Journal of
Risk and Financial Management

Article

Production Efficiency and Income Distribution with Competition Induced by Antitrust Measures

Peter Josef Stauvermann [1] and Ronald Ravinesh Kumar [2,*]

1 Department of Global Business and Economics, Changwon National University, Changwon 51142, Republic of Korea; pjsta@changwon.ac.kr
2 Department of Economics and Finance, The Business School, Saigon South Campus, RMIT University, Ho Chi Minh City 700000, Vietnam
* Correspondence: ronald.kumar@rmit.edu.vn

Abstract: We use a three-sector overlapping generations model to examine the efficiency characteristics and income distribution in the long-run steady-state equilibrium. Assuming two sectors produce intermediate goods within an oligopolistic competition, we explore the implications for production efficiency and income distribution, given an increase in competition induced by antitrust measures. Our analysis presents the possibility of steady-state welfare under imperfect competition surpassing that of perfect competition when declining competition leads to a redistribution of income from older to younger generations. Nevertheless, greater competition (within oligopoly competition) consistently results in a more equitable income distribution.

Keywords: oligopolistic competition; market power; overlapping generations model; two-sector model

check for updates

Citation: Stauvermann, Peter Josef, and Ronald Ravinesh Kumar. 2023. Production Efficiency and Income Distribution with Competition Induced by Antitrust Measures. *Journal of Risk and Financial Management* 16: 399. https://doi.org/10.3390/jrfm16090399

Academic Editors: Svetlozar (Zari) Rachev and Apostolos G. Christopoulos

Received: 9 August 2023
Revised: 3 September 2023
Accepted: 4 September 2023
Published: 6 September 2023

Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

There has been a decline in the labor income share in many developed countries over the last 40 years (Autor et al. 2017, 2019; Elsby et al. 2013; Karabarbounis and Neiman 2014, 2018). A decline in labor's share of income occurs when wages tend to grow slower than productivity, which is the output per hour of work. It must be noted that some productivity gains are absorbed by capital (Dao et al. 2017). The decline in the labor share is largely attributed to rapid progress in information and communication technology and the expansion of global value chains supported by automation. As noted by the IMF report (Dao et al. 2017; IMF 2017), both technology and global integration explain about 75 percent of the decline in the labor share in Germany and Italy, and about 50 percent in the United States. In the case of emerging markets, the decline in the labor share is attributed to the shift in production to more capital-intensive activities.

Moreover, Manyika et al. (2019) argue that the decline in the labor share of income goes beyond globalization and technological advancement. Using the USA as a case study, they highlight that the decline in the labor share of income since 1999 was due to: (a) super cycles and boom–bust periods (33%) (in extractive and real estate sectors), rising depreciation and shifts to intangibles and intellectual property products (IPP) capital (26%), superstar effects and consolidation (18%), capital substitution and technology (12%), and globalization and labor bargaining power (11%).

According to Velasquez (2023), the decline in the labor share can be attributed to various possible explanations. However, the two narratives that have garnered the most attention are the so-called 'capital accumulation' narrative, initially highlighted by Rognlie (2015), and the 'market power' narrative.

The underlying concept of the capital accumulation narrative posits that shifts in the rate of capital accumulation or increased capital productivity have altered income distribution, favoring capital over labor. This explanation holds true only if capital and labor are considered cross-substitutes, a condition met when the elasticity of substitution between capital and labor exceeds one. Nonetheless, Velasquez (2023) demonstrates that the elasticity of substitution in the USA falls within the range of 0.6 to 1, a finding consistent with the results of Herrendorf et al. (2015), Oberfield and Raval (2021), and several others (Knoblach et al. 2019; Knoblach and Stöckl 2020). Consequently, the credibility of the capital accumulation narrative becomes questionable.

The second prominent argument is the market power narrative, which suggests that firms' market power has increased in recent decades (Akcigit et al. 2021; Bajgar et al. 2019; De Loecker et al. 2020; Philippon 2019; Barkai 2020; Barkai and Benzell 2018; Affeldt et al. 2021; Velasquez 2023; Zingales 2012). While the former authors support their claims with empirical research, Zingales provides a range of political–economic explanations, including lobbying, corruption, regulatory capture, and other factors, to account for the rise in firms' market power.

In this study, we investigate whether antitrust measures aimed at promoting market competition can effectively improve production efficiency, enhance long-run welfare, and achieve a more equitable distribution of income between workers and firm owners. The approach used in our study is inspired by Laitner (1982), although in the case of Laitner's model, there is only one oligopolistic sector and the members of the older generation own the firms and receive the associated profits. In our model, we consider a simple overlapping generation (OLG) model with two oligopolistic intermediate goods sectors, a competitive final goods sector, and firms owned by the younger generation. Laitner's (1982) model was reexamined by Stauvermann and Kumar (2022) with the assumption that the younger generation owns the firms, however, with one oligopolistic sector and one perfectly competitive sector in the economy.

In this paper, we assume the Cournot–Nash behavior of firms and allow for a sufficient increase in the number of firms to achieve outcomes such as those expected in perfectly competitive markets. An advantage of this approach is that it allows a direct comparison between oligopolistic and perfectly competitive economies. Moreover, an advantage of Cournot–Nash competition is that it provides a stylized framework for formalizing antitrust measures. We assume that an antitrust authority can directly influence the number of firms in a market. Therefore, under Cournot–Nash behavior, an increase in the number of firms corresponds to greater competition, less market concentration, and a lower price markup. The approach used in this study considers two oligopolistic sectors, thus allowing us to analyze the implications for incomes, income distribution, and welfare when the government intervenes in either one or both of the sectors simultaneously. The rest of the paper is organized as follows. In Section 2, we present a review of the literature that addresses oligopolies within a dynamic general equilibrium framework. Section 3 focuses on the production side of the economy, initially in a static setting, and then examines production efficiency. In Section 4, we integrate the production side into a simple OLG model (Diamond 1965) to determine the long-run steady-state equilibrium. In Section 5, we analyze the effects on the factor incomes and income distribution given an increase in the number of firms in one oligopolistic sector versus an increase in the number of firms in both sectors. The long-run welfare effects resulting from the increase in the number of firms are investigated in Section 6. Finally, Section 7 provides concluding remarks.

2. Literature Review

In contrast to the extensive literature on monopolistic competition, there are relatively few papers that explore oligopolies within a general equilibrium framework (i.e., Neary 2002, 2010, 2016, or D'Aspremont et al. 1989, 1990, 1991, 1995). Furthermore, the number of papers addressing oligopolies in a dynamic general equilibrium framework is even more limited due to the presence of technical challenges (Hart 1982, 1985; Neary 2002, 2016). A key challenge is how to prevent oligopolies, which hold market power in the goods market, from exerting market power in the factor markets.

One of the pioneering attempts to integrate an oligopoly into an overlapping generations (OLG) model, similar to the Diamond (1965) model, was made by Laitner (1982).

Laitner considered two sectors: a perfectly competitive sector and an oligopolistic sector. Both sectors utilized an identical Cobb–Douglas technology. The goods produced in the perfectly competitive sector were used for consumption and investment, while the goods produced in the oligopolistic sector were exclusively used for consumption. Laitner further assumed that firms in the oligopolistic sector were owned by members of older generations who purchased firm shares using their savings, which diverted funds and subsequently crowd-out investments in productive capital.

In their paper, D'Aspremont et al. (1995) investigated the role of fiscal policy in a dynamic overlapping generations (OLG) model. Building upon their previous work in D'Aspremont et al. (1989, 1990), the authors focused on an economy characterized by imperfect competition, with labor as the sole input factor, money as a means of storing value, and a single good. Their study was based on an OLG model where firms act as Cournot oligopolists. The authors demonstrated that fiscal policy can enhance the efficiency of the economy by stimulating output and employment. Moreover, they illustrated how fiscal policy can be utilized to redistribute income in favor of the less affluent. Additionally, the authors incorporated the government's ability to employ fiscal policy to redistribute wealth across generations and to provide public goods. They established that fiscal policy can influence the market power of firms, thereby significantly impacting the equilibrium of the economy, and effectively guide the economy to a more efficient equilibrium. Furthermore, they revealed the potential of fiscal policy for promoting income redistribution in favor of disadvantaged individuals. The paper emphasizes that fiscal policy is a vital tool for macroeconomic management in economies with imperfect competition. Although D'Aspremont et al.'s model does not incorporate capital as an input, as with our study, they consider firms owned by the younger generation.

Kumar and Stauvermann (2020) constructed a one-good OLG model with an AK production function and a constant capital-to-output ratio that encompasses various identical oligopolistic sectors to examine the impact of simultaneously increasing the number of firms across all oligopolistic sectors. However, they introduced variations in the savings behavior of workers and firm owners in their respective studies. Kumar et al. (2022) investigated how different savings patterns can influence the overall dynamics of the model. These different studies (with different savings behavior and land markets) show that increasing competition may lead to a lower growth rate, albeit with a fairer intragenerational income distribution. Stauvermann and Kumar (2022) adopted a general neoclassical production and utility function for a broader examination of the interactions between oligopolistic market structures and economic variables. Consistent with the previous studies, they found that increasing market concentration leads to a more unequal intragenerational distribution of income and, in some cases, to a higher level of steady-state income. Accordingly, they concluded the possibility of antitrust measures reducing incomes in the long-run steady-state equilibrium.

The long-term effects of increasing market concentration on wages and capital incomes remain uncertain. It is unclear whether these factor incomes will experience a decline or an increase as market concentration intensifies. This ambiguity underscores the complex and multifaceted nature of oligopolistic markets and hence the need for further investigation. The underlying intuition behind the ambiguity of the results is that an increasing number of firms induce both short-run and long-run effects. The short-run effect results from a more efficient allocation of factor inputs, and leads to increased wages and interest rates. However, the long-run effect leads to an intergenerational redistribution of income, where the old generation or capital owners benefit from the decline in profits. If the old generation benefits more strongly from the decline in profits relative to the young, then the total savings and the steady-state capital stock will decline. The upcoming sections extend the analysis of Stauvermann and Kumar (2022) by considering a more comprehensive scenario. Specifically, we explore the case of two oligopolistic sectors, each characterized by different levels of market concentration. Furthermore, we highlight the intricacies involved in enhancing efficiency through the implementation of antitrust measures.

3. The Model

We now proceed by initially introducing a static representation of the model, focusing on production efficiency and income distribution. Subsequently, we integrate this aspect into an overlapping generation setting, inspired by the Diamond (1965) model, and analyze the long-term implications arising from the model.

The general setup of the model assumes a competitive final goods sector, which utilizes two intermediate goods as production inputs. These intermediate goods are produced within two sectors characterized by an oligopolistic market structure. The firms in both intermediate goods sectors utilize capital and labor as input factors for production. Additionally, we assume that the capital and labor markets are competitive.

