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HTAGN-Enhanced Factor Graph Optimization for Shipborne GNSS/INS/Gyrocompass Integrated Navigation

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Abstract

Shipborne Global Navigation Satellite System/Inertial Navigation System (GNSS/INS) integrated navigation based on Factor Graph Optimization (FGO) is increasingly used for maritime positioning, but its performance can be limited in low-dynamic sailing scenarios. In conventional GNSS/INS FGO without direct heading measurements, the heading state is mainly propagated by inertial measurements. During prolonged straight-line or low-maneuver sailing, position-related constraints provide only weak heading correction, which may lead to cumulative heading errors. In addition, existing learning-based GNSS outage compensation methods often introduce pseudo-GNSS observations with fixed covariance, making it difficult to represent the direction-dependent and time-varying uncertainty of data-driven predictions. To address these issues, this paper proposes a heteroscedastic TCN-Assisted GRU Network (HTAGN)-enhanced FGO framework for shipborne GNSS/INS/gyrocompass integrated navigation. Within the unified framework, a gyrocompass heading factor is constructed to provide persistent GNSS-independent heading constraints, and an HTAGN is designed to generate pseudo-GNSS position increments during GNSS outages. The direction-dependent standard deviations predicted by HTAGN are further mapped to the covariance matrix of pseudo-GNSS position factors, enabling confidence-adaptive factor weighting in the optimization process. The framework was validated on a single real-world sea trial with artificially simulated GNSS outages. The gyrocompass heading factor reduced the heading Root Mean Square Error (RMSE) by 84.6% compared with conventional GNSS/INS FGO. During 90 s and 180 s outages, the proposed method reduced the horizontal positioning RMSE by 63.7% and 52.5%, respectively, relative to the fixed-covariance TAGN baseline, demonstrating improved heading estimation and short-term positioning continuity under the tested conditions.

Keywords: shipborne integrated navigation; factor graph optimization; gyrocompass heading factor; heteroscedastic prediction; GNSS outages



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1. Introduction

The development of intelligent ships and advanced maritime operations has placed increasing demands on shipborne navigation systems [1–3]. Compared with vehicular navigation, shipborne navigation is characterized by long-duration operation, low-dynamic motion, prolonged straight-line sailing, slow turns, and persistent disturbances from wind, waves, and currents. These characteristics create two practical challenges for shipborne integrated navigation. Under low-dynamic motion, heading errors are difficult to correct using Global Navigation Satellite System (GNSS) position constraints alone, which may

lead to cumulative heading drift. In addition, GNSS degradation or outages in ports, coastal waters, and bridge areas can weaken absolute position constraints and cause rapid position error accumulation. Therefore, improving heading observability and maintaining positioning continuity under GNSS outages has become an important requirement for robust shipborne integrated navigation.

The integration of GNSS and the Inertial Navigation System (INS) provides a common framework for shipborne navigation by combining the long-term positioning accuracy of GNSS with the short-term continuity of INS. To further improve nonlinear state estimation and multi-source constraint fusion, Factor Graph Optimization (FGO) has gained increasing attention in GNSS/INS integrated navigation. Compared with filter-based methods, FGO estimates navigation states by jointly optimizing historical states and measurement factors, providing a flexible framework for heterogeneous sensor constraints [4,5]. Existing studies have improved FGO-based navigation through Inertial Measurement Unit (IMU) preintegration [6], continuous-time multi-sensor fusion [7], and open-source implementation [8]. FGO has also been extended to marine and shipborne platforms, including shipborne GNSS/INS and ship GNSS/DR integration [9,10], AUV navigation [11], PPP/GNSS/INS integration and intelligent-ship INS/GPS fusion [12,13], and RIMU/GNSS navigation [14]. However, the above challenges have not been fully addressed in existing shipborne GNSS/INS FGO frameworks, especially under low-dynamic and long-endurance navigation conditions.

In conventional GNSS/INS FGO, this heading drift problem is exacerbated because corrections rely solely on position-inertial coupling, which is often too weak under low-dynamic motion to continuously suppress gyroscope-bias-induced errors. Previous studies have explored external heading aids based on velocity-derived heading, dual-antenna GNSS, and magnetometers [15–17]. However, velocity-derived heading becomes unreliable at low speed, dual-antenna heading depends on GNSS availability, and magnetic heading is vulnerable to shipborne ferromagnetic interference. In contrast, a shipborne gyrocompass provides an external and GNSS-independent absolute heading observation. It can therefore continue to constrain the heading state when GNSS observations are degraded or unavailable. Constructing a dedicated gyrocompass heading factor within a shipborne GNSS/INS factor graph is a practical way to enhance heading observability, but this integration remains insufficiently explored.

During GNSS outages, position error divergence and suboptimal pseudo-observation weighting become the dominant concern. When GNSS measurements are unavailable, the INS loses external absolute position corrections, and position errors accumulate with velocity errors, inertial sensor biases, and attitude errors. This problem is particularly relevant to shipborne navigation, where long-duration operation, low-dynamic motion, large inertia, and slow maneuvering responses make reliable external correction and motion-state modeling more challenging. To mitigate outage-induced error divergence, data-driven methods have been introduced to estimate missing GNSS-related information or position increments from IMU measurements and navigation states. CNN-GRU-based methods have been used to predict position increments and construct learned pseudo-observations during GNSS outages [18,19]. LSTM-based and other neural-network-based methods have also been adopted for intelligent virtual sensing, GNSS/INS compensation, and FGO-assisted positioning [20–23]. In marine navigation, temporal models have been explored for ship attitude prediction and GNSS outage compensation, indicating their potential for modeling ship motion characteristics [24,25]. However, most existing learning-assisted compensation methods still provide deterministic predictions or pseudo-observations with fixed covariance. This fixed-weighting scheme cannot capture the direction-dependent and time-varying uncertainty of predicted position increments, where the prediction errors in

different North-East-Down (NED) frame directions may vary with ship motion states and maneuvering conditions. Consequently, low-confidence predictions may carry the same weight as high-confidence ones, degrading the overall optimization performance.

Recent studies have explored uncertainty-aware navigation fusion through different approaches. Huo et al. combined GRU-SA-based pseudo-measurement prediction with covariance-adaptive FGO for GNSS outage compensation [26]. Zhang et al. used deep learning to predict satellite visibility and pseudorange errors in urban GNSS environments [27], while Liu et al. developed an adaptive robust EKF for Arctic tightly coupled GNSS/INS navigation by dynamically adjusting observation-noise covariance [28]. Among these studies, Ref. [26] is most closely related to the present work because both methods generate pseudo-GNSS constraints during complete GNSS outages. The distinction of this study lies in jointly predicting the mean and direction-dependent standard deviation of each NED position increment through heteroscedastic regression. The increment-level covariance is further propagated over the outage interval to characterize the uncertainty of the accumulated pseudo-position and is subsequently mapped to the pseudo-GNSS factor information matrix for time-varying confidence-adaptive weighting. Therefore, the contribution is an integrated probabilistic pseudo-GNSS generation, uncertainty propagation, and adaptive factor-weighting framework rather than merely applying a TCN-GRU model or heteroscedastic regression to shipborne data.

These considerations motivate a unified shipborne GNSS/INS/gyrocompass FGO framework that combines a gyrocompass heading factor with HTAGN-derived uncertainty-aware pseudo-GNSS factors. Accordingly, this paper proposes an HTAGN-enhanced FGO method in which the gyrocompass factor provides persistent GNSS-independent heading constraints, while HTAGN supplies pseudo-GNSS position increments and their associated uncertainties during GNSS outages. These constraints are jointly optimized to improve heading stability and positioning continuity. The main contributions of this paper are summarized as follows:

- (1) A gyrocompass heading factor is incorporated into the HTAGN-enhanced FGO framework for shipborne GNSS/INS/gyrocompass integrated navigation. By introducing persistent GNSS-independent heading observations, this factor enhances heading-state observability and suppresses heading error accumulation under low-dynamic conditions.
- (2) An HTAGN-based probabilistic pseudo-GNSS prediction model is proposed for complete GNSS outages. It jointly predicts the mean and direction-dependent standard deviation of each NED position increment, providing both pseudo-GNSS information and sample-dependent uncertainty.
- (3) An uncertainty-propagation and confidence-adaptive factor weighting mechanism is developed. The increment-level covariance is accumulated over the outage interval and mapped to the pseudo-GNSS factor information matrix for time-varying adaptive weighting.

The remainder of this paper is organized as follows. Section 2 presents the GNSS/INS/gyrocompass factor graph model and optimization framework. Section 3 describes the HTAGN model and the adaptive fusion of pseudo-GNSS observations. Section 4 reports the experimental validation and discusses the results based on real ship trial data. Finally, Section 5 concludes the paper.

2. GNSS/INS/Gyrocompass Factor Graph Optimization Model

This section presents a shipborne GNSS/INS/gyrocompass factor graph optimization model, which provides a unified optimization framework for integrating the HTAGN model in the subsequent section. First, the system state and bias state are defined. Then, the residual models of the IMU preintegration factor, IMU bias factor, gyrocompass head-

ing factor, GNSS position factor, and marginalization factor are established. Finally, the multi-source observation fusion problem is formulated as a maximum a posteriori (MAP) estimation problem, and the corresponding nonlinear least-squares optimization objective is constructed.

2.1. GNSS/INS/Gyrocompass Factor Graph Framework

A GNSS/INS/gyrocompass factor graph is constructed for the ship GNSS/INS/gyrocompass integrated navigation system, as shown in Figure 1. Different node and edge types are used to illustrate the factor graph structure. The circular nodes represent navigation states, including position, velocity, and attitude, while the bias states are separately illustrated as described below. The square nodes denote measurement or constraint factors. Specifically, f^C , f^{GNSS} , f^{pre} , f^{bias} and f^M represent the gyrocompass heading factor, GNSS position factor, IMU preintegration factor, IMU bias factor, and marginalization factor, respectively. The directed edges indicate the connectivity between factors and their associated state variables. The system state is defined as $\mathcal{X} = \{x_1, x_2, \dots, x_k\}$, where x_k denotes the navigation state at epoch k . It is given by: $x_k = (p_k^n, v_k^n, q_{b,k}^n, b_{g,k}, b_{a,k})$, where p_k^n and v_k^n represent the ship's position and velocity in the navigation frame n , respectively; $q_{b,k}^n$ represents the attitude quaternion from the body frame b to the navigation frame n ; and $b_{g,k}$ and $b_{a,k}$ represent the gyroscope and accelerometer biases, respectively.

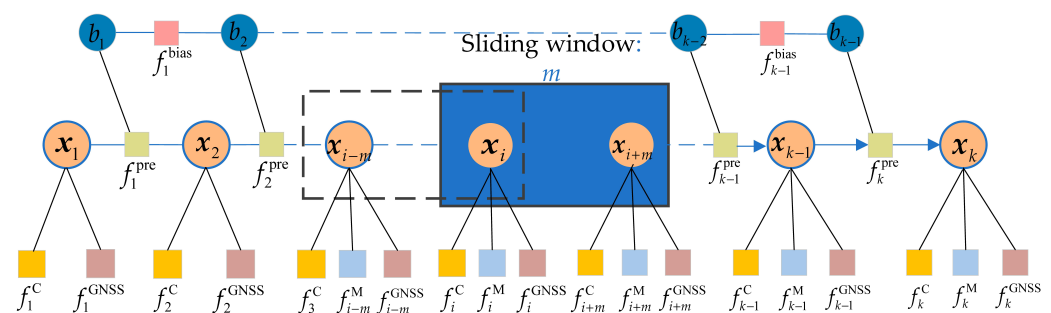


Figure 1. GNSS/INS/gyrocompass factor graph framework. Circular nodes denote navigation states, bias nodes denote IMU bias states, square nodes represent measurement or constraint factors, and edges indicate the connections between factors and their corresponding state variables.

