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Better Not Forget: On the Memory of S&P 500 Survivor Stock Companies

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Abstract: This study explores the dependency structure of S&P 500 survivor stocks. Using a hand-collected sample of stocks that survived in the S&P 500 since March 1957, we employ rescaled/range analysis to investigate survivors. First, we find nonlinearities in the return processes of survivor stocks due to Paretian tails. Second, the return processes of very long-lived outliers exhibit long-term memories with Hurst exponents that significantly exceed one half on average. Third, sample-split tests reveal that the memory on average has virtually not changed over time—that is, survivor stocks do not forget. Fourth, and last, the long-term memory of survivor stocks appears to be unrelated to their exposures to traditional asset pricing risk factors.

Keywords: asset pricing; S&P 500 index; survivor stocks; Hurst exponent; Paretian tails

JEL Classification: G10; G12; G15; G19

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1. Introduction

The S&P 500 stock index is widely considered to be an important indicator of both the U.S. economy and the world economy. Constituent companies must pass strict criteria imposed by the Index Committee:[1] (1) primarily U.S. based, (2) market capitalization exceeding USD 8.2 billion, (3) highly liquid shares[2], (4) public trading of 50% or more of its outstanding shares, (5), positive earnings in the most recent quarter, and (6) a positive sum for the previous four quarters' earnings. Only very successful companies can fulfill these requirements. Chen and Lin (2018) observed that member companies benefit from reductions in financial constraints and a lower cost of equity. Other potential advantages are a reduction in information asymmetry due to greater scrutiny by investors, increased investor recognition as an industry leader, and a decrease in shadow costs (Denis et al. 2003; Chen et al. 2004; Cai 2007; Baran and King 2012; Chan et al. 2013). Over time, most constituent companies eventually do not pass these criteria and are dropped from the index. Relevant to the present study, Standard & Poor's published, on 2 March 2007, a list of companies that have been in the S&P 500 index since the index was launched in March 1957.

It is noteworthy that only 17% of the original constituent firms have survived over 50 years. According to West (2017), approximately half of U.S. publicly traded companies disappear within 10 years. Additionally, based on power laws, the mortality curve showed that the number of companies that have "died" after 50 years is virtually 100%.[3] Hence, it can be argued that companies that survive more than 50 years are outliers. West (2017) inferred that these long-lived outliers afford an opportunity to better understand the aging of companies. The fact that S&P 500 long-term survivor companies not only survived periods of severe economic stress over time but remained constituent S&P 500 firms is extraordinary.

Despite their exceptional survival ability, few studies have examined the general characteristics of these long-lived outliers. One study by Siegel and Schwartz (2006) investigates the long-term returns of the original S&P 500 constituent companies from March 1957 to December 2003. They find that their buy-and-hold returns outperformed those of the continuously updated S&P 500 index used by investment professionals. Another more recent study by Grobys (2022), based on a sample from March 1957 to December 2019, examines the risk-adjusted average excess return of a portfolio of S&P 500 survivor stocks by applying Fama and French (2015, 2018) multifactor models. The author finds that the portfolio of S&P 500 survivor stocks outperformed the S&P 500 index even after controlling for well-established asset pricing risk factors, which strongly supports the findings of Siegel and Schwartz (2006). Moreover, Grobys (2022) documents that, relative to the S&P 500 index, survivor companies tend to be, on average, small value stocks that exhibit high profitability and conservative capital investment. Paradoxically, further findings indicate that the returns of the survivor stocks portfolio are negatively correlated with momentum factor returns, which suggests that their returns more closely mimic losers rather than winners in momentum portfolios. Investigating the volatility process of the portfolio of S&P 500 survivor stocks provided evidence that, despite the fact that the general index consists of considerably more companies, the portfolio of survivor stocks is less exposed to extreme events compared to the general index.

Motivated by West (2017), who recommends future research on the aging process of long-lived outlier firms, the present paper explores the memory of return processes of S&P 500 survivor stock companies. Examining dependency structures across financial asset returns has been the subject of intense academic research. Unfortunately, as pointed out from Mandelbrot (2008), correlation-based methods perform poorly in the presence of Paretian tails and other nonlinearities. For this reason, we follow Mandelbrot (2008) by employing rescaled range (R/S) analysis to investigate the dependency structures of S&P 500 survivor stocks. While efficient stock markets typically exhibit Hurst exponents of $H \approx 0.50$, statistically significant deviations from this benchmark can provide novel insights on the dependency structure and memory features of the return processes of these long-lived outliers.

We contribute to the scant literature in this area in a number of important ways. First, we hypothesize that firm survival is a function of the complex interaction of multiple factors. Whereas earlier studies focus on abnormal returns (Siegel and Schwartz 2006) and derive portfolio-level characteristics from factor loadings for well-established risk factors (Grobys 2022), we examine the dependency structures in the return processes of these long-lived companies. Assuming that markets are efficient, strategic enterprise management decisions should be reflected in the memory of a company's return generating process. Second, most studies in finance-related research use correlation-based methods to examine potential dependency structures of financial assets. As Mandelbrot (2008) notes, the Nobel prize in Economics has been awarded half a dozen of times for finance research based on the concept of correlation. However, a problem exists in applying correlation-based methods, especially in the presence of nonlinearities. Lux and Alfarano (2016) document that Paretian tails appear to be a stylized fact for financial market data. More specifically, financial assets typically exhibit Paretian tails with power-law exponents close to $\alpha \approx 3$. The authors argue that the approximate cubic form of the power law of returns appears to be accepted as a universal feature of practically all types of financial markets, including equity and futures markets as well as foreign exchange (FX) and precious metal markets. This study contributes to this literature by: (1) examining whether these long-lived outliers exhibit Paretian tails; and (2) estimating the economic magnitude of power-law exponents. In this respect, statistically significant Paretian tails would suggest the presence of nonlinearities and, subsequently, invalidate the application of correlation-based methods to assess the dependency structures of financial assets.

Using a sample on long-term survivors from September 1934 until December 2019, we find that the returns of survivor stocks exhibit Paretian tails. Further results indicate that the cross-sectional Hurst exponent of survivor stocks exceeds $H = 0.5$ by a substantial margin, which suggests that price persistence appears to be a stylized fact of these long-lived outliers. Third, subsample analyses provide evidence that cross-sectional price persistence, as measured by the cross-sectional Hurst exponent, is virtually invariant across time (i.e., high reliability in this metric). Fourth, and last, we do not find any clear link between price persistence and exposures of survivors' returns to traditional asset pricing risk factors. While Hurst exponents are invariant over time, changes in factor loadings suggest that survivor stocks experienced higher distress risk in the second subsample.

