. Algorithms of Inverse Beamforming

2.1. Array Received Data Model

The basic assumptions for the array and received data model are presented as follows:

1. The array receives target signals along with ambient noise. The noise is assumed to be additive white Gaussian noise (AWGN). The noise components at different array elements are statistically independent of one another, and the noise is also independent of useful signals. Meanwhile, the propagation medium is regarded as lossless, linear, non-dispersive, homogeneous and isotropic.
2. Far-field assumption: The adopted array is a planar array lying on the same plane as the incident direction of signals. The wavefront of far-field signals impinging on the array can be approximated as a plane wave.

The far-field assumption holds when the distance between the signal source and the array is sufficiently large. If the distance from the acoustic source exceeds the Fraunhofer distance of the array $2D^2/\lambda$ (where D denotes the array aperture and λ represents the signal wavelength), the curvature of the incident wavefront can be neglected, and the plane wave approximation is valid. For underwater acoustic towed linear arrays, the array aperture is on the order of tens of meters, while targets and interference sources are generally located in the far field several kilometers away. Therefore, the plane wave assumption is applicable to the scenarios discussed in this paper. Under this assumption, the differences of a single signal arriving at different array elements are merely time delays related to element positions and incident angles, which lays the foundation for subsequent beamforming and inverse beamforming processing.

The output of array received data is expressed as follows [20]:

$$x_n(t) = \sum_{p=1}^P g_n(\theta_p) s_p[t - \tau_n(\theta_p)] + e_n(t) \quad (1)$$

where $g_n(\theta_p)$ denotes the amplitude gain of the n th array element for the incident signal from direction θ_p ; $\tau_n(\theta_p)$ represents the time delay of the signal from direction θ_p received by the n th element relative to the reference element. Let P be the number of spatial signal sources and N be the total number of array elements. $s_p(t)$ stands for the signal radiated by the p th ($p = 1, 2, \dots, P$) far-field point source, and $e_n(t)$ is the additive noise at the n th array element. It is assumed that the number of spatial signal sources P satisfies $P \leq N$.

3. The array is configured as a standard Uniform Linear Array (ULA). The geometric size of each array element is much smaller than the wavelength of the incident plane wave, so the amplitude gain is set to 1. In addition, the inter-element spacing of the receiving array is far larger than the physical size of array elements, and the mutual coupling between elements can be neglected. Let d denote the inter-element spacing of the ULA. The first element of the array is taken as the reference element. For the p th signal incident from direction θ_p , the time delay at the n th element relative to the reference element is expressed as:

$$\tau_n(\theta_p) = (n - 1) \frac{d}{c} \cos \theta_p \quad (2)$$

where c is the speed of sound in water. Figure 1 illustrates the schematic diagram of plane waves impinging on a uniform linear array. The positive direction of the x -axis is defined as the direction of 0° .

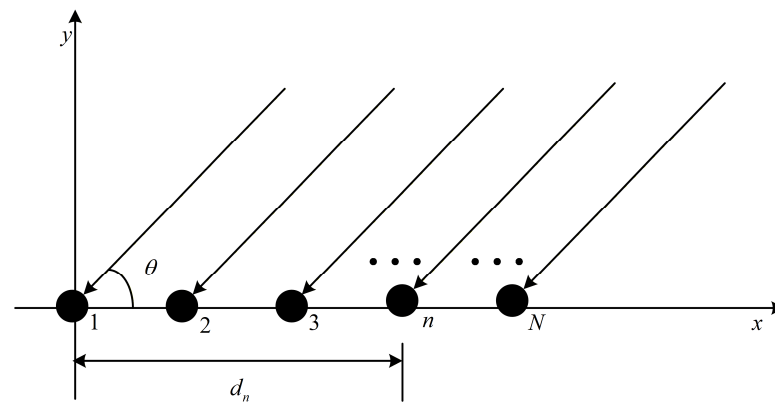


Figure 1. ULA under plane wave incidence.

It can be seen from Equation (2) that the time delays of the p th signal across all array elements form a set of phase relationships uniquely corresponding to the incident direction. The array manifold vector for a given direction is constructed by arranging the phase factors of each element relative to the reference element in element order. As the incident direction varies continuously, the set spanned by all manifold vectors is defined as the array manifold. Fully characterizing the spatial response of the array to incoming waves from different directions, the array manifold serves as the fundamental basis for designing weight vectors of beamformers and reconstructing interference in inverse beamforming.

To quantitatively evaluate the performance of interference suppression and target signal gain, this paper adopts the interference-to-noise ratio (INR) and signal-to-noise ratio (SNR) to describe input signal conditions. These metrics are defined as the ratio of interference power and target power to noise power, respectively, and are measured in decibels (dB). In the element domain, strong interference and weak targets are spatially overlapped, making them indistinguishable at a single array element. After spatial filtering via beamforming, signal energy from different directions is separated into independent beam channels. This facilitates subsequent extraction of interference features in the beam domain and interference cancellation in the element domain.

2.2. Wideband Beamforming

For an N -element array, the signals received by each array element are firstly sampled to obtain discrete signals, which can be expressed as:

$$x(l) = x(t)|_{t=lT_s}, \quad l = 1, 2, \dots \quad (3)$$

where l denotes the time-domain sample index, and T_s is the sampling period.

Let the long sequence of sample data received by the n th array element be defined as $x_n(1), x_n(2), \dots, x_n(l), \dots$. The data of each array element is divided into frames, with each frame containing a segment of data. Assuming the frame length is L , the Discrete Fourier Transform (DFT) is computed for the following discrete frequency points:

$$f_k = \begin{cases} f_s k / L & k = 0, 1, \dots, L/2 - 1 \\ f_s (k - L) / L & k = L/2, \dots, L - 1 \end{cases} \quad (4)$$

where $k(k = 0, 1, 2, \dots, L - 1)$ represents the frequency bin index. The frame length L is set as an even integer, ensuring that $L/2$ is an integer for symmetric frequency bin division in DFT.

The frequency-domain data vector of the m th frame is written as:

$$\mathbf{X}^{(m)}(k) = [X_1^{(m)}(k), X_2^{(m)}(k), \dots, X_N^{(m)}(k)]^T \quad (5)$$

where $k = 0, 1, 2, \dots, L-1$. For any vector $\mathbf{v}$, $\mathbf{v}^T$ denotes the transpose operation of the vector.

Let $w_n(k)$ be the complex weight of the n th array element at frequency f_k , and the narrowband weight vector is denoted as $\mathbf{w}(k) = [w_1(k), w_2(k), \dots, w_N(k)]^T$.

The beam-domain output at each frequency bin is calculated as:

$$Y^{(m)}(k) = \mathbf{w}^H(k) \mathbf{X}^{(m)}(k) \quad (6)$$

For any vector $\mathbf{v}$, $\mathbf{v}^H$ denotes the conjugate transpose operation of the vector.

The time-domain output with a length of L points can be acquired by performing the Inverse Discrete Fourier Transform on the frequency-domain beam data sequence:

$$y^{(m)}(l) = \frac{1}{L} \sum_{k=0}^{L-1} Y^{(m)}(k) e^{j2\pi kl/L} \quad (7)$$

Different from narrowband signals, the radiated noise of underwater targets and tugboat interference are both wideband signals occupying a certain frequency range. Narrowband beamforming assumes that signal energy concentrates on a single frequency bin, and its weight vector is designed merely for this frequency. Nevertheless, the array manifold varies across different frequencies for wideband signals. If weighting is implemented based on a single frequency bin, consistent spatial directivity cannot be maintained over the entire frequency band. Accordingly, wideband beamforming generally adopts the sub-band processing scheme: the wideband signal is decomposed into multiple narrow sub-bands for beamforming respectively, and then the processing results of all sub-bands are integrated.

This paper adopts the frequency-domain wideband beamforming architecture. Its basic procedures are as follows. First, the time-domain signals received by each array element are framed and transformed into the frequency domain via DFT. At each frequency bin, the narrowband weight vector is constructed according to the corresponding array manifold to complete beamforming. Finally, the beam-domain outputs of all frequency bins are fused across the frequency band to generate the final wideband beam output.

The selection of frame length N requires a trade-off between frequency resolution and time resolution. A larger N leads to higher frequency resolution and narrower sub-bands, which makes the narrowband assumption more valid, whereas each frame covers a longer time duration and imposes stricter requirements on signal stationarity. A smaller N produces the opposite effect.

In addition, compared with the time delay and summation operations required for each array element and each beam direction in time-domain wideband beamforming, frequency-domain processing leverages the Fast Fourier Transform to reduce the computational complexity from the order of point-by-point convolution to log-linear order. It presents prominent advantages in computational efficiency for scenarios with multiple beams and frequency bins, which is a major reason this paper selected the frequency-domain architecture as the processing framework for inverse beamforming.