3.1. The Final Goods Sector

The perfectly competitive final goods sector produces a quantity Q and operates with a Cobb–Douglas production technology, where it uses the quantities Q_i, $i = 1, 2$ as input factors.

$$Q = B(Q_1)^{\beta}(Q_2)^{1-\beta} \tag{1}$$

where $B > 0$ and $0 < \beta < 1$. The profit maximization problem of a representative firm in the final goods sector can be written as:

$$\max_{Q_1, Q_2} p_Q Q - p_1 Q_1 - p_2 Q_2, \tag{2}$$

where p_Q is the price of the final goods, and p_1 and p_2 are the prices of the intermediate goods, respectively. Using the resulting first-order conditions and the zero-profit condition, we derive the inverse demand functions for both intermediate goods:

$$p_1(Q_1) = p_Q \beta B \left(\frac{Q_2}{Q_1}\right)^{1-\beta} \tag{3}$$

$$p_2(Q_2) = p_Q (1-\beta) B \left(\frac{Q_1}{Q_2}\right)^{\beta} \tag{4}$$

From above, the price ratio of the intermediate goods can be written as:

$$\frac{p_1}{p_2} = \frac{\beta}{(1-\beta)} \frac{Q_2}{Q_1}. \tag{5}$$

3.2. The Intermediate Goods Sectors

The model presented here is a stylized version of the more general model presented in Appendix A.1 with the aim to reduce the amount of paperwork. We start with the sector that produces intermediate good 1. In this sector, we assume a symmetric oligopoly comprising n_1 firms. Legal barriers to entry prevent potential firms from entering the oligopolistic market. Each firm i in this sector utilizes capital $K_{1,i}$, which fully depreciates within one period, and labor $L_{1,i}$ as input factors. The production technology employed by these firms follows a Cobb–Douglas function:

$$Q_{1,i} = A(K_{1,i})^{\alpha}(L_{1,i})^{1-\alpha}, \forall i = 1, \ldots, n_1 \tag{6}$$

where $A > 0$, $0 < \alpha < 1$ is the capital share, $K_1 = \sum_{i=1}^{n_1} K_{1,i}$, $L_1 = \sum_{i=1}^{n_1} L_{1,i}$ and $Q_1 = \sum_{i=1}^{n_1} Q_{1,i}$. We assume that n_1 firms engage in a Nash–Cournot oligopoly competition and that the labor and capital market are perfectly competitive. Then the profit maximization problem of firm i in sector 1 can be written as:

$$\max_{L_{1,i}, K_{1,i}} p_1(Q_{1,i},\ Q_{1,-i})Q_{1,i} - wL_{1,i} - RK_{1,i}. \tag{7}$$

After applying the usual procedures to determine the Nash–Cournot equilibrium and aggregation (see Appendix A.1), we obtain the following first-order conditions (FOCs) regarding the wage rate w and interest factor R:

$$p_1\left(\frac{n_1-1}{n_1}\right)(1-\alpha)A(K_1)^{\alpha}(L_1)^{-\alpha}=w. \tag{8}$$

$$p_1\left(\frac{n_1-1}{n_1}\right)\alpha A(K_1)^{\alpha-1}(L_1)^{1-\alpha}=R. \tag{9}$$

In sector 1, the aggregate profits are the difference between the revenue in sector 1, $p_1(Q_1)Q_1$, and the total costs of production of this sector:

$$\Pi_1=\left(1-\frac{n_1-1}{n_1}\right)p_1A(K_1)^{\alpha}(L_1)^{1-\alpha}=\frac{p_1Q_1}{n_1}. \tag{10}$$

Because of the fact that the marginal costs of a homogenous production function are equal to one measured in terms of output, the price of intermediate good 1 is given by:

$$p_1^*=\frac{n_1}{n_1-1}. \tag{11}$$

We assume that the production function used in sector 2 is the same as described by (6). We repeat the procedures as above for sector 2 and obtain the following necessary conditions:

$$p_2\left(\frac{n_2-1}{n_2}\right)(1-\alpha)A(K_2)^{\alpha}(L_2)^{-\alpha}=w. \tag{12}$$

$$p_2\left(\frac{n_2-1}{n_2}\right)\alpha A(K_2)^{\alpha-1}(L_2)^{1-\alpha}=R. \tag{13}$$

Accordingly, the aggregate profit earned in sector 2 is:

$$\Pi_2=\left(1-\frac{n_2-1}{n_2}\right)p_2A(K_2)^{\alpha}(L_2)^{1-\alpha}=\frac{p_2Q_2}{n_2}. \tag{14}$$

and the equilibrium price of good 2 is:

$$p_2^*=\frac{n_2}{n_2-1}. \tag{15}$$

3.3. *Static Market Equilibrium*

From Equations (8), (9), (12), and (13) and the symmetry assumptions we can conclude that the capital intensity of all intermediate goods firms are equal, i.e.,:

$$\frac{K_1}{L_1}=\frac{K_{1,i}}{L_{1,i}}=\frac{K_2}{L_2}=\frac{K_{2,j}}{L_{2,j}},\ \forall i=1,\dots,n_1 \text{ and } \forall j=1,\dots,n_2. \tag{16}$$

Now, we can define:

$$\frac{K_2}{K_1}=\frac{L_2}{L_1}=\theta. \tag{17}$$

Furthermore, we define $\sum_{i=1}^{2}K_i=K$ and $\sum_{i=1}^{2}L_i=L$. Then, it follows that:

$$K_1=\frac{K}{1+\theta},\ L_1=\frac{L}{1+\theta},\ K_2=\frac{\theta K}{1+\theta},\ L_2=\frac{\theta L}{1+\theta} \tag{18}$$

We use the price ratio from Equation (5), the prices (Equations (10) and (13)), and the definition of θ to obtain the allocation of input factors:

$$\theta^*(n_1, n_2) = \frac{p_1^*}{p_2^*}\frac{(1-\beta)}{\beta} = \frac{n_1}{(n_1-1)}\frac{(n_2-1)}{n_2}\frac{(1-\beta)}{\beta}. \tag{19}$$

If both intermediate sectors were perfectly competitive, the optimal allocation would be $\theta^{opt} = \frac{(1-\beta)}{\beta}$. Any deviation from this allocation indicates a production inefficiency resulting from imperfect competition. Taking the derivative of $\theta^*(n_1, n_2)$ with respect to n_1 and n_2 yields:

$$\frac{\partial\theta^*(n_1, n_2)}{\partial n_1} = -\frac{1}{(n_1-1)^2}\frac{(n_2-1)}{n_2}\frac{(1-\beta)}{\beta} < 0 \text{ and } \frac{\partial\theta^*(n_1, n_2)}{\partial n_2} = \frac{n_1}{(n_1-1)}\frac{1}{(n_2)^2}\frac{(1-\beta)}{\beta} > 0. \tag{20}$$

Lemma 1. *If the number of firms increases in one oligopolistic sector, the capital and labor utilized in that sector will also increase, while the capital and labor employed in the other oligopolistic sector will decline.*

This means that if one intermediate goods sector becomes more competitive, it will attract capital and labor from the other intermediate goods sector. The reason is simply that having more firms in a sector leads to lower product prices, resulting in increased demand for the goods produced in that sector. Consequently, this will have implications for the output ratio between the two sectors. To examine this, we define a pseudo production function Z = $A(K)^\alpha(L)^{1-\alpha}$, and then the outputs of sectors 1 and 2 can be written as output in terms of the pseudo production function:

$$Q_1^* = A\left(\frac{K}{1+\theta^*}\right)^\alpha\left(\frac{L}{1+\theta^*}\right)^{1-\alpha} = \frac{Z}{1+\theta^*}, \tag{21}$$

$$Q_2^* = A\left(\frac{\theta^* K}{1+\theta}\right)^\alpha\left(\frac{\theta^* L}{1+\theta^*}\right)^{1-\alpha} = \frac{\theta^* Z}{1+\theta^*}. \tag{22}$$

From (20) and (21) it follows that $\frac{Q_2^*}{Q_1^*} = \theta^*$.

Proposition 1. *If the number of firms in one sector increases, the output of this sector increases, while the output of the other sector will decline.*

If the number of firms in sector 1 increases, the price of the intermediate goods produced in sector 1 will decrease. The lower price will incentivize the firms in the final goods sector to substitute intermediate goods produced in sector 2 with those produced in sector 1. As a result, the demand for intermediate good 2 will decline, while the demand for intermediate good 1 will increase.

The output produced in the final goods sector Q^* can be calculated by inserting Equations (21) and (22) into production function (1):

$$Q^* = B(Q_1^*)^{1-\beta}(Q_2^*)^\beta = \frac{B(\theta^*)^{1-\beta}}{1+\theta^*}Z = \frac{B(\theta^*)^{1-\beta}}{1+\theta^*}AK^\alpha L^{1-\alpha}, \tag{23}$$

Differentiating the quantity of final goods with respect to the number of firms gives us:

$$\begin{aligned} \frac{\partial Q^*}{\partial n_1} &= \frac{(n_2-n_1)B(\theta^*)^{1-\beta}n_2(1-\beta)\beta^2 Z}{n_1(n_1n_2-(1-\beta)n_1-n_2\beta)^2}\begin{cases} > 0, \text{ if } n_2 > n_1 \\ \leq 0, \text{ if} n_2 \leq n_1 \end{cases}, \text{ and} \\ \frac{\partial Q^*}{\partial n_2} &= \frac{(n_1-n_2)B(\theta^*)^{1-\beta}(1-\beta)\beta^2 Z(n_1-1)}{(n_2-1)(n_1n_2-(1-\beta)n_1-n_2\beta)^2}\begin{cases} > 0, \text{ if } n_1 > n_2 \\ \leq 0, \text{ if} n_1 \leq n_2 \end{cases}. \end{aligned} \tag{24}$$

Proposition 2. *The quantity of final goods increases with an increasing number of firms in only one intermediate goods sector if the absolute value of the difference $|n_2 - n_1|$ decreases. Otherwise, the quantity of final goods decreases.*

The proof can be derived directly from the inequalities (see Equation (24)). However, this proposition has significant implications for the short-run consequences of antitrust policy. It is important to note that increasing competition in one sector may lead to a decline in the final product's quantity or real national income. In other words, the common assumption that antitrust policy always enhances efficiency can be misleading if the antitrust authority regulates the 'wrong' industry. The reason, as noted in this model, is that the optimal ratio of intermediate goods, θ^{opt}, is always achieved when the price markup is equal in both sectors.[1] This also implies a paradoxical recommendation in cases where competition cannot be enhanced in a particular sector. In such situations, it is preferable, from the perspective of allocative efficiency, to reduce competition in the other sector rather than increasing it.

