

## A Co-Evolution Model of Competitive Mobile Platforms: Technoeconomic Perspective

Perambur Neelakanta<sup>1</sup> and Raef Yassin<sup>2</sup>

Florida Atlantic University, Department of Computer and Electrical Engineering & Computer Science,  
<sup>1</sup>neelakan@fau.edu, <sup>2</sup>yassin@fau.edu

Received 13 January 2011; received in revised form 10 April 2011; accepted 18 May 2011

### Abstract

A model depicting competitive technoeconomics of business structures specific to mobile-platforms is developed. The underlying co-evolution of large, competing enterprises of mobile-platforms that face customer-churning due to application-preferences and pricing structures in the deregulated ambient is viewed in the perspectives of nonlinear logistic systems akin to that of biological ecosystems. Relevant considerations are decided by and embodied with several stochastically-interacting subsystems. Hence, the temporal dynamics of competition/co-evolution of known competitors in the mobile-platform market, like Android, Symbian and iPhone is depicted by a novel model posing dichotomy of prey-predator flip-flops in the market; and, an asymptotic projection of *ex post* computations of underlying technoeconomics into the *ex ante* region would correspond to futuristic forecasts on the performance of test platforms. Further, computed results are exemplified with a sample calculation and associated sensitivity details.

**Keywords:** Mobile-platform, Co-evolution, Competition, Prey-predator model, Technoeconomic forecasting

## 1 Introduction

Developed in this paper is a co-evolution model that describes the growth/decay dynamics across competitive, (mobile-platform)-centric business structures. It is based on relevant analogy of dichotomous prey-predator states observed in bio-ecology; and, the modeling is done to provide an apt description of competitive market structures in Mobile OS industry. Appropriate model-validation of the proposed method is done *via* a set of data pertinent to real-world competitors in the mobile-platform market, such as Android, Rim and iPhone. An asymptotic projection of underlying technoeconomics into the *ex ante* region can offer a meaningful forecasting of associated market issues.

The mobile platforms mentioned above, broadly refer to the gamut of hardware and software environments developed for smart-phones, handheld devices (like PDAs), tablet computers and other information appliances that are *smart* by virtue of included embedded systems, and/or other mobile and wireless devices. Typically, iPhone, Android, Symbian, Palm and Windows Mobile are examples of mobile platforms. In essence, a mobile operating system, also known as a *mobile OS*, a *mobile platform*, or a *handheld operating system*, refers to an operating system that controls a mobile device or information appliance. It is based on principles similar to traditional operating systems like Windows, Mac OS, or Linux that control a desktop computer or laptop. But Mobile OS systems are simpler and concerned with wireless versions of broadband and local connectivity, mobile multimedia formats, and different input methods.

The mobile OS is conceived to facilitate *via* software platforms Internet-like end-to-end architectures in digital mobile environment. This Internet-mobile convergence inculcates an open and modular concept in the Internet architecture outmoding the classical, closed and integrated architectures of the telecommunication company (telco). Thus, unlike classical telecommunication, modern and/or next-generation perspectives of telco industry including mobile platforms portray a digital ecosystem [1]-[2], [11]-[12]. Admob [3]-[4] has recently released a report giving details on mobile Internet usage across the world and it points out that in the recent past usage of smartphones has surged and accounts for maximum mobile usage [14]-[15].

### 1.1 Evolution of Technoeconomic Systems: Bio-system Analogy

The concept of biological ecology rightly fits to describe the temporal evolution of technoeconomics [5]-[6], [18] in systems like telecommunication companies (telcos) that include Mobile OS services. That is, the relationships that exist inherently between and among organisms in a biological ecosystem can be analogously adopted to model the technoeconomics of the systems that represent a community composed of multiple, independent individuals and/or organizations (enterprises) sharing a common, business-centric mission and responsibility. Relevant efforts, at least partly apply to automated and intelligent workflows, for example in mobile platform context, securely exchanging protected information and optimally using digital media. Further, seamless functioning as a single, dynamic and complex unit with no single subunit being totally in charge in controlling the process is implicit in such operations. More so, similar to biological ecosystems that model the inter-relatedness of various interactions among biological species, the information technology (IT) and/or Mobile-OS centric industries and businesses also fit into an intersection of variety of social, economic, and even political ecosystems.

The biotic components depicting species like plants, animals, bacteria etc. of the bio-ecosystem correspond to *endogenous* factors in the technoeconomics of systems implemented or produced from, within the IT-centric computational/communication efforts. They refer to all variables that are determined within the model. Contrast to endogenous variable, an *exogenous* variable is directly amenable for manipulation by the programmer/service provider; and, in a broad sense such exogenous considerations can be equated to the abiotic influences of the bio system such as the habitat (ponds, lakes, sea, desert etc.) and weather features like temperature, rain, snow etc.

