

Article

A Sovereign Environmental Wealth Index: A Financial Framework for Measuring and Managing Sustainability Risk

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Abstract

The increasing importance of climate change, natural resource constraints, and sustainability-related risks has created a growing need for quantitative measures that connect environmental performance with economic and financial decision-making. Existing environmental and ESG indicators provide valuable assessments of sustainability conditions, but they are generally designed as ranking or reporting measures rather than dynamic financial indices suitable for risk modeling, portfolio analysis, and long-term economic evaluation. This paper proposes a sovereign-level environmental wealth framework that translates environmental performance into financially interpretable time-series indices. Using fourteen World Development Indicators (WDIs) covering environmental conditions and environmentally relevant economic characteristics for ten major economies, we construct Dollar Environmental Financial Indices (DEFIs). The proposed framework combines standardized environmental information with economic capacity, represented by GDP per capita, to measure the economic value associated with national environmental performance. The resulting indices are not intended to represent directly traded financial securities, but rather synthetic environmental wealth benchmarks that allow sustainability-related risks to be analyzed using established tools from financial economics. We further construct a Global Dollar Environmental Financial Index (GDEFI), which represents the common global component of environmental performance and serves as a benchmark for evaluating systematic environmental exposure across countries. Using robust regression, dynamic econometric models, and volatility analysis, we estimate environmental beta coefficients and examine how country-level environmental conditions respond to global environmental movements. Estimated environmental betas range from -0.708 (Australia) to 1.617 (China), while the explanatory power of the global benchmark reaches an adjusted R^2 of 0.886 for the United Kingdom, demonstrating substantial cross-country heterogeneity in environmental exposure, risk dynamics, and resilience. To demonstrate the usefulness of the proposed framework, we apply risk-adjusted performance measures, including Jensen's alpha, Sharpe ratio, Sortino ratio, and the Rachev ratio, together with mean-variance and CVaR-based portfolio optimization methods. These analyses show that environmental exposures exhibit distinct risk-return and tail-risk characteristics, highlighting potential diversification benefits in sustainability-oriented decision frameworks. Maximum-likelihood factor analysis further indicates that a three-factor structure provides an adequate representation of the common variation in environmental performance across countries. Finally, we provide a conceptual illustration of how environmental index-based derivatives could support future sustainability risk management. While DEFIs are not currently tradable assets,



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the proposed framework establishes a bridge between environmental measurement and financial modeling by allowing environmental risk to be evaluated through the analytical tools traditionally applied to financial markets.

Keywords: environmental wealth index; sovereign sustainability risk; Global Dollar Environmental Financial Index (GDEFI); natural capital; environmental risk; sustainable finance; portfolio optimization; factor analysis

1. Introduction

Growing concerns about climate change, resource depletion, and environmental degradation have intensified the global demand for quantitative frameworks capable of measuring the economic and financial implications of environmental performance. Environmental conditions increasingly influence long-term economic growth, sovereign risk, production capacity, regulatory costs, and investment decisions. While widely used indicators such as the Environmental Performance Index (EPI), Ecological Footprint Index (EFI), Green GDP, Environmental Sustainability Index (ESI), and sovereign ESG measures provide valuable assessments of sustainability conditions, they are primarily designed as descriptive ranking and reporting tools. These measures generally do not provide a dynamic financial framework that links environmental conditions with economic value creation, risk exposure, and allocation-based decision-making.

Sustainable investment has expanded rapidly in recent years, driven by institutional investors' interest in measuring climate-related risks, satisfying sustainability requirements, and allocating capital toward environmentally resilient economies. However, a persistent challenge remains: the absence of transparent, scalable, and financially interpretable environmental indices that connect environmental outcomes with economic capacity and risk measurement. Existing environmental and ESG indicators provide important information about environmental states, but they are generally not constructed as time-series indices suitable for financial modeling, risk analysis, and dynamic comparison across countries. In contrast, the framework proposed in this paper differs from traditional environmental indicators by (i) embedding environmental and economically relevant sustainability measures into a monetary representation, (ii) generating dynamic country-level time series that capture changes in environmental wealth, and (iii) enabling the application of financial econometric, portfolio optimization, asset-pricing, and risk-management methodologies. These differences allow the proposed Dollar Environmental Financial Indices (DEFIs) and the Global Dollar Environmental Financial Index (GDEFI) to serve not merely as descriptive environmental indicators, but as synthetic environmental wealth benchmarks that quantify a country's exposure to sustainability-related economic risks.

To address this gap, we construct a set of dollar-denominated environmental financial indices (DEFIs) for a group of major economies using fourteen World Development Indicators (WDIs) that capture both environmental conditions and environmentally relevant economic activity. Our objective is not to claim that these indices represent directly traded financial securities, but rather to develop financially interpretable environmental wealth indices that translate sustainability conditions into an economic measurement framework. By combining standardized environmental information with GDP per capita, the proposed approach connects environmental performance with a country's economic capacity, producing dynamic time-series measures that can be evaluated using econometric, portfolio-theoretic, and asset-pricing tools.

This construction enables us to define a global benchmark, the Global Dollar Environmental Financial Index (GDEFI), which aggregates country-level environmental wealth indices into a unified measure of global environmental performance. Conceptually, each country represents a component within a global portfolio of environmental exposure, similar to how country-level economic or market indices contribute to broader international benchmarks. This interpretation provides a framework for applying classical financial tools—such as the Capital Asset Pricing Model (CAPM), Jensen’s alpha, efficient frontier analysis, and dynamic risk modeling—to study environmental risk transmission, systematic exposure, and diversification opportunities.

Although the GDEFI is computed as an aggregation of country-level DEFIs, it should not be interpreted as a simple arithmetic environmental score. By construction, the GDEFI summarizes the common global component of environmental and sustainability-related economic performance, capturing shared environmental shocks, climate-related pressures, cross-country spillovers, and coordinated policy responses. In the same way that broad market indices represent systematic economic forces through the aggregation of individual securities, the GDEFI represents a systematic environmental factor underlying cross-country environmental dynamics. Treating the GDEFI as a benchmark for global environmental exposure provides economic meaning to the factor structure of the system and allows us to examine whether environmental conditions contain a non-diversifiable component similar to systematic risk in traditional financial markets. This interpretation motivates the regression analysis that follows, where country-level DEFIs are evaluated relative to the global benchmark to estimate environmental beta and Jensen’s alpha.

The connection to asset-pricing models is particularly relevant. If environmental performance influences long-term economic stability, resource efficiency, regulatory uncertainty, or exposure to climate-related shocks, then environmental conditions may represent a systematic source of macro-financial risk. Within this framework, environmental beta measures the sensitivity of a country’s environmental wealth index to global environmental movements. Using regressions between country-level DEFIs and the GDEFI, we examine whether countries exhibit different levels of exposure to common environmental dynamics and whether deviations from the benchmark provide evidence of country-specific environmental advantages or vulnerabilities. This approach connects sustainable finance with asset-pricing theory while recognizing that environmental indices represent synthetic risk measures rather than conventional traded assets.

Beyond risk-exposure analysis, the proposed index system enables the exploration of diversification opportunities across different sources of environmental risk. Using mean–variance optimization and alternative risk measures, we examine how environmental exposures may be combined across countries to reduce uncertainty and improve sustainability-oriented allocation decisions. Although environmental wealth indices are not currently tradable instruments, this analysis demonstrates how such indices could provide the foundation for future financial products and environmental risk-management frameworks. In this context, we also discuss how derivative-based instruments, such as put options written on environmental benchmarks, could conceptually provide protection against severe environmental deterioration.

Recent research has continued to strengthen the connection between sustainability, environmental risk, and financial decision-making from several complementary perspectives. Ullah et al. (2025) examine how green finance, technological innovation, and financial risk jointly influence sustainability indicators, highlighting the importance of financial development and technological progress in improving long-run environmental performance. Focusing on the G7 economies, Zeren et al. (2025) investigate the determinants of ESG-related sustainability uncertainty, demonstrating that macroeconomic conditions and

climate-related uncertainty play important roles in shaping sustainability dynamics. At the corporate level, Singh and Kushwaha (2025) develop a qualitative framework for evaluating corporate social accountability through weather-index insurance policies, emphasizing the role of climate-risk management in supporting stakeholder-oriented sustainability strategies. Likewise, Aziz and Cek (2025) show that board gender diversity contributes to corporate financial performance through its interaction with sustainability practices, climate risk management, and internal control systems, further illustrating the growing integration of environmental considerations into financial decision-making.

Although these studies significantly advance the understanding of sustainability, ESG performance, climate risk, and green finance, they primarily focus on identifying the determinants or consequences of environmental performance at the macroeconomic or corporate level. In contrast, the present paper develops a sovereign-level environmental wealth framework that transforms multidimensional environmental information into dynamic, financially interpretable time-series indices. By introducing the Dollar Environmental Financial Index (DEFI) and the Global Dollar Environmental Financial Index (GDEFI), we establish a framework that enables the application of financial econometric methods, environmental beta estimation, portfolio optimization, factor analysis, and conceptual derivative pricing to evaluate systematic environmental exposure, cross-country sustainability dynamics, and environmental risk allocation. Consequently, this study extends the existing sustainability literature by bridging environmental measurement with modern financial risk management through a unified environmental wealth index framework.

Accordingly, the present study shifts the focus from explaining sustainability outcomes toward constructing a financially interpretable environmental wealth framework that enables dynamic risk analysis, sovereign-level performance evaluation, and sustainability-oriented portfolio applications.

The proposed framework is developed to address three principal research questions. First, can environmental performance be transformed into a financially interpretable sovereign environmental wealth index that preserves meaningful environmental information while supporting quantitative financial analysis? Second, to what extent do country-level environmental wealth indices exhibit systematic exposure to global environmental dynamics, and can such exposure be characterized using established asset-pricing and financial econometric methods? Third, do sovereign environmental wealth indices exhibit systematic risk, diversification, and latent factor structures that provide useful insights for sustainability risk measurement, portfolio allocation, and environmental policy evaluation?

The present study makes three principal contributions to the sustainable finance and environmental economics literature. First, we introduce a new class of dollar-denominated environmental wealth indices that integrate environmental indicators with economic capacity to create financially interpretable sustainability measures. Second, we construct a global environmental benchmark (GDEFI) and examine its behavior using asset-pricing and econometric methods to evaluate systematic environmental exposure across countries. Finally, we analyze portfolio diversification, efficient frontiers, risk-adjusted performance measures, factor structures, and potential environmental risk-management applications, demonstrating how the proposed framework can support sustainable investment analysis and policy evaluation.

