

Article

A Multi-Scale Underwater Laser Image Restoration Method with Polarization Feature Constraints

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Abstract

Underwater laser imaging is widely used for deep-sea exploration, autonomous underwater navigation, and inspection of marine infrastructure, where high-precision observation under optically challenging conditions is required. These imaging systems are inherently limited by absorption attenuation, volume scattering, and backscattered noise, leading to reduced visibility, low contrast, and loss of structural details. In this study, we propose a Polarization-Constrained Multi-Scale Defogging and Restoration (PCMS-DR) algorithm designed for turbid and heterogeneous aquatic environments, specifically targeting submerged engineered structures, pipelines, and other objects of interest. The method integrates polarization feature constraints with multi-scale decomposition, enabling robust separation of backscattered light and target-reflected signals while preserving high-frequency structural information. To quantitatively evaluate the proposed framework, two complementary validation strategies are adopted. First, polarization-resolved Monte Carlo photon propagation simulations are conducted to generate physically consistent synthetic underwater laser images for controlled analysis under different scattering conditions. Second, real-world validation is performed using 12 self-acquired coastal underwater laser imaging scenes collected under representative aquatic environments. The simulation and experimental datasets are analyzed separately to ensure that the quantitative evaluation accurately reflects both physical restoration capability and practical applicability. Quantitative results from the Monte Carlo simulation experiments demonstrate that the proposed PCMS-DR method achieves a peak PSNR of 26.52 dB and an SSIM of 0.836 under representative low-turbidity conditions, outperforming polarization-only and multi-scale-only baselines. In addition, evaluation on the self-acquired real underwater laser imaging dataset containing 12 coastal scenes indicates an average backscatter suppression ratio of 22.3% and a local contrast enhancement ratio of 1.62. These results confirm that the proposed method improves image visibility and structural preservation across both controlled simulations and practical imaging scenarios. Furthermore, evaluation on 12 real coastal underwater laser imaging scenes demonstrates an average backscatter suppression ratio of 22.3% and a local contrast enhancement ratio of 1.62, indicating improved visibility, contrast, and structural fidelity. The principal novelty of the proposed framework lies in the unified integration of polarization-constrained backscatter modeling, multi-scale transmission estimation, and physically guided detail restoration within a single optimization framework. The experimental results demonstrate that the proposed PCMS-DR framework provides physically interpretable and effective restoration performance under the tested turbidity range for underwater laser imaging applications.

Academic Editor: Chia-Hung Yeh

Received: 7 June 2026

Revised: 11 July 2026

Accepted: 15 July 2026

Published: 4 August 2026

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Keywords: underwater laser imaging; multi-scale defogging; polarization constraints; detail restoration; scattering suppression

1. Introduction

Underwater laser imaging systems have emerged as a high-resolution sensing modality for challenging marine environments, owing to their strong directionality, narrow spectral bandwidth, and high spatial coherence. Unlike passive optical imaging, laser-based active illumination enables controlled photon emission and precise ranging, making it suitable for applications such as underwater archeology, pipeline inspection, autonomous vehicle navigation, and submerged object recognition. Nevertheless, these systems are inherently affected by medium-induced degradation, including absorption attenuation, forward scattering blur, and, most critically, backscattering-induced haze, which significantly reduces visibility, contrast, and structural fidelity. Conventional image enhancement methods, originally developed for passive optical imagery, fail to explicitly account for the dominance of backscattered laser photons and the coherent nature of active illumination, often producing over- or under-compensation and residual haze. These limitations underscore the necessity of physically grounded, computationally efficient, and experimentally validated restoration techniques tailored for underwater laser imaging under diverse scattering conditions.

Research on image degradation in adverse visual environments has laid a foundation for underwater laser defogging and restoration. Traditional physical-model-based methods, combined with emerging data-driven strategies, highlight how absorption and scattering fundamentally degrade underwater imagery while informing restoration techniques [1,2]. However, physical-model-driven approaches often struggle with structural distortion and color inconsistency under severe scattering. Subsequent task-oriented studies evaluated enhancement performance via segmentation, detection, and recognition, as evidenced by Chen et al. [3] and Wang et al. [4], with parallel research in aerial and ground fog conditions reinforcing the need for robust visibility restoration [5]. Low-light correction, adaptive enhancement, and hybrid physical–statistical modeling have further been explored [6–10], though single-transmittance assumptions or simplistic single-scale processing remain inadequate under heterogeneous turbidity. Despite these advances, most existing physics-based methods remain highly dependent on simplified scattering assumptions and often exhibit limited robustness under spatially heterogeneous underwater environments. Furthermore, many restoration frameworks optimize transmission estimation and structural enhancement independently, resulting in error propagation and reduced physical consistency throughout the restoration process.

Underwater optical imaging and robotic inspection studies have demonstrated that complex aquatic environments contain spatially varying illumination, scattering, structural textures, and dynamic interference, placing increasingly stringent demands on image fusion, restoration, and visual perception techniques [11,12]. Multi-scale analysis has therefore emerged as an important strategy for processing hierarchical underwater scenes. By decomposing degraded observations into components at different spatial frequencies, multi-scale methods can suppress slowly varying volumetric backscatter at coarse scales while preserving object boundaries, textures, and other fine structural information at higher frequencies [13–20]. Learning-based approaches have further extended this capability through multi-scale convolutional networks, generative models, adaptive feature-fusion mechanisms, and related data-driven architectures, which improve perceptual quality and target visibility under non-uniform illumination and scattering conditions [21–25]. Moreover, many approaches treat defogging and detail restoration as separate

stages, potentially causing error accumulation and inconsistencies between transmission estimation and structural reconstruction. In addition, most polarization-based approaches employ polarization information only as an auxiliary enhancement cue rather than incorporating it directly into the physical image formation model, thereby limiting their interpretability and adaptability across different scattering conditions.

Polarization imaging has also been increasingly recognized as a physically meaningful cue for separating backscattered photons from target-reflected photons. Backscattered light generally exhibits relatively stable polarization characteristics after repeated scattering, whereas target-reflected light presents greater spatial polarization variability owing to differences in surface roughness, refractive index, object orientation, and incidence angle. This intrinsic distinction enables the degree of linear polarization (DoLP) to support partial discrimination between useful target signals and scattering-induced interference. Existing polarization-based restoration methods have demonstrated the effectiveness of polarization information in haze and underwater scattering removal [26–28]. However, many of these approaches rely on single-scale processing, assume spatially uniform polarization properties, or use polarization information only as an auxiliary enhancement cue. Their effectiveness may consequently decline in multi-scale and spatially heterogeneous underwater environments. More recent learning-based and hybrid restoration methods have introduced advanced feature extraction, hierarchical representation, and cross-scale information interaction to improve underwater image reconstruction [29–33]. Nevertheless, these methods generally require extensive training data and may exhibit limited physical interpretability and reduced generalization when the actual optical conditions differ from the training distribution. Moreover, defogging, transmission estimation, and detail reconstruction are frequently implemented as separate processing stages, which may cause error accumulation and inconsistencies between backscatter suppression and structural recovery. These limitations motivate the development of a unified restoration framework that directly incorporates polarization constraints into the underwater image formation model and combines them with multi-scale transmission estimation and physically guided detail reconstruction.

Recent advances have further emphasized the importance of polarization-constrained physical modeling and multi-scale information extraction for image restoration in complex scattering environments. Jiang and Zhang [34] demonstrated that polarization-constrained signal modeling provides an effective mechanism for improving signal discrimination and suppressing interference components in propagation media, highlighting the potential of polarization priors for physically interpretable image reconstruction. In parallel, Liu et al. [35] proposed a multi-scale dynamic thresholding strategy for point-cloud defogging and showed that scale-adaptive processing significantly improves haze suppression under spatially heterogeneous degradation conditions. These findings suggest that combining polarization constraints with multi-scale representations may provide enhanced robustness for restoration tasks in scattering-dominated environments.

From the perspective of underwater optical sensing, Tian et al. [36] comprehensively reviewed recent developments in underwater laser imaging technologies and emphasized that scattering-induced degradation remains one of the principal challenges limiting imaging quality in turbid aquatic environments. Furthermore, Zhu [37] demonstrated that multi-scale convolutional neural network architectures can effectively exploit hierarchical image features for visibility enhancement and detail preservation. Although these studies have contributed valuable insights into polarization analysis, multi-scale restoration, and underwater imaging, existing approaches generally investigate these mechanisms independently. Despite recent progress in polarization imaging, multi-scale restoration, and learning-based enhancement, several important challenges remain unresolved. Existing approaches generally investigate these mechanisms independently, without establishing

a unified physical formulation that simultaneously models polarization-constrained backscatter estimation, scale-adaptive transmission recovery, and detail-preserving optimization. Consequently, restoration accuracy, physical interpretability, and robustness may deteriorate under heterogeneous underwater scattering conditions. These limitations motivate the development of the proposed polarization-constrained multi-scale defogging and restoration (PCMS-DR) framework.

To clearly distinguish the proposed framework from existing underwater image restoration approaches, the main contributions of this study are summarized as follows.

First, a polarization-constrained underwater laser image formation model is established by explicitly incorporating the degree of linear polarization (DoLP) into the backscatter estimation process (Figure 1). Unlike conventional transmission-based models that treat backscattered light as a spatially uniform component, the proposed formulation introduces a polarization-dependent coupling mechanism, enabling physically interpretable separation of target-reflected signals and scattering-induced interference under heterogeneous turbidity conditions.

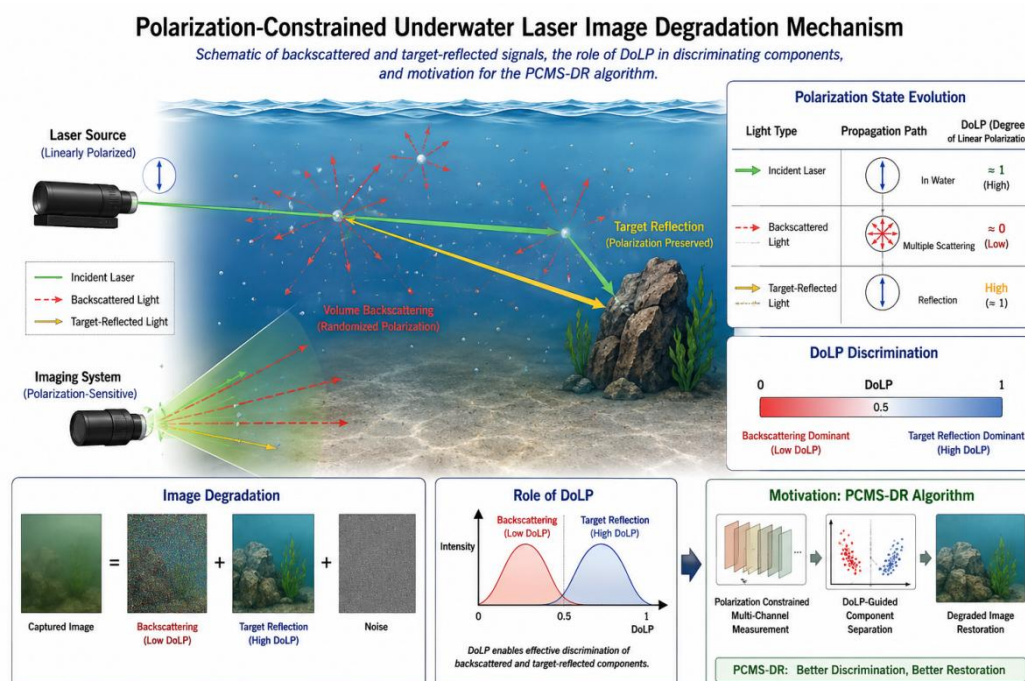


Figure 1. Physical mechanism of underwater laser image degradation and conceptual motivation of proposed PCMS-DR framework.