To describe the temporal evolution of IMU biases, the bias states contained in the full navigation state are denoted separately as $\mathcal{B} = \{b_1, b_2, \dots, b_k\}$, where $b_k \in \mathbb{R}^6$ denotes the IMU bias state at epoch k , with $b_k = (b_{g,k}, b_{a,k})$. It should be emphasized that $\mathcal{B}$ does not introduce an additional set of independent optimization variables. Instead, b_k denotes the same physical gyroscope and accelerometer bias terms already included in the full state x_k . In Figure 1, the bias states are drawn as separate nodes only for graphical clarity, which is a common representation in factor-graph-based inertial navigation. This separate depiction explicitly shows the bias random-walk constraints between adjacent epochs; otherwise, this temporal constraint would be hidden inside the state node x_k . Therefore, the bias nodes in Figure 1 should be understood as a graphical decomposition of the bias states of x_k , rather than as duplicated or independent bias variables.

Through this framework, the IMU preintegration factor characterizes the relative motion between adjacent states based on inertial preintegration theory, while the IMU bias factor models the temporal evolution of inertial sensor biases using a random-walk process. The gyrocompass heading factor introduces absolute heading observations into the factor graph to enhance heading-state observability, whereas the GNSS position factor provides absolute position constraints to correct the position drift accumulated through INS propagation. The marginalization factor preserves prior information from historical states

within the sliding window. By jointly modeling these factors, the proposed framework enables unified fusion of inertial, position, and heading information for multi-source state estimation.

2.2. IMU Preintegration Factor

The IMU preintegration factor is used to summarize high-rate inertial measurements between two adjacent keyframes and to establish relative motion constraints in the factor graph. Let the set of IMU measurements over the time interval from t_{k-1} to t_k be defined as:

$$\mathcal{Z}_k^{\text{imu}} = \left\{ \mathbf{f}_m^b, \boldsymbol{\omega}_m^b \right\}_{t_{k-1}:t_k} \quad (1)$$

where $\mathbf{f}_m^b$ and $\boldsymbol{\omega}_m^b$ denote the specific force and angular velocity measurements provided by the IMU, respectively. By integrating the bias-corrected IMU measurements, the relative position, velocity, and attitude increments between two adjacent keyframes can be obtained as:

$$\Delta \mathbf{p}_{k-1,k}, \quad \Delta \mathbf{v}_{k-1,k}, \quad \Delta \mathbf{q}_{k-1,k} \quad (2)$$

Using these preintegrated quantities, the IMU preintegration factor can be expressed as:

$$f_k^{\text{pre}}(\mathbf{x}_{k-1}, \mathbf{x}_k, \mathcal{Z}_k^{\text{imu}}) \quad (3)$$

This factor constrains the position, velocity, and attitude propagation between two adjacent states $\mathbf{x}_{k-1}$ and $\mathbf{x}_k$. Compared with conventional strapdown INS mechanization, IMU preintegration incorporates high-rate inertial measurements as relative constraints between keyframes. Therefore, repeated integration of raw IMU measurements during iterative optimization can be avoided, while the continuity of state propagation and the inertial information within the sliding window are preserved.

Furthermore, the residual of the IMU preintegration factor consists of the position residual, velocity residual, and attitude residual:

$$\mathbf{r}_k^{\text{pre}} = (\mathbf{r}_{p,k}, \mathbf{r}_{v,k}, \mathbf{r}_{q,k}) \quad (4)$$

where $\mathbf{r}_{p,k}$, $\mathbf{r}_{v,k}$ and $\mathbf{r}_{q,k}$ denote the position, velocity, and attitude residuals between the preintegrated prediction and the state to be optimized, respectively. Accordingly, the IMU preintegration factor can be modeled as:

$$f_k^{\text{pre}}(\mathbf{x}_{k-1}, \mathbf{x}_k, \mathcal{Z}_k^{\text{imu}}) = \exp\left(-\frac{1}{2} \|\mathbf{r}_k^{\text{pre}}\|_{\boldsymbol{\Sigma}_k^{\text{pre}}}^2\right) \quad (5)$$

where $\boldsymbol{\Sigma}_k^{\text{pre}}$ is the covariance matrix of IMU preintegration, which describes the uncertainty caused by IMU measurement noise and bias error propagation during the integration.

The covariance-weighted preintegration factor is used to impose continuous motion constraints within the GNSS/INS/gyrocompass factor graph. Since the IMU preintegrated quantities depend on the bias linearization point, the bias state is jointly estimated during optimization, and first-order bias correction terms are introduced to compensate for the influence of bias variations on the preintegration residuals [29].

2.3. IMU Bias Factor

The gyroscope and accelerometer biases are usually modeled as random-walk processes. Their discrete-time models can be expressed as:

$$\mathbf{b}_{g,k} = \mathbf{b}_{g,k-1} + \mathbf{n}_{bg} \quad (6)$$

$$\mathbf{b}_{a,k} = \mathbf{b}_{a,k-1} + \mathbf{n}_{ba} \quad (7)$$

where $\mathbf{b}_{g,k}$ and $\mathbf{b}_{a,k}$ denote the gyroscope and accelerometer biases at epoch k , respectively; $\mathbf{n}_{bg}$ and $\mathbf{n}_{ba}$ denote the corresponding random-walk driving noises. Since the bias variation between adjacent state nodes is driven only by random-walk noise, the residual of the IMU bias factor is defined as:

$$\mathbf{r}_k^{\text{bias}} = \begin{bmatrix} \mathbf{b}_{g,k} - \mathbf{b}_{g,k-1} \\ \mathbf{b}_{a,k} - \mathbf{b}_{a,k-1} \end{bmatrix} \quad (8)$$

The corresponding covariance matrix is given by:

$$\Sigma_k^{\text{bias}} = \begin{bmatrix} \mathbf{Q}_{bg}\Delta t & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{ba}\Delta t \end{bmatrix} \quad (9)$$

where $\mathbf{Q}_{bg}$ and $\mathbf{Q}_{ba}$ denote the noise intensity matrices of the gyroscope and accelerometer bias random walk, respectively. Accordingly, the IMU bias factor can be expressed as:

$$f_k^{\text{bias}}(\mathbf{b}_{k-1}, \mathbf{b}_k) = \exp\left(-\frac{1}{2}\left\|\mathbf{r}_k^{\text{bias}}\right\|_{\Sigma_k^{\text{bias}}}^2\right) \quad (10)$$

The IMU bias factor provides a temporal-continuity prior for the bias states and constrains their random-walk variations between adjacent epochs. Although it does not provide absolute correction by itself, it works with the IMU preintegration factor, GNSS observations, and gyrocompass measurements to estimate and compensate for inertial sensor biases, thereby reducing the long-term influence of bias errors on state propagation.

2.4. Gyrocompass Heading Factor

In conventional GNSS/INS integrated navigation, the heading state is mainly propagated by integrating IMU angular rate measurements and is affected by gyroscope bias and measurement noise. As a result, heading errors may gradually accumulate over time. To enhance heading observability, gyrocompass measurements are introduced, and a one-dimensional gyrocompass heading factor is constructed to constrain the yaw component of the attitude state.

Let the gyrocompass heading observation at epoch k be denoted as z_k^C . The gyrocompass observation model is expressed as:

$$z_k^C = h_C(\mathbf{x}_k) + n_k^C \quad (11)$$

where $z_k^C = \psi_{C,k}$, and $h_C(\mathbf{x}_k) = \psi(\mathbf{q}_{b,k}^n)$. Here, $\psi_{C,k}$ denotes the heading angle measured by the gyrocompass, $\psi(\mathbf{q}_{b,k}^n)$ denotes the heading angle extracted from the attitude state, and n_k^C denotes the gyrocompass measurement noise, which is assumed to satisfy:

$$n_k^C \sim \mathcal{N}(0, R_k^C) \quad (12)$$

where R_k^C is the variance of the gyrocompass heading measurement noise. Since the heading angle is periodic, the factor residual is defined as:

$$r_k^C = \text{wrap}(z_k^C - h_C(\mathbf{x}_k)) \quad (13)$$

The operator $\text{Wrap}()$ normalizes the heading residual to a specified angular interval, thereby avoiding residual jumps caused by the heading periodicity. Accordingly, the gyrocompass heading factor can be expressed as:

$$f_k^C(x_k) = \exp\left(-\frac{1}{2}\|r_k^C\|_{R_k^C}^2\right) \quad (14)$$

The gyrocompass heading factor suppresses heading drift by enforcing consistency between the measured heading and the heading predicted from the system state. It mainly constrains the yaw component of the attitude state and does not directly constrain the roll and pitch angles.

2.5. GNSS Position Factor

In the GNSS/INS/gyrocompass factor graph framework, the GNSS position solution is used as an external observation to provide absolute position constraints for the system. Let the GNSS position observation at epoch k be denoted as z_k^{GNSS} . The GNSS observation model is expressed as:

$$z_k^{\text{GNSS}} = h_{\text{GNSS}}(x_k) + n_k^{\text{GNSS}} \quad (15)$$

where $h_{\text{GNSS}}(x_k)$ denotes the predicted position extracted from the system state x_k , and n_k^{GNSS} denotes the GNSS measurement noise, which is assumed to satisfy:

$$n_k^{\text{GNSS}} \sim \mathcal{N}(0, \mathbf{R}_k^{\text{GNSS}}) \quad (16)$$

The residual of the GNSS position factor is defined as the difference between the observed position and the predicted position:

$$r_k^{\text{GNSS}} = z_k^{\text{GNSS}} - h_{\text{GNSS}}(x_k) \quad (17)$$

Accordingly, the GNSS position factor at epoch k can be written as:

$$f_k^{\text{GNSS}}(x_k) = \exp\left(-\frac{1}{2}\|z_k^{\text{GNSS}} - h_{\text{GNSS}}(x_k)\|_{\mathbf{R}_k^{\text{GNSS}}}^2\right) \quad (18)$$

where $\mathbf{R}_k^{\text{GNSS}}$ is the covariance matrix of the GNSS measurement noise. By introducing the GNSS position factor, absolute position information is incorporated into the factor graph to correct INS propagation errors and suppress position drift over time.

2.6. Marginalization Factor

In sliding-window factor graph optimization, marginalization is used to remove old states while preserving their historical information as a prior constraint. The state set within the sliding window is denoted as $\mathcal{X}^M$ and divided into the retained state set $\mathcal{X}_r$ and the marginalized state set x_m . After the relevant factors within the window are linearized around the current linearization point, the state x_m is eliminated through marginalization, resulting in a prior constraint on $\mathcal{X}_r$. This prior constraint can be represented as a marginalization factor:

$$f^M(\mathcal{X}_r) = \exp\left(-\frac{1}{2}\|r^M\|_{\Sigma^M}^2\right) \quad (19)$$

where r^M denotes the prior residual generated after marginalization, and Σ^M denotes the corresponding prior covariance matrix. In practical implementation, marginalization is usually performed by applying the Schur complement to the linearized normal equations, which absorbs the information of the marginalized states into the prior residual and prior covariance associated with the retained states [30].

The marginalization factor preserves historical measurement information while controlling the computational complexity of sliding-window optimization.