The next section reviews our data. Section 3 provides background discussion. Section 4 presents the results of our statistical analyses. The last section provides the conclusion.

2. Background Discussion

In his seminal study, entitled "Portfolio selection", published in the *Journal of Finance*, Markowitz (1952) proposes a methodology to construct optimal portfolios. To provide an illustrative example, let us consider two assets A and B. Denoting the return-generating processes of A and B at time t as Ret_t^A and Ret_t^B, and provided $Ret_t^A \sim N(\mu_A, \sigma_A^2)$ and $Ret_t^B \sim N(\mu_B, \sigma_B^2)$, the correlation of these assets denoted as $\rho\left(Ret_t^A, Ret_t^B\right)$ is calculated by using the following equation:

$$\rho\left(Ret_t^A, Ret_t^B\right) = \frac{COV\left(Ret_t^A, Ret_t^B\right)}{\sqrt{VAR\left(Ret_t^A\right)}\sqrt{VAR\left(Ret_t^B\right)}}$$

Note that the variance is an integral part of this metric. In a recent study, Grobys (2023) points out that variances, covariances, and correlations are commonly used in finance studies for point estimation, hypothesis testing, portfolio optimization, risk management, etc. For example, in portfolio diversification the lack of correlation is important for minimizing portfolio risk. Modern portfolio theory, in the spirit of Nobel prize laureate Markowitz (1952), argues that the expectation of portfolio P consisting of assets A and B is calculated with

$$E\left(Ret_t^P\right) = w\mu_A + (1-w)\mu_B,$$

where w denotes the weight of wealth invested in A, and μ_A (μ_B) is the expected return of asset A (B). The portfolio variance, denoted as $VAR\left(Ret_t^P\right)$, is then

$$\begin{aligned} VAR\left(Ret_t^P\right) &= w^2 VAR\left(Ret_t^A\right) + (1-w)^2 VAR\left(Ret_t^B\right) \\ &+2w(1-w)COV\left(Ret_t^A, Ret_t^B\right). \end{aligned}$$

According to Markowitz's approach to portfolio selection for any given $VAR\left(Ret_t^A\right)$ and $VAR\left(Ret_t^B\right)$, portfolio variance $VAR\left(Ret_t^P\right)$ decreases as $COV\left(Ret_t^A, Ret_t^B\right)$ decreases, and assuming there are no negatively correlated assets, the portfolio risk is minimized if $\rho\left(Ret_t^A, Ret_t^B\right) = 0$. In his seminal study, this methodological framework is extended by accounting for multiple assets. In this regard, Mandelbrot (2008) observes:

> "Thus, with Markowitz's math, for each level of risk you contemplate you can devise an efficient portfolio that will yield the highest possible profit. And for each level of profit you target, there is an efficient portfolio with the lowest possible risk. If you plot all these portfolios on a graph, they form a smooth, rising curve: go-go and risky portfolios towards the top, boring and safe ones down below". (Mandelbrot 2008, p. 65)

There are, of course, advancements in the correlation-based approach pioneered by Markowitz (1952). For instance, Hatemi-J and El-Khatib (2015) argue that Markowitz's approach is successful in finding a portfolio that offers a minimum level of risk in the set comprising all available portfolios. The authors argue that this approach might not result in maximizing the expected return per unit of risk and, hence, propose an alternative approach for finding the necessary weights for portfolio diversification based on maximizing the risk adjusted return subject to the budget constraint. Their findings indicate that the resultant portfolio generates about a 1% increase in the risk adjusted return compared to the traditional approach. Whereas Hatemi-J and El-Khatib (2015) provide a closed form solution for a portfolio that consists of only two assets, Hatemi-J et al. (2022) extend their work by deriving an exact solution for the risk adjusted return of a portfolio that consists of any potential number of assets subject to the budget restriction. Applying their approach to total share prices indices for the three largest financial markets in the world—that is, the US, the Euro area, and China covering the period January 1999 to April 2019—the authors show that their approach is superior to benchmark models.[4]

The different approaches to portfolio optimization share one important commonality: correlation must be defined. The concept of correlation is not only used for portfolio optimization but for measuring (co)dependencies in the time dimension of assets. Often-used statistical tests are, for instance, (i) the Ljung–Box test for estimating the autocorrelation of returns (Ljung and Box 1978), (ii) the variance ratio test (Lo and Mackinlay 1988), (iii) the automatic variance test (AVR) (Choi 1999), and (iv) the BDS test (Broock et al. 1996). In his seminal 1963 study, Mandelbrot showed that the variance is not defined for cotton price changes. As shown previously, an undefined variance implies that the correlation is not defined; hence, methodologies based on correlation are not applicable. Even though Lux and Alfarano (2016) documented that the pertinent literature gradually converged to the insight that for most financial assets the tail exponents are about $\alpha \approx 3$, which implies that the theoretical variance exists, Taleb (2020) conjectured that, if the kurtosis is undefined, the second moment is unstable. A key manifestation of an undefined kurtosis is sample-specificity of reported research findings. Consequently, we cannot work with the variance even if it exists in the theoretical distribution.

Whereas correlation-based methodologies deliver sample-specific results, Mandelbrot (2008) contends that one of the principal virtues of the R/S statistic is that, unlike many common statistical tests, " ... it makes no assumption about how the original data are organized—a critical point when studying something like stock prices for which evidence abounds that the conventional assumptions are flatly wrong". (Mandelbrot 2008, p. 298) Hence, using the R/S statistic to assess dependency structures of financial assets should yield more reliable results as opposed to correlation-based methodologies.

3. Data

Following Grobys (2022), we gathered Standard & Poor's list of survivor companies in the S&P 500 index from March 1957 to March 2007. The list of these survivors is publicly available on the internet.[5] Details of data collection are discussed in Grobys (2022). Our total sample consists of 92 original constituent companies.[6] Using these companies, we collected CRSP monthly return series for all survivor companies with data available from September 1934 to December 2019 (i.e., 1024 consecutive monthly returns). This data restriction enables sufficient monthly observations for implementing R/S analysis to derive Hurst exponents. Overall, 34 out of 92 original constituent companies met these data requirements.[7]

4. Statistical Analyses

In this section, we conduct empirical analyses of sample firms to test for the presence of Paretian tails, memory of these long-lived outliers, and impacts of exposures to asset pricing factors on long-term memory.