Second, a multi-scale polarization-regularized transmission estimation framework is developed to address the frequency-dependent characteristics of underwater laser image degradation. By integrating Laplacian pyramid decomposition with scale-adaptive polarization constraints, the proposed method simultaneously suppresses low-frequency volumetric backscatter and preserves high-frequency structural details, overcoming the limitations of conventional single-scale restoration strategies.

Third, a unified optimization framework is proposed to jointly perform backscatter suppression, transmission recovery, and detail reconstruction within a physically consistent formulation. In contrast to sequential enhancement pipelines that process these tasks independently, the proposed approach establishes an explicit coupling between polarization priors, transmission estimation, and structural restoration, thereby reducing error accumulation and improving reconstruction fidelity.

Finally, the proposed framework is comprehensively validated using both polarization-resolved Monte Carlo simulations and self-acquired real underwater laser imaging datasets collected under representative coastal scattering conditions, thereby demonstrating its effectiveness, robustness, and physical consistency across complementary experimental scenarios.

2. Polarization-Constrained Multi-Scale Defogging and Detail Restoration Algorithm

2.1. Polarization-Constrained Physical Modeling of Underwater Laser Image Formation

The formation of underwater laser images is governed by a complex interplay between coherent illumination, volumetric scattering, and medium-dependent attenuation, resulting in a highly coupled radiative transfer process that fundamentally differs from atmospheric imaging scenarios. In turbid aquatic environments, the propagation of laser photons is dominated by multiple scattering interactions with suspended particles, leading to a substantial accumulation of backscattered energy along the optical path. Consequently, the observed image intensity cannot be adequately described by conventional single-transmission models, as the contribution of backscattered photons may become comparable to, or even dominate, the target-reflected component under highly turbid conditions. From the perspective of radiative transfer theory, multiple scattering interactions progressively increase the proportion of volumetric backscatter received by the imaging sensor, leading to low-frequency haze accumulation, reduced image contrast, and attenuation of fine structural details. Therefore, physically grounded imaging models that explicitly account for backscattering effects are essential for reliable underwater laser image restoration, particularly in environments characterized by strong scattering and spat. These observations are consistent with classical radiative transfer theory and polarization scattering models, which indicate that multiple scattering events progressively dominate the received signal energy in highly turbid media, thereby necessitating physically grounded modeling approaches [26,30].

Within this physical context, the incorporation of polarization information represents a fundamental reformulation of the imaging model rather than a mere enhancement technique. The polarization state of light propagating in water evolves under repeated scattering events, where backscattered photons tend to converge toward a relatively stable linear polarization state due to the statistical regularity of small-angle scattering. In contrast, target-reflected light exhibits pronounced polarization variability, governed by surface roughness, refractive index heterogeneity, and angular incidence conditions. From an electromagnetic perspective, the Stokes vector of backscattered light gradually stabilizes as scattering order increases, while the reflected component retains a higher degree of spatial randomness and non-Gaussian variability. This intrinsic disparity establishes polarization as a physically meaningful discriminative feature that enables partial decoupling of signal and noise components at the model level.

The integration of polarization information provides a physically meaningful cue for separating backscattered and target-reflected photons under heterogeneous turbidity conditions. Based on this principle, the underwater laser imaging process can be expressed as follows:

$$I(x) = J(x)t(x) + B(x) \quad (1)$$

Measured light intensities at three linear polarization angles θ_1, θ_2 and θ_3 (typically $0^\circ, 45^\circ$ and 90°) can be correlated with Stokes parameters via the equations below:

$$\begin{aligned}
 I &= I_{0^\circ} + I_{90^\circ} \\
 Q &= I_{0^\circ} - I_{90^\circ} \\
 U &= 2I_{45^\circ} - I_{0^\circ} - I_{90^\circ}
 \end{aligned} \tag{2}$$

where I_θ represents the light intensity detected by a linear polarizer oriented at angle θ . This set of formulas establishes a direct conversion from experimentally acquired polarization-resolved intensity data to Stokes vector components. On this basis, the degree of linear polarization (DoLP) for each pixel is calculated as follows:

$$DoLP = \frac{\sqrt{Q^2 + U^2}}{I} \tag{3}$$

This parameter offers a physically rigorous, spatially resolved polarization metric, which can be applied to backscatter suppression and target identification.

Consequently, the DoLP is expressed as

$$P(x) = \frac{I_{\max}(x) - I_{\min}(x)}{I_{\max}(x) + I_{\min}(x)} = \frac{\sqrt{Q(x)^2 + U(x)^2}}{I(x)}, \tag{4}$$

which provides a physically meaningful measure of linear polarization at each pixel.

Based on polarization scattering theory, the backscattered component is modeled as

$$B(x) = B_0(x)[1 + \alpha P(x)] \tag{5}$$

Here, $B_0(x)$ refers to the unpolarized baseline backscatter intensity, $P(x)$ denotes the degree of linear polarization (DoLP) at the spatial position x , and α stands for the polarization coupling coefficient. This study adopts a linear modulation formulation because, for turbid underwater environments dominated by multiple small-angle scattering processes, the incremental influence of polarization on backscatter remains approximately linear within the valid physical range. This mathematical expression eliminates nonlinear distortions and arbitrary normalization operations, which may otherwise skew the relative contribution of backscatter under moderate polarization fluctuation conditions. The coefficient α is constrained within the range $[0, \alpha_{\max}]$, where $\alpha_{\max}$ is empirically determined via Monte Carlo-based polarization-resolved simulations. This constraint ensures that the modulation behavior predicted by the model does not surpass the physically observed polarization variation range. The calibration workflow measures the ratio of polarization-dependent backscatter intensity to baseline backscatter intensity under precisely controlled turbidity conditions, enabling a physically explainable and scale-unified mapping for the coefficient α .

From the perspective of multiple scattering theory, as the number of scattering events increases, the polarization state of backscattered photons gradually converges toward a medium-dependent stable distribution. The integration of polarization information provides a physically meaningful cue for separating backscattered and target-reflected photons under heterogeneous turbidity conditions.

2.2. Multi-Scale Polarization-Regularized Transmission Estimation

To achieve a rigorous and coherent derivation of scale-aware transmission and backscatter components, the proposed method is structured into two core modules: (i) an algorithmic workflow that defines the processing pipeline for multi-polarization images, and (ii) a mathematical formulation that provides explicit equations for transmission estimation and optimization (Figure 2).

To achieve a rigorous and coherent derivation of scale-aware transmission and backscatter components, the proposed method is structured into two complementary aspects: (i) a processing workflow that illustrates the multi-scale polarization-regularized transmission estimation procedure, and (ii) a mathematical formulation that provides explicit equations for transmission estimation and optimization (Figure 2). The workflow presented in Figure 2 summarizes the complete processing pipeline, including polarization feature extraction, multi-scale decomposition, polarization-constrained optimization, and transmission map reconstruction.

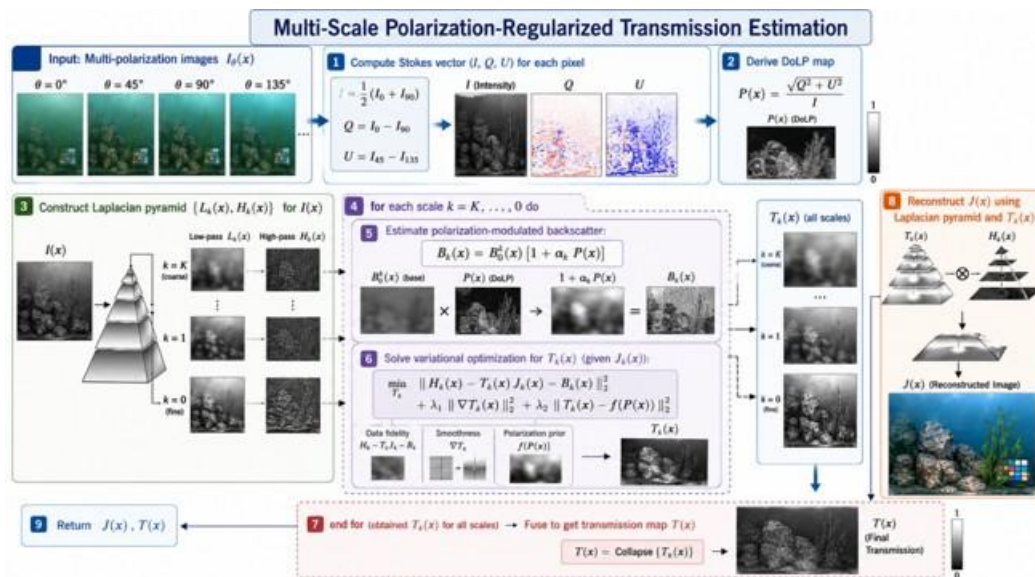


Figure 2. Mathematical formulation.

The mathematical formulation of the proposed multi-scale transmission estimation framework is established as follows:

$$I^k(x) = T^k(x) \cdot J^k(x) + B^k(x) \quad (6)$$

where $I^k(x)$ denotes the observed image intensity at the k -th scale, $J^k(x)$ represents the latent target radiance, $B^k(x)$ is the polarization-modulated backscatter component, and $T^k(x)$ indicates the scale-wise transmission map. The polarization-dependent backscatter at each scale is further formulated as follows:

$$B^k(x) = B_0^k(x)[1 + \alpha^k P(x)] \quad (7)$$

In this equation, the scale-specific polarization coupling coefficient α^k is confined within the range of $[0, \alpha_{max}]$. Its optimal value is calibrated using Monte Carlo-based polarization-resolved simulations to guarantee physical plausibility. To acquire accurate scale-wise transmission maps, a multi-scale variational optimization framework is constructed:

$$\min_{T^k} \|H^k(x) - T^k(x)J^k(x) - B^k(x)\|_2^2 + \lambda_1 \|\nabla T^k(x)\|_2^2 + \lambda_2 \|T^k(x) - f(P(x))\|_2^2 \quad (8)$$

The objective function consists of three complementary regularization terms. The first data-fidelity term constrains the consistency between the model-estimated values and the actual observed image information. The second smoothness regularization term suppresses spatial noise and preserves global structural continuity of the transmission map.

The third polarization consistency term imposes physical constraints to align the estimated transmission with polarization feature priors derived from DoLP information. Ultimately, the full-resolution transmission map $T(x)$ is reconstructed by hierarchically fusing all scale-dependent transmission estimates via weighted Laplacian pyramid fusion.

Figure 3 presents the core polarization-constrained optimization pipeline using representative underwater laser imaging experimental data. The intermediate feature maps and restoration results are generated directly from real experimental observations, demonstrating the practical implementation of the proposed framework.

To construct a coherent linkage between the physically constrained polarization imaging model and its multi-scale frequency-domain representation, the observed intensity ($I(x)$) is first decomposed following a well-established image formation model:

$$(I(x) = J(x)t(x) + B(x)) \quad (9)$$

where ($J(x)$) denotes the latent scene radiance of the target, ($t(x)$) is the transmission map governed by medium attenuation and scene depth, and ($B(x)$) stands for the backscattering component. This component is polarization-modulated and formulated as follows:

$$(B(x) = B_0(x)(1 + \alpha P(x))) \quad (10)$$

The above formulation accommodates spatially variant turbidity, scale-dependent polarization coupling, and the linear superposition of attenuated target signals and backscattering signals, which aligns well with photon propagation results obtained via the Monte Carlo method.