In general, considering the cyber infrastructure of telecommunications supporting IT, it is often attempted to comprehend how the data (information) flows along many steps in a chain and across vagaries of interactive steps (protocols). Self-organizing tug-of-war of goal-seeking computational iterations controls the rate of growth analogues to say, photosynthesis in biological sense of ecology; and corresponding rate of fall/decay is similar to biological decomposition. Further, the rate at which resources of interacting units are reused in telco context corresponds to recycling of nutrients in biosystems and retiring selectively of obsolete and redundant components across telco network is akin to natural selection in biology.

### 1.2 Scope of the Present Study

The present paper exclusively addresses an example case-study and feasibility of modeling of the growth/decay dynamics of competitive (mobile-platform)-centric systems in terms of technoeconomic considerations specified *via* observed traffic shares. Relevant evolution and devolution issues are modelled using the bio-ecology analog of dichotomous prey-predator growth dynamics [5]-[6], [18]. Hence the conceived scope of this study converges to address the following deliverables described in the subsequent sections:

- Viewing the digital mobile communication platforms analogous to evolving biological ecosystems
- Identifying specific mobile platform services that had faced in the recent past, dynamically varying technoeconomic performance in their operations
- Marking the events of ups and downs in the technoeconomic performance of the businesses across the aforesaid temporal dynamics
- Developing a nonlinear system model to describe the underlying dynamics
- Introducing dichotomous (flip-flop) evolution (growth) and devolution (decay) considerations in the dynamics pursued
- Using real-world economic data available (on Symbian, Android, iPhone etc.), validating the model developed
- Presenting a discussion on the heuristics of the theoretical model developed, which conforms to real-world technoeconomic performance details
- Making relevant conclusions on the observations made.

## 2 Co-evolution of Competitive Mobile OS Business Structures

The conceived pursuit of modeling the business structure of mobile digital communication in a competitive market ambient is based on co-evolution concept described below. For example, it is possible to consider the growth prospects of mobile platform industry, where, relevant (growth) parameters refer to the underlying techno- and/or technoeconomic-centric considerations and activities. Such activities normally involve both activity-reinforcing and activity-impeding processes negotiated by (augmentative and annihilating) information-flow between interacting subunits controlled by the associated cybernetics of self-organizing mechanism [7] dictating the technical/business progress (or deterioration) of co-existing units. The growth/decay then conforms to co-evolution considerations [11], [16].

Co-evolution in Nature refers to a joint progression of evolution/growth of closely-associated and coexisting species. Mostly such co-evolution is competitive and in a broader sense involves predator–prey relationships with an evolutionary advance in the predator, for instance, triggering an evolutionary response in the prey. Often, such prey-predator states may also flip-flop along the time-scale. It is attempted in this study to model the aforesaid co-evolution paradigm with a dichotomous pre-predator suite applied to competing Mobile OS environment involving nonlinear logistics similar to relevant biological ecosystem considerations. In this perspective, the proposed model in this study conforms to a complex system embodied with several stochastically interacting subsystems; and, the model is specifically applied to describe the technoeconomic co-evolution fitted to competitive corporate businesses like mobile platforms [5]-[6], [18].

Further, telco-like enterprises can be viewed as complex systems as indicated by Neelakanta and Deecharoenkul [9]. Relevant evolution models that can be ascribed to these business structures portray the growth and futuristic welfare of the industry in conformance to the prevailing competitive market *vis-à-vis* the underlying application needs/demands, service expansions, revenue growth, decision-making strategies and customer population/preferences etc. Also, pertinent to the cyberspace of Mobile OS service provisioning, the co-evolution models can lead to predicting the survival of underlying businesses implicated by interacting and competing aspects of customer expectations, variety in services (applications), compliance with government regulations, relative customer-churning, revenue/return-on-investment (RoI) etc.

In short, developed in this study is an algorithmic approach that describes the stochastic growth of (mobile-platform)-centric technoeconomics as a co-evolution paradigm under a competitive environment involving a dichotomous pre-predator suite. Three Mobile OS competitors, for example, are modeled as dichotomous structures that switch randomly to the role of being prey-to-predator (or *vice versa*) along the time-scale based on *ad hoc* decisions invoked by the participant competitors. This alternating or flip-flopping dispositions of dichotomy between prey- and predator-states in an observed time-frame considered as the *ex post* regime is expected to offer an asymptotic projection of relevant technoeconomics by forecasting the performance in the *ex ante* region of the time-scale. Proposed strategy is justified with real-world mobile platform data available in a competitive market structure.