4.1. Do the Returns on Long-Lived S&P 500 Outliers Exhibit Paretian Tails?

To estimate the economic magnitudes of the Paretian tails, we employed a data-driven method by Clauset et al. (2009). We began by estimating the following power-law function:

$$p(x) = Cy^{-\alpha}, \quad (1)$$

where $C = (\alpha - 1)y_{MIN}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, $y = |x|$ denotes the respective absolute return of a survivor stock provided $y \in \{\mathbb{R}_+ | y_{MIN} \leq y < \infty\}$, y_{MIN} is the minimum absolute return value that is governed by the power law process, and α is the magnitude of the firm-specific tail exponent.[8] To simplify notation, we avoided using an index i to reference survivor stock i. Next, following White et al. (2008) and Clauset et al. (2009), who pointed out that maximum likelihood estimation (MLE) performs best for estimating power law exponents, tail exponents are estimated as:

$$\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} \ln\left(\frac{y_i}{y_{MIN}}\right)\right)^{-1}, \quad (2)$$

where $\hat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding y_{MIN}, and other notation is as before. As Clauset et al. (2009) observed, an essential issue is how to determine the corresponding values for α and the cutoff to accurately estimate the probability density functions.

Based on Equation (2), the MLE estimator depends on the chosen cutoff and, subsequently, there are different MLE estimators from which to choose. In this regard, the authors comment that it is common practice to employ the $\hat{\alpha}/y_{MIN}$-plot and choose the value for y_{MIN} beyond which $\hat{\alpha}$ is stable. However, this approach is somewhat subjective and can be sensitive to noise or fluctuations in the tail of the distribution. For this reason, the authors proposed a data-driven method relying on optimizing the Kolmogorov–Smirnov (KS) distance equal to the maximum distance between the cumulative density functions (CDFs) of the data and the fitted model:

$$D = MAX_{y \geq y_{MIN}} |S(y) - P(y)|, \quad (3)$$

where $S(y)$ is the CDF of the data for the observation with value at least y_{MIN}, and $P(y)$ is the CDF for the power law model that best fits the data in the region $y \geq y_{MIN}$.[9] The estimate $\hat{y}_{MIN}$ is the value of y_{MIN} that minimizes D. The question arises whether the selected values for $\hat{\alpha}$ are statistically plausible. To answer this question, we employed a goodness-of-fit (GoF) test by Clauset et al. (2009). Specifically, these authors developed a GoF test that minimizes the distance between the power-law model and empirical data. Employing the parameter vector $(\hat{\alpha}, \hat{y}_{MIN})$ to optimize D, their GoF test generated a p-value that quantifies the plausibility of the power-law null hypothesis. Specifically, this test compares D with distance measurements for comparable synthetic data sets drawn from the hypothesized model, and the p-value is defined as the fraction of the synthetic distances that are larger than the empirical distance. Given a significance level of 5%, the power law null hypothesis is not rejected as the difference between the empirical data and the model can be attributed to statistical fluctuations.

Our results for 1024 monthly observations from September 1934 to December 2019 are reported in Table A1 in the Appendix A. As shown there, the estimated power-law exponents of survivor stocks vary between $\hat{\alpha} = 3.09$ and $\hat{\alpha} = 5.94$. Moreover, GoF tests reject the power-law null hypothesis for only three stocks in our sample (p-values < 0.05); however, the vast majority of survivor stocks are subject to Paretian tails.[10] Furthermore, the percentage of sample observations governed by power-law processes varies between 5% and 32%. Specifically, for 4 out of 34 stocks, more than 20% of sample observations, are governed by some power-law process generating fat tails in the return distributions and thus allowing for extreme events.

Because Clauset et al. (2009) showed that power-law exponents are normally distributed, the cross-sectional sample average of the power-law exponents must also be normally distributed. Here, the cross-sectional sample average is estimated at $\bar{\hat{\alpha}} = 4.20$ with a t-statistic of 29.52 indicating statistical significance at any level. The 99% confidence interval is estimated at $\bar{\alpha} \in [3.84; 4.57]$. Due to the presence of Paretian tails indicating nonlinearities, we infer that applications of correlation-based methods to examine potential dependency structures in return processes are questionable.

4.2. Is the Memory of Long-Lived Outliers Different from the Overall Equity Market?

4.2.1. Re-Scaled Range Analysis for the Overall Data Sample

Mandelbrot (2008) argues that one of the principal virtues of the R/S statistic is that, in contrast to many common statistical tests, " … it makes no assumption about how the original data are organized—a critical point when studying something like stock prices for which evidence abounds that the conventional assumptions are flatly wrong". (Mandelbrot 2008, p. 298) The R/S statistic is defined as

$$R/S_k = \frac{MAX_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \bar{x}_T\right) - MIN_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \bar{x}_T\right)}{\left[\frac{1}{T}\sum_j\left(x_j - \bar{x}_T\right)^2\right]^{1/2}}, \quad (4)$$

where average return $\bar{x}_T$ is calculated over the entire sample period T. For each subsample cluster j, the adjusted range $MAX_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \bar{x}_T\right) - MIN_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \bar{x}_T\right)$ is divided by the standard deviation $\left[\frac{1}{T}\sum_j\left(x_j - \bar{x}_T\right)^2\right]^{1/2}$. This difference is computed for $k \in \{4, 8, 16, 32, 64, 128, 256, 512\}$. Specifically, the estimate of the range from peak to trough in the accumulated deviations is simply computed by the differences between the corresponding maximum and minimum for each given k, which is the numerator in the above equation. The denominator is simply the standard deviation in the entire time series. Using Equation (4), we can estimate the Hurst exponent measuring the memory of an asset as follows:

$$\ln\left(\frac{R}{S}\right)_k = \ln(C) + H\ln(k) + u, \quad (5)$$

where $u \sim IID(0, \sigma_u)$. Hence, the estimated Hurst exponent $\hat{H}$ is obtained using log–log regression. Since there are $T = 1024$ observations and eight clusters for k, the reference test statistic is distributed as $t(6)$ with critical value for a 5% significance level corresponding to 1.94. According to Mandelbrot (2008), if the data were independent, the ratio between numerator and denominator should correspond to a Hurst exponent of $H = 0.50$. Moreover, $H > 0.50$ implies long-term dependence; that is, a long memory of the stochastic process in which the data are persistent. On the other hand, $H < 0.50$ implies anti-persistence, which is characterized by the tendency to revert back at a faster pace. For each survivor company, we tested the following hypothesis: $H_0 : H = 0.50$ versus $H_1 : H > 0.50$.

