

Article

An Interpolation-Free Near-Field Frequency-Domain Beamforming Algorithm for Uniform Linear Array Sonar

Zhibin Yue , Jinsong Tang *, Heping Zhong , Haoran Wu , Han Li  and Meng Zhao

Naval University of Engineering, Wuhan 430033, China; d23182407@nue.edu.cn (Z.Y.); 1209041071@nue.edu.cn (H.Z.); 1920191082@nue.edu.cn (H.W.); d24182404@nue.edu.cn (H.L.); 2420241051@nue.edu.cn (M.Z.)

* Correspondence: 0909041066@nue.edu.cn

Abstract

To address the issue of phase mismatch, mainlobe broadening, image defocusing, and high computational complexity caused by spherical wavefront propagation in near-field imaging of uniform linear array (ULA) imaging sonar, this paper proposes an interpolation-free near-field frequency-domain beamforming algorithm for uniform linear arrays. Under the considered operating conditions, the exact range history is approximated as the sum of a range-dependent quadratic term and a range-independent linear term with respect to the element position. The quadratic term produces negligible envelope migration; therefore, only its phase needs to be compensated. By contrast, the linear term produces more significant range migration, which is corrected together with its associated phase through range-domain and azimuth-domain FFT processing. Under the adopted range-history approximation and discrete sampling conditions, this implementation avoids the explicit point-by-point fractional-delay interpolation required by the backprojection (BP) algorithm. The proposed method substantially reduces the computational cost while maintaining near-field focusing performance comparable to that of the BP algorithm. For the simulated data, the runtime is reduced from 58.23 s to 0.41 s, with only a 0.1% broadening of the azimuth impulse-response width and a 0.11 dB degradation in the peak sidelobe ratio. For the measured lake-trial data, the runtime is reduced from 50.18 s to 0.29 s.

Keywords: uniform linear array (ULA); near-field frequency-domain beamforming; underwater acoustic signal processing; interpolation-free; computational efficiency

1. Introduction

Sonar imaging systems are important tools for underwater detection, navigation, and target recognition, and their imaging quality directly affects the effectiveness of underwater missions [1–10]. Among various array configurations, the uniform linear array (ULA) is widely used because of its simple structure and ease of implementation [11,12]. However, when a target is located in the near-field region of the array, acoustic waves propagate as spherical wavefronts rather than plane wavefronts, and the conventional far-field plane-wave assumption no longer holds. When conventional far-field beamforming methods are directly applied to near-field scenarios, the curvature of the spherical wavefront cannot be properly compensated. The resulting phase mismatch leads to mainlobe broadening, elevated sidelobe levels, and image defocusing [13–15]. Therefore, near-field beamforming for uniform linear arrays is a key technique for improving the near-field resolution of imaging sonar.



Academic Editor: Chia-Hung Yeh

Received: 11 July 2026

Revised: 27 July 2026

Accepted: 4 August 2026

Published: 7 August 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

Extensive research has been conducted on near-field beamforming for uniform linear arrays. However, existing beamforming methods all have different limitations when meeting the requirements of ULA near-field processing [16]. Time-domain delay-and-sum (DAS) beamforming is the most intuitive method. It compensates for propagation-path differences by applying appropriate time delays and then coherently sums the delayed signals [17,18]. Although DAS is simple and intuitive in principle, it has high computational complexity and is difficult to use for real-time processing.

Frequency-domain beamforming converts propagation delays into frequency-dependent phase rotations, thereby reducing the computational cost of broadband array processing. Early studies developed efficient frequency-domain digital beamforming [19], CZT-based three-dimensional acoustic imaging [20], and a dedicated CZT beamforming method for near-field 3-D sonar imaging [21]. These studies demonstrate that CZT is not inherently unsuitable for near-field imaging. It performs effectively when range migration can be approximated by a linear or locally linear scaling relationship; however, its performance may degrade in the presence of strong range dependence or wideband coupling. To address these issues, Nguyen et al. investigated the effects of FFT length, window functions, and frame overlap on STFT-domain beamforming [22]. Yan et al. combined matched filtering, array beamforming, and spatially variant apodization in forward-looking sonar to improve image sidelobe performance [23]. Ning et al. employed range-block and frequency-subband processing to achieve CZT-based range-migration correction [24]. Therefore, CZT remains applicable to near-field or strongly coupled imaging problems, but its performance depends on the accuracy of the local scaling model and the effective control of residual phase errors.

Another class of methods is based on range-history approximation, such as the Fresnel approximation and various Taylor-expansion-based methods. These methods can reduce the requirement for parameter-table storage and simplify the delay calculation. Chen et al. investigated optimization techniques for approximating the near-field range history using polynomial expansion [25]. However, although such methods reduce parameter-storage overhead, they usually cannot significantly reduce the overall computational complexity of the beamforming process, and therefore their real-time performance is insufficient. In addition, parameter optimization for range-history approximation is range-dependent and generally relies on scene precomputation, which limits its adaptability to dynamically changing environments.

Fast beamforming algorithms based on multilevel hierarchical decomposition and merging strategies reduce computational complexity by dividing the array into subarrays at different levels, processing them hierarchically, and then combining the results level by level [26,27]. However, the sidelobe performance of these methods is significantly affected by the choice of beam spacing at each processing level. The interpolation operations required between adjacent processing stages also introduce additional computational burden and potential numerical errors. Moreover, to obtain consistent imaging quality over the entire field of view, the algorithm parameters usually need to be carefully tuned.

To address the above problems, this paper proposes an interpolation-free near-field frequency-domain beamforming algorithm for uniform-linear-array imaging sonar. Starting from a physical interpretation, the proposed algorithm approximates the exact range history as the sum of a range-dependent quadratic term and a range-independent linear term with respect to the array-element position. This decomposition enables an efficient frequency-domain implementation that avoids explicit point-by-point fractional-delay interpolation under the adopted range-history approximation and discrete sampling conditions. The term “interpolation-free” refers to the discrete beamforming implementation and does not imply the absence of modeling or discretization approximations. In the specific implementation,

the algorithm first performs range pulse compression on the raw echo data to achieve range focusing. Then, a quadratic-term compensation function is multiplied in the time domain to correct the range-dependent phase error. Next, a range-domain fast Fourier transform and an azimuth-domain fast Fourier transform (FFT) are performed successively, and the range migration correction and phase compensation caused by the linear term of the array-element position are completed in the two-dimensional frequency domain. Finally, a range-domain inverse fast Fourier transform (IFFT) converts the signal back to the time domain to generate the final two-dimensional focused image. Under the adopted range-history approximation and discrete sampling conditions, the proposed algorithm does not require the point-by-point fractional-delay interpolation as in the backprojection (BP) algorithm. The proposed algorithm is implemented solely by phase multiplications, FFT/IFFT, and frequency-dependent spatial-bin mapping. It maintains near-field focusing performance comparable to that of the Backprojection (BP) algorithm while significantly reducing computational complexity.