Next, we determine the price of the final goods by applying the zero-profit condition of the final goods sector. This condition states that the revenue of the final goods sector, $p_Q Q^*$, must equal the production costs or, i.e., the expenditures on intermediate goods, $p_1^* Q_1^* + p_2^* Q_2^*$. Therefore, taking Q^* as the real national income, the price of the final goods can be calculated from Equations (11), (15), and (21)–(23) as:

$$p_Q^* = \frac{p_1^* + \theta^* p_2^*}{B(\theta^*)^{1-\beta}} = \frac{(p_1^*)^{\beta}(p_2^*)^{1-\beta}}{B\beta^{\beta}(1-\beta)^{1-\beta}} = \frac{\left(\frac{n_1}{n_1-1}\right)^{\beta}\left(\frac{n_2}{n_2-1}\right)^{1-\beta}}{B\beta^{\beta}(1-\beta)^{1-\beta}}. \tag{25}$$

Differentiating the price level with respect to the number of firms gives us:

$$\frac{\partial p_Q^*}{\partial n_1} = -\frac{\beta}{(n_1-1)n_1} p_Q^* < 0,\ \frac{\partial p_Q^*}{\partial n_2} = \frac{1-\beta}{(n_2-1)n_2} p_Q^* < 0 \text{ and } \left.\frac{\partial p_Q^*}{\partial n_2}\right|_{n_1=n_2=n} = \frac{p_Q^*}{n-1} < 0. \tag{26}$$

Lemma 2. *If one or both markets experience an increase in competition, the price level or price of the final output will decrease.*

If the intermediate goods sectors were perfectly competitive, the price level would reach its equilibrium at the competitive price level $p_Q^{pc} = \frac{1}{B\beta^{\beta}(1-\beta)^{1-\beta}}$, where the superscript pc indicates perfect competition.

To ascertain the nominal incomes of various groups within the economy, including workers, capital owners, and firm owners, we utilize Equations (8)–(16) for this purpose:

$$wL = (1-\alpha)Z, \tag{27}$$

$$RK = \alpha Z, \tag{28}$$

$$\Pi = \Pi_1 + \Pi_2 = \left[\left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}\right] Z. \tag{29}$$

After examining Equations (27) and (28), it is evident that the presence of oligopolies does not impact the nominal incomes of workers and capital owners. These nominal incomes align with those of workers and capital owners in an economy featuring perfectly competitive intermediate goods sectors. However, it is important to note that this does not hold true when considering real incomes, which are obtained by dividing the nominal income by the price level p_Q^*:

$$w^r L = (1-\alpha)\frac{Z}{p_Q^*}, \tag{30}$$

$$R^r K = \alpha\frac{Z}{p_Q^*}, \tag{31}$$

$$\Pi_1^r = \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*}\frac{Z}{p_Q^*}, \tag{32}$$

$$\Pi_2^r = \left(\frac{1}{n_1-1}\right)\frac{\theta^*}{1+\theta^*}\frac{Z}{p_Q^*}, \tag{33}$$

$$\Pi^r = \Pi_1^r + \mathit{\Pi}_1^r = \left[\left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}\right]\frac{Z}{p_Q^*} \tag{34}$$

Evidently, the real labor and real capital incomes will increase when the number of firms in one or both intermediate goods sectors decrease due to a declining price level (see Lemma 1). However, when it comes to profits, the reaction differs. The response of profits in each sector when the number of firms increases in that sector is given by:

$$\frac{\partial \mathit{\Pi}_1^r}{\partial n_1} = -\left(\frac{Z}{p_Q^*}\right)\frac{n_2\beta\big((n_2+\beta-1)(n_1^2+(1-\beta)n_1-1)+n_2\beta^2\big)}{(n_1-1)n_1(n_1n_2-(1-\beta)n_1-n_2\beta)^2} < 0, \tag{35}$$

$$\frac{\partial \mathit{\Pi}_2^r}{\partial n_2} = -\left(\frac{Z}{p_Q^*}\right)\frac{n_1(1-\beta)\Big((n_2+\beta-2)n_2(n_1-\beta)+n_2(1-\beta)^2\Big)}{(n_2-1)n_2(n_1n_2-(1-\beta)n_1-n_2\beta)^2} < 0, \tag{36}$$

Notably, the profits decrease when the number of firms increases in the sector, as two opposing effects come into play. On the one hand, the declining price level leads to an increase in profits, while on the other hand, the decreasing price markup reduces the profits, with the latter effect outweighing the former.

To examine the response of profits in one sector when the number of firms in the other sector increases, we obtain the following conditions:

$$\frac{\partial \mathit{\Pi}_1^r}{\partial n_2} = (n_1-n_2)\left(\frac{Z}{p_Q^*}\right)\frac{(1-\beta)\beta^2}{(n_2-1)(n_1n_2-(1-\beta)n_1-n_2\beta)^2}\begin{cases}\geq 0, \text{ if } n_1 \geq n_2\\ < 0, \text{ if } n_1 < n_2\end{cases}, \tag{37}$$

$$\frac{\partial \mathit{\Pi}_2^r}{\partial n_1} = (n_2-n_1)\left(\frac{Z}{p_Q^*}\right)\frac{(1-\beta)^2\beta}{(n_1-1)(n_1n_2-(1-\beta)n_1-n_2\beta)^2}\begin{cases}\geq 0, \text{ if } n_2 \geq n_1\\ < 0, \text{ if } n_2 < n_1\end{cases}. \tag{38}$$

As noted, the results are ambiguous and contingent upon the disparity between the numbers of firms in the two sectors. If one sector is more competitive than the other, the owners of firms in that sector stand to gain from an increase in competition in the other sector. Conversely, if one sector is less competitive compared to the other sector, firm owners in that sector will witness a decline in their profits when the number of firms increases in the other sector. These outcomes arise from the adjustments in prices of intermediate goods and the resultant changes in demand for these goods. However, these results have implications concerning the political economy of antitrust policy. If the antitrust authority implements antitrust measures in the sector that is relatively less competitive, its actions will garner support from firms in the sector that is relatively more competitive. Now we consider the aggregate profits:

$$\frac{\partial \Pi^r}{\partial n_1} = -\left(\frac{Z}{p_Q^*}\right)\frac{\beta\big(n_2^2(\beta^2+\beta(n_2-2)+n_2(n_2-1)+1)-n_1n_2\big(2\beta^2+\beta(n_2-2)+n_2\big)+n_2^2\beta^2\big)}{(n_1-1)n_1(n_1n_2-(1-\beta)n_1-n_2\beta)^2} < 0, \tag{39}$$

$$\frac{\partial \Pi^r}{\partial n_2} = -\left(\frac{Z}{p_Q^*}\right)\frac{(1-\beta)\left(n_1^2\left(n_2^2 - n_2(2-\beta) + (1-\beta)^2\right) - n_1 n_2 \beta(n_2 - 2(1-\beta)) + n_2^2\beta^2\right)}{(n_2-1)n_2(n_1 n_2 - (1-\beta)n_1 - n_2\beta)^2} < 0. \tag{40}$$

As noted from Equations (39) and (40), an increase in the number of firms in one or both sectors will result in a decrease in aggregate profits of the intermediate goods sector(s).

3.4. Production Efficiency and Distribution of Income

A well-known consequence of imperfect competition is that oligopolistic firms decrease the equilibrium quantity of goods and establish a price higher than marginal costs. As a result, there is a distorted allocation of resources compared to perfect competition. In this model, the optimal production structure of intermediate goods is described by the allocation of intermediate goods in perfectly competitive markets. This allocation is determined by the following ratio:

$$\theta^{opt} = \frac{(1-\beta)}{\beta}. \tag{41}$$

However, in imperfect competition, the ratio is given by:

$$\theta^* = \frac{n_1}{(n_1-1)}\frac{(n_2-1)}{n_2}\frac{(1-\beta)}{\beta}. \tag{42}$$

This implies that if $n_1 \neq n_2$, the market outcome will be distorted. Assuming that labor and capital are supplied at inelastic factor prices, then the only source of distortion arises from the imbalanced ratio of intermediate goods. In other words, if the distortions are identical or the number of firms is equal in both intermediate sectors, the quantity of final goods is independent of the market structure. However, if the number of firms differs, the production of final goods will decrease.

Proposition 3. *If input factors supplied are factor price inelastic, oligopolies in the intermediate goods markets are efficient in terms of factor allocation, provided that the market power in all intermediate sectors is identical.*

A proof is provided in Appendix A.2.

Proposition 4. *Discretionary antitrust measures aimed at increasing competition can enhance production efficiency only if the price markups in the two intermediate sectors move closer to each other.*

Proposition 4 carries significant implications for antitrust policy, emphasizing the importance of antitrust authorities possessing comprehensive information on all markets before implementing regulations on specific markets. Furthermore, it highlights the necessity of applying antitrust measures in sectors characterized by the highest price markups. Failing to do so could result in undesirable outcomes and a decline in production efficiency.

Another crucial implication of market power is its impact on income distribution. In a static context, to address the distributive consequences of oligopolies requires examination of the income shares of various income groups. These income shares are obtained by dividing Equations (30)–(32) by the quantity of final output (23). Consequently, the labor income share, the capital income share, and the profit share are determined as follows:

$$LS = \frac{(1-\alpha)(n_1 n_2 - (1-\beta)n_1 - n_2\beta)}{n_2 n_1}, \tag{43}$$

$$CS = \frac{\alpha(n_1 n_2 - (1-\beta)n_1 - n_2\beta)}{n_2 n_1}, \tag{44}$$

$$PS = \frac{n_1(1-\beta) + n_2\beta}{n_2 n_1}. \tag{45}$$

Taking the partial derivatives of the income shares with respect to the number of firms in sector 1 and sector 2, respectively, yields the following results:

$$\frac{\partial LS}{\partial n_1} = \frac{(1-\alpha)\beta}{n_1^2} > 0 \text{ and } \frac{\partial LS}{\partial n_2} = \frac{(1-\alpha)(1-\beta)}{n_2^2} > 0, \tag{46}$$

$$\frac{\partial CS}{\partial n_1} = \frac{\alpha\beta}{n_1^2} > 0 \text{ and } \frac{\partial LS}{\partial n_2} = \frac{\alpha(1-\beta)}{n_2^2} > 0, \tag{47}$$

$$\frac{\partial PS}{\partial n_1} = -\frac{\beta}{n_1^2} < 0 \text{ and } \frac{\partial PS}{\partial n_2} = -\frac{1-\beta}{n_2^2} < 0. \tag{48}$$

Proposition 5. *Antitrust measures implemented to promote competition will lead to an increase in both the labor income share and capital income share, but a decrease in the profit income share.*

On the one hand, these considerations demonstrate that antitrust measures have the potential to enhance efficiency, but they must be executed with some caution. The notion that increasing competition in a single sector will invariably result in improved production efficiency is erroneous. This is because augmenting competition in markets where competition is already relatively intense leads to a decrease in production efficiency.