While environmental indices are not currently traded assets, we include a conceptual option-pricing illustration in Section 6 to demonstrate how environmental derivatives could potentially function if such markets developed in the future. This example is not presented as a primary empirical contribution, but rather illustrates how transforming environmental performance into a dynamic financial index allows standard risk-management techniques—such as downside protection through put options—to be adapted for sustainability-

related risks. This discussion places the DEFI framework within the broader development of climate finance and environmental risk-transfer mechanisms.

These contributions address an important gap between environmental measurement and financial economics. Existing sustainability indicators provide valuable information about environmental conditions, but they generally lack the structure required for dynamic risk modeling, portfolio analysis, and systematic exposure measurement. By expressing environmental performance through financially interpretable indices, the proposed framework provides a bridge between environmental economics and modern financial risk management.

The remainder of the paper is organized as follows. Section 2 introduces the fourteen WDIs used to construct the environmental financial indices and discusses their economic and environmental relevance. Section 3 develops the DEFI methodology by transforming standardized environmental indicators into country-level dollar-denominated environmental wealth indices and constructing the global benchmark GDEFI. Section 4 analyzes systematic environmental exposure through regression models, environmental beta estimation, and Jensen's alpha. Section 5 examines diversification patterns and optimal environmental allocations using mean–variance and CVaR-based portfolio methods. Section 6 provides a conceptual illustration of environmental derivative pricing and potential hedging applications. Section 7 identifies latent global and country-specific drivers of environmental comovement. Finally, Section 8 concludes by discussing the implications of environmental financial indices for sustainable investment, risk management, and policy design.

2. World Development Economic and Environmental Indicators

Environmental and economic indicators play a central role in evaluating a nation's environmental quality, resource utilization, sustainability performance, and long-run economic resilience. The increasing recognition that environmental conditions influence economic productivity, financial stability, and sovereign risk has motivated the development of integrated measures that combine ecological and economic information. In this study, we rely on fourteen World Development Indicators (WDIs) from the World Bank that jointly characterize environmental conditions, natural resource dependence, energy transition, and economic capacity.

The selection of WDIs is motivated by the concept of environmental wealth, where a country's long-term economic value is influenced not only by traditional measures of output but also by the quality and sustainability of its natural resources. Unlike conventional environmental rankings, which primarily summarize current ecological conditions, our objective is to construct a dynamic framework that links environmental performance with economic exposure. Therefore, the selected indicators represent both environmental characteristics and the economic mechanisms through which environmental conditions may influence long-term financial and sustainability risks.

On the environmental side, we include CO₂ emissions, nitrous oxide emissions, methane emissions, forest area, agricultural land, arable land, land area, and surface area. Carbon dioxide emissions represent one of the primary measures of climate impact and reflect a country's contribution to greenhouse gas accumulation. Nitrous oxide and methane emissions capture additional sources of climate-related risk because these gases have significant global warming potential and are closely associated with industrial activity, agriculture, and energy production. Forest area represents natural capital by measuring ecological resources associated with biodiversity protection, carbon sequestration capacity, and climate resilience. Agricultural land and arable land describe the intensity and structure of land utilization, providing information about food production capacity, resource dependence, and potential environmental pressures. Land area and surface area provide

geographical context by capturing the scale over which environmental resources and risks are distributed.

On the economic and sustainability side, we include GDP per capita, adjusted net savings, the energy intensity level of primary energy consumption, renewable energy consumption, and total natural resource rents. GDP per capita measures economic capacity and represents the monetary scale through which environmental performance is linked to economic value creation. Adjusted net savings provides a sustainability-adjusted measure of wealth accumulation by accounting for investment in human capital, depreciation of physical capital, depletion of natural resources, and environmental damages. Energy intensity captures the efficiency with which economies transform energy consumption into economic output, while renewable energy consumption reflects progress toward cleaner and more sustainable energy systems. Total natural resource rents measure the economic contribution derived from natural resources, linking environmental assets directly to national income generation.

Together, these fourteen indicators provide a multidimensional representation of the relationship between environmental conditions and economic performance. The framework intentionally combines environmental variables with economically relevant sustainability measures because environmental risks influence financial systems through multiple channels, including resource availability, energy efficiency, climate exposure, and long-term productive capacity. This integrated structure distinguishes the proposed approach from traditional environmental scores by creating a foundation for constructing dynamic environmental wealth indices suitable for financial risk analysis.

Although the indicators are combined into a single index in the following section, each component captures a distinct aspect of environmental or economic sustainability. The aggregation methodology provides a transparent benchmark representation of environmental wealth, while the empirical analysis further examines cross-country differences, common global factors, and country-specific environmental dynamics.

Detailed definitions and data sources for all indicators are provided in the Appendix A following [IBRD \(2022\)](#).

3. Construction of Environmental Wealth Indices

In this section, we construct country-level Dollar Environmental Financial Indices (DEFIs) and aggregate them into a global environmental benchmark. The objective of this framework is to transform multidimensional environmental information into a financially interpretable measure of environmental wealth that captures the interaction between sustainability performance and economic capacity. The resulting indices are not intended to represent directly traded securities; rather, they provide synthetic environmental wealth measures that allow environmental risks and cross-country sustainability dynamics to be studied using financial and econometric methodologies.

The analysis uses fourteen World Development Indicators (WDIs) for ten major economies—the United States, China, Japan, Germany, the United Kingdom, India, Brazil, Australia, France, and Canada—covering the period 2007–2021 from the World Bank ([IBRD, 2022](#)). Let F_{klt} denote the value of the k th World Development Indicator for country l in year t , where $k = 1, \dots, 14$, $l = 1, \dots, 10$, and $t = 2007, \dots, 2021$. Each indicator represents a component of environmental conditions, resource utilization, or sustainability-related economic activity. Figure 1 illustrates the evolution of the U.S. development indicators over the sample period, while Figure 2 presents the log returns of the exponentially transformed Dollar Environmental Financial Index (DEFI) introduced below.

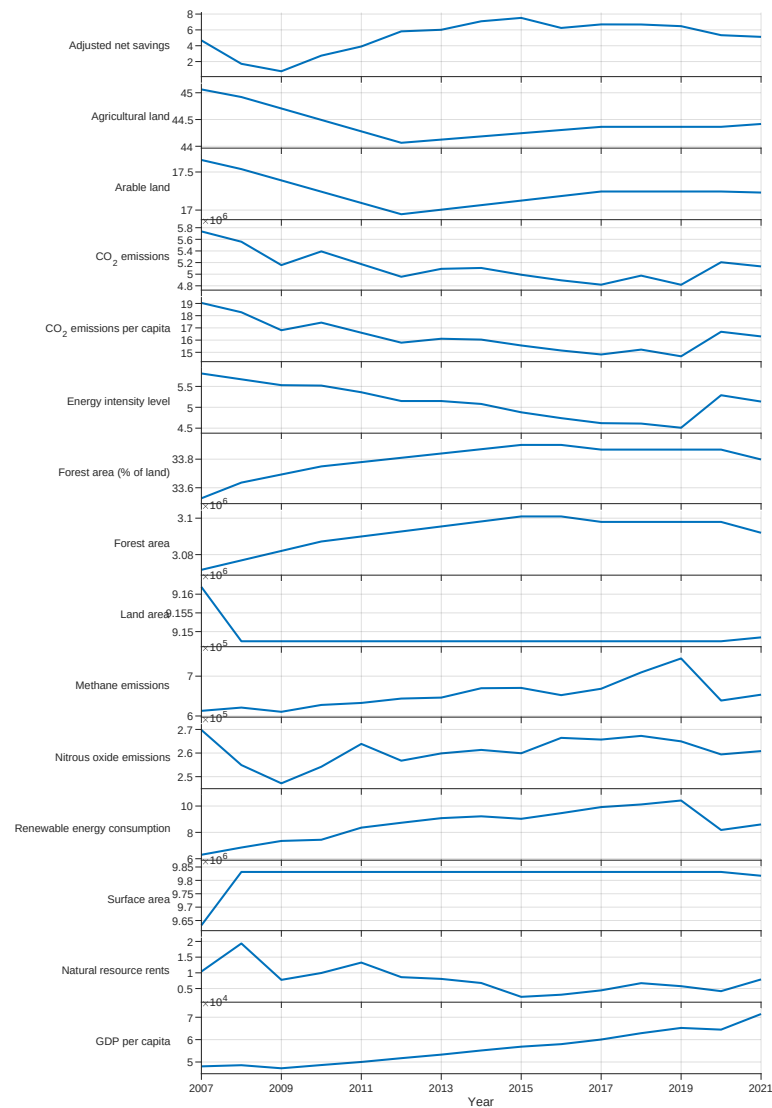


Figure 1. U.S. development indicators according to World Bank reports (IBRD, 2022) from 2007 to 2021.

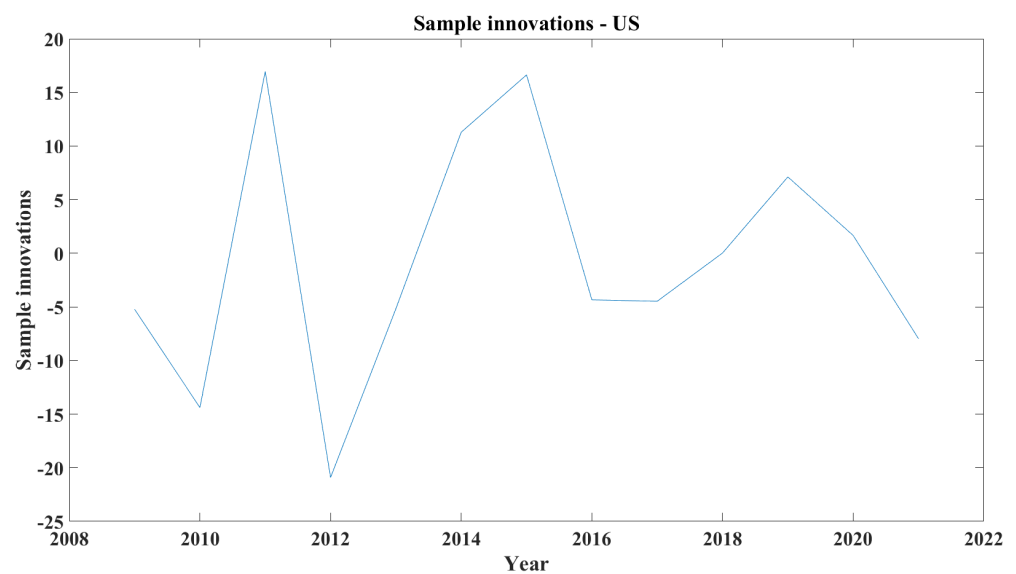


Figure 2. Log returns of the exponentially transformed Dollar Environmental Financial Index (DEFI) satisfying the constraints in Equation (5).