2.7. Factor Graph Optimization Process

In the GNSS/INS/gyrocompass integrated navigation factor graph framework, the constraint relationships between the system states and multi-source observations are jointly described by the factors introduced above. By jointly modeling these factors, the state estimation problem can therefore be transformed as a MAP estimation problem. The optimal state estimate is given by:

$$\hat{\mathcal{X}} = \underset{\mathcal{X}}{\operatorname{argmax}} p(\mathcal{X} | \mathcal{Z}^{\text{imu}}, \mathcal{Z}^{\text{GNSS}}, \mathcal{Z}^{\text{C}}) \quad (20)$$

where $\mathcal{Z}^{\text{GNSS}}$ denotes the GNSS observation set, and $\mathcal{Z}^{\text{C}}$ denotes the gyrocompass observation set. Based on the factorization property of factor graphs, the joint posterior probability can be factorized as:

$$p(\mathcal{X} | \mathcal{Z}^{\text{imu}}, \mathcal{Z}^{\text{GNSS}}, \mathcal{Z}^{\text{C}}) \propto f^{\text{M}} \prod_{k=1}^K f_k^{\text{pre}}(x_{k-1}, x_k) f_k^{\text{bias}}(b_{k-1}, b_k) \prod_{k \in \mathcal{G}} f_k^{\text{GNSS}}(x_k) \prod f_k^{\text{C}}(x_k) \quad (21)$$

where $\mathcal{G}$ denotes the set of epochs at which GNSS observations are available. When GNSS observations are unavailable, the corresponding GNSS position factors are removed. Under the Gaussian noise assumption, the MAP estimation problem can be transformed into the following nonlinear least-squares problem:

$$\hat{\mathcal{X}} = \underset{\mathcal{X}}{\operatorname{argmin}} \left\{ \sum_{k=1}^K \left(\|r_k^{\text{pre}}\|_{\Sigma_k^{\text{pre}}}^2 + \|r_k^{\text{bias}}\|_{\Sigma_k^{\text{bias}}}^2 \right) + \sum_{k \in \mathcal{G}} \|r_k^{\text{GNSS}}\|_{\mathbf{R}_k^{\text{GNSS}}}^2 + \sum \|r_k^{\text{C}}\|_{\mathbf{R}_k^{\text{C}}}^2 + \|r^{\text{M}}\|_{\Sigma^{\text{M}}}^2 \right\} \quad (22)$$

The nonlinear least-squares problem is solved iteratively. During optimization, the IMU preintegration and bias factors provide relative inertial constraints between adjacent states, the GNSS position factor provides absolute position constraints when GNSS observations are available, and the gyrocompass heading factor continuously provides heading constraints. In addition, marginalization converts historical state information into prior constraints, thereby maintaining the continuity of sliding-window optimization.

3. HTAGN-FGO-Based Shipborne Integrated Navigation Method

This section presents an HTAGN-FGO method for shipborne GNSS/INS/gyrocompass integrated navigation. Based on the GNSS/INS/gyrocompass factor graph framework, an HTAGN-based pseudo-GNSS observation model is introduced for GNSS outage compensation. The model predicts position increments and their uncertainties to construct adaptive pseudo-GNSS factors, enabling uncertainty-aware pseudo-GNSS fusion and improving navigation continuity under the evaluated outage scenarios.

3.1. HTAGN Model

3.1.1. Definition of Model Inputs and Outputs

The input–output design of the HTAGN determines its role in GNSS outage compensation. In this study, the GNSS position increment between adjacent epochs is selected as the prediction target and is defined as:

$$\Delta p_k^{\text{GNSS}} = p_k^{\text{GNSS}} - p_{k-1}^{\text{GNSS}} \quad (23)$$

HTAGN takes historical navigation feature sequences as input and jointly outputs the predicted mean and direction-dependent standard deviation of the position increment:

$$\mathbf{y}_k = (\boldsymbol{\mu}_k, \boldsymbol{\sigma}_k) \quad (24)$$

where $\boldsymbol{\mu}_k = [\mu_{n,k}, \mu_{E,k}, \mu_{D,k}]$ denotes the predicted position increment in the NED directions, and $\boldsymbol{\sigma}_k = [\sigma_{n,k}, \sigma_{E,k}, \sigma_{D,k}]$ denotes the corresponding direction-dependent standard deviation, which characterizes the uncertainty of pseudo-GNSS position increment prediction.

As shown in Figure 2, The input features are jointly constructed from IMU measurements and FGO-derived navigation states. The IMU specific force f_{ib}^b and angular velocity ω_{ib}^b characterize the short-term inertial motion of the ship, while the FGO-derived velocity $\mathbf{v}^n$ and heading angle ψ provide motion-state priors constrained by historical multi-source observations. Therefore, the feature vector at the j -th IMU timestamp within the k -th GNSS epoch is defined as:

$$\mathbf{z}_{k,j} = [f_{ib,k,j}^b, \omega_{ib,k,j}^b, \mathbf{v}_{k,j}^n, \psi_{k,j}] \quad (25)$$

where f_{ib}^b and ω_{ib}^b denote the specific force and angular velocity measured by the IMU, respectively. Each sample-level feature vector contains ten dimensions, including three-axis specific force, three-axis angular velocity, three-axis velocity, and heading angle. The specific force and angular velocity are obtained by dividing the measured velocity increments and angular increments by the IMU sampling interval. The remaining components consist of the north, east, and down velocities and the heading angle in radians at the corresponding IMU timestamp.

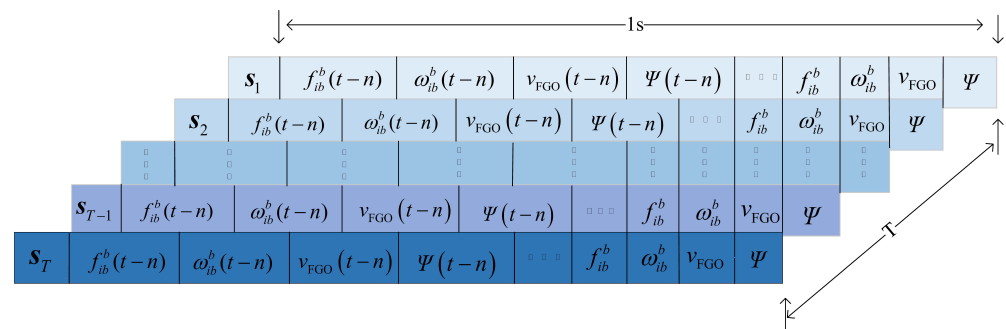


Figure 2. Construction of the HTAGN input sequence.

During the training stage, the 1 Hz FGO-derived velocity and heading states are linearly interpolated to the 100 Hz IMU timestamps to ensure temporal alignment with the inertial measurements. Therefore, each IMU sample within the same GNSS epoch is associated with the interpolated navigation state corresponding to its timestamp. During online inference, these navigation quantities are obtained from the current INS propagation or FGO-estimated navigation state.

The 100 sample-level feature vectors within each 1 s interval are concatenated in chronological order to construct the epoch-level feature vector:

$$\mathbf{s}_k = [\mathbf{z}_{k,1}, \mathbf{z}_{k,2}, \dots, \mathbf{z}_{k,100}] \in \mathbb{R}^{1 \times 1000} \quad (26)$$

The epoch-level feature vectors from the latest 16 consecutive GNSS-aligned intervals are then stacked chronologically to form the fixed-size HTAGN input tensor:

$$\mathbf{X}_k = [\mathbf{s}_{k-15}^T, \mathbf{s}_{k-14}^T, \dots, \mathbf{s}_k^T]^T \in \mathbb{R}^{16 \times 1000} \quad (27)$$

Before being fed into HTAGN, each input feature dimension is normalized to the range $[-1, 1]$ using a MinMax scaler fitted only on the training dataset. The same scaling parameters are applied to validation, testing, and online inference data to maintain consistent feature distributions.

3.1.2. Architecture of the HTAGN Model

Figure 3 shows the architecture of the proposed HTAGN model, which consists of a GRU primary encoder, a TCN auxiliary encoder, a subsequence alignment module, a feature consistency constraint module, and a heteroscedastic prediction module. The model takes historical navigation sequences as input. The GRU encoder captures long-term temporal dependencies, while the TCN encoder extracts local short-term dynamics caused by wind and wave disturbances, speed fluctuations, and attitude variations. By combining these two branches, HTAGN can represent both global temporal evolution and local motion perturbations in ship navigation sequences.

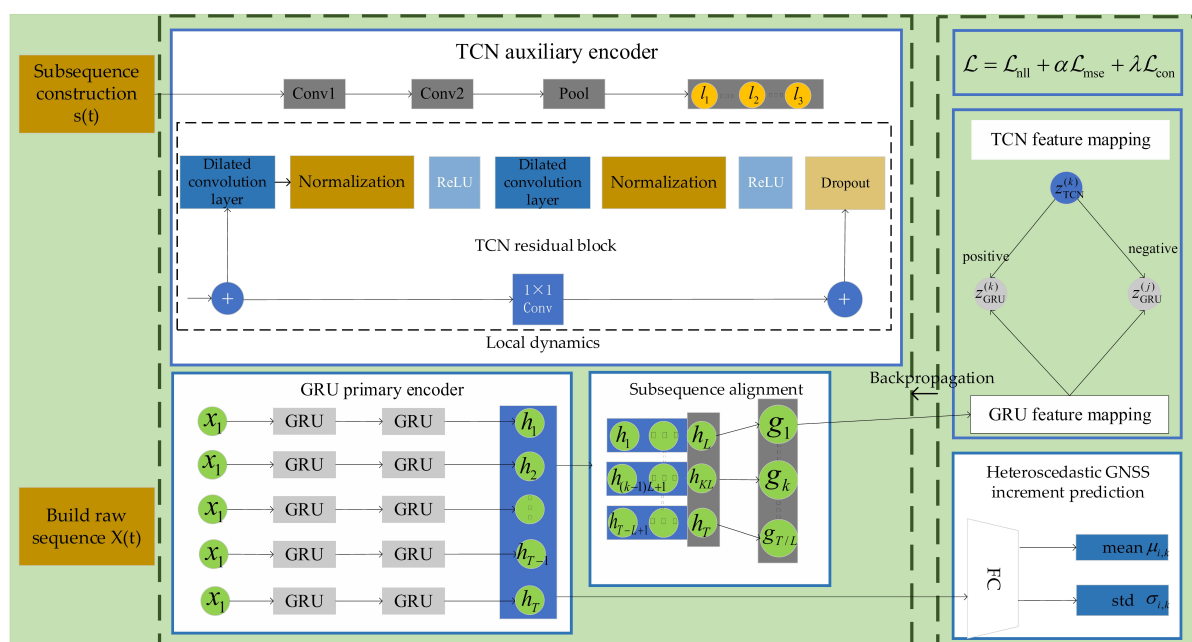


Figure 3. Overall architecture of the proposed HTAGN model.

Specifically, the TCN auxiliary encoder employs causal and dilated convolutions to enlarge the receptive field while preserving temporal causality. Residual connections, batch normalization, and dropout are further introduced to improve training stability and generalization. The GRU primary encoder uses its gating mechanism to selectively retain relevant historical information and learn the mapping from IMU measurements and FGO-derived navigation states to GNSS position increments. To reduce the feature mismatch between the two branches, the subsequence alignment module divides the input sequence into consecutive subsequences and aligns TCN features with GRU hidden states over the same time span. The feature consistency constraint module further projects the two types of features into a shared embedding space, enhancing within-subsequence similarity while preserving discriminability across different time segments.

The heteroscedastic output module uses fully connected layers to jointly predict the mean and direction-dependent standard deviation of GNSS position increments. During GNSS outages, the predicted mean is used to generate pseudo-GNSS observations, while the predicted standard deviation is used to construct the observation covariance of pseudo-GNSS factors. Unlike deterministic TAGN or CNN-GRU models that

output only position increment estimates, HTAGN provides uncertainty-aware pseudo-observations with direction-dependent confidence. This enables adaptive factor weighting and provides the basis for the subsequent heteroscedastic loss function and pseudo-GNSS factor construction.

3.1.3. Heteroscedastic Prediction and Pseudo-GNSS Modeling

To characterize the direction-dependent reliability of HTAGN predictions, GNSS position increment prediction is formulated as a heteroscedastic Gaussian regression problem. Given the historical input sequence $\mathbf{X}_k$, the true GNSS position increment in each NED direction is assumed to follow a conditional Gaussian distribution:

$$y_{i,k}|\mathbf{X}_k \sim \mathcal{N}(\mu_{i,k}, \sigma_{i,k}^2), i \in \{N, E, D\} \quad (28)$$

Based on this assumption, the heteroscedastic Gaussian negative log-likelihood loss is defined as:

$$\mathcal{L}_{\text{nll}} = \frac{1}{3} \sum_{i \in \{N, E, D\}} \left[\frac{(y_{i,k} - \mu_{i,k})^2}{2\sigma_{i,k}^2} + \frac{1}{2} \ln \sigma_{i,k}^2 \right] \quad (29)$$

This loss function jointly constrains the position increment prediction error and the predicted uncertainty. When the prediction error is large, the model can reduce the influence of the corresponding sample on the training objective by increasing σ_i^2 . Meanwhile, the $\ln \sigma_i^2$ term prevents the uncertainty from increasing without bound, thereby encouraging the model to learn a reasonable prediction confidence level.