On the other hand, antitrust measures will consistently lead to a decline in the profit share and an increase in the labor and capital income shares. Up to this point, we assumed that capital and labor are given. However, in the subsequent sections, we relinquish this assumption.

4. An OLG Model with Two Oligopolistic Markets

For simplicity, we assume that there are two types of individuals, differing only in that some individuals are firm owners of the oligopolistic firms, while others are workers. Additionally, we assume that only the number of workers, denoted as L, grows at a constant rate of g_N, while the number of firm owners remains fixed to the number of oligopolistic firms. Both groups of individuals have a lifespan of two periods.

During the working age, the firm owners manage their firms, and upon retirement, they pass on the right to run a firm to their child while living on their savings (including interest income). Thus, firms are assumed to be the same as family firms or family businesses (Bertrand and Schoar 2006) such as Walmart, BMW, Volkswagen, LG, Ford, Samsung, SK Holdings, Tyson Foods, Hyundai Motor, Dell, etc. According to Kelley et al. (2020), 75% of entrepreneurs and 81% of firm owners co-own and/or co-manage their firms with family members.

On the other hand, the workers supply their workforce inelastically and engage in work during the first period of their lives. In the second period, they live off their savings. It is assumed that all individuals are equipped with a log-linear utility function of the following form:

$$U\left(c_t^1, c_{t+1}^2\right) = \ln q_t^1 + \frac{1}{1+\mu} ln q_{t+1}^2, \tag{49}$$

where $\mu > 0$ is the subjective discount factor, q_t^1 the consumption of final goods in the first period of life, and q_{t+1}^2 the consumption of final goods in the second period of life.

The respective budget constraints are given by:

$$p_Q^* q_t^1 = y_t^k - s_t^k \text{ and } p_Q^* q_{t+1}^2 = R_{t+1} s_t^k, \text{ for } i = w, c, \tag{50}$$

where y_t^w is the wage income of a worker and y_t^c the profit income of the firm owner.

By substituting Equation (42) into (41), differentiating with respect to savings, and solving the first-order conditions for savings, we obtain the optimal savings as:

$$s_t^k = \frac{y_t^k}{2+\mu} = s y_t^k, \tag{51}$$

where $s \equiv \frac{1}{2+\mu}$ represents the constant marginal rate of savings. Referring to Equation (27), the aggregate savings of workers can be expressed as follows:

$$L_t s_t^w = s(1-\alpha)Z_t, \tag{52}$$

Utilizing Equation (29), the total savings of all firm owners involved in sector 1 and sector 2 can be calculated as follows:

$$n_1 s_t^{\Pi_1} = s\left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*}Z_t, \tag{53}$$

and

$$n_2 s_t^{\Pi_2} = s\left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}Z_t. \tag{54}$$

Now, assuming that final goods can also be utilized for investment purposes and considering that the resulting capital stock fully depreciates within one period, we can determine the capital stock. By adding the savings from Equations (52)–(54), the resulting capital market clearing condition is expressed as follows:

$$L_t s_t^w + n_1 s_t^{\Pi_1} + n_2 s_t^{\Pi_2} = s\left[(1-\alpha) + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}\right] A K_t^{\alpha} L_t^{1-\alpha} = p_Q^* K_{t+1} \tag{55}$$

Defining $z_t = \frac{Z_t}{L_t} = A k_t^{\alpha}$, we can reformulate the capital market clearing condition in per capita variables, as follows:

$$s\left[(1-\alpha) + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}\right] A k_t^{\alpha} = p_Q^* k_{t+1}(1+g_N). \tag{56}$$

where k_t is the capital intensity.

Inserting $k^* = k_t = k_{t+1}$ in (56), we determine the equilibrium capital intensity:

$$k^* = \left(\frac{sA\left[(1-\alpha) + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}\right]}{p_Q^*(1+g_N)}\right)^{\frac{1}{1-\alpha}} = \left(\frac{sA\left[(1-\alpha) + \frac{(1-\beta)n_1 + n_2\beta}{n_1 n_2 - (1-\beta)n_1 - n_2\beta}\right]}{\left(\frac{\left(\frac{n_1}{n_1-1}\right)^{\beta}\left(\frac{n_2}{n_2-1}\right)^{1-\beta}}{B\beta^{\beta}(1-\beta)^{1-\beta}}\right)(1+g_N)}\right)^{\frac{1}{1-\alpha}}. \tag{57}$$

The equilibrium is locally stable (see Appendix A.3).

5. Comparative Statics and Distribution of Income

Now, let us analyze the long-run effects of antitrust measures on the steady-state equilibrium. As before, the antitrust measure being considered is an increase in the number of firms in one sector. The impact of increasing n_1 or n_2 on the steady-state capital intensity can be determined as follows:

$$\frac{\partial k^*}{\partial n_1}\begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_2(1-\beta)(n_2-n_1)n_1}{(n_1 n_2 - (1-\beta)n_1 - \beta n_2)^2} \\ < 0, \text{ if } \alpha > \frac{n_2(1-\beta)(n_2-n_1)n_1}{(n_1 n_2 - (1-\beta)n_1 - \beta n_2)^2} \end{cases} \text{ and } \frac{\partial k^*}{\partial n_2}\begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_1\beta(n_1-n_2)n_2}{(n_1 n_2 - (1-\beta)n_1 - \beta n_2)^2} \\ < 0, \text{ if } \alpha > \frac{n_1\beta(n_1-n_2)n_2}{(n_1 n_2 - (1-\beta)n_1 - \beta n_2)^2} \end{cases}. \tag{58}$$

Proposition 6. *If the number of firms in sector 1 increases, then the steady-state capital intensity will only rise if* $\alpha \leq \frac{n_2(1-\beta)(n_2-n_1)n_1}{(n_1 n_2-(1-\beta)n_1-\beta n_2)^2}$*, and if the number of firms in sector 2 increases, then the steady-state capital intensity will only increase if* $\alpha \leq \frac{n_1\beta(n_1-n_2)n_2}{(n_1 n_2-(1-\beta)n_1-\beta n_2)^2}$.

The ambiguity of these two results stems from two effects. Firstly, an increase in the number of firms in one sector can lead to a more efficient allocation of resources and an increase in the income of the younger generation. This occurs when the number of firms in the sector with a higher price markup increases. Secondly, there is an effect caused by income redistribution from the young to the old generation, who do not save. An increase in the number of firms in one sector results in a decline in profits, an increase in wages, an increase in the interest rate, and consequently, an increase in the income of the older generation.

If the intergenerational redistributive effect does not significantly reduce the income of the younger generation, such that the improved resource allocation can compensate for their income loss, the capital intensity will increase. However, if the intergenerational redistributive effect diminishes the income of the younger generation to a large extent, the capital intensity will decline.

The magnitude of the intergenerational redistributive effect is influenced by the production elasticity of capital α. A smaller value of α corresponds to a smaller intergenerational redistributive effect. To emphasize and isolate the intergenerational redistributive effect, we assume that $n_1 = n_2 = n$, and we adjust Equation (57) accordingly. Consequently, the capital intensity can be expressed as follows:

$$k^*|_{n_1=n_2=n} = k^{**} = \left(\frac{sAB\beta^{\beta}(1-\beta)^{1-\beta}}{(1+g_n)}\right)^{\frac{1}{1-\alpha}} \left(\frac{(1-\alpha)(n-1)}{n} + \frac{1}{n}\right)^{\frac{1}{1-\alpha}} \tag{59}$$

By differentiating capital intensity with respect to the number of firms in both sectors, we obtain:

$$\frac{\partial k^{**}}{\partial n} = -\frac{\alpha k^{**}}{n(1-\alpha)(n(1-\alpha)+\alpha)} < 0. \tag{60}$$

Proposition 7. *If the market structure is identical in both intermediate goods markets, a proportional increase in the number of firms in both sectors will consistently result in a decline in the capital intensity.*

This implies that when the factor allocation is efficient and the number of firms increases proportionally in both sectors, the decrease in market power is accompanied by a redistribution of incomes from the younger to the older generation. Consequently, the lower incomes of the younger generation result in smaller savings, leading to a lower capital intensity.

Next, we examine the response of real factor incomes in per capita terms to an increase in the number of firms in one intermediate goods sector. The steady-state real factor incomes per capita can be expressed as follows:

$$WI^*_{real} = w^*_{real} = \frac{(1-\alpha)A(k^*)^{\alpha}}{p^*_Q}, \tag{61}$$

$$CI^*_{real} = R^*_{real}k^* = \frac{\alpha A(k^*)^{\alpha}}{p^*_Q}, \tag{62}$$

$$PI^*_{real} = \pi^*_{real} = \left(\left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*} + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*}\right)\frac{A(k^*)^{\alpha}}{p^*_Q}, \tag{63}$$

Next, we explore how the incomes respond to an increase in the number of firms. For simplicity, we begin by differentiating $\frac{z^*}{p_Q^*} = \frac{A(k^*)^\alpha}{p_Q^*}$ with respect to the number of firms:

$$\frac{\partial\left(\frac{z^*}{p_Q^*}\right)}{\partial n_1} = \frac{\overbrace{\alpha A(k^*)^{\alpha-1}\frac{\partial k^*}{\partial n_1}p_Q^*}^{+/-} - \overbrace{A(k^*)^\alpha \frac{\partial p_Q^*}{\partial n_1}}^{-}}{\left(p_Q^*\right)^2} \lesseqgtr 0, \tag{64}$$

$$\frac{\partial\left(\frac{z^*}{p_Q^*}\right)}{\partial n_2} = \frac{\overbrace{\alpha A(k^*)^{\alpha-1}\frac{\partial k^*}{\partial n_2}p_Q^*}^{+/-} - \overbrace{A(k^*)^\alpha \frac{\partial p_Q^*}{\partial n_2}}^{-}}{\left(p_Q^*\right)^2} \lesseqgtr 0. \tag{65}$$

Unfortunately, the reaction is ambiguous due to the implications of Lemma 1 and Proposition 5. However, a more detailed analysis reveals that:

$$\frac{\partial\left(\frac{z^*}{p_Q^*}\right)}{\partial n_1}\begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2n_2^2} \\ < 0, \text{ if } \alpha > \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2n_2^2}\end{cases}. \tag{66}$$

