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Quantum-Inspired Bio-Optimization for Robust Received Signal Strength-Based Underwater Target Localization in Noisy Stratified Acoustic Environments

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Abstract

Robust underwater target localization is essential for marine sensing and monitoring. However, RSS-based localization is affected by ray bending caused by sound-speed variations, anchor-node position perturbations, and non-line-of-sight (NLOS) propagation biases, resulting in a nonlinear and noise-sensitive estimation problem. To address these challenges, this paper proposes a Quantum-Inspired Animated Oat Optimization (QIAOO)-based localization algorithm for stratified underwater acoustic environments. First, a multi-error-coupled RSS localization model is established by jointly considering acoustic ray propagation effects, anchor-node perturbations, and NLOS propagation biases. Subsequently, the QIAOO algorithm is developed to solve the resulting nonlinear optimization problem, where quantum-inspired initialization, elite opposition-based learning, and a dynamic Cauchy–Gaussian mutation mechanism are incorporated to enhance search diversity and convergence performance. In addition, the Cramér–Rao Lower Bound (CRLB) is derived under the same localization model to provide a theoretical performance benchmark. Simulation results demonstrate that the proposed method outperforms benchmark algorithms in terms of localization accuracy and robustness under various scenarios.

Keywords: underwater target localization; received signal strength signal processing; stratified propagation; non-line-of-sight (NLOS) bias; anchor perturbation; quantum-inspired bio-optimization



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1. Introduction

Robust RSS-based target localization is a fundamental component of intelligent sensing and monitoring in underwater wireless sensor networks (UWSNs) [1,2]. For convenience, the abbreviations used in this manuscript are listed in Appendix A1. Compared with other range-based localization techniques, RSS-based localization provides a cost-effective and energy-efficient solution for resource-constrained UWSNs, as it does not require sophisticated hardware or strict time synchronization. However, achieving high robustness in practical underwater environments remains difficult due to complex acoustic propagation characteristics [3]. Specifically, variations in the sound speed profile (SSP) cause acoustic signals to propagate along curved paths rather than straight lines, rendering simple Euclidean distance models inaccurate [4]. Furthermore, ocean currents and mooring motions

often lead to anchor-node position perturbations, while underwater obstacles like reefs or marine organisms introduce non-line-of-sight (NLOS) propagation biases [5]. These coupled impairments result in a nonlinear and noise-sensitive observation model, where conventional localization methods often suffer from significant ranging errors without an effective mitigation strategy [6]. Therefore, robust target localization in complex underwater environments has become a research hotspot in underwater sensing and marine information systems.

Existing underwater localization techniques can generally be divided into range-free and range-based methods according to whether distance-related measurements are used. Compared with range-free schemes, range-based methods usually provide higher localization accuracy and have therefore attracted considerable attention [7,8]. Typical range-based measurements include time of arrival (TOA) [9], time difference of arrival (TDOA) [10], RSS [11,12], and angle of arrival (AOA) [13]. Based on these technologies, extensive research has been conducted on localization. For instance, Mei et al. derived a closed-form TOA-based localization solution in stratified underwater environments, where both ideal and perturbed anchor-node positions were considered [4]. Yang et al. proposed a weighted AOA-based localization method to improve localization accuracy in multipath environments by estimating the asymptotic variance of AOA errors and designing corresponding weighting coefficients [14]. Zhou et al. developed a three-dimensional AOA localization model by incorporating correlated noise and Gaussian prior information [15]. Zou et al. studied a TDOA-based joint localization and unknown propagation speed estimation problem, and solved the corresponding Maximum Likelihood (ML) estimation problem using Semidefinite Programming (SDP) in the presence of sensor position errors [16]. Although these methods can achieve satisfactory localization accuracy in specific scenarios, TOA- and TDOA-based schemes generally require accurate time synchronization among nodes [17], while AOA-based schemes depend on complex array hardware and angle measurement devices [18]. These requirements may increase the deployment cost and energy consumption of resource-constrained underwater sensing systems. In contrast, RSS-based localization has attracted increasing attention because it requires relatively simple hardware, does not rely on strict time synchronization, and can be implemented with low computational and energy overhead.

However, the localization models established based on the aforementioned ranging techniques are inherently nonlinear and sensitive to noise. Furthermore, when the aforementioned environmental impairments are taken into account, traditional iterative estimation methods, such as the Gauss–Newton method and least squares (LS) [19,20], often suffer from poor localization accuracy and limited robustness due to their strong dependence on the selection of initial values. As a result, Meta-heuristic optimization algorithms provide a feasible alternative for solving complex non-convex problems because of their global search capability. For instance, Ge et al. proposed a robust localization algorithm that combines distributed estimation with Particle Swarm Optimization (PSO) to reduce large errors caused by uncertainties in underwater acoustic communication and sampling processes [21]. Mei et al. developed an RSS ranging model considering stratified underwater structures and incorporated quantum mechanisms and multiple search strategies into Harris Hawks Optimization (HHO), resulting in a Quantum-Enhanced Harris Hawks Optimization (QEHHO) algorithm for complex non-convex localization problems [22]. Jin et al. designed an improved PSO algorithm for TDOA-based localization by dynamically adjusting inertia weights and learning factors according to individual fitness [23]. These studies demonstrate the potential of bio-inspired optimization in underwater localization and noisy signal processing. Nevertheless, many existing swarm-intelligence methods still suffer from insufficient population diversity, uneven initial search-space coverage, pre-

mature convergence, and an imbalance between global exploration and local exploitation. These limitations become more pronounced when the localization objective is constructed under multi-source coupled errors. For example, the Grey Wolf Optimizer (GWO) [24] and Sparrow Search Algorithm (SSA) [25] commonly rely on random population initialization, which may lead to uneven search-space coverage and insufficient population diversity. Moreover, their evolutionary mechanisms may fail to adaptively balance global exploration and local exploitation in complex error-coupled search spaces, resulting in premature convergence and degraded localization accuracy. Therefore, it is necessary to develop a more robust bio-inspired optimization framework for RSS-based underwater signal processing in noisy acoustic environments.

Motivated by the above observations, this paper proposes a quantum-inspired bio-optimization framework for robust RSS-based underwater target localization under stratified propagation, anchor-node perturbations, and NLOS biases. First, an RSS-based localization model is constructed by incorporating ray tracing theory under an isogradient SSP, so that the curved acoustic propagation path can be compensated in the observation model. Anchor-node position perturbations and NLOS biases are further incorporated into the model as error terms to better describe practical underwater localization environments. Then, an improved Animated Oat Optimization (AOO) algorithm [26], termed Quantum-Inspired Animated Oat Optimization (QIAOO), is developed to solve the resulting nonlinear localization problem. In the proposed QIAOO, Quantum Good Point Set (QGPS) initialization is used to improve initial population coverage, Elite Oppositional-Based Learning (EOBL) is introduced to enhance the ability to escape local optima, and a dynamic Cauchy–Gaussian mixed mutation (CGMM) strategy is designed to maintain population diversity and improve local refinement. In addition, the Cramér–Rao lower bound (CRLB) is derived under the same multi-error-coupled model to provide a theoretical benchmark for evaluating the achievable localization accuracy. Simulation results demonstrate that, in multi-error underwater environments, the proposed QIAOO-based localization scheme achieves higher localization accuracy and stronger robustness than other benchmark methods.

The main contributions of this paper are summarized as follows:

- Multi-error-coupled RSS localization model: A stratification-aware RSS localization model is developed for underwater target localization by incorporating ray tracing under an isogradient SSP, anchor-node position perturbations, and NLOS biases into a unified mathematical framework.
- Quantum-inspired bio-optimization algorithm: A QIAOO algorithm is proposed to solve the resulting nonlinear and non-convex localization problem. Quantum Good Point Set initialization, Elite Oppositional-Based Learning, and dynamic Cauchy–Gaussian mixed mutation are integrated to improve population diversity, global exploration, and local exploitation.
- CRLB-based performance evaluation: The CRLB is derived under the proposed multi-error-coupled RSS model to provide a theoretical benchmark for evaluating the localization performance under stratified propagation, anchor-node perturbations, and NLOS biases.
- Robustness validation in noisy underwater environments: Simulation experiments are conducted under different anchor-node numbers, noise levels, NLOS bias intensities, and anchor-node perturbation variances. The results verify that the proposed QIAOO-based localization scheme achieves superior localization performance and stronger robustness compared with mathematical localization algorithms and swarm intelligence algorithms.

The remainder of this paper is organized as follows. Section 2 presents the RSS-based localization model under sound speed stratification, anchor-node perturbations, and NLOS biases. Section 3 introduces the original AOO algorithm and describes the improvement strategies used in the proposed QIAOO algorithm. Section 4 derives the CRLB under the proposed multi-error-coupled RSS model. Section 5 presents the simulation results and comparative analyses under different underwater conditions. Finally, Section 6 concludes this paper and discusses future research directions.

2. Problem Formulation

2.1. Acoustic Ranging Model Based on RSS

In UWSNs, underwater node positions may deviate from their nominal locations due to ocean-current disturbances, mooring motion, and deployment errors. Suppose that N anchor nodes with known but erroneous observed positions and one target node to be localized are deployed. To ensure the solvability of the localization problem, the anchor nodes are assumed to be non-collinear. Let the observed and true positions of the i^{th} anchor node be denoted by $\mathbf{t}_i = [t_{i1}, t_{i2}, t_{i3}]^T$ and $\tilde{\mathbf{t}}_i = [\tilde{t}_{i1}, \tilde{t}_{i2}, \tilde{t}_{i3}]^T$, respectively, where $i = 1, \dots, N$. The relationship between the observed and true positions is modeled as $\tilde{\mathbf{t}}_i = \mathbf{t}_i + \boldsymbol{\varepsilon}_i$, where $\boldsymbol{\varepsilon}_i$ represents the anchor-position perturbation. It is assumed that $\boldsymbol{\varepsilon}_i$ follows a zero-mean Gaussian distribution, i.e., $\boldsymbol{\varepsilon}_i \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{t,i})$. For the isotropic perturbation case, $\boldsymbol{\Sigma}_{t,i} = \delta_i^2 \mathbf{I}_3$, where δ_i^2 is the perturbation variance. Furthermore, it is assumed that all nodes are equipped with depth pressure sensors, enabling them to obtain their depth information. The main variables and symbols involved in the localization model are summarized in Appendix A2.