### 2.1 Co-evolution of Competing Species

The co-evolution paradigm attributed to technoeconomic context described in [11] and [16] is based on the following principle: Considering the dynamics of two competing species say, two corporations *A* and *B*, it can be expressed by

a coupled system of (two) logistic relations. They specify the onset of *activation* (or *expanding*) phase being followed by a state of *inhibition* (or *contracting*) phase in the evolution (growth/decay) prospects of both corporate businesses. This can be explained as follows: In a non-competitive situation, any expansion (positive evolution or growth) trend of a corporate business is decided by the existing (current) levels of consumers supporting the underlying business activity and the prevailing growth rate; however, resource limitations may bring the system to a contracting phase implicated by supply-demand considerations. Hence, the growth will level off eventually. The same description applies, if a stand-alone business begins with a decaying (devolutionary) trend due to poor corporate decisions, it may settle down at an asymptotic minimum level; or, with timely prudent decisions may allow the business to take a positive growth-trend.

That is, in the event of a competitive ambient between two businesses *A* and *B*, the evolving (or devolving) trends will be dictated by the intervening smart and crucial decisions taken by both parties *vis-à-vis* phases of competitive market situations driven by the competitors. Then, the *theory of equality* would apply in presuming that each decision made by a competing enterprise is as good, intelligent and rational as that of other competing units. As such, any evolutionary prospects of a corporation and the decisions made are often confronted with the odds of competitor's plays consistent with decisions and evolutionary aspects of that competitor. Thus, in a competitive business environment, the associated evolutionary considerations may follow a dichotomous prey-predator paradigm with an alternating role (state) of being a prey or a predator (or *vice versa*) assumed by each of the two competing corporations, *A* and *B*.

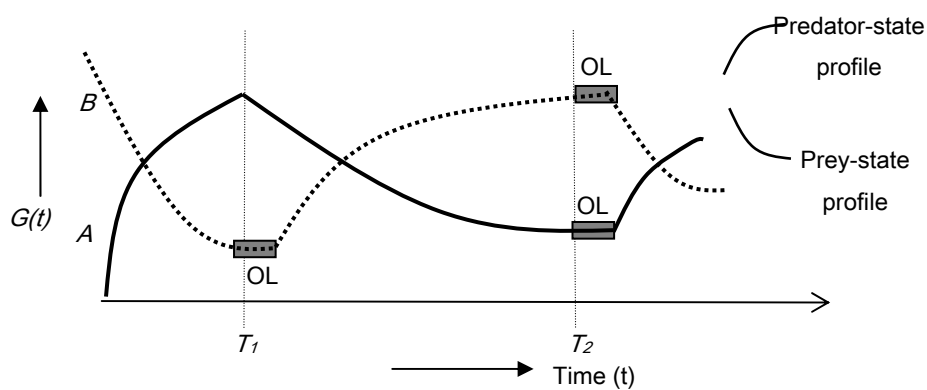


Figure 1: Growth function  $G(t)$  displaying the random flip-flop (dichotomous) states of two corporate species *A* and *B*

The dichotomy of states is modeled as the biological predator/prey (or *vice versa*) competition. The profiles of predator-state and prey-state are bounded between an upper level ( $G_{max}$ ) and a lower level ( $G_{min}$ ) respectively. The dynamics of evolutions (or devolutions) in the business profiles of *A* and *B* may overlap and this refractory region is indicated as OL.  $T_1$  and  $T_2$  are event-specific instants (epochs) at which the state transition takes place. The dynamics of temporal evolutions (or devolutions) specified by a function  $G(t)$  as regard to the competing business profiles of *A* and *B* can be illustrated as in Figure 1. Essentially, the occurrence of dichotomy (flip-flop) of states makes the corporate species *A* and *B* to assume the role of a predator or prey (or *vice versa*). Such flip-flops are event-specific and are randomly disposed. That is, based on the business strategies of the corporations and customer reaction, for example, the state reversal would occur at two epochs  $T_1$  and  $T_2$  as shown in Figure 1. Suppose *A* has made some smart decisions aggressive enough to be designated as a *predator* at a certain instant of time, pushing its business activity into a positive trait of evolution. Then, as a consequence, the competitor *B* becomes a *prey* and may suffer thereof market-losses indicated by a devolutionary trend in its business. However, at a later phase/instant of this downward market-trend, competitor *B* as mentioned earlier may exercise some business changes through prudent decisions that counter the market-losses; hence, its devolutionary trend may shift its slope to positive side and becomes evolutionary. That is, the competitor *B* emerges out of its role of being a prey and proceeds to be a predator. As a result, now the market of *A* may start showing a downward trend and therefore, will start assuming a status of being a prey. This flip-flopping dichotomy along the time-scale would continue until an equilibrium of both (*A* and *B*) finding sustained markets is reached; or one (or both!) corporations may go out of business. The flip-flop model of Figure 1 is indicated to describe the competitive growth dynamics of digital subscriber lines (DSL) in the context New Zealand's Telecom [11]-[16].