The remainder of this paper is organized as follows: Section 2 establishes the near-field geometric model of a uniform-linear-array imaging sonar, derives the exact range history, and gives a reasonable approximation. Section 3 describes the proposed interpolation-free near-field frequency-domain beamforming algorithm in detail. Section 4 compares the computational complexity of two algorithms. Section 5 verifies the effectiveness of the proposed algorithm through simulations and measured data. Section 6 concludes the paper.

2. Approximate Range History of ULA Imaging Sonar

2.1. Geometric Model and Exact Range History

The imaging geometry of a uniform-linear-array imaging sonar is shown in Figure 1. The transmitting element is located at the origin $(0, 0)$, and the receiving array consists of N elements with an inter-element spacing of d . The receiving elements are arranged along the X -axis and are symmetrically distributed with respect to the transmitting element.

Consider a stationary point target located at the polar coordinate (r, θ) , where r denotes the distance from the target to the transmitting element (the origin), and θ denotes the azimuth angle of the target relative to the array broadside direction (the Y -axis).

Assume that the receiving elements are indexed from left to right along the X -axis as $i = 1, 2, 3, \dots, N$. The abscissa of the i -th receiving element is then expressed as

$$d_i = (i - 1)d - \left(\frac{N - 1}{2}\right)d = \left(i - \frac{N + 1}{2}\right)d \quad (1)$$

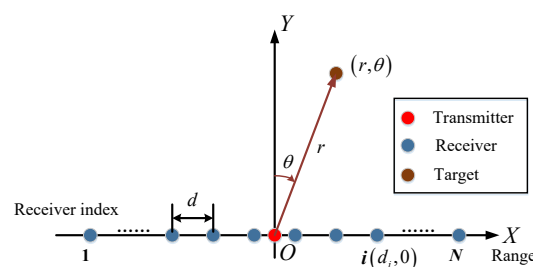


Figure 1. Schematic diagram of imaging geometry for ULA imaging sonar.

The two-way propagation path consists of the one-way path from the transmitting element to the target, whose distance is r , and the one-way path from the target to the i -th receiving element. Therefore, the exact range history of the i -th receiving element is

$$R_i(r, \theta) = r + \sqrt{r^2 + d_i^2 - 2rd_i \sin \theta} \quad (2)$$

Equation (2) is nonlinear with respect to r and d_i because of the square-root term, reflecting the spherical-wavefront propagation characteristics in the near-field region.

2.2. Approximation of the Exact Range History

Let $x = \frac{d_i^2}{r^2} - \frac{2d_i \sin \theta}{r}$. When the target range r is much larger than the array aperture, i.e., $r \gg d_i$ for all receiving elements, we have $|x| \ll 1$. By applying a Taylor-series expansion to Equation (2) and retaining the first-order term of x , the approximate range history is obtained as

$$R'_i(r, \theta) = 2r + \frac{d_i^2}{2r} - d_i \sin \theta \quad (3)$$

The three terms in Equation (3) have clear physical meanings:

1. The first term $2r$ represents the two-way distance between the transmitter and the target under the monostatic assumption. This term is independent of the receiving-element position and determines range focusing.
2. The second term $d_i^2/(2r)$ is the quadratic term of the array-element position and is inversely proportional to the target range r . It quantifies extra propagation offset induced by near-field spherical wave curvature: wavefront curvature varies across array elements, creating path-length deviations relative to the plane-wave model. Since this term scales inversely with r , its impact intensifies for targets close to the array.
3. The third term $-d_i \sin \theta$ is the linear term of the array-element position and is independent of the target range r . It is range-invariant and depends only on the sine of the target azimuth angle. This term represents the range offset caused by target azimuth variation under the plane-wave assumption, corresponding to the arrival-time difference across the array aperture for a plane wave incident from angle θ .

This decomposition forms the basis of the proposed implementation. As quantified in Section 2.3, the quadratic term produces an envelope displacement much smaller than one range-resolution cell. Its envelope contribution can therefore be neglected, whereas its phase contribution must still be compensated. By contrast, the range-independent linear term produces significant envelope migration and can be processed efficiently using FFT-based operations.

2.3. Range Migration Analysis

The system simulation parameters are listed in Table 1. The range resolution of the sonar system is $\rho_r = c/(2B) = 0.015$ m. The range offsets caused by each term are analyzed below. For the linear term of the array-element position, at the extreme azimuth angle $\theta = 45^\circ$ and for the edge receiving element ($|d_i| = (N - 1)d/2 = 0.0635$ m), the maximum range offset is

$$\Delta R_{\text{linear}}^{\max} = |d_i \sin \theta|_{\max} = 0.0635 \times \sin(45^\circ) \approx 0.0449 \text{ m} \quad (4)$$

This range offset is approximately three times the range resolution cell ($\rho_r = 0.015$ m), clearly exceeding one resolution cell. Therefore, range-cell migration correction must be performed to avoid image defocusing and resolution degradation. When the azimuth-angle span is large, the range offset caused by the plane-wave-related linear term of the array-element position often exceeds one range resolution cell and thus requires range migration correction.

The quadratic-term $d_i^2/(2r)$ corresponds to an image-range displacement $\Delta r_{q,i} = d_i^2/(4r)$. The range resolution is defined as $\rho_r = c/(2B)$. Thus, the ratio of the envelope migration caused by the quadratic term to one range resolution cell is given by

$$\eta_{q,i}(r) = \frac{\Delta r_{q,i}}{\rho_r} = \frac{Bd_i^2}{2rc} \quad (5)$$

The maximum migration occurs at the minimum target range and the edge receiving element. Therefore, the maximum envelope offset introduced by the quadratic term is expressed as

$$\eta_q^{\max} = \frac{Bd_{\max}^2}{2cr_{\min}} \quad (6)$$

For the parameters in Table 1, $d_{\max} = 0.0635$ m and $r_{\min} = 10$ m, yielding

$$\Delta r_q^{\max} = \frac{0.0635^2}{4 \times 10} \approx 1.01 \times 10^{-4} \text{ m}, \quad \eta_q^{\max} \approx 6.72 \times 10^{-3} \quad (7)$$

Thus, the quadratic-term envelope migration is less than 0.7% of one range resolution cell and is negligible under the considered system parameters. However, its maximum phase variation is

$$|\phi_q|_{\max} = \frac{\pi f_0 d_{\max}^2}{cr_{\min}} \approx 0.718 \text{ rad}, \quad (8)$$

where f_0 denotes the center frequency. For this reason, the quadratic phase term must be retained and compensated in the subsequent processing.