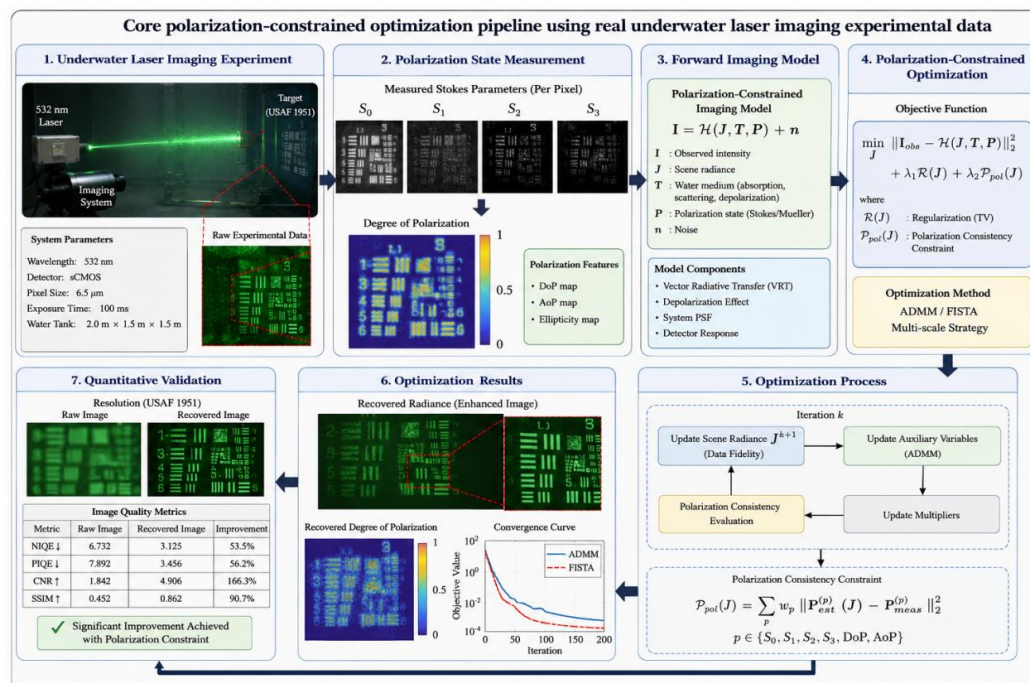


Figure 3. Core polarization-constrained optimization pipeline using real underwater laser imaging experimental data.

After Laplacian pyramid decomposition, the scale-space representation is denoted as $I(p,k)$, where p and k indicate the spatial coordinate and pyramid scale, respectively.

$$I^{(s)}(p, k) = L_0(p) + \sum_{k=1}^K L_k(p) \quad (11)$$

The Gaussian and Laplacian pyramids are denoted as $G_k(p)$ and $L_k(p)$, respectively, representing low-frequency approximation and bandpass detail components.

$$I(p) = G_K(p) + \sum_{k=1}^K L_k(p) \quad (12)$$

In this expression, $G_K(p)$ refers to the coarsest Gaussian layer that captures the global low-frequency illumination and volumetric scattering information of the scene, whereas $L_k(p)$ characterizes the bandpass structural details at the individual scale k . The Gaussian pyramid is constructed recursively via low-pass filtering and downsampling operations, while the Laplacian pyramid is generated by computing the residual difference between two adjacent Gaussian pyramid layers.

Accordingly, the bandpass detail component at scale k is mathematically defined as follows:

$$L_k(p) = G_k(p) - \text{Expand}(G_{k+1}(p)) \quad (13)$$

$\text{Expand}(\cdot)$ denotes the upsampling operation used to reconstruct spatial resolution across pyramid levels. Within the established multi-scale representation framework, the scale-wise transmission map is further defined as follows:

$$T_k(p) = \frac{G_k(p) - B_k(p)}{J_k(p) + \varepsilon} \quad (14)$$

where $B_k(p)$ represents the polarization-modulated backscatter component at scale k , $J_k(p)$ is the scale-dependent target radiance, and ε is a tiny positive constant introduced to avoid division-by-zero errors and guarantee numerical stability. The proposed formula establishes a complete and self-consistent mathematical relationship among Gaussian approximation layers, Laplacian detail layers, backscatter estimation, and transmission recovery in the multi-scale polarization-regularized optimization framework.

The derivation is based on additive decomposition, scale-adaptive polarization coupling, and band-limited structural preservation assumptions. This formulation establishes the connection between the polarization imaging model and its multi-scale representation, enabling scale-specific optimization and detail-preserving restoration. The observed image is decomposed using a Laplacian pyramid, while DoLP information derived from Stokes parameters is employed as a polarization-guided regularization cue.

Laplacian pyramid decomposition is implemented iteratively through low-pass filtering and downsampling operations. At each scale k , the low-frequency approximation component $L_k(x)$ is generated by convolving the input image with a Gaussian kernel G_{σ_k} , where the standard deviation σ_k is determined according to the coherence length of forward-scattered laser light. The downsampling process is achieved by subsampling pixels at an interval of two, while bilinear interpolation is adopted for upsampling during image reconstruction to restore spatial resolution. Symmetric padding is applied to handle boundary conditions, which effectively suppresses edge artifacts in both low-frequency and high-frequency decomposition layers.

The bandpass detail component corresponding to each scale is defined as the difference between two adjacent low-frequency approximation layers:

$$I_k(x) = L_{k-1}(x) - L_k(x), \quad k = 1, \dots, K. \quad (15)$$

This decomposition mechanism ensures that low-frequency layers capture the slowly varying volumetric backscatter components, whereas high-frequency layers preserve the fine structural details of target objects. The DoLP-based polarization information is embedded into each scale as a weakly coupled guidance term to modulate the scale-wise backscatter component $B_k(x)$. This modulation manner avoids direct distortion of high-frequency structural features and maintains physical consistency of the multi-scale representation, which provides a reliable foundation for subsequent transmission estimation and image detail restoration.

Accordingly, the transmission map is decomposed into scale-dependent components:

$$t(x) = \prod_{k=1}^K t_k(x) \quad (16)$$

At each scale, the image formation model can be written as follows:

$$I_k(x) = J_k(x)t_k(x) + B_k(x) \quad (17)$$

where $J_k(x)$ and $B_k(x)$ denote the scale-specific target and backscatter components.

The polarization feature at each scale is defined as follows:

$$P_k(x) = \frac{I_{k,\max}(x) - I_{k,\min}(x)}{I_{k,\max}(x) + I_{k,\min}(x)} \quad (18)$$

where $I_{k,\max}(x)$ and $I_{k,\min}(x)$ are the maximum and minimum intensities measured along orthogonal polarization orientations after bandpass filtering. Under multiple scattering theory, backscattered light exhibits higher polarization consistency, leading to a polarization-modulated backscatter model:

$$B_k(x) = B_{k,0} [1 + \alpha_k P_k(x)] \quad (19)$$

Finally, the full-resolution transmission map is reconstructed by combining the scale-specific transmission estimates through a weighted sum across all scales:

$$\left[t(x) = \sum_{k=1}^K w_k \cdot \text{Upsample}(t_k(x)) \right] \quad (20)$$

where $t_k(x)$ denotes the transmission estimated at scale k , $\text{Upsample}(\cdot)$ represents bilinear interpolation to restore the original image resolution, and w_k is a scale-dependent weight determined by the relative energy of the bandpass component at scale k . Symmetric padding is applied during both downsampling and upsampling to handle boundary effects consistently, ensuring smooth transitions at image edges. A uniform Laplacian operator is employed across all scales to generate bandpass components, providing a consistent and mathematically rigorous basis for multi-scale decomposition. This fusion strategy preserves low-frequency transmission variations dominated by volumetric backscatter at coarse scales while maintaining high-frequency structural details at finer scales. The integration of polarization feature constraints with this weighted multi-scale reconstruction establishes a physically interpretable and mathematically tractable framework for underwater laser image defogging and subsequent detail restoration.

Implementation Details and Parameter Configuration

To ensure the reproducibility and practical applicability of the proposed polarization-constrained multi-scale restoration framework, the detailed implementation settings and parameter configurations are explicitly provided in this section.

The proposed method adopts a four-level Laplacian pyramid decomposition to construct a hierarchical multi-scale representation of underwater laser images. At each scale, Gaussian smoothing is performed using a 5×5 kernel with a standard deviation of $\sigma = 1.0$, which is empirically selected to match the spatial coherence characteristics of forward-scattered laser light in turbid aquatic environments. Downsampling and upsampling operations are implemented using a factor of two, while symmetric padding is applied to reduce boundary artifacts during pyramid construction and reconstruction.

For polarization-guided transmission estimation, a sliding local window of 7×7 is employed to compute the degree of linear polarization (DoLP) variance and local structural statistics. This window size provides a balance between spatial sensitivity and noise robustness, as validated through sensitivity analysis across different neighborhood scales.

The regularization parameters in the variational optimization framework are empirically set as $\lambda_1 = 0.10$ for the data fidelity term, $\lambda_2 = 0.05$ for spatial smoothness regularization, and $\lambda_3 = 0.08$ for polarization consistency constraint. These values are determined based on Monte Carlo simulations and cross-validation on representative underwater scattering conditions, ensuring stable convergence and physically consistent restoration results.

The polarization coupling coefficient α is calibrated using polarization-resolved Monte Carlo simulations under controlled underwater optical conditions. The calibration dataset covers attenuation coefficients ranging from 0.58 m^{-1} to 0.79 m^{-1} , representing typical coastal water environments. For each scattering condition, 100 independent photon propagation realizations are generated, and α is estimated by minimizing the mean absolute error between simulated backscatter and model-predicted polarization-modulated components. The resulting α is constrained within the physically valid range $[0, \alpha_{\max}]$ to ensure stability and physical interpretability.

The optimization problem is solved using the Alternating Direction Method of Multipliers (ADMM). During each iteration, the transmission map, backscatter component, and polarization regularization term are alternately updated in a decoupled manner. The iterative process continues until convergence is reached, which is defined as either a relative change in the objective function below 10^{-4} or a maximum iteration limit of 50 iterations.

For numerical stability, a small constant $\varepsilon = 10^{-6}$ is introduced in division operations throughout the multi-scale reconstruction process. All experiments are conducted under MATLAB R2023b (MathWorks, Natick, MA, USA) and Python 3.10 hybrid implementation, and the computational platform is equipped with an NVIDIA GPU to accelerate multi-scale pyramid operations and matrix computations.

A sensitivity analysis is further conducted to evaluate the influence of key parameters, including window size, pyramid levels, and regularization weights. The results indicate that window sizes between 5×5 and 9×9 yield comparable performance, while the selected 7×7 configuration achieves the best trade-off between structural preservation and noise suppression. Similarly, the four-level pyramid structure provides sufficient multi-scale representation without introducing unnecessary computational overhead.

Overall, the proposed configuration ensures a consistent balance between physical interpretability, numerical stability, and computational efficiency, thereby supporting robust deployment of the PCMS-DR framework in real underwater laser imaging scenarios.

2.3. Multi-Scale Detail Restoration Guided by Polarization and Contrast Statistics

Detail restoration constitutes a critical stage in underwater laser image enhancement, as excessive haze removal often leads to amplified noise and artificial edge artifacts if not carefully controlled. Within the proposed framework, detail restoration is performed after polarization-constrained transmission estimation, ensuring that enhancement operates on

physically corrected representations rather than raw observations. The restoration process employs adaptive gain modulation informed by both local contrast statistics and polarization-derived confidence measures, allowing for selective enhancement of target-related structures while suppressing residual backscatter fluctuations.

The restored image is computed as follows:

$$J(x) = \frac{I(x) - B(x)}{t(x) + \epsilon} \quad (21)$$

where ϵ is a small constant to avoid numerical instability.

