

Article

Self-Organized Fencing Control of Multi-AUV Systems Under Limited Sensing and Nonuniform Acoustic Communication Delays

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Abstract

This paper formulates a self-organized dynamic fencing problem for multiple AUVs under finite target-sensing range and bounded nonuniform acoustic communication delays. Here, self-organization means that the fence is generated by local interactions without assigning fixed angular slots, virtual leaders, or persistent vehicle roles. A minimal observer-assisted attraction-repulsion fencing controller is proposed for AUV implementation, comprising target-AUV radial attraction–repulsion, AUV–AUV distance attraction–repulsion computed from timestamp-aligned delayed neighbor states, a state-only target-motion observer, and a label-free bearing-coverage repulsion that acts only on oversized target-centered angular gaps. To address acoustic delay without delaying physical execution, each AUV stores its own state history and evaluates pairwise relative geometry at the timestamp carried by the received neighbor packet. The resulting command is applied at the current time. The analysis shows that timestamp alignment converts acoustic delay into a bounded geometric perturbation and establishes collision avoidance, radial confinement, angular-gap contraction, target tracking, and packet-range preservation on a locally order-consistent regular fencing interval. The theorem does not claim global entry from arbitrary non-enclosing configurations. Numerical simulations, including a five-degree-of-freedom ocean-current robustness test without current feedforward compensation and an evasive-target stress test with four rapid finite-acceleration turns, demonstrate self-organized entry and maintenance of compact convex-hull fencing in the tested cases.

Keywords: autonomous underwater vehicle; multi-AUV system; dynamic fencing control; label-free coordination; communication delay; timestamp alignment



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1. Introduction

Multi-AUV cooperative control has become an important research topic in ocean observation, seabed inspection, underwater security, and target tracking [1–6]. Compared to a single AUV, a cooperative AUV group can cover larger spatial regions, provide redundant sensing, and improve the robustness of underwater missions. However, the underwater environment also makes cooperation substantially more difficult than terrestrial or aerial swarm control. Acoustic communication is low-bandwidth and delayed, exteroceptive

sensing is range-limited and noisy, and low-level vehicle tracking is affected by hydrodynamic uncertainty and actuator limits. These characteristics imply that cooperative AUV control should not be treated as a direct extension of surface or aerial multi-agent control.

Recent surveys have clarified the state of the art in multi-AUV path planning, formation control, task allocation, and underwater search. Hadi et al. [7] reviewed path planning and formation control methods for multiple AUVs and emphasized the coupling among motion planning, communication constraints, and vehicle dynamics. Yan et al. [8] further summarized formation-control architectures for multiple AUVs and highlighted the need to consider uncertain dynamics and communication constraints in practical cooperative missions. Wang et al. [9] studied task allocation for multi-AUV systems and showed that mission-level cooperation remains strongly affected by communication restrictions and environmental uncertainty. Wang et al. [10] surveyed multi-AUV underwater search for multiple targets, pointing out that searching, tracking, and formation maintenance are usually solved separately even though they are tightly coupled in practical missions. From a broader ocean-robotics perspective, Liu et al. [11] summarized swarm unmanned-vehicle technologies in shallow ocean environments and identified communication, localization, and environment disturbance as key limiting factors. These studies indicate that the next step for multi-AUV cooperation is not only to design better formation controllers but also to formulate task-oriented cooperative geometries that explicitly reflect sensing and communication limitations.

Formation control has been extensively studied for multi-AUV systems. Chen et al. [12] proposed a fusion control strategy for multi-AUV formation and cluster search, integrating formation keeping with cooperative search. Communication effects have also been considered. Li et al. [13] addressed formation control under communication delay, packet discreteness, and packet loss using prediction and compensation mechanisms. Event-triggered and predictive control strategies have been introduced to reduce unnecessary communication and improve robustness. For example, event-triggered model predictive control has been used for multi-AUV formation obstacle avoidance [14], while distributed dynamic predictive control has been investigated for searching and hunting multi-AUV targets in unknown environments [15]. These studies provide useful foundations, but most of them still focus on maintaining a prescribed formation or a tracking trajectory. They do not directly answer how a group of AUVs should maintain a time-varying protective boundary around a target when target information is delayed and only partially available.

Cooperative hunting and encirclement control are more closely related to the fencing task. In cooperative hunting, multiple robots move around a target so that the target is tracked, surrounded, or intercepted. Zhao et al. [16] proposed a cooperative hunting method for a multi-AUV swarm in a weak information environment by improving artificial potential fields. Recent work has further explored target search, tracking, and encirclement keeping in three-dimensional Internet of Underwater Things scenarios [17]. Learning-based methods, including distributional soft actor-critic and hierarchical multi-agent reinforcement learning, have also been used for underwater target pursuit and tracking [18,19]. In parallel, the general robotic-swarm literature has reviewed collaborative hunting as a problem involving search, pursuit, encirclement, and capture [20]. Nevertheless, a gap remains between cooperative hunting and engineering-oriented AUV fencing. Hunting-oriented methods often emphasize successful pursuit or encirclement, whereas fencing requires maintaining a stable, safe, and recoverable boundary under long-duration sensing and communication constraints. In this sense, fencing control should be viewed as a persistent safety-and-coverage problem rather than a transient interception problem.

The target-fencing literature provides an important theoretical basis for this study. Chen [21] proposed a cooperative target-fencing protocol with attraction, repulsion, and rotation terms, proving that multiple vehicles can fence a stationary target into their convex hull without collision or singleton collapse. Subsequent studies extended fencing to moving targets with unknown velocity [22,23], regular-polygon formations [24], label-free moving-target fencing [25], and connectivity-preserving target fencing [26]. These studies clarify the geometric essence of fencing, especially the importance of label-free ordering and pairwise repulsion. However, most of them are developed for generic multi-agent systems or idealized vehicle models and assume that relative position information is instantaneously available or consistently sampled. For AUVs, acoustic delay breaks this assumption: using the current self-state together with a delayed neighbor state creates a fictitious relative vector that never existed physically. This observation motivates the timestamp-aligned formulation in this paper.

Safety-critical cooperation has also attracted attention in marine robotics. Obstacle avoidance and formation reconfiguration methods have been developed for multi-AUV systems using improved artificial potential fields, affine transformation, and consensus mechanisms [27,28]. Delay-aware cooperative tracking has been studied for multi-AUV systems with Markovian switching topologies [29], while velocity- and input-constrained distributed tracking has also been investigated in networked transportation systems [30]. Recent AUV studies have further addressed cooperative obstacle avoidance [31], data-driven cooperative trajectory tracking with input saturation [32], and prescribed-performance H_∞ cooperation for underactuated vehicles [33]. These developments strengthen the implementation basis for constrained cooperative motion, but they do not formulate convex-hull fencing under finite target sensing and state-only delayed acoustic packets. Control barrier functions and safety filters provide a principled way to enforce state constraints, but simply applying an existing barrier-function or model-predictive-control framework to AUVs does not by itself constitute a sufficient contribution. For dynamic fencing, the main difficulty is to define the fence geometry, handle delayed neighbor information consistently, and preserve a functional enclosing boundary without assigning fixed fence slots. Therefore, this paper focuses on the fencing-specific problem formulation and on a direct timestamp-aligned controller. As summarized in Table 1, the unresolved combination is finite-range target sensing, nonuniform acoustic delays, state-only packets, label-free enclosure, collision avoidance, connectivity preservation, and execution through a five-degree-of-freedom AUV model.

The objective of this paper is to develop a self-organized dynamic fencing control framework for multiple AUVs under limited sensing and delayed acoustic communication. The term dynamic fencing is used to describe a cooperative control task in which AUVs form and maintain a moving enclosing boundary around a target or region while satisfying vehicle safety, sensing, and communication constraints. The term label-free means that no AUV is assigned to a fixed angular slot, virtual point, or leader-follower role on the fence. In this paper, self-organized further emphasizes that the boundary is generated by direct target–AUV and AUV–AUV attraction–repulsion interactions rather than by a centralized allocation of fence positions. Vehicle identities are used only for communication and onboard computation; the fencing objective itself is invariant under permutations of AUV labels. However, simulation and geometric analysis show that radial attraction–repulsion and pairwise distance regulation alone are not sufficient to guarantee convex-hull enclosure of a moving target from non-enclosing initial conditions. A minimal additional mechanism is needed to close oversized bearing gaps while preserving the label-free nature of the fence. The present work therefore keeps the attraction–repulsion mechanism as the core fencing mechanism and adds only two task-specific elements: a state-only target-motion observer

and a label-free bearing-coverage repulsion term. The observer supports target-centered local computation without forwarding target estimates, whereas the bearing-coverage term supplies the missing geometric mechanism required for convex-hull enclosure.

Table 1. Direct comparison with the most closely related enclosing, formation-control, and target-fencing studies.

Study	Task and Vehicle Model	Limited Target Sensing	Communication Impairment	Target-Estimate Forwarding	Angular Assignment	CA	CP
[1]	Moving-target enclosing; underactuated AUV	No; target velocity unavailable	Topology changes and link interruptions	Dynamic-observer architecture; state-only payload not specified	Self-organized; no fixed slots	NR	No
[13]	Leader-follower formation; AUV kinematic model	Not applicable	Delay, packet discreteness, and dropout	Leader position, velocity, and heading are transmitted	Prescribed leader-follower roles	No	No
[21]	Stationary-target fencing; single-integrator agents	No	No communication impairment	No observer or acoustic-packet model	No prescribed angular slots	Yes	No
[22,23]	Moving-target fencing; second-order agents	No	No communication impairment	No acoustic-packet model	No prescribed angular slots	Yes	No
[24]	Regular-polygon fencing; second-order agents	No	No communication impairment	Target-velocity observer; no acoustic-packet model	Label-free regular polygon	Yes	No
[25]	Rigid moving-target fencing; second-order agents	No	No communication impairment	Dynamic regulator; no acoustic-packet model	Explicitly label-free	Yes	No
[26]	General-target fencing; double-integrator agents	No	No communication impairment	No acoustic-packet model	No prescribed angular slots	Yes	Yes
Present work	Dynamic fencing; five-DOF AUV plant	Yes; finite sensing range	Bounded nonuniform delay; no packet loss	No; timestamp and AUV position/velocity only	Label-free oversized-gap regulation	Yes	Yes

CA: collision avoidance; CP: connectivity preservation; NR: not reported as a formal closed-loop guarantee. “No” under limited target sensing means that finite-range or intermittent target detection is not part of the stated problem. Where a cited study does not define an acoustic packet payload, the table does not infer a state-only communication architecture.

Dynamic fencing differs from related cooperative-control tasks in that it is a persistent boundary-integrity problem. It requires not only encirclement but also the ability to keep a moving target enclosed while the AUVs avoid collision and preserve the acoustic links needed by the fence. The main difficulties are that the fence must remain label-free, the vehicles may start from safe but non-enclosing positions, the target motion induces persistent tracking lag, and delayed acoustic packets must be used without creating physically inconsistent relative positions. Existing formation-control methods usually rely on prescribed virtual points or leader-follower assignments, while encirclement and hunting methods often assume sufficiently fresh target information and focus on transient pursuit. Existing target-fencing protocols clarify the geometric mechanism of attraction–repulsion fencing, but their direct use with all-neighbor distance interactions can leave a large angular sector uncovered; in that case, the target may remain outside the convex hull even though radial and pairwise-distance errors are bounded. To close this gap, this paper proposes a timestamp-aligned minimal attraction–repulsion controller with a state-only target observer and label-free bearing-coverage repulsion. The bearing term acts only on the two AUVs that bound an oversized target-centered angular gap, so it is not a fixed-slot or leader-follower controller. Each AUV reconstructs AUV–AUV pairwise geometry at the packet timestamp using its own state history, applies the command at the current time,

and uses a local observer to reduce moving-target lag without transmitting target estimates. The local analysis establishes maintenance once the stated regular fencing conditions hold, while the reported simulations provide numerical evidence of self-organized entry from the tested safe, non-enclosing deployments under bounded nonuniform acoustic delays.

The main contributions of this paper are summarized as follows.

- (1) To the best of the authors' knowledge, this is the first formulation of multi-AUV self-organized dynamic fencing under limited target sensing and nonuniform acoustic communication delays. Unlike conventional formation, hunting, or idealized target-fencing problems, the formulation considers safe non-enclosing initial conditions, target enclosure, collision avoidance, acoustic connectivity, and convex-hull containment in an AUV-relevant setting. The subsequent theorem is stated as a local maintenance result, whereas self-organized entry is assessed numerically.
- (2) A direct minimal attraction–repulsion controller is developed without fixed angular slots, leader assignment, additional constraint-projection layers, or rotational anti-degeneracy terms. The controller remains a desired horizontal velocity command. It keeps target–AUV and AUV–AUV attraction–repulsion as the core mechanism and adds only two necessary components: a state-only target-motion observer for limited sensing and a label-free bearing-coverage repulsion that closes uncovered angular gaps left by pure pairwise distance interactions. Acoustic packets contain only sender timestamp, position, and velocity.
- (3) The study establishes delay-coupled parameter feasibility conditions for dynamic fencing. Timestamp alignment bounds the nonuniform-delay geometry error, the state-only observer bounds the moving-target tracking residual, and an observer-error-to-bearing-error relation connects estimated bearing coverage to true convex-hull enclosure. On an order-consistent regular fencing interval, the resulting analysis proves collision avoidance, radial confinement, angular-gap contraction, convex-hull maintenance, practical target tracking, and acoustic connectivity.

Notations: Bold symbols denote vectors or matrices. The Euclidean norm is denoted by $\|\cdot\|$, and $(\cdot)^T$ denotes transpose. For a scalar x , $[x]_+ = \max\{x, 0\}$. The operator $\text{sat}_{u_{\max}}(\mathbf{x})$ limits the Euclidean norm of $\mathbf{x}$ to $u_{\max}$. The notation $\text{co}\{\cdot\}$ denotes the convex hull, $\text{int}(\cdot)$ denotes set interior, and $\text{dist}(x, \mathcal{S})$ denotes the Euclidean distance from x to a set $\mathcal{S}$. The function $\text{atan2}(\cdot, \cdot)$ returns the four-quadrant angle. Cyclic angular differences are understood modulo 2π .

2. Problem Formulation

2.1. AUV Network and Dynamics

Consider a group of $N \geq 3$ AUVs indexed by $\mathcal{V} = \{1, \dots, N\}$, and let $\mathbf{p}_i = [x_i, y_i]^T$ denote the horizontal position of AUV i in the inertial frame $\{I\}$. Acoustic communication among AUVs is represented by a time-varying directed arrival graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, where $\mathcal{E}(t)$ records successful packet deliveries. All AUVs broadcast synchronously at $t_\ell = \ell T_b$, $\ell \in \mathbb{N}_0$, where $T_b > 0$ is the broadcast period. A packet sent by AUV j at t_ℓ reaches AUV i at $t_{ij,\ell}^r = t_\ell + \tau_{ij,\ell}$ only when the physical arrival-range test is satisfied:

$$t_{ij,\ell}^r = t_\ell + \tau_{ij,\ell}, \quad (j, i) \in \mathcal{E}(t_{ij,\ell}^r) \iff \|\mathbf{p}_j(t_\ell) - \mathbf{p}_i(t_{ij,\ell}^r)\| \leq R_c. \quad (1)$$

Here, R_c is the acoustic communication radius and $\tau_{ij,\ell}$ is the nonuniform propagation delay of the corresponding transmission. Thus, (1) defines an arrival event rather than a distance test that the receiver must evaluate before a packet is available. Between arrival events, the controller uses the retained-packet buffers defined below; consequently, the buffered-

neighbor set need not equal the instantaneous arrival graph. The link delay is allowed to differ across ordered pairs and broadcast instants, while sharing the uniform bound

$$0 \leq \tau_{ij,\ell} \leq \bar{\tau}, \quad (2)$$

where $\bar{\tau}$ is the common propagation-delay upper bound.