Table 1. System simulation parameters.

Parameter	Value
Center frequency (kHz)	850
Bandwidth (kHz)	50
Element spacing (m)	0.001
Number of receiving elements	128
Range span (m)	(10, 60)
Angular span (°)	(−45, 45)

2.4. Approximation Error Analysis

To quantify the range-history approximation error and determine the valid range of the proposed algorithm, the range-history approximation error is defined as the difference between the exact range history and the approximate range history:

$$\begin{aligned} \Delta R_i(r, \theta) &= R_i(r, \theta) - R'_i(r, \theta) \\ &= \sqrt{r^2 + d_i^2 - 2rd_i \sin \theta} - r - \frac{d_i^2}{2r} + d_i \sin \theta \end{aligned} \quad (9)$$

Let $L_i(r, \theta) = \sqrt{r^2 + d_i^2 - 2rd_i \sin \theta}$ denote the one-way propagation distance from the target to the i -th receiving element. Using the identity $L_i^2 - r^2 = d_i^2 - 2rd_i \sin \theta$, the error of the approximate range history can be rewritten as

$$\begin{aligned} \Delta R_i(r, \theta) &= L_i - r - \frac{d_i^2}{2r} + d_i \sin \theta \\ &= L_i - r - \frac{L_i^2 - r^2}{2r} \\ &= -\frac{(L_i - r)^2}{2r} \end{aligned} \quad (10)$$

According to the reverse triangle inequality

$$|L_i(r, \theta) - r| \leq |d_i| \leq D, \quad D = \frac{(N-1)d}{2}, \quad (11)$$

where D is the array half-aperture. Since $r \geq r_{\min}$, the absolute approximation error of the range history for any receiving array element satisfies

$$|\Delta R_i(r, \theta)| \leq \frac{D^2}{2r} \leq \frac{D^2}{2r_{\min}} \quad (12)$$

Substitution of the parameters in Table 1 yields the following result

$$|\Delta R_i(r, \theta)| \leq 2.0161 \times 10^{-4} \text{ m} < \frac{\lambda}{8} = 2.2059 \times 10^{-4} \text{ m} \quad (13)$$

This rigorous upper bound applies to all receiving array elements and all target positions within the considered range–azimuth region. It guarantees that the range-history approximation error of every receiving element within the 90° detection sector is less than $\lambda/8$, satisfying the imaging precision requirements [28].

To intuitively demonstrate the spatial variation in the error, Figure 2 shows contour plots of the wavelength-normalized absolute range history approximation error of the first receiving element versus target range and azimuth. It can be observed that the approximation error remains low near the array normal, and generally rises as the azimuth shifts toward both edges of the detection sector. Larger errors are mainly concentrated in regions with short target ranges and large absolute azimuth angles.

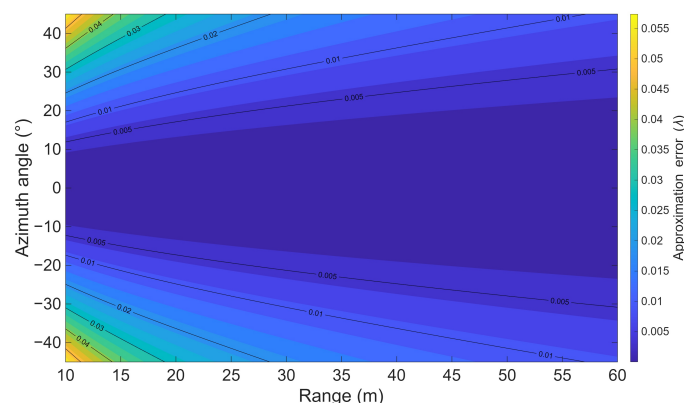


Figure 2. Range history approximation error.

3. Proposed Frequency-Domain Beamforming Algorithm

Based on the approximate range history established in Section 2.2, the complete procedure of the proposed interpolation-free near-field frequency-domain beamforming algorithm for ULA is shown in Figure 3.

The algorithm consists of three sequential processing stages:

1. Range pulse compression and quadratic-term compensation: First, matched filtering, i.e., pulse compression, is applied to the raw echo data of each receiving element to achieve range focusing. Then, in the range-time domain, the range-compressed data are multiplied by the quadratic-term compensation function $H_{\text{second}}(d_i, r)$ to correct the range-dependent phase error caused by near-field spherical-wavefront curvature.
2. Range-domain FFT and azimuth-domain FFT: A one-dimensional FFT is performed along the range dimension for each receiving element, transforming the data into the range-frequency domain. Then, for each range-frequency bin, a one-dimensional

FFT is performed along the azimuth, or element, dimension. By converting spatial sampling among the array elements into the angular domain, this operation simultaneously completes range-cell migration correction and azimuth focusing.

3. Range-domain IFFT: An IFFT is performed along the range-frequency dimension to transform the focused data back into the range-time domain, yielding the final two-dimensional focused sonar image.

The signal transformations in these three stages are derived in detail below. Assume that the sonar transmits a Linear Frequency-Modulated (LFM) pulse with center frequency f_0 , bandwidth B , and pulse width T_p . The acoustic wavelength is $\lambda = c/f_0$, where c is the sound speed in water. Let τ denote the fast-time variable. Consider a stationary point target located at polar coordinate (r, θ) in the imaging scene.

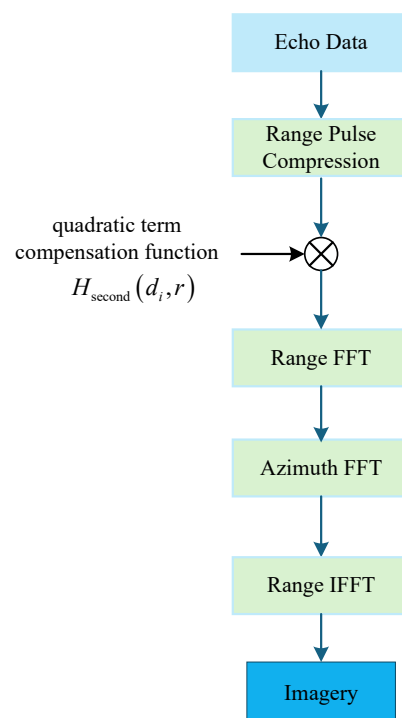


Figure 3. Algorithm flowchart.

After demodulation to baseband, the raw echo signal received by the i -th receiving element can be expressed as

$$s_i^{\text{raw}}(\tau; r, \theta) = A \cdot p\left(\tau - \frac{R_i(r, \theta)}{c}\right) \exp\left[-j \frac{2\pi f_0 R_i(r, \theta)}{c}\right] \quad (14)$$

where A is a complex amplitude factor including target reflectivity and propagation attenuation. It is treated as a constant and omitted in the following derivation. $p(\tau)$ is the complex envelope of the transmitted LFM pulse, and f_0 denotes the center frequency.