Since the WDIs are reported in different units and scales, direct aggregation is not meaningful. To ensure comparability across countries, each indicator is normalized relative to the cross-sectional total for each year t :

$$F_{N,kl} = \frac{F_{klt}}{\sum_{j=1}^L F_{kjt}}, \quad k = 1, \dots, 14, l = 1, \dots, 10, t = 2007, \dots, 2021. \quad (1)$$

where $L = 10$ denotes the total number of countries in the sample.

This normalization converts each indicator into a dimensionless measure representing a country's relative position within the selected group of countries. The objective is not to measure absolute environmental performance in isolation, but rather to construct a comparable cross-country framework for evaluating relative environmental wealth and sustainability characteristics. The normalization prevents indicators with large numerical scales or countries with larger absolute values from mechanically dominating the index construction while preserving meaningful differences in environmental and economic characteristics across countries. We acknowledge that this relative normalization does not capture common global shifts affecting all countries simultaneously. Alternative standardization procedures, including logarithmic transformations, Z-score normalization, or other scaling approaches, may provide complementary perspectives and constitute valuable directions for future robustness analyses.

The normalized indicators are then aggregated to construct the Environmental Financial Index (EFI):

$$DEFI_{lt} = \frac{1}{14} \sum_{k=1}^{14} F_{N,kl}, \quad l = 1, \dots, 10. \quad (2)$$

The equal-weight specification provides a transparent and easily reproducible benchmark that avoids imposing subjective preferences regarding the relative importance of different environmental dimensions. Because there is currently no universally accepted economic theory prescribing the relative contribution of individual environmental indicators to sovereign environmental wealth, equal weighting offers a neutral starting point that minimizes arbitrary modeling assumptions. Similar approaches are widely adopted in the construction of composite sustainability and environmental indices when objective weighting schemes are unavailable. Nevertheless, we acknowledge that alternative weighting methodologies—including principal component analysis (PCA), entropy-based methods, expert-based weighting, or machine-learning approaches—may yield different index compositions. Evaluating the sensitivity of the DEFI framework to alternative weighting schemes represents an important direction for future research.

By construction, $EFI_{lt} \in (0, 1)$, where larger values represent stronger relative environmental-economic performance within the selected country universe. The EFI itself is a normalized environmental measure; however, because it is dimensionless, additional economic scaling is required to connect environmental performance with financial interpretation.

To incorporate economic valuation, the EFI is scaled by GDP per capita to define the Dollar Environmental Financial Index (DEFI):

$$DEFI_{lt} = EFI_{lt} \times GDP_{lt}, \quad t = 2007, \dots, 2021. \quad (3)$$

The motivation for this transformation follows the concept of environmental wealth and natural capital valuation. The use of GDP per capita should not be interpreted as implying that economic development determines environmental performance. Rather, GDP per capita serves as an economic valuation mechanism that translates relative environ-

mental performance into a financially interpretable measure of sovereign environmental wealth. Conceptually, the EFI measures the quality of environmental performance, whereas GDP per capita represents the economic capacity associated with that performance. Consequently, the DEFI reflects the economic value associated with environmental wealth rather than environmental quality alone or the market value of a traded financial asset. The objective of the proposed framework is therefore not to construct another environmental ranking index, but to develop a synthetic environmental wealth measure that enables environmental conditions to be analyzed within a financial and macroeconomic framework.

Expressing environmental performance in dollar terms creates a financially interpretable index that can be analyzed using tools traditionally developed for economic and financial time series. The DEFI framework therefore provides a bridge between environmental measurement and financial risk analysis by allowing sustainability-related exposure, volatility, and cross-country dependence to be quantified.

To capture aggregate environmental dynamics across countries, we construct the Global Dollar Environmental Financial Index (GDEFI) as an equally weighted combination of country-level DEFIs:

$$GDEFI_t = \frac{1}{10} \sum_{l=1}^{10} DEFI_{lt}, \quad t = 2007, \dots, 2021. \quad (4)$$

Because DEFI is defined using GDP per capita rather than total GDP, equal weighting prevents large economies from dominating the benchmark solely because of population size or aggregate output. The GDEFI therefore represents the common global component of environmental wealth dynamics across the selected countries and serves as a global benchmark for measuring systematic environmental exposure.

In the following sections, the GDEFI is used as a reference index to evaluate environmental beta, country-specific deviations, diversification potential, and common factors driving international environmental dynamics. This interpretation is analogous to the role of broad economic and sustainability benchmarks, while recognizing that DEFIs represent synthetic environmental wealth indices rather than directly traded assets.

Financial and Econometric Modeling of Environmental Indices

To analyze the dynamic behavior of environmental wealth indices, we embed the proposed DEFI framework within a financial econometric setting. The objective is not to assume that environmental indices represent directly traded assets, but rather to study whether environmental performance, when expressed as a monetary time-series index, exhibits measurable risk dynamics similar to those observed in financial and macroeconomic variables. This perspective allows tools from asset pricing, volatility modeling, and risk management to be adapted for the analysis of sustainability-related uncertainty. The objective is not to claim that environmental indices satisfy all characteristics of financial asset prices, but rather to investigate whether their temporal dynamics can be represented using established stochastic models that facilitate quantitative analysis of environmental uncertainty and sustainability-related risk.

Following the financial index modeling framework of Schoutens (2003) and Duffie (2010), we first apply an exponential transformation to the DEFI time series. For each country and each year $t = t_0, \dots, 2021$, we define

$$\begin{aligned}
 f(x) &= ae^{bx}, \\
 f\left(\min_{l,t} DEFI_t(l)\right) &= 0, \\
 f\left(\max_{l,t} DEFI_t(l)\right) &= 1,
 \end{aligned} \tag{5}$$

The parameters (a, b) are chosen so that the transformed series is normalized to the unit interval while preserving the ordering of the original DEFI observations. This transformation provides a common scale across countries and improves comparability of environmental wealth dynamics, similar to the normalization procedures used in ESG scoring and sustainability benchmarks (Scatigna et al., 2021). For the U.S. DEFI, the estimated transformation parameters are $a = 0.0037$ and $b = 0.0001$. The exponential specification preserves positivity and generates a smooth index representation suitable for analyzing relative changes over time.

Given the transformed series $f(DEFI_t(l))$, we define the environmental index return for each country $l = 1, \dots, 10$ and for the global benchmark as

$$R_t(l) = \log \frac{f(DEFI_t(l))}{f(DEFI_{t-1}(l))}, \quad t = t_0, \dots, t_{15}. \tag{6}$$

The variable $R_t(l)$ measures the proportional change in the transformed environmental wealth index. Similar return-based transformations are commonly applied in financial and economic time-series analysis because they focus on relative dynamics, improve comparability across units, and reduce scale dependence (Duffie, 2010). In the DEFI framework, these returns represent changes in environmental-economic conditions rather than returns from a directly traded security.

To capture temporal dependence in environmental index movements, we model each country's transformed return series using an autoregressive specification:

$$R_t(l) = \phi_0 + \phi_1 R_{t-1}(l) + z_t + \theta_1 z_{t-1}, \quad t = 2008, \dots, 2021, \tag{7}$$

where ϕ_0 and θ_1 are parameters controlling the mean dynamics, and z_t represents an innovation process with conditional variance σ_t^2 . Specifically, $z_t = \sigma_t \epsilon_t$, where ϵ_t denotes an independent standardized innovation. This specification separates changes in the average environmental index dynamics from variations in uncertainty over time.

Environmental and climate-related risks may exhibit periods of stability followed by periods of increased uncertainty caused by regulatory changes, economic shocks, climate events, or transitions in energy systems. To capture this time-varying uncertainty, we consider three widely used conditional volatility models: autoregressive conditional heteroskedasticity (ARCH), generalized ARCH (GARCH), and exponential GARCH (EGARCH). These models allow the conditional variance of environmental index changes to evolve dynamically rather than remain constant.

The ARCH model captures volatility dependence through previous shocks, the GARCH model incorporates persistence in conditional volatility, and the EGARCH model allows asymmetric responses between positive and negative changes. Although originally developed for financial markets, these models have become broadly applicable for studying uncertainty in economic, environmental, and risk-related time series.

Model parameters are estimated using maximum likelihood, and model selection is based on the Akaike information criterion (AIC) and Bayesian information criterion (BIC). These criteria provide a systematic approach for comparing alternative dynamic specifications.¹

It is important to recognize that the available annual sample (2007–2021) is relatively short for estimating conditional volatility models. ARCH/GARCH-type models are traditionally developed for higher-frequency financial data, where volatility clustering can be estimated more precisely. Consequently, the volatility estimates presented here should not be interpreted as definitive forecasts of environmental uncertainty. Rather, they serve as parsimonious dynamic representations that allow us to compare alternative descriptions of time-varying variability within the DEFI framework and to construct a coherent simulation environment for the subsequent risk analysis. The principal objective is methodological illustration rather than precise volatility forecasting, and the empirical findings should therefore be interpreted with appropriate caution.

A particularly important specification is the GARCH(1,1) model:

$$\begin{aligned}\sigma_t &= \frac{z_t}{\epsilon_t}, \\ \sigma_t^2 &= \alpha_0 + \alpha_1 z_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad t = t_0, \dots, t_{30},\end{aligned}\quad (8)$$

where α_0 , α_1 , and β_1 determine the level and persistence of conditional volatility (Tsay, 2005). The GARCH framework captures volatility clustering, meaning that periods of elevated uncertainty tend to persist over time. Such persistence is particularly relevant for environmental risk because climate-related shocks, regulatory changes, and sustainability transitions often have long-lasting economic and environmental consequences.

We estimate AR(1)–ARCH(1), AR(1)–GARCH(1,1), and AR(1)–EGARCH(1,1) models (Models 1–3) for each country’s transformed DEFI series. The purpose of this analysis is to characterize the dynamic behavior and uncertainty structure of environmental wealth indices rather than to infer the existence of a directly tradable environmental market. For each country, the preferred specification is selected based on AIC and BIC values. Table 1 reports the model comparison results across countries.

Table 1. Estimated dynamic model comparison for the log returns of environmental indices based on the AIC and BIC (Model 1: AR(1)–ARCH(1), Model 2: AR(1)–GARCH(1,1), and Model 3: AR(1)–EGARCH(1,1)).