Let $\{B_i\}$ denote the body-fixed frame attached to AUV i . This paper considers the common five-degree-of-freedom AUV model in which roll motion is neglected because it is usually strongly stabilized by the vehicle restoring moment during normal cruising. For AUV i , let $\eta_i = [x_i, y_i, z_i, \theta_i, \psi_i]^T$ and $v_i = [u_i^b, v_i^b, w_i^b, \omega_{\theta,i}, \omega_{\psi,i}]^T$ be inertial-frame pose and body-fixed velocity, respectively, where η_i contains position, pitch, and yaw, and v_i contains body-frame surge, sway, heave, pitch rate, and yaw rate. Then, the five-degree-of-freedom dynamics can be written as [7,34]

$$\dot{\eta}_i = J_i(\eta_i)v_i, \quad (3)$$

$$M_i\dot{v}_i + C_i(v_i)v_i + D_i(v_i)v_i + g_i(\eta_i) = \tau_i + d_i(t), \quad (4)$$

where $J_i(\eta_i)$ is the kinematic transformation matrix that maps the body-fixed velocity v_i in $\{B_i\}$ to the inertial-frame pose rate $\dot{\eta}_i$ in $\{I\}$, $M_i = M_i^T > 0$ is the inertia matrix, C_i is the Coriolis and centripetal matrix, D_i is the hydrodynamic damping matrix, g_i is the restoring-force vector, τ_i is the generalized force and moment input, and d_i represents bounded unmodeled dynamics. Here, it is assumed that $\|d_i(t)\| \leq \bar{d}_i$.

The fencing task is defined by the horizontal projection of (3) and (4). Let $p_i = [x_i, y_i]^T$ be the horizontal position already contained in the pose η_i . The fencing controller generates a desired horizontal velocity command, which is tracked by the five-degree-of-freedom AUV plant through a low-level controller. For analysis, the induced horizontal closed-loop response is written as

$$\dot{p}_i = u_i + w_i(t), \quad (5)$$

where u_i is the desired horizontal velocity generated by the fencing controller, $v_i = \dot{p}_i$ is the actual horizontal velocity obtained from the five-degree-of-freedom kinematics, and w_i is a bounded lumped residual satisfying $\|w_i\| \leq \bar{w}_i$. The residual represents the mismatch between the guidance command and the horizontal velocity actually produced by (3) and (4), including low-level tracking lag, actuator saturation, hydrodynamic damping, and unmodeled dynamics. Thus, the analysis is expressed in horizontal task coordinates, but the AUV model remains the five-degree-of-freedom plant.

2.2. Limited Sensing and Delayed Acoustic Communication Model

Let $p_0(t)$ and $v_0(t)$ denote the target position and velocity. The target dynamics are assumed to satisfy

$$\begin{aligned} \dot{p}_0 &= v_0, \\ \dot{v}_0 &= a_0. \end{aligned} \quad (6)$$

Then, we make the following assumption:

Assumption 1. The target velocity and acceleration are bounded as $\|v_0\| \leq \bar{v}_0$ and $\|\dot{v}_0\| \leq \bar{a}_0$ for some positive constants $\bar{v}_0$ and $\bar{a}_0$.

This paper considers nonuniform communication delays but not packet loss after the physical range test in (1) has been satisfied. All AUVs use a common mission-time

coordinate for packet time-stamping and local-history indexing. Before deployment, their system clocks are synchronized to a common surface time reference and, during the mission, communication-layer two-way acoustic timing exchanges maintain this common timebase. Thus, the packet sent at t_ℓ carries the timestamp t_ℓ and is received at $t_{ij,\ell}^r = t_\ell + \tau_{ij,\ell}$ in the same time coordinate used by AUV i to store its state history. At a generic controller time t , the notation $s_{ij}(t)$ denotes the timestamp of the newest accepted packet retained in the j -to- i receive buffer. It equals t_ℓ for that packet and is held until a packet with a larger timestamp is accepted. AUV i can therefore retrieve its own state at exactly $s_{ij}(t)$ before constructing aligned relative geometry. If a residual clock offset is not negligible in a practical deployment, it must be included as an additional delay uncertainty when selecting $\bar{\tau}$ and the associated safety margins; this extension is outside the synchronized-clock operating condition considered here. Each AUV also has finite-range sensing. AUV i can directly measure the target when $\|p_0 - p_i\| \leq R_s$, where R_s is the sensing radius. When the target is detected, the onboard sensor of AUV i provides the raw noisy target-position measurement

$$y_{0,i}(t) = p_0(t) + n_{0,i}(t), \quad (7)$$

where $y_{0,i}$ is the target-position measurement, $\|n_{0,i}(t)\| \leq \bar{n}_0$ is bounded sensing noise for some constant $\bar{n}_0 > 0$. Define the direct-sensing indicator and the sensing-AUV set as

$$\sigma_i(t) = \begin{cases} 1, & \|p_0(t) - p_i(t)\| \leq R_s, \\ 0, & \|p_0(t) - p_i(t)\| > R_s. \end{cases}$$

The set of AUVs directly sensing the target is denoted by $\mathcal{V}_s(t) = \{i \in \mathcal{V} \mid \sigma_i(t) = 1\}$. Non-sensing AUVs do not receive target estimates or raw target measurements from neighbors. They maintain their own target-motion observers and use only delayed neighbor AUV states as a state-mediated compensation signal.

Assumption 2. *There exist constants $T_g > 0$, $T_o > 0$, and $\Delta_o \in (0, T_o]$ such that, for every $t \geq 0$, at least one AUV directly senses the target during $[t, t + T_g]$, and the union of the directed acoustic graphs over the same interval contains a spanning tree rooted at such a sensing AUV. In addition, each AUV satisfies the recurrent local-sensing condition*

$$\int_t^{t+T_o} \sigma_i(s) ds \geq \Delta_o, \quad i \in \mathcal{V}.$$

Thus, every local observer receives direct target measurements intermittently, although not necessarily at every instant.

Note that the spanning-tree clause is a mission-level range-connectivity condition; it does not transfer a target estimate through the network. The recurrent local-sensing clause is the condition used by the state-only observer analysis. It does not assign leaders or fixed fence positions. Because a state-only packet contains only AUV motion states, an AUV that never directly senses the target cannot eliminate an arbitrary absolute target-position error from neighbor velocities alone.

Each AUV maintains a target-motion observer corrected by onboard target measurements and updated by local sensing correction and delayed-neighbor-velocity-mediated compensation. The local sensing-correction part is

$$\begin{aligned} \dot{\hat{p}}_{0,i} &= \hat{v}_{0,i} + l_p \sigma_i(t) (y_{0,i} - \hat{p}_{0,i}), \\ \dot{\hat{v}}_{0,i} &= l_v \sigma_i(t) (y_{0,i} - \hat{p}_{0,i}), \end{aligned} \quad (8)$$

where $\hat{p}_{0,i}$ and $\hat{v}_{0,i}$ are the observer outputs, namely the locally estimated target position and velocity, and $l_p, l_v > 0$ are observer gains. The measurement $y_{0,i}$ in (7) is the observer input. The factor $\sigma_i(t)$ makes the correction term active only when AUV i directly senses the target. Define the observer errors as $e_{p,i} = \hat{p}_{0,i} - p_0$ and $e_{v,i} = \hat{v}_{0,i} - v_0$. During direct-sensing intervals without consensus coupling, the local error dynamics are

$$\begin{aligned}\dot{e}_{p,i} &= e_{v,i} - l_p e_{p,i} + l_p n_{0,i}, \\ \dot{e}_{v,i} &= -l_v e_{p,i} + l_v n_{0,i} - a_0.\end{aligned}\quad (9)$$

The linear part of (9) is Hurwitz for $l_p, l_v > 0$. This sensing-correction channel is therefore input-to-state stable with respect to sensing noise and target acceleration. The complete state-only observer and the resulting computable error bounds are given in Section 3.

Each packet broadcast at t_ℓ contains only the transmission timestamp and the sender AUV state at that timestamp:

$$m_j(t_\ell) = (t_\ell, p_j(t_\ell), v_j(t_\ell)). \quad (10)$$

where m_j denotes the acoustic message packet sent by AUV j . The clock-synchronization exchanges are handled by the communication layer and do not alter the state-only payload in (10). No target estimate, target measurement, fixed fence reference, slot assignment, or leader command is transmitted.

For every ordered pair (j, i) , receiver i maintains one buffer containing the newest accepted tuple $(s_{ij}, p_j(s_{ij}), v_j(s_{ij}))$. When a packet $m_j(t_\ell)$ satisfying (1) arrives, it replaces the buffered tuple only if $t_\ell > s_{ij}$; a delayed duplicate or an out-of-order packet with $t_\ell \leq s_{ij}$ is discarded. The accepted tuple is held between packet arrivals. Thus, a late packet never moves the local timestamp backward and never causes a retroactive change of the command. Define the retained-packet age and its worst-case bound by

$$a_{ij}(t) := t - s_{ij}(t), \quad \bar{a}_F := T_b + \bar{\tau}.$$

Under periodic broadcasting, the propagation-delay bound, and successful range-feasible delivery, the newest buffer satisfies $0 \leq a_{ij}(t) \leq \bar{a}_F$. The retained-neighbor set used by the observer and the distance interaction is therefore

$$\mathcal{N}_i^r(t) := \{j \neq i \mid \text{buffer } (j, i) \text{ is valid and } a_{ij}(t) \leq \bar{a}_F\}.$$

AUV i stores its own state history over $[t - \bar{a}_F, t]$. At each controller update, it uses the tuple currently held in the buffer and retrieves its own state at the same timestamp. Hence, for $j \in \mathcal{N}_i^r(t)$, AUV i reconstructs the time-aligned pairwise relative position

$$p_{ij}^a(t) = p_i(s_{ij}) - p_j(s_{ij}). \quad (11)$$

Then, the associated timestamp-aligned distance is $d_{ij}^a(t) = \|p_{ij}^a(t)\|$. Importantly, d_{ij}^a is evaluated only after a packet has passed the physical arrival test and has been accepted into the buffer; it is a controller variable and is never used to decide whether the packet can be received. For delayed pairwise geometry, the same packet timestamp is used for the self and neighbor positions, so a current self state is not mixed with a delayed neighbor state. The delayed neighbor velocity $v_j(s_{ij})$ is also used in the state-only observer. The control input is not physically delayed. The aligned geometry (11) is used only to compute the direct fencing command, and the resulting command is applied at the current time.

For the bearing-coverage calculation, AUV i does not receive a target estimate, a target bearing, or a precomputed angular order from any neighbor. Let $\mathcal{N}_i^F(t) := \mathcal{N}_i^r(t)$ denote

the fresh retained-neighbor set used for coverage. For each $j \in \mathcal{N}_i^F(t)$, AUV i forms the locally aligned neighbor position

$$\tilde{\mathbf{p}}_{j|i}(t) = \mathbf{p}_j(s_{ij}) + \mathbf{v}_j(s_{ij})(t - s_{ij}),$$

and uses its own target-center estimate $\hat{\mathbf{p}}_{0,i}(t)$ as the center for bearing computation. The local bearing set is therefore built from $\mathbf{p}_i(t)$, $\hat{\mathbf{p}}_{0,i}(t)$, and the received state-only neighbor packets only. Different AUVs are not assumed to have identical target estimates or packet ages. For general sampled operation, their local cyclic orders may also differ; the local theorem explicitly restricts attention to the order-consistent regular interval in which the quantified bearing-separation margin prevents such a disagreement.

The information set $\mathcal{I}_i(t)$ consists of the local sensing flag, the current and stored local observer state, the stored self-state history over $[t - \bar{a}_F, t]$, and the newest accepted state-only packets $\{(s_{ij}, \mathbf{p}_j(s_{ij}), \mathbf{v}_j(s_{ij})) : j \in \mathcal{N}_i^T(t)\}$.

2.3. Dynamic Fencing Objective

The dynamic fencing objective is to make the AUV group self-organize from a safe, connected initial configuration into a moving, enclosing boundary around the target. This objective is defined for the true target-relative geometry, while the controller in Section 3 uses only the information set $\mathcal{I}_i(t)$. For task definition and performance evaluation, define $\mathbf{q}_i = \mathbf{p}_i - \mathbf{p}_0$ and $\rho_i = \|\mathbf{q}_i\|$, where $\mathbf{q}_i$ and ρ_i are the true target-relative position and radial distance of AUV i , respectively. The radial part of the fence is specified by an admissible annulus and by the zero-force radius of the target–AUV attraction–repulsion interaction. The desired radius is $\rho_d = \rho_0$, where $\rho_{\min} < \rho_0 < \rho_{\max}$ is the nominal target–AUV equilibrium distance. The moving target does not make ρ_d time-varying. Instead, the ability to maintain this radius is determined by the target-speed compensation error, acoustic-delay error, AUV–AUV interaction margin, and available control authority. The admissible radial domain is $\rho_{\min} \leq \rho_i \leq \rho_{\max}$, where $\rho_{\min}$ prevents excessive target proximity and $\rho_{\max}$ prevents fence rupture.

Note that the enclosing part of the fence is label-free and convex-hull based. Define the convex-hull distance $D_h(t) = \text{dist}(\mathbf{p}_0(t), \text{co}\{\mathbf{p}_1(t), \dots, \mathbf{p}_N(t)\})$. The target is exactly enclosed if $D_h(t) = 0$, namely if $\mathbf{p}_0(t)$ belongs to the AUV convex hull. A strict interior enclosure additionally requires a positive hull margin. For analysis, let $\theta_i(t)$ be the target-centered bearing of AUV i , and sort the bearings as $\theta_{(1)} \leq \dots \leq \theta_{(N)}$ on $[0, 2\pi)$. The cyclic bearing gaps are

$$\Delta\theta_k = \begin{cases} \theta_{(k+1)} - \theta_{(k)}, & k = 1, \dots, N-1, \\ \theta_{(1)} + 2\pi - \theta_{(N)}, & k = N. \end{cases}$$

If all AUVs have positive radius and $\max_k \Delta\theta_k < \pi$, then every line through the target has AUVs on both sides; therefore the target lies in the interior of the AUV convex hull. Conversely, if a bearing gap is larger than π , all AUVs lie in a closed half-plane whose boundary passes through the target, so convex-hull enclosure is impossible. Hence the fencing problem is equivalent to controlling both radial distances and the largest bearing gap. This paper does not require the target to be inside the initial convex hull. Instead, the initial condition is required to be safe, sensing-rooted connected, and nondegenerate. Nondegenerate means that the AUVs are not all located on an invariant collinear or single-ray configuration with respect to the target.

Let $d_{ij}(t) = \|\mathbf{p}_i(t) - \mathbf{p}_j(t)\|$ denote the true inter-AUV distance. The AUV group must also satisfy inter-vehicle collision avoidance and local acoustic-connectivity constraints. Specifically, $d_{ij}(t) \geq d_{\min}$ is required for $i \neq j$, and every fence-supporting packet transmitted after the self-organization transient must satisfy the physical arrival-range condition

(1). Let $\mathcal{E}_f(t_\ell)$ denote the set of directed AUV–AUV transmissions used to maintain the enclosing boundary at broadcast instant t_ℓ ; in simulation the corresponding undirected pairs are obtained from the convex-hull boundary edges.