Based on the aforementioned system model, the target node acquires RSS measurements from the anchor nodes. For the i^{th} anchor node, the RSS measurement model can be formulated as [27]:

$$P_{ri} = P_0 - 10n \log_{10} \frac{d_i}{d_0} - \alpha_f(d_i - d_0) + m_i + \eta_i, \quad (1)$$

where P_{ri} is the received signal power associated with the i^{th} anchor node; P_0 denotes the received power at the reference distance d_0 ; n is the path loss exponent; d_0 represents the reference distance, typically set as 1 m; d_i is the propagation distance between the i^{th} anchor node and the target node; η_i is the RSS measurement noise following a zero-mean Gaussian distribution with variance σ_i^2 , i.e., $\eta_i \sim \mathcal{N}(0, \sigma_i^2)$; and m_i denotes the NLOS bias. In this work, the NLOS bias is modeled as $m_i = \tilde{m}_i + \tau_i$, where $\tilde{m}_i$ is the nominal NLOS bias and τ_i is the corresponding uncertainty term. It is assumed that τ_i follows a zero-mean Gaussian distribution with variance β_i^2 , i.e., $\tau_i \sim \mathcal{N}(0, \beta_i^2)$. The absorption coefficient α_f can be determined using Thorp's formula as a function of the acoustic frequency f [28]:

$$\alpha_f = 0.11 \frac{f^2}{1 + f^2} + 44 \frac{f^2}{4100 + f^2} + 2.75 \times 10^{-4} f^2 + 0.003. \quad (2)$$

2.2. Underwater Isogradient SSP Model

Spatial variations in underwater temperature, salinity, and depth lead to the stratification of sound speed, which is usually characterized by the SSP. Under this influence, as the propagation distance increases, the discrepancy between the actual curved propagation path and the assumed Euclidean distance becomes increasingly significant. Therefore, conventional Euclidean distance-based ranging models cannot accurately characterize the true acoustic propagation distance, resulting in degraded localization accuracy, as shown in Figure 1. To address this issue, this paper introduces ray tracing theory and Snell's law

of refraction, and employs an isogradients SSP model to characterize the propagation paths of underwater acoustic signals.

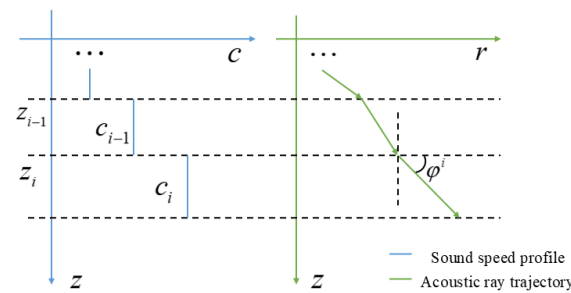


Figure 1. Schematic diagram of the influence of depth-dependent sound speed on acoustic propagation paths.

In a stratified environment, assuming that the sound speed C varies linearly with depth z , the functional relationship is expressed as:

$$C(z) = az + b, \quad (3)$$

where a denotes the sound speed gradient, which characterizes the rate of change of sound speed with respect to depth; z is the depth variable; b represents the sound speed at the reference depth $z = 0$; and $C(z)$ is the acoustic propagation speed at depth z .

According to Snell's law and ray tracing theory, the propagation of underwater acoustic signals in a stratified environment satisfies:

$$\frac{\cos \varphi}{C(z)} = \frac{\cos \varphi^u}{C(u_3)} = \frac{\cos \varphi^i}{C(\tilde{t}_{i3})} = k, \quad (4)$$

where k is a constant; φ denotes the grazing angle of the acoustic signal; $\varphi^u, \varphi^i \in [-\pi/2, \pi/2]$ represent the grazing angles at the target node and the i^{th} anchor node, respectively, as shown in Figure 2, since the grazing angle is defined relative to the horizontal direction, the interval $[-\pi/2, \pi/2]$ is sufficient to characterize all possible upward and downward acoustic propagation paths while avoiding directional ambiguity; and $C(u_3)$ and $C(\tilde{t}_{i3})$ are the sound speeds at depths u_3 and $\tilde{t}_{i3}$, respectively. By integrating along the acoustic propagation path, we obtain:

$$\tan \varphi = \frac{\partial z}{\partial r}, \quad \sin \varphi = \frac{\partial z}{\partial l}, \quad C(z) = \frac{\partial l}{\partial t}, \quad (5)$$

where l denotes the arc length along the acoustic propagation path; t is the propagation time of the underwater acoustic signal; and r is the horizontal distance between the i^{th} anchor node and the target node, satisfying $r = \sqrt{(u_1 - \tilde{t}_{i1})^2 + (u_2 - \tilde{t}_{i2})^2}$. By rearranging the terms and differentiating with respect to φ , we obtain:

$$\partial z = -\frac{1}{ak} \sin \varphi \partial \varphi, \quad (6)$$

By substituting Equation (6) into Equation (5), the compensation form for curved path propagation under the stratification effect can be derived as:

$$l_i = -(au_3 + b) \frac{\varphi^u - \varphi^i}{a \cos \varphi^u}. \quad (7)$$

where l_i denotes the length of the curved propagation path between the i^{th} anchor node and the target node.

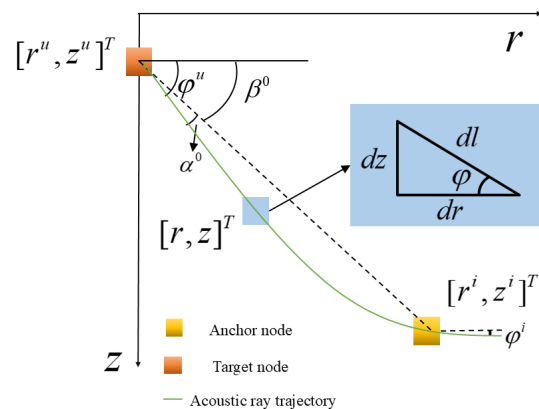


Figure 2. Schematic diagram of underwater acoustic signal propagation.

For notational simplicity, $d_0 = 1$ m is adopted in the following derivation, and the corresponding constant absorption term is absorbed into P_0 . By substituting Equation (7) into Equation (1), the RSS measurement model can be reformulated as:

$$P_{ri} = P_0 - 10n \log_{10} l_i - \alpha_f l_i + m_i + \eta_i, \quad (8)$$

It should be clarified that Equation (1) describes the conventional RSS propagation model under ideal conditions, whereas Equation (8) represents the RSS propagation model that accounts for curved propagation paths induced by stratified sound speed profiles. Therefore, Equation (1) serves as the theoretical foundation of the proposed model, while Equation (8) is employed for localization and performance evaluation throughout this work.

2.3. Mathematical Model of the Localization Algorithm

By rearranging the terms in Equation (8) and applying an exponential transformation, we have:

$$10^{\frac{P_{ri}}{10n}} l_i = 10^{\frac{P_0 - \alpha_f l_i + \bar{m}_i}{10n}} 10^{\frac{\eta_i + \tau_i}{10n}}, \quad (9)$$

Under the assumption of small noise for η_i and τ_i relative to the received signal power, the term $10^{(\eta_i + \tau_i)/10n}$ in Equation 9 can be approximated using a first-order Taylor series expansion as follows:

$$\mu_i l_i \approx \lambda_i \left(1 + \frac{\ln 10}{10n} (\eta_i + \tau_i) \right), \quad (10)$$

where $\mu_i = 10^{\frac{P_{ri}}{10n}}$ and $\lambda_i = 10^{\frac{P_0 - \alpha_f l_i + \bar{m}_i}{10n}}$. This first-order approximation holds under reasonable noise conditions where η_i and τ_i are relatively small compared to the received signal power. Notably, this first-order approximation is valid for the investigated noise levels because the resulting modeling bias is mathematically calculated to be less than 1% of the ranging distance, which is negligible compared to the environmental measurement noise. However, it should be acknowledged that for extremely high noise levels, the neglected second-order terms in the Taylor expansion may introduce a slight estimation bias.

Based on the aforementioned approximation, the objective function with respect to the target position $\mathbf{u}$ is formulated as follows:

$$\arg \min_{\mathbf{u}} \sum_{i=1}^N (\mu_i l_i - \lambda_i)^2, \quad (11)$$

Let $\mathbf{y} = [u_1, u_2, u_3, \chi]^T$, where $\chi = \mathbf{u}^T \mathbf{u}$. By squaring and rearranging each term in Equation (11), we obtain:

$$\arg \min_{\mathbf{y}} \|\mathbf{A}\mathbf{y} - \mathbf{B}\|^2, \quad (12)$$

$$\text{where } \mathbf{A} = \begin{bmatrix} 2\mu_1^2 t_{11} & 2\mu_1^2 t_{12} & 2\mu_1^2 t_{13} & -\mu_1^2 \\ 2\mu_2^2 t_{21} & 2\mu_2^2 t_{22} & 2\mu_2^2 t_{23} & -\mu_2^2 \\ \vdots & \vdots & \vdots & \vdots \\ 2\mu_N^2 t_{N1} & 2\mu_N^2 t_{N2} & 2\mu_N^2 t_{N3} & -\mu_N^2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mu_1^2 \|\mathbf{t}_1\|^2 - \lambda_1^2 \\ \mu_2^2 \|\mathbf{t}_2\|^2 - \lambda_2^2 \\ \vdots \\ \mu_N^2 \|\mathbf{t}_N\|^2 - \lambda_N^2 \end{bmatrix}.$$

3. Proposed Improvements: The QIAOO Algorithm

3.1. Original AOO Algorithm

AOO is a nature-inspired swarm intelligence meta-heuristic algorithm inspired by the dispersal behavior of wild oat seeds. By mimicking dynamic processes such as rolling, dispersing, and ejecting, the algorithm facilitates individual position updates within the search space to achieve a balance between global exploration and local exploitation. The corresponding mathematical formulation is given as follows.

Each individual in the population represents a parameter vector to be estimated. For simplicity, the variables r_{init} , r_W , r_e , r_o , etc., denote random scalars or vectors independently drawn from a uniform distribution over the interval $[0, 1]$.

During the initialization phase, the AOO algorithm randomly generates an initial population within the search space to ensure comprehensive coverage and population diversity. Specifically, random initialization is performed within the predefined feasible region as follows:

$$y_{i,j} = LB_j + r_{\text{init}} \cdot (UB_j - LB_j), \quad i = 1, 2, \dots, N_p, j = 1, 2, \dots, D, \quad (13)$$

where D denotes the dimension of the search space; N_p represents the population size; UB_j and LB_j correspond to the upper and lower bounds of the j^{th} dimension, respectively.

During the movement phase, the movement of AOO is governed by a dynamic decay parameter $\alpha(t)$, which simulates energy dissipation during motion.

$$\alpha(t) = q \cdot \exp\left(-\frac{m_{\text{seed}} \cdot e}{L} \cdot \frac{t}{T_{\text{max}}}\right) \quad (14)$$

where m_{seed} denotes the mass of the seed; L is the length of the seed's main awn; e represents the eccentricity coefficient during rolling; t and T_{max} are the current and maximum number of iterations, respectively; and q is the dynamic adjustment factor. In the initial stage of iterations when t is small, the parameter $\alpha(t)$ maintains a high value, granting the seeds robust rolling and dispersal capabilities, which facilitates global exploration. As the number of iterations increases, $\alpha(t)$ gradually decreases, simulating the seeds stabilizing due to energy dissipation, thereby enhancing the algorithm's exploitation precision in localized regions.

In the exploration phase, the individual position update incorporates the population mean, the current best individual, and the individual's own position information, sup-

plemented by a random perturbation term. The position update for an individual in the $(t + 1)^{\text{th}}$ iteration is expressed as [26]:

$$y_{t+1}(i) = \begin{cases} \frac{1}{N_p} \sum_{q=1}^{N_p} y_t(q) + W, & \text{if } \text{mod}(i, N_p/10) = 0, \\ y_{\text{best}} + W, & \text{if } \text{mod}(i, N_p/10) = 1, \\ y_t(i) + W, & \text{otherwise,} \end{cases} \quad (15)$$

where $y_{t+1}(i)$ denotes the i^{th} individual in the $(t + 1)^{\text{th}}$ iteration; y_{best} is the current best individual in the population; $W = \frac{q}{\pi} \times (2 \times r_W - 1) \otimes \mathbf{UB}$ represents the random dispersal term simulating the influence of the external environment; and $\otimes$ denotes element-wise multiplication.