The structure of the frequency-domain wideband beamformer is shown in Figure 2.

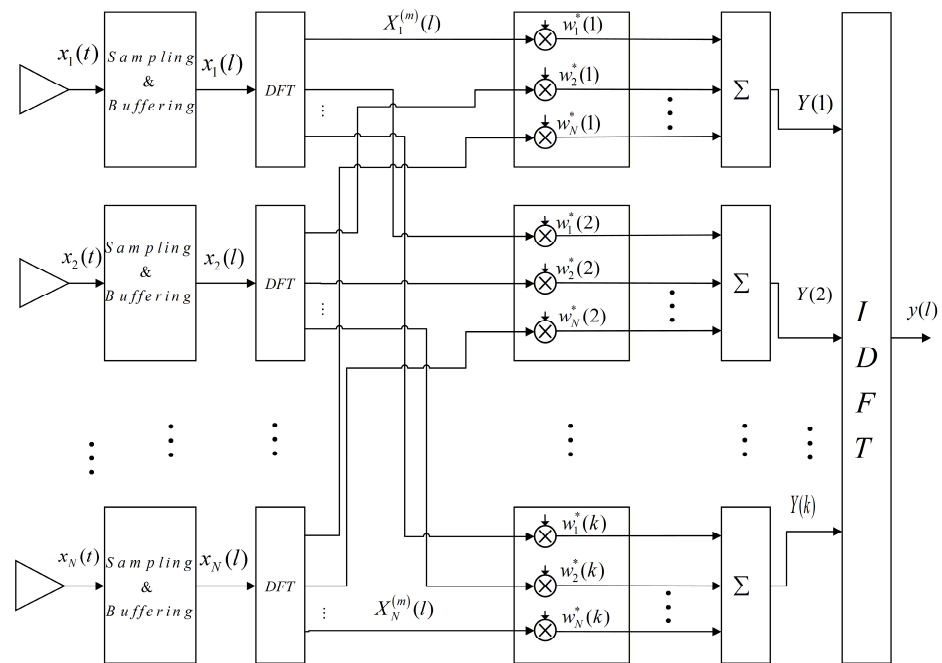


Figure 2. Block diagram of DFT processing for array signals.

2.3. Inverse Beamforming Algorithm and Its Improvement

2.3.1. Principle of Step-by-Step Cancellation for Stationary Random Processes

Let the random process $X(t)$ have a mean of $m_X(t)$ and a variance of $\sigma_X^2(t)$, and the random process $Y(t)$ have a mean of $m_Y(t)$ and a variance of $\sigma_Y^2(t)$. The difference between $X(t)$ and $Y(t)$ is also a random process, whose second raw moment is expressed as:

$$E[(X(t) - Y(t))^2] = \sigma_X^2(t) + \sigma_Y^2(t) - 2\text{Cov}[X(t), Y(t)] + (m_X(t) - m_Y(t))^2 \quad (8)$$

where $E(\bullet)$ denotes the mathematical expectation and $\text{Cov}(\bullet)$ represents the covariance.

If two zero-mean random processes have inconsistent second-order statistical characteristics, the second raw moment in Equation (8) is non-zero. By contrast, if their second-order statistical characteristics are identical, the second raw moment equals zero. This constitutes the core principle for the step-by-step cancellation of coherent sources.

2.3.2. Wideband IBF Implementation for Uniform Linear Array

According to Equation (2), the array manifold of a standard uniform linear array (ULA) is derived as: $\mathbf{a}(\theta_p, k) = [1e^{-j\frac{2\pi f_k d \cos \theta_p}{c}} \dots e^{-j\frac{2\pi f_k (N-1)d \cos \theta_p}{c}}]^T$.

The weight vector of the conventional beamformer is given by:

$$\mathbf{w}(\theta_p, k) = \mathbf{a}(\theta_p, k) / \sqrt{N} \quad (9)$$

The output of conventional beamforming at direction θ_p and frequency bin f_k is written as: $b(\theta, k) = \sum_n w_n^*(\theta, k) X_n^{(m)}(k) = \sum_n a_n^*(\theta, k) X_n^{(m)}(k) / \sqrt{N}$, where $w_n(\theta_p, k)$ is the weight corresponding to the n th array element, and $a_n(\theta_p, k)$ denotes the n th entry of the array manifold vector, and $X_n^{(m)}(k)$ represents the data received by the n th array element at frequency f_k . For any scalar or complex variable v , v^* represents its complex conjugate.

The beam-domain data corresponding to the interference direction contains almost complete amplitude and phase information of interference signals. Accordingly, the amplitude and phase features of interference can be extracted from such beam-domain data. To reconstruct element-domain interference data reversely from beam-domain interference

outputs, an inverse beamformer derived from conventional beamforming is adopted. The corresponding flowchart is illustrated in Figure 3.

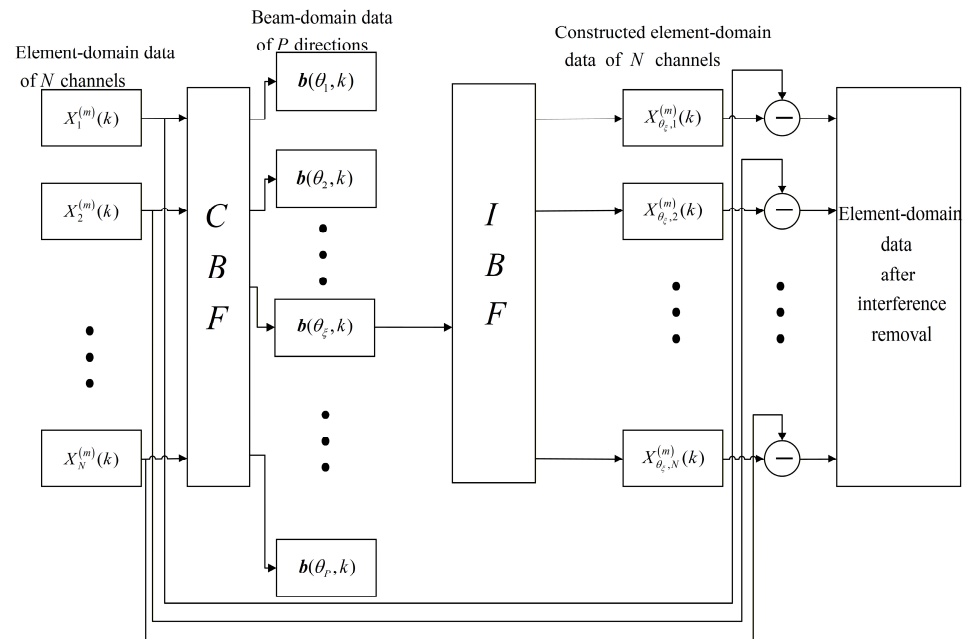


Figure 3. Flowchart of inverse beamforming.

Reference [21] obtains the uniquely determined vector for the inverse process based on the Moore–Penrose axiom. From the perspective of conventional beamforming, the weight vector for the inverse process of conventional beamforming towards the interference direction θ_{ξ} is derived from Equation (9) as: $w'(\theta_{\xi}, k) = a(\theta_{\xi}, k) / \sqrt{N}$.

The reconstructed interference signal in direction θ_{ξ} is:

$$\mathbf{X}_{\theta_{\xi}}^{(m)}(k) = \mathbf{w}'(\theta_{\xi}, k) \mathbf{b}(\theta_{\xi}, k) = \frac{\mathbf{a}(\theta_{\xi}, k)}{N} \sum_{n=1}^N \left[\mathbf{a}_n^*(\theta_{\xi}, k) X_n^{(m)}(k) \right] \quad (10)$$

The element-domain interference data at frequency f_k can be expressed in the form of a column vector for all array elements: $\kappa(k) \mathbf{a}(\theta_{\xi}, k)$, where $\kappa(k) = \frac{1}{N} \sum_{n=1}^N \left[X_n^{(m)}(k) \mathbf{a}_n^*(\theta_{\xi}, k) \right]$ stands for the beam-domain data of direction θ_{ξ} obtained via conventional beamforming at frequency f_k .

Expanding Equation (10) yields:

$$\kappa(k) \mathbf{a}(\theta_{\xi}, k) = \frac{1}{N} \sum_{n=1}^N \left[X_n^{(m)}(k) \mathbf{a}(\theta_{\xi}, k) \varphi(n) \right] \quad (11)$$

where $\varphi(n) = e^{j \frac{2\pi f_k (n-1)d \cos \theta_{\xi}}{c}}$.