The mission objective and the analytical claim are distinguished here. The mission objective is self-organized entry from a safe connected non-enclosing deployment followed by compact-fence maintenance. The theorem in Section 3 proves the maintenance part only after the trajectory has entered an order-consistent regular interval. The distributed command $\mathbf{u}_i(t) = \Phi_i(\mathcal{I}_i(t))$ is designed without assigning fixed angular slots, leaders, or prescribed target-relative reference points. Its desired mission-level outcome is that there exists an entry time $T_{\text{ent}} \geq 0$ for which

$$\begin{aligned} \max_{i \in \mathcal{V}} |\rho_i(t) - \rho_d| &\leq \varepsilon_\rho^*, & t &\geq T_{\text{ent}}, \\ D_h(t) &\leq \varepsilon_h^*, & t &\geq T_{\text{ent}}, \\ \max_k \Delta\theta_k(t) &\leq \pi - \varepsilon_\theta^*, & t &\geq T_{\text{ent}}, \\ \rho_{\min} &\leq \rho_i(t) \leq \rho_{\max}, & t &\geq T_{\text{ent}}, \\ \|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| &> d_{\min}, & i &\neq j, \quad t \geq 0, \\ \|\mathbf{p}_j(t_\ell) - \mathbf{p}_i(t_{ij,\ell}^t)\| &\leq R_c, & (j, i) &\in \mathcal{E}_f(t_\ell), \quad t_\ell \geq T_{\text{ent}}, \\ \mathbf{p}_0(t) &\in \text{co}\{\mathbf{p}_1(t), \dots, \mathbf{p}_N(t)\} & \text{if } \varepsilon_h^* &= 0, \end{aligned} \quad (12)$$

where ε_ρ^* , ε_h^* , and $\varepsilon_\theta^* > 0$ are practical residual or margin constants induced by target motion, state-only observer error, acoustic delay, saturation, and low-level tracking residuals. The entry time T_{ent} is a mission-performance quantity, not a prescribed design parameter. Section 3 establishes the corresponding local maintenance properties on a verified regular interval; it does not assert that every safe non-enclosing deployment reaches that interval. In the ideal zero-delay, fully observed, and error-free case the radial and hull-distance residuals vanish locally.

3. Main Results

The controller is designed as an explicit horizontal velocity command for the AUV autopilot. The design keeps the target-AUV and AUV-AUV attraction-repulsion mechanism as the main driving field, adds a state-only observer so that every AUV has a local target-centered geometry estimate, and adds only one label-free coverage term to close oversized angular gaps.

3.1. State-Only Target-Motion Observer

The local observer in (8) is extended with state-mediated velocity compensation over delayed acoustic links. Define the delayed neighbor-velocity average available to AUV i as

$$\bar{\mathbf{v}}_{\mathcal{N}_i^t}^a(t) = \begin{cases} \sum_{j \in \mathcal{N}_i^t} a_{ij} \mathbf{v}_j(s_{ij}), & \mathcal{N}_i^t \neq \emptyset, \\ \mathbf{0}, & \mathcal{N}_i^t = \emptyset, \end{cases} \quad (13)$$

where $a_{ij} \geq 0$ and $\sum_{j \in \mathcal{N}_i^t} a_{ij} = 1$ when $\mathcal{N}_i^t$ is nonempty. The state-only observer is

$$\begin{aligned} \dot{\hat{\mathbf{p}}}_{0,i} &= \hat{\mathbf{v}}_{0,i} + l_p \sigma_i(\mathbf{y}_{0,i} - \hat{\mathbf{p}}_{0,i}), \\ \dot{\hat{\mathbf{v}}}_{0,i} &= l_v \sigma_i(\mathbf{y}_{0,i} - \hat{\mathbf{p}}_{0,i}) - (1 - \sigma_i) \lambda_o (\hat{\mathbf{v}}_{0,i} - \bar{\mathbf{v}}_{\mathcal{N}_i^t}^a), \end{aligned} \quad (14)$$

where $\lambda_o > 0$ is the state-mediated velocity compensation gain. The estimates $\hat{\mathbf{p}}_{0,i}$ and $\hat{\mathbf{v}}_{0,i}$ are used only for target-centered local computation. The term $\bar{\mathbf{v}}_{\mathcal{N}_i^t}^a$ uses only delayed

neighbor AUV velocities from (10); it is not target-estimate forwarding. When AUV i directly senses the target, the velocity update is driven by local measurement correction. When AUV i does not directly sense the target, the state-mediated term drives $\hat{v}_{0,i}$ toward the neighbor-velocity signal without the biased steady state produced by unequal damping and injection gains. It is used as a motion-prediction input during sensing gaps, not as a source of absolute target-position information.

Let $\mathbf{m}_i(t) = \bar{\mathbf{v}}_{\mathcal{N}_i}^a(t) - \mathbf{v}_0(t)$ denote the state-mediated velocity mismatch. Since the weights in (13) are convex and Assumption 5 gives $\|\mathbf{v}_j(s_{ij})\| \leq \bar{v}_A$, a directly computable conservative bound is

$$\|\mathbf{m}_i(t)\| \leq \bar{e}_m := \bar{v}_A + \bar{v}_0.$$

A less conservative value may be used when a verified bound on the delayed neighbor-target velocity residual is available. The following explicit switched-observer condition is used instead of inferring observer boundedness from recurrent sensing alone.

Assumption 3 (Checkable observer regularity). *Choose a scaling length $\ell_o > 0$. There exist a symmetric matrix $\mathbf{P}_o > 0$ and constants $\lambda_0 \geq 0$ and $\lambda_1 > 0$ such that, for*

$$\mathbf{A}_0 = \begin{bmatrix} 0 & \ell_o^{-1} \\ 0 & -\lambda_0 \end{bmatrix}, \quad \mathbf{A}_1 = \begin{bmatrix} -l_p & \ell_o^{-1} \\ -l_v \ell_o & 0 \end{bmatrix},$$

the inequalities $\mathbf{A}_0^T \mathbf{P}_o + \mathbf{P}_o \mathbf{A}_0 \preceq 2\lambda_0 \mathbf{P}_o$ and $\mathbf{A}_1^T \mathbf{P}_o + \mathbf{P}_o \mathbf{A}_1 \preceq -2\lambda_1 \mathbf{P}_o$ hold, and $\lambda_1 \Delta_o > \lambda_0(T_o - \Delta_o)$.

This is a finite-dimensional gain-and-dwell-time test. It can be checked once for the chosen $(l_p, l_v, \lambda_o, T_o, \Delta_o)$ and does not introduce an additional communicated variable. For the gains used in Section 4, one explicit certificate is $\ell_o = 1$, $\mathbf{P}_o = \begin{bmatrix} 33/56 & -1/2 \\ -1/2 & 361/182 \end{bmatrix}$, $\lambda_0 = 0.30$, and $\lambda_1 = 0.20$. Direct substitution gives $\mathbf{A}_0^T \mathbf{P}_o + \mathbf{P}_o \mathbf{A}_0 \preceq 0.60 \mathbf{P}_o$ and $\mathbf{A}_1^T \mathbf{P}_o + \mathbf{P}_o \mathbf{A}_1 = -\mathbf{I}_2 \preceq -0.40 \mathbf{P}_o$. Thus the dwell-time test is met whenever $\Delta_o/T_o > 0.60$. The numerical regular-time proxy in Section 4 additionally requires all AUVs to be sensing, and therefore satisfies this certificate on the reported local maintenance interval.

Lemma 1 (State-only observer boundedness). *Under Assumptions 1–3, suppose that $\|\mathbf{m}_i(t)\| \leq \bar{e}_m$, $\|\mathbf{n}_{0,i}(t)\| \leq \bar{n}_0$, and $\|[\ell_o^{-1} \mathbf{e}_{p,i}^T(0), \mathbf{e}_{v,i}^T(0)]^T\| \leq E_0$. Then, the observer (14) is practically bounded. In particular, with*

$$\alpha_o := \frac{\lambda_1 \Delta_o - \lambda_0(T_o - \Delta_o)}{T_o} > 0, \quad c_o := \sqrt{\frac{\lambda_{\max}(\mathbf{P}_o)}{\lambda_{\min}(\mathbf{P}_o)}} e^{\lambda_0 T_o},$$

and

$$\bar{b}_o := \sqrt{\left(\frac{l_p \bar{n}_0}{\ell_o}\right)^2 + (l_v \bar{n}_0 + \lambda_o \bar{e}_m + \bar{a}_0)^2},$$

one has

$$\left\| \begin{bmatrix} \ell_o^{-1} \mathbf{e}_{p,i}(t) \\ \mathbf{e}_{v,i}(t) \end{bmatrix} \right\| \leq c_o e^{-\alpha_o t} E_0 + \frac{c_o \bar{b}_o}{\alpha_o} \leq \bar{z}_o := c_o E_0 + \frac{c_o \bar{b}_o}{\alpha_o}.$$

Consequently, the fencing analysis may use the computable bounds

$$\|\hat{\mathbf{p}}_{0,i} - \mathbf{p}_0\| \leq \bar{e}_p := \ell_o \bar{z}_o, \quad \|\hat{\mathbf{v}}_{0,i} - \mathbf{v}_0\| \leq \bar{e}_v := \bar{z}_o. \quad (15)$$

Proof. Let $\mathbf{z}_i = [\ell_o^{-1} \mathbf{e}_{p,i}^T, \mathbf{e}_{v,i}^T]^T$. When $\sigma_i = 0$ and $\sigma_i = 1$, respectively, the homogeneous parts of the scaled error dynamics are $\mathbf{A}_0 \otimes \mathbf{I}_2$ and $\mathbf{A}_1 \otimes \mathbf{I}_2$. The additive term has norm at

most $\bar{b}_o$. Assumption 3 gives $\dot{V}_o \leq 2\lambda_0 V_o$ during non-sensing intervals and $\dot{V}_o \leq -2\lambda_1 V_o$ during sensing intervals for $V_o = \mathbf{z}_i^T (\mathbf{P}_o \otimes \mathbf{I}_2) \mathbf{z}_i$. The recurrent-sensing inequality in Assumption 2 therefore yields a decay factor no larger than $\exp[-2(\lambda_1 \Delta_o - \lambda_0(T_o - \Delta_o))]$ over each interval of length T_o . The stated c_o and α_o bound the resulting transition matrix. Variation of constants gives the displayed bound. This result also shows why an AUV that never directly senses the target is outside the theorem: state-only neighbor velocities do not, by themselves, identify an arbitrary absolute target position. $\square$

3.2. Minimal Observer-Assisted Attraction–Repulsion Controller

AUV i computes $\hat{\mathbf{q}}_i = \mathbf{p}_i - \hat{\mathbf{p}}_{0,i} = [\hat{q}_{i,x}, \hat{q}_{i,y}]^T$ and $\hat{\rho}_i = \|\hat{\mathbf{q}}_i\|$. To avoid singularity when the estimated target-relative vector is small, define

$$\hat{\rho}_{i,\epsilon} = \max\{\hat{\rho}_i, \epsilon_\rho\}, \quad 0 < \epsilon_\rho < \rho_{\min} - \bar{e}_p.$$

The target–AUV radial interaction is

$$\mathbf{u}_{T,i} = \beta_T(\hat{\rho}_i) \frac{\hat{\mathbf{q}}_i}{\hat{\rho}_{i,\epsilon}}, \quad (16)$$

where

$$\beta_T(r) = -k_T(r - \rho_d) + k_B[\rho_B^- - r]_+ - k_B[r - \rho_B^+]_+, \quad \rho_{\min} < \rho_B^- < \rho_d < \rho_B^+ < \rho_{\max},$$

and $k_T, k_B > 0$. The first term is the original target–AUV attraction–repulsion field. The two boundary-recovery branches only reinforce its outward action near $\rho_{\min}$ and inward action near $\rho_{\max}$; they add no communication variable, reference slot, projection, or optimization layer. This term regulates the radius without imposing a time-varying desired circle.

For AUV–AUV interaction, AUV i uses timestamp-aligned relative vectors $\mathbf{p}_{ij}^a$ from (11). The propagation-delay- and packet-holding-tightened safe distance is

$$d_s = d_{\min} + 2\bar{v}_A \bar{a}_F + \epsilon_s, \quad (17)$$

where $\epsilon_s > 0$ is a design margin. The distance interaction is

$$\mathbf{u}_{A,i} = \sum_{j \in \mathcal{N}_i^\tau} \beta_A(d_{ij}^a) \frac{\mathbf{p}_{ij}^a}{d_{ij}^a}, \quad (18)$$

with

$$r_\epsilon(r) = \max\{r, d_s + \epsilon_r\}$$

and

$$\beta_A(r) = -k_A(r - d_A) + k_R \left[\frac{1}{r_\epsilon(r) - d_s} - \frac{1}{\mu - d_s} \right]_+, \quad (19)$$

where $d_s < \mu < d_A$ and $\epsilon_r > 0$. The first term is a simple distance attraction–repulsion around d_A , while the second term is activated only near the delay-tightened safety distance. The small regularization ϵ_r keeps the implemented repulsion finite when delayed data momentarily place d_{ij}^a inside the tightened buffer; the physical safety proof is still written for the true boundary $d_{ij} = d_{\min}$. Equation (18) is evaluated only with timestamp-aligned neighbor geometry.

Distance attraction–repulsion alone can regulate radius and spacing without guaranteeing target enclosure. To close uncovered angular sectors while preserving the state-only packet, AUV i constructs a local bearing list from

$$\mathcal{B}_i(t) = \{i\} \cup \mathcal{N}_i^F(t), \quad M_i(t) = |\mathcal{B}_i(t)|.$$

For $j = i$, set $\tilde{\mathbf{p}}_{j|i}(t) = \mathbf{p}_i(t)$. For every $j \in \mathcal{B}_i(t)$, AUV i computes

$$\hat{\mathbf{q}}_{j|i}^F(t) = \tilde{\mathbf{p}}_{j|i}(t) - \hat{\mathbf{p}}_{0,i}(t), \quad \hat{\theta}_{j|i}^F(t) = \text{atan2}(\hat{q}_{j|i,y}^F(t), \hat{q}_{j|i,x}^F(t)).$$

The pairs $(\hat{\theta}_{j|i}^F, j)$, $j \in \mathcal{B}_i(t)$, are sorted lexicographically. Thus, the identifier is used only to break an exact equal-bearing tie and never assigns a fixed angular slot. Let π_i^F be the resulting local cyclic permutation of $\mathcal{B}_i(t)$ and define

$$\hat{\Delta}\theta_{i,k}^F = \begin{cases} \hat{\theta}_{\pi_i^F(k+1)|i}^F - \hat{\theta}_{\pi_i^F(k)|i}^F & k = 1, \dots, M_i - 1, \\ \hat{\theta}_{\pi_i^F(1)|i}^F + 2\pi - \hat{\theta}_{\pi_i^F(M_i)|i}^F & k = M_i. \end{cases}$$

If a local gap exceeds $\theta_d + \theta_\epsilon$, where $\theta_d = 2\pi/N$ and $\theta_\epsilon > 0$, only a boundary AUV of that locally computed gap applies the corresponding tangential component. The implemented coverage term is

$$\mathbf{u}_{C,i}(t) = \sum_{k=1}^{M_i(t)} k_C [\hat{\Delta}\theta_{i,k}^F - \theta_d - \theta_\epsilon]_+ \boldsymbol{\zeta}_{i,k}^F(t), \quad (20)$$

where $k_C > 0$ and

$$\boldsymbol{\zeta}_{i,k}^F(t) = \begin{cases} \mathbf{R}_{\pi/2} \hat{\mathbf{q}}_i(t) / \hat{\rho}_{i,\epsilon}(t), & i = \pi_i^F(k), \\ -\mathbf{R}_{\pi/2} \hat{\mathbf{q}}_i(t) / \hat{\rho}_{i,\epsilon}(t), & i = \pi_i^F(k+1), \\ \mathbf{0}, & \text{otherwise,} \end{cases}$$

with cyclic interpretation $\pi_i^F(M_i + 1) = \pi_i^F(1)$. Here $\mathbf{R}_{\pi/2}$ is the planar 90° rotation matrix. At each discrete controller instant, the local permutation is recomputed from the currently held admissible packet set and is held until the next controller update. Because the controller sampling period is positive, cyclic-order changes cannot generate Zeno switching. At an exact equal-bearing event, the lexicographic identifier rule gives a unique sampled implementation; the local analysis below excludes such order-change events through the explicit separation condition in Assumption 5.

Every quantity in (20) is obtained from AUV i 's local observer, current self state, synchronized packet timestamps, and state-only neighbor packets. Therefore $\mathbf{u}_{C,i} = \Phi_{C,i}(\mathcal{I}_i)$ is distributed under the declared information set. The sampled implementation permits different local packet ages and target-center estimates. The local theorem does not treat an arbitrary disagreement of cyclic permutations as a small perturbation; instead, Assumption 5 states the explicit separation condition under which the locally sorted permutations agree up to a cyclic shift.