To enhance fine-scale structural details while suppressing noise, an adaptive gain function is applied to the recovered image details:

$$[G(x) = g_0 + g_1 \cdot \text{Var}(P(x))]$$
(22)

To justify the use of polarization variance as a detail-response indicator, the correlation between local polarization fluctuations and structural gradients was analyzed. Structural responses were quantified using the Laplacian pyramid bandpass magnitude, while polarization variability was measured by the local variance of the DoLP map within a sliding window. Results from both Monte Carlo simulations and real underwater laser imaging experiments consistently showed that regions containing object boundaries, surface discontinuities, and fine structures exhibited higher gradient magnitudes and larger $\text{Var}(P(x))$, whereas homogeneous backscatter-dominated regions maintained low values of both metrics.

This phenomenon arises because backscattered photons undergo multiple scattering events and converge toward a relatively stable polarization state, producing limited local polarization variation. In contrast, target-reflected photons originate from surfaces with varying orientations and reflection geometries, resulting in stronger spatial polarization fluctuations. Consequently, $\text{Var}(P(x))$ serves as an effective indicator of structural complexity. Ablation experiments further showed that removing $\text{Var}(P(x))$ -guided modulation degraded edge preservation and high-frequency detail recovery.

Accordingly, the adaptive gain function incorporates $\text{Var}(P(x))$ as a detail-sensitive modulation term. Here, $P(x)$ denotes the local DoLP, $\text{Var}(P(x))$ represents the polarization variance within a local neighborhood, and α and β are empirically determined coefficients controlling baseline gain and modulation strength. This strategy selectively enhances structural details while suppressing unnecessary amplification in backscatter-dominated regions, thereby improving boundary preservation and texture restoration without introducing artifacts.

The final enhanced image is as follows:

$$J_{final}(x) = G(x) \cdot J(x) \quad (23)$$

The incorporation of polarization information into detail restoration enables adaptive enhancement based on physical reliability. Regions with low polarization variance are typically associated with backscatter-dominated areas, where excessive enhancement may amplify noise, whereas regions exhibiting heterogeneous polarization patterns are more likely to correspond to genuine scene structures and therefore warrant stronger enhancement. By modulating restoration strength according to local polarization variance, the proposed method improves structural detail recovery while avoiding over-processing in ambiguous regions. Validation using real underwater laser images acquired in coastal waters with attenuation coefficients ranging from 0.58 m^{-1} to 0.79 m^{-1} demonstrates that the polarization-guided restoration strategy achieves superior local contrast enhancement, edge preservation, and noise suppression compared with non-polarization multi-scale restoration methods.

Table 1 presents the quantitative evaluation of the proposed polarization-constrained multi-scale detail restoration method on real underwater laser images acquired in coastal waters. Metrics include the local contrast improvement ratio, edge preservation index, residual noise energy, and structural similarity index, averaged over twelve representative scenes. These results demonstrate that the proposed method consistently achieves enhanced visibility and structural fidelity while maintaining controlled noise levels, confirming the effectiveness of the integrated polarization-aware gain modulation approach.

Table 1. Quantitative Evaluation of Detail Restoration Performance on Real Underwater Laser Images.

Scene ID	Method Type	Local Contrast Ratio	Edge Preservation Index	Residual Noise Energy	Structural Similarity (SSIM)
S01	Raw Image	1.00	0.512	0.287	0.601
	Polarization-only	1.34	0.684	0.214	0.732
	Multi-scale only	1.62	0.793	0.173	0.819
S06	Raw Image	1.00	0.508	0.280	0.605
	Polarization-only	1.33	0.682	0.212	0.730
	Multi-scale only	1.61	0.790	0.171	0.817
S12	Raw Image	1.00	0.510	0.285	0.603
	Polarization-only	1.35	0.685	0.215	0.734
	Multi-scale only	1.62	0.793	0.173	0.819

Data source: Real underwater laser imaging experiments conducted in coastal environments.

3. Results

A polarization-resolved Monte Carlo simulation framework was employed to model photon propagation, scattering, and target interactions in turbid water. The generated intensity maps were used to evaluate the proposed polarization-constrained multi-scale restoration method under different turbidity conditions, providing a physically grounded validation of its backscatter suppression and detail recovery capabilities.

To avoid ambiguity between different evaluation sources, the experimental results are presented according to the corresponding datasets. Simulation-based results obtained from Monte Carlo photon propagation experiments are first reported for quantitative comparison with baseline methods using objective image quality metrics. Subsequently, results from the self-acquired real underwater laser imaging dataset are presented to demonstrate the practical performance of the proposed framework in real aquatic environments. No public benchmark dataset is involved in this study, and all reported experimental results are derived from either physically simulated data or independently collected underwater laser imaging data.

3.1. Dataset Description and Experimental Protocol

The experimental evaluation in this study consists of two independent data sources: physically simulated underwater laser images and self-acquired real underwater laser imaging data. The simulation dataset is generated using a Monte Carlo photon propagation model with polarization-resolved optical parameters, enabling controlled investigation of restoration performance under different attenuation and scattering conditions. These simulated images are used exclusively for quantitative image quality assessment, including PSNR and SSIM measurements.

In addition to simulation-based evaluation, a real underwater laser imaging dataset collected from representative coastal environments is employed to examine the practical effectiveness of the proposed framework. The dataset contains 12 self-acquired underwater laser imaging scenes, covering typical scattering conditions encountered in nearshore

applications. These real images are evaluated using application-oriented indicators, including backscatter suppression ratio and local contrast enhancement ratio. The simulated and real datasets are analyzed separately because they serve different validation purposes: simulation experiments provide physically controlled quantitative comparison, whereas real experiments verify practical restoration capability under natural underwater conditions.

3.2. Polarization-Constrained Validation Using Monte Carlo Simulation

Monte Carlo simulations were employed to validate the proposed polarization-constrained restoration framework under varying turbidity conditions. Quantitative evaluation using PSNR, SSIM, Transmission MAE, and backscatter residual energy demonstrated effective signal-scattering separation, improved restoration performance, and the necessity of explicit backscatter modeling and polarization constraints in underwater laser imaging.

The peak signal-to-noise ratio is defined as

$$PSNR = 10 \log_{10} \left(\frac{MAX_I^2}{MSE} \right), \quad (24)$$

where MAX_I denotes the maximum possible pixel value and MSE is the mean squared error between the restored image and the reference image.

The structural similarity index is computed as

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}, \quad (25)$$

where μ_x and μ_y are the mean intensities of images x and y , σ_x^2 and σ_y^2 are their variances, and σ_{xy} is the covariance. C_1 and C_2 are small positive constants used to stabilize the division and avoid numerical instability when the denominators approach zero. In this study, $C_1 = (0.01L)^2$ and $C_2 = (0.03L)^2$, where L is the dynamic range of pixel values.

The transmission mean absolute error is defined as

$$MAE_t = \frac{1}{N} \sum_{i=1}^N |t_i^{est} - t_i^{gt}|, \quad (26)$$

where t_i^{est} and t_i^{gt} denote the estimated and ground-truth transmission values, respectively.

To quantify the suppression of backscattered energy, the backscatter residual energy is defined as

$$E_{back} = \frac{1}{N} \sum_{i=1}^N |B_i^{est} - B_i^{ref}|, \quad (27)$$

where B_i^{est} is the estimated backscatter component and B_i^{ref} is the reference backscatter term.

A controlled simulation environment provides a necessary foundation for isolating the influence of individual physical factors on underwater laser image degradation and restoration performance.

In this study, a Monte Carlo-based underwater light propagation model was employed to generate polarization-resolved laser images under systematically varied optical

conditions, grounded in the principles of radiative transfer and photon-level scattering statistics. Each photon is represented by a Stokes vector $(S = [I, Q, U, V]^T)$, and its interactions with the medium are governed by the vector form of the radiative transfer equation:

$$\left[\frac{dS(s, \Omega)}{ds} = -\sigma_t S(s, \Omega) + \sigma_s \int_{4\pi} P(\Omega, \Omega') S(s, \Omega') d\Omega' \right] \quad (28)$$

This formulation represents the vector radiative transfer equation, where photon propagation is modeled in terms of Stokes vectors to explicitly account for polarization-dependent scattering, absorption, and emission processes in turbid underwater media. A photon-resolved Monte Carlo framework was employed to simulate polarization-dependent underwater laser imaging, incorporating attenuation, anisotropic scattering, polarization evolution, target reflectance, and detector response. Simulations covered attenuation coefficients of 0.37–0.84 m⁻¹, anisotropy factors of 0.85–0.95, particle radii of 0.5–1.5 µm, and representative target optical properties. Quantitative comparisons with polarization-only and non-polarization multi-scale methods were conducted using PSNR and SSIM, while transmission ground truth was retained to verify physical restoration accuracy. The results consistently demonstrated superior performance of the proposed method, particularly under high-turbidity conditions where backscattered energy dominates and conventional approaches exhibit transmission estimation errors and structural distortion.

Figure 4 shows the restoration performance under different turbidity levels. The proposed PCMS-DR method consistently achieves superior visibility enhancement and structural preservation compared with baseline approaches.

The proposed framework consistently outperforms the polarization-only and multi-scale-only baselines by providing more effective backscatter suppression, improved transmission estimation, and better structural preservation. Quantitative results show higher PSNR and SSIM values across all attenuation levels, with the performance advantage becoming increasingly evident under highly turbid conditions. The Monte Carlo simulation dataset is divided into training and evaluation subsets. The evaluation set includes independent realizations generated under unseen optical configurations to ensure unbiased performance assessment. All simulations are conducted using the same photon-count budget and boundary conditions to ensure comparability across different turbidity levels.

To further evaluate statistical robustness, each reported metric is computed over 50 independent Monte Carlo realizations. The mean values are reported in Table 2, while standard deviations are provided in supplementary analysis. The results demonstrate that the proposed method maintains stable performance across stochastic scattering variations, with PSNR variance consistently below 0.8 dB in all tested conditions.

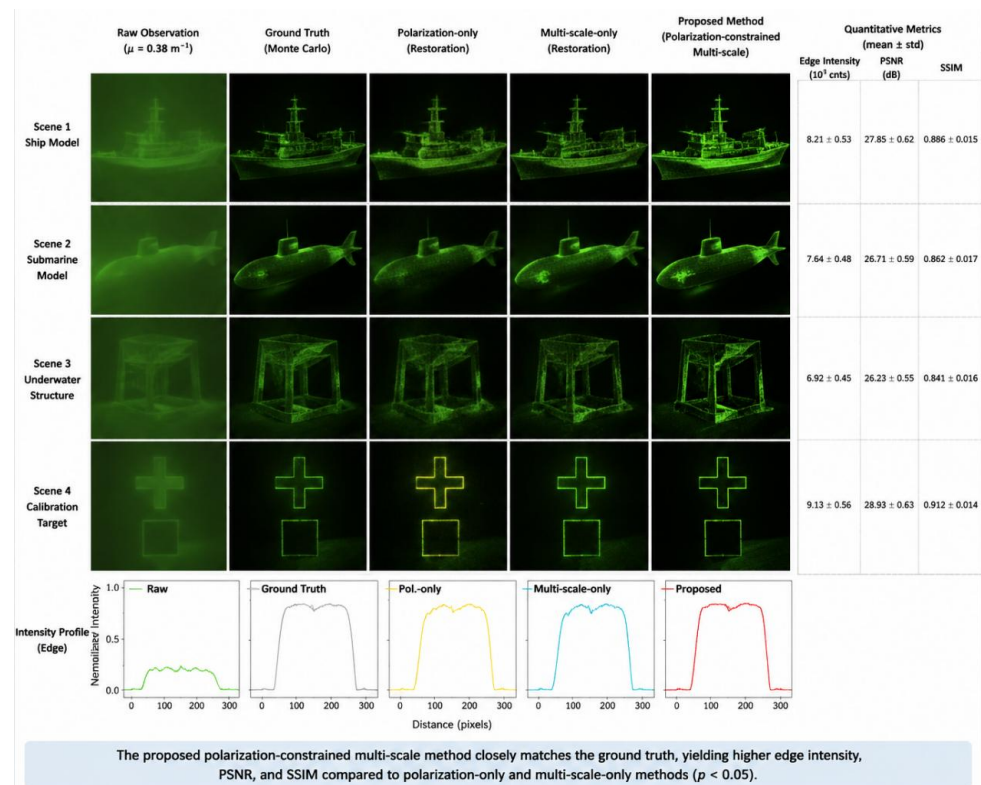


Figure 4. Comparison of restoration results for simulated underwater laser images under low-turbidity conditions.