The results of the long-term dependence tests for the overall sample are reported in Table 1, whereas the corresponding tests for subsamples are reported in Tables 2 and 3. Hurst exponents in bold figures indicate statistical significance at the 5% level. We observed from Table 1 that 30 out of 34 survivor companies exhibit Hurst exponents that are statistically significantly greater than 0.50. As mentioned earlier, from a more general perspective, we can compute the average Hurst exponent across all survivor stocks. We find that the cross-sectional sample average is estimated at $\bar{\hat{H}} = 0.597$. In Table 4, we report the corresponding descriptive statistics for the cross-sectional Hurst exponent. We cannot reject the null hypothesis that the cross-sectional Hurst exponent is distributed as normal, i.e., the p-value of the Jarque–Bera test statistic equals 0.67. Additionally, the

99% confidence interval for is estimated at $\overline{H} \in [0.582; 0.612]$. Testing the hypothesis that $H_0 : \overline{H} = 0.50$ versus $H_1 : \overline{H} > 0.50$, the corresponding test statistic is $\hat{\lambda} = 16.95 > 1.65$ (p-value 0.0000). Using a significance level of 5%, we clearly reject the null hypothesis that $\overline{H} = 0.50$ for the universe of survivor companies, which implies statistical long-term dependence manifested in the return processes of survivor stocks.[11] This interesting finding is in line with earlier research on high-tech stocks. As Mandelbrot discusses:

> " . . . Peters of Pan Agora, reported in 1994 what appeared to be a complete, logical system of variation of *H* by asset type. High-tech stocks had high dependence and *H* values; stable utility shares had *H* values closer to those of a random walk. That meant the high-tech stocks were more volatile than conventional analysis tells us. Peters went on to argue that, for an investor, that made them a better bet because their price trends could be more easily perceived". (Mandelbrot 2008, p. 263)

Consistent with Peters, it could be argued that survivor companies are better bets for investors simply due to the fact that their price trends are more prominent than those for other stocks.

Table 1. Estimated Hurst exponents of S&P 500 survivor firms: evidence from the sample September 1934 to December 2019.

No.	1	2	3	4	5	6	7	8	9	10
H	**0.57**	**0.56**	**0.61**	**0.67**	**0.63**	**0.63**	0.53	**0.61**	**0.61**	**0.63**
Std. Dev	0.02	0.02	0.02	0.01	0.01	0.01	0.03	0.02	0.02	0.01
t-statistic	3.36	2.80	5.05	15.64	10.44	12.02	1.07	6.94	6.46	9.67
No.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
H	0.54	**0.64**	**0.60**	**0.63**	**0.61**	0.54	0.55	**0.57**	**0.59**	**0.58**
Std. Dev	0.03	0.01	0.03	0.02	0.02	0.02	0.03	0.02	0.02	0.02
t-statistic	1.38	13.18	3.86	7.49	6.28	1.89	1.69	3.66	4.62	3.51
No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
H	**0.61**	**0.63**	**0.60**	**0.59**	**0.58**	**0.64**	**0.60**	**0.60**	0.55	**0.59**
Std. Dev	0.02	0.01	0.02	0.02	0.02	0.01	0.01	0.02	0.03	0.02
t-statistic	6.74	15.13	5.96	3.86	4.14	14.71	8.37	5.85	1.74	5.92
No.	**31**	**32**	**33**	**34**						
H	**0.57**	**0.64**	**0.61**	**0.59**						
Std. Dev	0.02	0.00	0.02	0.01						
t-statistic	2.99	28.37	4.76	6.51						

Using Standard & Poor's press release form for 2 March 2007, wherein the index provider published a list of survivor companies in the S&P 500 index from March 1957 to March 2007, 34 survivor stock companies where identified as having available monthly stock return data in the CRSP database from September 1934 to December 2019. Using 1024 monthly observations, the Hurst exponents of these stock companies are estimated by using the R/S statistic defined as:

$$R/S_k = \frac{MAX_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \overline{x}_T\right) - MIN_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \overline{x}_T\right)}{\left[\frac{1}{T}\sum_j\left(x_j - \overline{x}_T\right)^2\right]^{1/2}},$$

where average return $\overline{x}_T$ is calculated over the whole sample period T. For each subsample cluster, the difference between variance x_j over that period and average return $\overline{x}_T$ is calculated while keeping a running total of all the differences as the time period lengthens to period k. This difference value is computed for $k \in \{4, 8, 16, 32, 64, 128, 256, 512\}$ and then the maximum of all differences (MAX) is identified. The estimate of the range from peak to trough in the accumulated deviations is computed by the differences between the corresponding maximum and minimum, which is the numerator of the equation above. The denominator is simply the standard deviation of the overall time series. If the data are independent, the ratio between numerator and denominator according to Mandelbrot (2008) should be 1:2, which corresponds to a Hurst exponent of $H = 0.50$. Moreover, $H > 0.50$ implies long-term dependence; that is, a long memory of the stochastic process in which the data are persistent. On the other hand, $H < 0.50$ implies anti-persistence, which is characterized by the tendency to keep back on themselves. The theory posits:

$$\ln\left(\frac{R}{S}\right)_k = \ln(C) + H\ln(k) + u,$$

where $u \sim IID(0, \sigma_u)$. Hence, the estimated Hurst exponent H is obtained using log–log regression. This table reports the estimated Hurst exponents (H) for each survivor company and the test statistic for the corresponding hypothesis test:

$$H_0 : H = 0.50 \text{ versus } H_1 : H > 0.50.$$

Since we have $T = 1024$ observations and eight clusters for k, the reference test statistic is distributed as $t(6)$ with a critical value for a 5% significance level corresponding to 1.94. Hurst exponents in bold figures indicate statistical significance on at least a 5% level.

4.2.2. Re-Scaled Range Analysis for Two Nonoverlapping Subsamples

To explore whether the memory in the universe of survivor companies has been subject to change over time and related reliability of this metric, we split the overall subsample into two nonoverlapping subsamples of equal length. Specifically, the first subsample is from September 1934 to May 1977, whereas the second subsample is from June 1977 to December 2019. Again, we use the R/S statistic as in Equation (4) with $\in \{4, 8, 16, 32, 64, 128, 256\}$. Since there are $T = 512$ observations and seven clusters for k, the reference test statistic is distributed as $t(5)$ with critical value for a 5% significance level corresponding to 2.02. For each subsample and the return process of every survivor stock, we test the hypothesis $H_0 : H = 0.50$ versus $H_1 : H > 0.50$.