The concept of dichotomy in state-reversal of prey-to-predator (or *vice versa*) as characterized by Neelakanta and Sardenberg in [11] is summarized below:

- The state reversal (flip-flop) occurs randomly: With the corporate structures indicated above, this translates to unexpected epochs of service strategies implemented by *A* or *B* via their decision plays
- The duration of being in a particular state by *A* or *B* is also nondeterministic

- However, the deployment of decision-policy and changes in the service structure can be expected from *A* or *B* when the market of individual corporation shows a downward trend with revenue losses or when the market trend shows a positive direction with increases in revenue
- In the dichotomy of states of *A* and *B* observed, there could be overlapping refractory time-frames marked as OL in Figure 1.

Careful handling of technoeconomics in competing businesses and industries such as in Mobile OS context would require diligent planning and prudent approaches. Further, in the context of competitive and deregulated environment, meaningful forecasts on technological needs as well as financial necessities are crucial so as to sustain the *well being* of the industry through retention of a satisfied customer base with a *willingness-to-pay* (WP) attitude as well as for realizing a comfortable *return-on-investment* (RoI) [8]. Consistent analytical considerations and computational pursuits of the model on the dynamics of competitive technoeconomic evolution are developed here on the basis of a practical set of (real-world) data observed in mobile platform market. Since the model indicated is specific to the data set already observed, it conforms to the *ex post* regime of time; and using this growth model, a forecast strategy can be exercised to project the business economics into the *ex ante* time-frame.

### 3 Dynamics of Technoeconomic Growth Function

Again considering the technoeconomic evolution as a function of time, relevant growth model of the dynamics  $G(t)$  versus  $t$ , can be specified in terms of  $G(t)$  expressed as a nonlinear logistic function of time. Such nonlinear prescriptions are adopted in classical models mostly using empirical considerations as described by Neelakanta and Baeza in [8]. However, representation *via* rigorous analytical considerations of an evolutionary phenomenon infested with random inputs should also duly account for the interaction of associated stochastic parameters. Conceiving such a model was originally developed by Neelakanta and De Groff as presented in [10]. It refers to the context of neural information growth and the corresponding growth function  $G(t)$  is expressed in [10] *via* the so-called Bernoulli-Langevin function,  $L_Q(t)$ , (which will be detailed later). Further, the concept of using  $G(t) \equiv L_Q(t)$  to describe the growth dynamics in technoeconomics context has been posted by Neelakanta and Baeza as an exercise in [8]. As a sequel, Neelakanta and Sardenberg have exemplified in [13] a model depicting customer reaction to Internet pricing. Subsequently, description of the growth/decay processes using  $G(t) \equiv L_Q(t)$  has been judiciously adopted in [11] and [16] in the context of technoeconomic business structures by analogously equating the underlying evolutionary dynamics to those encountered in biological systems. The model of [11] and [16] describes the competitive growth dynamics of digital subscriber lines (DSL) in the service structure of New Zealand's Telecom mentioned earlier. Currently, the analytical format of  $G(t) \equiv L_Q(t)$  is again adopted to model the competitive business economics of Mobile OS systems following the procedure reported in [11] and [16]. The approach pursued thereof is as follows:

In a competitive technoeconomic structure, the growth/decay dynamics would normally involve variables pertinent to a set of endogenous parameters similar to biotic parameters in the bio-ecosystem; and, a set of exogenous variables akin to abiotic influences as defined by Neelakanta and Sardenberg in [13] and elaborated in [16]. Broadly, such variables can be specified as follows: (i) Endogenous variable set that impedes the growth:  $\{u_{IN}(t)\}_i$   $i = 1, 2, \dots, I$ ; (ii) exogenous variable set that impedes the growth:  $\{u_{IX}(t)\}_j$   $j = 1, 2, \dots, J$ ; (iii) endogenous variable set that promotes the growth:  $\{v_{PN}(t)\}_k$   $k = 1, 2, \dots, K$ ; and (iv) exogenous variable set that promotes the growth:  $\{v_{PX}(t)\}_\ell$   $\ell = 1, 2, \dots, L$ .