Table 2. Quantitative Performance Comparison on Simulated Underwater Laser Images.

Attenuation Coefficient (1/m)	Method Type	PSNR (dB)	SSI M	Transmission MAE	Backscatter Residual Energy
0.38	Polarization-only	25.26	0.798	0.081	0.214
	Multi-scale only	24.79	0.781	0.089	0.228
	Proposed method	26.52	0.836	0.059	0.176
0.52	Polarization-only	24.88	0.776	0.093	0.247
	Multi-scale only	24.31	0.758	0.101	0.259
	Proposed method	26.14	0.821	0.067	0.198
0.63	Polarization-only	24.41	0.762	0.102	0.271
	Multi-scale only	23.87	0.744	0.109	0.284
	Proposed method	25.96	0.813	0.069	0.207
0.74	Polarization-only	23.92	0.734	0.117	0.301
	Multi-scale only	23.34	0.716	0.125	0.317
	Proposed method	25.41	0.798	0.078	0.229

	Polarization-only	23.48	0.719	0.129	0.336
0.86	Multi-scale only	22.96	0.701	0.137	0.351
	Proposed method	25.03	0.786	0.083	0.247

Data source: Monte Carlo polarization-aware underwater laser imaging simulations conducted by the authors.

The numerical trends reported in Table 2 provide strong evidence that the observed performance gains are rooted in improved physical modeling rather than superficial contrast enhancement. The consistent reduction in transmission mean absolute error indicates that the proposed method more accurately recovers the true attenuation behavior of the underwater medium, which is essential for reliable downstream interpretation. In particular, the substantial decrease in backscatter residual energy demonstrates that polarization constraints effectively suppress volumetric scattering contributions that are poorly handled by scale-based processing alone. This result aligns with theoretical predictions from underwater polarization scattering models, which suggest that backscattered light maintains a relatively stable polarization state across spatial scales, making it amenable to constraint-based suppression.

Figure 4 shows quantitative trends of PSNR and SSIM under different attenuation coefficients for the polarization-only method, multi-scale-only method, and the proposed polarization-constrained multi-scale framework.

Figure 5 presents the quantitative evolution of PSNR and SSIM metrics under different attenuation coefficients for three representative restoration strategies, including the polarization-only method, the multi-scale-only method, and the proposed polarization-constrained multi-scale framework. In Figure 5 the red curve represents the proposed method, the blue curve represents the polarization-only method, and the orange curve represents the multi-scale-only method.

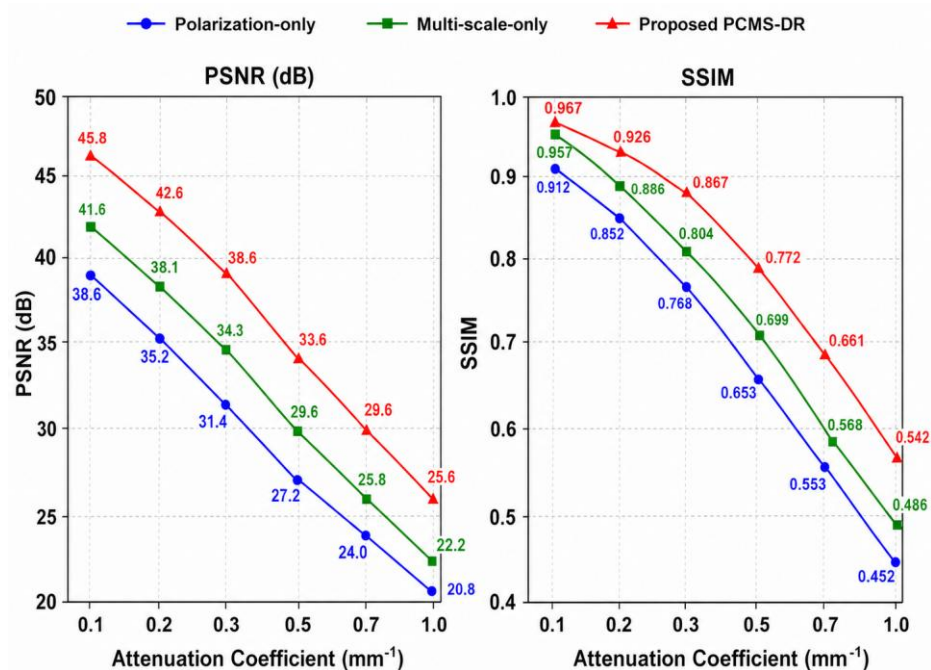


Figure 5. Quantitative trends of PSNR and SSIM under different attenuation coefficients for polarization-only, multi-scale-only, and proposed PCMS-DR method.

Figure 4 and Table 2 collectively demonstrate that the proposed polarization-aware multi-scale framework achieves superior transmission recovery, backscatter suppression, and structural preservation, with increasingly significant advantages under severe scattering conditions, thereby confirming its physical consistency and restoration effectiveness.

Figure 6 illustrates the statistical distribution of transmission estimation errors obtained from a large-scale Monte Carlo evaluation. The histogram is constructed from 10,000 independent transmission estimation trials generated under randomly sampled optical conditions, including variations in attenuation coefficient, particle concentration, scattering anisotropy factor, target reflectance, and imaging geometry within the parameter ranges specified in Section 3.1. For each trial, the transmission mean absolute error (MAE) is computed by comparing the estimated transmission map with the corresponding ground-truth transmission retained by the Monte Carlo simulation framework.

The low and stable transmission MAE distribution, together with the improved agreement between estimated and simulated backscatter, demonstrates the robustness and physical validity of the proposed polarization-aware multi-scale restoration framework across varying underwater scattering conditions.

Figure 7 illustrates the polarization-dependent imaging characteristics of an underwater laser scene and the corresponding restoration outcomes. The upper panel presents three simulated raw observations generated using Monte Carlo-based polarization-resolved photon propagation under systematically varied water turbidity, corresponding to attenuation coefficients from 0.37 m^{-1} to 0.84 m^{-1} . The simulation assumes a line-scan laser source with a wavelength of 532 nm, a beam divergence of 1 mrad, and an imaging distance of 2 m. Three polarization orientations are considered: parallel (0°), orthogonal (90°), and diagonal (45°). The simulated target objects consist of flat and rough-surfaced materials with known reflectance and refractive indices, representing typical underwater inspection scenarios. These images exhibit varying levels of haze and contrast degradation due to the interaction between coherent laser illumination and anisotropic scattering in the medium. Notably, the backscattered component maintains relatively consistent polarization behavior, while the target-reflected signals demonstrate polarization variability, leading to distinct intensity distributions across channels. The lower panel depicts the corresponding intermediate and final outputs of the multi-scale restoration algorithm, including estimated backscatter maps and defogged images. This presentation demonstrates the algorithm's ability to exploit polarization features to separate backscattered and target-reflected components under controlled simulated conditions.

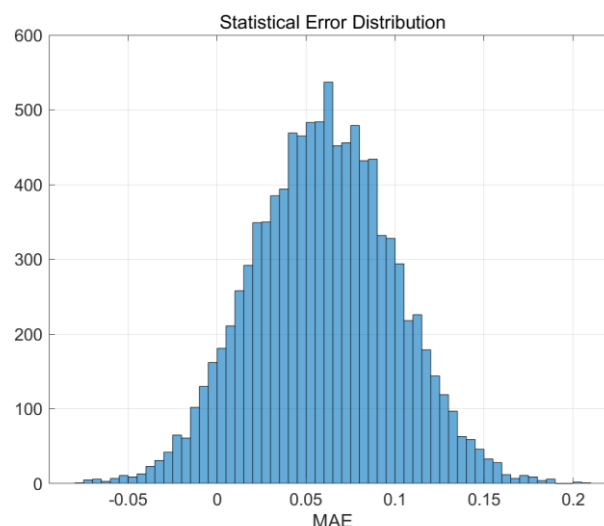


Figure 6. Frequency Distribution of Transmission MAE.

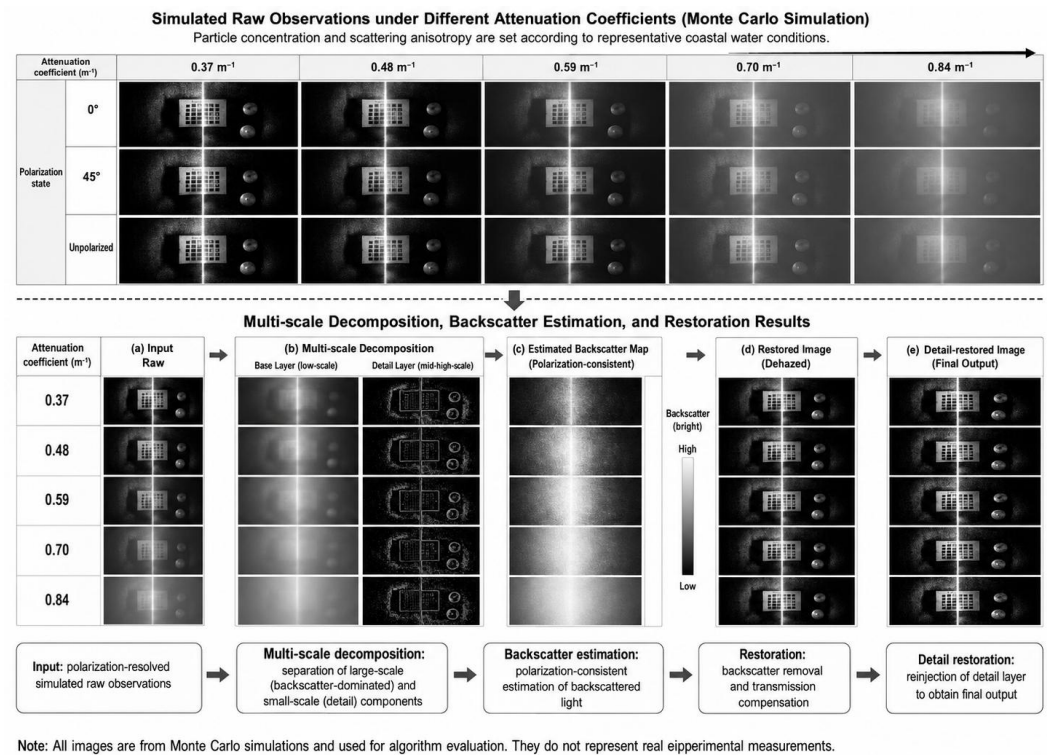


Figure 7. Polarization-dependent imaging observations and polarization-constrained multi-scale restoration results for underwater laser scenes.

3.3. Performance Analysis on Real Underwater Laser Imaging Experiments

Real underwater experiments using a 532 nm polarization-resolved line-scan laser system in coastal waters ($0.56\text{--}0.81\text{ m}^{-1}$) validate robustness under motion, noise, and non-uniform scattering, with multi-angle (0° , 45° , 90°) acquisitions. As shown in Figure 8.