In the exploitation phase, individuals adopt different update strategies—including rolling and ejection—depending on the environmental constraints. Specifically, the rolling mechanism facilitates a fine-grained search through local perturbations to improve solution precision, while the ejection mechanism employs large step-size updates to enhance the ability to escape local optima. Assuming these two mechanisms occur with equal probability, the position update in obstacle-free conditions is primarily executed via the rolling mechanism, which is formulated as:

$$y_t(i) = y_{\text{best}} + R + q \cdot \text{Levy}(D) \otimes y_{\text{best}}, \quad (16)$$

where $R = (m_{\text{seed}} \cdot e + L^2) \cdot \frac{r_a(-A, A)}{D}$ is the fine-tuning parameter for the rolling mechanism; $r_a(-A, A)$ is a random matrix with elements sampled from the range $[-A, A]$; $\text{Levy}(D) = 0.01 \times \frac{\phi \cdot \psi}{\|v\|^{1/\zeta}}$ denotes the Lévy flight mechanism [29], which is employed to enhance the diversity of position updates; ϕ is a random coefficient uniformly distributed in the interval $[0, 1]$, representing the population mean position; v denotes the current velocity vector; and ψ signifies the step-size scale parameter that controls the search range, formulated as:

$$\psi = \left(\frac{\Gamma(1 + \zeta) \cdot \sin\left(\frac{\pi \cdot \zeta}{2}\right)}{\Gamma\left(\frac{1 + \zeta}{2}\right) \cdot \zeta \cdot 2^{\left(\frac{\zeta - 1}{2}\right)}} \right)^{\frac{1}{\zeta}}, \quad (17)$$

where ζ is the stability index of the Lévy flight, which governs the randomness and diversity of the step size. In this paper, it is set to $\zeta = 1.5$. $\Gamma(\cdot)$ denotes the Gamma function, which extends the factorial operation to non-integer arguments.

When individuals encounter obstacles during the dispersal process, the ejection mechanism is triggered to facilitate leaps across the obstructed regions. The position update formula for the ejection mechanism is expressed as:

$$y_t(i) = y_{\text{best}} + G + q \cdot \text{Levy}(D) \otimes y_{\text{best}}, \quad (18)$$

where $G = \frac{2 \cdot \rho \cdot x^2 \cdot \sin(2\omega)}{m_{\text{seed}}} \cdot \frac{r_b(-B, B)}{D} \cdot (1 - \alpha)$ is the parameter controlling position fine-tuning in the rolling mechanism; $r_b(-B, B)$ is a random matrix with elements sampled from $[-B, B]$; $\rho = 0.5 + 0.5 \times r_e$ denotes the elasticity coefficient of the seed's main awn in the ejection mechanism; $x = 3 \times \frac{r_e}{D}$ represents the deformation (change in length) of the main awn during the energy storage phase; $\omega = \pi \cdot r_e$ is the projection angle formed

between the ejection trajectory and the horizontal plane; $\alpha = \frac{1}{\pi} \times e^{\frac{r'_e}{T_{\text{max}}}}$ signifies the air

drag coefficient during the ejection movement; and $e \frac{r'_e}{T_{\max}}$ is a random value in the range of 0 to T ; $\otimes$ denotes element-wise multiplication.

In summary, although the AOO algorithm achieves a certain balance between global exploration and local exploitation, it still faces limitations in complex optimization landscapes. The stochastic initialization strategy may lead to an uneven distribution of the initial population, thereby reducing the quality of early-stage exploration. In addition, the high randomness in the position update mechanism increases the risk of premature convergence to local optima. To address these issues, multiple enhancement strategies are incorporated into the AOO framework to improve search performance, convergence accuracy, and the ability to escape local optima.

3.2. Improved Population Initialization Strategy

Good Point Set (GPS) [30]: To address the local aggregation issue of the initial population in the search space of the AOO algorithm, Good Point Set theory is introduced to optimize the initialization process described in Equation (13).

Assume that GD is the unit hypercube in the D -dimensional Euclidean space. Given a good point $h \in GD$, the good point set $H_{N_p}(k)$ is defined as:

$$H_{N_p}(k) = \{\{k \cdot h_1\}, \{k \cdot h_2\}, \dots, \{k \cdot h_D\}\}, \quad k = 1, 2, \dots, N_p, \quad (19)$$

where $\{\cdot\}$ denotes the fractional part of a real number; $h \equiv (h_1, \dots, h_D)$ is the generator vector of the good point set, which is specifically expressed as:

$$h_j = \left\{ 2 \cos \left(\frac{2\pi j}{p} \right) \right\}, \quad j = 1, 2, \dots, D, \quad (20)$$

where p is the smallest prime number satisfying $(p - D/2) \geq D$.

Utilizing the generated good point sequences, the population initialization in Equation (13) is optimized as follows:

$$y_{i,j} = LB_j + H_{N_p,j}(i) \cdot (UB_j - LB_j), \quad i = 1, 2, \dots, N_p, \quad j = 1, 2, \dots, D, \quad (21)$$

where $H_{N_p,j}(i)$ denotes the j^{th} component of the i^{th} individual in the good point sequence.

This strategy enables a quasi-uniform distribution in the search space, thereby expanding the coverage of initial feasible solutions and enhancing the global exploration capability of the algorithm.

Quantum Computing [31]: To further augment population diversity, a Quantum Good Point Set initialization strategy is proposed. This strategy utilizes the GPS sequence as the quantum phase input and generates a double-sized initial population through probability amplitude mapping. In quantum computing, the state of a qubit is represented as a linear combination of the basis states $\iota_0|0\rangle$ and $\iota_1|1\rangle$. The quantum state $|\mathbb{Z}\rangle$ is defined as:

$$|\mathbb{Z}\rangle = \iota_0|0\rangle + \iota_1|1\rangle, \quad (22)$$

where ι_0 and ι_1 are probability amplitudes satisfying the normalization condition $|\iota_0|^2 + |\iota_1|^2 = 1$. By utilizing the phase angle θ , the qubit state can be parameterized as:

$$[\iota_0, \iota_1]^T = [\cos \theta, \sin \theta]^T \quad (23)$$

For the i^{th} individual in a D -dimensional space, its quantum probability amplitude matrix $\mathbb{Q}_i$ is defined as:

$$\mathbb{Q}_i = \begin{bmatrix} \cos \theta_{i,1} & \cos \theta_{i,2} & \dots & \cos \theta_{i,D} \\ \sin \theta_{i,1} & \sin \theta_{i,2} & \dots & \sin \theta_{i,D} \end{bmatrix}. \quad (24)$$

To ensure a uniform distribution of the quantum phases, the phase angle θ is constructed using the good point sequence generated by Equation (20), as follows:

$$\theta_{i,j} = 2\pi \cdot H_{N_p,j}(i), \quad i = 1, \dots, N_p, j = 1, \dots, D. \quad (25)$$

Utilizing the mapping mechanism of quantum probability amplitudes, each quantum agent yields two distinct solutions: the cosine-based solution $y_{i,j}^{(c)}$ and the sine-based solution $y_{i,j}^{(s)}$. These are subsequently transformed into the feasible range $[LB, UB]$ as follows:

$$y_{i,j}^{(c)} = \frac{1}{2} [UB_j(1 + \cos \theta_{i,j}) + LB_j(1 - \cos \theta_{i,j})], \quad (26)$$

$$y_{i,j}^{(s)} = \frac{1}{2} [UB_j(1 + \sin \theta_{i,j}) + LB_j(1 - \sin \theta_{i,j})]. \quad (27)$$

Subsequently, the elite oppositional-based learning in Equation (28) and the hybrid mutation operations in Equations (31) and (32) are all performed within the solution space formed by Equations (26) and (27). This strategy expands the initial population size to $2N_p$. The proposed Quantum Good Point Set initialization method not only preserves the low-discrepancy property of the Good Point Set but also significantly enhances individual diversity through the dual-state mapping mechanism.

3.3. Improved Position Update Strategy

Elite Oppositional-Based Learning (EOBL): To prevent stagnation in local optima, EOBL is integrated to maintain population diversity and enhance global search capability.

Within the current search space, the oppositional individual $\hat{y}_{i,j}$ corresponding to the current solution is generated via the oppositional-based search mechanism as follows:

$$\hat{y}_{i,j} = (LB_j + UB_j) - y_{i,j}. \quad (28)$$

To prevent a decline in population quality due to the stochastic nature of blind oppositional searching, elite guidance from the current best individual is utilized. By performing a convex combination of the oppositional solution and the global optimal solution, the elite oppositional individual $y'_{i,j}$ is constructed as follows:

$$y'_{i,j} = r_o \hat{y}_{i,j} + (1 - r_o) y_{\text{best},j}, \quad (29)$$

where $y_{\text{best},j}$ denotes the j component of the global best individual in the current population.

A greedy selection mechanism is then employed to update the individuals, ensuring a continuous improvement in population quality:

$$y_i^{t+1} = \begin{cases} y'_i, & \text{if } f(y'_i) < f(y_i^t), \\ y_i^t, & \text{otherwise.} \end{cases} \quad (30)$$

The Elite Oppositional-Based Learning mechanism leverages the spatial distribution information of the current best individual to guide the oppositional search direction, thereby avoiding population degradation caused by blind oppositional searching.

Cauchy–Gaussian Mixed Mutation (CGMM): To prevent population homogenization, the global best individual y_{best} is perturbed via a dynamic Cauchy–Gaussian mixed mutation strategy.

The mathematical expression of the Cauchy mutation is given as follows:

$$y_{\text{best},C}^t = y_{\text{best}} \otimes (1 + \hbar_1) \quad (31)$$

where $\hbar_1 \sim \text{Cauchy}(0, 1)$ is the random mutation factor.

Secondly, a Gaussian distribution is introduced. By exploiting its high-probability density around zero, fine-grained local search can be achieved in the neighborhood of the current best solution. The mathematical expression of the Gaussian mutation is given as follows:

$$y_{\text{best},G}^t = y_{\text{best}} \otimes (1 + \hbar_2) \quad (32)$$

where $\hbar_2 \sim \text{Gaussian}(0, 1)$ is a random perturbation factor sampled from a normal distribution.