The coverage actions are intrinsically coupled. To expose this structure, consider the ideal current-state frame with a fixed true cyclic order and define $\theta_c := \theta_d + \theta_\epsilon$ and $r_k := [\Delta\theta_k - \theta_c]_+$. The AUV occupying ordered position k bounds gaps $k-1$ and k ; hence, its net ideal tangential coverage command is $k_C(r_k - r_{k-1})\mathbf{e}_{\theta,(k)}$, where $\mathbf{e}_{\theta,(k)}$ is its true unit tangent and the indices are cyclic. Thus, actions generated by adjacent active gaps may cancel at one AUV, but that cancellation is exactly represented by the difference $r_k - r_{k-1}$.

The stability analysis below therefore treats the complete vector of active gaps rather than differentiating one maximum gap in isolation.

The observer provides the target-motion compensation,

$$\mathbf{u}_{V,i} = \hat{\mathbf{v}}_{0,i}. \quad (21)$$

The raw controller is

$$\mathbf{u}_i^{\text{raw}} = \mathbf{u}_{T,i} + \mathbf{u}_{A,i} + \mathbf{u}_{C,i} + \mathbf{u}_{V,i}. \quad (22)$$

The command sent to the AUV autopilot is

$$\mathbf{u}_i = \text{sat}_{u_{\max}}(\mathbf{u}_i^{\text{raw}}). \quad (23)$$

Equations (22) and (23) form the complete direct velocity command.

3.3. Closed-Loop Analysis

The following assumptions state the scope of the closed-loop result. Define the local bearing-reconstruction error and its angular counterpart by

$$\varepsilon_F := 2\bar{v}_A \bar{a}_F + \bar{e}_p, \quad \bar{\theta}_F := \arcsin\left(\frac{\varepsilon_F}{\rho_{\min}}\right).$$

The delayed closed loop is initialized after a communication buffer-fill interval. Specifically, each position history $\mathbf{p}_i(s)$ is continuously differentiable on $s \in [-\bar{a}_F, 0]$ and satisfies $\|\dot{\mathbf{p}}_i(s)\| \leq \bar{v}_A$, $d_{ij}(s) > d_{\min}$, and $\rho_{\min} < \rho_i(s) < \rho_{\max}$ throughout that interval. At $t = 0$, every link used by the controller has a newest accepted packet whose timestamp lies in $[-\bar{a}_F, 0]$ and whose arrival satisfies (1). Because $\bar{a}_F = T_b + \bar{\tau} \geq \bar{\tau}$, this convention includes the required safe physical history over $[-\bar{\tau}, 0]$ and also covers the additional packet-holding interval introduced by periodic broadcasting. The time origin of the analysis is taken after this finite buffer-fill phase.

Assumption 4. *The state history and receive buffers satisfy the initialization convention above. For the self-organization mission, the deployment configuration is collision-safe, sensing-rooted connected, and nondegenerate; the target need not initially lie in the AUV convex hull. This mission-level condition is used for the numerical entry tests, but it is not by itself the initial condition of the local maintenance theorem below.*

Assumption 5 (Local regular fencing operation). *Let $[t_0, t_1)$ be a nonempty interval after the buffer-fill phase. At t_0 , $d_{ij} > d_{\min}$, $\rho_{\min} < \rho_i < \rho_{\max}$, and $\|\dot{\mathbf{p}}_i\| \leq \bar{v}_A$. Let $\mathcal{K}_R$ be a compact neighborhood of the trajectory before any first contact with the collision or radial boundaries, separated from the interaction singularities. On $[t_0, t_1)$, the observer obeys (15) with $\bar{e}_p < \rho_{\min}$ and $\varepsilon_F < \rho_{\min}$, every ordered pair has a valid retained state-only packet of age at most $\bar{a}_F = T_b + \bar{\tau}$, the speed bound $\|\dot{\mathbf{p}}_i\| \leq \bar{v}_A$ holds, and the physical range satisfies $2\rho_{\max} + \bar{v}_A \bar{\tau} < R_c$. In addition, for some $\theta_{\text{sep}} > 0$, the true cyclic bearing gaps satisfy $\min_k \Delta\theta_k \geq 2\bar{\theta}_F + \theta_{\text{sep}}$. Packets satisfying (1) are delivered within $\bar{\tau}$, no additional packet loss is considered, and the latest-timestamp buffer rule of Section 2.2 is used. Thus, on $[t_0, t_1)$ every local coverage frame contains all AUV states and has the same cyclic order as the true frame, up to a cyclic shift. The distance d_{ij}^a is evaluated only after packet acceptance and is used solely by the interaction gain; it is not an availability condition.*

The separation condition is not a fixed-slot requirement. It excludes only unresolved near-tie bearing configurations on the interval where a common ordering is needed for the coupled-gap proof. The observer, packet-freshness, speed, and separation conditions are

premises of the local result. The first-exit argument below proves that, while those premises hold, collision and radial boundaries cannot be the first cause of leaving the regular regime. Self-organized entry from a non-enclosing deployment is assessed by simulation rather than claimed as a global consequence of this assumption.

For later use, let u_i^0 denote the zero-delay, unsaturated, observer-error-free command generated by (16)–(22). Define the net radial margins

$$\gamma_\rho^+ = \min_i \inf_{\rho_i = \rho_{\max}} \left[-\frac{q_i^T}{\rho_i} u_i^0 \right], \quad \gamma_\rho^- = \min_i \inf_{\rho_i = \rho_{\min}} \left[\frac{q_i^T}{\rho_i} u_i^0 \right], \quad (24)$$

and the physical safety margin

$$\gamma_s = \inf_{d_{ij} = d_{\min}} n_{ij}^T (u_i^0 - u_j^0), \quad n_{ij} = \frac{p_i - p_j}{d_{ij}}.$$

For the coupled bearing-coverage dynamics, define the cycle constant $\lambda_N := 1/[N(N-1)]$ and the nominal contraction rate $c_\theta := k_C \lambda_N / \rho_{\max}$. The origin of λ_N and the fact that it remains valid when several gaps are simultaneously active are established in Lemma 5. A bearing computed about a local target estimate has the pointwise error bound

$$|\hat{\theta}_{j|i} - \theta_j| \leq \bar{\theta}_e := \arcsin\left(\frac{\bar{e}_p}{\rho_{\min}}\right),$$

and the local state-only bearing reconstruction error is bounded by $\bar{\theta}_F$. On $\mathcal{K}_R$, let $L_C > 0$ be a local Lipschitz constant of the single-valued coverage field with the cyclic order fixed by Assumption 5, and define $\bar{\delta}_C := L_C \varepsilon_F$. Let $L_T > 0$ be a local Lipschitz constant of the target–AUV interaction on $\mathcal{K}_R$, and let $\bar{\delta}^s$ satisfy $\|\delta_i^s\| \leq \bar{\delta}^s$ for the saturation residual $\delta_i^s = \text{sat}_{u_{\max}}(u_i^{\text{raw}}) - u_i^{\text{raw}}$. A sufficient fleet-level bound on the nonideal target-relative velocity perturbation is

$$\bar{\xi} := L_T \bar{e}_p + \bar{e}_v + \max_i L_i^T \bar{a}_F + \bar{\delta}_C + \max_i \bar{w}_i + \bar{\delta}^s,$$

where L_i^T is the timestamp-alignment constant established in Lemma 3. Define the largest tangential component of the ideal distance-interaction field by

$$\bar{u}_{A,\theta} := \max_i \sup_{\mathcal{K}_R} |e_{\theta,i}^T u_{A,i}^0|, \quad e_{\theta,i} := \frac{R_{\pi/2} q_i}{\rho_i},$$

and use the computable aggregate gap-rate bound

$$\bar{\xi}_\theta := \frac{2\sqrt{N}}{\rho_{\min}} (\bar{u}_{A,\theta} + \bar{\xi}).$$

The term $\bar{u}_{A,\theta}$ accounts explicitly for the tangential component of the attraction–repulsion distance field, while $\bar{\xi}_\theta$ also contains observer error, timestamp mismatch, saturation, and low-level tracking residuals. These quantities are derived from the operating set and the nominal field; they are introduced only to state the gain margins compactly.

Assumption 6. *The controller gains are selected so that*

$$\min\{\gamma_\rho^+, \gamma_\rho^-\} > \bar{\xi}, \quad \gamma_s > 2\bar{\xi}, \quad \frac{\bar{\xi}_\theta}{c_\theta} < \pi - \theta_c.$$

The first two inequalities make the radial and collision-avoidance margins dominate the nonideal velocity perturbation. The last inequality makes the coupled bearing-coverage contraction dominate the complete gap-rate perturbation and leaves a strictly positive enclosure margin.

Remark 1. Assumptions 5 and 6 are local feasibility conditions rather than global restrictions. Assumption 5 specifies the compact operating region in which the AUV speed, the state-only observer error, and the acoustic range are bounded. The conditions $\bar{e}_p < \rho_{\min}$ and $\varepsilon_F < \rho_{\min}$ ensure that the current and local state-only target-centered bearings are well defined and that the bearing-error bounds used in Lemmas 2 and 4 are finite; they do not require exact target-state reconstruction or identical local target estimates. The range condition $2\rho_{\max} + \bar{v}_A \bar{\tau} < R_c$ is conservative but directly checkable. It combines radial confinement with the maximum sender displacement during acoustic propagation and therefore guarantees the arrival test (1) without introducing an additional topology controller. This inequality is sufficient, not necessary. If it is violated while the actual fence-supporting transmissions still satisfy $\|\mathbf{p}_j(t_\ell) - \mathbf{p}_i(t_{ij,\ell}^r)\| \leq R_c$, the controller can continue to operate, and immediate instability is not implied. Once a required transmission fails (1), however, no new state-only packet enters the corresponding receive buffer. If its age then exceeds $\bar{a}_F$, that neighbor is removed from $\mathcal{N}_i^r$ and the associated delayed interaction cannot be evaluated. The theorem then no longer guarantees bounded packet age, sensing-rooted connectivity, activation of pairwise repulsion, bearing-gap contraction, or convex-hull containment. A transient range failure may cause only temporary performance degradation if the required spanning tree and safety-critical buffers remain fresh, whereas persistent failures may partition the retained-neighbor graph, enlarge observer and coverage errors, allow the target to leave the convex hull, and invalidate the proved collision margin. A real mission should therefore monitor packet age and retained-graph connectivity and reduce the fence radius or operating speed, or initiate a fleet-reconfiguration procedure, before the range margin is exhausted. Assumption 6 is a gain-margin requirement: the radial restoring action, close-range repulsion, and bearing-coverage action must dominate the combined perturbation caused by observer error, timestamp-alignment delay, saturation, and low-level tracking residuals. These margins are standard in practical constrained multi-agent control and can be verified from the selected controller gains and mission envelope.

Lemma 2 (Distributed implementation of the coverage term). Under Assumption 5, the command (20) is a distributed map $\mathbf{u}_{C,i} = \Phi_{C,i}(\mathcal{I}_i)$. Let $\mathbf{u}_{C,i}^0(\mathbf{p}, \mathbf{p}_0)$ denote the ideal zero-age coverage field evaluated with the true cyclic order. With the local state-only bearing reconstruction uncertainty ε_F , on $\mathcal{K}_R$,

$$\text{dist}(\mathbf{u}_{C,i}(t), \mathbf{u}_{C,i}^0(\mathbf{p}(t), \mathbf{p}_0(t))) \leq L_C \varepsilon_F = \bar{\delta}_C, \quad (25)$$

where $L_C > 0$ is a piecewise-local Lipschitz constant of the regularized coverage field on the compact operating set.

Proof. For the packet set accepted by AUV i , every $\tilde{\mathbf{p}}_{ji}$ is reconstructed from a packet of the form (10), while $\hat{\mathbf{p}}_{0,i}(t)$ and the current tangential direction are obtained from AUV i 's local observer and self state. Sorting these locally available pairs and applying (20) therefore requires no target estimate, target bearing, or ordering message from another AUV. This proves $\mathbf{u}_{C,i} = \Phi_{C,i}(\mathcal{I}_i)$.

Because $0 \leq t - s_{ij} \leq \bar{a}_F$ and $\|\mathbf{v}_j(s_{ij})\| \leq \bar{v}_A$,

$$\begin{aligned} & \left\| [\tilde{\mathbf{p}}_{ji}(t) - \hat{\mathbf{p}}_{0,i}(t)] - [\mathbf{p}_j(t) - \mathbf{p}_0(t)] \right\| \\ & \leq \|\mathbf{p}_j(s_{ij}) - \mathbf{p}_j(t)\| + \|\mathbf{v}_j(s_{ij})\|(t - s_{ij}) + \|\hat{\mathbf{p}}_{0,i}(t) - \mathbf{p}_0(t)\| \\ & \leq 2\bar{v}_A \bar{a}_F + \bar{e}_p = \varepsilon_F. \end{aligned}$$

The condition $\varepsilon_F < \rho_{\min}$ keeps every locally reconstructed bearing well defined. Moreover, $\min_k \Delta\theta_k \geq 2\bar{\theta}_F + \theta_{\text{sep}}$ implies that the local bearing perturbations cannot reverse the true cyclic order. Hence, every local permutation equals the true permutation up to a cyclic shift, and the locally implemented coverage field is single-valued and locally Lipschitz on $\mathcal{K}_R$. The preceding geometry bound then gives (25). The lexicographic tie rule remains part of the sampled implementation, but equal-bearing and order-change events are outside the local maintenance theorem rather than being treated as a uniformly small perturbation. $\square$

Lemma 3 (Timestamp alignment with discrete packet holding). *Under the synchronized mission-time condition, the discrete packet process in Section 2.2, (2), and $\|\dot{\mathbf{p}}_i\| \leq \bar{v}_A$, every retained packet used at controller time t satisfies $0 \leq t - s_{ij}(t) \leq \bar{a}_F$ and*

$$\|\mathbf{p}_{ij}^a(t) - (\mathbf{p}_i(t) - \mathbf{p}_j(t))\| \leq 2\bar{v}_A\bar{a}_F. \quad (26)$$

Moreover, if $d_{ij}(t) \leq d_{\min} + \epsilon_s$, then $d_{ij}^a(t) \leq d_s$, where d_s is defined in (17). On any compact operating subset, the timestamp-aligned AUV–AUV interaction can be written as

$$\mathbf{u}_{A,i}(t) = \mathbf{u}_{A,i}^0(t) + \delta_{A,i}^\tau(t), \quad \|\delta_{A,i}^\tau(t)\| \leq L_i^\tau \bar{a}_F, \quad (27)$$

where $\mathbf{u}_{A,i}^0$ is the corresponding zero-age AUV–AUV interaction and $L_i^\tau > 0$ is a local Lipschitz constant on the considered compact set.