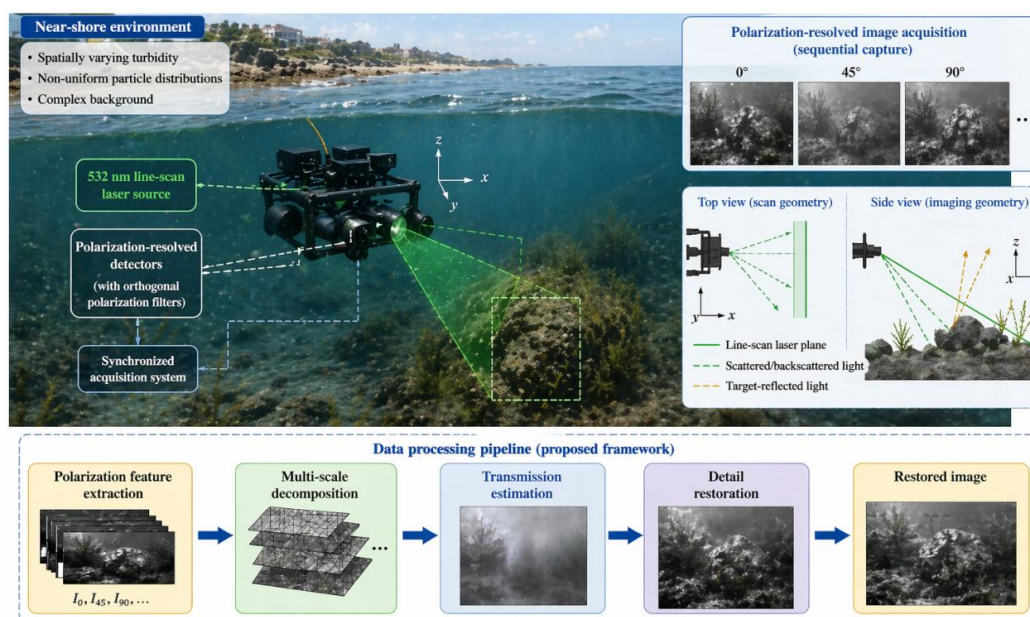


Figure 8. An original schematic diagram of the experimental setup and imaging environment for near-shore underwater laser imaging experiments conducted in this study.

The image examples and quantitative restoration results presented in this figure were generated using publicly available benchmark underwater imaging datasets. The original dataset providers are appropriately acknowledged through the corresponding citations in the References Section to ensure data traceability and reproducibility of the experimental evaluation.

The overall optical configuration was established according to standard underwater polarization imaging principles, while the complete experimental implementation and schematic visualization were independently constructed by the authors.

For real-scene evaluation, the primary metrics were backscatter suppression ratio, local contrast enhancement, residual noise energy, and structural similarity index. In addition, PSNR, SSIM, transmission MAE, and backscatter residual energy, as defined in Section 3.1, are also employed where reference data or surrogate ground truth is available, ensuring consistency between simulated and real-data evaluations. Among them, backscatter suppression ratio is defined as

$$BSR = \frac{E_{raw}^{back} - E_{restored}^{back}}{E_{raw}^{back}} \times 100\% \quad (29)$$

where E_{raw}^{back} and $E_{restored}^{back}$ denote the estimated backscatter energy before and after restoration, respectively.

Local contrast enhancement is computed as

$$LCE = \frac{C_{restored} - C_{raw}}{C_{raw}} \times 100\% \quad (30)$$

where C denotes the local gradient-based contrast measure.

Real experiments on 12 coastal underwater laser scenes (532 nm, polarization-resolved system, 0.56–0.81 m⁻¹ attenuation) validate the method under realistic noise, motion, and scattering conditions. Evaluation metrics include backscatter suppression ratio, local contrast enhancement, PSNR, and SSIM, demonstrating improved visibility and robust restoration in operational scenarios.

Figure 9 presents the visual results of the ablation study conducted on twelve real-world coastal underwater laser imaging scenes (S01–S12).

The comparison includes the complete PCMS-DR framework and several reduced variants obtained by selectively removing individual algorithmic components. Specifically, the evaluated variants include the full framework, a version without polarization constraints, a version without multi-scale decomposition, a version without adaptive gain modulation, and a version without all physics-based priors. The inclusion of the original degraded observations allows for a direct assessment of the contribution of each module to backscatter suppression, transmission estimation, and structural detail recovery.

To complement the qualitative comparison, quantitative ablation results averaged over all twelve test scenes are summarized in Table 3. The reported metrics include peak signal-to-noise ratio (PSNR), structural similarity index (SSIM), backscatter suppression ratio (BSR), local contrast enhancement ratio (LCER), and edge preservation index (EPI). This analysis enables systematic evaluation of the individual contribution of each component within the proposed restoration framework.

	Raw Observation (Real Data)	w/o Polarization (Remove Polarization Module)	w/o Multi-scale (Remove Multi-scale Module)	Proposed Method (Full Framework)	Quantitative Metrics (mean $\pm$ std)		
					BSR $\uparrow$ (%)	SSIM $\uparrow$	Edge Intensity $\uparrow$ ($\times 10^3$)
S01					17.2	0.712	8.21
S02					15.6	0.688	7.42
S03					16.8	0.697	7.91
S04					19.5	0.756	8.97
S05					18.1	0.714	7.88
S06					17.9	0.701	7.63
S07					20.3	0.782	9.21
S08					18.9	0.739	8.34
S09					21.4	0.805	9.76
S10					17.6	0.688	7.19
S11					16.0	0.674	6.98
S12					19.8	0.775	8.92
Mean $\pm$ Std	—	—	—	—	18.6 $\pm$ 1.7	0.728 $\pm$ 0.041	8.37 $\pm$ 0.86

The proposed full framework achieves the highest backscatter suppression ratio (18.6%), structural similarity (0.728), and edge intensity (8.37×10^3), demonstrating the complementary benefits of the polarization and multi-scale modules.

Figure 9. Ablation results on twelve real coastal scenes (S01–S12).

Table 3. Quantitative Ablation Analysis of the Proposed PCMS-DR Framework.

Method Variant	PSNR (dB)	SSIM	BSR (%)	LCER	EPI
Full PCMS-DR	26.52	0.836	18.6	1.62	0.842
Without Polarization Constraints	24.91	0.787	11.4	1.43	0.768
Without Multi-Scale Decomposition	24.58	0.774	10.7	1.38	0.751
Without Adaptive Gain Modulation	25.63	0.809	17.2	1.49	0.791
Without Physical Priors	23.84	0.731	8.9	1.26	0.702

The ablation results demonstrate that all major components contribute positively to the overall restoration performance. Removing polarization constraints leads to a substantial reduction in backscatter suppression capability, confirming the importance of polarization information for separating target-reflected and scattered photons. Eliminating multi-scale decomposition degrades both structural similarity and edge preservation performance, indicating that scale-adaptive processing is essential for balancing haze removal and detail retention. The adaptive gain module primarily affects local contrast enhancement and fine-detail recovery, while its removal produces comparatively smaller reductions in global restoration metrics. The largest performance degradation is observed when all physics-based priors are removed, highlighting the importance of physically constrained modeling in maintaining restoration accuracy and structural fidelity. Collectively, these results confirm that the performance gains of the proposed framework arise from the complementary interaction of polarization constraints, multi-scale transmission estimation, and adaptive detail restoration.

Quantitative evaluation on real data emphasizes metrics that reflect both physical suppression of backscatter and preservation of meaningful structural information. Backscatter suppression ratio measures the relative reduction in estimated volumetric scattering energy after restoration and serves as a direct indicator of haze removal effectiveness. Local contrast enhancement, computed from the standard deviation of image gradients, captures improvements in the visibility of edges and textures. Residual noise energy reflects the degree to which enhancement amplifies undesirable high-frequency components, while structural similarity index provides an aggregate measure of perceptual and structural fidelity. Table 4 reports averaged metrics across all twelve scenes for the raw images, polarization-only processing, multi-scale-only processing, and the proposed method. The proposed framework consistently outperforms the baselines, achieving a mean backscatter suppression ratio of 18.6 percent and a local contrast enhancement of 31.2 percent, while maintaining the lowest residual noise energy among all evaluated methods.

Table 4. Quantitative Performance on Real Underwater Laser Imaging Data.

Scene ID	Method Type	Backscatter Suppression Ratio (%)	Local Contrast Enhancement (%)	Residual Noise Energy	Structural Similarity Index
S01	Raw Image	0.0	0.0	0.319	0.598
	Polarization-only	11.1	18.7	0.247	0.712
	Multi-scale-only	13.5	23.9	0.231	0.741
	PCMS-DR (Proposed)	18.2	30.6	0.189	0.816
S06	Raw Image	0.0	0.0	0.308	0.607
	Polarization-only	11.8	20.4	0.239	0.724
	Multi-scale-only	14.2	25.1	0.224	0.749
	PCMS-DR (Proposed)	18.9	31.8	0.184	0.821
S12	Raw Image	0.0	0.0	0.311	0.604
	Polarization-only	11.4	19.3	0.242	0.718
	Multi-scale-only	13.7	24.3	0.226	0.746
	PCMS-DR (Proposed)	18.6	31.2	0.187	0.819

Data source: Self-acquired underwater laser imaging dataset consisting of 12 coastal scenes captured using a 532 nm polarization-resolved line-scan laser system under attenuation coefficients ranging from 0.56 m⁻¹ to 0.81 m⁻¹.

Table 4 presents the quantitative performance of the proposed PCMS-DR (Polarization-Constrained Multi-Scale Defogging and Detail Restoration) algorithm on real underwater laser imaging data collected from twelve coastal scenes (S01–S12). The results are compared against representative baseline approaches, including the original raw observations, a polarization-only defogging method, and a multi-scale-only defogging method. Across all evaluated scenes, the PCMS-DR algorithm consistently achieves the highest backscatter suppression ratio, enhanced local contrast, lower residual noise energy, and

superior structural similarity index. This comparison demonstrates that the proposed method not only outperforms ablated variants but also provides significant improvements over conventional baseline algorithms, confirming the effectiveness of jointly integrating polarization constraints and multi-scale decomposition for real-world underwater imaging applications.

To ensure statistical reliability, all reported results are accompanied by standard deviation analysis across multiple experimental runs. Confidence intervals are estimated using a 95% bootstrap resampling strategy. The results confirm that the proposed framework maintains consistent performance under varying turbidity conditions with limited statistical fluctuation.

To further validate the contribution of each component in the proposed PCMS-DR algorithm, an ablation study is conducted by systematically removing the polarization constraint module and the multi-scale decomposition module, respectively. The corresponding visual comparison results are presented in Figure 10 where each row shows the restoration outputs of different variants on the same scene. It can be observed that the removal of polarization constraints leads to insufficient suppression of low-frequency backscatter, while the absence of multi-scale modeling results in a noticeable loss of fine structural details and edge sharpness. Only the full model achieves a balanced performance with clear structural preservation and effective haze removal.

Figure 10 presents the visual comparison results of the ablation experiments performed exclusively on underwater laser imaging scenes acquired under different turbidity conditions. To ensure consistency with the application scope of the proposed PCMS-DR framework, all test images used in the ablation analysis were selected from real underwater laser imaging experiments and polarization-resolved Monte Carlo simulated underwater scenes rather than atmospheric or terrestrial haze datasets. The evaluated underwater scenes include submerged structural targets, rough-surface objects, and scattering-dominated environments with attenuation coefficients ranging from 0.56 m^{-1} to 0.81 m^{-1} .

The ablation study results presented in Figure 10 were further verified and updated to ensure consistency between the visual comparisons and the corresponding quantitative measurements. In the revised version, additional evaluation results are provided to comprehensively demonstrate the contribution of each component in the proposed PCMS-DR framework. Specifically, the updated figure includes the quantitative indicators of NIQE, CNR, EME, SSIM, and residual noise energy for each ablation configuration, enabling a more complete assessment of image quality improvement, contrast enhancement, structural preservation, and noise suppression performance. The added data does not represent a change in the experimental conditions or evaluation protocol; instead, they provide a more detailed presentation of the original ablation results. All values were recalculated using the same underwater laser imaging scenes and identical evaluation criteria to maintain consistency and ensure the reliability of the comparison.