	AIC			BIC		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
US	18.959	20.959	21.290	21.219	23.784	24.114
Brazil	29.362	31.362	25.261	31.622	34.187	28.085
Germany	30.849	31.803	23.499	33.109	34.628	26.323
India	15.352	16.545	6.4929	17.611	19.370	9.3176
UK	44.244	46.244	39.406	46.504	49.069	42.230
Canada	44.724	46.718	43.304	46.984	49.543	46.129
France	36.781	38.101	36.897	39.041	40.926	39.722
Australia	40.424	42.420	41.536	44.983	49.536	44.935
China	−16.228	−14.228	−10.800	−13.968	−11.403	−7.9757
Japan	29.272	32.206	27.869	31.532	35.031	30.693
World	24.683	26.683	24.266	26.943	29.508	27.091

To examine possible future dynamics and dependence among environmental wealth indices, we use the selected time-series specifications as scenario-generation models rather than forecasting models. The estimated conditional volatility processes provide a convenient mechanism for generating internally consistent environmental uncertainty scenarios, while the multivariate NIG distribution captures cross-country dependence, skewness, and heavy-tailed behavior. Accordingly, the simulated paths should be interpreted as

hypothetical stress scenarios designed to study environmental risk rather than statistical forecasts of future environmental conditions.

The simulated scenarios generate possible future paths for the transformed DEFI indices and allow the evaluation of uncertainty, downside risk, and cross-country dependence. These simulations should be interpreted as environmental risk scenarios rather than forecasts of traded asset prices. This scenario-based approach is consistent with stress-testing methodologies used in financial and sustainability risk management (Jacobson et al., 2005; Orgeldinger, 2006).

The combination of dynamic volatility models and multivariate NIG innovations provides a flexible framework for studying the behavior of environmental wealth indices under uncertainty. By modeling transformed DEFI changes, volatility persistence, and cross-country dependence, this structure supports the subsequent analysis of systematic exposure, portfolio allocation, factor dynamics, and conceptual environmental risk-hedging applications.

In summary, we represent country-level environmental performance through financially interpretable environmental wealth indices by transforming DEFIs into normalized dynamic time-series processes, modeling their changes with ARCH/GARCH-type volatility structures, and capturing their cross-country dependence through a multivariate NIG distribution. This econometric framework provides a foundation for analyzing environmental risk exposure, uncertainty, diversification potential, and possible future risk-management applications using tools adapted from financial economics.

4. Regression Analysis and Environmental Exposure of DEFIs with Respect to the GDEFI

A central motivation for constructing the Global Dollar Environmental Financial Index (GDEFI) is to identify the common component of environmental wealth dynamics across countries. Although the GDEFI is computed through the aggregation of country-level DEFIs, it should not be interpreted as a simple environmental score. Instead, the GDEFI represents the common global component of environmental wealth dynamics and serves as the benchmark for measuring systematic environmental exposure across countries. The GDEFI therefore serves as a reference factor representing global environmental exposure.

Analyzing the relationship between each country-level DEFI and the GDEFI allows us to measure how strongly national environmental wealth dynamics respond to common global environmental movements. In this framework, the slope coefficient β_g is interpreted as an environmental beta, measuring the sensitivity of a country's DEFI to changes in the global environmental benchmark.

The intercept α_g , following the intuition of Jensen's alpha, measures persistent deviations in a country's environmental wealth performance relative to the global environmental benchmark after adjusting for systematic environmental exposure. Unlike the traditional CAPM interpretation, these deviations should not be viewed as abnormal financial returns. Instead, they quantify whether a country's environmental wealth evolves more favorably or less favorably than would be expected given its exposure to common global environmental dynamics.

These deviations may reflect country-specific characteristics, including environmental policies, economic structure, resource dependence, technological development, or sustainability transitions.

To estimate this relationship, we employ the regression model

$$r_{g,t} = \alpha_g + \beta_g r_t + \epsilon_{g,t} \quad (9)$$

where $r_{g,t}$ and r denote the log changes of the DEFI for country g and the GDEFI, respectively. Because environmental and economic time series may contain nonlinear behavior, extreme observations, and non-normal fluctuations, ordinary least squares (OLS) estimates may be sensitive to outliers. We therefore employ robust regression (RR), which reduces the influence of extreme observations and provides more stable parameter estimates for long-horizon environmental and economic data (Hu et al., 2021; Knez & Ready, 1997). The transformed DEFI series used in this section are obtained from the dynamic framework introduced in Section Financial and Econometric Modeling of Environmental Indices.

Table 2 reports the estimated relationships between country-level DEFIs and the GDEFI. The environmental beta coefficients β_g are statistically significant for most countries, indicating that national environmental wealth indices share important common movements with the global benchmark. Countries such as the United States, United Kingdom, Canada, France, Germany, China, and India exhibit positive environmental beta values, suggesting stronger alignment with global environmental dynamics.

Table 2. Regression estimates of environmental beta (β_g) and Jensen's alpha (α_g), measuring systematic environmental exposure and country-specific deviations in environmental wealth performance relative to the Global Dollar Environmental Financial Index (GDEFI).

Country	α_g	p -Value	β_g	p -Value	R-Squared
Australia	0.273	0.049	−0.708	0.012 *	0.369
Brazil	0.120	0.443	0.228	0.428	−0.026
Canada	0.142	0.047	0.476	0.000 ***	0.809
China	−0.466	0.087	1.617	0.017 *	0.340
France	0.111	0.158	0.545	0.000 ***	0.752
Germany	0.027	0.792	0.715	0.001 ***	0.558
India	−0.189	0.309	0.974	0.024 *	0.304
Japan	0.144	0.242	0.480	0.013 *	0.364
UK	0.154	0.009 ***	0.475	0.000 ***	0.886
US	0.018	0.767	1.009	0.000 ***	0.847

* $p \leq 0.05$, *** $p \leq 0.001$.

Australia exhibits a negative estimated environmental beta, suggesting that its environmental wealth dynamics differ from the common global environmental component represented by the GDEFI. This result may reflect differences in economic structure, resource dependence, environmental policy, or other country-specific characteristics. The adjusted R^2 values vary considerably across countries, highlighting that the degree of integration with global environmental dynamics is heterogeneous. Some countries are strongly associated with the global environmental component, whereas others retain larger country-specific environmental characteristics.

After examining the relationship between national DEFIs and the global environmental benchmark, we further investigate whether changes in environmental wealth indices are associated with movements in domestic equity markets. This analysis explores the potential connection between environmental conditions and broader financial-market behavior. For each country, we regress the log returns of the corresponding stock-market index on changes in the country's DEFI. The selected equity benchmarks include the S&P/ASX 200 (Australia), Bovespa Index (Brazil), S&P/TSX Composite Index (Canada), Shanghai Composite Index (China), CAC 40 (France), DAX (Germany), BSE Sensex (India), Nikkei 225 (Japan), FTSE 100 (United Kingdom), and S&P 500 (United States).

Table 3 reports the regression results. The estimated relationships suggest that environmental wealth dynamics are more closely associated with equity-market movements in some countries than others. In particular, the United States, Japan, and Germany ex-

hibit statistically significant positive coefficients, indicating stronger connections between environmental-economic conditions and market performance. These findings suggest that economies with more developed sustainability reporting systems and environmental policies may exhibit stronger links between environmental performance and financial outcomes.

Table 3. Regression estimates examining the relationship between country-level DEFI changes and domestic equity-market returns.

Country	α	p -Value	β	p -Value	R-Squared
Australia	0.194	0.144	−1.193	0.095	0.150
Brazil	0.046	0.750	0.897	0.103	0.140
Canada	−0.051	0.848	2.379	0.100	0.143
China	0.376	0.000 ***	0.091	0.650	0.064
France	0.059	0.806	1.253	0.300	0.013
Germany	0.066	0.570	1.906	0.002 **	0.521
India	0.345	0.004 **	−0.033	0.920	0.082
Japan	−0.079	0.569	2.227	0.002 **	0.538
UK	−0.094	0.774	−1.655	0.526	0.046
US	0.006	0.952	1.676	0.001 ***	0.550

** $p \leq 0.01$, *** $p \leq 0.001$.

For China and India, the estimated relationships are weaker, suggesting that additional macroeconomic, industrial, or policy factors may dominate short-term equity-market behavior. Similarly, Australia, Brazil, Canada, France, and the United Kingdom show lower explanatory power, indicating that environmental wealth measures represent only one component among many factors influencing financial markets.

It is important to emphasize that insignificant intercept estimates do not reduce the usefulness of the DEFI framework. Unlike traditional asset-pricing applications where Jensen's alpha represents abnormal financial returns, the interpretation in this setting relates to deviations from the global environmental benchmark. These results further illustrate that the estimated intercept should not be interpreted as abnormal financial performance in the traditional CAPM sense. Instead, statistically significant positive or negative values indicate persistent country-specific environmental wealth dynamics that cannot be explained solely by exposure to the common global environmental factor represented by the GDEFI.

Overall, the regression results demonstrate that the GDEFI provides a useful benchmark for evaluating systematic environmental exposure across countries. The heterogeneity observed in beta coefficients, explanatory power, and country-specific deviations highlights the ability of the DEFI framework to distinguish between global environmental dynamics and local environmental-economic characteristics. These findings provide the empirical foundation for the subsequent analyses of environmental portfolio allocation, risk-adjusted performance measurement, factor decomposition, and the conceptual derivative-pricing framework developed later in the paper.

Performance Evaluation Metrics

Performance evaluation metrics provide a systematic framework for evaluating the stability, uncertainty, and risk characteristics of dynamic time-series processes. In the context of environmental wealth indices, these metrics allow us to compare how countries' DEFIs behave after accounting for different dimensions of environmental and economic risk. Traditional measures such as standard deviation, beta, value at risk (VaR), conditional value at risk (CVaR), and drawdown quantify different aspects of uncertainty, while performance ratios such as the Sharpe ratio, Sortino ratio, Jensen's alpha, and the Rachev ratio provide interpretable measures of risk-adjusted behavior (Cheridito & Kromer,

2013). Together, these metrics provide complementary measures of environmental stability, resilience, and downside vulnerability, allowing policymakers, environmental agencies, researchers, and sustainability practitioners to compare environmental wealth dynamics using a unified quantitative framework.

In our analysis, four primary metrics are considered: Jensen's alpha, the Sharpe ratio, the Sortino ratio, and the Rachev ratio.

Although these performance measures were originally developed for evaluating financial assets, within the present framework they are interpreted as descriptive measures of environmental wealth dynamics rather than indicators of investment performance. Their purpose is to summarize complementary dimensions of environmental stability, systematic exposure, downside risk, and resilience after environmental conditions have been translated into financially interpretable indices. Consequently, the interpretation of these measures differs from that in traditional financial applications and should be understood within the context of environmental wealth assessment.

Jensen's alpha measures the average deviation of a country's environmental wealth performance relative to the Global Dollar Environmental Financial Index after adjusting for systematic environmental exposure (Jensen, 1968). Within the DEFI framework, alpha should not be interpreted as abnormal financial returns because DEFIs are synthetic environmental wealth indices rather than traded financial assets. Instead, Jensen's alpha measures persistent over- or under-performance in environmental wealth relative to the global environmental benchmark after controlling for environmental beta. Positive values indicate that a country's environmental wealth evolves more favorably than would be expected given its systematic environmental exposure, whereas negative values indicate relatively weaker environmental wealth performance after accounting for global environmental dynamics.