Proof. Let $s = s_{ij}(t)$ be the timestamp of the newest accepted packet held by AUV i . The common mission-time coordinate makes s the same physical epoch in the self-history and in the transmitted neighbor state. Periodic broadcasting, bounded propagation delay, and the latest-timestamp acceptance rule give

$$0 \leq t - s \leq T_b + \bar{\tau} = \bar{a}_F.$$

Since $\mathbf{p}_{ij}^a = \mathbf{p}_i(s) - \mathbf{p}_j(s)$,

$$\mathbf{p}_{ij}^a - (\mathbf{p}_i(t) - \mathbf{p}_j(t)) = [\mathbf{p}_i(s) - \mathbf{p}_i(t)] - [\mathbf{p}_j(s) - \mathbf{p}_j(t)].$$

Each bracket is bounded by $\int_s^t \|\dot{\mathbf{p}}_\ell(\xi)\| d\xi \leq \bar{v}_A\bar{a}_F$, which proves (26). Because the repulsive branch in (19) is regularized by ϵ_r , the implemented interaction is locally Lipschitz on every compact operating subset. Therefore, the geometry error in (26) produces a command perturbation bounded by a local Lipschitz constant times $\bar{a}_F$. If $d_{ij}(t) \leq d_{\min} + \epsilon_s$, then (26) gives

$$d_{ij}^a(t) \leq d_{ij}(t) + 2\bar{v}_A\bar{a}_F \leq d_{\min} + 2\bar{v}_A\bar{a}_F + \epsilon_s = d_s.$$

Thus, the propagation-delay- and packet-holding-tightened repulsive branch is activated before the true physical distance can reach the safety boundary. $\square$

Lemma 4. *If $\rho_j \geq \rho_{\min}$ and $\|\hat{\mathbf{p}}_{0,i} - \mathbf{p}_0\| \leq \bar{e}_p < \rho_{\min}$, define $\hat{\theta}_{j|i}$ as the bearing of $\mathbf{p}_j - \hat{\mathbf{p}}_{0,i}$. Then, $|\hat{\theta}_{j|i} - \theta_j| \leq \bar{\theta}_e = \arcsin(\bar{e}_p/\rho_{\min})$. For the locally reconstructed state-only bearing vector, the corresponding bound is $|\hat{\theta}_{j|i}^F - \theta_j| \leq \bar{\theta}_F := \arcsin(\varepsilon_F/\rho_{\min})$. Under Assumption 5, the separation margin converts these pointwise bounds into a common cyclic order. Independently, if the true bearings satisfy $\max_k \Delta\theta_k < \pi$, then,*

$$\mathbf{p}_0 \in \text{int co}\{\mathbf{p}_1, \dots, \mathbf{p}_N\}.$$

If additionally $\rho_i \leq \rho_{\max}$ for all i , then the enclosing hull is compact and satisfies

$$\text{diam}(\text{co}\{\mathbf{p}_1, \dots, \mathbf{p}_N\}) \leq 2\rho_{\max}. \quad (28)$$

Proof. The true and locally estimated target-relative vectors of AUV j have endpoint separation at most $\bar{e}_p$ and the true vector length is at least $\rho_{\min}$. The largest possible angular deviation is therefore $\arcsin(\bar{e}_p/\rho_{\min})$. Replacing $\bar{e}_p$ by the local state-only reconstruction bound ε_F from Lemma 2 gives the bearing bound for $\hat{\theta}_{ji}^F$. Two adjacent true bearings remain separated by at least $2\bar{\theta}_F + \theta_{\text{sep}}$, so the perturbed order cannot reverse. Translate the coordinate frame so that $\mathbf{p}_0 = \mathbf{0}$. If $\mathbf{0}$ were not in the interior of the AUV convex hull, a separating line through $\mathbf{0}$ would exist such that all AUV positions lie in one closed half-plane. The bearings of all AUVs would then be contained in an interval of angular width at most π , leaving a complementary cyclic gap of width at least π . This contradicts $\max_k \Delta\theta_k < \pi$. Hence, $\mathbf{0}$ lies in the interior of the convex hull. Finally, for any two AUVs in the radial annulus, $\|\mathbf{p}_i - \mathbf{p}_j\| \leq \|\mathbf{p}_i - \mathbf{p}_0\| + \|\mathbf{p}_j - \mathbf{p}_0\| \leq 2\rho_{\max}$. Taking the supremum over all points in the convex hull gives (28). $\square$

Lemma 5 (Coupled bearing-gap contraction). *Under Assumptions 5 and 6, let*

$$\theta_c = \theta_d + \theta_\varepsilon, \quad r_k(t) = [\Delta\theta_k(t) - \theta_c]_+, \quad \mathbf{r} = [r_1, \dots, r_N]^T,$$

and define the coupled gap energy and its norm by

$$V_\theta(t) = \frac{1}{2} \mathbf{r}^T \mathbf{r}, \quad W_\theta(t) = \sqrt{2V_\theta(t)} = \|\mathbf{r}(t)\|.$$

Along the sampled state-only implementation (20) on the order-consistent interval $[t_0, t_1)$,

$$D^+ W_\theta(t) \leq -c_\theta W_\theta(t) + \bar{\xi}_\theta. \quad (29)$$

Consequently,

$$W_\theta(t) \leq e^{-c_\theta(t-t_0)} W_\theta(t_0) + \frac{\bar{\xi}_\theta}{c_\theta} (1 - e^{-c_\theta(t-t_0)}), \quad (30)$$

and

$$\max_k \Delta\theta_k(t) \leq \theta_c + W_\theta(t).$$

Proof. First consider an open order cell in which the true cyclic ordering is fixed. Let $\mathbf{H} \in \mathbb{R}^{N \times N}$ be the incidence matrix of the directed cycle whose k th edge points from ordered AUV (k) to ordered AUV ($k+1$). Thus, column k has $+1$ at its left boundary and -1 at its right boundary. The AUV at ordered position k is the left boundary of gap k and the right boundary of gap $k-1$. Therefore, its two active-gap commands combine as

$$\mathbf{u}_{C,(k)}^0 = k_C(r_k - r_{k-1})\mathbf{e}_{\theta,(k)}, \quad \mathbf{e}_{\theta,(k)} = \frac{\mathbf{R}_{\pi/2}\mathbf{q}_{(k)}}{\rho_{(k)}}.$$

Because a tangential velocity $v_{\theta,(k)}\mathbf{e}_{\theta,(k)}$ produces the bearing rate $v_{\theta,(k)}/\rho_{(k)}$, define

$$D_\rho = \text{diag}\left(\frac{k_C}{\rho_{(1)}}, \dots, \frac{k_C}{\rho_{(N)}}\right).$$

Since the unwrapped gap vector satisfies $\Delta = -H^T \theta$, the ideal coverage contribution to the coupled gap dynamics is

$$\dot{\Delta}_C = -H^T D_\rho H r. \quad (31)$$

Equation (31) retains every neighboring-gap action. In particular, no separate decrease is assigned to an individual maximum gap when the two actions acting on a common AUV cancel.

The tangential component of the ideal distance interaction and all nonideal implementation terms give an additive gap-rate perturbation $d_\theta(t)$. By the definitions preceding Assumption 6,

$$\dot{\Delta} \in -H^T D_\rho H r + d_\theta(t), \quad \|d_\theta(t)\| \leq \bar{\xi}_\theta.$$

Indeed, each noncoverage bearing-rate contribution is bounded by $(\bar{u}_{A,\theta} + \bar{\xi})/\rho_{\min}$, each cyclic gap rate is the difference of two such bearing rates, and the Euclidean norm of the resulting N -gap perturbation is no greater than $2\sqrt{N}(\bar{u}_{A,\theta} + \bar{\xi})/\rho_{\min}$.

The scalar map $x \mapsto \frac{1}{2}[x]_+^2$ is continuously differentiable. Hence, within the current order cell,

$$\begin{aligned} D^+ V_\theta &\leq -r^T H^T D_\rho H r + r^T d_\theta \\ &\leq -\frac{k_C}{\rho_{\max}} \|Hr\|^2 + \bar{\xi}_\theta \|r\|. \end{aligned} \quad (32)$$

At least one component of r is zero. Otherwise every gap would satisfy $\Delta\theta_k > \theta_c$, which would imply $2\pi = \sum_k \Delta\theta_k > N\theta_c = 2\pi + N\theta_e$, a contradiction. Let $r_j = 0$. For any i , summing the consecutive differences along a path of at most $N - 1$ cycle edges from j to i and applying the Cauchy–Schwarz inequality gives

$$r_i^2 \leq (N - 1) \|Hr\|^2.$$

After summing over i ,

$$\|Hr\|^2 \geq \lambda_N \|r\|^2, \quad \lambda_N = \frac{1}{N(N - 1)}.$$

Substitution into (32) yields

$$D^+ V_\theta \leq -c_\theta W_\theta^2 + \bar{\xi}_\theta W_\theta.$$

For $W_\theta > 0$, division by W_θ gives (29); at $W_\theta = 0$, the same inequality holds in the upper-Dini comparison sense.

Simultaneous maximum gaps are already represented by distinct components of r and require no separate maximum-gap argument. The separation condition in Assumption 5 excludes equal-bearing and order-change events on $[t_0, t_1)$, so the same coupled calculation applies throughout that interval. The scalar comparison lemma gives (30), and $r_k \leq \|r\|$ gives the stated maximum-gap bound. No claim is made here for a uniform contraction inequality across a local-order crossing. $\square$

Theorem 1 (Local practical fencing maintenance). *Suppose Assumptions 1–3, 5 and 6 hold on an order-consistent regular interval $[t_0, t_1)$. Under the state-only observer (14) and the timestamp-aligned controller (16)–(23), no collision or radial boundary $d_{ij} = d_{\min}$, $\rho_i = \rho_{\min}$, or $\rho_i = \rho_{\max}$*

can be the first exit from the local regular regime while the observer, packet-freshness, speed, and order-separation premises remain valid. Moreover, for $t \in [t_0, t_1)$,

$$W_\theta(t) \leq e^{-c_\theta(t-t_0)} W_\theta(t_0) + \frac{\bar{\xi}_\theta}{c_\theta} (1 - e^{-c_\theta(t-t_0)}). \quad (33)$$

For any $0 < \varepsilon_\theta^* < \pi - \theta_c - \bar{\xi}_\theta/c_\theta$, define $a_\theta := \pi - \theta_c - \bar{\xi}_\theta/c_\theta - \varepsilon_\theta^*$. If $t_0 + T_\theta < t_1$, where

$$T_\theta = \begin{cases} 0, & W_\theta(t_0) \leq a_\theta, \\ c_\theta^{-1} \ln\left(\frac{W_\theta(t_0)}{a_\theta}\right), & W_\theta(t_0) > a_\theta, \end{cases}$$

then $\max_k \Delta\theta_k(t) \leq \pi - \varepsilon_\theta^*$ and $D_h(t) = 0$ for every $t \in [t_0 + T_\theta, t_1)$. The enclosing hull has diameter at most $2\rho_{\max}$, and every required state packet satisfies the physical arrival test. This theorem establishes local entry and maintenance only while the order-consistent regular interval exists; it does not establish global entry from an arbitrary safe connected non-enclosing deployment or persistence through a bearing-order crossing.

Proof. Consider a first possible time at which a collision or radial boundary is reached, while the non-geometric premises in Assumption 5 remain valid. Before that time, the trajectory is in the interior of the corresponding safety and radial constraints, so the local bounds and Lipschitz constants on $\mathcal{K}_R$ apply. Define $q_i = p_i - p_0$ and $\tilde{q}_i = p_i - \hat{p}_{0,i}$. From (5), (21), (15), Lemma 2 and Lemma 3,

$$\dot{q}_i = u_i^0 + \xi_i, \quad \|\xi_i\| \leq L_T \bar{e}_p + \bar{e}_v + \max_i L_i^r \bar{a}_F + \bar{\delta}_C + \max_i \bar{w}_i + \bar{\delta}^s = \bar{\xi}.$$

The target-velocity term therefore cancels the target drift up to the bounded observer residual.

On $d_{ij} = d_{\min}$, Lemma 3 activates the delay-tightened repulsion before the physical boundary and gives

$$D^+ d_{ij} = n_{ij}^T (u_i^0 - u_j^0) + n_{ij}^T (\xi_i - \xi_j) \geq \gamma_s - 2\bar{\xi} > 0.$$

Thus the safety component cannot be crossed. Similarly, at $\rho_i = \rho_{\max}$ and $\rho_i = \rho_{\min}$, respectively,

$$\dot{\rho}_i \leq -\gamma_\rho^+ + \bar{\xi} < 0, \quad \dot{\rho}_i \geq \gamma_\rho^- - \bar{\xi} > 0.$$

The radial component of the regular set is therefore inward pointing at both boundaries.

On $[t_0, t_1)$, Assumption 5 fixes the true cyclic order and Lemma 2 makes the delayed local coverage field a bounded perturbation of the ideal ordered field. Lemma 5 therefore yields (33). Solving the scalar comparison inequality for $W_\theta(t) \leq a_\theta + \bar{\xi}_\theta/c_\theta$ gives the stated T_θ and hence, $\max_k \Delta\theta_k \leq \pi - \varepsilon_\theta^*$.

Lemma 4 converts this strict true bearing-gap margin into $p_0 \in \text{int co}\{p_1, \dots, p_N\}$ and $D_h = 0$. Radial confinement gives the hull-diameter bound. Finally, for a packet sent at t_ℓ and received at $t_{ij,\ell}^r$,

$$\|p_j(t_\ell) - p_i(t_{ij,\ell}^r)\| \leq \bar{v}_A \bar{\tau} + 2\rho_{\max} < R_c,$$

which proves the physical arrival condition and completes the local result. $\square$

Theorem 1 is deliberately narrower than the original global claim. It proves what the state-only packet model can support without assuming a common transmitted target center: bounded delayed perturbations and fencing maintenance in a verified order-regular

regime. The non-enclosing deployment-to-fence transition remains a self-organization behavior demonstrated by the simulations, not a global theorem claim.

4. Simulation Results

Numerical simulations were conducted to evaluate the proposed timestamp-aligned minimal state-only observer-assisted attraction–repulsion controller with label-free bearing coverage, denoted by MIN-AR-COV-TA. They have two distinct purposes: to demonstrate self-organized entry from a non-enclosing safe deployment, which is outside the local theorem, and to audit the conditions and maintenance behavior after a regular fencing interval has formed. The Simulation parameters can be checked in Table 2.

Table 2. Simulation parameters.

Parameter	Symbol	Value
Number of AUVs	N	6
Target–AUV zero-force radius	ρ_d	6 m
Radial bounds	$[\rho_{\min}, \rho_{\max}]$	[4, 8] m
Minimum safety distance	$d_{\min}$	1.5 m
Sensing radius	R_s	8.5 m
Communication radius	R_c	20 m
Acoustic delay	τ_{ij}	0.2–1.2 s
State broadcast period	T_b	0.20 s
Maximum retained-packet age	$\bar{a}_F$	1.40 s
Maximum speed	$u_{\max}$	1.00 m/s
Nominal radial gain	k_T	0.50 s^{-1}
Boundary-recovery gain	k_B	1.25 s^{-1}
Boundary-recovery radii	$[\rho_B^-, \rho_B^+]$	[4.5, 7.5] m
AUV spacing gain	k_A	0.035 s^{-1}
AUV repulsion gain	k_R	$0.35 \text{ m}^2/\text{s}$
Bearing-coverage gain	k_C	1.20 m/s
Observer gains	(l_p, l_v)	$(1.40 \text{ s}^{-1}, 0.65 \text{ s}^{-2})$
State-mediated velocity gain	λ_o	0.12 s^{-1}
Repulsion activation distance	μ	5.0 m
Delay safety margin	ϵ_s	0.25 m
Repulsion regularizer	ϵ_r	10^{-4} m
Radial regularizer	ϵ_ρ	0.50 m
Coverage dead angle	θ_e	2°
Observer speed limit	$\hat{v}_{\max}$	0.90 m/s

State packets were broadcast every $T_b = 0.20 \text{ s}$. For the coverage term, each AUV sorted only its locally received state-only neighbor packets together with its current self state about its own target estimate, applied the identifier rule only for exact bearing ties, and used its current local tangent for immediate command execution. Thus the numerical implementation followed the distributed construction in (20) and did not use a common target estimate, transmitted target bearing, or transmitted angular order. All observer, distance-interaction, and coverage calculations discard a retained packet once its age exceeds $\bar{a}_F$; no stale packet is used by any control component. Unlike a first-order point-mass simulation, each AUV was propagated by the five-degree-of-freedom model (3)–(4). The fencing controller only generated the desired horizontal velocity command, and a low-level controller converted this command into generalized forces and moments. Six AUVs were required to form a compact dynamic fence around a moving target in the

horizontal plane. The target moved from the left-middle region to the right side along a curved trajectory whose adjacent bends are wider than the compact fence scale,

$$\mathbf{p}_0(t) = \begin{bmatrix} -35 + 0.075t \\ 4 + 13 \sin(0.012t) \end{bmatrix} \text{ m.}$$

Reproducibility Settings

The complete numerical implementation is supplied as Supplementary File S1. The code was executed in MATLAB R2015b with the deterministic seed `rng(11)`. At $t = 0$, the target position is $\mathbf{p}_0(0) = [-35, 4]^T$ m. The initial AUV poses used in all three scenarios are listed in Table 3.