Subtracting Equation (10) from Equation (5) gives:

$$\mathbf{X}^{(m)'}(k) = \mathbf{X}^{(m)}(k) - \kappa(k) \mathbf{a}(\theta_{\xi}, k) = \frac{1}{N} \sum_{n=1}^N \left[X_n^{(m)}(k) \mathbf{a}(\theta_{\xi}, k) \odot \boldsymbol{\varepsilon} \right] \quad (12)$$

where $\boldsymbol{\varepsilon}$ denotes an $N \times 1$ column vector. Its specific definition is presented as follows: $\boldsymbol{\varepsilon}(i) = \begin{cases} -e^{j \frac{2\pi f_k (n-1)d \cos \theta_{\xi}}{c}}, & i \neq n \\ (N-1)e^{j \frac{2\pi f_k (n-1)d \cos \theta_{\xi}}{c}}, & i = n \end{cases}$, $i = 1, 2, \dots, N$, where $\mathbf{v}_1 \odot \mathbf{v}_2$ represents

the Hadamard product of two equal-dimension vectors $\mathbf{v}_1, \mathbf{v}_2$, which operates element by element.

Applying conventional beamforming (CBF) to Equation (12) yields the interference-suppressed beam-domain data as follows:

$$\mathbf{b}^{(m)'}(k) = \frac{\mathbf{a}^H(\theta_{\xi}, k)}{N^2} \sum_{n=1}^N \left[X_n^{(m)}(k) \mathbf{a}(\theta_{\xi}, k) \odot \boldsymbol{\varepsilon} \right] = \sum_{n=1}^N \left\{ [1 - D(\theta_{\xi})] X_n^{(m)}(k) \psi(n) \right\} \quad (13)$$

where $D(\theta_{\xi}) = \left| \frac{1}{N} \frac{\sin\left(\frac{N\pi df_k}{c} (\cos\theta_{\xi} - \cos\theta_p)\right)}{\sin\left(\frac{\pi df_k}{c} (\cos\theta_{\xi} - \cos\theta_p)\right)} \right|$ is the directivity function with $\psi(n) = e^{j\frac{2\pi f_k(n-1)d \cos\theta_p}{c}}$. The result of Equation (8) equals zero when $\theta_p = \theta_{\xi}$.

The interference power in direction θ_{ξ} is calculated as:

$$\rho(\theta_{\xi}) = \sum_{k=1}^L \left| \sum_{n=1}^N \left\{ [1 - D(\theta_{\xi})] X_n^{(m)}(k) \psi(n) \right\} \right|^2 \quad (14)$$

It can be concluded from Equation (9) that the wideband frequency-domain inverse beamforming fails to reconstruct ideal interference signals and essentially only performs phase shifting on the original element-domain data. This explains why the original IBF algorithm forms a null in the interference direction, while prominent residual power still exists in the region adjacent to the null. To achieve accurate reconstruction of ideal interference signals, an modified IBF algorithm is proposed in Section 2.3.3.

2.3.3. Modified Inverse Beamforming (MIBF) Algorithm

To reconstruct more accurate interference signals, Equation (12) is rearranged into the following form: $\mathbf{A}\mathbf{X}^{(m)''}(k) = N\mathbf{b}$, where $\mathbf{A} = [\mathbf{a}^*(\theta_{\xi} - \lambda, k), \dots, \mathbf{a}^*(\theta_{\xi} - 1, k), \mathbf{a}^*(\theta_{\xi}, k), \dots, \mathbf{a}^*(\theta_{\xi} + \mu, k)]^T$, $\mathbf{X}^{(m)''}(k) = [X_1^{(m)''}(k), X_2^{(m)''}(k), \dots, X_n^{(m)''}(k), \dots, X_N^{(m)''}(k)]^T$, $\mathbf{b} = [b(\theta_{\xi} - \lambda), \dots, b(\theta_{\xi} - 1), b(\theta_{\xi}), \dots, b(\theta_{\xi} + \mu)]^T$.

Then, we obtain

$$\mathbf{X}^{(m)''}(k) = \mathbf{N}\mathbf{A}^+\mathbf{b} \quad (15)$$

In the formula, $X_n^{(m)''}(k)$ denotes the signal reconstructed via least squares in Equation (15), from which all strong interference components have been eliminated; theoretically, only weak target signals remain. $\mathbf{A}^+$ is the generalized inverse matrix of $\mathbf{A}$. Both λ and μ are constants, and the sum of λ and μ equals the row number of matrix $\mathbf{A}$. By subtracting the least-squares result in Equation (15) from the array observation vector in Equation (5), we can obtain interference-free signals $\mathbf{X}^{(m)''}(k)$. The above signal processing framework constitutes the proposed Modified Inverse Beamforming (MIBF) algorithm.

By adjusting the row number of the array manifold matrix $\mathbf{A}$ via different parameter settings, the width of the interference notch after suppression can be broadened. While maintaining the local signal-to-noise ratio of weak target signals, the residual interference power in other directions can be further reduced. When $\lambda < \mu$, the interference on the right side of the interference direction is suppressed to a greater extent; when $\lambda > \mu$, stronger suppression is achieved for the interference on the left side. Increasing the sum of λ and μ will deepen the formed interference notch. When $\lambda = \mu = 0$, MIBF yields the same performance as the original IBF.

2.3.4. Parameter-Adaptive Optimization for MIBF

As indicated in Equation (15), the interference suppression performance of MIBF is determined by the row number of the array manifold matrix $\mathbf{A}$, which is jointly governed by parameters λ and μ . Physically, λ and μ represent the numbers of adjacent beam directions extended to the left and right with the interference direction as the center, respectively.

Let the beam index corresponding to the interference direction be J . The subset of beam directions selected by MIBF is expressed as:

$$\Theta_{\lambda,\mu} = \{\theta_{J-\lambda}, \dots, \theta_J, \dots, \theta_{J+\mu}\} \quad (16)$$

The number of elements in this subset equals the row number of matrix A . Larger values of λ and μ lead to a higher order of A , a wider and deeper interference notch. Asymmetric broadening of the notch on both sides of the interference direction can be realized when $\lambda = \mu = 0$. Nevertheless, in previous theoretical analysis and sea trials, λ and μ were manually set based on experience, without quantitative criteria matching the interference azimuth, interference intensity and prior information of targets. When the angular interval between interference and weak targets is small, excessively large λ and μ will cause an over-wide interference notch and mistakenly suppress target signals, while overly small values will result in insufficient suppression of residual power. Both cases degrade the local signal-to-noise ratio (SNR) gain. Therefore, it is necessary to develop a parameter-adaptive optimization mechanism for MIBF to determine λ and μ online according to actual observation scenarios.

1. Construction of Composite Cost Function

The core of parameter-adaptive optimization is to establish an objective functional that quantitatively evaluates interference suppression performance and weak target protection. In this paper, the selection of parameters (λ, μ) is formulated as a constrained optimization problem. A trade-off is made between suppressing residual interference power and protecting weak target signals, and the composite cost function is constructed as follows:

$$J(\lambda, \mu) = \alpha P_{\text{res}} - \beta \text{SNR}_{\text{local}} + \gamma R_{\text{tgt}} \quad (17)$$

where α , β and γ are non-negative weight factors for each term, which adjust the relative priority of residual power suppression, target gain and target protection respectively.

The first term is the residual power term, which quantifies the completeness of interference suppression. It is defined as the mean value of spatial spectrum power (in dB) within the evaluation region on both sides of the interference direction excluding the notch region, and is truncated by a lower limit to avoid over-optimization in numerical calculation:

$$P_{\text{res}} = \max \left\{ \frac{1}{|\Omega_J|} \sum_{\theta \in \Omega_J} 10 \lg P_{\text{MIBF}}(\theta; \lambda, \mu), P_{\text{floor}} \right\} \quad (18)$$

where $P_{\text{MIBF}}(\theta; \lambda, \mu)$ denotes the output power of MIBF at direction θ with given parameters (λ, μ) ; P_{floor} is the preset lower power limit, which prevents the residual power term from dominating the optimization after interference is fully suppressed. The evaluation region Ω_J is defined as a symmetric neighborhood on both sides of the interference direction excluding the mainlobe of the notch:

$$\Omega_J = [\theta_J - \Delta_2, \theta_J - \Delta_1] \cup [\theta_J + \Delta_1, \theta_J + \Delta_2] \quad (19)$$

where Δ_1 and Δ_2 represent the inner and outer boundary offsets of the evaluation region relative to the interference direction. Δ_1 skips the main notch region, while Δ_2 limits the range of the neighborhood, ensuring that only the residual power immediately outside the notch is counted in the residual power term.