The underlying growth regulation is decided by a pair of controllers, one exercising antagonistic (impeding) influence *via* negative information-flux or entropy and the other providing promoting actions (*via* positive information or negentropic flow); and, the interacting control dynamics on the net data-flow would eventually cause a logistic growth profile for  $G(t)$ . That is, the interacting subsets being themselves invariably nonlinear functions of independent variables involved, a change observed in any of the functions (for a given incremental change in the independent variable) would largely depend on the already prevailing value of that function [7]-[10]. Further, the variables in question could be largely stochastic and partly deterministic. As such, a stochastic (nonlinear) differential equation can be specified for the dynamics under discussion; and, the resulting solution for  $G(t)$ , as indicated before, would correspond to the Bernoulli-Langevin function,  $L_Q(t)$  as specified by Neelakanta and De Groff in [7] and [10]. Explicitly  $L_Q(t)$  is given by  $(1 + 1/2Q)\coth[(1 + 1/2Q)t] - (1/2Q)\coth[(1/2Q)t]$ . Further, when  $Q = 1/2$ ,  $L_{Q=1/2}(t) = \tanh(t)$ , which defines the upper-bound on the disorder; and, as  $Q \rightarrow \infty$ ,  $L_{Q \rightarrow \infty}(t) = [\coth(t) - (1/t)]$ , which corresponds to the lower-bound of the disorder statistic. Also in the limiting case as  $Q \rightarrow 0$ ,  $L_{Q \rightarrow 0}(t) = \text{sgn}(t)$  depicting the scenario wherein the participant subsystems do not interact with each other. Further, the function  $L_Q(t)$  describes the logistic growth in the context of stochastic variables being present with the stochastic feature of disorder set by  $Q$ . That is, the associated statistical disorder is dictated by  $Q$ ; and  $Q$  is limited to the range  $(1/2 < Q < \infty)$ .

In terms of the *theory of interaction statistics* [7] and [10],  $(1/2 < Q < \infty)$  explicitly decides the extent of nonlinear interaction. Such interactions span the regime of disorder being totally isotropic when  $Q \rightarrow 1/2$ ; and when the randomness corresponds to being totally anisotropic,  $Q \rightarrow \infty$ . Lastly, when  $Q \rightarrow 0$ , it implies a total non-interaction (that is, mutually non-relying) state of the parameters/variables that globally decide the underlying activity. Lastly, the Bernoulli-Langevin function has a slope that ranges from 1 to 1/3 at the origin corresponding to the range of  $Q$  between 1/2 to  $\infty$ .

In formulating the nonlinear growth of  $G(t)$ , it is also necessary to consider certain exogenously-infused variables that may influence implicitly the growth or decay in addition to the functional sets:  $\{u_{IN}(G, t), u_{IX}(G, t)\}$  and  $\{v_{PN}(G, t), v_{PX}(G, t)\}$  mentioned earlier. Again, such influences could either be favoring or retarding the growth (or decay) along the time-frame of observation and are depicted as  $P_X(t)$  and  $R_X(t)$  respectively. Hence, derived in [13], [16] is a nonlinear differential equation that describes the (exogenous and endogenous) data-balance relations in the concerned system; and, by applying the chain-rule of total differentiation on the variables, relevant dynamics of  $G(t)$  is deduced as follows:

$$\frac{dG(t)}{dt} = \frac{\left\{ \frac{\partial G}{\partial v_{PN}} \times \frac{\partial v_{PN}}{\partial t} \right\} + \left\{ \frac{\partial G}{\partial v_{PX}} \times \frac{\partial v_{PX}}{\partial t} \right\} - \left\{ \frac{\partial G}{\partial u_{IN}} \times \frac{\partial u_{IN}}{\partial t} \right\} + \left\{ \frac{\partial G}{\partial u_{IX}} \times \frac{\partial u_{IX}}{\partial t} \right\}}{\left[ 1 - \left\{ \frac{\partial F_X}{\partial G} \right\} \right]} \quad (1)$$

where,  $\left[ 1 - \left\{ \frac{\partial F_X}{\partial G} \right\} \right] \equiv \left[ 1 - \left\{ \frac{\partial P_X}{\partial G} - \frac{\partial R_X}{\partial G} \right\} \right]$  and, the entity  $(P_X - R_X)$  is jointly set equal to  $F_X$  considering the heuristics that the net result of superposed nonlinear functions is itself nonlinear [7], [10]. The algorithm of Equation (1) adopted in [13] and [16] is applied exclusively in the present study to describe the dynamics of mobile platform economics. Again, as described in [16], the associated nonlinear functions of Equation (1) are expressed via Bernoulli-Langevin function as indicated below:

That is,  $F_X(G) = L_{q_G}(G)$ ;  $G(u_{IX}) = L_{q_{u_{IX}}}(u_{IX})$ ;  $G(u_{IN}) = L_{q_{u_{IN}}}(u_{IN})$ ;  $G(v_{PX}) = L_{q_{v_{PX}}}(v_{PX})$  and  $G(v_{PN}) = L_{q_{v_{PN}}}(v_{PN})$  where  $L_{q_s}(\bullet)$  denotes the Bernoulli-Langevin function with  $q_s$  being the order parameter ( $\equiv Q$ ) for the governing function indexed by the subscript  $s$  and this subscript  $s$  stands for  $G, v_{PN}, v_{PX}, u_{IN}$  or  $u_{IX}$  as appropriate. With necessary simplifications, Equation (1) can be rewritten as follows:

$$\frac{dG(t)}{dt} = [A_q] \frac{\partial v_{PN}}{\partial t} + [B_q] \frac{\partial v_{PX}}{\partial t} - [C_q] \frac{\partial u_{IN}}{\partial t} - [D_q] \frac{\partial u_{IX}}{\partial t} \quad (2)$$

where,  $A_{q_s} = L'_{q_{v_{PN}}}(v_{PN}) / [1 - L'_{q_G}(G)]$ ;  $B_{q_s} = L'_{q_{v_{PX}}}(v_{PX}) / [1 - L'_{q_G}(G)]$ ;  $C_{q_s} = L'_{q_{u_{IN}}}(u_{IN}) / [1 - L'_{q_G}(G)]$ ;  $D_{q_s} = L'_{q_{u_{IX}}}(u_{IX}) / [1 - L'_{q_G}(G)]$ . The prime on the functions indicates differentiation with respect to the argument. As per Equation (2), the net growth of information flux ( $G$ ) promoted (or demoted) is implicitly dictated by the dynamics (that is, the rate of change) of the quantities:  $v_{PN}, v_{PX}, u_{IN}$  and  $u_{IX}$ ; and, each of these rates is weighted appropriately by a nonlinear coefficient of the set  $\{A_{q_s}, B_{q_s}, C_{q_s}, D_{q_s}\}$ . Further, it can be observed that all these coefficients as seen, are in a general form,  $L'_{q_\rho}(\theta) / [1 - L'_{q_\kappa}(\phi)]$  where  $\rho$  and  $\kappa$  denote the index  $s$  as appropriate and the set  $\{\theta, \phi\}$  represents the variables involved as arguments.

Hence, for any arbitrary value of  $q_s \equiv Q$  between the states of disorder *extremum*, the coefficients  $A_{q_s}, B_{q_s}, C_{q_s}$  and  $D_{q_s}$  are decided by  $L_{q_s}(x; \theta, \phi)$  and its derivative with respect to the argument. It is indicated in [13] and [16] that  $L'_{q_s}(x; \alpha, \beta)$  would incline to follow approximately,  $1/z(q_s; x; \theta, \phi, \dots)$  law where  $z$  is a simple, arbitrary first-order function of its independent variables, namely,  $(q_s; x; \alpha, \beta, \dots)$ . The functional trend of  $1/z(q_s; x; \theta, \phi, \dots)$  shows that the dependence of the function  $L'_{q_s}(x; \theta, \phi)$  on  $(q_s; x; \theta, \phi)$  is significantly influenced for low values of the associated independent variables  $(q_s; x; \theta, \phi)$ . Further, it would asymptotically tend to zero for large values of the arguments,  $(x; \theta, \phi)$ . Also, for a random set of  $(q_s; x; \theta, \phi)$ , the nonlinear aspect  $L_{q_s}(x; \theta, \phi)$  versus stochastic variations in the regulatory variables would be significant and therefore, will be jagged for the low-range of  $q_s$  between  $(1/2$  and  $\infty)$ . However, at larger levels of such influences, the perceived variations would tend to cease depicting more or less an invariant state of  $L_{q_s}(\cdot)$  asymptotically reaching the terminal statistics of steady-state.

The limiting trends as above indicate that the coefficients in question may significantly influence the underlying processes only for low values of the associated independent variables  $(q_s; x; \theta$  and  $\phi)$ ; and, these coefficient functions would asymptotically tend to zero for large values of their arguments. Physically, this implies that the variability in the nonlinear aspect of the rate of change in regulatory variables would be significant for levels of endogenous and/or exogenous influences present during the early stages of temporal discourse of  $G(t)$ . However, at a later stage, that is, towards terminal dynamics of  $G(t)$ , such variations would tend to cease showing more or less an invariant state of flow of the participant fluxes involved. Hence, as dictated by the statistics of endogenous and exogenous variables, jagged variations can be observed in real-world technoeconomic growth during the initial phases of  $G(t)$ . This initial phase observed in the market can be regarded as the so-called *ex post* regime of system economics. However in practice, it is also of interest to know the growth profile that lies ahead. This amounts to forecasting, which is necessary so as to be prepared with resources in order to meet future demands in the market.