After matched filtering with the reference signal, the range-compressed signal of the i -th receiving element is

$$\begin{aligned} s_i(\tau; r, \theta) &= \text{sinc}\left\{B \left[\tau - \frac{R'_i(r, \theta)}{c}\right]\right\} \exp\left[-j \frac{2\pi f_0 R'_i(r, \theta)}{c}\right] \\ &= \text{sinc}\left\{B \left[\tau - \frac{2r + d_i^2/(2r) - d_i \sin \theta}{c}\right]\right\} \\ &\quad \times \exp\left[-j \frac{2\pi f_0 (2r + d_i^2/(2r) - d_i \sin \theta)}{c}\right] \end{aligned} \quad (15)$$

According to the analysis in Section 2.3, the range-cell migration caused by the quadratic term $d_i^2/(2r)$ is negligible relative to one range resolution cell. Therefore, the quadratic term can be separated from the sinc envelope and regarded only as a phase modulation term [29]:

$$s_i(\tau; r, \theta) \approx \text{sinc} \left\{ B \left[\tau - \frac{2r - d_i \sin \theta}{c} \right] \right\} \times \exp \left[-j \frac{2\pi f_0 \left(2r + \frac{d_i^2}{2r} - d_i \sin \theta \right)}{c} \right] \quad (16)$$

To eliminate the phase error introduced by near-field spherical-wavefront curvature, the following quadratic-term phase compensation function is constructed:

$$H_{\text{second}}(d_i, r) = \exp \left(j \frac{\pi f_0 d_i^2}{cr} \right) \quad (17)$$

Multiplying Equation (16) by $H_{\text{second}}(d_i, r)$ gives the compensated signal:

$$\begin{aligned} s_{i1}(\tau; r, \theta) &= s_i(\tau; r, \theta) \cdot H_{\text{second}}(d_i, r) \\ &= \text{sinc} \left\{ B \left[\tau - \frac{2r - d_i \sin \theta}{c} \right] \right\} \\ &\quad \times \exp \left[-j \frac{2\pi f_0 (2r - d_i \sin \theta)}{c} \right] \end{aligned} \quad (18)$$

After quadratic-term phase compensation, the signal model corresponds to a plane-wave propagation scenario. Using the Fourier-transform pair

$$\text{sinc}[B(\tau - \tau_i)] \longleftrightarrow \frac{1}{B} \text{rect}\left(\frac{f}{B}\right) \exp(-j2\pi f \tau_i) \quad (19)$$

Performing a Fourier transform on Equation (18) along the fast-time (range) dimension and neglecting the constant amplitude factor $1/B$, which exerts no influence on normalized imaging results, the range-frequency domain signal is obtained as follows:

$$\begin{aligned} S_i(f; r, \theta) &= \int s_{i1}(\tau; r, \theta) \exp(-j2\pi f \tau) d\tau \\ &= \text{rect}\left(\frac{f}{B}\right) \exp \left[-j \frac{4\pi(f_0 + f)r}{c} \right] \\ &\quad \times \exp \left[j \frac{2\pi(f_0 + f)d_i \sin \theta}{c} \right] \end{aligned} \quad (20)$$

In the above expression, the first exponential term represents the range-dependent phase associated with two-way propagation to range r , and the second exponential term is the element-dependent linear phase containing the target azimuth information.

For each fixed range-frequency sample f , the element-dependent phase in Equation (20) can be expressed through the normalized spatial frequency

$$\nu(f, \theta) = \frac{(f_0 + f)d \sin \theta}{c} \quad (21)$$

The azimuth-domain FFT coherently accumulates this sequence at the spatial frequency corresponding to $\nu(f, \theta)$. When the spatial frequency is exactly matched, the element-dependent phase is eliminated, yielding:

$$\tilde{S}S(f; r, \theta) = \text{rect}\left(\frac{f}{B}\right) \exp\left[-j\frac{4\pi(f_0 + f)r}{c}\right]. \quad (22)$$

The factor $\exp[-j4\pi f_0 r/c]$ is independent of the range-frequency variable f and affects only the complex phase of the focused range response. Since the final image is formed from the signal magnitude, this constant phase factor is omitted, giving

$$SS(f; r, \theta) = \text{rect}\left(\frac{f}{B}\right) \exp\left[-j\frac{4\pi fr}{c}\right] \quad (23)$$

Finally, applying an IFFT to Equation (23) along the range-frequency dimension yields the fully focused two-dimensional image:

$$sS(\tau; r, \theta) = \text{sinc}\left[B\left(\tau - \frac{2r}{c}\right)\right] \quad (24)$$

Equation (24) gives the focused range response at the matched output angle. The response is sinc-shaped and centered at the true target range r , while azimuth focusing is provided by the azimuth-domain FFT.

The main computational advantage of the proposed algorithm is that its core beam-forming operations are implemented using FFT-based transforms and pointwise phase multiplication, thereby avoiding explicit point-by-point fractional-delay interpolation under the adopted range-history approximation and discrete sampling conditions.

To provide a clearer implementation procedure and improve the reproducibility of the proposed algorithm, its main discrete processing steps are summarized in Algorithm 1. The inputs of the algorithm include raw echo data and preset array system parameters, while the output is the final spatially focused imaging result.

Algorithm 1 Interpolation-free near-field frequency-domain beamforming

Input: Raw echo matrix $\mathbf{X} \in \mathbb{C}^{N_{re} \times N_r}$; system and array parameters $\{f_0, c, d, d_i, r_n, f_k, \theta_m, N_a\}$.
Output: Focused image $\mathbf{I} \in \mathbb{R}^{N_\theta \times N_r}$.