The second term is the local SNR term of the target, i.e., the local SNR in the direction of the weak target. The proposed scheme guarantees that the target gain suffers no degra-

dation by sustaining the ratio of the output power in the target direction to the average noise power over the neighboring angular region.:

$$\text{SNR}_{\text{local}} = 10 \lg \frac{P_{\text{MIBF}}(\theta_{\Gamma}; \lambda, \mu)}{P_{\text{noise}}(\theta_{\Gamma}; \lambda, \mu)} \quad (20)$$

where θ_{Γ} is the direction of the weak target, and the denominator is the average noise power in the bilateral neighborhood of the target direction.

The third term is the target protection penalty term. When the expanded interference notch covers the target direction, the target power will decrease. Accordingly, the power drop of the target relative to the output of the original IBF is adopted as the penalty criterion, and a positive penalty is imposed only when the drop exceeds the allowable threshold ΔP_{tgt} :

$$\begin{cases} R_{\text{tgt}} = \max\{0, \Delta P_{\text{tgt}} - \delta\} \\ \Delta P_{\text{tgt}} = P_{\text{tgt}}^{\text{IBF}} - P_{\text{tgt}}(\lambda, \mu) \end{cases} \quad (21)$$

where $P_{\text{tgt}}(\lambda, \mu)$ is the output power in the target direction under parameters (λ, μ) ; $P_{\text{tgt}}^{\text{IBF}}$ is the target power in the spectrum output by the original IBF (i.e., $\lambda = \mu = 0$), which serves as the reference benchmark. ΔP stands for the target power drop, and ΔP_{tgt} is the maximum allowable power drop threshold δ . If an over-wide notch causes the target power drop to exceed δ , the penalty term increases linearly with the drop. This imposes a prominent penalty on parameter combinations that lead to false target suppression, so the optimization process suppresses interference while actively avoiding damage to weak target signals.

2. Algorithm 1: Parameter Optimization Based on Grid Search

Since λ and μ are non-negative integers, their value domain is a finite discrete set. Let the parameter search space consist of all integer pairs satisfying the constraints of λ and μ . Let the parameter search space be a set consisting of all integer pairs satisfying $0 \leq \lambda \leq \lambda_{\max}$ and $0 \leq \mu \leq \mu_{\max}$. The optimal parameters can be directly obtained by grid search, which traverses all combinations in this set and selects the one that minimizes the cost function:

$$(\lambda^*, \mu^*) = \underset{(\lambda, \mu) \in \mathcal{P}}{\operatorname{argmin}} J(\lambda, \mu) \quad (22)$$

where the set $\mathcal{P} = \{(\lambda, \mu) \mid \lambda \in [0, \lambda_{\max}], \mu \in [0, \mu_{\max}]\}$ represents the feasible search domain of the two optimization parameters. Grid search features global optimality and full interpretability. When the upper bounds of $\lambda_{\max}$ and $\mu_{\max}$ are small, its computational load is acceptable, making it a benchmark method for parameter optimization.

3. Algorithm 2: Efficient Parameter Optimization Based on Bayesian Optimization

Grid search suffers from low efficiency when the parameter search space expands or the computational cost of each cost function evaluation is high. To address this issue, Bayesian optimization is introduced as an efficient optimization strategy in this work. Taking the parameter vector $x = (\lambda, \mu)$ as the input and the cost function J as the optimization objective, Bayesian optimization adopts a Gaussian process as the surrogate model to regress and model the cost function surface.

Suppose m evaluations have been completed and an observation dataset is obtained. The posterior prediction of the Gaussian process for the cost function at any candidate point follows a normal distribution, with the posterior mean and posterior variance given by:

$$m_{\alpha}(x) = \mathbf{k}_{\alpha}^T (\mathbf{K}_{\alpha} + \sigma_{\varepsilon}^2 \mathbf{I})^{-1} \mathbf{J}_{\alpha} \quad (23)$$

$$s_{\alpha}^2(\mathbf{x}) = k(\mathbf{x}, \mathbf{x}) - \mathbf{k}_{\alpha}^T (\mathbf{K}_{\alpha} + \sigma_{\epsilon}^2 \mathbf{I})^{-1} \mathbf{k}_{\alpha} \quad (24)$$

where $\mathbf{K}_{\alpha}$ is the kernel matrix constructed from α observed points, $\mathbf{k}_{\alpha}(\mathbf{x})$ is the kernel vector between the candidate point $\mathbf{x}$ and all observed points, $\mathbf{J}_{\alpha}$ is the vector composed of observed cost values, σ_{ϵ}^2 denotes the observation noise variance, and $\mathbf{I}$ is the identity matrix.

Algorithm 1. Exhaustive grid search for the optimal notch parameters

Input search domain $\mathbb{P} = \{(\lambda, \mu) : \lambda \in [0, \lambda_{\max}], \mu \in [0, \mu_{\max}]\}$ $\triangleright$ Equation (22)
interference direction θ_J ; weak target direction θ_{Γ}
weight factors α, β, γ ; boundary offsets Δ_1, Δ_2
power floor P_{floor} ; allowable power drop δ ; reference power $P_{\text{tgt}}^{\text{IBF}}$

Output optimal pair (λ^*, μ^*) and minimal cost J^*

```

1       $J^* \leftarrow +\infty$ 
2      for  $\lambda = 0$  to  $\lambda_{\max}$  do
3          for  $\mu = 0$  to  $\mu_{\max}$  do
4               $\Theta_{\lambda, \mu} \leftarrow \{\theta_{J-\lambda}, \dots, \theta_J, \dots, \theta_{J+\mu}\}$   $\triangleright$  Equation (16)
5              obtain  $P_{\text{MIBF}}(\theta; \lambda, \mu)$  by applying MIBF with  $\Theta_{\lambda, \mu}$   $\triangleright$  Equation (15)
6               $\Omega_J \leftarrow [\theta_J - \Delta_2, \theta_J - \Delta_1] \cup [\theta_J + \Delta_1, \theta_J + \Delta_2]$   $\triangleright$  Equation (19)
7               $P_{\text{res}} \leftarrow \max \left\{ \frac{1}{|\Omega_J|} \sum_{\theta \in \Omega_J} 10 \lg P_{\text{MIBF}}(\theta; \lambda, \mu), P_{\text{floor}} \right\}$   $\triangleright$  Equation (18)
8               $\text{SNR}_{\text{local}} \leftarrow 10 \lg \frac{P_{\text{MIBF}}(\theta_{\Gamma}; \lambda, \mu)}{P_{\text{noise}}(\theta_{\Gamma}; \lambda, \mu)}$   $\triangleright$  Equation (20)
9               $\Delta P_{\text{tgt}} \leftarrow P_{\text{tgt}}^{\text{IBF}} - P_{\text{tgt}}(\lambda, \mu)$ 
10              $R_{\text{tgt}} \leftarrow \max\{0, \Delta P_{\text{tgt}} - \delta\}$   $\triangleright$  Equation (21)
11              $J(\lambda, \mu) \leftarrow \alpha P_{\text{res}} - \beta \text{SNR}_{\text{local}} + \gamma R_{\text{tgt}}$   $\triangleright$  Equation (17)
12             if  $J(\lambda, \mu) < J^*$  then
13                  $J^* \leftarrow J(\lambda, \mu); (\lambda^*, \mu^*) \leftarrow (\lambda, \mu)$ 
14             end if
15         end for
16     end for
17     substitute  $(\lambda^*, \mu^*)$  into Equation (15) to complete the MIBF
interference cancellation
18     return  $(\lambda^*, \mu^*), J^*$ 

```

The kernel function characterizes the smoothness of the cost surface. The Matérn 5/2 kernel, which shows good adaptability to the parameter space, is adopted in this paper:

$$k(\mathbf{x}, \mathbf{x}) = \left(1 + \frac{\sqrt{5}r}{\ell} + \frac{5r^2}{3\ell^2}\right) e^{-\frac{\sqrt{5}r}{\ell}} \quad (25)$$

where r is the Euclidean distance between two parameter points, and ℓ is the length-scale parameter of the kernel that controls the correlation range of the cost surface.

In each iteration, an acquisition function is constructed based on the posterior distribution to select the next evaluation point. The Expected Improvement (EI) criterion is used herein, which selects the parameter point with the maximum expected improvement over the current minimum cost. Its analytical expression is:

$$\text{EI}(\mathbf{x}) = (J^* - q_m - \xi)\Phi(z) + s_m\phi(z) \quad (26)$$

where $z = \frac{J^* - q_m(\mathbf{x}) - \xi}{s_m(\mathbf{x})}$, J^* is the minimum cost obtained from existing observations, ξ is a tuning parameter to balance exploration and exploitation, and $\Phi(\cdot)$ and $\phi(\cdot)$ represent the

cumulative distribution function and probability density function of the standard normal distribution, respectively.