When considering the derivative of $\frac{z^*}{p_Q^*}$ with respect to the number of firms in the second sector (sector 2), we obtain:

$$\frac{\partial\left(\frac{z^*}{p_Q^*}\right)}{\partial n_2}\begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2n_2^2} \\ < 0, \text{ if } \alpha > \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(3-2\beta)n_2+(1-\beta)^2\right)-2n_1n_2(\beta^2-\beta+\beta n_2)+\beta^2n_2^2}\end{cases}. \tag{67}$$

Unfortunately, it is ambiguous whether $\frac{z^*}{p_Q^*}$ rises or declines with an increasing number of firms. As a result, the impact of antitrust measures aimed at increasing competition on steady-state real wage and real interest incomes is also ambiguous. These incomes, being linear in $\frac{z^*}{p_Q^*}$, can either rise or fall depending on the specific circumstances. From the outcomes (65) and (66), we can directly derive the reactions of the wage and capital incomes on changes in n_1 and n_2:

$$\frac{\partial\left(\frac{(1-\alpha)A(k^*)^\alpha}{p_Q^*}\right)}{\partial n_1}\begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2n_2^2} \\ < 0, \text{ if } \alpha > \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2n_2^2}\end{cases}, \tag{68}$$

$$\frac{\partial\left(\frac{(1-\alpha)A(k^*)^\alpha}{p_Q^*}\right)}{\partial n_1}\begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2n_2^2} \\ < 0, \text{ if } \alpha > \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(3-2\beta)n_2+(1-\beta)^2\right)-2n_1n_2(\beta^2-\beta+\beta n_2)+\beta^2n_2^2}\end{cases}. \tag{69}$$

and

$$\frac{\partial\left(\frac{\alpha A(k^*)^\alpha}{p_Q^*}\right)}{\partial n_1}\begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2n_2^2} \\ < 0, \text{ if } \alpha > \frac{n_1n_2(n_1n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2n_2^2}\end{cases}, \tag{70}$$

$$\frac{\partial\left(\frac{\alpha A(k^*)^\alpha}{p_Q^*}\right)}{\partial n_1}\begin{cases}\geq 0, \text{ if } \alpha \leq \frac{n_1 n_2(n_1 n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(1-\beta)2n_2+(1-\beta)^2\right)-2n_1 n_2\left(\beta^2-\beta+\left(\frac{1}{2}+\beta\right)n_2\right)+\beta^2 n_2^2} \\ < 0, \text{ if } \alpha > \frac{n_1 n_2(n_1 n_2-(1-\beta)n_1-\beta n_2)}{n_1^2\left(2n_2^2-(3-2\beta)n_2+(1-\beta)^2\right)-2n_1 n_2(\beta^2-\beta+\beta n_2)+\beta^2 n_2^2}\end{cases}. \tag{71}$$

Concerning the response of real aggregate steady-state profits to changes in the number of firms, we observe the following outcomes:

$$\frac{\partial\pi^*_{real}}{\partial n_1} = Y\frac{A(k^*)^\alpha}{n_1 p_Q^*}\left(\varepsilon_{Y,n_1} + \varepsilon_{\frac{1}{p_Q^*},n_1} + \varepsilon_{z,n_1}\right) < 0, \tag{72}$$

$$\frac{\partial\pi^*_{real}}{\partial n_2} = Y\frac{A(k^*)^\alpha}{n_2 p_Q^*}\left(\varepsilon_{Y,n_2} + \varepsilon_{\frac{1}{p_Q^*},n_2} + \varepsilon_{z,n_2}\right) < 0, \tag{73}$$

where $\varepsilon_{Y,n_i} = \frac{\partial Y}{\partial n_i}\frac{n_i}{Y} < 0$, $\varepsilon_{\frac{1}{p_Q^*},n_i} = \frac{\partial\left(\frac{1}{p_Q^*}\right)}{\partial n_2}\frac{n_i}{\left(\frac{1}{p_Q^*}\right)} > 0$, and $\varepsilon_{z,n_i} = \frac{\partial z}{\partial n_i}\frac{n_i}{z} \lesseqgtr 0$ $i = 1,2$ are elasticities and $Y = \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*} + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*}$. (See details in Appendix A.4).

The derivative is negative due to the relatively large absolute value of ε_{Y,n_i} compared to the positive effects of the inverse price level and the absolute value of the output effect (assuming the latter is positive). However, the results become more evident when we assume that price markups in both oligopolistic sectors are identical ($n_1 = n_2 = n$). In such a case, $\frac{z^*}{p_Q^*}$ can be expressed as follows:

$$\left.\frac{z^*}{p_Q^*}\right|_{n_1=n_2=n} = \frac{A(k^{**})^\alpha B\beta^\beta(1-\beta)^{1-\beta}}{n}. \tag{74}$$

Differentiating Equation (73) with respect to the number of firms gives us:

$$\frac{\partial\left(\left.\frac{z^*}{p_Q^*}\right|_{n_1=n_2=n}\right)}{\partial n} = \frac{z^*}{p_Q^*}\frac{(n-\alpha(2n-1))}{n(1-\alpha)(n-\alpha(n-1))}\begin{cases} > 0, \text{ if } \alpha < \frac{n}{2n-1} \\ \leq 0, \text{ if } \alpha \geq \frac{n}{2n-1}\end{cases}. \tag{75}$$

Accordingly, wage and capital incomes will only grow if the production elasticity of capital is sufficiently small. Moreover:

$$\frac{\partial\left(\pi^*_{real}\big|_{n_1=n_2=n}\right)}{\partial n} = -\frac{z^*}{p_Q^*}\frac{\left(n(1-\alpha)^2+\alpha\right)}{n((1-\alpha)n+\alpha)} < 0. \tag{76}$$

Based on the earlier assumptions, it is evident from derivative (76) that the real profit per capita will inevitably decline with an increasing number of firms in both markets.

6. Steady-State Welfare

Regarding the welfare analysis, it is evident that we should focus solely on dynamic equilibria that are efficient in a static sense. In other words, the number of firms must be equal in both intermediate sectors. However, we only analyze the steady-state welfare of workers since a Pareto improvement cannot be achieved due to the decline in the production of Q as the number of firms increases. Therefore, differentiating (23) with respect to the number of firms yields:

$$\left.\frac{\partial\left(\frac{Q^*}{L_t}\right)}{\partial n}\right|_{n_1=n_2=n} = -\frac{\alpha^2}{(1-\alpha)(n(1-\alpha)+\alpha)}\frac{A(k^{**})^\alpha}{p_Q^*} < 0. \tag{77}$$

By utilizing (48) in conjunction with (50), we obtain the steady-state utility of a representative worker:

$$U^*\left(q_t^{1*}, q_{t+1}^{2*}\right) = ln\left(\frac{1+\mu}{2+\mu}(1-\alpha)\frac{A(k^*)^{\alpha}}{p_Q^*}\right) + \frac{1}{1+\mu} ln\left(\frac{1}{2+\mu}R^*(1-\alpha)\frac{A(k^*)^{\alpha}}{p_Q^*}\right), \quad (78)$$

where $R^* = \frac{\alpha(2+\mu)(1+g_n)(n-1)}{(n-1)(1-\alpha)+1} p_Q^*$.

Using the result of Equation (75), differentiating Equation (78) with respect to the number of firms, and solving the first-order condition for n, yields the following:

$$\frac{\partial U^*\left(q_t^{1*}, q_{t+1}^{2*}\right)}{\partial n} = \frac{n(2+\mu) + \alpha(1+\mu-n(3+2\mu)) - (n-1)\alpha^2}{(1+\mu)(n(1-\alpha)+\alpha)(1-\alpha)n(n-1)} \begin{cases} \geq 0, \text{ if } \alpha \leq \frac{1+\mu-n(2\mu+3)+\sqrt{\mu^2(2n-1)^2+2\mu\left(8n^2-7n-1\right)+17n^2-14n+1}}{2(n-1)} \\ < 0, \text{ if } \alpha > \frac{1+\mu-n(2\mu+3)+\sqrt{\mu^2(2n-1)^2+2\mu\left(8n^2-7n-1\right)+17n^2-14n+1}}{2(n-1)} \end{cases}. \quad (79)$$

Proposition 8. *If the number of firms are identical in both intermediate goods sectors and the number of firms increase, then the impact on the steady-state welfare is ambiguous, because the change in welfare depends on the production elasticity of capital.*

Note that this result implies that steady-state welfare will always increase if the production elasticity of capital is sufficiently close to zero. This suggests that the intergenerational distribution effect resulting from the increase in the number of firms plays a decisive role in determining whether steady-state welfare will increase or decrease. However, it is noteworthy that it can be demonstrated that welfare and per capita income under imperfect competition may exceed the level of welfare and income under perfect competition.

7. Conclusions

In this paper, we examined an OLG model in which the entire economy is influenced by imperfect competition, specifically through the inclusion of two oligopolistic sectors. By assuming that members of the younger generation own the firms with market power, a key finding emerges: the existence of oligopolies induces both an intergenerational and intragenerational redistribution of income, from the older to the younger generation and from workers to firm owners, respectively.

The approach adopted in this paper permits direct comparisons of the outcomes and steady-state equilibrium between an economy under imperfect competition and an economy under perfect competition. Such a comparison allows for a comprehensive assessment of the effects of imperfect competition on income distribution and welfare.

Moreover, we note that although less competition in an economy leads to a more unequal distribution of income, increasing competition inappropriately may lead to less efficiency, a lower national income, and reduced welfare in the short- and long-run. Therefore, arbitrarily enhancing competition in a specific industry or sector can yield unintended consequences. In other words, if the number of firms increases in one sector, (i.e., moving closer to a perfectly competitive market) more than the other sector, the efficiency of the factor allocation declines, and consequently reduces the level of production and decreases welfare. This outcome raises questions about the appropriateness of current antitrust policies applied in many developed countries. Antitrust authorities typically focus on individual markets without considering the broader implications on the entire economy. It also appears that investigations by antitrust authorities sometimes have an arbitrary nature or are initiated based on populist demands from policymakers. The simple model presented in this paper demonstrates that for antitrust authorities to act appropriately, they require extensive information, although this is often concealed by firms. While we do not favor justifying oligopolies in terms of welfare and income, we highlight the challenging problems that arise due to the existence of imperfect competition. Oligopolies contribute to an unfair distribution of income and wealth, even if their presence allows for a higher standard of living. Clearly, there is a political trade-off that needs to be addressed.

Undoubtedly, our results are based on a few restrictive assumptions. Specifically, we assume that savings are interest inelastic and labor supply is wage inelastic. However, it can be argued that most empirical investigations on the wage elasticity of labor supply and interest elasticity of savings estimate relatively low values. Moreover, to ensure that our model is tractable, we use simple functions for analysis. Hence, it remains to be seen in future research whether the findings can be generalized.