- 1: Perform frequency-domain pulse compression to obtain $s_{pc}(i, n)$.
- 2: $s_c(i, n) \leftarrow s_{pc}(i, n) \exp(j\pi f_0 d_i^2 / (cr_n))$
- 3: $S_c(i, k) \leftarrow \text{FFT}_n\{s_c(i, n)\}$
- 4: $S_a(q, k) \leftarrow \text{fftshift}_q\{\text{FFT}_i^{(N_a)}[S_c(i, k)]\}$
- 5: $n_c \leftarrow (N_{re} - 1)/2$
- 6: $S_a^s(q, k) \leftarrow S_a(q, k) \exp(j2\pi\nu_q n_c)$, where $\nu_q = (q - 1 - N_a/2)/N_a$.
- 7: **for** $k = 1, \dots, N_r$ **do**
- 8: **for** $m = 1, \dots, N_\theta$ **do**
- 9: $\nu_{m,k} \leftarrow (f_0 + f_k)d \sin \theta_m / c$
- 10: $q_{m,k} \leftarrow \text{round}(N_a \nu_{m,k} + N_a/2 + 1)$
- 11: $S_\theta(m, k) \leftarrow S_a^s(q_{m,k}, k)$
- 12: **end for**
- 13: **end for**
- 14: $s_{img}(m, n) \leftarrow \text{IFFT}_k\{S_\theta(m, k)\}$
- 15: $I(\theta_m, r_n) \leftarrow |s_{img}(m, n)|$
- 16: **return** $\mathbf{I}$

As shown in Algorithm 1, the proposed method consists mainly of range pulse compression, range-dependent near-field phase compensation, range- and azimuth-domain FFT processing, array phase-center correction, frequency-dependent matched-bin selection, and range-domain IFFT. For each range-frequency sample and output angle, the required

normalized spatial frequency is mapped directly to the nearest bin of the oversampled azimuth FFT, without a weighted combination of adjacent FFT bins. Therefore, the term “interpolation-free” refers to the absence of explicit fractional-delay interpolation and weighted interpolation between adjacent azimuth-FFT bins in the discrete beamforming implementation; it does not imply the absence of modeling or discretization approximations.

4. Computational Complexity

4.1. Complexity of the Proposed Algorithm

This section analyzes the computational complexity of the proposed algorithm in detail by counting the number of complex multiplications required at each processing stage. Let N_{re} denote the number of receiving elements, N_r the number of range samples, and N_a the number of azimuth-angle samples, which is usually equal to the azimuth FFT size and satisfies $N_a \geq N_{re}$. The input data dimension is $N_{re} \times N_r$.

The beamforming part of the proposed algorithm consists of the following computational stages:

1. Quadratic-term phase compensation: The $N_{re} \times N_r$ data matrix is multiplied element by element with a precomputed compensation matrix, requiring $N_{re}N_r$ complex multiplications.
2. Range-domain FFT: An N_r -point FFT is performed once for each of the N_{re} receiving elements, requiring $N_{re} \times \frac{N_r}{2} \log_2 N_r$ complex multiplications.
3. Azimuth-domain FFT: An N_a -point FFT is performed once for each of the N_r range cells, requiring $N_r \times \frac{N_a}{2} \log_2 N_a$ complex multiplications.
4. Range-domain IFFT: An N_r -point IFFT is performed once for each of the N_a azimuth cells, requiring $N_a \times \frac{N_r}{2} \log_2 N_r$ complex multiplications.

Summing all stages gives the total computational complexity of the proposed algorithm:

$$\begin{aligned} C_{\text{proposed}} &= N_{re}N_r + \frac{N_{re}N_r}{2} \log_2 N_r + \frac{N_rN_a}{2} \log_2 N_a + \frac{N_aN_r}{2} \log_2 N_r \\ &= N_{re}N_r + \frac{(N_{re} + N_a)N_r}{2} \log_2 N_r + \frac{N_rN_a}{2} \log_2 N_a \end{aligned} \quad (25)$$

4.2. Complexity of the BP Algorithm

The BP algorithm is used as the standard reference for imaging-quality comparison. For each receiving element and each azimuth angle, the BP algorithm calculates the corresponding range index according to the exact range history and obtains the signal value at the required delay through interpolation. Specifically, for each receiving element, the BP processing procedure includes the following:

1. Calculating the range history $R_i(r, \theta_m)$ for each of the N_a azimuth angles and each of the N_r range samples.
2. Resampling the range-compressed data at the exact delay using M -point interpolation, typically $M = 8$ for sinc interpolation. Each element requires $M \cdot N_aN_r$ complex multiplications.
3. Performing time-domain phase multiplication for coherent beamforming summation. Each element requires N_aN_r complex multiplications.

Therefore, the total computational complexity of the BP algorithm is

$$C_{\text{BP}} = N_{re} \cdot (M \cdot N_aN_r + N_aN_r) = (M + 1)N_{re}N_aN_r \quad (26)$$

When 8-point sinc interpolation is used, i.e., $M = 8$, $C_{\text{BP}} = 9N_{re}N_aN_r$.

4.3. Complexity Comparison and Discussion

For practical sonar imaging scenarios, the typical parameters satisfy $N_a \gg 1$, $N_r \gg 1$, and $N_a \gg \log_2 N_r$. Under these conditions, the asymptotic complexity orders are

$$\begin{aligned} C_{\text{BP}} &= \mathcal{O}(N_{re} N_a N_r) \\ C_{\text{proposed}} &= \mathcal{O}(N_r N_a \log_2 N_a + N_r N_a \log_2 N_r) \end{aligned} \quad (27)$$

The proposed algorithm reduces computational complexity by restructuring the beam-forming procedure rather than by reducing the number of sampling points. In the BP algorithm, the exact propagation delay must be calculated for every element–angle–range combination, followed by interpolation and coherent summation. Consequently, its computational complexity increases linearly with the product $N_{re} N_a N_r$. In the proposed framework, the range-dependent quadratic phase term is compensated through phase multiplication, whereas the range-independent linear phase term is processed jointly using the range-domain FFT and the azimuth-domain FFT. By replacing the repeated pointwise interpolation and element-by-element accumulation required by conventional BP with reusable batch FFT operations, the dominant computational complexity is reduced from $\mathcal{O}(N_{re} N_a N_r)$ to $\mathcal{O}[N_r N_a (\log_2 N_r + \log_2 N_a)]$.

To provide a numerical comparison, consider the following representative parameters: $N_{re} = 128$, $N_r = 6144$, and $N_a = 512$. Substituting them into the complexity expressions gives

$$\begin{aligned} C_{\text{BP}} &= 9 \times 128 \times 512 \times 6144 = 3.62 \times 10^9 \\ C_{\text{proposed}} &= 128 \times 6144 + \frac{(512 + 128) \times 6144}{2} \log_2 6144 \\ &\quad + \frac{512 \times 6144}{2} \log_2 512 \\ &= 7.86 \times 10^5 + 2.46 \times 10^7 + 1.42 \times 10^7 \\ &\approx 3.97 \times 10^7 \end{aligned} \quad (28)$$

Table 2 summarizes the computational-complexity comparison under several parameter configurations, showing that the proposed algorithm continuously achieves approximately an 60–140-fold complexity reduction compared with the BP algorithm, depending on the parameter setting.