In each iteration, the parameter point that maximizes the expected improvement is selected as the next evaluation point:

$$\mathbf{x}_{m+1} = \underset{\mathbf{x} \in \mathcal{P}}{\operatorname{argmax}} \operatorname{EI}(\mathbf{x}) \quad (27)$$

The cost value of this point is calculated by substituting it into MIBF, and the observation set and the surrogate model are updated accordingly. The iteration terminates when the preset number of evaluations is reached. By balancing the exploration of unevaluated regions and exploitation of known optimal regions via the predicted mean and uncertainty of the surrogate model, Bayesian optimization can approach the global optimum with far fewer evaluations than grid search.

Algorithm 2. Gaussian-process Bayesian optimization with the EI acquisition function

Input search space $\mathbb{P} = \{(\lambda, \mu) : \lambda \in [0, \lambda_{\max}], \mu \in [0, \mu_{\max}]\}$ ▷ Equation (22)
 Matérn 5/2 kernel $k(\cdot, \cdot)$ with length-scale ℓ ▷ Equation (25)
 observation noise variance σ_ε^2 ; exploration parameter ξ
 initial design size m_0 ; evaluation budget M ($m_0 < M$)

Output optimal pair (λ^*, μ^*) and minimal cost J^*

- 1 select m_0 initial points $\mathbf{x}_1, \dots, \mathbf{x}_{m_0} \in \mathbb{P}$, $\mathbf{x}_i = (\lambda_i, \mu_i)$
- 2 **for** $i = 1$ **to** m_0 **do**
- 3 $J(\mathbf{x}_i) \leftarrow$ substitute $\mathbf{x}_i$ into MIBF and evaluate by Equations (15)–(21)
- 4 **end for**
- 5 $m \leftarrow m_0$; $\mathbf{J}_m \leftarrow [J(\mathbf{x}_1), \dots, J(\mathbf{x}_m)]^T$
- 6 **while** $m < M$ **do**
- 7 $\mathbf{K}_m \leftarrow [k(\mathbf{x}_i, \mathbf{x}_j)]_{m \times m}$, $1 \leq i, j \leq m$ ▷ Equation (25)
- 8 $J^* \leftarrow \min\{J(\mathbf{x}_1), \dots, J(\mathbf{x}_m)\}$ ▷ incumbent minimum
- 9 **for all** candidate $\mathbf{x} \in \mathbb{P} \setminus \{\mathbf{x}_1, \dots, \mathbf{x}_m\}$ **do**
- 10 $\mathbf{k}_m(\mathbf{x}) \leftarrow [k(\mathbf{x}, \mathbf{x}_1), \dots, k(\mathbf{x}, \mathbf{x}_m)]^T$
- 11 $q_m(\mathbf{x}) \leftarrow \mathbf{k}_m^T(\mathbf{x})(\mathbf{K}_m + \sigma_\varepsilon^2 \mathbf{I})^{-1} \mathbf{J}_m$ ▷ Equation (23)
- 12 $s_m^2(\mathbf{x}) \leftarrow k(\mathbf{x}, \mathbf{x}) - \mathbf{k}_m^T(\mathbf{x})(\mathbf{K}_m + \sigma_\varepsilon^2 \mathbf{I})^{-1} \mathbf{k}_m(\mathbf{x})$ ▷ Equation (24)
- 13 $z \leftarrow \frac{J^* - q_m(\mathbf{x}) - \xi}{s_m(\mathbf{x})}$
- 14 $\operatorname{EI}(\mathbf{x}) \leftarrow (J^* - q_m(\mathbf{x}) - \xi)\Phi(z) + s_m(\mathbf{x})\phi(z)$ ▷ Equation (26)
- 15 **end for**
- 16 $\mathbf{x}_{m+1} \leftarrow \underset{\mathbf{x} \in \mathbb{P}}{\operatorname{argmax}} \operatorname{EI}(\mathbf{x})$ ▷ Equation (27)
- 17 $J(\mathbf{x}_{m+1}) \leftarrow$ substitute $\mathbf{x}_{m+1}$ into MIBF and evaluate by Equations (15)–(21)
- 18 $\mathbf{J}_{m+1} \leftarrow [\mathbf{J}_m^T, J(\mathbf{x}_{m+1})]^T$; $m \leftarrow m + 1$
- 19 **end while**
- 20 $i^* \leftarrow \operatorname{argmin}_{1 \leq i \leq M} J(\mathbf{x}_i)$; $(\lambda^*, \mu^*) \leftarrow \mathbf{x}_{i^*}$; $J^* \leftarrow J(\mathbf{x}_{i^*})$
- 21 substitute (λ^*, μ^*) into Equation (15) to complete the MIBF interference cancellation
- 22 **return** $(\lambda^*, \mu^*), J^*$

4. Overall Algorithm Flow

The overall flow of MIBF integrated with parameter-adaptive optimization is summarized as follows: First, conventional beamforming (CBF) is used to acquire the azimuth and power information of interference and then determine the beam indices of interference J

and weak target Γ . Second, the spatial spectrum output of the original IBF ($\lambda = \mu = 0$) is taken as the reference benchmark for target power. Third, either grid search (Algorithm 1) or Bayesian optimization (Algorithm 2) is selected according to computational resources to solve for the optimal parameter (λ^*, μ^*) that minimizes the composite cost function $J(\lambda, \mu)$. Finally, the optimal parameters are substituted into Equation (15) to complete the interference cancellation of the MIBF algorithm.

This mechanism enables MIBF to automatically configure asymmetric notch shapes according to the relative azimuth of interference and targets. It fully suppresses residual power while avoiding false suppression of weak target signals, thereby eliminating the reliance on manual parameter tuning for notch configuration.

3. Performance Analysis

3.1. Simulation Results of the Original IBF Algorithm

To systematically verify the performance of the modified IBF (MIBF) algorithm and its parameter-adaptive optimization mechanism, numerical simulations were first carried out under ideal array conditions. A standard uniform linear array (ULA) was adopted, with the main simulation parameters listed in Table 1. Both strong interference and weak targets were modeled as mutually independent zero-mean Gaussian wideband random signals occupying the same frequency band. The ambient noise was modeled as additive white Gaussian noise (AWGN) that is statistically independent across array elements.

Table 1. Simulation parameter settings.

Parameter	Value	Parameter	Value
Number of array elements	80	Sampling frequency	8000
Element spacing	0.5 m	DFT points	4096
Speed of sound	1500 m/s	Processing frequency band	1000–1500 Hz
Interference azimuth	13°	Interference-to-Noise Ratio (INR)	20 dB
Target azimuth	27°	Signal-to-Noise Ratio (SNR)	0 dB
Beam scanning range	0°~180°		

Taking the null-to-null beamwidth of the beam pattern as the reference, the noise power was recorded and averaged by extending 2° outwards from the left and right valley points, as shown in Figure 4. The local signal-to-noise ratio (local SNR) is defined as the ratio of the peak power of the weak target to the average noise power. Variations in the target local SNR following strong interference suppression are quantified and summarized in Table 2, while the spatial spectrum simulation results of wideband frequency-domain IBF, CBF, and the benchmark MVDR adaptive beamformer are jointly plotted in Figure 5.

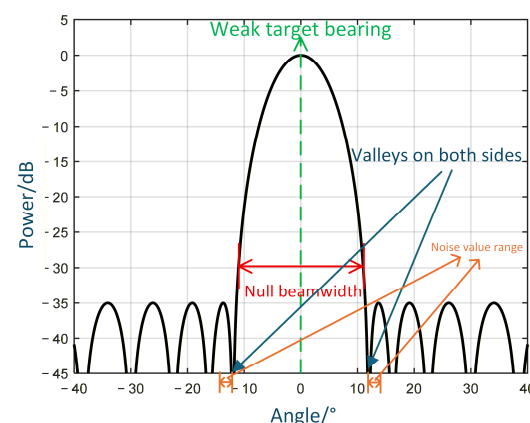
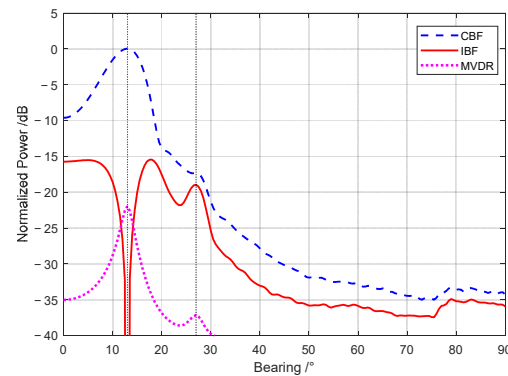


Figure 4. Range for noise power calculation.