Table 3. Initial AUV and observer states used in the simulations.

AUV i	$x_i(0)$ (m)	$y_i(0)$ (m)	$z_i(0)$ (m)	$\theta_i(0)$ (rad)	$\psi_i(0)$ (rad)
1	−37.0188	11.5342	0	0	1.8326
2	−38.9853	8.7495	0	0	2.2689
3	−41.5254	7.0429	0	0	2.7053
4	−40.4164	3.0449	0	0	−2.9671
5	−40.6687	−0.7566	0	0	−2.4435
6	−37.2573	−2.2020	0	0	−1.9199

All body-fixed velocities are initialized as $\mathbf{v}_i(0) = \mathbf{0}$. Every local observer is initialized by $\hat{\mathbf{p}}_{0,i}(0) = \mathbf{p}_0(0)$ and $\hat{\mathbf{v}}_{0,i}(0) = \mathbf{0}$. The analysis time origin is taken after the buffer-fill phase described in Section 2.2; numerically, each ordered receive buffer is initialized with the corresponding sender state at the initial sample, and the prehistory is held at the initial state.

Equal weights are used in the delayed-neighbor velocity average, $a_{ij} = 1/|\mathcal{N}_i^\tau|$ for $j \in \mathcal{N}_i^\tau$. For a packet broadcast at t_ℓ , the ordered-link delay is

$$\tau_{ij,\ell} = \tau_{\min} + \frac{\tau_{\max} - \tau_{\min}}{2} [1 + \sin(0.005t_\ell + 0.61i + 1.27j)],$$

where $\tau_{\min} = 0.2$ s and $\tau_{\max} = 1.2$ s. Packets are broadcast every $T_b = 0.2$ s. A receiver processes a packet at the first controller sample not earlier than its computed arrival time, accepts it only when the physical range test in (1) is satisfied and its timestamp is newer than the retained timestamp, and otherwise holds the latest accepted packet. The resulting maximum retained-packet age is $\bar{a}_F = T_b + \tau_{\max} = 1.4$ s. The fleet-level speed bound used for delay tightening is $\bar{v}_A = 1.00$ m/s, giving $d_s = d_{\min} + 2\bar{v}_A\bar{a}_F + \epsilon_s = 4.55$ m. The raw simulated horizontal-speed peaks are 0.671, 0.726, and 0.731 m/s in the calm-water, ocean-current, and evasive-target cases, respectively, and hence remain below this bound.

The deterministic target-position measurement noise is

$$\mathbf{n}_{0,i}(t) = 0.15 \begin{bmatrix} \sin(0.063t + 0.31i) \\ \cos(0.041t + 0.47i) \end{bmatrix} \text{ m,}$$

so each component has amplitude 0.15 m and $\|\mathbf{n}_{0,i}\| \leq 0.15\sqrt{2}$ m. No additional stochastic process disturbance is injected. The regularization and coverage parameters omitted from the original description are $\epsilon_s = 0.25$ m, $\epsilon_r = 10^{-4}$ m, $\epsilon_\rho = 0.50$ m, and $\theta_\epsilon = 2^\circ$.

For all AUVs, the five-degree-of-freedom plant uses

$$\begin{aligned} M_i &= \text{diag}(35, 45, 55, 8, 10), \\ C_i(v_i) &= \mathbf{0}, \\ D_i(v_{r,i})v_{r,i} &= \mathbf{d}_l \odot v_{r,i} + \mathbf{d}_q \odot |v_{r,i}| \odot v_{r,i}, \\ \mathbf{d}_l &= [18, 24, 30, 8, 8]^T, \quad \mathbf{d}_q = [6, 8, 10, 3, 3]^T, \\ g_i(\eta_i) &= [0, 0, 20z_i, 12\theta_i, 0]^T, \end{aligned}$$

where $\odot$ denotes componentwise multiplication and $v_{r,i}$ is the water-relative body velocity. Setting $C_i = \mathbf{0}$ is the low-speed diagonal simulation parameterization used here, not a change to the general model in (4).

Let $v_{d,i} = \min\{\|u_i\|, u_{\max}\}$, $\psi_{d,i} = \text{atan2}(u_{i,y}, u_{i,x})$, $e_{\psi,i} = \text{wrap}(\psi_{d,i} - \psi_i)$, and $r_{d,i} = \text{sat}_{0.7}(e_{\psi,i}/\Delta t)$. The low-level generalized-force command is

$$\tau_i = \begin{bmatrix} 45(v_{d,i} - u_i^b) \\ -35v_i^b \\ -35z_i - 25w_i^b \\ -25\theta_i - 15\omega_{\theta,i} \\ 30e_{\psi,i} + 18(r_{d,i} - \omega_{\psi,i}) \end{bmatrix},$$

with componentwise force and moment limits $|\tau_i| \leq [60, 45, 80, 25, 25]^T$. If $\|u_i\| \leq 10^{-6}$ m/s, the current yaw is retained as $\psi_{d,i}$.

The five-degree-of-freedom plant is integrated using the explicit midpoint second-order Runge–Kutta method with $\Delta t = 0.2$ s. Observer states are updated by forward Euler at the same sampling period, and the guidance command and generalized forces are held over each integration step. Linear interpolation of the stored plant history is used only to evaluate the receiver position at a packet's continuous arrival time. The numerical convex-hull tolerance is 0.20 m, and post-settling metrics are evaluated over $t \geq 0.45T = 270$ s. All trajectory figures use the raw simulated state histories; no temporal smoothing or curve filtering is applied. These settings, the ocean-current field, and the evasive-target parameters are all declared in Supplementary File S1.

The initial AUV positions were placed in a nonuniform one-sided configuration relative to the target. Thus the target was not inside the initial AUV convex hull, while the initial pairwise distances satisfied $d_{ij}(0) > d_{\min}$ and the range graph was connected. The target–AUV zero-force radius was $\rho_d = 6$ m, with local-maintenance bounds $\rho_{\min} = 4$ m and $\rho_{\max} = 8$ m. The self-organization transient is allowed to leave this annulus; it is therefore not presented as a direct validation of the local radial-invariance theorem. The reported $T_{\text{reg}}^{\text{num}}$ identifies the later time after which the observed trajectory remains in the numerical regular regime. The nominal AUV–AUV zero-force distance was $d_A = 2\rho_d \sin(\pi/N)$. The minimum safety distance was $d_{\min} = 1.5$ m, the sensing radius was $R_s = 8.5$ m, and the acoustic communication radius was $R_c = 20$ m. The nonuniform acoustic delay varied in $[0.2, 1.2]$ s for different ordered links. Packet loss was not modeled. The maximum AUV speed was $u_{\max} = 1.00$ m/s, the observer gains were $l_p = 1.40 \text{ s}^{-1}$, $l_v = 0.65 \text{ s}^{-2}$, and $\lambda_o = 0.12 \text{ s}^{-1}$, and the sampling period was $\Delta t = 0.2$ s. The reduced finite sensing radius activates the state-only prediction branch during the early deployment transient. The longest individual no-sensing intervals are 25.6 s, 33.0 s, and 16.6 s in the calm-water, current, and evasive cases, respectively. The corresponding full-trajectory maximum local position-observer errors are reported in Tables 4 and 5. The later $T_{\text{reg}}^{\text{num}}$ values are intentionally reported separately because

the analytical local-maintenance conditions are assessed only after the observer error, sensing, radius, and bearing-separation proxies are simultaneously recovered.

Table 4. Dynamic fencing performance of MIN-AR-COV-TA under nonuniform acoustic delays, with and without the simulated ocean current. T_{tol} is the persistent numerical-tolerance entry time, T_{exact} requires persistent $D_h = 0$ and $\max_k \Delta\theta_k < \pi$, $T_{\text{reg}}^{\text{num}}$ is the observed entry time of the stated numerical regular proxy, and $T_{\text{ns}}^{\text{max}}$ is the longest contiguous local no-sensing interval.

Scenario	T_{tol} (s)	T_{exact} (s)	$T_{\text{reg}}^{\text{num}}$ (s)	$D_{h,\text{max}}$ (m)	$\Delta\theta_{\text{max}}$ (deg)	E_ρ (m)	E_c (m)	$d_{\text{min}}^{\text{obs}}$ (m)	$e_{p,\text{max}}$ (m)	$T_{\text{ns}}^{\text{max}}$ (s)	$\max \ v_c\ $ (m/s)
Calm water	5.6	6.0	28.8	0.00	71.50	0.66	0.40	3.06	3.55	25.6	0.000
Ocean current	6.0	6.6	36.2	0.00	72.33	0.66	0.48	3.06	3.84	33.0	0.179

Table 5. Evasive-target stress-test results under ocean current and nonuniform acoustic delays.

Scenario	Turns	$\max \ v_0\ $ (m/s)	$\max \ \dot{v}_0\ $ (m/s ²)	T_{tol} (s)	T_{exact} (s)	$T_{\text{reg}}^{\text{num}}$ (s)	$D_{h,\text{max}}$ (m)	$\Delta\theta_{\text{max}}$ (deg)	E_ρ (m)	E_c (m)	$d_{\text{min}}^{\text{obs}}$ (m)
Evasive target + current	4	0.273	0.047	6.0	6.6	408.6	0.00	78.64	0.85	0.93	3.06

To test environmental robustness beyond the nominal no-current analysis, an additional five-degree-of-freedom simulation was performed in a spatially and temporally varying horizontal ocean current. The current field, expressed in the inertial frame, was

$$v_c(x, y, t) = \begin{bmatrix} -0.12 + 0.05 \sin(0.015t + 0.035y) \\ 0.06 \cos(0.012t + 0.035x) \end{bmatrix} \text{ m/s,}$$

where the mean stream opposes the principal positive- x target motion and $\|v_c\| \leq 0.181$ m/s. Let $J_{p,i}$ denote the 3×3 translational block of J_i . In the plant integration, the current was converted to the body frame as $v_{c,i} = [J_{p,i}^{-1}[v_c^T, 0]^T; 0; 0]$ and the hydrodynamic damping was evaluated using the water-relative velocity $v_{r,i} = v_i - v_{c,i}$ [34]. The fencing controller, observer, acoustic-delay realization, and low-level controller were otherwise unchanged. In particular, no current measurement or feedforward current compensation was supplied to any AUV.

The evaluation metrics distinguish numerical self-organization from the theorem's local maintenance scope. The radial error E_ρ evaluates practical radius regulation, the center-lag error E_c evaluates fleet tracking, the observed minimum distance $d_{\text{min}}^{\text{obs}}$ evaluates collision safety, the post-settling convex-hull error $D_{h,\text{max}}$ evaluates containment, and the post-settling maximum bearing gap $\Delta\theta_{\text{max}}$ evaluates label-free angular coverage. T_{tol} is the first time from which D_h remains below the numerical tolerance, whereas T_{exact} additionally requires exact numerical hull containment and a true maximum bearing gap below π . The numerical regular time $T_{\text{reg}}^{\text{num}}$ is reported separately and is not identified with the analytical t_0 . These metrics are defined as

$$\begin{aligned}
E_\rho &= \sqrt{\frac{1}{N(T - T_s)} \int_{T_s}^T \sum_{i=1}^N (\rho_i - \rho_d)^2 dt}, \\
E_c &= \sqrt{\frac{1}{T - T_s} \int_{T_s}^T \left\| \frac{1}{N} \sum_{i=1}^N \mathbf{p}_i - \mathbf{p}_0 \right\|^2 dt}, \\
D_{h,\max} &= \max_{t \in [T_s, T]} \text{dist}(\mathbf{p}_0(t), \text{co}\{\mathbf{p}_1(t), \dots, \mathbf{p}_N(t)\}), \\
\Delta\theta_{\max} &= \max_{t \in [T_s, T]} \max_k \Delta\theta_k(t), \\
d_{\min}^{\text{obs}} &= \min_{t \in [0, T]} \min_{i \neq j} \|\mathbf{p}_i(t) - \mathbf{p}_j(t)\|, \\
T_{\text{tol}} &= \inf \left\{ t \geq 0 : \max_{s \in [t, T]} D_h(s) \leq \varepsilon_h^{\text{num}} \right\}, \\
T_{\text{exact}} &= \inf \left\{ t \geq 0 : \max_{s \in [t, T]} D_h(s) \leq 10^{-10}, \max_{s \in [t, T]} \max_k \Delta\theta_k(s) < \pi \right\}.
\end{aligned}$$

where $T_s = 0.45T$ is the post-settling evaluation start used for the reported angular-gap, convex-hull, radial, and center-lag metrics, and $\varepsilon_h^{\text{num}} = 0.20$ m is used only for the numerical tolerance metric. In the script, $T_{\text{reg}}^{\text{num}}$ is the first time after which the true radii remain in $[4, 8]$ m, $D_h = 0$, $\max_k \Delta\theta_k < \pi$, the minimum bearing gap is at least 5° , the maximum local position-observer error is at most 0.50 m, and all AUVs are sensing. It is a transparent numerical proxy for, not a proof of, the full analytical regular-set conditions. The 5-DOF consistency checks also report the maximum depth error, maximum pitch angle, and maximum normalized generalized-force saturation ratio, denoted by $\|\tau\|_\infty / \tau_{\max}$.

Table 4 compares calm-water execution with the ocean-current scenario using the same MIN-AR-COV-TA gains and the same nonuniform delayed acoustic links. Persistent tolerance entry occurs at $T_{\text{tol}} = 5.6$ s in calm water and 6.0 s under current; the stricter numerical condition $D_h = 0$ with $\max_k \Delta\theta_k < \pi$ persists from $T_{\text{exact}} = 6.0$ s and 6.6 s, respectively. The later numerical regular proxies are 28.8 s and 36.2 s. In both cases $D_{h,\max} = 0$ over the post-settling interval and the maximum bearing gaps are 71.50° and 72.33° , below 180° . The reduced sensing range produces maximum local no-sensing intervals of 25.6 s and 33.0 s, during which the state-only velocity-mediated prediction branch is active. The full-trajectory maximum position-observer errors are 3.55 m and 3.84 m, whereas the later regular-time proxy requires the error to have fallen below 0.50 m. The post-settling radial errors are 0.66 m in both cases, the center-lag errors are 0.40 m and 0.48 m, and the minimum observed inter-AUV distance is 3.06 m. These results demonstrate compact numerical containment with state-only acoustic packets; they are not presented as proof of global entry or of the full analytical separation margin.

Figure 1 visualizes the complete raw motion of the target and AUVs. The AUV trajectories are shown with a uniform color to keep the dense trajectory field readable, and the representative fence snapshots are drawn only by boundary lines connecting the AUVs. The snapshots are selected after the numerical regular time in the ocean-current case and show a compact fence rather than a large filled hull. Figure 2 shows that the post-settling largest true bearing gap remains below 180° , which is the geometric enclosure condition. Figure 3 reports post-settling radial regulation and fleet-center tracking; the curves are numerical evidence of practical performance and are not used to infer global radial invariance during the preceding entry transient. Figure 4 records observer performance together with the sensing count. With $R_s = 8.5$ m, the count drops below six during the early transient and later returns to six after the compact fence forms; hence the state-only

observer branch is exercised without transmitting any target estimate. The panel also makes clear that the local theorem is relevant only after the separate regular-time proxy is recovered. Figure 5 verifies that the observed distance remains above $d_{\min}$ throughout the simulations. Figure 6 confirms that the horizontal fencing task was executed through the five-degree-of-freedom AUV dynamics rather than by direct position integration. Since the task is horizontal, depth and pitch remain at their commanded equilibrium values. The maximum normalized generalized-force ratio reaches 1.00, showing that the compact maneuver consumes the available low-level control authority. The results should therefore be interpreted as practical closed-loop numerical performance under actuator saturation, not as ideal kinematic trajectories.