To enhance the stability of mean prediction, an auxiliary mean-squared error term for the position increment is further introduced during training:

$$\mathcal{L}_{\text{mse}} = \frac{1}{3} \sum_{i \in \{N, E, D\}} (y_{i,k} - \mu_{i,k})^2 \quad (30)$$

Since the TCN and GRU branches capture different temporal characteristics, i.e., local dynamic patterns and long-term sequential dependencies, their latent features may exhibit distribution discrepancies during feature extraction. Therefore, a contrastive feature consistency loss is introduced to explicitly regulate the relationship between the two feature spaces. Specifically, the GRU and TCN features extracted from the same subsequence are treated as positive pairs, while features from different subsequences are regarded as negative pairs. By maximizing the similarity between positive feature pairs and separating negative pairs, the contrastive constraint encourages the two branches to learn complementary yet consistent motion representations. The contrastive loss is defined as:

$$\mathcal{L}_{\text{con}} = -\frac{1}{K} \sum_{k=1}^K \log \frac{\exp(\text{sim}(\mathbf{z}_{\text{TCN}}^k, \mathbf{z}_{\text{GRU}}^k) / \tau)}{\sum_{j=1}^K \exp(\text{sim}(\mathbf{z}_{\text{TCN}}^k, \mathbf{z}_{\text{GRU}}^j) / \tau)} \quad (31)$$

where $\mathbf{z}_{\text{TCN}}^k$ and $\mathbf{z}_{\text{GRU}}^k$ denote the projected TCN and GRU features of the k -th subsequence, respectively; $\text{sim}()$ denotes the cosine similarity function; and τ is the temperature parameter. The sensitivity analysis was conducted using a 90 s GNSS outage scenario from the validation dataset, where the model performance was evaluated based on the horizontal RMSE of the recovered trajectory. Table 1 presents the sensitivity of HTAGN to the contrastive temperature parameter τ . The model achieves the lowest validation horizontal RMSE when $\tau = 0.1$. A smaller value of $\tau = 0.05$ produces sharper similarity discrimination, which slightly reduces training stability, whereas larger values weaken the contrastive

constraint and lead to moderate performance degradation. The results indicate that the model is moderately sensitive to the temperature parameter, and an appropriate temperature is important for balancing feature discrimination and training stability. Therefore, $\tau = 0.1$ was adopted in the final model.

Table 1. Sensitivity analysis of the contrastive temperature parameter τ .

Temperature τ	Validation RMSE-H (m)	Best Epoch
0.05	2.932	212
0.1	2.816	274
0.2	3.480	298
0.5	4.581	323

Combining the heteroscedastic prediction loss, the mean prediction constraint, and the multi-scale feature consistency constraint, the overall training objective of HTAGN is defined as:

$$\mathcal{L} = \mathcal{L}_{\text{nll}} + \alpha \mathcal{L}_{\text{mse}} + \lambda \mathcal{L}_{\text{con}} \quad (32)$$

where α is the weight of the auxiliary mean-error term, and λ is the weight of the contrastive constraint loss. In this study, α and λ are set to 0.02 and 0.05, respectively.

During GNSS outages, the position increment mean output by HTAGN is used to generate pseudo-GNSS observations. Let $\mathbf{p}_k^{\text{last}}$ denote the reliable position before GNSS outages. Then, the pseudo-GNSS position at the $k + j$ outage epoch can be recursively expressed as:

$$\mathbf{z}_{k+j}^{p,\text{GNSS}} = \mathbf{p}_k^{\text{last}} + \sum_{s=1}^j \boldsymbol{\mu}_{k+s} \quad (33)$$

where $\boldsymbol{\mu}_{k+s}$ denotes the predicted position increment mean at the $k + s$ outage epoch. This recursive accumulation describes the generation of pseudo-position observations during continuous GNSS outages.

Meanwhile, the direction-dependent uncertainties output by HTAGN are propagated over the GNSS outage period to characterize the uncertainty of the accumulated pseudo-GNSS position. Since the pseudo-position is recursively generated by accumulating multiple predicted position increments, the covariance of the pseudo-GNSS observation is obtained by accumulating the uncertainty of each predicted increment:

$$\mathbf{R}_{k+j}^{p,\text{GNSS}} = \sum_{s=1}^j \mathbf{R}_{k+s}^{\Delta p} \quad (34)$$

where $\mathbf{R}_{k+s}^{\Delta p}$ denotes the covariance matrix of the position increment predicted by HTAGN at the $k + s$ outage epoch, which is constructed as:

$$\mathbf{R}_{k+s}^{\Delta p} = \begin{bmatrix} \sigma_{n,k+s}^2 & 0 & 0 \\ 0 & \sigma_{E,k+s}^2 & 0 \\ 0 & 0 & \sigma_{D,k+s}^2 \end{bmatrix} \quad (35)$$

The cross-correlation between adjacent predicted increments is not explicitly modeled in the current framework. Instead, an independent increment approximation is adopted as a practical uncertainty propagation strategy. Accordingly, the resulting accumulated covariance should be interpreted as an approximate uncertainty representation rather than the exact covariance of the accumulated prediction error. This approximation avoids introducing an additional temporal cross-covariance model and maintains the computational efficiency required for real-time factor graph optimization. Although the overlapping

16 s historical input windows may introduce temporal correlations among consecutive predictions, the predicted direction-dependent standard deviations still provide sample-dependent relative confidence information for adaptive pseudo-GNSS weighting under the evaluated conditions. With the propagated uncertainty incorporated into the pseudo-GNSS observation covariance, the corresponding information matrix used in factor graph optimization is obtained as:

$$\Omega_{k+j}^{p,\text{GNSS}} = \left(\mathbf{R}_{k+j}^{p,\text{GNSS}} \right)^{-1} \quad (36)$$

Therefore, the residual of the pseudo-GNSS factor can be written as:

$$\mathbf{r}_{k+j}^{p,\text{GNSS}} = \mathbf{p}_{k+j}^n - \mathbf{z}_{k+j}^{p,\text{GNSS}} \quad (37)$$

Its weighted residual form is:

$$\left\| \mathbf{r}_{k+j}^{p,\text{GNSS}} \right\|_{\mathbf{R}_{k+j}^{p,\text{GNSS}}}^2 = \left(\mathbf{r}_{k+j}^{p,\text{GNSS}} \right)^T \Omega_{k+j}^{p,\text{GNSS}} \mathbf{r}_{k+j}^{p,\text{GNSS}} \quad (38)$$

Through this modeling, the direction-dependent uncertainties predicted by HTAGN are propagated throughout the GNSS outage period and explicitly incorporated into the covariance matrix of the pseudo-GNSS factor. As the outage interval increases, the accumulated uncertainty gradually increases, leading to a corresponding reduction in the pseudo-observation constraint weight. Therefore, low-confidence long-term predictions are prevented from imposing excessively strong constraints on the FGO optimization. This uncertainty-aware covariance propagation directly links HTAGN prediction confidence with the FGO information matrix and distinguishes the proposed method from fixed-noise TAGN-based pseudo-GNSS fusion.

3.2. HTAGN-FGO Training and Inference Process

Using the HTAGN-based pseudo-GNSS increment prediction model described above, the proposed method operates in two phases: training and inference.

As shown in Figure 4, GNSS observations are available during the training phase. Factor graph optimization is first performed using IMU and GNSS measurements to obtain integrated navigation state estimates. Taking the GNSS epoch as the time reference, IMU measurements within each GNSS cycle are aligned with the corresponding FGO-derived states and reorganized into per-epoch feature units. These units are then arranged over multiple consecutive GNSS epochs to construct a fixed-length input window for HTAGN, enabling the model to learn the mapping from historical navigation features to GNSS position increments and prediction uncertainties.

When a GNSS outage occurs, the system switches to the inference phase, as illustrated in Figure 4. Since GNSS observations are unavailable, buffered multi-epoch IMU measurements and FGO-derived states, including velocity and heading, are fed into HTAGN using the same data format as in the training phase. HTAGN outputs the predicted position increments and corresponding standard deviations. The predicted increments are accumulated from the last reliable GNSS position to generate pseudo-GNSS position observations, while the predicted standard deviations are converted into the observation covariance and information matrix of the pseudo-GNSS factor. In this way, the pseudo-GNSS factor provides absolute position constraints during GNSS outages, suppressing INS-only error accumulation and improving positioning continuity and stability.

Although HTAGN is trained using FGO-derived states obtained under normal GNSS availability, the inference stage relies on states corrected by recursively generated pseudo-GNSS factors, which may introduce a potential train–inference distribution discrepancy. This effect is limited because HTAGN does not directly use absolute position as an input;

instead, its inputs mainly consist of short-term IMU measurements and locally estimated motion states, which characterize vessel dynamics. Furthermore, the uncertainty-aware pseudo-GNSS factor mitigates recursive error feedback by dynamically adjusting the observation confidence. As prediction uncertainty increases, the accumulated covariance is enlarged and the corresponding constraint weight is reduced, preventing low-confidence pseudo-observations from dominating FGO optimization. The 90 s and 180 s GNSS outage experiments inherently evaluate this closed-loop inference process, since the predicted pseudo-GNSS observations are recursively generated and fused into FGO. The stable positioning performance under increasing outage interval demonstrates that the proposed framework can effectively suppress recursive error accumulation during GNSS outages.

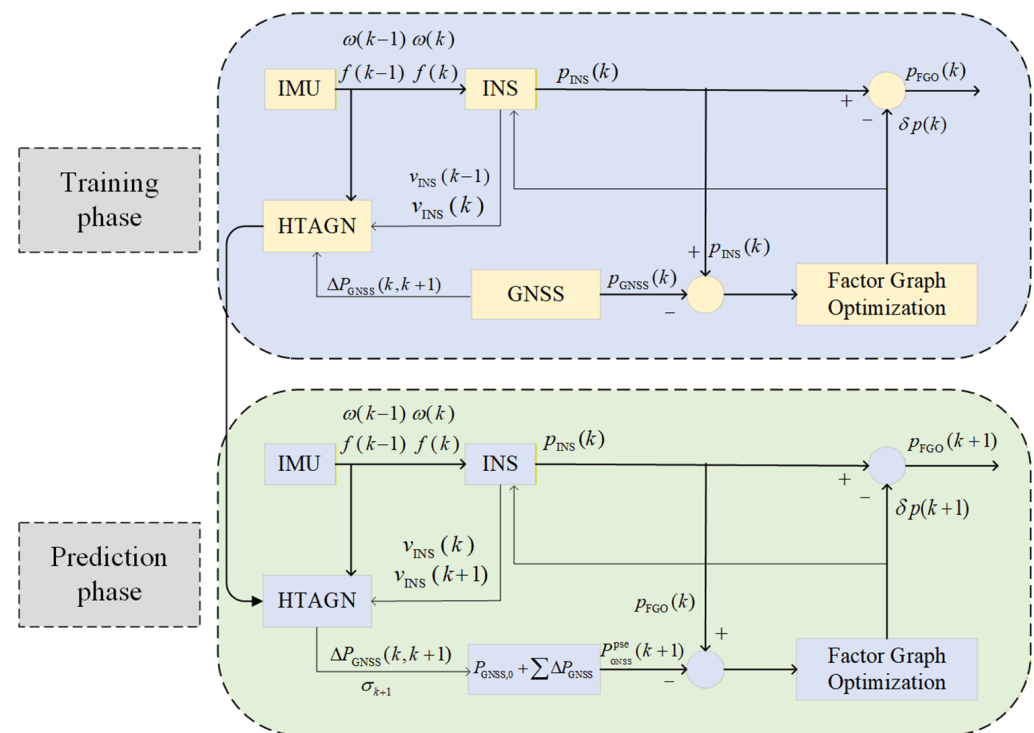


Figure 4. HTAGN-FGO workflow.

4. Results and Discussion

4.1. Experimental Setup

For reproducibility, the implementation details and parameter settings of the proposed FGO framework used in the experiments are provided as follows. The FGO was implemented using Ceres Solver with a fixed-lag sliding window containing 40 consecutive 1 s IMU preintegration intervals and 41 navigation states, corresponding to a temporal span of 40 s. The graph was optimized at 1 Hz, with a maximum of 40 nonlinear iterations at each epoch. The nonlinear least-squares problem was solved using the Levenberg–Marquardt method, together with Sparse Normal Cholesky factorization and the SuiteSparse backend.