From the results shown in Table 2 (Table 3) we see that 33(34) out of 34(34) survivor stocks exhibit estimated Hurst exponents that are significantly larger than 0.50 in the first (second) subsample. In unreported results to conserve space, the second subsample's $\hat{H}$ estimate for 21 out of 34 stocks falls into the 95% confidence interval for the corresponding estimates of the first subsample. As before, from a more general perspective, we estimated the cross-sectional Hurst exponents for both subsamples. The cross-sectional sample average for the first subsample is estimated at $\bar{\hat{H}} = 0.626$. From Table 4, we observed that the null hypothesis that the cross-sectional Hurst exponents are normally distributed cannot be rejected for both subsamples, as p-values for the Jarque–Bera test statistics are estimated at 0.45 and 0.42, respectively. For the first subsample, the 99% confidence interval is estimated at $\bar{H} \in [0.608; 0.643]$. The cross-sectional sample average for the second subsample is estimated at $\bar{\hat{H}} = 0.604$. For the second subsample, the 99% confidence interval for is estimated at $\bar{H} \in [0.593; 0.615]$. Even if the point estimate for the second subsample (viz., $\bar{\hat{H}} = 0.604$) does not fall into the confidence interval for the first subsample, we see that both confidence intervals are overlapping to a relatively high degree. We interpret this evidence to mean that the memory of the population of survivor companies has virtually not changed over time.

Table 2. Estimated Hurst exponents of S&P 500 survivor firms: evidence from the sample September 1934 to May 1977.

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
H	**0.60**	**0.61**	**0.67**	**0.70**	**0.68**	**0.67**	**0.56**	**0.61**	**0.60**	**0.64**
Std. Dev	0.02	0.02	0.01	0.01	0.01	0.01	0.03	0.03	0.02	0.01
t-statistic	5.07	4.45	19.95	16.69	25.15	25.98	2.22	4.14	4.57	9.79
No.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
H	**0.58**	**0.66**	**0.64**	**0.66**	**0.66**	**0.57**	**0.59**	**0.60**	**0.65**	**0.62**
Std. Dev	0.03	0.01	0.02	0.02	0.01	0.03	0.03	0.02	0.01	0.02
t-statistic	3.22	12.87	8.03	9.00	19.97	2.57	3.39	4.91	11.95	6.71
No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
H	**0.61**	**0.68**	**0.61**	**0.62**	0.56	**0.68**	**0.64**	**0.61**	**0.61**	**0.60**
Std. Dev	0.02	0.01	0.02	0.02	0.04	0.01	0.02	0.02	0.03	0.03
t-statistic	5.23	21.88	5.96	5.53	1.59	21.05	8.32	7.01	4.18	2.90
No.	**31**	**32**	**33**	**34**						
H	**0.60**	**0.67**	**0.66**	**0.56**						
Std. Dev	0.02	0.01	0.01	0.03						
t-statistic	4.34	26.41	11.39	2.32						

Using Standard & Poor's press release form for 2 March 2007, wherein the index provider published a list of survivor companies in the S&P 500 index from March 1957 to March 2007, 34 survivor stocks companies where identified having available monthly stock return data in the CRSP database from September 1934 to May 1977. Using 512 monthly observations, the Hurst exponents of these stock companies are estimated by using the R/S statistic defined as:

$$R/S_k = \frac{MAX_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \bar{x}_T\right) - MIN_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \bar{x}_T\right)}{\left[\frac{1}{T}\sum_j\left(x_j - \bar{x}_T\right)^2\right]^{1/2}},$$

where average return $\bar{x}_T$ is calculated over the whole sample period T. For each subsample cluster, the difference between variance x_j over that period and average return $\bar{x}_T$ is calculated while keeping a running total of all the differences as the time period lengthens to period k. This difference value is computed for $k \in \{4, 8, 16, 32, 64, 128, 256\}$ and then the maximum of all differences (MAX) is identified. The estimate of the range from peak to trough in the accumulated deviations is computed by the differences between the corresponding maximum and minimum, which is the numerator of the equation above. The denominator is simply the standard deviation of the overall time series. If the data are independent, the ratio between numerator and denominator according to Mandelbrot (2008) should be 1:2, which corresponds to a Hurst exponent of $H = 0.50$. Moreover, $H > 0.50$ implies long-term dependence; that is, a long memory of the stochastic process in which the data are persistent. On the other hand, $H < 0.50$ implies anti-persistence, which is characterized by the tendency to keep back on themselves. Theory posits:

$$\ln\left(\frac{R}{S}\right)_k = \ln(C) + H\ln(k) + u,$$

where $u \sim IID(0, \sigma_u)$. Hence, the estimated Hurst exponent H is obtained using log–log regression. This table reports the estimated Hurst exponents (H) for each survivor company and the test statistic for the corresponding hypothesis test:

$$H_0 : H = 0.50 \text{ versus } H_1 : H > 0.50.$$

Since we have $T = 1024$ observations and eight clusters for k, the reference test statistic is distributed as $t(5)$ with critical value for a 5% significance level corresponding to 1.94. Hurst exponents in bold figures indicate statistical significance on at least a 5% level.

Table 3. Estimated Hurst exponents of S&P 500 survivor firms: evidence from the sample June 1977 to December 2019.

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
H	**0.58**	**0.58**	**0.63**	**0.60**	**0.62**	**0.62**	**0.58**	**0.61**	**0.63**	**0.64**
Std. Dev	0.02	0.02	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.01
t-statistic	4.47	3.96	10.01	7.41	8.71	7.63	3.23	7.28	6.39	9.79
No.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
H	**0.58**	**0.62**	**0.64**	**0.55**	**0.62**	**0.57**	**0.62**	**0.57**	**0.59**	**0.62**
Std. Dev	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.02
t-statistic	4.46	6.15	6.46	2.08	7.20	2.71	6.46	2.69	3.44	6.71
No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
H	**0.65**	**0.58**	**0.58**	**0.62**	**0.62**	**0.62**	**0.58**	**0.63**	**0.60**	**0.60**
Std. Dev	0.02	0.02	0.03	0.02	0.01	0.02	0.02	0.02	0.02	0.03
t-statistic	7.39	3.69	2.85	5.61	11.69	6.68	4.57	8.51	3.94	2.90
No.	**31**	**32**	**33**	**34**						
H	**0.59**	**0.62**	**0.62**	**0.60**						
Std. Dev	0.03	0.01	0.03	0.02						
t-statistic	3.76	11.49	3.71	6.39						