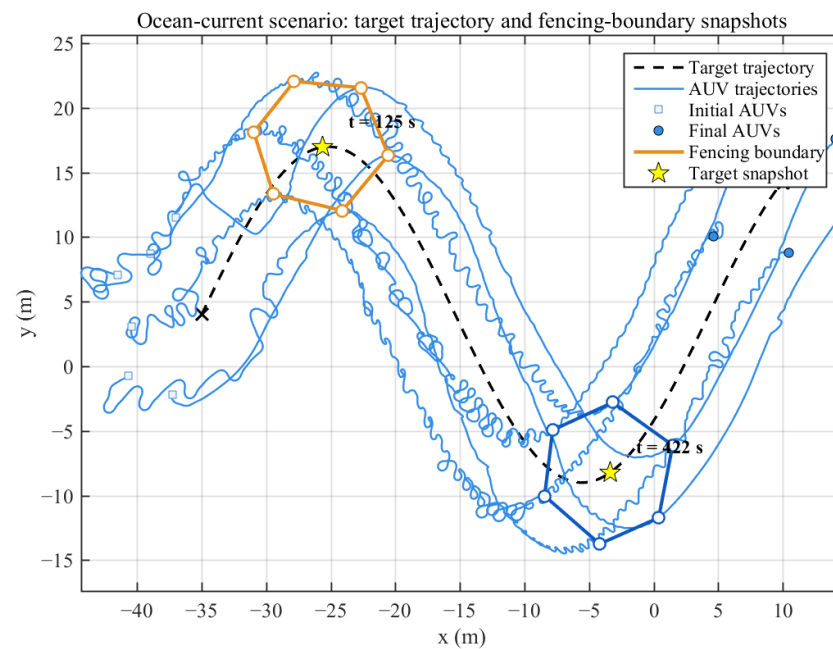


Figure 1. System trajectory of the proposed controller with uniform-color AUV trajectories and unfilled fencing-boundary snapshots.

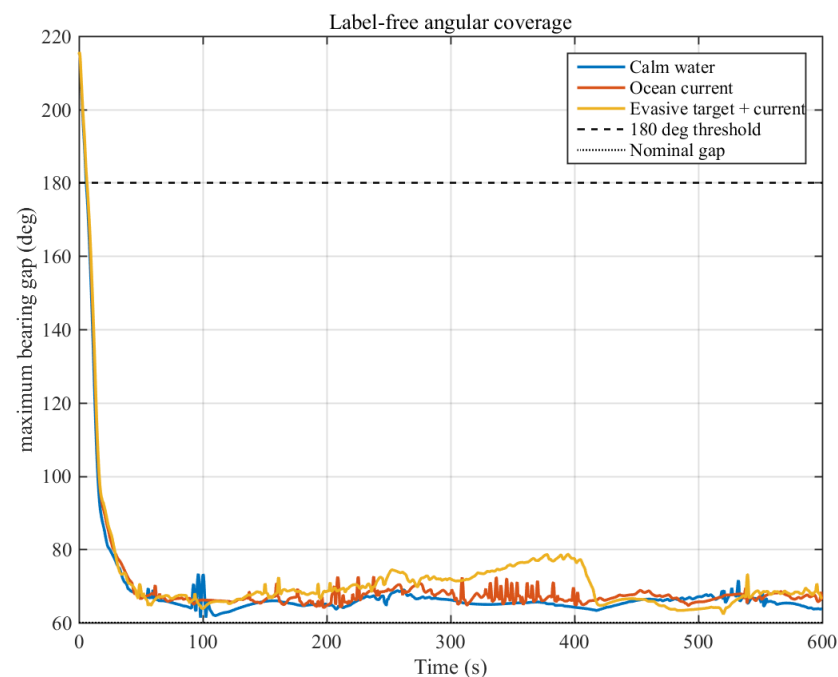


Figure 2. Maximum target-centered bearing gap under label-free bearing-coverage repulsion.

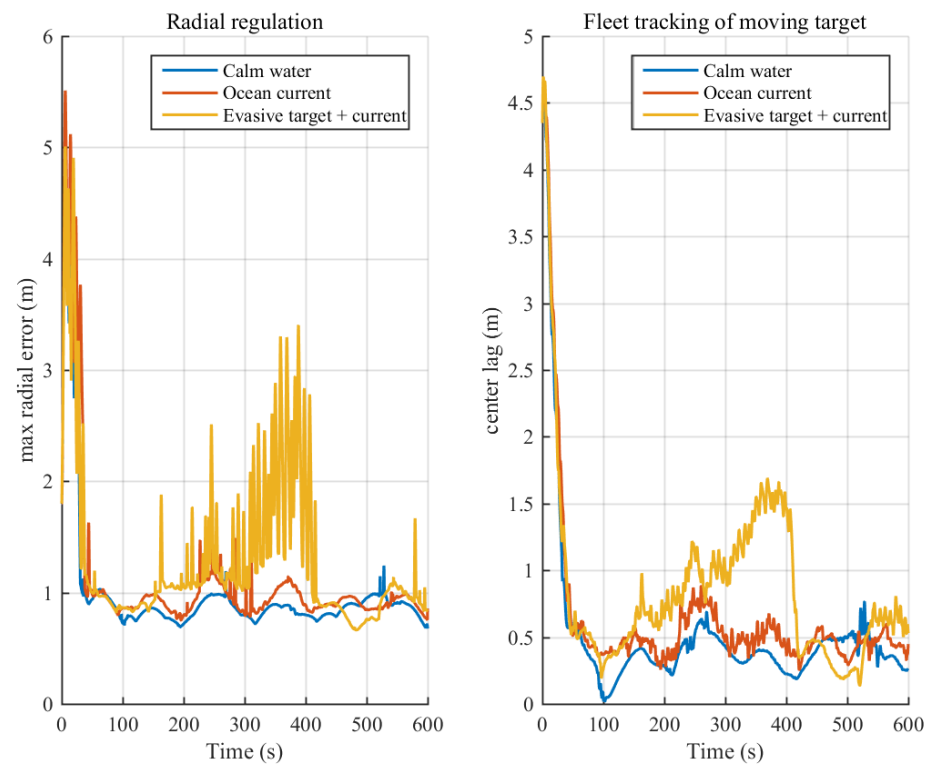


Figure 3. Radial regulation and fleet-center tracking errors.

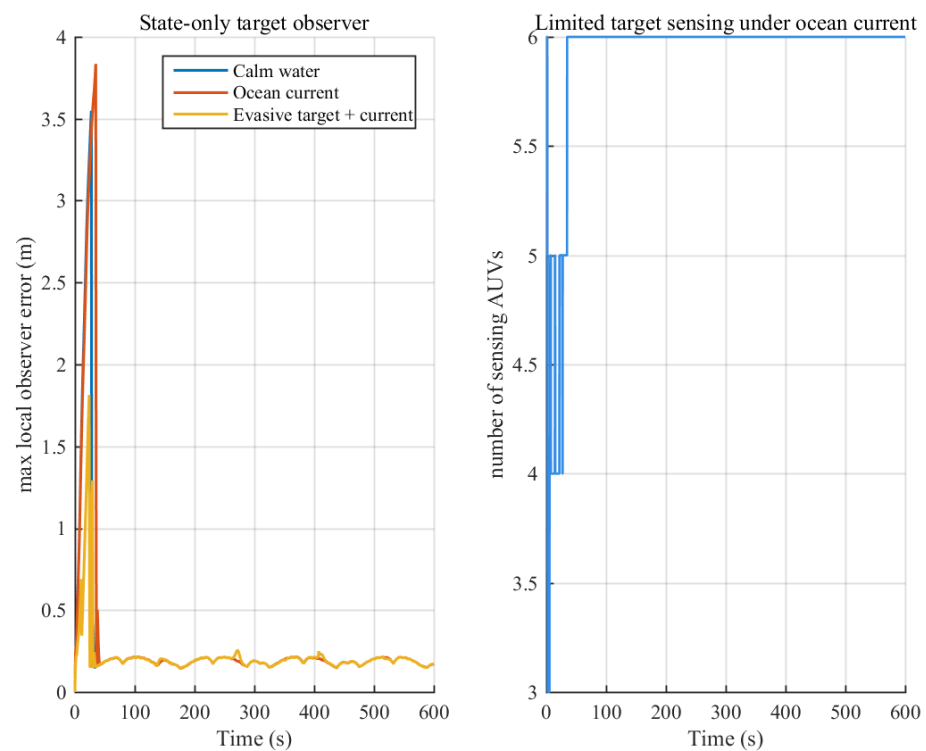


Figure 4. State-only observer performance and number of directly sensing AUVs.

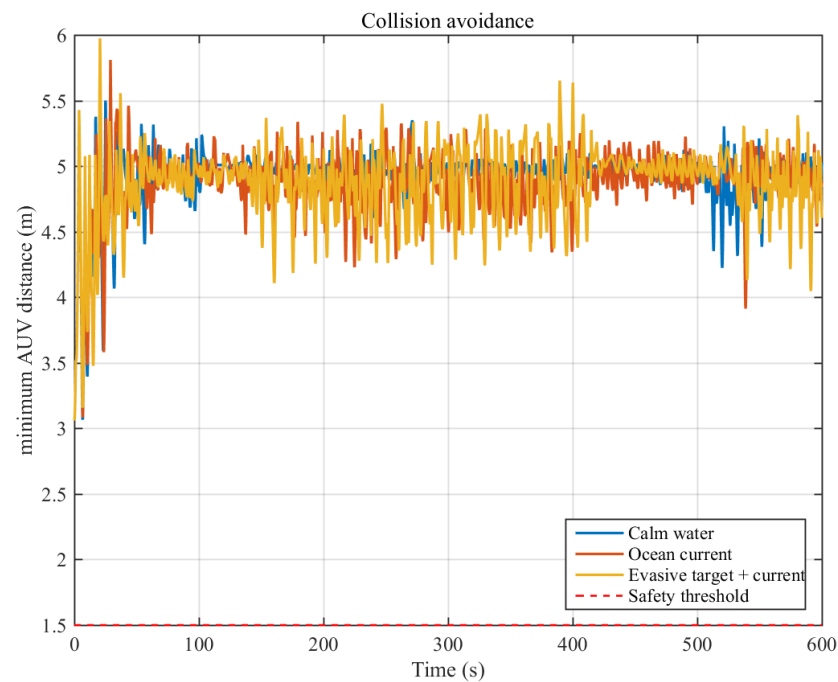


Figure 5. Minimum inter-AUV distance during the mission.

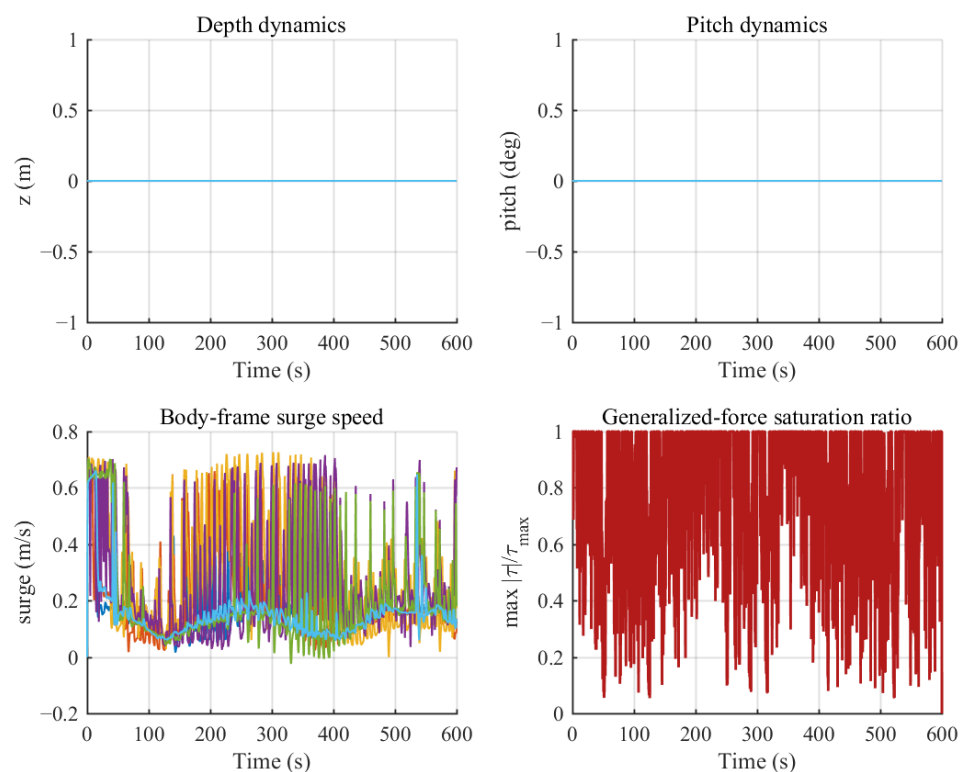


Figure 6. Representative five-degree-of-freedom AUV states and generalized-force saturation ratio.

Figure 7 visualizes the added robustness test. The two unfilled fencing boundaries in Figure 7a retain the target inside a compact AUV convex hull while the current magnitude varies up to 0.180 m/s. Figure 7c shows the expected increase in fleet-center lag relative to calm water, whereas Figure 7d confirms that the safety margin is preserved throughout the mission. This numerical test evaluates the robustness of the existing closed-loop implementation; it does not add an ocean-current compensation term to the proposed fencing law or extend the nominal stability theorem to arbitrary currents.

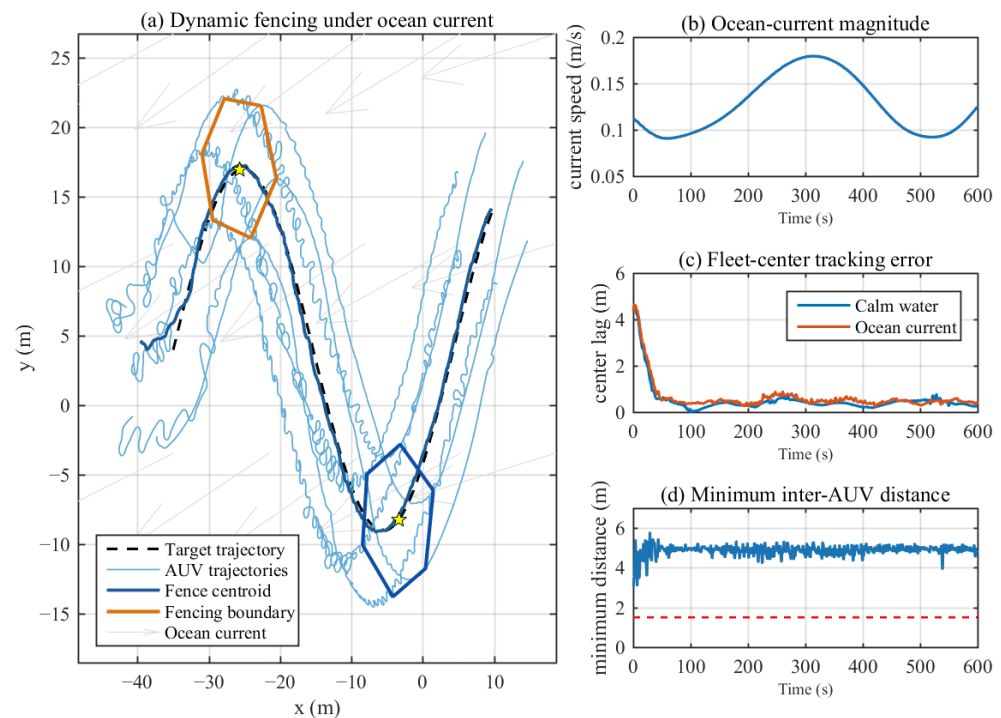


Figure 7. Ocean-current robustness test of the proposed controller. (a) The spatially and temporally varying current field, target trajectory, all AUV trajectories, fence-centroid trajectory, and two compact fencing-boundary snapshots. (b) Current magnitude sampled at the fleet centroid. (c) Fleet-center tracking error for the calm-water and current cases. (d) Minimum inter-AUV distance under the current. The controller receives no current measurement or feedforward compensation.