Table 2. Computational-complexity comparison under different parameter configurations, measured by the number of complex multiplications

N_{re}	N_r	N_a	BP ($\times 10^9$)	Proposed Algorithm ($\times 10^7$)	Ratio
64	4096	256	0.60	1.05	57.1
128	4096	512	2.42	2.68	90.3
128	6144	512	3.62	3.97	91.2
256	8192	1024	19.3	14.3	134.9

As shown in the table, as the number of elements N_{re} and the number of azimuth samples N_a increase, the complexity advantage of the proposed algorithm becomes more pronounced. This is because the complexity of the BP algorithm increases linearly with both parameters, whereas the dependence of the proposed algorithm on N_{re} appears only in non-dominant terms. This makes the proposed algorithm particularly suitable for high-resolution sonar systems with larger arrays and finer angular sampling.

In addition to its low raw computational complexity, the proposed algorithm has further practical advantages. Since it uses only FFTs and elementwise multiplications, it

can be parallelized and hardware-accelerated using optimized FFT libraries and parallel hardware architectures.

5. Experimental Results and Discussion

5.1. Simulation Experiment

This section verifies the effectiveness of the proposed algorithm through point-target simulation. The simulation parameters of the forward-looking sonar system are listed in Table 3. A stationary point target is placed in the detection scene at coordinate (15 m, 30°). These parameters place the target in the near-field region of the array ($2L^2/\lambda \approx 18.4$ m, where $L = 0.127$ m is the array aperture), ensuring that the spherical-wavefront effect is significant.

Table 3. Simulation parameters of the forward-looking sonar system.

Parameter	Value
Center frequency (kHz)	850
Bandwidth (kHz)	50
Pulse width (s)	0.001
Element spacing (m)	0.001
Number of receiving elements	128
Range span (m)	(10, 60)
Angular span (°)	(−45, 45)

Figure 4 shows the two-dimensional imaging results obtained by the reference BP algorithm and the proposed algorithm, respectively. Both images are displayed with a 40 dB dynamic range. It can be seen that both algorithms achieve good focusing of the point target, and the visual quality is highly similar. Neither result shows defocusing, geometric distortion, or obvious artifacts. The experimental results indicate that, in the near-field region, the proposed algorithm can maintain imaging quality comparable to that of the reference BP algorithm.

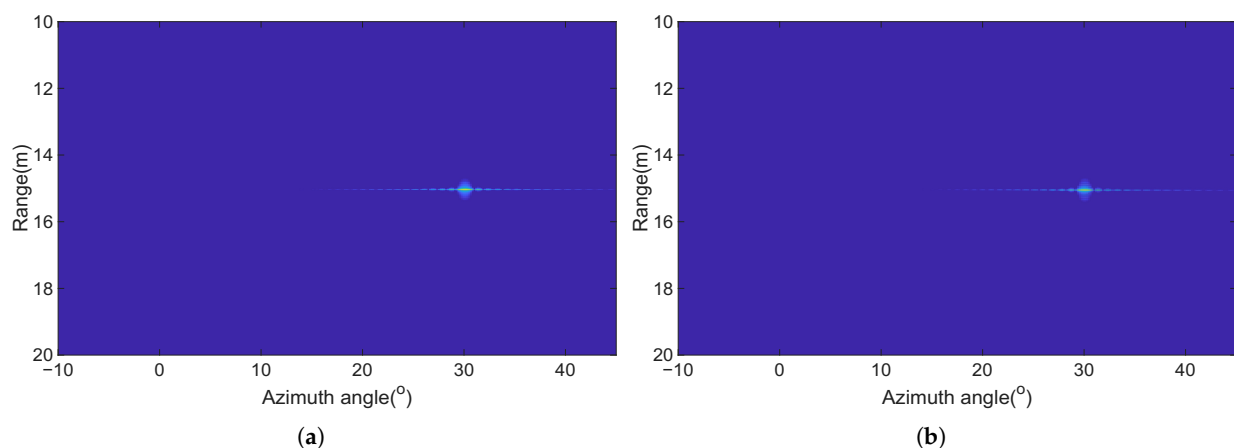


Figure 4. Simulation imaging results. (a) BP algorithm; (b) Proposed algorithm.

To quantitatively compare the imaging performance of the two algorithms, the angular profile of the target is selected for analysis. Figure 5 shows the angular profiles obtained by the two algorithms, both normalized to their respective peaks and expressed in decibels. As can be seen from Figure 5, compared with the BP result, the azimuth-profile mainlobe width of the proposed method is only slightly broadened, and the sidelobe level increases by only 0.11 dB. The overall shape of the angular profile is highly consistent with the BP

reference result. This high consistency indicates that the approximation and frequency-domain processing in the proposed algorithm introduce only negligible degradation in angular focusing performance.

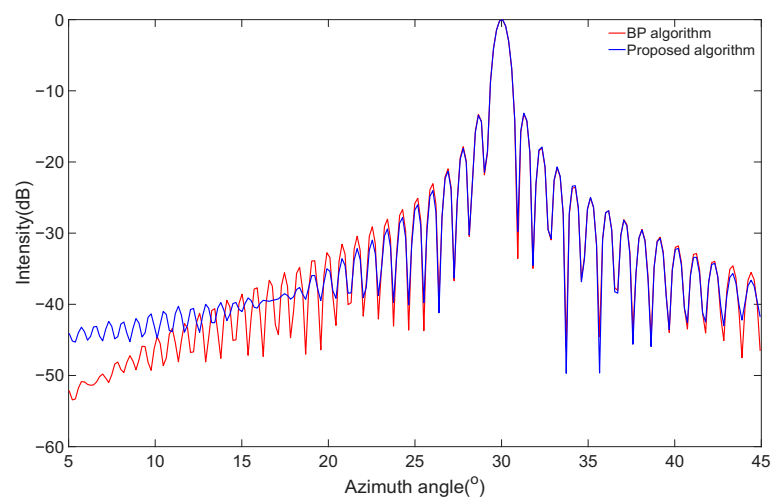


Figure 5. Comparison of angular profiles

For a comprehensive quantitative evaluation, Table 4 summarizes key performance metrics calculated from the azimuth profiles, including the impulse response width (IRW) and the peak sidelobe ratio (PSLR). In addition, the computation time of each algorithm is listed to quantify the improvement in computational efficiency. All experiments were conducted in MATLAB (v2025b) on the same hardware platform, equipped with an AMD Ryzen 9 9950X 16-core processor and 32 GB RAM. Both algorithms used the same input data, numerical precision, and timing procedure, and neither implementation used GPU acceleration. Reasonable vectorization and precomputation were applied where permitted by each algorithmic structure. Since the two methods rely on different computational operations, their absolute runtimes may also be affected by the efficiency of the corresponding MATLAB routines. The timing results reflect only the relative performance differences of the algorithms under the same experimental environment. The results in Table 4 show that the proposed algorithm is basically equivalent to the BP reference algorithm in imaging quality: the difference in IRW is only 0.001° , corresponding to a relative deviation of about 0.1%; the difference in PSLR is 0.11 dB, which is a negligible small deviation. In terms of computational efficiency, the runtime of the proposed algorithm is only 0.41 s, whereas that of the BP algorithm is 58.23 s, yielding an acceleration of about 142 times and nearly two orders of magnitude improvement in processing speed. This result verifies that the proposed algorithm can significantly reduce computational cost while maintaining high-fidelity imaging performance.