Using Standard & Poor's press release form for 2 March 2007, wherein the index provider published a list of survivor companies in the S&P 500 index from March 1957 to March 2007, 34 survivor stocks companies where identified having available monthly stock return data in the CRSP database from June 1977 to December 2019. Using 512 monthly observations, the Hurst exponents of these stock companies are estimated by using the R/S statistic defined as

$$R/S_k = \frac{MAX_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \bar{x}_T\right) - MIN_{1\le k\le T}\sum_{j=1}^{k}\left(x_j - \bar{x}_T\right)}{\left[\frac{1}{T}\sum_j\left(x_j - \bar{x}_T\right)^2\right]^{1/2}},$$

where average return $\bar{x}_T$ is calculated over the whole sample period T. For each subsample cluster, the difference between variance x_j over that period and average return $\bar{x}_T$ is calculated while keeping a running total of all the differences as the time period lengthens to period k. This difference value is computed for $k \in \{4, 8, 16, 32, 64, 128, 256\}$ and then the maximum of all differences (MAX) is identified. The estimate of the range from peak to trough in the accumulated deviations is computed by the differences between the corresponding maximum and minimum, which is the numerator of the equation above. The denominator is simply the standard deviation of the overall time series. If the data are independent, the ratio between numerator and denominator according to Mandelbrot (2008) should be 1:2, which corresponds to a Hurst exponent of $H = 0.50$. Moreover, $H > 0.50$ implies long-term dependence; that is, a long memory of the stochastic process in which the data are persistent. On the other hand, $H < 0.50$ implies anti-persistence, which is characterized by the tendency to keep back on themselves. Theory posits:

$$\ln\left(\frac{R}{S}\right)_k = \ln(C) + H\ln(k) + u,$$

where $u \sim IID(0, \sigma_u)$. Hence, the estimated Hurst exponent H is obtained using log–log regression. This table reports the estimated Hurst exponents (H) for each survivor company and the test statistic for the corresponding hypothesis test:

$$H_0 : H = 0.50 \text{ versus } H_1 : H > 0.50.$$

Since we have $T = 1024$ observations and eight clusters for k, the reference test statistic is distributed as $t(5)$ with critical value for a 5% significance level corresponding to 1.94. Hurst exponents in bold figures indicate statistical significance on at least a 5% level.

Table 4. The cross-sectional distribution of Hurst exponents.

Sample	Whole Sample	First Subsample	Second Subsample
Mean	0.60	0.63	0.60
Median	0.60	0.62	0.61
Maximum	0.67	0.70	0.65
Minimum	0.53	0.56	0.55
Std. Dev.	0.03	0.04	0.02
Skewness	−0.20	0.07	−0.29
Kurtosis	2.37	1.94	2.06
Jarque–Bera	0.80	1.62	1.72
p-value	0.67	0.45	0.42

This table reports the descriptive statistics for cross-sectional Hurst exponents estimated for 34 survivor stocks in the S&P 500 index. The sample period is from September 1939 to December 2019. First and second subsample periods are from September 1939 to May 1977 and from June 1977 to September 2019. The reported p-values correspond to the Jarque–Bera test statistic, which assumes normality under the null hypothesis.

4.3. Is the Long-Term Memory Manifested in Exposures to Asset Pricing Risk Factors?

Lastly, we provide tests of whether the documented long-term memory of survivor stocks is reflected in exposures to well-known asset pricing risk factors. Following standard practice, we estimated factor risk exposures to the market factor (*MKT*), size factor (*SMB*), value factor (*HML*), and momentum factor (*MOM*) by regressing the excess returns for each survivor stock (R_i^{ex}) on the Carhart (1997) four-factor model that adds a momentum factor to the Fama and French (1992) three-factor model:

$$R_{i,t}^{ex} = a_i + b_i MKT_t^{ex} + c_i SMB_t + d_i HML_t + e_i MOM_t + \epsilon_{it}, \quad (6)$$

where ϵ_{it} is an IID-distributed error term. Data were downloaded from Kenneth French's website.

Panel A of Table A2 reports the descriptive statistics for the cross-sectional point estimates for the first subsample (September 1939−May 1977), and Panel B reports the results for the second subsample (June 1977−September 2019). Panel A shows that the sample average of the intercept term equals 0.37% per month (or 4.4% per year), which is economically meaningful. Hence, the risk factors do not explain a significant portion of the excess returns of survivor stocks. This finding confirms Grobys (2022), who finds that survivor companies generate significant excess returns after risk-adjusting the payoffs using various well-known asset pricing factor models. Depending on the factor model, in the sample period July 1963 to December 2019, survivors exhibited between 0.21% and 0.43% risk-adjusted monthly average payoffs.

It is noteworthy that the beta risk exposure with respect to the market risk factor is 1.00. Thus, on average, survivor stocks generated excess returns without being leveraged against the market risk factor. Additionally, the results from the first subsample indicate that, with the exception of the market factor, survivor stocks are not significantly exposed to any asset pricing risk factor. Contrary to these findings, the results in the second subsample suggest that survivor stocks exhibit, on average, a high book-to-market ratio. In this regard, the significant positive exposure (i.e., $\hat{d} = 0.29$) to this factor implies that survivor stock returns positively co-move with value stock returns as opposed to growth stock returns. Some researchers have conjectured that value stocks have relatively higher distress risk than other stocks (e.g., see Griffin and Lemmon 2002). If so, we infer that survivor stocks experienced higher distress risk in the second subsample due to changing economic conditions. The lower intercept term in the second subsample (i.e., lower risk-adjusted) is consistent with this explanation.

We should point out that estimates obtained from the regression model in Equation (6) are derived from the concept of correlation, which requires linear dependency structures to deliver reliable results. As an example, let us consider the point estimator for b_i from Equation (6), which we can rewrite as follows:

$$b_i = \frac{COV(MKT_t^{ex}, R_{it}^{ex})}{VAR(MKT_t^{ex})} = \frac{\rho_{MKT_t^{ex}, R_{it}^{ex}} \sqrt{VAR(R_{it}^{ex})}}{\sqrt{VAR(MKT_t^{ex})}}, \quad (7)$$

where $\rho_{MKT_t^{ex}, R_{it}^{ex}}$ denotes the correlation between excess market factor returns and excess stock returns for the ith survivor stock. Since the results in Table A1 suggest that survivor stocks are exposed to Paretian tails, or nonlinearity, the concept of correlation is unreliable. Hence, it is not surprising that no clear pattern emerges on how long-term memory, as measured by the Hurst exponent, is reflected in some risk factor exposures. Whereas exposures to risk factors change across subsamples, the average Hurst exponent lacks this kind of serious variation across samples and therefore provides a higher level of reliability.