The Sharpe ratio evaluates the relationship between average index changes and total variability (W. F. Sharpe, 1966, 1994), while the Sortino ratio focuses specifically on downside deviations and therefore provides additional insight into unfavorable environmental movements (Sortino & Price, 1994; Sortino et al., 2001). The Rachev ratio (or R-ratio), defined as the relationship between expected extreme positive outcomes and expected extreme negative outcomes, measures the balance between upside potential and downside tail exposure (Biglova et al., 2004; Stoyanov et al., 2007). A higher Rachev ratio indicates greater resilience against extreme adverse movements. Together, these four measures provide a multidimensional assessment of environmental index stability, risk exposure, and sustainability-related uncertainty.

Table 4 summarizes the four performance measures for the ten country-level DEFIs. Several important patterns emerge. China and India exhibit the highest Jensen's alpha values and strong Sharpe and Sortino ratios, indicating large positive movements relative to their overall variability.² However, both countries display negative Rachev ratios, suggesting that, despite relatively strong average environmental performance, their estimated tail-risk characteristics are less favorable according to the Rachev ratio.

Germany, Canada, and Japan show positive alpha values but lower Sharpe and Sortino ratios, indicating more moderate environmental index dynamics. In contrast, the UK, Japan, and Canada exhibit the strongest Rachev ratios, suggesting more favorable tail-risk characteristics and greater resilience against extreme negative movements. These results demonstrate that countries with relatively moderate average performance may still exhibit strong stability when evaluated from a downside-risk perspective.

Table 4. Combined performance metrics for each country's DEFI: Jensen's alpha, Sharpe ratio, Sortino ratio, and Rachev ratio ($\alpha = \beta = 0.5$).

Country	Jensen's α	Sharpe	Sortino	Rachev
Australia	–	0.3767	0.6212	0.3404
Brazil	−0.636	0.1963	0.3346	0.5862
Canada	0.769	0.0041	0.0048	0.9889
China	2.646	1.8453	111.53	−0.5550
France	−0.007	0.0725	0.0861	0.7723
Germany	1.095	0.2870	0.4071	0.4048
India	1.616	1.0527	2.1476	−0.2435
Japan	1.181	0.0002	0.0003	0.9994
UK	−0.309	−0.0195	−0.0224	1.0587
US	−0.191	0.2575	0.3679	0.4637

Brazil, France, Australia, and the United States present mixed profiles, characterized by moderate risk-adjusted measures and intermediate Rachev ratios. Their DEFI dynamics indicate neither excessive instability nor dominant risk-adjusted performance, placing them between high-growth/high-risk and highly stable environmental profiles.

Overall, the four performance measures provide complementary information regarding different dimensions of environmental wealth dynamics. While some countries exhibit stronger average environmental performance, others display greater resilience to downside environmental risk or more favorable tail-risk characteristics. Together, these measures illustrate how the DEFI framework can distinguish different environmental risk profiles and provide a richer understanding of sustainability-related uncertainty than any single metric alone.

The observation that several countries exhibit modest or weak performance metrics does not reduce the usefulness of the DEFI framework; rather, it reflects the environmental and economic characteristics that the index is designed to capture. Measures such as the Sharpe ratio, Sortino ratio, and Rachev ratio evaluate stability, asymmetry, and tail behavior of environmental index dynamics. Lower values indicate greater uncertainty, instability, or downside exposure within the environmental wealth process.

Therefore, weaker performance metrics should not be interpreted as a failure of the index construction. Instead, they provide valuable information about differences in environmental resilience across countries. The objective of the DEFI framework is not to identify countries with the highest financial returns, but to transform environmental conditions into measurable signals that reveal sustainability-related risks and opportunities. The observed heterogeneity across countries demonstrates the ability of DEFI to distinguish relatively stable environmental trajectories from those characterized by greater vulnerability and extreme downside exposure.

5. Environmental Risk Allocation and Efficient Frontier Analysis

A key objective of this paper is to examine whether environmental wealth indices exhibit uncertainty, dependence, and diversification patterns similar to those observed in traditional financial systems. Although DEFIs are not directly traded financial securities, representing environmental performance through dynamic time-series indices allows the application of quantitative allocation methods to study how sustainability-related exposures interact across countries.

The purpose of this analysis is therefore not to prescribe investment strategies, but to demonstrate that environmental performance can be organized within a risk-allocation framework. By applying portfolio optimization concepts to DEFIs, we examine the struc-

ture of global environmental uncertainty and identify how combinations of country-level environmental exposures may improve stability and reduce downside vulnerability.

Modern portfolio theory, introduced by Markowitz (1952), provides a mathematical framework for allocating weights across multiple uncertain components in order to optimize the relationship between expected performance and risk. In traditional applications, these components represent financial assets. In the DEFI framework, they represent country-level environmental wealth indices whose dynamics reflect changes in sustainability conditions, environmental quality, and economic exposure. The objective is to determine the allocation weights w that minimize overall uncertainty while achieving a target level of environmental index performance r_p .

Two widely adopted optimization frameworks are considered: mean–variance optimization and mean–CVaR optimization. Mean–variance optimization minimizes overall variability, while mean–CVaR optimization minimizes conditional value-at-risk ($\text{CVaR}_{p,\alpha}$), a coherent risk measure that focuses on extreme downside outcomes (Artzner et al., 1999; Uryasev & Rockafellar, 2001). Let

$$r(t) = (r_1(t), r_2(t), \dots, r_n(t))$$

denote the vector of DEFI changes across countries, with corresponding expected values

$$\bar{r} = (\bar{r}_1, \dots, \bar{r}_n).$$

The classical Markowitz optimization problem is

$$\min_w w' \Sigma w \quad \text{subject to} \quad \bar{r}' w = r_p, \quad \sum_{i=1}^n w_i = 1, \quad (10)$$

where Σ denotes the covariance matrix of DEFI changes, $w' \Sigma w$ measures total environmental index variability, $\bar{r}' w = r_p$ specifies the target level of environmental index performance, and $\sum_i w_i = 1$ ensures that the environmental exposure weights are fully allocated.

A useful representation of the efficient frontier (EF) is obtained by expressing the optimization as a tradeoff between expected index performance and uncertainty. Let $E(r_p)$ denote expected DEFI growth and $V(r_p)$ represent the chosen risk measure (variance, $\text{CVaR}_{p,0.05}$, or $\text{CVaR}_{p,0.01}$). The optimization problem can be written as

$$\min_w [-\gamma E(r_p) + (1 - \gamma) V(r_p)] \quad \text{subject to} \quad \sum_{i=1}^n w_i = 1, \quad (11)$$

where $\gamma \in [0, 1]$ determines the relative importance assigned to expected environmental index growth versus uncertainty reduction. When $\gamma = 0$, the solution corresponds to the minimum-risk allocation. Increasing γ places greater emphasis on higher expected index growth while accepting additional uncertainty.

In the context of environmental indices, we consider three measures of uncertainty: standard deviation, $\text{CVaR}_{p,0.05}$, and $\text{CVaR}_{p,0.01}$. Standard deviation captures overall fluctuations, whereas CVaR focuses on extreme downside movements. This distinction is particularly important for environmental applications because severe downside movements may represent periods of environmental deterioration, sustainability challenges, or increased climate-related exposure.

Figure 3 presents the efficient frontier constructed using the DEFI framework. The frontier is obtained by solving Equation (11) over the grid $\gamma = 0, 0.01, \dots, 0.99$. The results indicate that uncertainty increases as greater emphasis is placed on higher environmental index growth. China exhibits relatively high DEFI growth together with greater variability,

whereas countries such as India and Canada display lower average dynamics. Among moderate-growth countries, Japan demonstrates relatively lower variability compared with France, suggesting a more stable environmental wealth trajectory. Brazil exhibits weaker performance combined with higher uncertainty, reflecting greater environmental-economic instability.

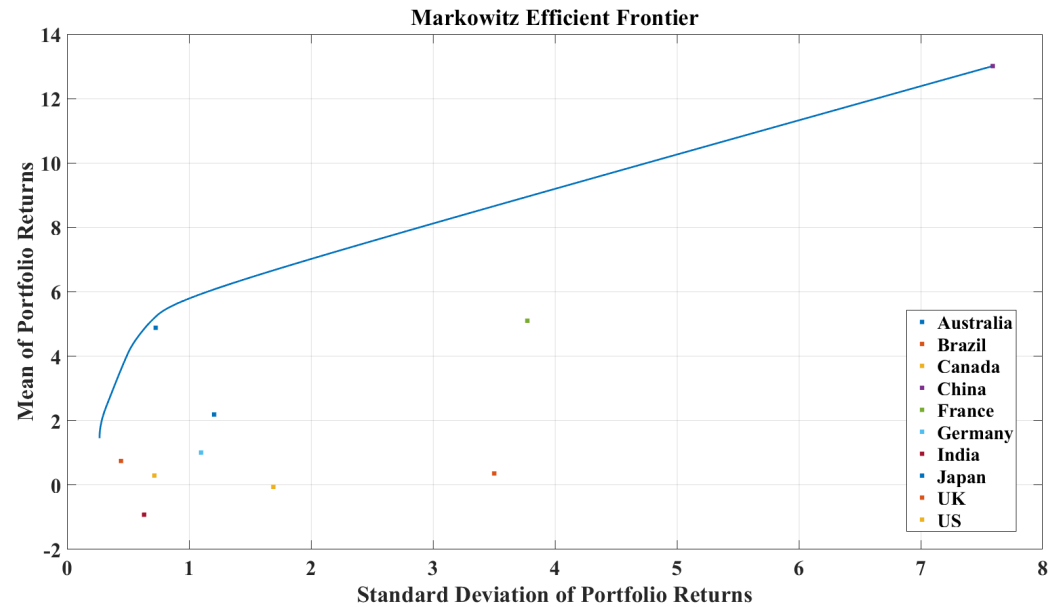


Figure 3. Markowitz efficient frontier for the global DEFI.

Figure 4a,b presents efficient frontiers based on $\text{CVaR}_{p,0.05}$ and $\text{CVaR}_{p,0.01}$. The $\text{CVaR}_{p,0.01}$ frontier emphasizes more severe downside scenarios and provides insight into extreme environmental risk exposure. Although the overall structure remains similar to the mean–variance frontier, incorporating CVaR highlights differences among countries in terms of vulnerability to extreme environmental movements.