The local NED frame was established using the first valid GNSS position. The initial attitude, velocity, gyroscope bias, and accelerometer bias were specified by the corresponding initialization parameters, while subsequent states were initialized by IMU preintegration from the preceding optimized state.

The covariance of the IMU preintegration factors was propagated from the calibrated gyroscope angle random walk, accelerometer velocity random walk, and IMU bias-noise parameters. For each GNSS position factor, an epoch-dependent covariance matrix was constructed using the north, east, and down standard deviations provided in the ob-

servation file. The covariance of each pseudo-GNSS factor was constructed from the direction-dependent standard deviations predicted by HTAGN. The standard deviation of the gyrocompass heading factor was set to 0.3° . A prior was imposed on the gyroscope and accelerometer biases of the newest state in the window, with its standard deviations set equal to the corresponding calibrated bias standard deviations. The marginalization prior inherited its information matrix from the marginalized residuals and therefore did not require an independently specified covariance.

A Huber loss with a scale parameter of 5.0 was applied to the GNSS, pseudo-GNSS, and gyrocompass factors. No robust loss was applied to the IMU preintegration, bias-prior, or marginalization factors.

4.2. Effect of the Gyrocompass Factor on Heading Accuracy

To verify the applicability of the proposed gyrocompass heading factor in real maritime scenarios, field experiments were conducted aboard the Haixun 156 experimental vessel. The CHCNAV CGI-830 GNSS/INS high-precision system (Shanghai Huace Navigation Technology Ltd., Shanghai, China) is used as the reference system to provide post-processed high-accuracy positioning results for trajectory evaluation. The CHCNAV CGI-430 tightly coupled GNSS/INS system (Shanghai Huace Navigation Technology Ltd., Shanghai, China) is deployed as the onboard navigation unit, providing GNSS measurements at 1 Hz and IMU data at 100 Hz for the proposed factor graph framework. In addition, a BlueSun110 fiber-optic gyrocompass (Beijing BlueSun Technology Co., Ltd., Beijing, China) is installed onboard to provide absolute heading observations with 1 Hz update rate, 0.1° static accuracy, and 0.3° dynamic accuracy, which are used to construct the gyrocompass heading factor.

All sensors were rigidly installed on the same vessel and time-synchronized through a unified onboard data acquisition and processing platform to ensure the consistency of multi-source navigation data. The lever-arm effect between the GNSS antenna and the IMU was compensated using the calibrated installation offset. Specifically, the lever arm was expressed in the IMU body frame as $l_{ant}^b = [0, -1.32, 0]^T$ m, and the corresponding GNSS antenna position was compensated in the GNSS factor formulation.

The gyrocompass measurements were compensated using pre-established device-level calibration parameters, including a 21 s time synchronization delay and a 0.985° mounting misalignment angle. These parameters were obtained before the sea trial and were not estimated from the evaluated trajectory.

The time synchronization of all sensors was performed using the onboard acquisition system. GNSS and IMU measurements were aligned at integer-second epochs, while the 100 Hz IMU data were interpolated to the corresponding timestamps of other navigation measurements. The residual synchronization error between the GNSS and IMU measurements was less than 1 ms.

All measurements were transformed into a unified local NED frame. The navigation frame was initialized using the first GNSS position under the WGS84 Earth-centered Earth-fixed coordinate system. The IMU body frame was transformed to the local NED frame using the calibrated attitude transformation, and the gyrocompass heading observations were converted through the calibrated yaw alignment before being introduced into the factor graph. The experimental vessel platform is shown in Figure 5.

The detailed IMU specifications of the CHCNAV CGI-430 and CGI-830 systems are summarized in Table 2.

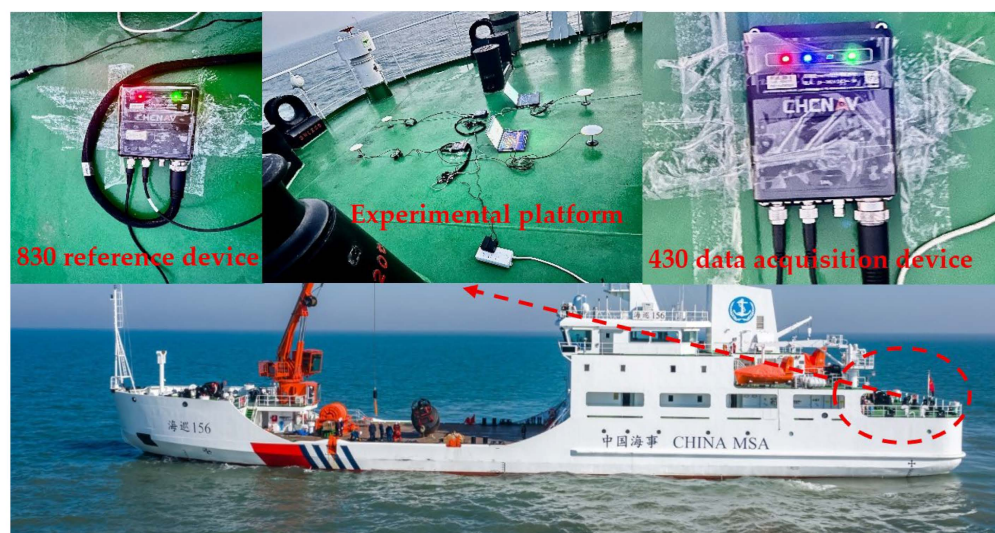


Figure 5. Maritime vessel experimental platform. The red circle indicates the installation location of the onboard navigation sensors.

Table 2. IMU specifications of the experimental vessel.

Sensor	Parameter	CHCNAV CGI-430	CHCNAV CGI-830
Gyroscope	Bias stability (deg/h)	4	1
	Angular random walk (deg/sqrt(h))	0.1	0.05
Accelerometer	Bias stability (mGal)	49	49
	Velocity random walk (m/s/sqrt(h))	0.035	0.03

On 8 November 2025, navigation data were collected aboard the Haixun 156 vessel during a voyage from Tianjin Port to Caofeidian Port. The ship trajectory is shown in Figure 6. The selected 4000 s segment, corresponding to the GPS seconds-of-week range from 529,200 s to 533,200 s, was mainly located in open waters. During this period, the vessel sailed approximately straight at a nearly constant speed under autopilot control, with only small speed fluctuations and local disturbances induced by wind and waves. Therefore, the dataset reflects the low-dynamic and locally perturbed motion characteristics of ships in real maritime environments. The best-performing HTAGN checkpoint was selected according to the validation loss under a simulated 90 s GNSS outage starting at 532,200 s within the independent validation set of 532,000–532,400 s. This validation outage did not overlap with the test set beginning at 532,400 s or any subsequent outage evaluations.

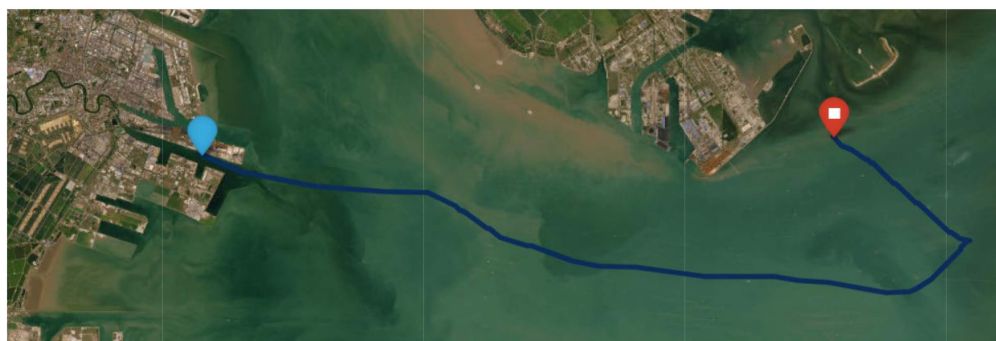


Figure 6. Trajectory of the experimental vessel.

To evaluate the effect of the gyrocompass heading factor on heading estimation, a comparative experiment was conducted using the above real ship trial data. Two schemes were designed under the same factor graph optimization framework: a baseline GNSS/INS FGO scheme without the gyrocompass factor and a GNSS/INS/gyrocompass FGO scheme with the gyrocompass heading factor. In both schemes, the IMU preintegration factor, GNSS position factor, IMU bias factor, marginalization factor, and optimization parameters were kept consistent; only the gyrocompass heading constraint was changed. By comparing the heading error curves and statistical indicators over the entire voyage segment, the ability of the gyrocompass factor to suppress heading error accumulation under low-dynamic ship navigation conditions was evaluated. The heading solution provided by the independent CGI-830 integrated navigation system was used directly as the reference for heading performance evaluation and was not involved in the FGO optimization process. Before constructing the gyrocompass heading observations, the gyrocompass measurements were compensated using pre-established device-level calibration parameters, including a 21 s time synchronization delay and a 0.985° mounting misalignment angle. These parameters were obtained before the sea trial and were not estimated from the evaluated trajectory; therefore, no calibration interval within the 529,200–533,200 s segment was used. The experimental results are shown in Figures 7 and 8.

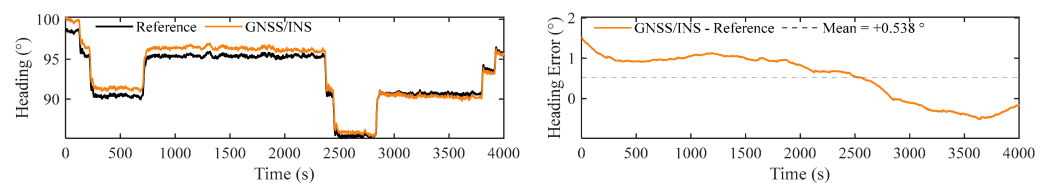


Figure 7. Heading comparison without the gyrocompass heading factor.

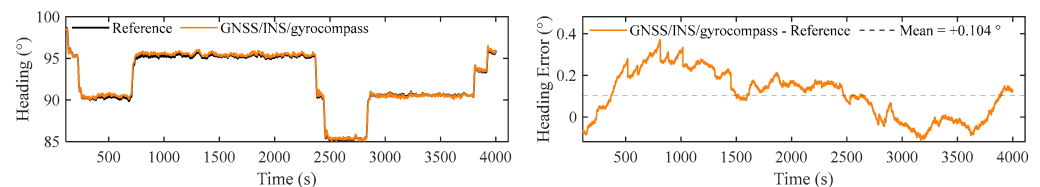


Figure 8. Heading comparison with the gyrocompass heading factor.

In the following analysis, the mean value denotes the average signed heading error (bias). As shown in Figure 7, without the gyrocompass heading factor, the heading estimated by the integrated navigation system generally follows the reference heading variation. However, the heading error shows a cumulative drift trend over time. The error remains mostly positive in the first half of the voyage, but gradually decreases after approximately 2500 s and eventually becomes negative. The mean heading error over the entire voyage segment is $+0.538^\circ$, with an RMSE of 0.786° , indicating cumulative drift with a positive mean error rather than a stable constant bias. This is mainly because GNSS observations in the conventional GNSS/INS factor graph provide position constraints but lack direct heading observations. Under low-dynamic and approximately constant-speed sailing conditions, heading drift caused by gyroscope bias cannot be fully suppressed.

As shown in Figure 8, after introducing the gyrocompass heading factor, the GNSS/INS/gyrocompass solution agrees more closely with the reference heading. The heading error remains within a small range without obvious long-term accumulation. Compared with the baseline scheme without the gyrocompass factor, the mean heading error decreases from $+0.538^\circ$ to $+0.104^\circ$, and the RMSE decreases from 0.786° to 0.121° , indicating a significant improvement in heading accuracy. These results demonstrate that the gyrocompass heading factor enhances heading observability under low-dynamic ship

navigation conditions and effectively suppresses heading drift caused by gyroscope bias and integration errors.