To achieve an adaptive balance between global exploration and local exploitation throughout the evolutionary process, linear dynamic weights based on the iteration count t are introduced to construct the hybrid mutated individual y_{mut}^t :

$$y_{\text{mut}}^t = \frac{t}{T_{\text{max}}} y_{\text{best},G}^t + \left(1 - \frac{t}{T_{\text{max}}}\right) y_{\text{best},C}^t. \quad (33)$$

Upon the completion of the mutation operation, a greedy selection mechanism is implemented to update the global best individual:

$$y_{\text{best}}^{t+1} = \begin{cases} y_{\text{mut}}^t, & \text{if } f(y_{\text{mut}}^t) < f(y_{\text{best}}^t) \\ y_{\text{best}}^t, & \text{else} \end{cases} \quad (34)$$

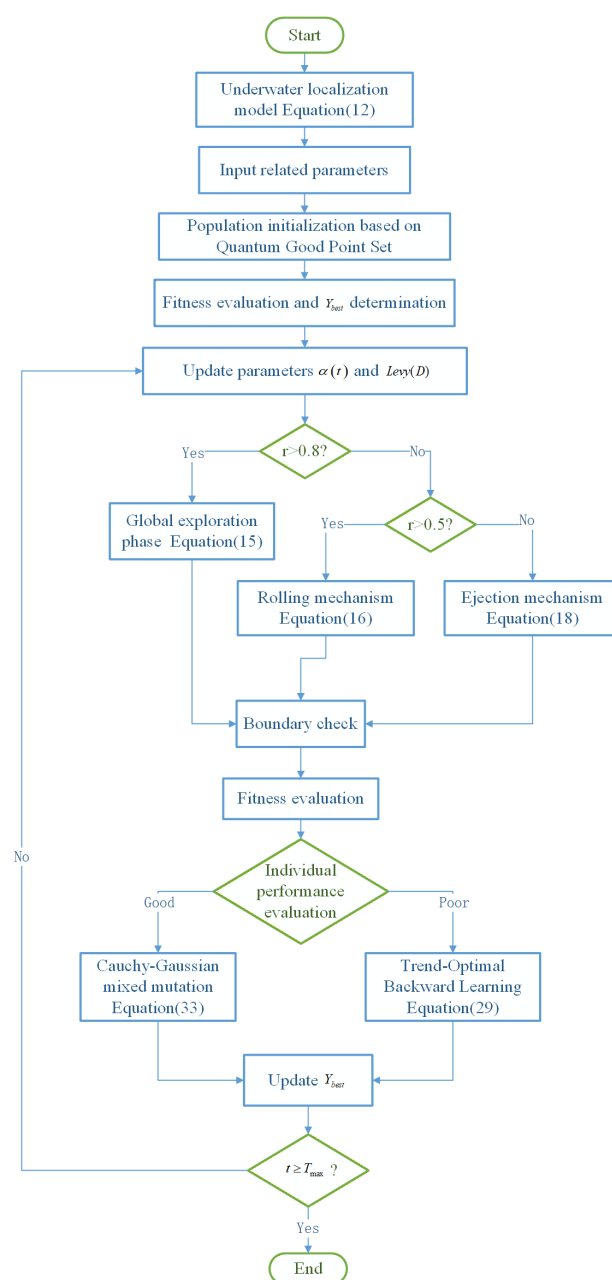
Unlike the update mechanism applied only to ordinary individuals, the proposed mutation strategy directly perturbs the global best solution y_{best} . Combined with the greedy selection mechanism (Equation (34)), the algorithm updates the best position only when a better solution is found. This ensures that the previously obtained optimal information is preserved, while enabling fine-grained exploitation of the current search region and enhancing the ability to escape local optima. Since the above subsections involve multiple optimization mechanisms, a concise summary of their key functions is provided in Table 1 to improve readability.

The search behavior of QIAOO is governed by Equation (14), which regulates the transition from global exploration to local exploitation. This specific combination of strategies is designed to address the multi-error-coupled challenges in underwater localization: (1) During the initialization stage, the low-discrepancy property of QGPS enables a more uniform spatial distribution than random sampling, which is crucial for effectively capturing ray-bending effects in complex stratified media. (2) To mitigate the impact of anchor-node position perturbations, the elite-guided mechanism in EOBL directs the population toward the true global optimum, thereby effectively reducing the estimation shift caused by coordinate errors. (3) Meanwhile, the Cauchy mutation in CGMM provides large-step perturbations, preventing the population from being trapped in local optima induced by NLOS biases. This synergistic framework achieves a favorable balance between population diversity and search accuracy, making QIAOO effective for solving localization problems involving multiple coupled error sources.

Table 1. Summary of mechanisms and strategies in the proposed QIAOO algorithm.

Component	Key Mechanism	Function in Localization
4.1 Original AOO	Random Dispersal (Equation (15)) Rolling (Equation (16)) Ejection (Equation (18))	Global search in early stages Fine-grained local exploitation Leaping across obstructed regions
4.2 Initialization	QGPS (Equations (24)–(27))	Ensures spatial coverage in 3D space
4.3 Update Strategy	EOBL (Equation (29)) CGMM (Equation (33))	Prevents stagnation in local optima Balances diversity and refinement

The flowchart of the proposed underwater localization algorithm is shown in Figure 3, and the pseudocode of QIAOO is summarized in Algorithm 1:

**Figure 3.** Flowchart of the QIAOO Algorithm.

Algorithm 1 Quantum-Inspired Animated Oat Optimization (QIAOO)

```

1: Input: Population size  $N_p$ , Maximum iterations  $T_{\max}$ , Dimension  $D$ , Search bounds  $[LB, UB]$ .
2: Output: The estimated target position  $\hat{u}$  (Global best individual  $y_{best}$ ).
3: Step 1: Initialization
4: Initialize population  $Y$  using Quantum Good Point Set mapping (Equations (19)–(27)).
5: Calculate fitness  $f(y_i)$  for each agent and identify the initial global best  $y_{best}$ .
6: Step 2: Iterative Optimization
7: for  $t = 1$  to  $T_{\max}$  do
8:   Update dynamic energy parameter  $\alpha(t)$  (Equation (14)) and generate Lévy flight vector  $Levy(D)$ .
9:   for each agent  $i = 1$  to  $N_p$  do
10:    if  $\text{rand} > 0.8$  then
11:      Execute Exploration Phase (Random Dispersal) using Equation (15).
12:    else
13:      if  $\text{rand} > 0.5$  then
14:        Execute Exploitation Phase (Rolling mechanism) using Equation (16).
15:      else
16:        Execute Exploitation Phase (Ejection mechanism) using Equation (18).
17:      end if
18:    end if
19:  end for
20:  Perform boundary check and evaluate fitness for the entire population.
21:  Calculate average fitness  $f_{avg} = \text{mean}(|f(Y)|)$ .
22: Step 3: Integrated Improvement Strategies
23: for each agent  $i = 1$  to  $N_p$  do
24:   if  $f(y_i) > f_{avg}$  then
25:     Apply EOBL using Equation (29).
26:   else
27:     if  $f(y_i) \leq f_{avg}$  then
28:       Apply CGMM using Equation (33).
29:     end if
30:   end if
31:   Use greedy selection to update  $y_i$  if the new candidate is superior.
32: end for
33: Update the global best individual  $y_{best}$  and current best score.
34: end for
35: Return  $y_{best}$  as the final localized target position  $\hat{u}$ .

```

4. RSS-Based CRLB Under Anchor Node Errors and NLOS Biases

CRLB is widely adopted as a theoretical benchmark for evaluating localization performance [32]. Specifically, the inverse of the Fisher Information Matrix (FIM) provides a lower bound on the covariance matrix of any unbiased estimator. Furthermore, a scalar lower bound for the position estimation error can be obtained by taking the trace of the inverse FIM [33].

According to the observation model established in Section 3, the RSS observation vector is defined as $\mathbf{P}_r = [P_{r1}, P_{r2}, \dots, P_{rN}]^T$. The corresponding CRLB can be expressed as:

$$\text{CRLB} = \text{Tr}[\text{FIM}^{-1}], \quad (35)$$

where FIM denotes the Fisher information matrix associated with the unknown parameters.

Based on the definition of the FIM, the expression is given by:

$$\text{FIM} = -\mathbb{E}\left[\frac{\partial^2 \ln f(\Xi; \Lambda)}{\partial \Lambda \partial \Lambda^T}\right], \quad (36)$$

where the observation vector is denoted as $\Xi = [\mathbf{P}_r^T, \mathbf{T}^T, \mathbf{M}^T]^T$, $\mathbf{T} = [t_1^T, t_2^T, \dots, t_N^T]^T$, and $\mathbf{M} = [m_1, m_2, \dots, m_N]^T$. The parameter vector to be estimated is defined as $\Lambda = [\mathbf{u}^T, \tilde{\mathbf{T}}^T, \tilde{\mathbf{M}}^T]^T \in \mathbb{R}^{(3+3N+N) \times 1}$, where $\mathbf{u}$ is the target position, $\tilde{\mathbf{T}} = [\tilde{t}_1^T, \tilde{t}_2^T, \dots, \tilde{t}_N^T]^T$ contains the true anchor-node positions, and $\tilde{\mathbf{M}} = [\tilde{m}_1, \tilde{m}_2, \dots, \tilde{m}_N]^T$ contains the nominal NLOS biases. The variables $\tilde{\mathbf{T}}$ and $\tilde{\mathbf{M}}$ are treated as nuisance parameters in the CRLB analysis.

Based on the relationships between the parameters, the FIM can be constructed in the following block matrix form:

$$\text{FIM} = \begin{bmatrix} \mathcal{A} & \mathcal{B} & \mathcal{C} \\ \mathcal{B}^T & \mathcal{D} & \mathcal{F} \\ \mathcal{C}^T & \mathcal{F}^T & \mathcal{G} \end{bmatrix} \quad (37)$$

where $\mathcal{A}$ denotes the information sub-matrix associated with the target position $\mathbf{u}$; $\mathcal{B}$ represents the coupling sub-matrix between the target position and the true anchor-node positions; $\mathcal{C}$ represents the coupling sub-matrix between the target position and the NLOS biases; $\mathcal{D}$ is the information sub-matrix corresponding to the true anchor-node positions; $\mathcal{F}$ represents the coupling between the anchor-node positions and the NLOS biases; and $\mathcal{G}$ is the information sub-matrix corresponding to the NLOS bias variables.

For Gaussian RSS measurement noise, let $\mathbf{R}_r = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2)$ denote the RSS noise covariance matrix. The anchor-position perturbation covariance matrix is denoted by $\mathbf{R}_t = \text{blkdiag}(\Sigma_{t,1}, \Sigma_{t,2}, \dots, \Sigma_{t,N})$, and the NLOS-bias uncertainty covariance matrix is denoted by $\mathbf{R}_m = \text{diag}(\beta_1^2, \beta_2^2, \dots, \beta_N^2)$. Therefore, the sub-matrices in Equation (37) can be expressed as $\mathcal{A} = \left(\frac{\partial \mathbf{P}_r}{\partial \mathbf{u}^T}\right)^T \mathbf{R}_r^{-1} \left(\frac{\partial \mathbf{P}_r}{\partial \mathbf{u}^T}\right)$, $\mathcal{B} = \left(\frac{\partial \mathbf{P}_r}{\partial \mathbf{u}^T}\right)^T \mathbf{R}_r^{-1} \left(\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{T}}^T}\right)$, $\mathcal{C} = \left(\frac{\partial \mathbf{P}_r}{\partial \mathbf{u}^T}\right)^T \mathbf{R}_r^{-1} \left(\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{M}}^T}\right)$, $\mathcal{D} = \left(\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{T}}^T}\right)^T \mathbf{R}_r^{-1} \left(\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{T}}^T}\right) + \mathbf{R}_t^{-1}$, $\mathcal{F} = \left(\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{T}}^T}\right)^T \mathbf{R}_r^{-1} \left(\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{M}}^T}\right)$, and $\mathcal{G} = \left(\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{M}}^T}\right)^T \mathbf{R}_r^{-1} \left(\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{M}}^T}\right) + \mathbf{R}_m^{-1}$.