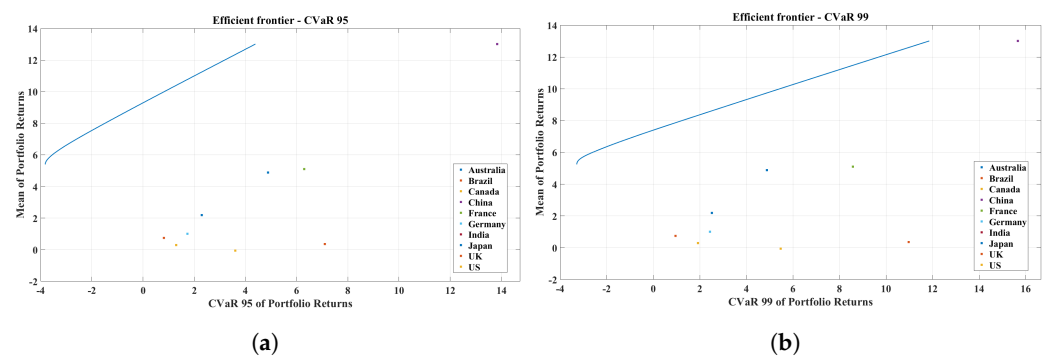


Figure 4. Conditional value-at-risk efficient frontiers: (a) $\text{CVaR}_{p,0.05}$ and (b) $\text{CVaR}_{p,0.01}$.

Overall, the comparison between mean–variance and mean–CVaR optimization demonstrates that environmental exposures contain different sources of uncertainty and diversification potential. Tail-sensitive measures provide additional information by identifying countries more exposed to extreme sustainability-related risks. These findings demonstrate that allocation-based risk methods can be adapted to environmental wealth indices to support sustainability risk analysis and long-term environmental decision-making.

6. Conceptual Option Pricing Framework for Environmental Indices

The purpose of this section is to provide a conceptual illustration of how environmental wealth indices could be incorporated into derivative-based risk-management frameworks. Although DEFI is not currently traded financial assets, representing environmental performance through a dynamic, financially interpretable environmental wealth index creates a natural framework for investigating how established financial risk-transfer methodologies could be adapted to environmental uncertainty.

If environmental conditions contain systematic components, as indicated by the environmental exposure analysis in previous sections, institutions may benefit from quantitative mechanisms designed to manage downside sustainability risks. Index-linked derivatives represent one possible future mechanism for transferring exposure to adverse environmental movements. The objective of this section is therefore not to propose or value an existing financial product, but to demonstrate how established derivative-pricing methodologies could theoretically be adapted once environmental risks are represented through measurable index processes.

Accordingly, this section should be interpreted as a conceptual extension of the proposed DEFI framework rather than one of the paper's primary empirical contributions. The objective is not to develop or validate a pricing model for an existing environmental security, but to illustrate that once sovereign environmental wealth is represented through a dynamic index, well-established derivative-pricing methodologies can be adapted to investigate potential sustainability risk-transfer mechanisms. We therefore present this analysis as a proof of concept intended to motivate future research on environmental financial products rather than as evidence for an existing environmental derivatives market.

Classical derivative-pricing models such as Black–Scholes–Merton, binomial and trinomial trees, and finite-difference methods (Duffie, 2010; Hull, 2003) often rely on assumptions of constant volatility and normally distributed innovations. However, environmental index dynamics may display nonlinear behavior, changing uncertainty, asymmetry, and heavy-tailed fluctuations. Therefore, more flexible time-series specifications are needed to represent these characteristics.

To capture these properties, we employ a discrete-time stochastic volatility framework based on GARCH(1,1) dynamics with Normal–Inverse Gaussian (NIG) innovations. This approach follows Duan (1995) and related extensions (Barone-Adesi et al., 2008; Chorro et al., 2012), allowing conditional volatility, asymmetry, and heavy-tailed behavior to be incorporated simultaneously.

Under the physical probability measure $\mathbb{P}$, the one-period DEFI change R_t follows an NIG distribution whose scale parameter evolves according to the conditional variance generated by the GARCH process. Since the resulting distribution allows deviations from normality, the classical Gaussian valuation framework is not directly applicable (Gerber & Shiu, 1994).

For the purpose of this conceptual pricing exercise, we construct a risk-adjusted probability measure using the Esscher transform introduced by Chorro et al. (2012). Under the transformed measure $\mathbb{Q}$, the distribution of R_t remains within the NIG family but with an adjusted asymmetry parameter $\beta + \sqrt{a_t}\theta_t$, where θ_t satisfies the Esscher condition

$$MGF(1 + \theta_t) = MGF(\theta_t)e^{r_t^f},$$

where $MGF(\cdot)$ represents the conditional moment-generating function of R_{t+1} . This transformation provides a mathematically consistent framework for generating risk-adjusted environmental index scenarios.

To illustrate the proposed framework, hypothetical index-linked contract values are estimated using Monte Carlo simulation under the risk-adjusted dynamics described above.

After estimating the GARCH(1,1)–NIG parameters and forecasting the conditional variance sequence $\{a_t\}$, we generate N simulated paths according to

$$DEFI_{t+1} = DEFI_t \exp(R_{t+1}), \quad t = 1, \dots, T,$$

where T represents the maturity horizon. The values of hypothetical European-style index-linked contracts with strike level K are approximated by

$$\hat{C}(t, T, K) = \frac{e^{-r'_t(T-t)}}{N} \sum_{i=1}^N \max(DEFI_T^{(i)} - K, 0), \quad (12)$$

$$\hat{P}(t, T, K) = \frac{e^{-r'_t(T-t)}}{N} \sum_{i=1}^N \max(K - DEFI_T^{(i)}, 0). \quad (13)$$

Figure 5 presents the resulting call- and put-style valuation surfaces for the DEFI framework. The put-style contracts increase with both strike level and maturity horizon, reflecting greater protection value against unfavorable environmental index movements. Longer horizons introduce additional uncertainty regarding future environmental conditions, increasing the value associated with downside protection.

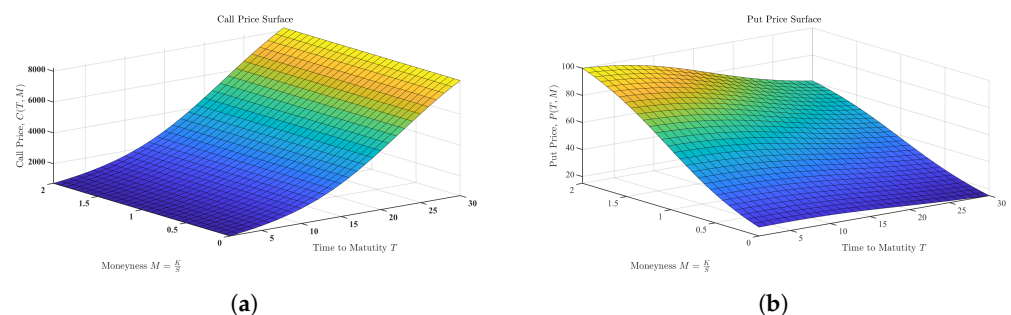


Figure 5. Hypothetical index-linked contract values for the US DEFI at time t for a given strike level K using a GARCH(1,1) model with NIG innovations: (a) call-style contract values and (b) put-style contract values. The color gradient represents the magnitude of the corresponding contract values, with warmer colors indicating higher values and cooler colors indicating lower values.

Figure 6 presents the implied-volatility surface obtained from the simulated values. The observed skew indicates that negative environmental movements carry greater uncertainty than positive movements, consistent with the asymmetric nature of environmental and climate-related risks. Higher implied volatility associated with downside scenarios reflects increased sensitivity to potential environmental deterioration.

Overall, this section should be interpreted solely as a methodological illustration rather than an empirical valuation of existing securities or evidence for an active environmental derivatives market. Its purpose is to demonstrate that, once environmental performance is represented through a dynamic sovereign environmental wealth index, concepts from derivative pricing and financial risk management can be adapted to investigate environmental uncertainty within a coherent quantitative framework. If sustainability-linked derivatives, climate-risk transfer instruments, or environmental insurance markets become more widely developed in the future, the proposed framework may provide a useful starting point for extending traditional financial pricing methodologies to environmental risk management.

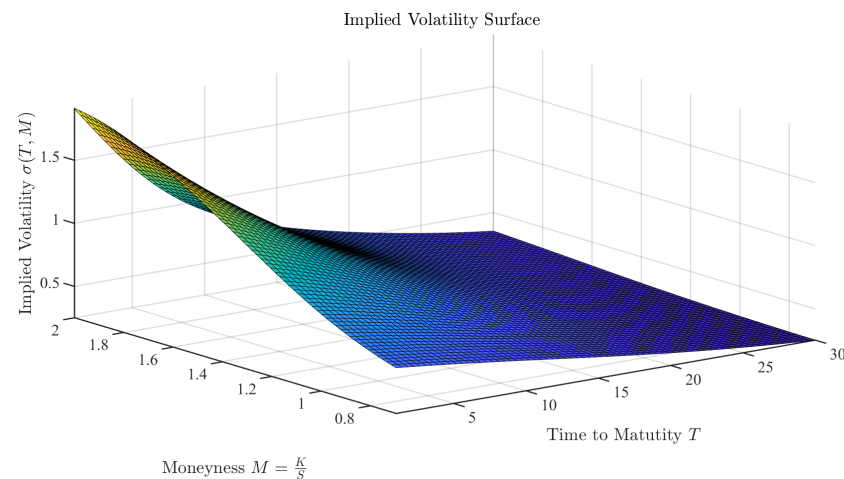


Figure 6. US DEFI implied volatilities versus the time to maturity (T) and moneyness ($M = S/K$), where S and K denote the DEFI index level and strike level, respectively, under a GARCH(1,1) model with NIG innovations. The color gradient represents the magnitude of the implied volatility surface, with warmer colors indicating higher implied volatilities and cooler colors indicating lower implied volatilities.

7. Factor Analysis

Factor analysis is a widely used statistical technique for identifying latent sources of variation that jointly influence a set of observed variables. In financial economics, factor models are commonly used to separate common sources of variation from idiosyncratic components. This decomposition provides insight into whether observed movements are driven primarily by broad systematic forces or by characteristics unique to individual units.

This methodology has a natural application in the analysis of environmental wealth indices. Environmental conditions are influenced by a combination of global forces, such as climate change, international regulations, technological transitions, and energy-market developments, as well as country-specific factors including policy choices, resource dependence, and economic structure. Applying factor analysis to the Dollar Environmental Financial Indices (DEFIs) allows us to identify common environmental drivers and separate them from country-specific sustainability dynamics.

In this paper, we construct DEFIs for ten major economies and use factor analysis to investigate the latent structure governing their cross-country comovement. Because each DEFI combines environmental indicators with economic capacity, the resulting factor structure captures the interaction between sustainability conditions and economic development. Factor loadings measure the sensitivity of each country's environmental wealth index to common global factors, while latent error variances represent country-specific environmental and economic components not explained by shared dynamics.