4.3. Comparative Experiment on GNSS Outage Compensation

To verify the applicability of the proposed HTAGN model in real maritime scenarios, the same experimental equipment and dataset as those in the heading error comparison experiment were used. Model training was conducted on a Windows 11 64-bit workstation equipped with an Intel Core i9-14900 CPU, 64 GB RAM, and an NVIDIA GeForce RTX 5080 GPU. The Adam optimizer was used to update the model parameters, and the learning rate was set to 0.001. The model training parameters are listed in Table 3. During training, the model checkpoint with the lowest validation horizontal RMSE under the 90 s GNSS outage condition was selected as the final model. Early stopping was adopted to prevent overfitting, and training was terminated when the validation metric did not improve for 50 consecutive epochs.

Table 3. Model training parameters.

Model Parameter	Value
Time step	16
Subsequence length	4
Number of primary prediction encoder layers	2
Number of hidden units in the primary prediction encoder	128
Number of auxiliary encoder channels	[256, 128]
Kernel size of the auxiliary encoder	3
Projection dimension	64
Dropout	0.2
Learning rate	0.001
Random seed	20
Batch size	128
Maximum number of epochs	500

Although HTAGN introduces additional uncertainty estimation components, the objective of the comparison is to evaluate the effectiveness of uncertainty-aware pseudo-GNSS weighting rather than the influence of increased model complexity.

To evaluate the effectiveness of the proposed HTAGN model for pseudo-GNSS increment prediction, GRU, TCN, and TAGN were selected as comparison models. GRU was adopted to evaluate recurrent temporal modeling, while TCN was used to investigate local dynamic feature extraction based on temporal convolution. TAGN combines the TCN and GRU branches through subsequence alignment and a feature-consistency constraint and serves as the deterministic fixed-covariance pseudo-GNSS baseline. Based on the same TAGN backbone, HTAGN further introduces a heteroscedastic uncertainty prediction head to enable uncertainty-aware adaptive weighting.

For a fair comparison, all models used the same input sequences, position-increment targets, data-partitioning strategy, optimizer, learning rate, batch size, maximum number of epochs, early-stopping strategy, and model-selection protocol. The architecture-specific hyperparameters, including the GRU hidden dimensions and TCN channel configurations, were selected according to validation-set performance. In particular, TAGN and HTAGN were maintained at nearly identical parameter scales to isolate the effect of heteroscedastic uncertainty modeling. The final model configurations and trainable parameter numbers are summarized in Table 4.

As shown in Table 4, HTAGN contains 1,466,502 trainable parameters, which is only 387 more than TAGN. These additional parameters are mainly introduced by the het-

eroscedastic uncertainty prediction branch. The comparable parameter scales between HTAGN and TAGN indicate that the performance improvement is not primarily caused by increased model complexity, but by the uncertainty-aware adaptive weighting mechanism.

Table 4. Architecture configurations and trainable parameters of comparison models.

Model	Architecture Configuration	Training Loss	Number of Trainable Parameters
GRU	Two-layer GRU with 128 hidden units	MSE loss	533,379
TCN	Two-layer TCN with channel sizes [256, 128] and kernel size 3	MSE loss	1,402,371
TAGN	TCN-assisted GRU with subsequence alignment and feature-consistency constraint	$\mathcal{L}_{\text{mse}} + \lambda \mathcal{L}_{\text{con}}$	1,466,115
HTAGN	TAGN + heteroscedastic uncertainty prediction head	$\mathcal{L}_{\text{nll}} + \alpha \mathcal{L}_{\text{mse}} + \lambda \mathcal{L}_{\text{con}}$	1,466,502

For model training and evaluation, the trajectory data from 0 s to 4000 s were chronologically divided into training, validation, and test sets with ratios of 70%, 10%, and 20%, respectively. The first 2800 s were used for training, 2800–3200 s for validation, and 3200–4000 s for testing. Two artificial GNSS outages were introduced in the test set, corresponding to 90 s (3200–3290 s) and 180 s (3460–3640 s) outage.

Figure 9 shows the positioning trajectories under the 90 s and 180 s GNSS outage conditions. Without GNSS updates, INS-only propagation suffers from continuous error accumulation caused by inertial sensor biases, attitude errors, and velocity error integration. The horizontal RMSE increases from 8.295 m for the 90 s outage to 28.144 m for the 180 s outage, while the maximum horizontal error increases from 20.783 m to 88.036 m. In comparison, all neural-network-aided methods reduce position drift to varying degrees.

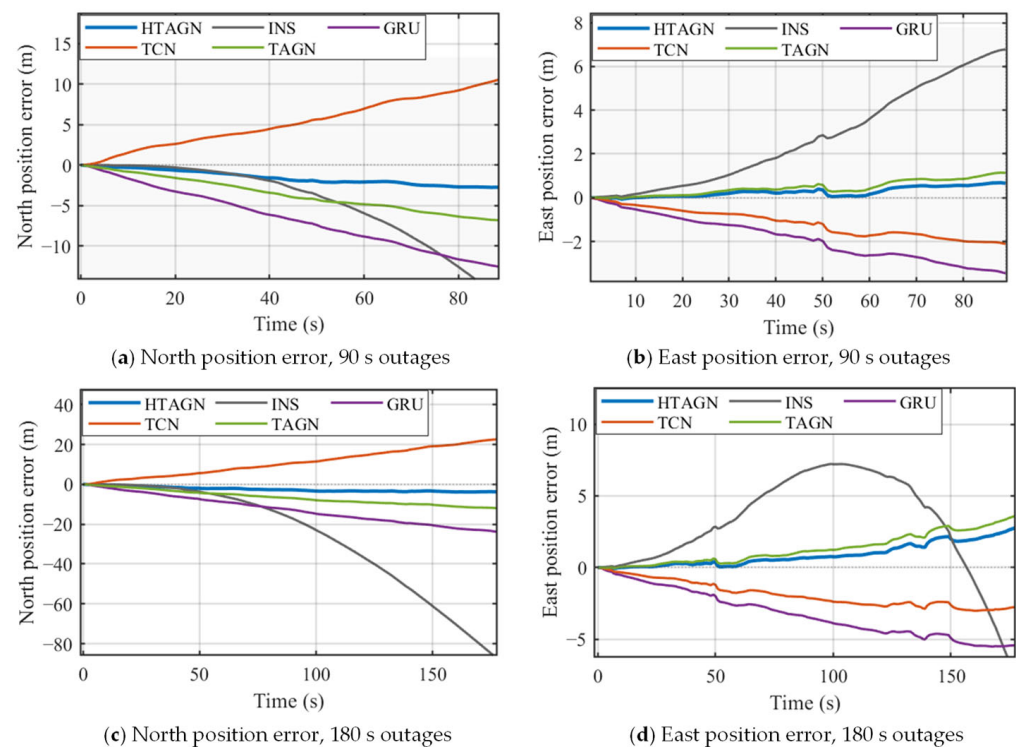


Figure 9. Time-varying position error curves during real-vessel GNSS outages.

As shown in Table 5, HTAGN achieves the best positioning performance under both GNSS outage durations. For the 90 s outages, its horizontal RMSE and maximum horizontal error are 1.817 m and 2.816 m, respectively, which are 63.7% and 59.1% lower than those of

the next-best TAGN baseline. For the 180 s outages, the corresponding values are 3.456 m and 4.614 m, representing reductions of 52.5% and 58.4%, respectively. The error curves show that HTAGN maintains small and stable errors in both the north and east directions, without the continuous divergence observed in INS-only propagation or the pronounced directional drift observed in GRU and TCN.

Table 5. Real-vessel position errors during GNSS outages.

Metric	Algorithm	GNSS 90 s Outages			GNSS 180 s Outages		
		δp_N (m)	δp_E (m)	δp_H (m)	δp_N (m)	δp_E (m)	δp_H (m)
RMSE	INS	7.537	3.465	8.295	26.885	8.324	28.144
	GRU	6.730	2.092	7.048	12.733	4.639	13.552
	TCN	5.612	1.205	5.740	10.977	2.160	11.187
	TAGN	4.971	0.558	5.002	6.551	3.179	7.281
	HTAGN	1.785	0.339	1.817	2.765	2.073	3.456
Maximum Absolute Error	INS	19.405	7.442	20.783	87.635	8.389	88.036
	GRU	12.486	3.389	12.938	23.540	5.420	24.156
	TCN	10.467	2.033	10.663	21.746	2.753	21.779
	TAGN	6.786	1.147	6.882	10.493	3.584	11.088
	HTAGN	2.731	0.687	2.816	3.705	2.750	4.614

This improvement can be attributed to two aspects. First, the TCN-assisted GRU structure enables HTAGN to capture both local dynamic variations and long-term temporal dependencies in ship motion, thereby improving the prediction accuracy of pseudo-GNSS position increments. This is particularly important for low-dynamic ship navigation, where motion changes are relatively slow but local disturbances caused by wind, waves, and speed fluctuations still exist. Second, the heteroscedastic prediction mechanism maps direction-dependent uncertainty to the covariance of pseudo-GNSS factors, thereby preventing unreliable predictions from imposing overly strong constraints on the factor graph. By adaptively adjusting the pseudo-observation weights according to prediction confidence, HTAGN achieves a better balance between error correction and constraint reliability. Overall, HTAGN provides higher positioning accuracy and better error stability under both short and longer GNSS outage conditions, validating the effectiveness of the proposed uncertainty-aware TCN-GRU-assisted factor graph method for shipborne GNSS outage compensation.

Figure 10 presents the north and east velocity errors during the 90 s and 180 s GNSS outages. Compared with the position errors, the velocity errors of all methods are much smaller and do not exhibit rapid divergence. Even when the outage interval increases, the velocity error curves mainly show small fluctuations or slow drift rather than continuously amplified divergence. This indicates that GNSS outages have a more pronounced impact on position accuracy than on short-term velocity accuracy.

This phenomenon is consistent with the error propagation mechanism of inertial navigation. Velocity errors are mainly affected by accelerometer bias, attitude errors, gravity projection errors, and initial velocity errors, which tend to vary relatively slowly under low-dynamic ship motion with mild speed changes. In contrast, position errors are obtained by integrating velocity errors over time. Therefore, even small velocity biases can gradually accumulate into significant position drift as the outage interval increases.

As shown in Table 6, INS-only propagation exhibits increasing velocity errors as the outage interval increases, reflecting the degradation of inertial estimation without GNSS updates. For the 90 s outages, all learning-based methods reduce velocity errors compared with INS-only propagation, with HTAGN achieving the best performance, reaching north

and east velocity RMSEs of 0.0172 m/s and 0.0159 m/s, respectively, corresponding to reductions of 85.9% and 88.9% compared with TAGN. For the 180 s outages, INS-only velocity errors further increase significantly, with north velocity RMSE reaching 0.5997 m/s. In contrast, HTAGN achieves the lowest north velocity RMSE of 0.0392 m/s and maintains stable performance across both axes, indicating consistent velocity estimation performance and reduced peak errors within the evaluated voyage under long-duration GNSS outages.