According to the RSS observation model in Equation (8), the Jacobian matrix of the RSS measurement vector with respect to the NLOS bias $\tilde{\mathbf{M}}^T$ is given by the identity matrix $\mathbf{I}_N$. Furthermore, the Jacobian matrix with respect to the target position $\mathbf{u}$ can be expressed as:

$$\frac{\partial \mathbf{P}_r}{\partial \mathbf{u}^T} = \left[\frac{\partial P_{r1}}{\partial u_1}, \frac{\partial P_{r1}}{\partial u_2}, \frac{\partial P_{r1}}{\partial u_3}, \dots, \frac{\partial P_{rN}}{\partial u_1}, \frac{\partial P_{rN}}{\partial u_2}, \frac{\partial P_{rN}}{\partial u_3} \right] \quad (38)$$

The corresponding partial derivatives are given by:

$$\frac{\partial P_{ri}}{\partial \mathbf{u}^T} = \left(-\frac{10n}{l_i \ln 10} - \alpha_f \right) \frac{\partial l_i}{\partial \mathbf{u}^T}, \quad (39)$$

where the partial derivatives of the distance with respect to the position components are expressed as:

$$\frac{\partial l_i}{\partial u_1} = -\xi \left(\kappa \frac{\partial \varphi^u}{\partial r} - \frac{\partial \varphi^i}{\partial r} \right) \frac{u_1 - \tilde{t}_{i1}}{r}, \quad (40)$$

$$\frac{\partial l_i}{\partial u_2} = -\xi \left(\kappa \frac{\partial \varphi^u}{\partial r} - \frac{\partial \varphi^i}{\partial r} \right) \frac{u_2 - \tilde{t}_{i2}}{r}, \quad (41)$$

$$\frac{\partial l_i}{\partial u_3} = -\xi \left(\kappa \frac{\partial \varphi^u}{\partial u_3} - \frac{\partial \varphi^i}{\partial u_3} \right) - \frac{\varphi^u - \varphi^i}{\cos \varphi^u}. \quad (42)$$

where $\xi = \frac{au_3 + b}{a \cos \varphi^u}$ and $\kappa = 1 + (\varphi^u - \varphi^i) \tan \varphi^u$.

From the geometry in Figure 2, performing the integration over φ yields:

$$\frac{u_3 - \tilde{t}_{i3}}{r} = -\frac{\cos \varphi^u - \cos \varphi^i}{\sin \varphi^u - \sin \varphi^i}, \quad (43)$$

$$\frac{b + au_3}{b + a\tilde{t}_{i3}} = \frac{\cos \varphi^u}{\cos \varphi^i} \quad (44)$$

By taking the partial derivatives of Equations (43) and (44) with respect to r and u_3 , respectively, we obtain:

$$\frac{\partial \varphi^u}{\partial r} = -\frac{u_3 - \tilde{t}_{i3}}{r^2} \frac{(\sin \varphi^u - \sin \varphi^i)^2}{1 - \cos(\varphi^u - \varphi^i)} - \frac{\partial \varphi^i}{\partial r}, \quad (45)$$

$$\frac{\partial \varphi^u}{\partial u_3} = \frac{1}{r} \frac{(\sin \varphi^u - \sin \varphi^i)^2}{1 - \cos(\varphi^u - \varphi^i)} - \frac{\partial \varphi^i}{\partial u_3}, \quad (46)$$

$$\frac{\partial \varphi^i}{\partial u_3} = \frac{b + au_3}{b + a\tilde{t}_{i3}} \frac{\sin \varphi^i}{\sin \varphi^u} \frac{\partial \varphi^u}{\partial u_3} - \frac{a}{b + a\tilde{t}_{i3}} \frac{\cos \varphi^i}{\sin \varphi^u}, \quad (47)$$

$$\frac{\partial \varphi^i}{\partial r} = \frac{b + au_3}{b + a\tilde{t}_{i3}} \frac{\sin \varphi^i}{\sin \varphi^u} \frac{\partial \varphi^u}{\partial r}. \quad (48)$$

To simplify the expressions, let

$$\begin{aligned} \mathcal{H}_1^u &= -\frac{u_3 - \tilde{t}_{i3}}{r^2} \frac{(\sin \varphi^u - \sin \varphi^i)^2}{1 - \cos(\varphi^u - \varphi^i)}, & \mathcal{H}_2^u &= -\frac{b + au_3}{b + a\tilde{t}_{i3}} \frac{\sin \varphi^i}{\sin \varphi^u}, \\ \mathcal{H}_3^u &= \frac{1}{r} \frac{(\sin \varphi^u - \sin \varphi^i)^2}{1 - \cos(\varphi^u - \varphi^i)}, & \mathcal{H}_4^u &= -\frac{a}{b + a\tilde{t}_{i3}} \frac{\cos \varphi^i}{\sin \varphi^u}. \end{aligned} \quad (49)$$

Then, the partial derivatives can be formulated as $\frac{\partial \varphi^u}{\partial r} = -\frac{\mathcal{H}_1^u \mathcal{H}_2^u}{1 - \mathcal{H}_2^u}$; $\frac{\partial \varphi^i}{\partial r} = \frac{\mathcal{H}_1^u}{1 - \mathcal{H}_2^u}$; $\frac{\partial \varphi^u}{\partial u_3} = \frac{\mathcal{H}_4^u - \mathcal{H}_2^u \mathcal{H}_3^u}{1 - \mathcal{H}_2^u}$; and $\frac{\partial \varphi^i}{\partial u_3} = \frac{\mathcal{H}_3^u - \mathcal{H}_4^u}{1 - \mathcal{H}_2^u}$.

Then, the Jacobian matrix of the RSS measurement vector with respect to the true anchor-node positions $\tilde{\mathbf{T}}^T$ can be expressed in the following block diagonal form:

$$\frac{\partial \mathbf{P}_r}{\partial \tilde{\mathbf{T}}^T} = \text{blkdiag} \left\{ \left(\frac{\partial P_{r1}}{\partial \tilde{\mathbf{t}}_1^T} \right), \dots, \left(\frac{\partial P_{rN}}{\partial \tilde{\mathbf{t}}_N^T} \right) \right\}, \quad (50)$$

where the partial derivatives with respect to the i^{th} anchor-node position components are given by:

$$\frac{\partial P_{ri}}{\partial \tilde{\mathbf{t}}_i^T} = \left(-\frac{10n}{l_i \ln 10} - \alpha_f \right) \frac{\partial l_i}{\partial \tilde{\mathbf{t}}_i^T}, \quad (51)$$

Furthermore, the partial derivatives of the distance with respect to the anchor positions are expressed as:

$$\frac{\partial l_i}{\partial \tilde{t}_{i1}} = -\zeta \left(\kappa \frac{\partial \varphi^u}{\partial r} - \frac{\partial \varphi^i}{\partial r} \right) \frac{\tilde{t}_{i1} - u_1}{r}, \quad (52)$$

$$\frac{\partial l_i}{\partial \tilde{t}_{i2}} = -\zeta \left(\kappa \frac{\partial \varphi^u}{\partial r} - \frac{\partial \varphi^i}{\partial r} \right) \frac{\tilde{t}_{i2} - u_2}{r}, \quad (53)$$

$$\frac{\partial l_i}{\partial \tilde{t}_{i3}} = -\xi \left(\kappa \frac{\partial \varphi^u}{\partial \tilde{t}_{i3}} - \frac{\partial \varphi^i}{\partial \tilde{t}_{i3}} \right). \quad (54)$$

Taking the partial derivatives of Equations (41) and (42) with respect to the anchor coordinate vector $\tilde{\mathbf{t}}_i$ yields:

$$\frac{\partial \varphi^i}{\partial r} = \frac{u_3 - \tilde{t}_{i3}}{r^2} \frac{(\sin \varphi^u - \sin \varphi^i)^2}{1 - \cos(\varphi^u - \varphi^i)} - \frac{\partial \varphi^u}{\partial r}, \quad (55)$$

$$\frac{\partial \varphi^u}{\partial \tilde{t}_{i3}} = -\frac{1}{r} \frac{(\sin \varphi^u - \sin \varphi^i)^2}{1 - \cos(\varphi^u - \varphi^i)} - \frac{\partial \varphi^i}{\partial \tilde{t}_{i3}}, \quad (56)$$

$$\frac{\partial \varphi^i}{\partial \tilde{t}_{i3}} = -\frac{a}{b + a\tilde{t}_{i3}} \frac{\cos \varphi^i}{\sin \varphi^i} + \frac{b + a\tilde{t}_{i3}}{b + au_3} \frac{\sin \varphi^u}{\sin \varphi^i} \frac{\partial \varphi^u}{\partial \tilde{t}_{i3}}, \quad (57)$$

$$\frac{\partial \varphi^i}{\partial r} = \frac{b + a\tilde{t}_{i3}}{b + au_3} \frac{\sin \varphi^u}{\sin \varphi^i} \frac{\partial \varphi^u}{\partial r}. \quad (58)$$

To simplify the expressions, let:

$$\begin{aligned} \mathcal{H}_1^t &= \frac{u_3 - \tilde{t}_{i3}}{r^2} \frac{(\sin \varphi^u - \sin \varphi^i)^2}{1 - \cos(\varphi^u - \varphi^i)}, & \mathcal{H}_2^t &= -\frac{b + a\tilde{t}_{i3}}{b + au_3} \frac{\sin \varphi^u}{\sin \varphi^i}, \\ \mathcal{H}_3^t &= -\frac{1}{r} \frac{(\sin \varphi^u - \sin \varphi^i)^2}{1 - \cos(\varphi^u - \varphi^i)}, & \mathcal{H}_4^t &= -\frac{a}{b + a\tilde{t}_{i3}} \frac{\cos \varphi^i}{\sin \varphi^i}. \end{aligned} \quad (59)$$

Then, the partial derivatives can be formulated as $\frac{\partial \varphi^i}{\partial r} = -\frac{\mathcal{H}_1^t \mathcal{H}_2^t}{1 - \mathcal{H}_2^t}$; $\frac{\partial \varphi^u}{\partial r} = \frac{\mathcal{H}_1^t}{1 - \mathcal{H}_2^t}$; $\frac{\partial \varphi^u}{\partial \tilde{t}_{i3}} = \frac{\mathcal{H}_3^t - \mathcal{H}_4^t}{1 - \mathcal{H}_2^t}$; and $\frac{\partial \varphi^i}{\partial \tilde{t}_{i3}} = \frac{\mathcal{H}_4^t - \mathcal{H}_2^t \mathcal{H}_3^t}{1 - \mathcal{H}_2^t}$.