A key question is whether a small number of latent factors can explain the majority of variation among country-level DEFIs. Identifying such factors provides evidence that environmental performance contains systematic components that extend beyond individual countries. This structure helps characterize how environmental risks and sustainability-related economic forces propagate across economies.

7.1. CAPM Motivation and Environmental Beta

The classical Capital Asset Pricing Model (CAPM), introduced by W. Sharpe (1964), provides one of the earliest examples of a factor-based framework for separating systematic

and idiosyncratic components. In its traditional financial setting, CAPM explains the return of an asset through exposure to a common market factor:

$$r_{it} = r_{ft} + \beta_i r_{mt} + \epsilon_{it}, \quad (14)$$

where r_{ft} is the risk-free rate, r_{mt} represents the market factor, β_i measures sensitivity to systematic movements, and ϵ_{it} represents the idiosyncratic component.

Although DEFIs are not directly traded assets, the intuition behind factor-based models can be adapted to environmental risk analysis. Environmental conditions are affected by common global forces such as climate change, resource constraints, regulatory transitions, and technological development. These shared forces create systematic environmental movements that influence multiple countries simultaneously.

Within the DEFI framework, the GDEFI serves as a global environmental benchmark rather than a traditional financial market portfolio. A country's environmental beta measures the sensitivity of its environmental wealth index to this common benchmark. Higher beta values indicate stronger exposure to global environmental dynamics, whereas lower beta values suggest that country-specific characteristics play a larger role.

Thus, the CAPM framework is used here as a conceptual motivation for measuring systematic exposure, not as a claim that environmental indices represent conventional financial assets. This interpretation allows environmental performance to be studied using the broader principles of factor modeling, risk decomposition, and macro-financial analysis.

7.2. General Factor Model

A general factor model represents the observed dynamics of N indices through a smaller number of underlying common factors. Let

$$\mathbf{r}_t = (r_{1t}, \dots, r_{Nt})^\top$$

denote the vector of country-level DEFI changes. The factor representation is given by

$$\mathbf{r}_t = \boldsymbol{\mu} + \boldsymbol{\beta} \mathbf{f}_t + \boldsymbol{\epsilon}_t, \quad (15)$$

where $\boldsymbol{\mu}$ represents average index dynamics, $\boldsymbol{\beta}$ is the $N \times m$ matrix of factor loadings, $\mathbf{f}_t$ is the vector of latent common environmental factors with $m < N$, and $\boldsymbol{\epsilon}_t$ contains country-specific components.

If the country-specific components are independent, their covariance matrix is

$$\mathbf{D} = \text{diag}(\sigma_1^2, \dots, \sigma_N^2).$$

The covariance structure becomes

$$\text{Cov}(\mathbf{r}_t) = \boldsymbol{\beta} \boldsymbol{\Sigma}_f \boldsymbol{\beta}^\top + \mathbf{D}, \quad (16)$$

where $\boldsymbol{\Sigma}_f$ represents the covariance matrix of the common environmental factors.

7.3. Observable and Latent Factor Approaches

Factor models can generally be classified into observable and latent approaches. Observable factor models rely on variables whose realizations are directly measurable. In financial economics, examples include the market factor in the Capital Asset Pricing Model (CAPM) and the size, value, profitability, and investment factors in the Fama–French factor models (Fama & French, 1992, 1993). In such models, factor sensitivities are esti-

mated by measuring how strongly an observed variable responds to predefined sources of systematic variation.

Although these models were originally developed for financial assets, their underlying principle—separating common systematic movements from individual-specific components—can be applied more broadly to environmental and economic systems. In the DEFI framework, observable factor analysis is used to examine how country-level environmental wealth indices respond to movements in the global environmental benchmark (GDEFI). Following Knez and Ready (1997), robust regression methods are employed when estimating these relationships to reduce the influence of extreme observations and obtain more stable estimates of environmental exposure.

Latent factor models provide a complementary perspective by identifying hidden sources of variation directly from the covariance structure of the observed data. Unlike observable models, latent approaches do not require the researcher to specify the factors in advance. Instead, they extract underlying forces that jointly explain the comovement among the indices. Within the DEFI system, these latent factors may represent broad sustainability forces such as global climate trends, energy transitions, international environmental policies, technological changes, or regional economic structures.

In this study, we estimate a three-factor latent structure for the ten country-level DEFI series using maximum-likelihood factor analysis. This approach allows us to determine whether environmental wealth dynamics are primarily driven by common global forces or by country-specific characteristics.

7.4. Empirical Factor Analysis of the DEFI System

A maximum-likelihood factor analysis is applied to the system of ten country-level DEFIs to identify the dominant sources of cross-country environmental comovement. Table 5 reports the estimated factor loadings and latent error variances associated with the three-factor specification.

Table 5. Estimated loadings and latent error variances for a three-factor model.

Country	β_{i1}	β_{i2}	β_{i3}	σ_i^2
Australia	−0.1748	−0.1489	−0.0818	0.9406
Brazil	0.1355	0.4476	0.1294	0.7646
Canada	0.1403	0.2787	0.9474	0.0050
China	0.6000	−0.1423	0.3845	0.4719
France	0.5708	0.3541	−0.0106	0.5487
Germany	0.2737	0.1680	−0.1917	0.8601
India	0.1214	0.7491	−0.2632	0.3549
Japan	0.0887	0.9399	0.3220	0.0050
UK	0.9581	0.2777	0.0021	0.0050
US	0.6536	0.0405	0.0752	0.5655

The likelihood ratio test produces a p -value of approximately 0.99, suggesting that the three-factor model provides an adequate representation of the covariance structure among the DEFIs. However, a statistically adequate factor representation does not imply that all countries are equally explained by global environmental components. The latent error variances reported in Figure 7 reveal substantial heterogeneity in the degree to which countries are influenced by common versus country-specific factors.

Australia, Germany, and Brazil display the largest idiosyncratic variances—0.94, 0.86, and 0.76, respectively—indicating that a substantial portion of their DEFI variation is associated with country-specific characteristics rather than shared global environmental

movements. These differences may reflect unique economic structures, natural resource dependence, geographic conditions, or domestic environmental policies.

By contrast, Canada, Japan, and the United Kingdom exhibit latent error variances close to zero, suggesting that their environmental wealth dynamics are largely captured by the identified common factors. This indicates stronger alignment with broader environmental and sustainability-related trends.

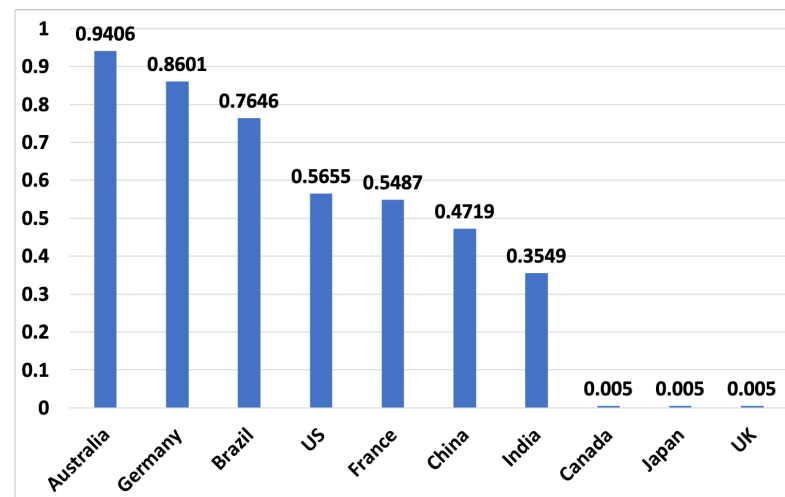


Figure 7. Country indices ordered by latent error variance.

To interpret the estimated factor structure, Figure 8 presents the loading vectors ($\beta_{i1}, \beta_{i2}, \beta_{i3}$) in a three-dimensional factor space. The first factor can be interpreted as a global environmental component, with countries such as France, China, the United Kingdom, the United States, and Germany exhibiting positive exposure. These countries share similar movements that may reflect their integration with global economic activity, international environmental agreements, and common sustainability challenges.

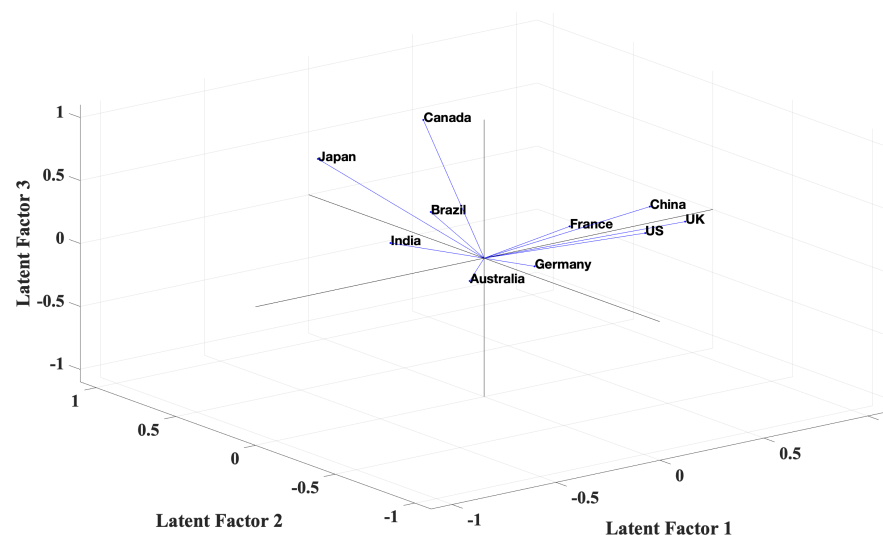


Figure 8. Three-dimensional factor-loading representation for the ten DEFIs.

The second factor captures structural differences in environmental dynamics. Australia, Japan, and India exhibit relatively stronger exposure to this component, suggesting that their DEFI movements are influenced by factors different from those captured by the global component. Such differences may arise from energy composition, industrial structure, geographic characteristics, or resource utilization patterns.

The third factor represents additional country-specific or regional environmental characteristics. Canada, Brazil, and Japan display meaningful loadings on this dimension, suggesting the presence of systematic environmental patterns that are not fully explained by the first two factors. Canada's combination of a low latent error variance and strong third-factor loading indicates that this component captures an important and persistent part of its environmental wealth dynamics.

Together, these findings demonstrate that environmental wealth dynamics are multidimensional. Some countries, including the United States, United Kingdom, France, China, and Germany, exhibit stronger exposure to common global environmental factors, whereas others, including Australia, Japan, India, and Brazil, display more distinct environmental trajectories.

The presence of heterogeneous factor structures highlights the importance of using a multi-factor approach when analyzing environmental indices. Environmental wealth dynamics are shaped simultaneously by global sustainability forces, regional structural differences, and country-level economic and environmental characteristics.