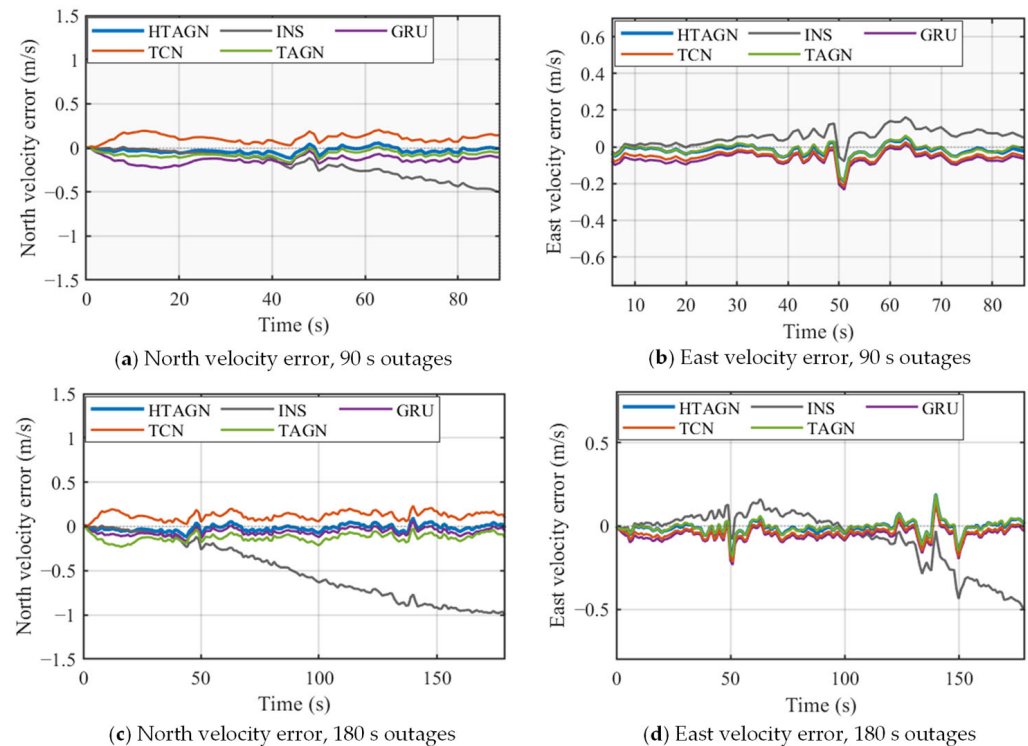


Figure 10. Time-varying velocity error curves during real-vessel GNSS outages.

Table 6. Real-vessel velocity errors during GNSS outages.

Metric	Algorithm	GNSS 90 s Outages		GNSS 180 s Outages	
		δv_N (m/s)	δv_E (m/s)	δv_N (m/s)	δv_E (m/s)
RMSE	INS	0.2591	0.2299	0.5997	0.1858
	GRU	0.2134	0.0244	0.1323	0.0608
	TCN	0.1695	0.1757	0.1312	0.0414
	TAGN	0.1220	0.1428	0.0711	0.0677
	HTAGN	0.0172	0.0159	0.0392	0.0468
Maximum Absolute Error	INS	0.5038	0.2735	1.0018	0.5072
	GRU	0.2642	0.2984	0.1547	0.1546
	TCN	0.1790	0.2021	0.1922	0.1973
	TAGN	0.2114	0.2984	0.2399	0.1801
	HTAGN	0.0232	0.0538	0.0923	0.1090

4.4. Ablation Study of the HTAGN Components

4.4.1. Effect of the Heteroscedastic Prediction Mechanism

To isolate the contribution of the heteroscedastic adaptive mechanism, an ablation experiment was conducted. In the HTAGN-fixed-std variant, the network architecture, loss function, and training strategy were kept identical to those of HTAGN. The only difference was in GNSS outage inference: the observation covariance of the pseudo-GNSS factor was replaced by a fixed empirical covariance constructed from the per-direction time-averaged predicted standard deviations over the training set. Therefore, the comparison

between HTAGN and HTAGN-fixed-std enables an independent evaluation of whether the predicted standard deviations are used for adaptive factor weighting. Figure 11 compares their positioning errors under the 180 s GNSS outages.

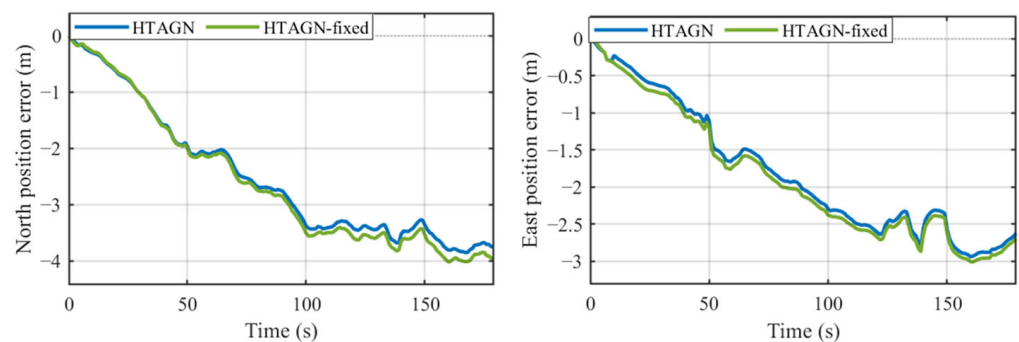


Figure 11. Ablation comparison of the heteroscedastic mechanism.

To verify that the performance improvement of the proposed HTAGN-FGO framework is not solely attributed to improved position-increment prediction accuracy, the raw prediction errors of HTAGN and HTAGN-fixed-std were evaluated before FGO fusion.

HTAGN-fixed-std and HTAGN use the same trained network, inference inputs, and predicted position increments. Their pre-fusion prediction results are therefore identical, as confirmed by Table 7, in which both methods achieve the same RMSE values in the north, east, and horizontal directions. Under this controlled setting, the difference in the FGO positioning results is attributable solely to the covariance weighting strategy. Further analysis shows that the performance gap between HTAGN and HTAGN-fixed-std increases from 25.0% under the 90 s outage to 36.6% under the 180 s outage. During the early stage of an outage, INS propagation errors and pseudo-observation biases remain relatively small, limiting the difference between adaptive and fixed weighting. As the outage duration increases, the fixed-covariance strategy cannot represent the time-varying prediction confidence and may assign excessive weights to low-confidence pseudo-GNSS observations. In contrast, HTAGN adaptively adjusts the pseudo-GNSS factor weights according to the predicted uncertainty, demonstrating the effectiveness of the proposed covariance-adaptive mapping mechanism.

Table 7. Prediction accuracy comparison before FGO fusion.

Method	RMSE-N (m)	RMSE-E (m)	RMSE-H (m)
HTAGN-fixed	0.0573	0.0591	0.0823
HTAGN	0.0573	0.0591	0.0823

4.4.2. Effect of Different HTAGN Components

To further investigate the contribution of different components in HTAGN, additional ablation experiments were conducted by individually removing the TCN auxiliary encoder, subsequence alignment module, contrastive feature consistency loss, and heteroscedastic uncertainty modeling mechanism. Specifically, the effects of the feature extraction architecture, cross-branch feature consistency constraint, and uncertainty-aware prediction strategy were evaluated separately. All variants were trained using the same input data, optimization settings, and evaluation protocol as the full HTAGN model. The results are summarized in Table 8.

Table 8 presents the component-wise ablation results of HTAGN under the 180 s GNSS outage condition. The complete HTAGN achieves the lowest horizontal RMSE of 3.456 m,

demonstrating the effectiveness of the proposed feature extraction, feature consistency, and uncertainty modeling strategies.

Table 8. Ablation study of different components in HTAGN.

Method	TCN	Alignment	Contrastive Loss	Heteroscedastic Loss	RMSE-N (m)	RMSE-E (m)	RMSE-H (m)
w/o TCN	No	Yes	Yes	Yes	15.730	1.574	15.809
w/o Alignment	Yes	No	Yes	Yes	11.739	1.090	11.789
w/o L_{con}	Yes	Yes	No	Yes	9.221	3.743	9.952
w/o L_{nll}	Yes	Yes	Yes	No	8.726	3.826	9.527
HTAGN	Yes	Yes	Yes	Yes	2.765	2.073	3.456

Removing the TCN auxiliary encoder leads to the largest increase in positioning error, indicating that the TCN branch plays a key role in extracting local dynamic motion patterns and improving position increment prediction accuracy. However, the contribution of the heteroscedastic mechanism should be evaluated from the perspective of uncertainty estimation and adaptive fusion rather than prediction accuracy alone.

The removal of the contrastive feature consistency loss and heteroscedastic loss increases the horizontal RMSE to 9.952 m and 9.527 m, respectively. These results indicate that the contrastive loss improves the consistency of latent representations between the TCN and GRU branches, while the heteroscedastic loss enhances uncertainty estimation and improves the reliability of adaptive pseudo-GNSS weighting. Overall, the ablation results verify that each component contributes to the final navigation performance, with the TCN encoder and temporal alignment providing the primary improvement and the consistency constraint and uncertainty modeling further enhancing the stability and effectiveness of the proposed framework under the evaluated conditions.

4.4.3. Training Stability Analysis

To further examine whether the observed performance improvement is affected by training randomness, a random seed sensitivity analysis was additionally conducted.

Specifically, TAGN and HTAGN were independently trained using five different random seeds (1–5), while keeping the training configurations, datasets, and evaluation settings unchanged. The mean and standard deviation of the horizontal RMSE under 90 s and 180 s GNSS outages are summarized in Table 9.

Table 9. Performance variability of TAGN and HTAGN under different random seeds.

Method	Random Seeds	90 s RMSE-H (m)	180 s RMSE-H (m)
TAGN	1–5	4.97 ± 0.21	7.21 ± 0.36
HTAGN	1–5	1.82 ± 0.18	3.46 ± 0.29

As shown in Table 9, HTAGN achieves substantially lower mean horizontal RMSE than TAGN under both outage durations. The relatively small inter-run standard deviations indicate limited performance variation caused by stochastic training. Moreover, the performance differences between the two methods are considerably larger than their corresponding inter-run variations, indicating that the improvement of HTAGN is unlikely to result from random initialization alone.

4.5. Empirical Evaluation of HTAGN-Predicted Uncertainty

To assess whether the direction-dependent standard deviations predicted by HTAGN reflect the empirical dispersion of the position-increment errors, an empirical coverage

analysis was conducted using the held-out 180 s GNSS outage. Specifically, the proportions of prediction errors falling within the predicted 1σ and 2σ intervals were calculated and compared with the corresponding Gaussian reference coverage probabilities of 0.683 and 0.954, respectively. This analysis provides an empirical assessment of the uncertainty estimates under the evaluated outage scenario.

Figure 12 presents the empirical coverage probabilities of the single-step uncertainty intervals predicted by HTAGN. The 1σ coverage probabilities for the north, east, and down directions are 0.69, 0.66, and 0.63, respectively, while the corresponding 2σ coverage probabilities are 0.99, 0.90, and 0.91. These results indicate that the predicted standard deviations provide useful relative confidence information, although directional calibration differences remain. Post-training variance calibration was not additionally applied in this study, because the predicted uncertainties are primarily used as sample-dependent confidence indicators rather than strict probabilistic guarantees. The under-coverage in the east and down directions suggests that the corresponding covariances may be underestimated, resulting in larger information weights and potentially over-confident pseudo-GNSS constraints. Additional calibration strategies, such as validation-set-based scaling or adaptive uncertainty correction, could be introduced to further improve probabilistic consistency.

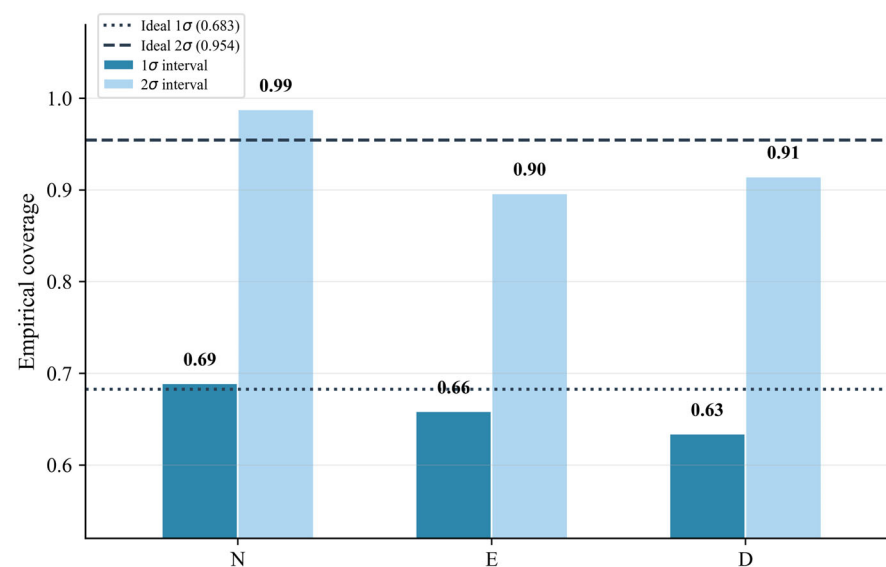


Figure 12. Empirical coverage analysis of HTAGN-predicted uncertainty intervals.