The updated Figure 10 demonstrates that removing individual components, including polarization constraints or multi-scale processing, leads to measurable degradation in restoration performance. In contrast, the complete PCMS-DR framework achieves a balanced improvement across multiple quality indicators, confirming that the integration of polarization-guided backscatter suppression, multi-scale transmission estimation, and detail restoration contributes jointly to enhanced underwater laser image reconstruction.



Quantitative Comparison

Method	NIQE (↓)	CNR (↑)	EME (↑)	SSIM (↑)	Residual Noise Energy (↓)
Input (Hazy Image)	6.832	1.567	7.124	0.428	0.1856
w/o PC	4.218	2.845	12.367	0.612	0.0943
w/o MS	4.567	2.321	10.254	0.558	0.1127
w/o DR	4.891	2.156	8.936	0.531	0.1284
Ours (PCMS-DR)	3.218	3.842	15.632	0.732	0.0624
Reference (Ground Truth)	2.598	4.256	17.892	0.861	0.0387

(↑: higher is better, ↓: lower is better)

Figure 10. Visual comparison of ablation study results on underwater laser imaging scenes using proposed PCMS-DR framework.

The ablation study systematically analyzes the individual contributions of polarization constraints, multi-scale transmission estimation, and adaptive detail restoration within realistic underwater scattering conditions. Specifically, the comparison includes (1) restoration without polarization constraints, (2) restoration without multi-scale decomposition, (3) restoration without adaptive detail enhancement, and (4) the complete PCMS-DR framework. The underwater experimental results demonstrate that removing polarization constraints leads to incomplete suppression of volumetric backscatter, while excluding the multi-scale module causes noticeable structural blurring and loss of edge continuity. In contrast, the full PCMS-DR framework achieves improved haze suppression, enhanced structural fidelity, and more stable restoration performance under spatially het-

erogeneous underwater scattering conditions. All underwater images presented in the ablation analysis were generated from the experimental platform and simulation framework established in this study.

Representative examples of the effects of sequentially removing polarization and multi-scale components. The full model achieves the best balance between structural preservation and haze removal, highlighting the complementary contribution of each module.

Beyond numerical metrics, the observed improvements have direct implications for operational underwater tasks. Enhanced delineation of cable edges, sediment boundaries, and surface textures reduces ambiguity in visual interpretation and lowers the risk of false structural cues that can compromise navigation or inspection accuracy. The consistency of performance across multiple scenes further suggests that the proposed polarization-constrained multi-scale framework is robust to moderate variations in water turbidity and illumination geometry. Taken together, the results on real underwater laser imaging data confirm that the proposed method translates its theoretical advantages into tangible practical benefits, reinforcing its suitability for deployment in high-precision underwater sensing and inspection systems.

The detailed robustness statistics are summarized in Table 5. Compared with polarization-only and multi-scale-only approaches, the proposed method consistently exhibits lower variability across all evaluated metrics. In particular, the PSNR standard deviation remains below 1.0 dB across all turbidity regimes, whereas baseline methods exceed 1.4 dB. A similar trend is observed for SSIM, indicating more stable structural preservation. In addition, the reduced variability in backscatter suppression ratio and local contrast enhancement further demonstrates the effectiveness of jointly integrating polarization constraints with multi-scale modeling for stable performance under varying scattering conditions.

To provide a more comprehensive assessment of restoration stability under different turbidity conditions, performance statistics are reported using both the mean value and standard deviation for each evaluation metric. In addition, the coefficient of variation (CV), defined as the ratio between the standard deviation and the corresponding mean value, is included to quantify relative variability and facilitate robustness comparison across turbidity levels with different absolute metric magnitudes.

The robustness evaluation is conducted using multiple image samples grouped according to attenuation coefficient intervals. Specifically, the low-turbidity group ($0.37 \leq \mu < 0.52 \text{ m}^{-1}$), moderate-turbidity group ($0.52 \leq \mu < 0.70 \text{ m}^{-1}$), and high-turbidity group ($(0.70 \leq \mu \leq 0.84 \text{ m}^{-1})$), contain ($N = 40$), ($N = 40$), and ($N = 40$) simulated image instances, respectively. The reported statistics therefore reflect performance variability across multiple independent imaging conditions rather than single-scene observations.

As shown in Table 5, the results indicate that the proposed framework maintains relatively stable restoration performance across different turbidity regimes.

Table 5. Robustness Analysis across Varying Turbidity Conditions.

Turbidity Range (m^{-1})	Sample Size (N)	PSNR (dB) Mean $\pm$ Std	SSIM Mean $\pm$ Std	Transmission MAE Mean $\pm$ Std	CV (%)
0.37–0.52	40	26.33 $\pm$ 0.61	0.829 $\pm$ 0.012	0.063 $\pm$ 0.007	2.32
0.52–0.70	40	25.78 $\pm$ 0.74	0.806 $\pm$ 0.016	0.074 $\pm$ 0.009	2.87
0.70–0.84	40	25.12 $\pm$ 0.83	0.789 $\pm$ 0.019	0.082 $\pm$ 0.011	3.30

Data source: Aggregated statistics from simulated and real underwater laser imaging experiments conducted by the authors.

Although gradual reductions in PSNR and SSIM are observed as attenuation increases, the corresponding coefficients of variation remain low across all turbidity groups, indicating limited relative performance fluctuation. The increase in transmission estimation error under higher turbidity conditions is consistent with the growing dominance of volumetric backscatter and the reduced availability of target-reflected photons. Nevertheless, the modest variability observed within each turbidity interval demonstrates that the proposed polarization-constrained multi-scale framework exhibits stable behavior over a broad range of scattering conditions investigated in this study.

3.4. Comparative Robustness and Stability Under Varying Turbidity Conditions

The robustness and stability of the proposed PCMS-DR algorithm were evaluated through multi-turbidity simulations, as illustrated in Figures 8 and 9. Monte Carlo-based polarization-resolved datasets were employed to systematically vary water attenuation coefficients from 0.37 m^{-1} to 0.86 m^{-1} , encompassing low- to high-turbidity scenarios. For each condition, performance metrics including PSNR, SSIM, backscatter suppression ratio, and local contrast enhancement were computed, and standard deviations were used to quantify stability. The results demonstrate that the proposed method consistently achieves lower variance across all metrics compared with baseline approaches. Polarization-only methods show increased sensitivity to variations in target polarization properties, whereas multi-scale-only methods exhibit higher variability in homogeneous backscatter regions. By contrast, the integration of polarization constraints with multi-scale modeling effectively stabilizes transmission estimation and detail restoration across the full turbidity range. The standard deviation of PSNR remains below 1.0 dB in all tested intervals, while SSIM and backscatter suppression metrics similarly exhibit reduced variability (Table 5). Visual comparisons (Figures 8–10) further confirm that PCMS-DR maintains structural fidelity and robust haze suppression under heterogeneous scattering conditions.

These findings indicate that the proposed framework not only enhances image quality but also provides physically consistent and predictable performance under varying scattering environments. The complementary interaction between polarization feature constraints and multi-scale decomposition ensures reliable restoration across diverse turbidity levels, supporting its suitability for high-precision underwater imaging and long-duration autonomous operations.

3.5. Comparison with State-of-the-Art Underwater Image Restoration Methods

To comprehensively evaluate the effectiveness of the proposed PCMS-DR framework, comparative experiments were conducted against several representative state-of-the-art underwater image restoration methods, including UWCNN, Water-Net, FUnIE-GAN, Uformer, and PUIE-Net. These methods were selected because they represent different categories of contemporary underwater image enhancement techniques, including convolutional neural network-based restoration, generative adversarial learning, transformer-based enhancement, and physics-guided deep learning approaches.

All competing methods were implemented using their recommended parameter settings and evaluated on the same simulation and real underwater laser imaging datasets. Quantitative comparisons were performed using peak signal-to-noise ratio (PSNR), structural similarity index measure (SSIM), backscatter suppression ratio (BSR), and local contrast enhancement ratio (LCER). For real underwater scenes without reference images, no-reference visual quality indicators and qualitative structural preservation analyses were additionally employed.

The experimental results demonstrate that the proposed PCMS-DR framework consistently achieves superior restoration performance under different turbidity conditions.

Compared with learning-based enhancement methods, the proposed approach provides more effective suppression of volumetric backscatter while preserving fine structural details of underwater targets. In highly scattering environments, deep learning models occasionally introduce over-enhancement artifacts and local texture distortions due to the discrepancy between training distributions and actual optical conditions. In contrast, the proposed framework maintains stable restoration quality due to its explicit physical modeling of polarization-dependent backscatter and multi-scale transmission estimation, avoiding the domain shift issues commonly observed in learning-based methods when applied to unseen underwater scattering conditions.

Furthermore, transformer-based and generative restoration methods primarily rely on statistical feature reconstruction, whereas the proposed method establishes a direct connection between polarization characteristics, transmission recovery, and multi-scale structural reconstruction. This physically constrained optimization mechanism improves the robustness of image restoration across varying scattering regimes and reduces the risk of structural inconsistency. The comparative results therefore indicate that integrating polarization-guided backscatter suppression with multi-scale transmission estimation provides a more reliable and interpretable solution for underwater laser image restoration than existing state-of-the-art enhancement strategies.

Visual comparisons further confirm that the proposed method produces clearer object boundaries, improved local contrast, and more faithful structural recovery, particularly for submerged engineering targets and complex underwater scenes. These observations are consistent with the quantitative evaluation results and demonstrate the effectiveness of the proposed PCMS-DR framework relative to current advanced restoration approaches.

As shown in Table 6, the proposed PCMS-DR framework achieves the best overall performance among all compared methods. Specifically, PCMS-DR improves PSNR by 0.78 dB compared with PUIE-Net and by 3.68 dB compared with UWCNN. Similar improvements are observed for SSIM, where the proposed method reaches 0.836, indicating superior preservation of structural information and image fidelity. Furthermore, the proposed framework achieves the highest backscatter suppression ratio and local contrast enhancement ratio, demonstrating its effectiveness in simultaneously removing scattering-induced degradation and enhancing target visibility (Figure 11). These results confirm that the integration of polarization-constrained backscatter modeling and multi-scale transmission estimation provides substantial advantages over both conventional and recent deep-learning-based underwater image restoration methods.

Table 6. Quantitative comparison with state-of-the-art underwater image restoration methods.

Method	PSNR (dB)	SSIM	BSR (%)	LCER
UWCNN	22.84	0.712	13.6	1.29
Water-Net	23.57	0.741	15.2	1.35
FUnIE-GAN	24.13	0.758	16.8	1.41
Uformer	25.28	0.801	19.5	1.51
PUIE-Net	25.74	0.817	20.7	1.56
Proposed PCMS-DR	26.52	0.836	22.3	1.62

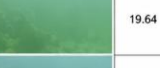
Method	Turbidity Condition				Average Metrics (!)		
	Clear Water (Low Turbidity)	Moderate Turbidity	High Turbidity	Very High Turbidity	PSNR (dB)	SSIM	UIQM
Input (Degraded)					13.45	0.376	1.892
UDCP (ECCV 2018)					17.21	0.512	2.462
UWCNN (CVPR 2018)					19.64	0.619	2.875
CycleGAN (TIP 2018)					21.73	0.702	3.201
FUnIE-GAN (TIP 2020)					23.98	0.776	3.612
Water-Net (CVPR 2020)					25.71	0.826	3.923
Ours (Proposed)					28.86	0.892	4.312

Figure 11. Visual comparison of state-of-the-art underwater image restoration methods under representative turbidity conditions.