Table 2. Comparison of Local SNR among CBF, IBF and MVDR.

	CBF	IBF	MVDR
Local SNR/dB	0	3.6	0.6

**Figure 5.** Comparison of spatial spectra among wideband frequency-domain IBF, CBF and MVDR. The black dashed lines denote the azimuth angles of interferences and targets.

As shown in the spatial spectrum curves of Figure 5, the conventional MVDR beamformer achieved only a local SNR of 0.6 dB under the identical wideband simulation setup. Despite its widespread adoption as a classic adaptive beamforming baseline, MVDR suffers severe performance degradation against strong directional interference, accompanied by obvious spectrum power leakage across the entire bearing range. By contrast, the original IBF delivered prominent performance superiority over MVDR and CBF. Although the baseline IBF only suppressed dominant interference at the exact interference arrival angle and retained residual interference power in off-interference azimuths, it still yielded substantially higher local SNR gains than MVDR. The blue dashed CBF curve exhibits extremely high sidelobe power across all off-target bearings, which severely elevates background noise power and deteriorates target detection capability. The magenta dotted MVDR curve forms shallow attenuation valleys around the interference azimuth yet cannot effectively suppress interference spillover, resulting in a low local SNR of merely 0.6 dB for the weak target. In comparison, the solid red IBF curve creates deeper attenuation for interference components and significantly reduces background normalized power, thereby yielding a much improved local SNR relative to MVDR and CBF. Even with residual interference power remaining in directions outside the main interference arrival angle, the original IBF is proven to outperform the mainstream MVDR beamformer in terms of weak target local SNR enhancement and interference power attenuation under strong interference scenarios.

3.2. Simulation Results of the Modified IBF Algorithm

The simulation conditions remain consistent with those in Section 3.1. The variations in the target local SNR after interference suppression under different λ and μ are recorded in Table 3. The simulation results of the modified wideband frequency-domain IBF are illustrated in Figure 6.

Table 3. Comparison of Local SNR among CBF, IBF and MIBF.

	CBF	IBF	MIBF
Local SNR/dB	0	3.6	6.7

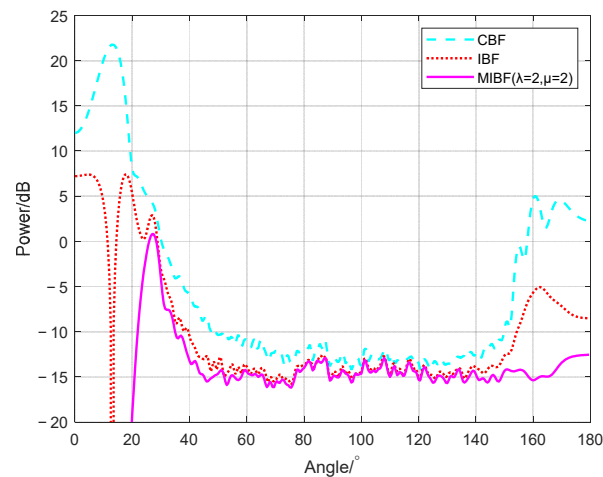


Figure 6. Comparison of spatial spectra among CBF, original IBF and MIBF.

In Figure 6, modified IBF refers to the proposed MIBF algorithm. It can be observed that MIBF can reconstruct interference signals much closer to the real ones. With the adopted parameter settings, MIBF basically eliminated the interference within the mainlobe and reduced the in-mainlobe power by approximately 30 dB. Meanwhile, the local SNR was further increased by 3.1 dB.

The regulation effect of parameters λ and μ on the shape of interference notches are further analyzed. Parameters λ and μ denote the numbers of extended beam directions on the left and right sides of the interference direction for constructing the array manifold matrix, respectively. When $\lambda = \mu$, the notch expands approximately symmetrically around the interference direction. When $\lambda > \mu$, the notch shifts to the left side of the interference with deeper suppression on the left; the opposite effect occurs when $\lambda < \mu$. The notch depth is mainly determined by the sum of λ and μ . A larger sum enables more accurate reconstruction of interference signals, resulting in a deeper notch and more thorough suppression of residual power. This asymmetric adjustable characteristic distinguishes MIBF from the original IBF. When a weak target is adjacent to the interference on one side, reducing the corresponding extension parameter can prevent the notch from covering the target direction, so as to suppress interference while protecting the weak target. However, the parameters in this section were set manually. The optimal matching between parameter values and the spatial layout of interference and targets will be solved via parameter-adaptive optimization in Section 2.3.4.

3.3. Influence of Different Parameter Settings on MIBF

Under the same simulation conditions as Section 3.1, different combinations of λ and μ were adopted to compare the interference suppression performance of MIBF.

The results in Figure 7 verify that the notch width after interference suppression can be adjusted by tuning λ and μ . A larger λ achieved stronger suppression on the left side of the interference direction, while a larger μ produced better suppression on the right side. Increasing the sum of λ and μ deepened the interference notch. Even with small parameter values, MIBF outperformed the original IBF. In addition, the capability of MIBF to eliminate residual power of the original IBF was enhanced with increases in λ and μ , achieving a residual power reduction of at least 30 dB, as shown in Figure 7.

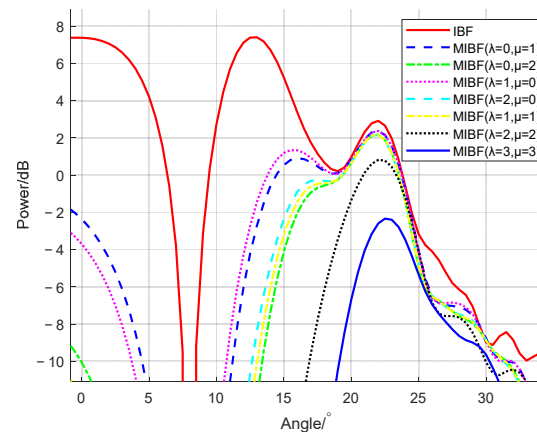


Figure 7. Comparison of spatial spectra of MIBF under different parameter settings.

3.4. Simulation Results of Parameter-Adaptive Optimization

The simulations in Sections 3.2 and 3.3 validate the effectiveness of MIBF, but the parameters λ and μ were still set manually. This section conducts simulation verification on the parameter-adaptive optimization mechanism proposed in Section 2.3.4. The simulation scenario followed the parameters in Table 1, with interference at 13° and target at 27° . The composite cost function was constructed according to Equation (17), where the weight coefficients were set as $\alpha = 1.0$, $\beta = 2.0$, $\gamma = 5.0$. The lower limit of residual power P_{floor} was -40 dB, and the allowable threshold of target power drop δ was 2 dB. The parameter search range was defined as $0 \leq \lambda \leq 14$ and $0 \leq \mu \leq 14$, discretized into a dense 15×15 grid that yielded 225 distinct parameter combinations.

Figure 8 presents the two-dimensional distribution of the composite cost function $J(\lambda, \mu)$ over the entire parameter space. An obvious low-value region can be observed. When λ and μ are too small, the residual interference power cannot be fully suppressed, leading to a high cost. When the two parameters are excessively large, the over-wide notch will cover the target direction and trigger the target protection penalty term, which also increases the cost. Grid search traversed all 225 combinations. The optimal parameter $(\lambda^*, \mu^*) = (3, 0)$ that minimizes the cost function was obtained, with the corresponding cost value being $J^* = -53.7$. The optimal solution is an asymmetric parameter combination, indicating that moderately expanding the notch to the left side of the interference (opposite to the target) achieves the best overall performance in this scenario. Table 4 quantitatively compares the computational overhead of grid search and Gaussian-process-based Bayesian optimization (BO) with the expected improvement acquisition function. Both tuning strategies identified the same global optimum $(\lambda^*, \mu^*) = (3, 0)$, yet BO drastically cut the number of cost function evaluations from 225 to merely 20 and achieved a speedup factor of $18.07\times$. Figure 9 further illustrates the convergence trajectory of the BO algorithm, where the minimal cost rapidly converges to the grid-search ground-truth optimum within only eight function queries, verifying the high search efficiency of the BO framework.

Table 4. Quantitative performance comparison between exhaustive grid search and Gaussian-process-based Bayesian optimization with EI acquisition function.