This period of interest lying ahead denotes an *ex ante* regime and it mostly implies the terminal statistics of the growth function under consideration [8].

In summary, based on the details of [13] and [16], the governing stochastic differential equations of Equations (1) and (2) of the underlying evolutionary process (growth process) yield convergent solutions that can be expressed in normalized scale in terms of the function  $L_Q(t)$ . The associated stochastic feature is set by the temporal disorder,  $Q$ . Likewise, a time-dependent decay process can be described by the function  $[1 - L_Q(t)]$  in normalized scale. Hence, the phenomenon of competitive evolution (growth) or devolution (decay) between A and B as indicated in [11] and [16] can be specified by the following dichotomous functional relations:

$$G_{A \text{ or } B(t)}|_{\text{Growth}} = \{\alpha_i L_{QA}(t) \times Y_{ti}(t) + \beta_j [1 - L_{QB}(t)] \times Y_{tj+\tau_j}(t)\}, \quad (t \geq 0)$$

$$G_{A \text{ or } B(t)}|_{\text{Decay}} = \{\gamma_i [1 - L_{QA}(t)] \times Y_{tj+\tau_j}(t) + \delta_j L_{QB}(t) \times Y_{tj}(t)\}, \quad (t \geq 0) \quad (3)$$

Where  $i$  and  $j$  are discrete instants where dichotomous transitions (flip-flopping) of prey/predator states occur and ( $\tau_i$  and  $\tau_j$ ) denote shifts in time-scale with respect to  $t_i$  and  $t_j$  at which the transitions take place. Further, the set  $\{\alpha_i, \beta_j, \gamma_i, \delta_j\}$  depicts the extents (levels) of evolution (growth) or devolution (decay) at the transitions. Thus, Equation (3) is a quantitative portrayal of Fig. 1. Further, considering the technoeconomics of competitive industry, the elements in the coefficient-set  $\{\alpha_i, \beta_j, \gamma_i, \delta_j\}$  are implicitly decided by the endogenous and exogenous parameters, namely  $\{u_{IN}, u_{IX}\}$ ,  $\{v_{PN}, v_{PX}\}$  and  $F_X$ ; and, Equation (3) depicting the dynamics of growth (or decay) of A and B denotes a doubly-stochastic process. That is, in the absence of dichotomous flipping, suppose the evolution and devolution states are denoted respectively by a nonlinear set corresponding to the growth-function,  $f(t) \rightarrow L_Q(t)$  and the decay-function,  $g(t) \rightarrow [1 - L_Q(t)]$ . Then, with the superposition of random dichotomous (flip-flop) transitions denoted by switching functions (signum functions),  $Y_{ti}(t)$  and  $Y_{tj+\tau_j}(t)$  on A and B, the dynamics of A and B is specified either as:  $G_A(t) = f(t) \times Y_{ti}(t)$  and  $G_B(t) = g(t) \times Y_{tj+\tau_j}(t)$ ; or as,  $G_A(t) = f(t) \times Y_{tj+\tau_j}(t)$  and  $G_B(t) = g(t) \times Y_{ti}(t)$ . Equation (3) formalizes this switching or state transition features.

## 4 Model Verification with Real-world Mobile Platform Data

With reference to real-world data concerning traffic share in US by Mobile OS platforms (of iPhone, Android and Symbian), the efficacy of the proposed model is verified in this study. Relevant data used in the computations refers to AdMob details on smartphone market as reported in [3]-[4], [14]-[15] and [17]. The data indicated therein illustrates relative competition between the Mobile OS devices in sharing the traffic with a flip-flop trend. A summary of the traffic/market trends of these devices are as follows:

- Competitively iPhone and Android share the *market-climbing* concurrent to a rapid increase in mobile web-browsing traffic.
- iPhone *versus* Android conform to *two-horse smart-phone OS race*, which implies a flip-flopping dichotomy in their market structure in traffic sharing. With the launch of iPhone 3G in 2008, for example, the iPhone market share quickly arose with a peak at the release of the iPhone 3GS. Correspondingly, the launch of new Android devices (as well as Palm's webOS-based Pre), *took some of the wind out of the iPhone's sails* at least temporarily.
- Relatively, the traffic shares of RIM and Windows Mobile have remained low due to rapid market- growths experienced by Android and iPhone.

Typical real-world data on actual profiles of Mobile OS traffic-share in US as available in [3]-[4], [14]-[15] and [17] are presented in Figure 2. It depicts actual profiles of the traffic-shares (in %) borne by Symbian, iPhone and Android over, a time-period of 16 months, from October 2008 through February 2010. This traffic-share is considered in the present study as an implicit economic index of growth (or decay) of underlying businesses. Further, shown in Figure 3, is a normalized version of Figure 2 where the traffic-share on y-axis is calibrated to depict the relative traffic-share of relevant Mobile OS devices; and, the x-axis extends from 0 to 1 covering the 16 months involved.