A further stress test was conducted with an evasive target under the same ocean-current field, nonuniform acoustic delays, five-degree-of-freedom AUV model, controller gains, and actuator limits. No gain was retuned for this case. To create sharp but physically admissible maneuvers, the target velocity was prescribed by four smooth finite-time changes

$$v_0(t) = v_b + \sum_{k=1}^4 \Delta v_k S_k(t), \quad S_k(t) = \frac{1}{2} \left[1 + \tanh\left(\frac{t - t_k}{T_e}\right) \right],$$

with $p_0(t) = p_0(0) + \int_0^t v_0(s) ds$, $p_0(0) = [-35, 4]^T$ m, $v_b = [0.09, 0.02]^T$ m/s, $T_e = 4$ s, and

$$t_{\text{turn}} = [140, 270, 410, 520]^T \text{ s},$$

$$[\Delta v_1, \Delta v_2, \Delta v_3, \Delta v_4]^T = \begin{bmatrix} -0.03 & 0.18 \\ 0.18 & -0.33 \\ -0.31 & 0.10 \\ 0.20 & 0.11 \end{bmatrix} \text{ m/s}.$$

This construction produces rapid heading changes and acceleration pulses while retaining continuous velocity and bounded acceleration, as required by Assumption 1. The resulting limits were $\max_t \|v_0(t)\| = 0.273$ m/s and $\max_t \|\dot{v}_0(t)\| = 0.047$ m/s².

Figure 8a shows the raw trajectories during all four heading changes. The acceleration pulses in Figure 8b generate visible tracking transients, while the stricter numerical containment condition persists from $T_{\text{exact}} = 6.6$ s. The post-settling maximum bearing gap is $78.64^\circ < 180^\circ$, the post-settling center-lag RMS value is 0.93 m, and the global minimum inter-AUV distance is 3.06 m, exceeding $d_{\min} = 1.5$ m. The largest local no-sensing interval is 16.6 s and the full-trajectory maximum local position-observer error is 1.81 m. The nu-

merical regular proxy is not recovered until $T_{\text{reg}}^{\text{num}} = 408.6$ s, which makes explicit that the local theorem is not invoked throughout the early transient or every evasive turn. Thus, this test provides numerical evidence of persistent containment and collision avoidance for the tested bounded maneuvers, not a global stability guarantee under arbitrary target accelerations.

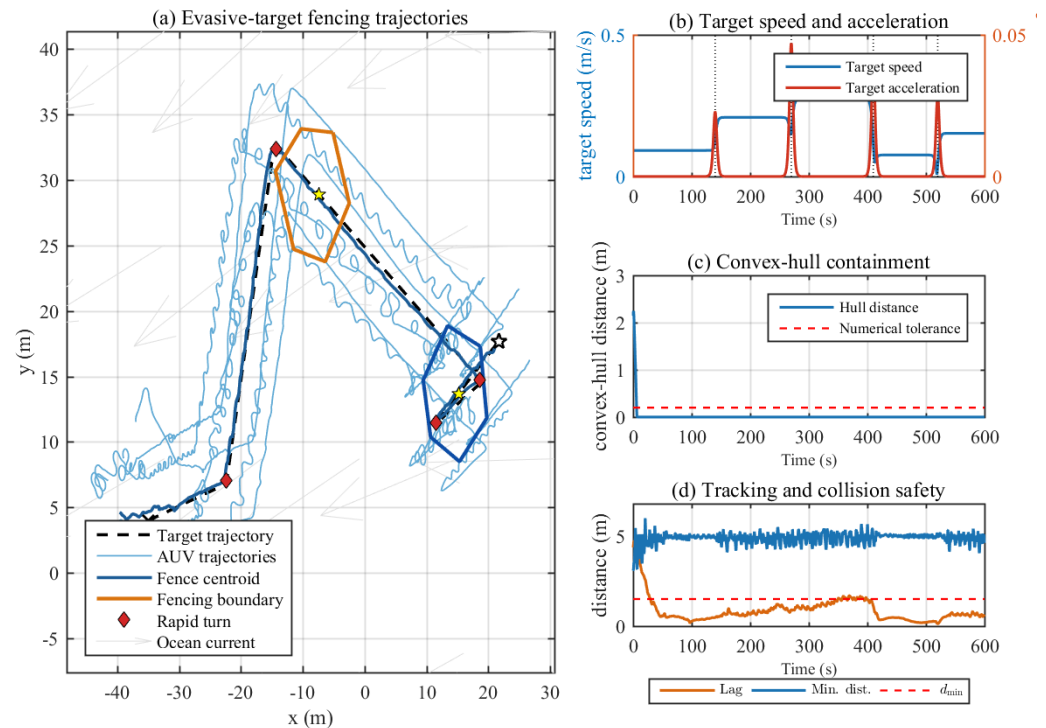


Figure 8. Evasive-target stress test under the spatially and temporally varying ocean current. (a) Target, all AUV, and fence-centroid trajectories, four rapid-turn instants, and two compact fencing-boundary snapshots. (b) Target speed and acceleration; the vertical lines mark the commanded rapid turns. (c) Convex-hull distance and numerical entry tolerance. (d) Fleet-center lag, minimum inter-AUV distance, and safety threshold.

Overall, the simulations demonstrate self-organized numerical entry, persistent convex-hull containment, post-settling bearing coverage, bounded tracking residuals, collision avoidance, and five-degree-of-freedom execution under state-only delayed acoustic communication. They also identify the later numerical regular intervals on which the local maintenance theorem is relevant. The current and evasive-target cases are empirical robustness evidence for the tested disturbances and maneuvers; they do not enlarge the theorem to global entry, long sensing outages, packet loss, arbitrary currents, or arbitrary bearing-order crossings.

5. Conclusions

This paper formulated and studied a self-organized, label-free dynamic fencing control problem for multi-AUV systems under limited target sensing and bounded nonuniform acoustic communication delays. A state-only target-motion observer was introduced so that AUVs can perform target-centered computation without transmitting target estimates, target measurements, or fixed reference points. The central delay-handling idea was timestamp alignment: when an AUV receives a delayed neighbor packet, it reads its own historical state at the same timestamp and computes pairwise AUV–AUV geometry from time-consistent data. Based on this model, a direct desired-velocity controller was developed using target–AUV radial attraction–repulsion, AUV–AUV distance attraction–

repulsion, target-motion observer compensation, delay-tightened proximity repulsion, and label-free bearing-coverage repulsion.

The closed-loop analysis showed that timestamp alignment converts nonuniform acoustic delay into a bounded geometric perturbation, while the bearing-coverage term gives a direct mechanism for reducing oversized target-centered angular gaps. Under explicit observer, delay, gain, and order-separation conditions, the analysis proves local collision avoidance, radial confinement, angular-gap contraction, convex-hull maintenance, packet-range preservation, and practical target tracking on a regular fencing interval. It does not claim global self-organization from arbitrary safe connected initial conditions. Numerical simulations from a compact non-enclosing initial configuration and using five-degree-of-freedom AUV dynamics showed that MIN-AR-COV-TA enters and maintains convex-hull fencing around the tested moving targets using state-only acoustic packets. An additional spatially and temporally varying ocean-current test, performed without current feedforward compensation, retained containment and collision avoidance while producing bounded tracking degradation. A second stress test with four rapid finite-acceleration target turns retained exact convex-hull enclosure after settling and preserved the collision margin without gain retuning. Future work will extend the framework to packet dropouts, stronger state-mediated observability analysis, and experimental validation with acoustic modems.

Supplementary Materials: The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/jmse14151440/s1>. Supplementary File S1: simulation source code `simulate_minimal_observer_fencing.m`, including the complete parameters and routines used to generate all reported simulation figures and performance metrics.

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Data Availability Statement: No experimental or field data were generated or analyzed in this study. The complete simulation source code, initial conditions, model and controller parameters, communication-delay and measurement-noise laws, disturbance settings, and numerical integration routines used to produce the reported results are provided as Supplementary File S1. The figures and performance metrics can be reproduced by running `simulate_minimal_observer_fencing.m`.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Yan, Z.; Yu, Y.; Xing, W.; Shi, Y. Dynamic Observer-Based Multi-AUV Enclosing Control for Target Without Velocity Measurement. *IEEE Trans. Instrum. Meas.* **2025**, *74*, 7508714.
2. Huang, Y.; Fang, W.; Chen, Z.; Li, Y.; Yang, C. Flocking of multiagent systems with nonuniform and nonconvex input constraints. *IEEE Trans. Autom. Control* **2022**, *68*, 4329–4335.
3. Song, Z.; Wu, Z.; Huang, H. Cooperative learning formation control of multiple autonomous underwater vehicles with prescribed performance based on position estimation. *Ocean Eng.* **2023**, *280*, 114635.
4. Huang, Y.; Duan, M.; Mo, L. Multiagent containment control with nonconvex states constraints, nonuniform time delays, and switching directed networks. *IEEE Trans. Neural Netw. Learn. Syst.* **2019**, *31*, 5021–5028.

5. Jiang, S.; Chi, P.; Yang, L.; Zhao, J.; Wang, Y. Plug-and-Play Distributed Fencing of Multi-Agent Systems for an Uncooperative Target Without Velocity Measurements. *IEEE Trans. Autom. Sci. Eng.* **2025**, *22*, 20098–20111.
6. Suryendu, C.; Subudhi, B. Formation control of multiple autonomous underwater vehicles under communication delays. *IEEE Trans. Circuits Syst. II Express Briefs* **2020**, *67*, 3182–3186.
7. Hadi, B.; Khosravi, A.; Sarhadi, P. A review of the path planning and formation control for multiple autonomous underwater vehicles. *J. Intell. Robot. Syst.* **2021**, *101*, 67.
8. Yan, T.; Xu, Z.; Yang, S.X.; Gadsden, S.A. Formation control of multiple autonomous underwater vehicles: A review. *Intell. Robot.* **2023**, *3*, 1–22.
9. Wang, C.; Mei, D.; Wang, Y.; Yu, X.; Sun, W.; Wang, D.; Chen, J. Task allocation for multi-AUV system: A review. *Ocean Eng.* **2022**, *266*, 112911.
10. Wang, L.; Zhu, D.; Pang, W.; Zhang, Y. A survey of underwater search for multi-target using multi-AUV: Task allocation, path planning, and formation control. *Ocean Eng.* **2023**, *278*, 114393.
11. Liu, G.; Chen, L.; Liu, K.; Luo, Y. A swarm of unmanned vehicles in the shallow ocean: A survey. *Neurocomputing* **2023**, *531*, 74–86.
12. Chen, Y.L.; Ma, X.W.; Bai, G.Q.; Sha, Y.; Liu, J. Multi-autonomous underwater vehicle formation control and cluster search using a fusion control strategy at complex underwater environment. *Ocean Eng.* **2020**, *216*, 108048.
13. Li, X.; Wang, Y.; Zhang, J.; Zhang, W. Formation control of multiple autonomous underwater vehicles under communication delay, packet discreteness and dropout. *J. Mar. Sci. Eng.* **2022**, *10*, 920.
14. Wang, L.; Xu, X.; Han, B.; Zhang, H. Multiple AUV formation obstacle avoidance control using event-triggered model predictive control. *J. Mar. Sci. Eng.* **2023**, *11*, 2016.
15. Li, X.; Wang, Y.; Zhang, J.; Zhang, W. Distributed dynamic predictive control for multi-AUV target searching and hunting in unknown environments. *Machines* **2022**, *10*, 366.
16. Zhao, Y.; Yan, Z.; Zhang, L.; Wang, C. A cooperative hunting method for multi-AUV swarm in weak information environment. *J. Mar. Sci. Eng.* **2022**, *10*, 1266.
17. Zhang, M.; Chen, H.; Cai, W. Collaborative hunting method of multi-AUV in three-dimensional Internet of Underwater Things: Searching, tracking, and encirclement keeping. *IEEE Internet Things J.* **2025**, *12*, 10958–10973.
18. Hou, Y.; Han, G.; Zhang, F.; Lin, C.; Peng, J.; Liu, L. Distributional soft actor-critic-based multi-AUV cooperative pursuit for maritime security protection. *IEEE Trans. Intell. Transp. Syst.* **2024**, *25*, 6049–6060.
19. Zhu, S.; Han, G.; Lin, C.; Tao, Q. Underwater target tracking based on hierarchical software-defined multi-AUV reinforcement learning: A multi-AUV advantage-attention actor-critic approach. *IEEE Trans. Mob. Comput.* **2024**, *23*, 13639–13653.
20. Cai, W.; Chen, H.; Zhang, M. A survey on collaborative hunting with robotic swarm: Key technologies and application scenarios. *Neurocomputing* **2024**, *598*, 128008.
21. Chen, Z. A cooperative target-fencing protocol of multiple vehicles. *Automatica* **2019**, *107*, 591–594.
22. Kou, L.; Huang, Y.; Chen, Z.; Xiang, J. Cooperative fencing control of multiple second-order vehicles for a moving target with and without velocity measurements. *Int. J. Robust Nonlinear Control* **2021**, *31*, 4602–4615.
23. Kou, L.; Chen, Z.; Xiang, J. Cooperative fencing control of multiple vehicles for a moving target with an unknown velocity. *IEEE Trans. Autom. Control* **2022**, *67*, 1008–1015.
24. Hu, B.B.; Chen, Z.; Zhang, H.T. Distributed moving target fencing in a regular polygon formation. *IEEE Trans. Control Netw. Syst.* **2022**, *9*, 210–218.
25. Hu, B.B.; Zhang, H. T.; Shi, Y. Cooperative label-free moving target fencing for second-order multi-agent systems with rigid formation. *Automatica* **2023**, *148*, 110788.
26. Pan, Z.; Chen, B.M. Cooperative target fencing of multiple vehicles for a general target with connectivity preservation and collision avoidance. *J. Syst. Sci. Complex.* **2024**, *37*, 136–151.
27. Pang, S.; Zhu, D.; Sun, B. Multi-AUV formation reconfiguration obstacle avoidance algorithm based on affine transformation and improved artificial potential field. *IEEE Trans. Autom. Sci. Eng.* **2024**, *21*, 2264–2279.
28. Wang, Y.; Zhu, D.; Pang, S.; Luo, X. A novel obstacle avoidance consensus control for multi-AUV formation system. *IEEE/CAA J. Autom. Sin.* **2023**, *10*, 1304–1318.
29. Wang, J.; Fan, Y.; Xu, H.; Xu, C.; Qu, Q. Cooperative tracking control of multiple autonomous underwater vehicles with Markovian jumping switching topologies and communication delays. *Optim. Control Appl. Methods* **2025**, *46*, 1449–1458.
30. Lin, P.; Huang, Y.; Zhang, Q.; Yuan, Z. Distributed velocity and input constrained tracking control of high-speed train systems. *IEEE Trans. Syst. Man Cybern. Syst.* **2020**, *51*, 7882–7888.
31. Zhang, Y.; Wang, Q.; Shen, Y.; Dai, N.; He, B. Multi-AUV cooperative control and autonomous obstacle avoidance study. *Ocean Eng.* **2024**, *304*, 117634.
32. Wu, C.; Karimi, H.R.; Shan, L.; Dai, Y. Data-driven iterative learning cooperative trajectory tracking control for multiple autonomous underwater vehicles with input saturation constraints. *J. Field Robot.* **2024**, *41*, 2475–2487.

33. Zhuo, J.; Tian, X.; Liu, H. Distributed cooperative H_∞ optimal control of underactuated autonomous underwater vehicles based on reinforcement learning and prescribed performance. *Ocean Eng.* **2024**, *312*, 119323.
34. Fossen, T. I. *Handbook of Marine Craft Hydrodynamics and Motion Control*; John Wiley & Sons: Hoboken, NJ, USA, 2011.

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