4.4. Limitations

This study employs Mandelbrot's (2008) R/S analysis, which is based on earlier works published in Mandelbrot (1963, 1969, 1971, 1972) and Mandelbrot and Wallis (1969). However, other related methodologies are available in the literature. For instance, Peng et al. (1994) proposed detrended fluctuation analysis (DFA), which is designed to effectively detect the long memory of signals with polynomial trend. Reviews of other methods for estimating long range dependencies are provided in Taqqu et al. (1995), Montanari et al. (1999), and Serinaldi (2010). However, as mentioned earlier, a major advantage of R/S statistic is that, in contrast to many common statistical tests, no assumption is made about how the original data are organized. Even though R/S analysis is a well-established and often-used methodology, future research is encouraged to replicate our findings using other methodologies. Since these analyses are beyond the scope of this paper, they are left for future research.

5. Conclusions

This study examined the dependency structures of the return processes of S&P 500 survivor stocks. To do this, we employed rescaled/range analysis proposed by Mandelbrot (2008). Unlike correlation-based methodologies, R/S analysis is a valid approach for measuring dependencies regardless of how the data are organized.

We found that survivor stock returns are organized in a non-linear manner, which is manifested in Paretian tails. Survivor stock returns exhibited, on average, long-term memory as evidenced by an average Hurst exponent significantly greater than one half. Splitting the overall subsample into two subsamples revealed that long-term memory was stable across time. We interpret this finding to mean that "survivor stocks do not forget". By contrast, exposures to traditional asset pricing risk factors were *not* stable across time. This instability is not surprising in view of the way stock return data are organized. As observed by Mandelbrot (2008), Peters of Pan Agora found that high-tech stocks had high dependence manifested in higher H values than other stocks, which makes them a better bet for investors due to more readily perceived price trends. Similarly, this study showed that survivor stocks exhibit high dependence manifested in H values. Consistent with Siegel and Schwartz (2006) and Grobys (2022), these findings help explain why they have been a good bet for investors. Given the fact that they remained in the S&P 500 index for more than 50 years, their excess risk-adjusted stock returns are partially attributable to their long memory.

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Appendix A

Table A1. Estimated power-law exponents for survivor stocks.

No.	1	2	3	4	5	6	7	8	9	10
$\hat{\alpha}$	**4.17**	**4.07**	**3.52**	**3.92**	**4.09**	**4.82**	**3.89**	**4.39**	**3.19**	**4.54**
y_{MIN}	0.12	0.11	0.17	0.08	0.10	0.09	0.10	0.08	0.09	0.11
p-value GoF	0.09	0.06	0.61	0.00	0.66	1.00	0.23	0.19	0.02	0.26
% (N)	0.09	0.14	0.13	0.15	0.08	0.06	0.17	0.13	0.28	0.10
No.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
$\hat{\alpha}$	**4.32**	**3.80**	**3.19**	**3.09**	**4.05**	**4.01**	**4.91**	**3.50**	**3.10**	**4.75**
y_{MIN}	0.13	0.08	0.07	0.11	0.09	0.11	0.19	0.06	0.07	0.11
p-value GoF	0.19	0.32	0.69	0.50	0.76	0.12	0.91	0.00	0.00	0.70
% (N)	0.10	0.20	0.21	0.15	0.18	0.13	0.05	0.29	0.32	0.08
No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
$\hat{\alpha}$	**4.56**	**3.87**	5.94	4.54	7.25	**4.44**	**3.48**	**3.70**	**4.20**	**3.99**
y_{MIN}	0.19	0.09	0.18	0.17	0.13	0.14	0.08	0.12	0.12	0.10
p-value GoF	0.69	0.06	0.83	0.88	0.98	0.65	0.06	0.18	0.76	0.87
% (N)	0.07	0.16	0.03	0.04	0.03	0.10	0.18	0.16	0.13	0.16
No.	**31**	**32**	**33**	**34**						
$\hat{\alpha}$	**4.51**	**5.27**	**4.45**	**3.38**						
y_{MIN}	0.16	0.13	0.19	0.11						
p-value GoF	0.85	0.81	0.51	0.82						
% (N)	0.06	0.06	0.07	0.18						

To estimate power-law exponent that govern Paretian tails of survivor stocks, we use the following power-law function:

$$p(y) = Cy^{-\alpha},$$

where $C = (\alpha - 1){y_{MIN}}^{\alpha-1}$ with $\alpha \in \{\mathbb{R}_+ | \alpha > 1\}$, $y = |x|$ denotes the respective absolute return of a survivor stock provided $y \in \{\mathbb{R}_+ | y_{MIN} \leq y < \infty\}$, y_{MIN} is the minimum absolute return value that is governed by the power law process, and α is the magnitude of the specific tail exponent. Following White et al. (2008) and Clauset et al. (2009), maximum likelihood estimation (MLE) is used for estimating power law exponents:

$$\hat{\alpha} = 1 + N\left(\sum_{i=1}^{N} \ln\left(\frac{y_i}{y_{MIN}}\right)\right)^{-1},$$

where $\hat{\alpha}$ denotes the MLE estimator, N is the number of observations exceeding y_{MIN}. The cutoff is determined by optimizing the Kolmogorov–Smirnov (KS) distance. Specifically, the Kolmogorov–Smirnov (KS) distance is the maximum distance between the cumulative density functions (CDFs) of the data and the fitted model as defined by:

$$D = MAX_{y \geq y_{MIN}} |S(y) - P(y)|,$$

where $S(y)$ is the CDF of the data for the observation with value at least y_{MIN}, and $P(y)$ is the CDF for the power law model that best fits the data in the region $y \geq y_{MIN}$. The estimate $\hat{y}_{MIN}$ is the value of y_{MIN} that minimizes D. This table reports the estimated power-law exponent for absolute monthly returns for our sample of survivor stocks from September 1934 to December 2019, the corresponding minimum value y_{MIN}, the p-value for the GoF test of the power-law null hypothesis, and the percentage of sample observations governed by some power-law process. Power-law exponents in bold figures indicate statistical significance on at least a 5% level.