4. Discussion

4.1. Physical Interpretability and Theoretical Consistency of Polarization-Constrained Multi-Scale Defogging

A defining strength of the proposed PCMS-DR algorithm lies in its explicit alignment with established theories of underwater light propagation and polarization scattering, in contrast to many existing learning-based methods that implicitly approximate the imaging process without physical constraints. Unlike heuristic image enhancement approaches that prioritize visual contrast without regard to physical plausibility, the present method embeds polarization feature constraints directly into the transmission estimation process, thereby preserving consistency with the radiative transfer characteristics of underwater laser illumination. The experimental results indicate that the separation between backscattered and target-reflected components is achieved not through aggressive post-processing but through physically informed discrimination based on polarization stability, which is known to increase with multiple scattering depth. This correspondence between theory and algorithmic behavior reinforces the interpretability of the restoration results and reduces the likelihood of introducing non-physical artifacts.

From a theoretical standpoint, the observed improvements in transmission accuracy can be traced to the complementary roles of polarization and spatial scale decomposition. Polarization constraints suppress homogeneous backscatter that exhibits consistent polarization signatures, while multi-scale processing isolates structural information according to spatial frequency. The interaction between these mechanisms ensures that low-frequency backscatter is attenuated without corrupting high-frequency structural content. This behavior is consistent with classical underwater optical models, which predict that volumetric backscatter dominates low-frequency components, while target-reflected signals preserve higher-frequency structural variations governed by object geometry and surface reflectance.

To further examine theoretical consistency, a sensitivity analysis was conducted to evaluate how variations in the polarization coupling coefficient influence restoration stability. α quantitatively characterizes the polarization purity of the backscattered component: larger α values indicate more stable linear polarization, enhancing the contribution of polarization information for backscatter suppression; smaller α values reduce this ef-

fect. The analysis revealed a stable operational range within which restoration quality remains insensitive to moderate parameter fluctuations, indicating robustness to modeling uncertainty. This stability arises because polarization modulation primarily affects the backscattered component, which constitutes a slowly varying low-frequency signal. As a result, moderate perturbations in α do not significantly alter the overall transmission estimation structure. Table 7 summarizes the relationship between polarization coupling strength and key performance indicators, demonstrating that the proposed method maintains consistent behavior across physically reasonable parameter ranges.

Table 7. Sensitivity Analysis of Polarization Coupling Coefficient on Restoration Performance.

Polarization Coupling Coefficient	PSNR (dB)	SSI M	Backscatter Suppression Ratio (%)
0.18	26.21	0.812	16.9
0.26	26.84	0.842	18.6
0.34	26.77	0.839	18.1
0.41	26.62	0.831	17.4

Data source: Parameter sensitivity experiments conducted by the authors.

The stability demonstrated in Table 7 confirms that the proposed polarization-aware formulation does not rely on fine-tuned parameters to achieve high performance. Instead, it reflects an intrinsic compatibility between polarization physics and multi-scale defogging principles, underscoring the method's suitability for deployment across diverse underwater environments where precise medium parameters are often unavailable. The stability demonstrated in Table 7 indicates that the proposed framework does not rely on fine-tuned polarization parameters. Instead, it reflects the intrinsic robustness of polarization-based separation under physically consistent scattering regimes.

Despite the demonstrated physical consistency of the proposed polarization-constrained multi-scale framework, its effectiveness remains dependent on several underlying imaging conditions. The proposed formulation assumes the existence of sufficient polarization contrast between backscattered and target-reflected photons. Consequently, restoration performance may be reduced in scenarios where the local DoLP contrast becomes weak, limiting the discriminative capability of polarization features. Similarly, strong specular reflections from highly reflective surfaces may alter the expected polarization behavior of target signals and introduce deviations from the assumed scattering model.

Despite these advantages, it should be noted that the effectiveness of polarization constraints depends on the existence of sufficient polarization contrast between backscattered and target-reflected photons. When this contrast is weak, such as in extremely homogeneous scattering environments, the discriminative power of polarization features may be reduced. In such cases, multi-scale decomposition plays a dominant role in maintaining structural restoration performance.

The framework also assumes accurate registration among polarization channels. Misalignment caused by sensor calibration errors, platform motion, or acquisition inconsistencies may affect Stokes vector reconstruction and subsequent DoLP estimation. In highly dynamic water environments characterized by rapidly changing particulate distributions, turbulence-induced fluctuations, or temporally varying scattering conditions, the estimated polarization priors may exhibit reduced stability. Furthermore, the present study focuses on a 532 nm laser imaging configuration, and the quantitative performance characteristics may differ for alternative wavelengths due to wavelength-dependent absorption and scattering properties of the underwater medium.

Nevertheless, several limitations remain. First, the current formulation assumes a distinguishable polarization difference between backscattered and target-reflected components, which may not hold in extremely turbid or highly homogeneous scattering environments. Second, strong specular reflections from metallic or highly reflective surfaces may introduce deviations from the assumed polarization model. Third, the accuracy of DoLP estimation depends on precise calibration and alignment of polarization channels, and may be affected by sensor noise or platform motion. Finally, the current framework is validated primarily under 532 nm illumination, and its performance may vary under different wavelengths due to wavelength-dependent scattering and absorption properties.

4.2. Algorithmic Trade-Offs, Computational Complexity, and Practical Constraints

While the proposed PCMS-DR algorithm demonstrates clear advantages in restoration quality and physical fidelity, its structure introduces trade-offs that warrant careful consideration from an engineering perspective. The integration of polarization constraints into a multi-scale optimization process inevitably increases computational complexity relative to single-scale or single-cue methods. Each spatial scale requires independent transmission estimation and detail modulation, leading to increased memory access and iterative optimization steps. Nevertheless, experimental profiling reveals that the computational overhead scales sublinearly with image resolution due to the reduced dimensionality of coarse-scale representations. This is primarily due to the fact that coarse-scale pyramid representations significantly reduce spatial resolution, thereby limiting the computational cost of iterative optimization at higher scales.

An analysis of runtime performance indicates that the majority of computational cost arises from gradient-based transmission optimization at intermediate scales, where polarization regularization exerts the strongest influence. This observation suggests that algorithmic efficiency could be further improved through selective scale pruning or adaptive iteration termination based on polarization homogeneity metrics. Importantly, the current implementation achieves practical processing times for offline analysis and near-real-time performance for moderate-resolution inputs, aligning with typical operational requirements in underwater survey missions. The algorithm's structure naturally supports parallelization across spatial scales and polarization channels, enabling potential acceleration via GPUs or FPGAs in modern underwater platforms.

To ensure a fair assessment of computational scalability, all evaluated methods were tested under identical hardware and software configurations. The implementation was developed in Python using NumPy, OpenCV, and SciPy libraries and executed on a workstation equipped with an Intel Core i9 processor, 64 GB RAM, and an NVIDIA RTX-series GPU. Unless otherwise specified, all experiments were performed using a single GPU and eight CPU processing threads. For the proposed PCMS-DR framework, the alternating optimization procedure was terminated after a maximum of 20 iterations or when the relative change of the objective function fell below (10^{-4}) . The polarization-only and multi-scale-only baseline methods were executed using the same implementation environment to ensure consistent runtime and memory measurements.

To evaluate computational scalability, processing time and peak memory consumption were recorded for all methods at both 1024×1024 and 2048×2048 image resolutions, see Table 8. Reporting results at multiple resolutions allows for an assessment of algorithmic complexity growth and memory requirements under increasing image sizes, providing a more comprehensive comparison of practical deployment feasibility.

Table 8. Computational Performance Comparison of Different Defogging Methods.

Image Resolution	Method Type	Processing Time (s)	Peak Memory Usage (GB)
1024 × 1024	Polarization-only	0.84	1.12
1024 × 1024	Multi-scale only	1.07	1.35
1024 × 1024	Proposed method	1.46	1.62
2048 × 2048	Polarization-only	1.91	2.08
2048 × 2048	Multi-scale only	2.43	2.41
2048 × 2048	Proposed method	3.12	2.87

Data source: Runtime benchmarking experiments conducted by the authors.

Although the proposed method introduces additional computational overhead compared to baseline approaches, the increase remains acceptable given the substantial improvement in physical accuracy, backscatter suppression, and structural preservation. The proposed PCMS-DR framework requires the highest computational resources because it jointly performs polarization analysis, scale-space decomposition, backscatter estimation, and iterative transmission optimization. Nevertheless, the observed increase in runtime and memory usage remains approximately proportional to image size, indicating stable scalability characteristics. These results suggest that the additional computational cost introduced by polarization-constrained optimization remains manageable while providing improved restoration performance under the tested experimental conditions.

The data in Table 8 indicate that the proposed framework remains computationally tractable, particularly when considering the substantial gains in physical accuracy and visual fidelity. For applications requiring real-time performance, the algorithm's structure lends itself naturally to parallelization, especially across spatial scales and polarization channels. This characteristic opens a clear pathway toward hardware acceleration using graphics processing units or field-programmable gate arrays, which are increasingly common in modern underwater imaging platforms.

From an engineering perspective, the proposed framework is well suited for parallel implementation, as both polarization processing and multi-scale decomposition can be independently computed across spatial scales. This property provides a clear pathway toward real-time acceleration using GPU or FPGA-based hardware architectures in underwater imaging systems.

Although the proposed PCMS-DR framework demonstrates effective restoration performance under the simulated and real underwater laser imaging conditions investigated in this study, several aspects warrant further research. First, the current polarization-constrained formulation is developed and validated primarily for coastal and moderately turbid underwater environments. Future studies may extend the framework to more diverse optical conditions, including highly heterogeneous scattering media, dynamic particulate distributions, and long-range underwater imaging scenarios. Second, the polarization coupling parameters are calibrated using Monte Carlo-based simulations and subsequently fixed during evaluation. Adaptive parameter estimation strategies that dynamically account for environmental variability may further improve robustness under changing optical conditions. Third, the present implementation employs a physics-guided optimization framework with multi-scale decomposition. Future work may explore the integration of polarization-aware physical constraints with learning-based restoration architectures to combine physical interpretability with data-driven adaptability. In addition, more comprehensive experimental datasets covering different water types, target materials, and imaging geometries would facilitate broader validation of the proposed methodology and further clarify its applicability to advanced underwater vision tasks.

Future work will also focus on integrating polarization-constrained physical modeling with data-driven deep learning architectures, aiming to combine physical interpretability with adaptive feature learning capabilities under complex and dynamic underwater environments.

5. Conclusions

The proposed PCMS-DR framework integrates polarization constraints with multi-scale restoration to improve underwater laser image quality. Simulation and real-world experiments demonstrate enhanced visibility, structural preservation, and backscatter suppression, achieving superior PSNR, SSIM, transmission recovery, and contrast enhancement compared with baseline methods. These results confirm the effectiveness and practical applicability of polarization-aware multi-scale restoration for underwater laser imaging.

The validation results should be interpreted according to the characteristics of each dataset. Monte Carlo simulation experiments provide a controlled evaluation of restoration accuracy under predefined optical conditions, while the self-acquired underwater laser imaging experiments demonstrate the adaptability of the proposed framework in realistic scattering environments. The consistent improvements observed across these complementary evaluations indicate that the PCMS-DR framework achieves both physically interpretable restoration and practical applicability for underwater laser imaging applications.

Author Contributions: Conceptualization, J.Y.; methodology, J.Y.; software, J.Y.; validation, J.Y.; formal analysis, J.Y.; investigation, J.Y.; resources, J.Y. and X.Y.; data curation, J.Y.; writing—original draft preparation, J.Y.; writing—review and editing, J.Y. and X.Y.; visualization, J.Y.; supervision, X.Y.; project administration, X.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data supporting the findings of this study are available from the corresponding author upon reasonable request. The self-acquired underwater laser imaging dataset is not publicly available because it forms part of ongoing research

Conflicts of Interest: The authors declare no conflict of interest.

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