Since the primary objective of the localization problem is the estimation accuracy of the target position $\mathbf{u}$, the influence of the nuisance parameters, i.e., the true anchor-node positions $\tilde{\mathbf{T}}$ and the NLOS biases $\tilde{\mathbf{M}}$, must be eliminated from the FIM. To simplify the derivation, all nuisance parameters are consolidated into a single block. Let the coupling matrix $\mathbf{J}_c$ and the nuisance information matrix $\mathbf{J}_n$ be defined as:

$$\mathbf{J}_c = \begin{bmatrix} \mathcal{B} & \mathcal{C} \end{bmatrix}, \quad \mathbf{J}_n = \begin{bmatrix} \mathcal{D} & \mathcal{F} \\ \mathcal{F}^T & \mathcal{G} \end{bmatrix}, \quad (60)$$

According to the block matrix inversion lemma, the Equivalent Fisher Information Matrix (EFIM) for the target position $\mathbf{u}$, denoted by $\mathbf{J}_e(\mathbf{u})$, can be expressed as:

$$\mathbf{J}_e(\mathbf{u}) = \mathcal{A} - \mathbf{J}_c \mathbf{J}_n^{-1} \mathbf{J}_c^T. \quad (61)$$

$\mathbf{J}_e(\mathbf{u})$ characterizes the equivalent Fisher information associated with the target position after eliminating the nuisance parameters. Finally, the theoretical lower bound of the position estimation mean-square error is defined as follows:

$$\text{CRLB}(\mathbf{u}) = \text{Tr}(\mathbf{J}_e^{-1}(\mathbf{u})) \quad (62)$$

For comparison with the Root Mean Square Error (RMSE) obtained from Monte Carlo simulations, the square root of $\text{CRLB}(\mathbf{u})$ is used. It should be noted that meta-heuristic algorithms like QIAOO may exhibit slight estimation biases due to their stochastic nature,

finite iteration counts, and the influence of environmental uncertainties. Consequently, the comparison between the localization RMSE and the CRLB should be interpreted as a theoretical performance benchmark, providing a reference for the algorithm's efficiency.

5. Simulation Results

To validate the effectiveness of the proposed QIAOO algorithm and the theoretical CRLB derived in Equation (62), numerical simulations were conducted using MATLAB 2023b on a Windows platform. Given that both anchor and target nodes may experience position variations due to marine environmental disturbances, random functions were utilized to model stochastic node fluctuations. The fixed simulation parameters adopted in this study are listed in Table 2 [34]. To demonstrate the superiority of the proposed QIAOO, comparative studies were performed against several benchmark algorithms, including Coarse-to-Fine Localization Method (CFLM) [34], LS, Nutcracker Optimization Algorithm (NOA) [35], QEHHO [22], AOO [26]. To ensure a fair comparison, all benchmark algorithms were configured using the same parameter settings as QIAOO. The internal parameters of all benchmark algorithms were configured according to the recommendations provided in their original publications, and all methods were terminated upon reaching the maximum number of iterations, $T_{\max}$. It should be noted that QEHHO enhances the search-based mechanism of HHO, whereas QIAOO leverages the energy dissipation and scattering model of AOO, which exhibits a more intuitive mathematical similarity to the exponential attenuation characteristics of RSS signals. Furthermore, while QEHHO primarily focuses on improving general search diversity, the integration of EOBL and CGMM in QIAOO is specifically designed to mitigate biases caused by NLOS conditions and to handle underwater anchor node perturbations. As a result, QIAOO demonstrates greater robustness in scenarios involving multiple coupled error sources. The RMSE was adopted as the primary metric to evaluate localization precision.

Table 2. Simulation parameters used in the underwater localization experiments.

Parameter	Value	Parameter	Value
P_0	−55 dBm	n	3
d_0	1 m	α_f	0.06 dB/m
a	0.1	b	1500 m/s
Monte Carlo runs	1000		

To theoretically evaluate the computational cost, the computational complexities of the proposed QIAOO algorithm and the benchmark algorithms are analyzed. The computational complexity of QIAOO mainly consists of two phases: (1) the initialization phase, which is used to generate the initial population and has a complexity of $O(N_p \cdot D)$; and (2) the iterative optimization phase, which requires $O(T_{\max} \cdot N_p \cdot D)$ operations. Therefore, the overall computational complexity of QIAOO can be expressed as $O(T_{\max} \cdot N_p \cdot D)$. The computational complexities of the other benchmark algorithms are summarized in Table 3.

Table 3. Computational complexity comparison of QIAOO and benchmark algorithms.

Algorithm	Algorithm Category	Complexity Order	Iteration-Based
LS	Analytical	$O(N \cdot D^2 + D^3)$	No
CFLM	Iterative–Analytical	$O(N \cdot D^2 + D^3)$	Yes
NOA	Meta-heuristic	$O(T_{\max} \cdot N_p \cdot D)$	Yes
QEHHO	Meta-heuristic	$O(T_{\max} \cdot N_p \cdot D)$	Yes
AOO	Meta-heuristic	$O(T_{\max} \cdot N_p \cdot D)$	Yes
QIAOO	Meta-heuristic	$O(T_{\max} \cdot N_p \cdot D)$	Yes

As shown in Table 3, although QIAOO incorporates multiple enhancement strategies to address complex underwater interference, its overall computational complexity remains of the same order as that of other metaheuristic algorithms and is lower than that of mathematical iterative methods. This demonstrates that QIAOO achieves a balance between localization accuracy and computational efficiency, making it well suited for robust target localization in resource-constrained underwater environments.

5.1. Ablation Study of QIAOO Components

To comprehensively evaluate the contribution of each enhancement mechanism, an ablation study was conducted by comparing five variants: AOO, AOO_QGPS, AOO_EOBL, AOO_CGMM, and QIAOO algorithm. As shown in the RMSE convergence curves in Figure 4, the AOO_QGPS variant exhibits a lower initial error compared to the original AOO algorithm, confirming that the Quantum Good Point Set initialization provides a superior starting point for searching in complex stratified acoustic environments. The AOO_EOBL variant demonstrates a faster convergence rate, proving its significant effectiveness in helping the algorithm escape from local optima induced by NLOS biases and anchor node perturbations. Additionally, the CGMM enhances the exploitation capability of the algorithm in the later stages, enabling it to achieve higher localization precision under noisy conditions. Overall, the complete QIAOO algorithm consistently outperforms all individual components in terms of both convergence speed and final accuracy, demonstrating that the proposed framework is not a simple hybrid scheme, but a tailored architecture where each component synergistically addresses the non-linear and multi-noise challenges of underwater RSS-based localization.

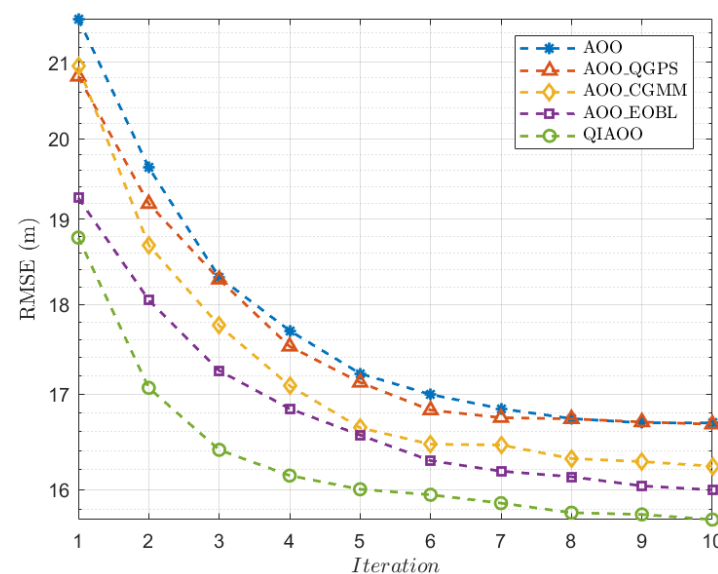


Figure 4. RMSE convergence curves for the ablation study with $N = 12$, $\sigma_i^2 = 3$, $SideLength = 100$ m, $\epsilon_i^2 = 3$, $\beta_i^2 = 3$, $T_{max} = 10$, and $N_p = 10$, demonstrating the independent contribution of each enhancement mechanism and the superiority of the QIAOO algorithm.

5.2. Convergence Analysis

Figure 5 illustrates the convergence evolution of the RMSE for the QIAOO algorithm under different population sizes N_p . As shown in the figure, QIAOO demonstrates excellent convergence stability and consistency across all tested scales. In the initial iterations ($T_{max} < 10$), the RMSE exhibits a steep decline, with the localization error shrinking from approximately 70 m to approximately 35 m, which strongly validates the algorithm's global exploration capability. As iterations proceed, the curves for all scales reach a steady state around the 100th generation, with the final localization error stabilized within the range

of 15 m to 20 m. It is observed that increasing the population size effectively smooths the convergence trajectory and further reduces the steady-state error; specifically, as N_p increases from 5 to 30, the final positioning accuracy is improved by approximately 38.1%.

Furthermore, comparative analysis reveals that, except for $N_p = 5$, the convergence trajectories for population sizes ranging from 10 to 30 are strikingly similar. This indicates that further increasing the population size yields diminishing marginal returns in terms of positioning precision. Notably, at $T = 10$ iterations, the localization error under these population scales has already converged to approximately 30 m. Considering the trade-off between localization performance and energy consumption in UWSNs, the marginal accuracy improvement obtained by further increasing the population size or iteration count cannot justify the additional computational and communication overhead. Therefore, a parameter combination of population size $N_p = 10$ and maximum iteration number $T_{\max} = 10$ is adopted in the subsequent simulations. This setting aims to achieve a favorable balance between localization performance and system energy consumption while maintaining sufficient positioning effectiveness.

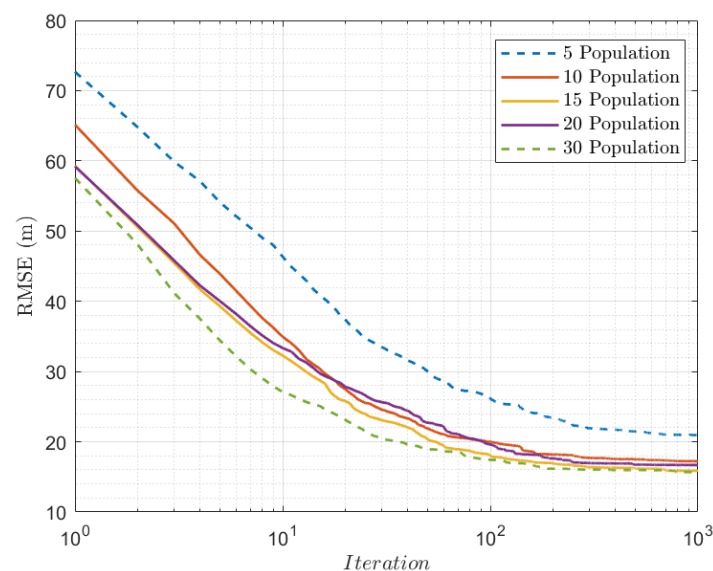


Figure 5. Convergence analysis of the RMSE under different population sizes N_p , indicating that QIAOO achieves stable and rapid convergence even with relatively small population scales.

5.3. Localization Performance Under Different Numbers of Anchor Nodes

Figure 6 illustrates the trend of RMSE for various algorithms as the number of anchor nodes N increases from 6 to 14. It can be observed that the localization errors of all evaluated algorithms exhibit a steady downward trend. This phenomenon is attributed to the fact that an increased number of anchor nodes provide richer spatial geometric constraints and observation information, thereby effectively suppressing the negative impact of complex underwater noise and NLOS biases on positioning accuracy.