Although Figure 12 evaluates single-step position-increment uncertainties, the covariance used in the pseudo-GNSS factor is obtained by accumulating multiple predicted increments. Therefore, the empirical coverage of the accumulated pseudo-position covariance was further evaluated for outage interval from 30 s to 180 s, as summarized in Table 10.

As shown in Table 10, the accumulated 2σ coverage ranges from 0.936 to 1.000 and remains close to or above the Gaussian reference value of 0.954 in most cases, indicating broadly conservative empirical coverage of the propagated uncertainty. The 1σ coverage exhibits larger directional variations, particularly in the east direction under longer outages, showing that perfect calibration is not achieved in all directions. Nevertheless, the accumulated covariance provides useful relative confidence information for adaptive pseudo-GNSS weighting under the evaluated conditions.

Table 10. Cumulative uncertainty coverage of pseudo-position covariance during GNSS outages. Samples denotes the number of valid, potentially overlapping accumulated prediction windows.

Outage Duration	Samples	1 σ Coverage (N/E/D)	2 σ Coverage (N/E/D)
30 s	143	0.755/0.944/0.811	0.965/1.000/0.979
60 s	137	0.693/0.883/0.701	0.949/1.000/1.000
90 s	131	0.672/0.870/0.618	0.947/1.000/0.977
120 s	125	0.672/0.872/0.632	0.936/1.000/0.992
150 s	119	0.647/0.739/0.647	0.958/1.000/1.000
180 s	113	0.681/0.575/0.664	0.947/1.000/1.000

4.6. Real-Time Feasibility Analysis of the HTAGN-FGO Framework

To evaluate the computational complexity and real-time feasibility of the proposed HTAGN-FGO framework, both the HTAGN inference latency and the complete navigation update latency were measured on an Intel Core i9-14900 CPU platform.

The HTAGN model contains 1,466,502 trainable parameters, corresponding to a model size of approximately 5.60 MB. The deployed inference model retains the same parameter number, since online operation only requires forward propagation with the trained weights. As shown in Figure 13, the mean forward inference latency is 2.18 ms, with a 95th-percentile latency of 2.68 ms. Considering the complete inference pipeline, including input preprocessing and output conversion, the end-to-end latency increases to 2.60 ms, with a 95th-percentile latency of 2.98 ms.

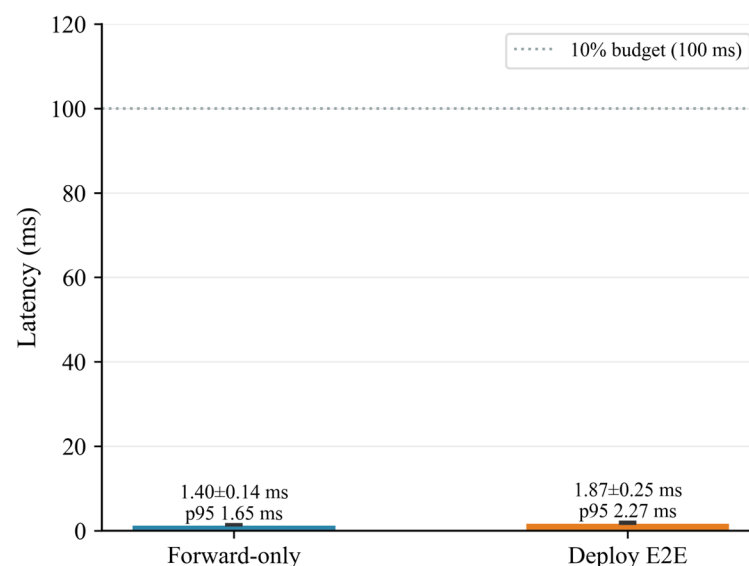


Figure 13. Inference latency evaluation of HTAGN deployment.

Although HTAGN inference introduces only limited computational overhead, the practical real-time performance depends on the complete navigation update cycle, including FGO optimization, HTAGN inference, and pseudo-GNSS factor construction. Therefore, the end-to-end processing latency of the complete HTAGN-FGO framework was further measured using sequential 1 Hz updates on the test dataset.

As summarized in Table 11, the complete HTAGN-FGO framework achieves an average processing time of 150.7 ms and a 95th-percentile latency of 253.9 ms. The maximum measured latency is 550.6 ms, and no update cycle exceeds the 1000 ms computational budget. The embedded HTAGN processing time is higher than the standalone deployment latency in Figure 13 because it additionally includes feature assembly, interface invocation, and data conversion within the complete navigation pipeline. These results indicate that

the proposed framework satisfies the computational timing requirement of 1 Hz shipborne navigation updates on the evaluated hardware platform, with the 95th-percentile latency occupying approximately 25.4% of the available update period.

Table 11. Computational latency of the complete HTAGN-FGO framework.

Component	Mean (ms)	95th Percentile (ms)
FGO optimization	133.0	239.3
Embedded HTAGN processing	6.1	8.4
Complete 1 Hz update cycle	150.7	253.9

For the FGO module, the computational complexity is controlled by the fixed-lag sliding window containing 40 IMU preintegration intervals. Through marginalization, the number of active states remains bounded during long-term operation, enabling stable computational requirements for continuous navigation estimation.

4.7. Within-Voyage Repeatability Analysis Under Multiple GNSS Outage Windows

To further evaluate the performance consistency of the proposed method across different GNSS outage locations within the same voyage, additional multiple-outage experiments were conducted on the test dataset collected from 532,400 s to 533,200 s. For each outage duration (90 s and 180 s), four non-overlapping outage intervals were selected at different locations along the test trajectory, with the outage start times set to 532,400 s, 532,600 s, 532,800 s, and 533,000 s, respectively. This configuration ensures that the selected outage scenarios cover different sailing segments rather than concentrating on a single local trajectory region. The same initialization procedure, model parameters, and evaluation metrics were used for all experiments. The horizontal positioning RMSE obtained from these outage scenarios was averaged, and the corresponding standard deviation was calculated to characterize the performance variation across different outage locations within the evaluated voyage. The experimental results are summarized in Table 12.

Table 12. Mean and standard deviation of positioning errors across four GNSS outage intervals.

Algorithm	GNSS 90 s Outages		GNSS 180 s Outages	
	RMSE-H (m)	Max-H (m)	RMSE-H (m)	Max-H (m)
INS	8.576 ± 1.279	21.756 ± 3.807	29.636 ± 3.743	93.215 ± 11.220
GRU	7.113 ± 0.742	13.942 ± 2.458	13.221 ± 1.371	25.184 ± 3.563
TCN	5.363 ± 0.371	10.982 ± 1.905	13.436 ± 1.069	24.156 ± 3.609
TAGN	4.381 ± 0.268	7.336 ± 1.636	7.792 ± 0.758	14.638 ± 2.606
HTAGN	1.834 ± 0.201	2.564 ± 1.157	3.487 ± 0.502	6.281 ± 2.321

As shown in Table 12, the proposed HTAGN method achieves the lowest average horizontal RMSE under both 90 s and 180 s GNSS outage conditions, indicating consistent positioning performance across the evaluated outage intervals. Compared with the baseline methods, HTAGN also exhibits smaller standard deviations in both RMSE-H and Max-H, suggesting relatively lower observed variation among the evaluated outage locations. These results indicate that the proposed uncertainty-aware pseudo-GNSS fusion strategy maintains consistent performance across different outage locations within the evaluated voyage. The individual RMSE-H and Max-H results for all outage intervals are provided in Supplementary Table S1.

Taken together, the experimental results provide complementary evidence for the effectiveness of HTAGN-FGO. The comparative and ablation experiments verify the contributions of the proposed network components, while the uncertainty calibration and fixed-

covariance comparison demonstrate the effectiveness of uncertainty-aware pseudo-GNSS weighting for FGO fusion. In addition, the multiple-outage and latency experiments indicate within-voyage performance consistency and real-time feasibility under the evaluated shipborne conditions. These results support the design rationale and practical effectiveness of the proposed framework. However, since the current validation is based on a single low-dynamic voyage, further evaluations under broader operating conditions are required.

4.8. Discussion on Generalization Under Varying Operating Conditions

The evaluated trajectory represents a relatively narrow operating distribution characterized by mild speed variations, limited maneuvering, and a fixed sensor configuration. Therefore, the current results primarily support interpolation under conditions similar to the tested voyage rather than unrestricted generalization to substantially different operating regimes.

Under stronger acceleration, turning, or speed changes, the relationships among specific force, angular velocity, velocity, heading, and position increments become more non-linear and time-varying. The TCN and GRU branches are designed to capture short-term motion transitions and long-term temporal dependencies, respectively, while subsequence alignment and feature-consistency constraints facilitate their complementary fusion. Nevertheless, motion patterns that lie well outside the training distribution may still increase prediction bias.

Sea-state variations may introduce additional roll, pitch, heave, vibration, and gravity-projection disturbances into the IMU measurements. The heteroscedastic branch can assign larger standard deviations to uncertain samples resembling those observed during training, thereby reducing the influence of low-confidence pseudo-GNSS factors. However, it does not guarantee reliable detection of previously unseen operating conditions. Severe out-of-distribution cases may therefore require additional uncertainty calibration, out-of-distribution monitoring, or online model adaptation.

Changes in vessel type or sensor configuration may also cause domain shifts owing to differences in hull dynamics, maneuvering characteristics, IMU noise, bias stability, sampling rates, sensor mounting, and gyrocompass accuracy. Although feature normalization can reduce scale differences, deployment on substantially different platforms may still require vessel-specific calibration, fine-tuning, or transfer learning. The predicted uncertainty should likewise be recalibrated before being used to construct pseudo-GNSS covariance matrices.

In addition, real GNSS degradation differs from the complete simulated outages considered in this study. Multipath, intermittent blockage, and non-line-of-sight reception may produce biased or discontinuous observations rather than complete signal loss. Practical implementation would therefore require GNSS quality monitoring, fault detection, and robust switching or adaptive weighting between real and pseudo-GNSS factors. Accordingly, broader sea trials are required to assess the general applicability of the proposed framework across different vessels, sea states, sensor grades, maneuvering regimes, and GNSS degradation patterns.

5. Conclusions

This paper presented an HTAGN-enhanced factor graph optimization method for shipborne GNSS/INS/gyrocompass integrated navigation. In the proposed framework, gyrocompass heading observations were incorporated as heading factors, and HTAGN-derived position increments with direction-dependent uncertainty were introduced as pseudo-GNSS position factors during GNSS outages. The method was evaluated using real

ship trial data, in which GNSS outages were artificially simulated to assess the performance under controlled 90 s and 180 s outage conditions.

The experimental results show that, under the tested low-dynamic sailing condition, the gyrocompass heading factor effectively reduced heading error accumulation compared with the conventional GNSS/INS FGO framework. In the simulated GNSS outage tests, the HTAGN-derived pseudo-GNSS factors improved horizontal positioning performance compared with the fixed-covariance TAGN baseline. The adaptive covariance mapping based on heteroscedastic standard deviations helped adjust the influence of pseudo-GNSS factors according to prediction confidence, reducing the risk of assigning overly strong weights to low-confidence predictions.

It should be noted that the validation in this study was conducted on limited real ship-trial data with artificially simulated GNSS outages. Therefore, the reported improvements mainly demonstrate the effectiveness of the proposed method under the specific vessel platform, sensor configuration, navigation trajectory, and outage conditions considered in this work. Since the simulated outages mainly represent complete GNSS signal loss, they cannot fully capture complex GNSS degradation scenarios, such as multipath effects, signal blockage, and degraded measurement quality. Furthermore, the applicability of the proposed framework to other vessels, sensor configurations, sea states, and realistic GNSS failure conditions remains to be further investigated.

Future work will focus on extending the experimental validation using additional voyages with diverse sailing speeds, maneuvering conditions, sensor configurations, and environmental conditions. Moreover, more realistic GNSS degradation scenarios, including partial signal degradation and measurement-quality variations, will be considered to further evaluate the adaptability and practical applicability of the proposed framework.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/jmse14151381/s1>, Table S1: Individual RMSE-H and Max-H results across four GNSS outage intervals.

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