Parameter Tuning Strategy	Total Cost Evaluations	Wall-Clock Time (s)	Optimal Cost J^*	Optimal Hyperparameter Pair (λ^*, μ^*)
15×15 Exhaustive Grid Search	225	283.39	-53.655	$(3, 0)$
GP-Bayesian Optimization (EI)	20	15.68	-53.655	$(3, 0)$

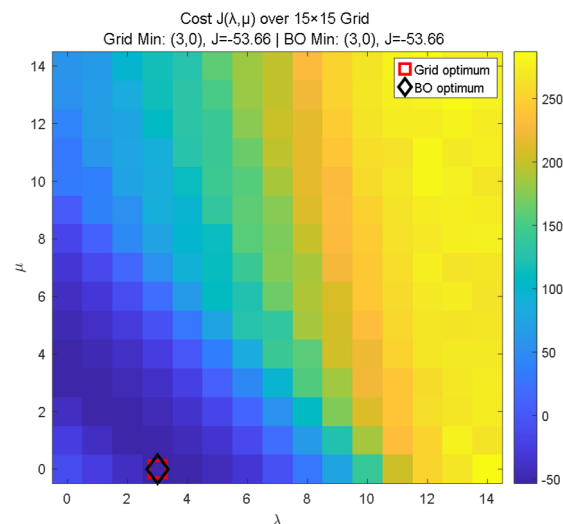


Figure 8. Distribution of the composite cost function.

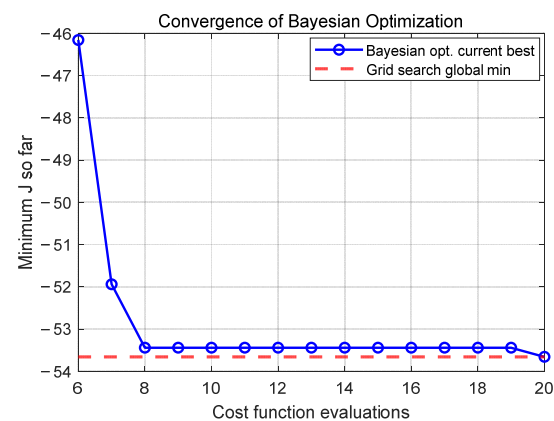


Figure 9. Convergence curve of the minimal composite cost J during Gaussian-process-based Bayesian optimization, compared against the global minimum obtained via exhaustive 15×15 grid search.

This optimal solution is an asymmetric parameter combination, indicating that moderately expanding the notch to the left side of the interference (opposite to the target) achieves the best overall performance in this scenario.

To further evaluate the adaptability of the adaptive optimization mechanism to different spatial layouts, the azimuth of the weak target was gradually changed from 33° to 21° , and the optimal parameters (λ^*, μ^*) were solved for each layout. The results are shown in Figure 10.

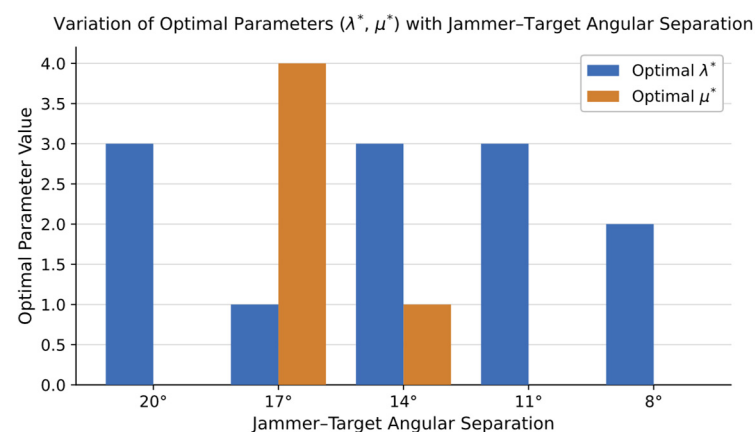


Figure 10. Variation in optimal parameters with interference-target angular separation.

It was found that the optimal parameters adjust adaptively with the angular separation between interference and target. Notably, for angular separations of 20° , 11° , and 8° , the optimal μ^* does not yield a valid non-zero solution (i.e., the value of optimal μ^* is zero and thus not plotted in the histogram). When the angular separation is large and the target is far from the notch coverage (e.g., 20°), only a large optimal λ^* is required to fully suppress residual power, while the optimal μ^* equals zero, meaning no extra expansion weight is needed on the target side. When the angular separation decreases to 17° and 14° , both non-zero optimal λ^* and μ^* can be obtained; the extension parameter μ^* on the target side is reduced adaptively as the angular interval shrinks to avoid notch coverage, resulting in an apparent asymmetric optimal solution. When the angular separation further narrows to 11° and 8° , the optimal μ^* returns to zero again, and only λ^* is adjusted to balance performance. The above results prove that the composite cost function can automatically balance interference suppression and target protection based on the prior azimuth of the target, and the proposed adaptive mechanism has good adaptability to various spatial layouts of interference and targets.

3.5. Performance Verification in Critical Scenarios

To test the performance of the adaptive optimization mechanism under extreme conditions, a critical simulation scenario was established. The interference was fixed at 13° with INR of 20 dB, while the weak target was moved to 21° , narrowing the angular separation down to 8° . In this case, the target fell within the coverage of the interference mainlobe, and no independent distinguishable spectral peak formed in the CBF spatial spectrum. This is a typical critical condition for evaluating the resolution capability of algorithms.

Figure 11 compares the normalized spatial spectra of different methods in this critical scenario. For CBF, only a slight fluctuation appeared on the falling edge of the interference mainlobe at the target azimuth, with the local SNR as low as -2.4 dB, making effective target detection impossible. The original IBF ($\lambda = \mu = 0$) formed a deep null at the interference direction and generated an identifiable spectral peak at the target position, raising the local SNR to 2.5 dB. However, adopting large symmetric parameters such as (3,3) or (4,4) will cause the over-wide notch to cover the adjacent target completely, making the target spectral peak disappear. In contrast, parameter-adaptive optimization selected asymmetric optimal parameters for this scenario. The notch was extended to the left side of the interference (away from the target) for enhanced interference suppression, while no extension was set on the right side to keep clear of the target. Consequently, a distinct spectral peak formed at 27° , and the local SNR improved to 7.9 dB.

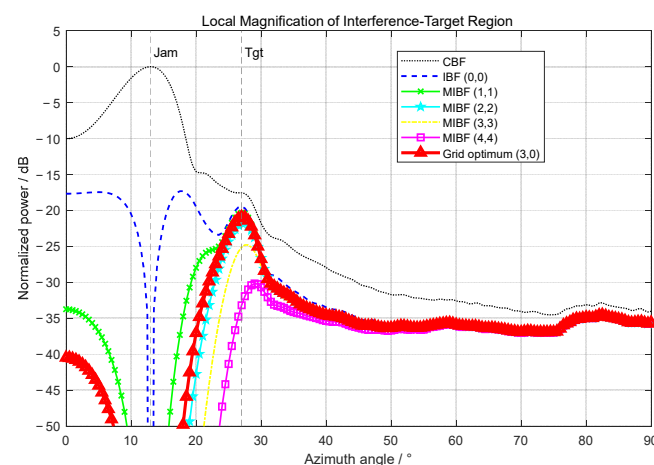


Figure 11. Comparison of spatial spectra of different methods in the critical scenario.

Table 5 summarizes the quantitative results in the critical scenario. The optimal parameters obtained by adaptive optimization improve the target local SNR by 10.3 dB compared with CBF and 5.4 dB compared with the original IBF. The proposed method can still recover the target when symmetric parameters fail completely. The results fully demonstrate that the accurate selection of asymmetric parameters is the key to the superior performance of MIBF when interference and target are closely spaced. The presented parameter-adaptive optimization mechanism realizes such accurate parameter selection, and its practical value is fully reflected in critical scenarios.

Table 5. Comparison of local SNR of different methods in the critical scenario.

Method	CBF	Original IBF	IBF (2, 2)	Adaptive (3, 0)
Local SNR/dB	−2.4	2.5	Target submerged	7.9
Gain vs. CBF/dB	-	4.9	-	10.3

3.6. Sea Trial Data Validation

Numerical simulations verify the algorithm performance under ideal array conditions, while practical marine environments differ greatly from ideal assumptions. Towed linear arrays suffer from array shape distortion induced by ocean current disturbance during navigation. The spatiotemporal variation of hydrological conditions makes acoustic propagation deviate from the ideal plane wave model, and marine ambient noise also exhibits non-stationary characteristics. To evaluate the applicability of MIBF in complex practical environments, the shallow-water measurement data from the fixed linear array in the S59 Event of the Swellex96 experiment was adopted for algorithm validation [22].