8. Discussion and Conclusions

The results of this study demonstrate that environmental performance, when represented through a dollar-denominated and financially interpretable index framework, exhibits measurable patterns of uncertainty, dependence, and systematic exposure. This finding supports the central objective of the paper: developing a quantitative framework that connects environmental conditions with economic value, sustainability risk, and financial decision-making tools without altering the underlying environmental interpretation. By constructing country-level Dollar Environmental Financial Indices (DEFIs) and the Global Dollar Environmental Financial Index (GDEFI), we provide a systematic approach for measuring environmental wealth dynamics and evaluating how environmental and sustainability-related forces differ across countries.

A key motivation for this framework is the growing need for transparent, comparable, and economically meaningful measures of environmental performance. Existing ESG and sustainability indicators provide valuable information, but they are often designed primarily as ranking or reporting systems and may have limited ability to support dynamic risk modeling and quantitative analysis. In contrast, DEFIs are constructed from standardized macroeconomic and environmental indicators and expressed in monetary terms by linking environmental performance with economic capacity. This structure creates dynamic environmental wealth indices that complement existing ESG frameworks by providing additional insight into environmental exposure, volatility, downside risk, and cross-country dependence.

The empirical analysis shows that country-level DEFIs exhibit meaningful relationships with the global environmental benchmark. Countries such as the United States, Germany, Japan, China, and others display significant environmental beta coefficients, indicating that environmental wealth dynamics contain common components associated with broader global sustainability trends. At the same time, differences in beta values, Jensen's alpha estimates, and tail-risk measures reveal substantial heterogeneity across countries. These findings demonstrate that environmental conditions contain both systematic and country-specific components, which can be analyzed through financial econometric and risk-management methods.

The performance evaluation and efficient frontier analyses further highlight the importance of considering environmental uncertainty from multiple perspectives. Measures such as the Sharpe ratio, Sortino ratio, and Rachev ratio reveal differences in stability, downside exposure, and tail behavior across countries. Similarly, mean-variance and mean-CVaR

optimization demonstrate how environmental exposures can be combined and evaluated within an allocation-based risk framework. These results do not imply that environmental indices are currently traded securities, but rather show that financial optimization tools can provide useful insights into sustainability risk allocation and environmental resilience.

The proposed framework also has broader implications for governments, environmental agencies, and international organizations responsible for sustainability assessment and environmental policy. Because the DEFI and GDEFI provide consistent, dynamic measures of sovereign environmental wealth, they may serve as complementary quantitative tools for monitoring long-term environmental performance, evaluating the effectiveness of climate and sustainability policies, identifying countries with heightened environmental vulnerability, and supporting evidence-based resource allocation. In addition, the framework may assist sovereign wealth funds, development institutions, and policymakers in incorporating environmental risk into long-term planning, sustainability reporting, and strategic investment decisions, thereby strengthening the integration of environmental considerations into economic policy and financial risk management.

Beyond traditional risk analysis, the conceptual derivative-pricing illustration demonstrates how environmental index-linked instruments could potentially support sustainability risk management if such markets were to develop in the future. While DEFIs are synthetic environmental wealth indices rather than directly tradable assets, contracts linked to environmental benchmarks could, in principle, provide mechanisms for transferring exposure to severe environmental deterioration. This discussion therefore highlights a possible direction for future research at the intersection of environmental economics, climate finance, and environmental risk-transfer mechanisms.

The factor analysis provides additional evidence that environmental wealth dynamics are multidimensional. The estimated three-factor structure identifies common global environmental components as well as regional and country-specific characteristics. Countries such as France, China, the United Kingdom, Germany, and the United States exhibit stronger connections with common environmental factors, whereas countries including India, Japan, Brazil, Australia, and Canada display more distinct patterns. These findings suggest that environmental wealth dynamics are simultaneously influenced by common global sustainability forces and country-specific economic, environmental, and policy characteristics.

Several limitations provide opportunities for future research. First, because DEFIs are synthetic environmental wealth indices rather than traded financial instruments, future studies could investigate methods for linking such indices to investable products, sustainability-linked securities, or climate-risk benchmarks. Second, expanding the dataset to include a larger number of countries and longer time periods would allow more detailed analysis of environmental risk transmission and improve the robustness of dynamic modeling approaches. Third, future research could explore alternative weighting methodologies, including data-driven approaches such as principal component analysis or machine-learning-based weighting schemes, to further examine the sensitivity of index construction. Future research may also investigate alternative normalization procedures, including logarithmic and Z-score standardization, together with alternative economic valuation mechanisms such as purchasing-power-parity-adjusted GDP or broader measures of national wealth, to further evaluate the robustness of the proposed environmental wealth framework. Finally, sector-level or firm-level extensions of the DEFI framework could provide additional insight into how environmental risks propagate from individual economic activities to national and global sustainability outcomes.

Another limitation concerns the relatively short annual sample used for estimating the conditional volatility models. Although ARCH/GARCH-type specifications provide a

flexible framework for representing time-varying uncertainty and constructing internally consistent environmental risk scenarios, their parameter estimates are naturally less reliable with annual observations than with higher-frequency financial data. Accordingly, the volatility results presented in this study should be interpreted primarily as a methodological illustration of dynamic environmental uncertainty rather than as precise volatility forecasts. Future research based on quarterly, monthly, or satellite-derived environmental observations would permit more robust estimation of conditional volatility processes, facilitate additional robustness analyses, and provide a more detailed characterization of sustainability-related risk dynamics.

More broadly, the proposed DEFI framework illustrates how concepts from financial economics can be adapted to enrich environmental assessment without requiring environmental performance to be represented by tradable securities. In this sense, the framework establishes a bridge between sustainability measurement and quantitative risk analysis that may support future methodological developments in sustainable finance and environmental economics.

In summary, this paper develops a unified framework for translating environmental performance into financially interpretable environmental wealth indices. By integrating the DEFI and GDEFI framework with econometric modeling, environmental beta estimation, risk-adjusted performance evaluation, portfolio optimization, factor analysis, and a conceptual derivative-pricing framework, we demonstrate how environmental conditions can be analyzed using quantitative tools traditionally applied in financial economics while preserving their environmental interpretation. The proposed framework bridges environmental measurement and financial risk analysis, providing a transparent foundation for evaluating sustainability-related risks, supporting long-term environmental and economic decision-making, and encouraging future methodological developments at the intersection of sustainable finance, environmental economics, and quantitative risk management.

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Appendix A. Environmental and Economic Indicators

This appendix provides definitions and interpretations of the fourteen World Development Indicators (WDIs) used in the construction of the Dollar Environmental Financial Indices (DEFIs). The selected indicators capture different dimensions of environmental quality, natural capital, resource utilization, energy transition, and economic capacity.

Together, they provide the foundation for measuring environmental wealth dynamics across countries.

Appendix A.1. Adjusted Net Savings, Including Particulate Emission Damage (Percentage of Gross National Income)

Adjusted net savings provide a sustainability-adjusted measure of national wealth accumulation by incorporating investments in productive capacity and the costs associated with environmental degradation. This indicator accounts for changes in human capital, depletion of natural resources, and damages associated with pollution. It reflects whether economic growth is achieved while maintaining long-term environmental and economic sustainability.

Appendix A.2. Agricultural Land (Percentage of Land Area)

Agricultural land represents the share of a country's land devoted to agricultural activities, including arable land, permanent crops, and permanent pastures. This indicator captures land-use intensity, agricultural capacity, and potential environmental pressures associated with food production and resource management.

Appendix A.3. Arable Land (Percentage of Land Area)

Arable land measures the portion of land suitable for temporary crop production. It reflects agricultural productivity potential, food security capacity, and the availability of land resources required to support economic and population growth.

Appendix A.4. CO₂ Emissions (kt)

Carbon dioxide emissions measure total emissions generated from fossil fuel combustion, industrial activities, and cement production. This indicator represents a country's contribution to greenhouse gas accumulation and provides information about carbon intensity and climate-related environmental pressures.

Appendix A.5. CO₂ Emissions (Metric Tons per Capita)

Carbon dioxide emissions per capita measure the average emissions produced per person within a country. This indicator adjusts total emissions by population size and provides a comparable measure of individual-level carbon intensity across economies.

Appendix A.6. Energy Intensity Level of Primary Energy

Energy intensity measures the amount of energy required to generate a unit of economic output. Lower energy intensity generally indicates greater energy efficiency and reflects progress toward more sustainable and resource-efficient economic activity.

Appendix A.7. Forest Area (Percentage of Land Area)

Forest area measures the proportion of national land covered by forests. Forest resources represent an important component of natural capital by supporting biodiversity, carbon sequestration, ecosystem stability, and climate resilience.

Appendix A.8. Land Area (sq. km)

Land area represents the total physical land resources available within a country, excluding major water bodies. This measure provides geographical context for evaluating resource availability, environmental capacity, and sustainability constraints.

Appendix A.9. Methane Emissions (kt of CO₂ Equivalent)

Methane emissions measure the release of methane expressed in carbon dioxide equivalent units. Methane is a major greenhouse gas associated with agriculture, energy production, and industrial processes. This indicator captures an important dimension of climate-related environmental pressure.

Appendix A.10. Nitrous Oxide Emissions (Thousand Metric tons of CO₂ Equivalent)

Nitrous oxide emissions represent greenhouse gas emissions primarily associated with agricultural activities, fertilizer use, and industrial processes. Due to its strong warming potential, this indicator is important for assessing climate-related sustainability risks.

Appendix A.11. Renewable Energy Consumption (Percentage of Total Final Energy Consumption)

Renewable energy consumption measures the share of energy obtained from renewable sources. It reflects progress toward cleaner energy systems, reduced dependence on fossil fuels, and transition toward lower-carbon economic development.

Appendix A.12. Surface Area (sq. km)

Surface area measures the total geographic size of a country, including land and inland water areas. This indicator provides additional geographical context for understanding environmental resources, population pressures, and economic activity distribution.

Appendix A.13. Total Natural Resources Rents (Percentage of GDP)

Total natural resource rents measure the economic value generated from natural resources relative to GDP. This indicator captures the role of natural assets in economic activity and highlights the importance of sustainable resource management.

Appendix A.14. GDP per Capita (Constant US Dollars)

GDP per capita measures average economic output per person and represents a country's economic capacity. Within the DEFI framework, GDP per capita provides the monetary scaling component that connects environmental performance with economic value, allowing environmental conditions to be expressed through a financially interpretable index.

Notes

- ¹ For comprehensive discussions of financial time-series modeling, see [Tsay \(2005\)](#) and [Hamilton \(2020\)](#).
- ² The exceptionally large Sortino ratio for China results from the very small estimated downside deviation during the sample period and should be interpreted as a relative stability measure rather than a prediction of future performance.

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