In comparing the performance of the algorithms, QIAOO achieves an RMSE of 4.46 m in the challenging scenario with a limited number of anchors $N = 6$, which is lower than the 5.11 m of the original AOO. Notably, this represents an 83.6% accuracy improvement over the traditional analytical LS method (27.24 m). As the number of anchor nodes increases to $N = 14$, the localization error of QIAOO further reduces to 3.85 m. The marginal decrease of only 0.61 m compared to $N = 6$ indicates that QIAOO possesses significant robustness to variations in anchor node scale, maintaining stable and high-precision performance across different node densities. Meanwhile, the RMSE curve of QIAOO consistently maintains the smallest gap to the theoretical CRLB across the entire

range. Furthermore, QIAOO demonstrates superior positioning performance compared to other meta-heuristic algorithms such as NOA and QEHHO. In conclusion, the QIAOO algorithm not only achieves higher localization accuracy but also demonstrates stronger adaptability to variations in anchor node density, making it more suitable for scenarios with constrained node deployment.

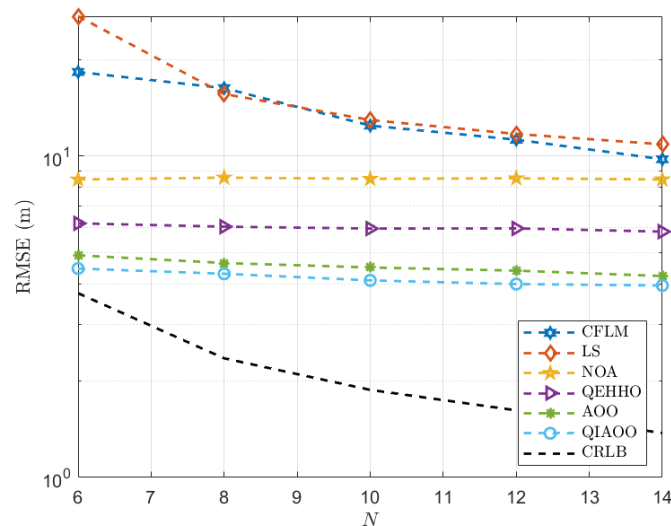


Figure 6. RMSE versus the number of anchors N with $\sigma_i^2 = 3$, $SideLength = 100$ m, $\epsilon_i^2 = 3$, $\beta_i^2 = 3$, $T_{max} = 10$, and $N_p = 10$, illustrating that QIAOO consistently follows the theoretical CRLB trend and maintains the highest localization accuracy as the number of anchors increases.

5.4. Localization Performance Under Different Side Lengths of the Monitoring Area

Figure 7 illustrates the evolution of the RMSE for various algorithms as the side length of the monitoring area increases. It can be observed that as the side length expands from 100 m to a large-scale range of 500 m, the localization errors of all algorithms exhibit a monotonic upward trend. This phenomenon is attributed to the expansion of the search space, which significantly increases the positional uncertainty of the target node, compounded by more severe environmental disturbances and cumulative NLOS biases over longer propagation distances.

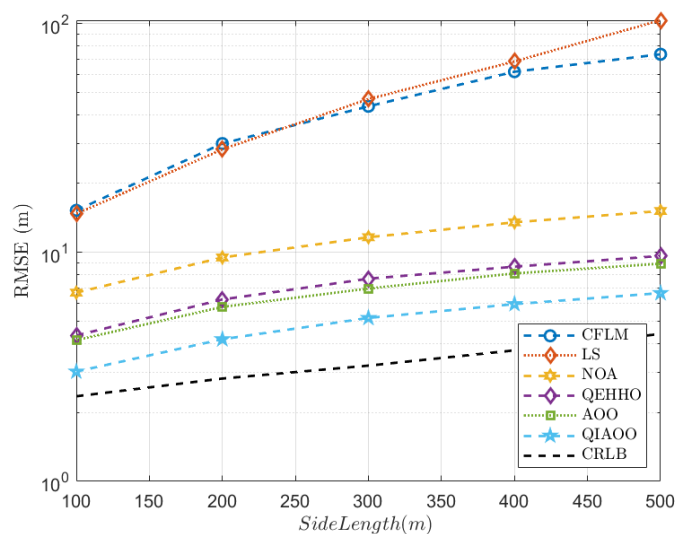


Figure 7. RMSE versus the monitoring area side length $SideLength$ with $\sigma_i^2 = 3$, $N = 12$, $\epsilon_i^2 = 3$, $\beta_i^2 = 3$, $T_{max} = 10$, and $N_p = 10$, showing that QIAOO provides superior robustness and lower error growth compared with benchmark algorithms in wide-area scenarios.

Comparative analysis demonstrates that the proposed QIAOO algorithm consistently exhibits significant superiority across different spatial scales. Its RMSE curve remains at the bottom of all competing algorithms and maintains the smallest gap to the theoretical CRLB benchmark. At a side length of 100 m, the localization error of QIAOO is approximately 3.01 m, while at the 500 m scale, QIAOO still manages to constrain the error to approximately 6.5 m, representing a 27% accuracy improvement over the original AOO algorithm (8.9 m). In contrast, the traditional LS and CFLM algorithms are adversely affected by non-linear errors, with their RMSE values surging to 103 m and 73.3 m at 500 m, respectively, exhibiting poor localization performance. Among the meta-heuristic algorithms, although NOA, and QEHHO outperform the analytical methods, their performance remains inferior to that of QIAOO in large-scale search spaces. Leveraging the high spatial coverage provided by the Quantum Good Point Set initialization and the dynamic adjustment of the search path by the hybrid mutation strategy, QIAOO effectively maintains superior localization precision. This confirms that the QIAOO algorithm can achieve robust and high-precision localization in highly dynamic and wide-area underwater scenarios.

5.5. Localization Performance Under Different Anchor Node Position Uncertainties

Figure 8 illustrates the trend of the RMSE for various algorithms as the anchor node position error increases. It can be observed that within the range from 10^{-3} to 10^0 , except for the theoretical CRLB, which rises with the increasing variance, the RMSE curves for all evaluated localization algorithms maintain a high degree of stability and remain largely unaffected by the error variance. This phenomenon can be explained by the fact that anchor-node perturbations have a limited impact on the RSS observation structure and overall geometric configuration, indicating that the algorithms are relatively insensitive to anchor uncertainties and therefore remain robust.

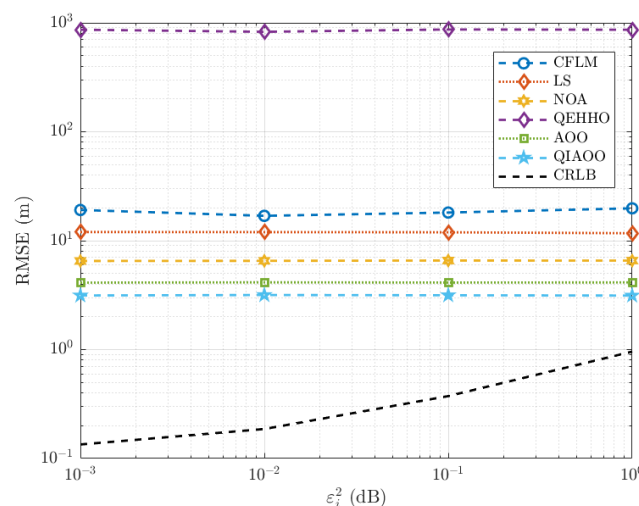


Figure 8. RMSE versus anchor-node position perturbation variance ε_i^2 with $\sigma_i^2 = 3$, $N = 12$, $SideLength = 100$ m, $\beta_i^2 = 3$, $T_{max} = 10$, and $N_p = 10$, confirming that QIAOO achieves high robustness against anchor coordinate inaccuracies.

Comparative analysis of the algorithms reveals that the proposed QIAOO algorithm consistently achieves the lowest localization error across the entire experimental range, with its RMSE curve being the closest to the theoretical CRLB. Specifically, the RMSE of QIAOO stabilizes at approximately 3.1 m, outperforming the original AOO algorithm (4.2 m). In contrast, the RMSE of the QEHHO algorithm exceeds 800 m, indicating that its positioning accuracy deteriorates severely in environments affected by anchor node perturbations due to the lack of an effective search mechanism. Furthermore, the performance of both LS

and CFLM is significantly inferior to that of QIAOO. The stability of the QIAOO algorithm across a wide range of anchor node errors is attributed to its joint modeling approach, which treats the true anchor node positions as nuisance parameters, ensuring that the algorithm maintains high robustness and localization precision in complex environments.

5.6. Localization Performance Under Different Levels of NLOS Biases

Figure 9 shows the variation trend of RMSE for each comparative algorithm as the NLOS bias β_i^2 increases. This phenomenon is attributed to the fact that NLOS bias introduces deviations into RSS measurements, thereby affecting distance estimation and consequently degrading localization accuracy. From the figure, as the NLOS interference intensity increases from 10^{-2} to 10^1 , the localization errors of all algorithms show an upward trend to varying degrees. In the low-bias scenario of 10^{-2} , the RMSE of QIAOO is only 2.33 m, which is 16.4% and 65.7% lower than the original AOO algorithm (2.79 m) and the NOA algorithm (6.8 m), respectively, showing performance closest to the theoretical CRLB. As the NLOS bias strengthens to the 10^1 scenario, a performance gap also emerges. The RMSE of LS and CFLM increases rapidly to 17.3 m and 25.06 m; because they cannot handle NLOS bias, their localization accuracy is seriously reduced. In contrast, QIAOO shows a very strong interference suppression capability, and its RMSE only grows slowly to 5.61 m, with an accuracy improvement of about 67.6% compared to the LS algorithm. It is worth noting that the error of the QEHHO algorithm always remains around 500 m, reflecting that the algorithm has experienced search failure in the NLOS bias environment. The experimental results prove that, compared to other intelligent optimization algorithms, QIAOO is more robust to changes in NLOS bias and can provide high-reliability localization assurance in complex underwater NLOS propagation environments.

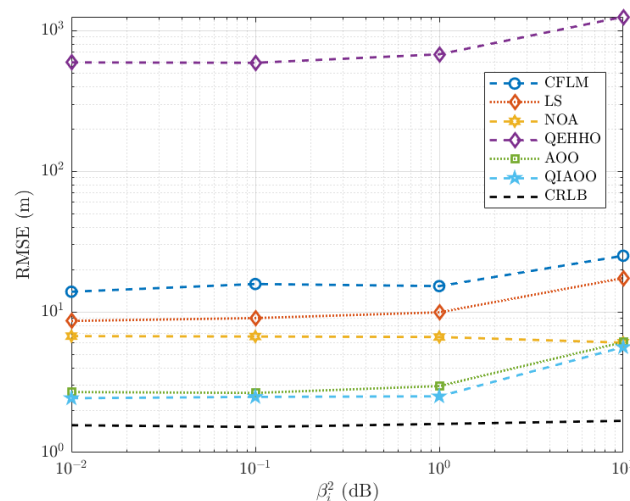


Figure 9. RMSE versus NLOS bias intensity β_i^2 with $\sigma_i^2 = 3$, $N = 12$, $SideLength = 100$ m, $\varepsilon_i^2 = 3$, $T_{max} = 10$, and $N_p = 10$, demonstrating that QIAOO achieves higher localization accuracy and remains closest to the theoretical CRLB.