This paper uses the field data collected by the northern fixed array in the Event S59 of the Swellex-96 sea trial to verify the performance of the proposed strong interference suppression algorithm in real marine environments. Similar to Event S5, the sound source was towed at a constant depth during the S59 Event. Different from Event S5, an additional interference source was included in S59.

The interference source started slightly to the east between two horizontal line arrays (HLAs) and moved northward along the 180 m isobath at a speed of approximately 5 knots (2.5 m/s). In the last 25 min of the event, the tow vessel turned to the port side and completed a circular maneuver, as shown in Figure 12.

R/P FLIP listed in Table 6 refers to a large floating instrument platform capable of carrying a wide range of atmospheric and oceanographic measuring instruments. The measured data collected from the 12:05 to the 12:15 during the experiment was selected as the test samples. Other vessels also passed through the detection area during this period. Comparative tests were conducted using CBF, the proposed MIBF and the MIBF integrated with parameter-adaptive optimization. We analyzed the interference null depth, residual power suppression capability, adjustable range of interference notches and local SNR gain of weak targets under non-stationary marine noise. The engineering practicability and environmental adaptability of the proposed algorithms are fully verified, and the results are presented in Figures 13–15.

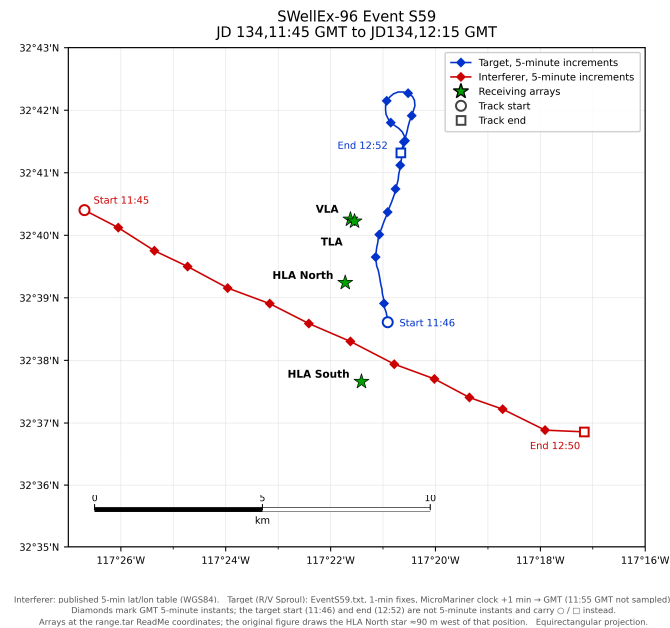


Figure 12. Situation map of Event S59 from the SWellEx-96 shallow-water acoustic experimental dataset [22].

Table 6. Real-time position of the interference source from the SWellEx-96 shallow-water acoustic experimental dataset [22].

Time (GMT)	Interferer Distance from R/P FLIP (m)	Interferer Bearing from R/P FLIP (Deg)
11:45	7937	272
11:50	6927	268
11:55	5917	261
12:00	5051	254
12:05	4185	241
12:10	3463	224
12:15	3319	202
12:20	3608	180
12:25	4474	163
12:30	5340	152
12:35	6350	146
12:40	7216	141
12:45	8514	137
12:50	9380	132

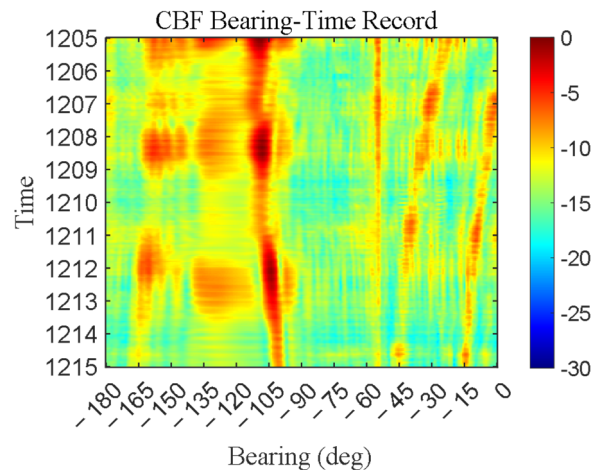


Figure 13. CBF bearing-time record.

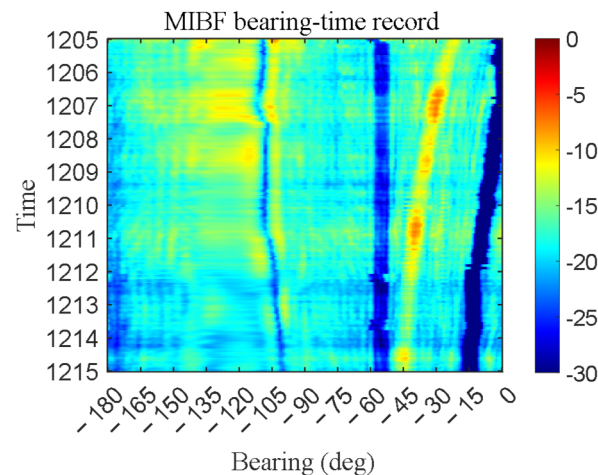


Figure 14. MIBF bearing-time record.

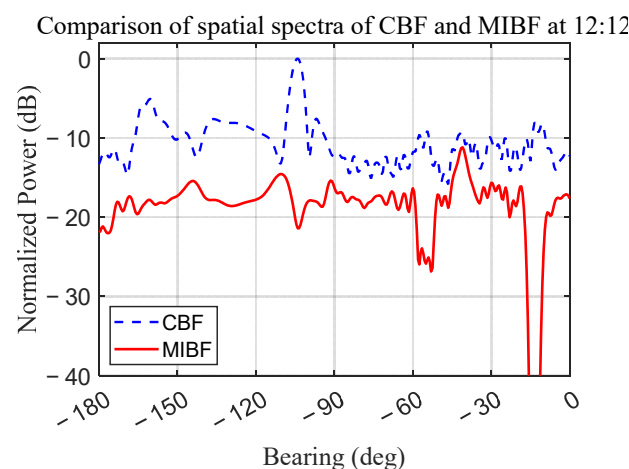


Figure 15. Comparison of spatial spectra of CBF and MIBF at 12:12.

Comparative analysis shows that the modified inverse beamforming (MIBF) achieved favorable interference suppression performance using actual sea trial data and mitigated interference effects originating from directions outside the interference arrival angle. The azimuth range of -30° to 0° corresponds to the incident direction of strong interferences configured in the simulation framework; the proposed MIBF algorithm accurately reconstructed interference components and formed deep nulls within this angular region to attenuate interference power, yielding the evident reduction of spatial spectrum amplitude observed in Figure 15. Relevant elaborations on this phenomenon have been provided in the caption of Figure 14 and the corresponding analytical section of the main text. Due to non-ideal hydrological conditions, the local signal-to-noise ratio (SNR) of conventional beamforming was 1.53 dB at the selected spatial spectrum snapshot, whereas the local SNR of the modified inverse beamforming rose to 2.68 dB. Although the local SNR gain for weak targets is inferior to the performance obtained from numerical simulations, the proposed algorithm is proven to suppress strong interferences effectively under realistic marine environments, and its practical efficacy is fully validated via measured sea trial data.

4. Conclusions

Aiming at the problem of weak target detection under strong underwater interference, this paper improves and extends the inverse beamforming algorithm. The main work and conclusions are summarized as follows:

The internal mechanism of the original IBF was theoretically analyzed. It forms a deep null at the interference direction but leaves strong residual power outside the null region, which is essentially caused by the failure to reconstruct ideal interference signals.

A modified inverse beamforming algorithm based on the least squares method is proposed. By adjusting the order of the array manifold matrix via parameters, the width and depth of interference notches can be flexibly controlled. Simulation results show that the proposed algorithm suppresses in-mainlobe power by at least 30 dB and achieves an additional local SNR gain of 2–3 dB.

To address the limitation of manual parameter tuning for notch configuration, a composite cost function was constructed to balance residual power suppression and weak target protection. A parameter-adaptive selection mechanism based on grid search and Bayesian optimization is proposed, which can automatically obtain the optimal asymmetric parameters according to the relative azimuth of interference and targets. In critical scenarios where interference and targets are closely spaced, this mechanism improves the target local SNR by up to 10.3 dB compared with CBF.

Sea trial data validated the effectiveness of the proposed algorithm for strong interference suppression under practical conditions with array shape distortion.

Future research will focus on two directions. First, array shape distortion correction will be combined with the modified IBF to further improve weak target gain in practical environments. Second, the parameter-adaptive optimization mechanism will be extended to complex scenarios with multiple interference sources, and we will develop cost functions and efficient optimization methods to collaboratively shape multiple interference notches.

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