Author Contributions: Methodology: P.J.S. and R.R.K.; formal analysis: P.J.S. and R.R.K.; writing—original draft preparation: P.J.S.; writing—review and editing: R.R.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Acknowledgments: We sincerely thank the anonymous reviewers for their useful suggestions and recommendations; all remaining errors are ours. Peter J. Stauvermann sincerely acknowledges the financial support of the Changwon National University 2023–2024.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Appendix A.1. The General Model

As mentioned in the main body of the paper, we address a problem originally raised by Hart (1982, 1985). Hart argues that it is a significant issue to assume perfectly competitive factor markets when only a few oligopolists dominate the goods market. In this section, we present an approach that bears similarities to the ideas of Neary (2002, 2010, 2016). The idea of Neary (2002, 2010, 2016) is that firms should possess market power in the goods market while being small enough in relation to the economy to act as price takers in the factor markets. We follow Stauvermann and Kumar (2022), who, although differing from Neary, also utilize this idea.

To incorporate this idea into the model, we introduce m preliminary goods sectors. Consequently, the model consists of three vertical levels of production: a final goods sector, two intermediate goods sectors (both markets are assumed to be perfectly competitive), and m industries producing preliminary goods required to produce the intermediate goods. The objective is to demonstrate that firms possess market power in the goods market while being sufficiently small in relation to the overall economy, allowing them to act as price takers in the factor markets.

To begin, we consider a perfectly competitive final goods market. The production of a certain quantity of final goods Q requires two intermediate goods Q_1 and Q_2 in the production process.

$$Q = B(Q_1)^{\beta}(Q_2)^{1-\beta}. \tag{A1}$$

Let p_Q denote the price of the final goods, and p_i (for $i = 1, 2$) represent the prices of the intermediate goods. Profit maximization in the perfectly competitive final goods market leads to the following inverted demand functions for the two intermediate goods.

$$p_1(Q_1) = \frac{\beta p_Q Q}{Q_1}, \tag{A2}$$

$$p_2(Q_2) = \frac{(1-\beta) p_Q Q}{Q_2}. \tag{A3}$$

We assume that these intermediate goods markets are also perfectly competitive. As a result, firms operating in the intermediate goods markets are price takers. They utilize a set of m preliminary goods to produce the intermediate goods. The quantity of intermediate goods Q_i (for $i = 1,2$) is produced using all m types of preliminary goods through a symmetric Cobb–Douglas production function with constant returns to scale.

$$Q_i = m\prod_{j=1}^{m}(x_{i,j})^{\frac{1}{m}}, \tag{A4}$$

where $x_{i,j}$ represents the quantity of pre-goods produced in the j-th sector of the pre-goods market. It is worth noting that, if $x_{i,j} = \overline{x}_{i,j}$, $\forall j = 1,\ldots,m$, then

$$Q_i = m\overline{x}_{i,j}. \tag{A5}$$

The profit maximization problem of a representative firm in the perfectly competitive intermediate goods sector is given by:

$$\Pi_i = p_i\left(m\prod_{j=1}^{m}(x_{i,j})^{\frac{1}{m}}\right) - \sum_{j=1}^{m} p_{i,j}x_{i,j}, \tag{A6}$$

The first-order condition derived from the profit maximization problem of a representative intermediate goods firm can be reformulated as the respective demand function of a pre-good j.

$$p_{i,j}(x_{i,j}) = \frac{p_i Q_i}{m x_{i,j}},\ \forall j = 1,\ldots,m \tag{A7}$$

Now, we assume that there are n_i oligopolistic firms operating in each of the m pre-good markets. All firms involved in pre-good production utilize an identical technology described by a well-behaved neoclassical production function.

$$x_{i,j,k} = F\left(K_{i,j,k}, L_{i,j,k}\right),\ \forall k = 1,\ldots,n_1 \tag{A8}$$

where it is assumed that $F(.,.)$ is linearly homogenous in its input capital $K_{i,j,k}$ and labor $L_{i,j,k}$. In the main text, we assume implicitly a Cobb–Douglas function:

$$x_{i,j,k} = AK_{i,j,k}^{\alpha}L_{i,j,k}^{1-\alpha},\ k = 1,\ldots,n_1,\ \text{respectively},\ k = 1,\ldots,n_2. \tag{A9}$$

Here we continue with a more general production function. We define the capital intensity as $k_{i,j,k} = \frac{K_{i,j,k}}{L_{i,j,k}}$. Then, the production function in per capita terms $f\left(k_{i,j,k}\right)$ fulfills the following requirements:

$$f'\left(k_{i,j,k}\right) > 0,\ f''\left(k_{i,j,k}\right) < 0,\ f(0) = 0,\ \lim_{k_{i,j,k}\to 0} f'\left(k_{i,j,k}\right) = \infty \text{ and } \lim_{k_{i,j,k}\to\infty} f'\left(k_{i,j,k}\right) = 0. \tag{A10}$$

It is important to note that this economy consists of $m(n_1 + n_2)$ symmetric oligopolies. In each of the m pre-good markets, n_1 and n_2 oligopolies compete, respectively. When the total number of oligopolies, $m(n_1 + n_2)$, is sufficiently large, it is reasonable to assume that each oligopolist behaves as a price taker in the factor markets. Additionally, we assume that the oligopolists exhibit Cournot–Nash behavior.

The profit function of an oligopolist k ($k = 1,\ldots,n_i,\ i = 1,\ 2$) in the pre-good markets i,j is given by:

$$\Pi_{i,j,k}\left(x_{i,j,k}, x_{-i,j,k}\right) = p(x_{i,j})x_{i,j.k} - RK_{i,j,k} - wL_{i,j,k}, \tag{A11}$$

where $x_{i,j,k} = F\left(K_{i,j,k}, L_{i,j,k}\right)$ and $x_{i,j} = \sum_{k=1}^{n_i} x_{i,j,k} = \sum_{k=1}^{n_i} F\left(K_{i,j,k}, L_{i,j,k}\right)$. The profits of the k-th firm in the i,j-th pre-good markets can be rewritten as:

$$\Pi_{i,j}\left(x_{i,j,k}, x_{-i,j,k}\right) = \frac{p_i Q_i}{m x_{i,j}} x_{i,j,k} - R K_{i,j,k} - w L_{i,j,k} = \frac{p_i Q_i}{m}\left(\frac{F\left(K_{i,j,k}, L_{i,j,k}\right)}{\sum_{k=1}^{n_i} F\left(K_{i,j,k}, L_{i,j,k}\right)}\right) - R K_{i,j,k} - w L_{i,j,k}. \quad \text{(A12)}$$

Maximizing (A12) with respect to the firm's capital and firm's labor force leads to following first-order conditions:

$$\frac{p_i Q_i}{m}\left[\frac{F_{K_{i,j,k}}\left(\sum_{k=1}^{n_i} F\left(K_{i,j,k}, L_{i,j,k}\right)\right) - F\left(K_{i,j,k}, L_{i,j,k}\right) F_{K_{i,j,k}}}{\left(\sum_{k=1}^{n_i} F\left(K_{i,j,k}, L_{i,j,k}\right)\right)^2}\right] - R = 0. \quad \text{(A13)}$$

$$\frac{p_i Q_i}{m}\left[\frac{F_{L_{i,j,k}}\left(\sum_{k=1}^{n_i} F\left(K_{i,j,k}, L_{i,j,k}\right)\right) - F\left(K_{i,j,k}, L_{i,j,k}\right) F_{L_{i,j,k}}}{\left(\sum_{k=1}^{n_i} F\left(K_{i,j,k}, L_{i,j,k}\right)\right)^2}\right] - w = 0. \quad \text{(A14)}$$

As noted above, all oligopolists are identical, and therefore the equilibrium outcome is symmetric. This implies that $x_{i,j} = \sum_{k=1}^{n_i} x_{i,j,k} = n_i \overline{x}_{i,j,k} = F\left(\frac{K_i}{n_i m}, \frac{L_i}{n_i m}\right)$, where K_i is the aggregate capital stock and L_i is the aggregate labor force used for the production of intermediate good i. Using this knowledge, the FOCs (A13) and (A14) become:

$$\frac{p_i Q_i}{m} F_{K_{i,j,k}}\left[\frac{n\overline{x}_{i,j,k} - \overline{x}_{i,j,k}}{\left(n_i \overline{x}_{i,j,k}\right)^2}\right] = \frac{p_i Q_i}{m}\frac{F_{K_{i,j,k}}}{n_i \overline{x}_{i,j,k}}\left[1 - \frac{1}{n_i}\right] = p_i F_{K_{i,j}}\left[\frac{n_i - 1}{n_i}\right] = R. \quad \text{(A15)}$$

$$\frac{p_i Q_i}{m} F_{L_{i,j,k}}\left[\frac{n\overline{x}_{i,j,k} - \overline{x}_{i,j,k}}{\left(n_i \overline{x}_{i,j,k}\right)^2}\right] = \frac{p_i Q_i}{m}\frac{F_{L_{i,j,k}}}{n_i \overline{x}_{i,j,k}}\left[1 - \frac{1}{n}\right] = p_i F_{L_{i,j,k}}\left[\frac{n - 1}{n}\right] = w. \quad \text{(A16)}$$

where we use $x_{i,j} = n\overline{x}_{i,j,k}$. Because the first derivative of a linear homogenous function is homogenous of degree zero ($F_{K_{i,j,k}} = F_{K_i}$ and $F_{L_{i,j,k}} = F_{L_i}$), we can write:

$$p_i\left(\frac{n_i - 1}{n_i}\right) F_{L_i} = p_i\left(\frac{n_i - 1}{n_i}\right)(1-\alpha) A (K_i)^{\alpha} (L_i)^{-\alpha} = w. \quad \text{(A17)}$$

$$p_i\left(\frac{n_1 - 1}{n_1}\right) F_{K_i} = p_i\left(\frac{n_i - 1}{n_i}\right)\alpha A (K_i)^{\alpha-1} (L_i)^{1-\alpha} = R. \quad \text{(A18)}$$

These FOCs are identical to the FOCs in the main text.