Table 4. Angular-profile parameters and imaging time.

Method	Computation Time (s)	IRW ($^\circ$)	PSLR (dB)
BP	58.23	0.930	−13.25
Proposed algorithm	0.41	0.931	−13.14

5.2. Measured Data

To further verify the proposed algorithm under real operating conditions, this paper uses lake-trial data from a forward-looking sonar experiment. The main sonar-system parameters are the same as those listed in Table 3. The imaging results obtained by processing the measured data using the BP algorithm and the proposed algorithm are

shown in Figure 6. To ensure a fair visual comparison, the two images are displayed using the same dynamic range and colormap settings.

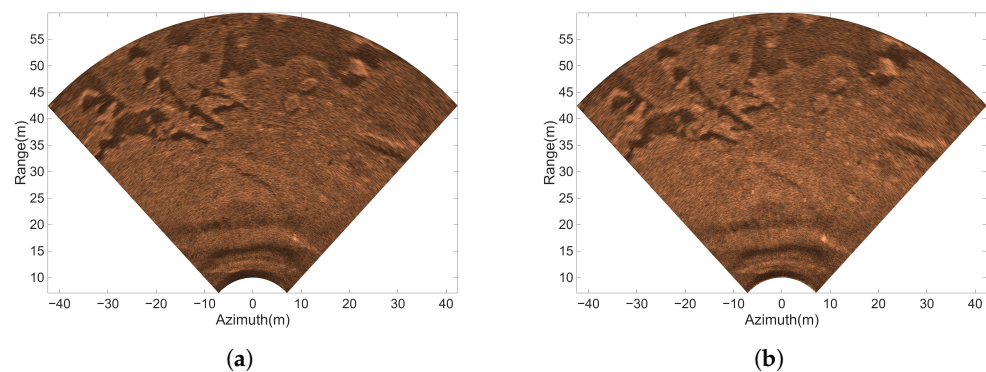


Figure 6. Lake-trial imaging results. (a) BP algorithm; (b) Proposed algorithm.

As shown in Figure 6, the imaging result generated by the proposed algorithm is basically consistent with that of the BP algorithm, with no obvious differences in feature location, scene morphology, or image contrast. Multiple bottom features are clearly visible in both images, with consistent positions and comparable intensity levels.

For the measured data, the computation time of the BP algorithm was 50.18 s, whereas that of the proposed algorithm was only 0.29 s, corresponding to an acceleration factor of approximately 173. The lake-trial data contain actual system noise, acoustic propagation effects, and environmental variations. Therefore, the high consistency between the imaging results obtained using BP and the proposed algorithm validates the effectiveness of the proposed method under practical operating conditions.

5.3. Practical Engineering Limitations and Real-Time Applicability

The proposed method consists mainly of range- and azimuth-domain FFT/IFFT operations and pointwise phase compensation. Its regular processing structure is well suited to parallel implementation on multicore CPUs, GPUs, and FPGAs. In practical systems, however, real-time throughput may be limited by memory bandwidth and data reorganization between the range- and azimuth-domain FFT stages. Depending on the hardware architecture, this transition may require strided memory access, blockwise transposition, or corner-turn buffering.

The method also relies on the validity of the adopted range-history approximation. At shorter ranges, with larger apertures, wider imaging sectors, or higher fractional bandwidths, neglected higher-order terms may cause residual phase errors and non-negligible envelope migration. If the quadratic-term migration is no longer small relative to a range-resolution cell, range-segmented correction or exact BP processing should be used at the cost of increased computation. Thus, practical performance depends on both data-movement efficiency and the validity of the range-history model.

6. Conclusions

To reduce the computational burden of conventional near-field beamforming for uniform linear array imaging sonar, this paper proposes an interpolation-free near-field frequency-domain beamforming algorithm. The exact range history is approximated as the sum of a range-dependent quadratic term and a range-independent linear term with respect to the element position. The quadratic term is compensated in the range-time domain, whereas the linear term is processed through range- and azimuth-domain FFTs to achieve range-migration correction and phase focusing. Under the adopted range-history approximation and discrete sampling conditions, the proposed processing chain avoids

the point-by-point fractional-delay interpolation required by the BP algorithm. Simulation and measured-data results demonstrate that the proposed method achieves near-field focusing performance comparable to that of BP while improving computational efficiency by approximately 150 times. Its regular FFT-based processing structure also offers favorable parallelization characteristics and is well suited to implementation on multicore CPUs, GPUs, and FPGAs.

The proposed method is currently intended for two-dimensional near-field imaging with a uniform linear array, and its accuracy depends on the validity of the adopted range-history approximation. Under more demanding conditions, such as shorter ranges, larger apertures, or wider imaging sectors, neglected higher-order terms may introduce residual phase errors and envelope migration. Future work will investigate higher-order compensation, range-segmented processing, and extension to planar arrays for three-dimensional near-field imaging.

Author Contributions: Conceptualization, J.T. and Z.Y.; methodology, J.T.; software, H.L.; validation, M.Z., Z.Y. and H.L.; formal analysis, Z.Y. and H.W.; investigation, H.Z.; resources, H.Z.; data curation, H.Z. and Z.Y.; writing—original draft preparation, Z.Y.; writing—review and editing, Z.Y.; visualization, H.L.; supervision, M.Z.; project administration, M.Z.; funding acquisition, J.T. and H.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China (Grant No. U2341227) and the Independent Research Project of Naval University of Engineering (Project No. 202650I020).

Data Availability Statement: The data presented in this paper are available after contacting the corresponding author.