5.7. Computational Time

Figure 10 illustrates the average computation time of each algorithm under four simulation scenarios ($N, \sigma_i^2, SideLength, \varepsilon_i^2$). As shown in the figure, the execution times of LS and CFLM are the shortest, in the range of 10^{-8} s to 10^{-6} s, which is attributed to the fact that they only involve algebraic operations for closed-form solutions without requiring iterations. In contrast, all intelligent optimization algorithms, including QIAOO, have higher computational overhead than the LS and CFLM algorithms due to population

evolution and multiple iterations, with average execution times ranging between 10^{-4} s and 10^{-1} s.

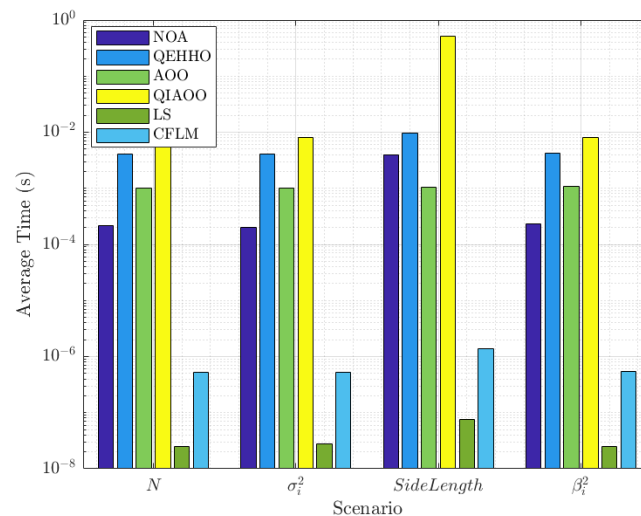


Figure 10. Computational time across different simulation scenarios, verifying that QIAOO maintains a favorable balance between high localization accuracy and real-time execution efficiency.

Among the intelligent optimization algorithms, the computation time of the proposed QIAOO algorithm is slightly higher than that of the original AOO algorithm, remaining around 10^{-2} s in most scenarios and increasing to within 0.5 s only in the SideLength scenario with the highest computational load. This additional computational overhead remains within an acceptable range, mainly due to the introduction of Quantum Good Point Set initialization, Elite Oppositional-Based Learning, and mixed mutation strategies in each iteration. Although these strategies improve search accuracy and enhance the ability to escape local optima, they also increase the computational complexity of each iteration. Considering the update-cycle requirements of practical underwater localization systems, this level of computational latency is sufficient to satisfy real-time application demands. In summary, the QIAOO algorithm achieves significant improvements in localization accuracy and robustness at a relatively low computational cost.

5.8. Discussion

The simulation results demonstrate that, compared with the benchmark algorithms, QIAOO consistently achieves higher localization accuracy and improved robustness. Specifically, compared with analytical methods (LS and CFLM), QIAOO more effectively compensates for the model mismatch caused by sound-speed-profile-induced ray bending. Compared with meta-heuristic algorithms (AOO, NOA, and QEHHO), the combination of QGPS and EOBL effectively prevents the search process from being trapped in symmetric local optima induced by NLOS biases.

Meanwhile, in most scenarios, the RMSE achieved by QIAOO closely follows the trend of the CRLB. However, several operational boundaries that may degrade the algorithm's performance were identified: (1) Noise limitation: when the measurement noise exceeds 10 dB, the localization error gradually deviates from the CRLB due to the deterioration of signal quality. (2) Geometric constraint: at least six non-collinear anchor nodes ($N = 6$) are required; otherwise, the search space becomes highly multimodal, making consistent convergence difficult to achieve. (3) Physical limitation: if the NLOS bias exceeds the actual propagation distance, the physical relationship between RSS and distance can no longer be reliably recovered.

Future work will focus on further improving the adaptability of the proposed framework to more complex and dynamic underwater environments.

6. Conclusions

To mitigate localization degradation caused by ray bending, anchor-node position perturbations, and NLOS biases in complex marine environments, this paper proposed a robust RSS-based localization scheme using the QIAOO algorithm. Specifically, by simultaneously considering the three major sources of interference in underwater localization, namely SSP-induced ray refraction, anchor-node position perturbations, and NLOS biases (which have often been treated separately in previous studies), a unified RSS-based localization framework was established to better represent practical marine environments, thereby bridging the gap in existing studies that inadequately address multi-error-coupled scenarios. Furthermore, the corresponding CRLB was derived to provide a theoretical benchmark for localization performance evaluation. Based on this framework, multiple enhancement strategies were incorporated into the original AOO algorithm to improve population diversity, global exploration capability, and local exploitation performance, thereby further enhancing the localization accuracy and robustness. In addition, the proposed method achieves high-precision and robust localization without relying on expensive hardware for time synchronization.

Simulation results demonstrate that the proposed QIAOO algorithm consistently outperforms the benchmark methods under various noise levels, anchor-node configurations, and NLOS conditions. Specifically, compared with the conventional analytical LS method, the proposed scheme achieves a maximum localization accuracy improvement of 83.6%. Moreover, the RMSE of QIAOO remains consistently lower than that of the original AOO algorithm (e.g., approximately 3.95 m when $N = 14$). As illustrated by the RMSE curves in Figures 6–9, the performance of QIAOO consistently approaches the theoretical CRLB. Furthermore, the average execution time remains at the millisecond level, demonstrating a favorable balance among localization accuracy, computational cost, and real-time applicability.

Although the proposed QIAOO algorithm exhibits robust localization performance under various underwater scenarios, there remains a certain gap between its localization accuracy and the theoretical CRLB. In practical deployments, several engineering challenges must be addressed, such as the sensitivity to SSP estimation errors and the real-time implementation on low-power UWSN nodes. These issues could be mitigated through periodic SSP recalibration and the use of computational offloading to surface gateways to preserve node battery life. Future work will focus on developing more adaptive multi-strategy fusion mechanisms to address these practical constraints and further improve algorithm performance. Moreover, since the current study is primarily based on simulations, future work will include field experiments in real marine environments to further validate the localization accuracy and robustness of the proposed scheme under complex and dynamic underwater conditions.

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Appendix A. List of Abbreviations

Table A1 summarizes the English terms and their abbreviations used throughout the manuscript.

Table A1. Summary of abbreviations used in this paper.

Full Name	Abbrev.	Full Name	Abbrev.
Received Signal Strength	RSS	Underwater Wireless Sensor Networks	UWSNs
Sound Speed Profile	SSP	Non-Line-of-Sight	NLOS
Time of Arrival	TOA	Time Difference of Arrival	TDOA
Angle of Arrival	AOA	Maximum Likelihood	ML
Semidefinite Programming	SDP	Least Squares	LS
Particle Swarm Optimization	PSO	Harris Hawks Optimization	HHO
Quantum-Enhanced Harris Hawks Optimization	QEHHO	Grey Wolf Optimizer	GWO
Sparrow Search Algorithm	SSA	Animated Oat Optimization	AOO
Quantum-Inspired Animated Oat Optimization	QIAOO	Quantum Good Point Set	QGPS
Elite Oppositional-Based Learning	EOBL	Cauchy–Gaussian mixed mutation	CGMM
Cramér–Rao Lower Bound	CRLB	Fisher Information Matrix	FIM
Equivalent Fisher Information Matrix	EFIM	Root Mean Square Error	RMSE
Coarse-to-Fine Localization Method	CFLM	Nutcracker Optimization Algorithm	NOA

Appendix B. Nomenclature

Table A2 summarizes the mathematical symbols used in this study.

Table A2. Summary of mathematical symbols and definitions.

Symbol	Definition	Symbol	Definition
N	Number of anchors	$\mathbf{u}$	Target position vector
$\mathbf{t}_i$	Observed i^{th} anchor position	$\tilde{\mathbf{t}}_i$	True i^{th} anchor position
$\boldsymbol{\varepsilon}_i$	Anchor position perturbation error	$\boldsymbol{\Sigma}_{t,i}$	Perturbation covariance matrix
δ_i^2	Anchor perturbation variance	P_{ri}	Received signal power at anchor i
P_0	Received power at reference distance d_0	d_0	Reference distance (typically 1 m)
d_i	Euclidean (straight-line) distance	n	Path loss exponent
α_f	Acoustic absorption coefficient	f	Acoustic frequency

Table A2. Cont.

Symbol	Definition	Symbol	Definition
η_i	RSS measurement noise	σ_i^2	RSS noise variance
m_i	NLOS propagation bias	$\hat{m}_i$	Nominal NLOS bias
τ_i	NLOS bias uncertainty term	β_i^2	NLOS uncertainty variance
$C(z)$	Sound speed at depth z	a	Sound speed gradient (SSP)
b	Sound speed at reference depth $z = 0$	φ	Grazing angle of acoustic ray
φ^u	Grazing angle at the target node	φ^i	Grazing angle at anchor i
k	Snell's Law refractive constant	l_i	Ray propagation arc length
r	Horizontal propagation distance	N_p	Population size
D	Search space dimensionality	$T_{\max}$	Maximum number of iterations
t	Current iteration index	LB_j, UB_j	Lower and upper search bounds
$\alpha(t)$	Dynamic energy decay parameter	m_{seed}	Mass of the oat seed
L	Length of the seed awn	e	Seed eccentricity coefficient
R	The fine-tuning parameter	r_{init}, r_W, r_e, r_o	Random scalars or vectors
q	Dynamic adjustment factor	y_{best}	Global best individual
W	Random dispersal term	ζ	Lévy flight stability index
ψ	Lévy flight scale parameter	h_j	GPS generator vector
GD	Unit hypercube in D -dimensional space	h	Good point in GD
$H_{N_p}(k)$	Generated good point set	$\{\cdot\}$	Fractional part of a real number
p	Smallest prime number for GPS	$ Z\rangle$	Quantum state (qubit)
$ 0\rangle, 1\rangle$	Basis states of a qubit	ι_0, ι_1	Quantum probability amplitudes
$\theta_{i,j}$	Phase angle for quantum mapping	$\mathbb{Q}_i$	Quantum probability amplitude matrix
$y_{i,j}^{(c)}$	Cosine-based quantum solution	$y_{i,j}^{(s)}$	Sine-based quantum solution
$y_{i,j}$	Oppositional individual in EOBL	$y_{i,j}$	Elite oppositional individual
r_o	Guided guidance random coefficient	$y_{best,j}$	j^{th} component of the global best
$y_{best,C}^t$	Cauchy-mutated best individual	$y_{best,G}^t$	Gaussian-mutated best individual
$\bar{h}_1$	Cauchy random mutation factor	$\bar{h}_2$	Gaussian random perturbation factor
y_{mut}^t	Hybrid mutated individual	$t/T_{\max}$	Dynamic weighting factor
$f(\cdot)$	Fitness (objective) function	FIM	Fisher Information Matrix
EFIM	Equivalent Fisher Info. Matrix	Λ	Parameter vector to be estimated
Ξ	Total observation vector	$\mathcal{A}, \mathcal{B}, \mathcal{C}$	FIM information sub-matrices
J_c	Coupling sub-matrix in FIM	$\mathbf{R}_r, \mathbf{R}_t, \mathbf{R}_m$	Error covariance matrices
$\mathcal{H}_1 \dots \mathcal{H}_4$	Jacobian auxiliary terms		

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