Accordingly, the aggregate profits resulting from the demand of intermediate goods sector i become:

$$\Pi_i = p_i\left(Q_i - \left(\frac{n_i - 1}{n_i}\right)\left(F_{L_i}(K_i, L_i) L_i + F_{K_i}(K_i, L_i) K_i\right)\right) = \frac{p_i Q_i}{n_i} = \frac{F(K_i, L_i)}{n_i}. \quad \text{(A19)}$$

Because of the symmetry, in the market equilibrium the following equalities hold: $K_{i,j} = \sum_{k=1}^{n_i} K_{i,j,k} = \frac{K_i}{m}$ and $L_{i,j} = \sum_{k=1}^{n_i} L_{i,j,k} = \frac{L_i}{m}$. Furthermore, the symmetry also leads to the results $K_{i,j,k} = \frac{K_i}{m n_i}$ and $L_{i,j,k} = \frac{L_i}{m n_i}$. It follows that the aggregate profits generated in a pre-good market j, are given by:

$$\Pi_{i,j} = \frac{p_i Q_i}{m n_i} = \frac{F(K_i, L_i)}{m n_i}, \ \forall j = 1, \ldots, m. \quad \text{(A20)}$$

and the profit of an oligopolist k in intermediate market i, j is:

$$\Pi_{i,j,k} = \frac{p_i Q_i}{n_i^2 m} = \frac{F(K_i, L_i)}{n_i^2 m}. \forall j = 1, \ldots, m \text{ and } \forall k = 1, \ldots, n_i. \tag{A21}$$

The aggregate profits in this economy are given by:

$$\Pi = n_1 m \left(\frac{p_1 Q_1}{n_1^2 m} \right) + n_2 m \left(\frac{p_2 Q_2}{n_2^2 m} \right) = \frac{p_1 Q_1}{n_1} + \frac{p_2 Q_2}{n_2}. \tag{A22}$$

Appendix A.2. Derivative of Output of Final Goods with Respect to n_1 and n_2

$$\frac{\partial Q^*}{\partial n_1} = Q^* \frac{\beta(1-\beta)}{n_1(n_1-1)(n_1 n_2 - (1-\beta)n_1 - n_2\beta)}(n_2 - n_1), \tag{A23}$$

$$\frac{\partial Q^*}{\partial n_2} = -Q^* \frac{\beta(1-\beta)}{n_2(n_2-1)(n_1 n_2 - (1-\beta)n_1 - n_2\beta)}(n_2 - n_1). \tag{A24}$$

The necessary conditions for a maximum of Q^* are fulfilled if both derivatives are zero. This is obviously the case if $n_2 = n_1$,

$$\left.\frac{\partial Q^*}{\partial n_1}\right|_{n_2=n_1} = \left.\frac{\partial Q^*}{\partial n_2}\right|_{n_2=n_1} = 0. \tag{A25}$$

Furthermore, the signs of the derivatives are as follows:

$$\frac{\partial Q^*}{\partial n_1} \begin{cases} > 0, \text{ if } n_2 > n_1 \\ < 0, \text{ if } n_2 < n_1 \end{cases}. \tag{A26}$$

$$\frac{\partial Q^*}{\partial n_2} \begin{cases} < 0, \text{ if } n_2 > n_1 \\ > 0, \text{ if } n_2 < n_1 \end{cases}. \tag{A27}$$

Equations (A26) and (A27) indicate that the efficiency will only be improved if the competition in the sector with more market power than the other sector increases. In contrast, if competition increases in the sector with less market power, the efficiency of production will decline.

Appendix A.3. Stability of the Steady-State Capital Intensity

$$k^* = \left(\frac{sA\left[(1-\alpha) + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}\right]}{p_Q^*(1+g_N)} \right)^{\frac{1}{1-\alpha}}$$

We define $\Omega = (1-\alpha) + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}$. Then, the capital market clearing condition can be written as:

$$s\Omega A k_t^\alpha - p_Q^* k_{t+1}(1+g_N) = 0. \tag{A28}$$

Total differentiation with respect to k_t and k_{t+1} and reformulating leads to the stability condition:

$$\frac{dk_{t+1}}{dk_t} = \frac{s\Omega A \alpha k_t^{\alpha-1}}{p_Q^*(1+g_N)} < 1. \tag{A29}$$

Inserting k^* for k_t delivers:

$$\left.\frac{dk_{t+1}}{dk_t}\right|_{k_t=k_{t+1}=k^*} = \alpha < 1. \tag{A30}$$

It also can be shown that the steady-state equilibrium is unique and globally stable.

Appendix A.4. The Derivative of Steady-State Capital Intensity with Respect to the Number of Firms

First, we consider the case that n_1 increases.

$$k^* = \left(\frac{sA\left[(1-\alpha) + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}\right]}{p_Q^*(1+g_N)} \right)^{\frac{1}{1-\alpha}}. \tag{A31}$$

To reduce the paperwork, we use a few definitions:

$$\Psi = \frac{sA\left[(1-\alpha) + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}\right]}{p_Q^*(1+g_N)} \tag{A32}$$

and

$$\Omega = (1-\alpha) + \left(\frac{1}{n_1-1}\right)\frac{1}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*}. \tag{A33}$$

Then we can write:

$$k^* = \left(\left(\frac{sA}{(1+g_N)}\right)\Omega\left(\frac{1}{p_Q^*}\right) \right)^{\frac{1}{1-\alpha}} = (\Psi)^{\frac{1}{1-\alpha}}, \tag{A34}$$

Furthermore, it is helpful to calculate the following derivatives:

$$\frac{\partial\Omega}{\partial n_1} = -\frac{\beta n_2^2}{(n_1 n_2 - (1-\beta)n_1 - n_2\beta)^2} < 0, \tag{A35}$$

$$\frac{\partial\left(\frac{1}{p_Q^*}\right)}{\partial n_1} = \left(\frac{1}{p_Q^*}\right)\frac{\beta}{(n_1-1)n_1} > 0. \tag{A36}$$

Using (A31)–(A34), the derivative of the steady-state capital intensity regarding the number of firms in sector one is:

$$\frac{\partial k^*}{\partial n_1} = \frac{(\Psi)^{\frac{\alpha}{1-\alpha}} sA\beta}{(1-\alpha)p_Q^*(1+g_N)(n_1 n_2 - (1-\beta)n_1 - n_2\beta)} \left(\frac{(1-\alpha)n_1 n_2 + \alpha((1-\beta)n_1 + n_2\beta)}{(n_1-1)n_1} - \frac{n_2^2}{n_1 n_2 - (1-\beta)n_1 - n_2\beta} \right). \tag{A37}$$

As we can see, the sign of the term in the brackets is not unique. To obtain a better insight, we consider the term in the brackets in detail,

$$\Phi_1 = \frac{(1-\alpha)n_1 n_2 + \alpha((1-\beta)n_1 + n_2\beta)}{(n_1-1)n_1} - \frac{n_2^2}{n_1 n_2 - (1-\beta)n_1 - n_2\beta}. \tag{A38}$$

We solve (A38) for α and can conclude as follows regarding the derivative $\frac{\partial k^*}{\partial n_1}$:

$$\frac{\partial k^*}{\partial n_1} \begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_2(1-\beta)(n_2-n_1)n_1}{(n_1 n_2 - (1-\beta)n_1 - \beta n_2)^2} \\ < 0, \text{ if } \alpha > \frac{n_2(1-\beta)(n_2-n_1)n_1}{(n_1 n_2 - (1-\beta)n_1 - \beta n_2)^2} \end{cases}. \tag{A39}$$

The condition (A39) also implies that $\frac{\partial k^*}{\partial n_1} < 0$, if $n_1 \geq n_2$, because the condition is then negative or zero and $\alpha > 0$ by definition.

Second, we consider the case that n_2 increases. Repeating the calculations above for n_2 delivers:

$$\frac{\partial k^*}{\partial n_2} = \frac{(\Psi)^{\frac{\alpha}{1-\alpha}} sA(1-\beta)}{(1-\alpha)p_Q^*(1+g_N)(n_1 n_2 - (1-\beta)n_1 - n_2\beta)}\left(\frac{(1-\alpha)n_1 n_2 + \alpha((1-\beta)n_1 + n_2\beta)}{(n_2-1)n_2} - \frac{n_1^2}{n_1 n_2 - (1-\beta)n_1 - n_2\beta}\right). \tag{A40}$$

Now the sign of the derivative depends on Φ_2, which is defined as:

$$\Phi_2 = \frac{(1-\alpha)n_1 n_2 + \alpha((1-\beta)n_1 + n_2\beta)}{(n_2-1)n_2} - \frac{n_1^2}{n_1 n_2 - (1-\beta)n_1 - n_2\beta}. \tag{A41}$$

Solving (A41) for α delivers this equation regarding the derivative $\frac{\partial k^*}{\partial n_2}$:

$$\frac{\partial k^*}{\partial n_2}\begin{cases} \geq 0, \text{ if } \alpha \leq \frac{n_1\beta(n_1-n_2)n_2}{(n_1n_2-(1-\beta)n_1-\beta n_2)^2} \\ < 0, \text{ if } \alpha > \frac{n_1\beta(n_1-n_2)n_2}{(n_1n_2-(1-\beta)n_1-\beta n_2)^2} \end{cases}. \tag{A42}$$

We investigate at least the case that $n_1 = n_2 = n$:

$$\left.\frac{\partial k^*}{\partial n_2}\right|_{n_1=n_2=n} = -\frac{\alpha(\Psi)^{\frac{\alpha}{1-\alpha}} sA(1-\beta)}{(1-\alpha)p_Q^*(1+g_N)(n^2-n)} < 0. \tag{A43}$$

Appendix A.5. Derivative of Total Profits per Capita π^*_{real} with Respect to n_1 and n_2

$$\pi^*_{real} = \left(\left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{1}{1+\theta^*}\right)\frac{A(k^*)^\alpha}{p_Q^*} = Y\frac{z}{p_Q^*}, \tag{A44}$$

where $Y = \left(\frac{1}{n_2-1}\right)\frac{\theta^*}{1+\theta^*} + \left(\frac{1}{n_2-1}\right)\frac{1}{1+\theta^*}$.