Table A2. Estimated risk factor exposures for survivor stocks.

Panel A. Regression Estimates for the First Subsample					
Point Estimate	***a***	***b***	***c***	***d***	***e***
Mean	0.37 ***	1.00 ***	0.07	0.02	−0.03 *
(t-statistic)	(8.25)	(26.24)	(1.14)	(0.06)	(−1.68)
Median	0.39	1.00	−0.02	−0.02	−0.06
Maximum	0.81	1.45	0.86	0.89	0.31
Minimum	−0.21	0.52	−0.61	−0.56	−0.19
Std. Dev.	0.26	0.22	0.37	0.36	0.11
Skewness	−0.20	−0.08	0.49	0.58	0.94
Kurtosis	2.52	2.43	2.79	3.04	3.93
Jarque–Bera	0.55	0.50	1.41	1.94	6.20
p-value	0.76	0.78	0.49	0.38	0.04
Panel B. Regression Estimates for the Second Subsample					
Point Estimate	***a***	***b***	***c***	***d***	***e***
Mean	0.22 ***	0.91 ***	−0.11 *	0.29 ***	−0.08 *
(t-statistic)	(3.91)	(16.09)	(−1.85)	(6.39)	(−1.85)
Median	0.24	0.91	−0.19	0.30	0.03
Maximum	0.91	1.72	1.32	0.97	0.17
Minimum	−0.45	0.33	−0.48	−0.41	−0.94
Std. Dev.	0.32	0.33	0.34	0.27	0.26
Skewness	−0.35	0.24	2.34	0.10	−1.54
Kurtosis	2.61	2.67	10.24	3.82	5.08
Jarque–Bera	0.91	0.49	105.23	1.01	19.59
p-value	0.64	0.78	0.00	0.60	0.00

Note: *** statistically significant on a 1% level. * statistically significant on a 10% level.

We estimate risk factor exposures of survivor stocks to the excess market factor (MKT_t^{ex}), size factor (SMB_t), value factor (HML_t), and momentum factor (MOM_t) by regressing the excess returns for each survivor stock i ($R_{i,t}^{ex}$) on the Fama and French (1992) three-factor model:

$$R_{i,t}^{ex} = a_i + b_i MKT_t^{ex} + c_i SMB_t + d_i HML_t + e_i MOM_t + \epsilon_{it},$$

where ϵ_{it} is an IID-distributed error term. Data were downloaded from Kenneth French's website. Panel A reports the descriptive statistics for the cross-sectional point estimates for the first subsample from September 1939 to May 1977. Panel B reports the corresponding figures for the second subsample from June 1977 to September 2019. The reported p-values correspond to the Jarque–Bera test statistic which assumes normality under the null hypothesis.

Notes

[1] As of February 2019 guidance.

[2] The value of a stock's market capitalization traded annually should be at least a quarter million dollars of its shares in each of the previous six months.

[3] West (2017) uses the term death in the context of companies that do not report sales anymore. Using this definition, companies can die due to various reasons, including mergers, splits, and liquidation.

[4] Another recent study is by Zhang et al. (Zhang et al.), who study the dynamic portfolio allocation problem using an interval type-2 fuzzy set to express and manipulate uncertainty. In doing so, the decision strategy with competitive-cum-compensatory is embedded in optimization. Their findings indicate that their proposed approach is more accurate than classical fuzzy sets in describing the uncertainty of asset information. Available at: https://www.globalpapermoney.com/s-p-releases-list-of-86-companies-in-the-s-p-500-since-1957-cms-1023 (accessed on 15 January 2023).

[5] See https://www.globalpapermoney.com/s-p-releases-list-of-86-companies-in-the-s-p-500-since-1957-cms-1023 (accessed on 15 January 2023).

[6] As detailed in Grobys (2022), in the data collection process, the survivor list of company names is matched to ticker symbols and stock returns in the CRSP database.

[7] Note that the goal with respect to the data collection is to identify those companies that have been in the index since the index was launched in 1957. Identifying survivor stocks is per se a challenge due to spin-offs, mergers, acquisitions, etc. (see Siegel and Schwartz 2006). Here we follow Grobys (2022) in selecting survivor stocks identified by the index provider S&P. The index provider published a press release in 2007 wherein the original constitute companies were listed. From this list, we used the approach detailed in Grobys (2022) to identify and match the sample of firms listed on S&P's release with corresponding stock companies, which resulted in a sample of 92 survivor stocks. From this sample of survivor stocks, we identified 34 stock companies with available data over 1024 consecutive months starting in September 1934.

[8] Note that the main objective for choosing the power-law function given by Equation (1) is to identify whether the return processes exhibit Paretian tails. In this regard, we follow previous literature. For instance, Lux and Alfarano (2016) highlighted that: "Focusing on absolute returns, |ret|, ... [is] one of its most frequently analyzed manifestations". (Lux and Alfarano 2016, p. 5). We consider this analysis as a necessary presumption for choosing R/S analysis as opposed to correlation-based methods. For modeling the overall return-generating process, Mandelbrot (2008) proposed a multifractal model of asset returns (MMAR). Modelling the entire stock return dynamics is outside the scope of our study and therefore left for future studies.

[9] Note that the selection of proper cutoffs is somewhat ambiguous as various estimation techniques can deliver different results. In this regard, Lux (2000) commented: "In view of these problems of implementations, the recent development of methods for data-driven selection of the tail sample constitutes an important advance". (Lux 2000, p. 646) In our study, we follow Lux by adopting a data-driven approach. While he used optimized mean squared error functions, we employed KS-distances, which offers the benefit of implementing a directly-related GoF test as proposed by Clauset et al. (2009).

[10] Taleb (2010) points out that a long time is needed for some fractal processes to reveal their properties, such that theoretical means are underestimated in finite samples. Hence, even though we do not find evidence for Paretian tails for some of the stocks *in-sample*, it is possible that Paretian tails will be manifested in subsequent *out-of-sample* samples.

[11] To explore the Hurst exponent for the overall U.S. equity market, for the sample period from September 1934 to December 2019, we downloaded 30 equal-weighted industry portfolios from Kenneth French's website (see https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, accessed on 15 January 2023). Subsequently, we constructed an equal-weighted market index based on an equal-weighted portfolio of industry portfolios. The estimated Hurst exponent for this sample is exactly $\hat{H} = 0.50$. This result is in line with earlier research that finds the returns of efficient equity markets are independently distributed.

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