Acknowledgments: Authors hereby declare that the ideas, data, results and content of the manuscript are all original creations of the author. Deepseek V4 Flash was only used during the translation and polishing of the manuscript.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Ayesha, N.; Vijayalakshmi, V.; Banu, S.; Tabassum, Z.; Fathima, A.; Madar, R.Y. Development of a Forward-Looking SONAR based System for Real-Time Underwater Object Detection and Mapping. In *Proceedings of the 2025 International Conference on Knowledge Engineering and Communication Systems (ICKECS)*; IEEE: New York, NY, USA, 2025; pp. 1–6.
2. Neupane, D.; Seok, J. A review on deep learning-based approaches for automatic sonar target recognition. *Electronics* **2020**, *9*, 1972. [CrossRef]
3. Cheng, C.; Zhou, X.; Gao, M.; Zong, Z.; Ji, Y.; Yu, B. Research on monopulse forward-looking high-resolution imaging algorithm based on adaptive iteration. *Def. Technol.* **2020**, *16*, 158–171. [CrossRef]
4. Huang, H.; Li, B.; Liu, J.; Liu, Z.; Wei, L.; Zhao, S. Sonar Image Underwater Target Recognition: A Comprehensive Overview and Prospects. *J. Electron. Inf. Technol.* **2024**, *46*, 1742–1760.
5. Klausner, N.H.; Azimi-Sadjadi, M.R. Performance prediction and estimation for underwater target detection using multichannel sonar. *IEEE J. Ocean. Eng.* **2019**, *45*, 534–546.
6. Wang, P.; Chi, C.; Liu, J.; Huang, H. Improving performance of three-dimensional imaging sonars through deconvolution. *Appl. Acoust.* **2021**, *175*, 107812. [CrossRef]
7. Ma, C.; Sun, D.; Mei, J.; Teng, T. Spatiotemporal two-dimensional deconvolution beam imaging technology. *Appl. Acoust.* **2021**, *183*, 108310. [CrossRef]
8. Li, Y.; Ma, M.; Zhao, C.; Zhou, Z. Forward-looking Imaging via Doppler Estimates of Sum-difference Measurements in Scanning Monopulse Radar. *J. Radars* **2021**, *10*, 131–142.
9. Zhou, T.; Si, J.; Wang, L.; Xu, C.; Yu, X. Automatic detection of underwater small targets using forward-looking sonar images. *IEEE Trans. Geosci. Remote Sens.* **2022**, *60*, 4207912. [CrossRef]
10. Karoui, I.; Quidu, I.; Legris, M. Automatic sea-surface obstacle detection and tracking in forward-looking sonar image sequences. *IEEE Trans. Geosci. Remote Sens.* **2015**, *53*, 4661–4669. [CrossRef]

11. Kulkarni, P.; Vaidyanathan, P.P. Non-integer arrays for array signal processing. *IEEE Trans. Signal Process.* **2022**, *70*, 5457–5472. [[CrossRef](#)]
12. Zhang, L.; Liu, W.; Langley, R.J. A class of constrained adaptive beamforming algorithms based on uniform linear arrays. *IEEE Trans. Signal Process.* **2010**, *58*, 3916–3922. [[CrossRef](#)]
13. Elbir, A.M.; Demir, Ö.T.; Mishra, K.V.; Chatzinotas, S.; Haardt, M. Near-field signal processing: Unleashing the power of proximity. *IEEE Signal Process. Mag.* **2025**, *42*, 20–32. [[CrossRef](#)]
14. Yang, D. Near-Field Beamforming Algorithm and Its Application Research. Master's Thesis, University of Electronic Science and Technology of China (UESTC), Chengdu, China, 2012.
15. He, T.; Pan, Q.; Liu, Y.; Liu, X.; Hu, D. Near-field beamforming analysis for acoustic emission source localization. *Ultrasonics* **2012**, *52*, 587–592. [[CrossRef](#)] [[PubMed](#)]
16. Zaheer, R.; Phung, V.Q.; Ahmad, I.; Aziz, A.; Habibi, D.; Rong, Y.; Hasan, W.K. A review on underwater beamforming: Techniques, challenges, and future directions. *J. Ocean Eng. Sci.* **2025**, *11*, 58–77. [[CrossRef](#)]
17. Moritz, N. Time and Frequency Domain SONAR Beam-Forming in the Near-Field. Ph.D. Thesis, Karlsruhe University of Applied Sciences, Karlsruhe, Germany, 2007.
18. Madhavanunni, A.; Panicker, M.R. Beam multiply and sum: A tradeoff between delay and sum and filtered delay multiply and sum beamforming for ultrafast ultrasound imaging. *Biomed. Signal Process. Control* **2023**, *85*, 104807. [[CrossRef](#)]
19. Maranda, B. Efficient digital beamforming in the frequency domain. *J. Acoust. Soc. Am.* **1989**, *86*, 1813–1819. [[CrossRef](#)]
20. Palmese, M.; Trucco, A. Three-dimensional acoustic imaging by chirp zeta transform digital beamforming. *IEEE Trans. Instrum. Meas.* **2009**, *58*, 2080–2086. [[CrossRef](#)]
21. Palmese, M.; Trucco, A. An efficient digital CZT beamforming design for near-field 3-D sonar imaging. *IEEE J. Ocean. Eng.* **2010**, *35*, 584–594. [[CrossRef](#)]
22. Nguyen, T.N.; Giang, T.Q.; Van Hoa, P.; Hung, D.V. Design of frequency-domain beamforming solution for passive sonar systems in shallow-water environments. *J. Mil. Sci. Technol.* **2025**, *108*, 31–39. [[CrossRef](#)]
23. Yan, L.; Yang, J.; Xu, F.; Piao, S. Low-sidelobe imaging method utilizing improved spatially variant apodization for forward-looking sonar. *Remote Sens.* **2024**, *16*, 2100. [[CrossRef](#)]
24. Ning, M.; Zhong, H.; Tang, J.; Wu, H.; Zhang, J.; Zhang, P.; Ma, M. A Novel Chirp-Z Transform Algorithm for Multi-Receiver Synthetic Aperture Sonar Based on Range Frequency Division. *Remote Sens.* **2024**, *16*, 3265. [[CrossRef](#)]
25. Chen, P.; Tian, X.; Chen, Y. Optimization of the digital near-field beamforming for underwater 3-D sonar imaging system. *IEEE Trans. Instrum. Meas.* **2009**, *59*, 415–424. [[CrossRef](#)]
26. Houston, K.M. A fast beamforming algorithm. In *Proceedings of the OCEANS'94*; IEEE: New York, NY, USA, 1994; Volume 1, pp. I/211–I/216.
27. Chen, Y.; Ye, Q.; Huang, H. A Fast Imaging Algorithm Using Multistage-Focusing Beamforming. *J. Appl. Acoust.* **2008**, *27*, 207–211.
28. Hawkins, D.W. Synthetic Aperture Imaging Algorithms: With Application to Wide Bandwidth Sonar. Ph.D. Thesis, University of Canterbury, Christchurch, New Zealand, 1996.
29. Ning, M.; Tang, J.; Wu, H.; Zhong, H.; Ma, M. An efficient subswath-subband Chirp Z-Transform algorithm for multiple-receiver SAS considering the differential range curvature. *IEEE Trans. Geosci. Remote Sens.* **2025**, *63*, 5207319. [[CrossRef](#)]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.