The respective derivative of total real profits becomes:

$$\frac{\partial \pi^*_{real}}{\partial n_1} = Y\frac{A(k^*)^\alpha}{n_1 p_Q^*}\left(\varepsilon_{Y,n_1} + \varepsilon_{\frac{1}{p_Q^*},n_1} + \varepsilon_{z,n_1}\right) < 0, \tag{A45}$$

where

$$\varepsilon_{Y,n_1} = -\frac{n_2^2\beta n_1}{(n_1n_2 - (1-\beta)n_1 - \beta n_2)((1-\beta)n_1 + \beta n_2)} < 0, \tag{A46}$$

$$\varepsilon_{\frac{1}{p_Q^*},n_1} = \frac{\beta}{n_1-1} > 0,$$

$$\varepsilon_{z,n_1} = \frac{\alpha\beta\left(n_1^2\left(\alpha(n_2-1+\beta)^2 + n_2(1-\beta)\right) - 2n_1n_2\left(\alpha\beta(n_2-1+\beta) + \frac{1}{2}n_2(1-\beta)\right) + \alpha n_2^2\beta^2\right)}{(n_1-1)(1-\alpha)[n_1(\alpha(n_2-1+\beta)-n_2) - n_2\alpha\beta][n_1n_2 - (1-\beta)n_1 - n_2\beta]} \lesseqgtr 0. \tag{A47}$$

and we obtain this equation regarding the number of firms in sector 2:

$$\frac{\partial \pi^*_{real}}{\partial n_2} = Y\frac{A(k^*)^\alpha}{n_2 p_Q^*}\left(\varepsilon_{Y,n_2} + \varepsilon_{\frac{1}{p_Q^*},n_2} + \varepsilon_{z,n_2}\right) < 0, \tag{A48}$$

where

$$\varepsilon_{Y,n_2} = \frac{\partial Y}{\partial n_2}\frac{n_2}{Y} = -\frac{n_1^2(1-\beta)n_2}{(n_1n_2 - (1-\beta)n_1 - \beta n_2)((1-\beta)n_1 + \beta n_2)} < 0, \tag{A49}$$

$$\varepsilon_{\frac{1}{p_Q^*},n_2} = \frac{\partial\left(\frac{1}{p_Q^*}\right)}{\partial n_2}\frac{n_2}{\left(\frac{1}{p_Q^*}\right)} = \frac{1-\beta}{n_2-1} > 0, \tag{A50}$$

$$\varepsilon_{z,n_2} = \frac{\partial z}{\partial n_2}\frac{n_2}{z} = \frac{\alpha(1-\beta)\left(n_1^2\left(\alpha(n_2-1+\beta)^2+n_2\beta\right)-2\beta n_1 n_2\left(\alpha(n_2-1+\beta)+\frac{1}{2}n_2\right)+\alpha n_2^2\beta^2\right)}{(n_2-1)(1-\alpha)[n_1(\alpha(n_2-1+\beta)-n_2)-n_2\alpha\beta][n_1 n_2-(1-\beta)n_1-n_2\beta]} \lesseqgtr 0. \quad \text{(A51)}$$

A very tedious and time-consuming analysis shows that $\frac{\partial \pi^*_{real}}{\partial n_1} < 0$ and $\frac{\partial \pi^*_{real}}{\partial n_2} < 0$.

Note

1 It is not clear whether this result remains valid when the labor supply and capital supply are not perfectly wage and interest inelastic, respectively.

References

Affeldt, Pauline, Tomaso Duso, Klaus P. Gugler, and Joanna Piechucka. 2021. *Market Concentration in Europe: Evidence from Antitrust Markets*. DIW Discussion Papers, No. 1930. Berlin: Deutsches Institut für Wirtschaftsforschung (DIW). Available online: https://ideas.repec.org/p/diw/diwwpp/dp1930.html (accessed on 29 June 2023).

Akcigit, Ufuk, Wenjie Chen, Federico J. Diez, Romain A. Duval, Philipp Engler, Jiayue Fan, Chiara Maggi, Marina M. Tavares, Daniel A. Schwarz, Ippei Shiabata, and et al. 2021. *Rising Corporate Market Power: Emerging Policy Issues*. IMF Staff Discussion Notes 2021/001. Washington, DC: International Monetary Fund.

Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John Van Reenen. 2017. Concentrating on the fall of the labor share. *American Economic Review* 107: 180–85. [CrossRef]

Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John Van Reenen. 2019. The Fall of the labor share and the rise of superstar firms. *Quarterly Journal of Economics* 135: 645–709. [CrossRef]

Bajgar, Matej, Giuseppe Berlingieri, Sara Calligaris, Chiara Criscuolo, and Jonathan Timmis. 2019. *Industry Concentration in Europe and North America*. No. 18. Paris: OECD Publishing. Available online: https://www.oecd-ilibrary.org/economics/industry-concentration-in-europe-and-north-america_2ff98246-en (accessed on 29 June 2023).

Barkai, Simcha. 2020. Declining labor and capital shares. *The Journal of Finance* 75: 2421–63. [CrossRef]

Barkai, Simcha, and Seth G. Benzell. 2018. *70 Years of US Corporate Profits*. New Working Paper Series No. 22; Chicago: Stigler Center for the Study of the Economy and the State University of Chicago, Booth School of Business. Available online: https://research.chicagobooth.edu/stigler/research/-/media/8ee68ec563aa4c70aa94897ee04f68b6 (accessed on 29 June 2023).

Bertrand, Marianne, and Antoinette Schoar. 2006. The role of family in family firms. *Journal of Economic Perspectives* 20: 73–96. [CrossRef]

Dao, Mai C., Mitali Das, Zsoka Koczan, and Weicheng Lian. 2017. Drivers of Declining Labor Share of Income. Insights and Analysis on Economics and Finance. IMF Blog, IMF. Available online: https://www.imf.org/en/Blogs/Articles/2017/04/12/drivers-of-declining-labor-share-of-income#:~:text=In%20advanced%20economies%2C%20about%20half,could%20be%20easily%20be%20automated (accessed on 30 August 2023).

D'Aspremont, Claude, Rodolphe Dos Santos Ferreira, and Louis-André Gérard-Varet. 1989. Unemployment in an extended oligopoly model. *Oxford Economic Papers* 41: 490–505. [CrossRef]

D'Aspremont, Claude, Rodolphe Dos Santos Ferreira, and Louis-André Gérard-Varet. 1990. Monopolistic competition and involuntary unemployment. *Quarterly Journal of Economics* 105: 895–919. [CrossRef]

D'Aspremont, Claude, Rodolphe Dos Santos Ferreira, and Louis-André Gérard-Varet. 1991. Imperfect competition, rational expectations and unemployment. In *Equilibrium Theory and Applications, Proceedings of the Sixth International Symposium in Economic Theory and Econometrics*. Edited by William A. Bamett, Benard Cornet, Claude D'Aspremont, Jean Gabszewicz and Andreu Mas-Colell. Cambridge: Cambridge University Press, pp. 353–81.

D'Aspremont, Claude, Rodolphe Dos Santos Ferreira, and Louis-André Gérard-Varet. 1995. Imperfect competition in an overlapping generations model: A case for fiscal policy. *Annales D'Economie et de Statistique* 37/38: 531–55.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger. 2020. The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics* 135: 561–644. [CrossRef]

Diamond, Peter A. 1965. National debt in a neoclassical growth model. *The American Economic Review* 55: 1126–50.

Elsby, Michael W. L., Bart Hobijn, and Ayşegül Şahin. 2013. The decline of the US labor share. *Brookings Papers on Economic Activity* 2013: 1–63. [CrossRef]

Hart, Oliver D. 1982. A model of imperfect competition with Keynesian features. *Quarterly Journal of Economics* 97: 109–38. [CrossRef]

Hart, Oliver D. 1985. Imperfect competition in general equilibrium: An overview of recent work. In *Frontiers of Economics*. Edited by Kenneth Arrow and Seppo Honkapohja. New York: Blackwell, pp. 100–50.

Herrendorf, Berthold, Christopher Herrington, and Akos Valentinyi. 2015. Sectoral Technology and Structural Transformation. *American Economic Journal: Macroeconomics* 7: 104–33. [CrossRef]

IMF. 2017. World Economic Outlook, April 2017: Gaining Momentum? IMF. Available online: https://www.imf.org/en/Publications/WEO/Issues/2017/04/04/world-economic-outlook-april-2017 (accessed on 30 August 2023).

Karabarbounis, Loukas, and Brent Neiman. 2014. The global decline of the labor share. *Quarterly Journal of Economics* 129: 61–103. [CrossRef]

Karabarbounis, Loukas, and Brent Neiman. 2018. *Accounting for Factor-Less Income*. NBER Working Paper 24404. Cambridge: NBER.
Kelley, Donna, William. B. Gartner, and Matt R. Allen. 2020. *Global Entrepreneurship Monitor Family Business Report*. Babson Park: Babson College Press.
Knoblach, Michael, and Fabian Stöckl. 2020. What determines the elasticity of substitution between capital and labor? A literature review. *Journal of Economic Surveys* 34: 847–75. [CrossRef]
Knoblach, Michael, Martin Roessler, and Patrick Zwerschke. 2019. The Elasticity of factor substitution between capital and labor in the US economy: A meta-regression. *Oxford Bulletin of Economics and Statistics* 82: 62–82. [CrossRef]
Kumar, Ronald R., and Peter J. Stauvermann. 2020. Economic and social sustainability: The influence of oligopolies on inequality and growth. *Sustainability* 12: 9378. [CrossRef]
Kumar, Ronald. R., Peter J. Stauvermann, and Frank Wernitz. 2022. The capitalist spirit and endogenous growth. *Journal of Risk and Financial Management* 15: 27. [CrossRef]
Laitner, John. 1982. Monopoly and long-run capital accumulation. *Bell Journal of Economics* 13: 143–57. [CrossRef]
Manyika, James, Jan Mischke, Jacques Bughin, Jonathan Woetzel, Mekala Krishnan, and Samuel Cudre. 2019. A New Look at the Declining Labor Share of Income in the United States. McKinsey Global Institute, May 22. Available online: https://www.mckinsey.com/featured-insights/employment-and-growth/a-new-look-at-the-declining-labor-share-of-income-in-the-united-states (accessed on 30 August 2023).
Neary, Peter J. 2016. International trade in general oligopolistic equilibrium. *Review of International Economics* 24: 669–98. [CrossRef]
Neary, Peter J. 2002. *The Road Less Travelled: Oligopoly and Competition Policy in General Equilibrium*. Centre for Economic Research Working Paper Series, No. WP02/22. Dublin: University College Dublin, Department of Economics.
Neary, Peter J. 2010. Two and a half theories of trade. *World Economy* 33: 1–19. [CrossRef]
Oberfield, Ezra, and Devesh Raval. 2021. Micro Data and Macro Technology. *Econometrica* 89: 703–32. [CrossRef]
Philippon, Thomas. 2019. *The Great Reversal: How America Gave Up on Free Markets*. Boston: Belknap Press: An Imprint of Harvard University Press.
Rognlie, Matthew. 2015. Deciphering the Fall and the Rise of the Net Capital Share. In *Brookings Papers on Economic Acitivity*. Berlin/Heidelberg: Spring, pp. 1–54.
Stauvermann, Peter J., and Ronald R. Kumar. 2022. Does more market competition lead to higher income and utility in the long run? *Bulletin of Economic Research* 74: 761–82. [CrossRef]
Velasquez, Agustin. 2023. Production Technology, Market Power, The Decline of Labor Share. IMF Working Paper no 2023/032. Available online: https://www.imf.org/-/media/Files/Publications/WP/2023/English/wpiea2023032-print-pdf.ashx (accessed on 30 August 2023).
Zingales, Luigi. 2012. *A Capitalism for the People*. New York: Basic Books.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.