The reason for considering the normalized profiles (Figure 3) is to track the dynamics of the growth/decay function consistent with Equation (3) where the model is based on the sigmoid  $L_Q(t)$ , which converges in its asymptotic limit to 1; that is, as  $t \rightarrow \infty$ ,  $L_Q(t) \rightarrow 1$ . Normalized data may, however, apparently restrict the model to describe the evolution/devolution for relative comparison only. On the other hand, once such relative comparison is elucidated by the present method, the actual growth/decay performance (in absolute scale) can be decided by denormalizing the results inasmuch as the normalization is done linearly using a constant weight.

From the dynamics of ups and downs in the traffic-share in Figures 2 and 3, one can also identify distinct points of inflexion (shown with a circle) wherever the slope of the growth-curve markedly changes. These points are identified here as *epochs* and are indicated in Figs. 2 and 3 as the epoch-set  $\{a, b, c, \dots, g\}$ .

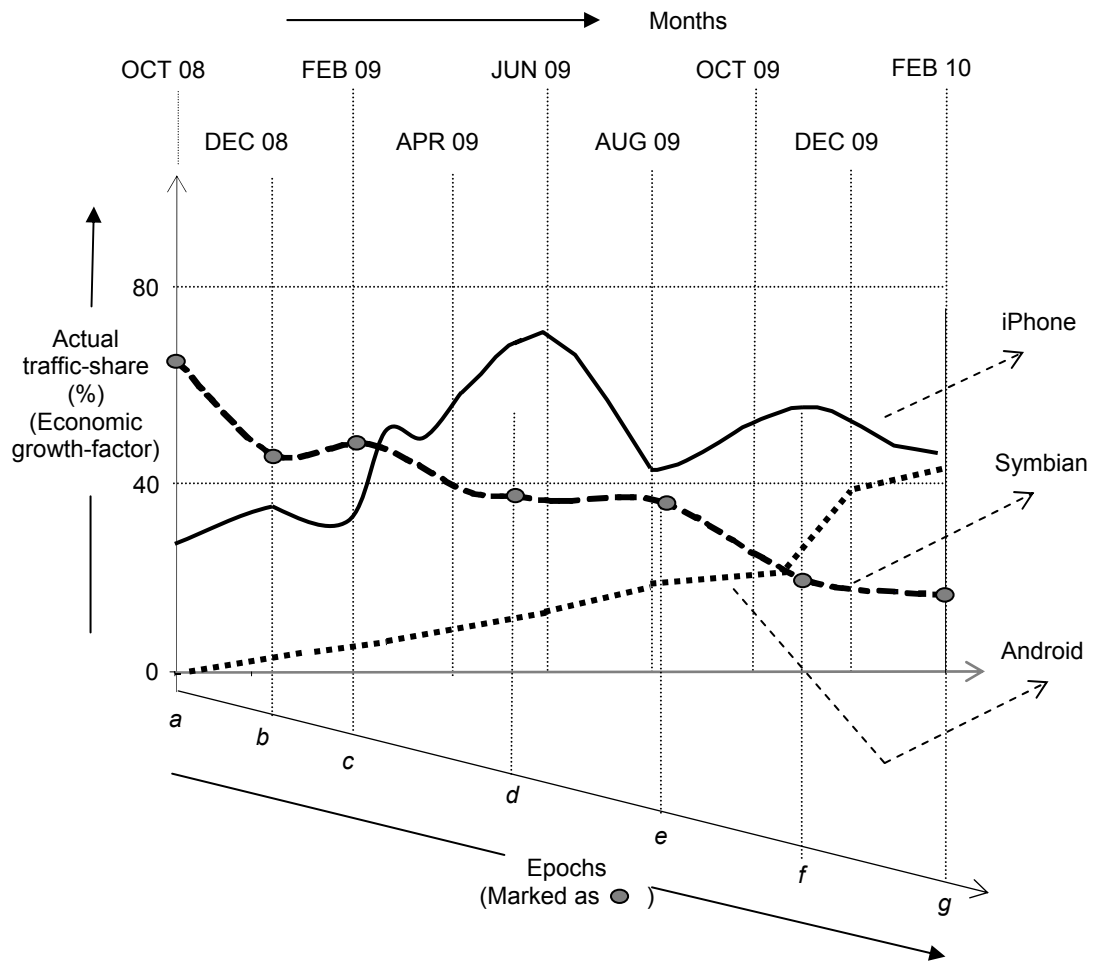


Figure 2: Mobile OS traffic-shares in the USA



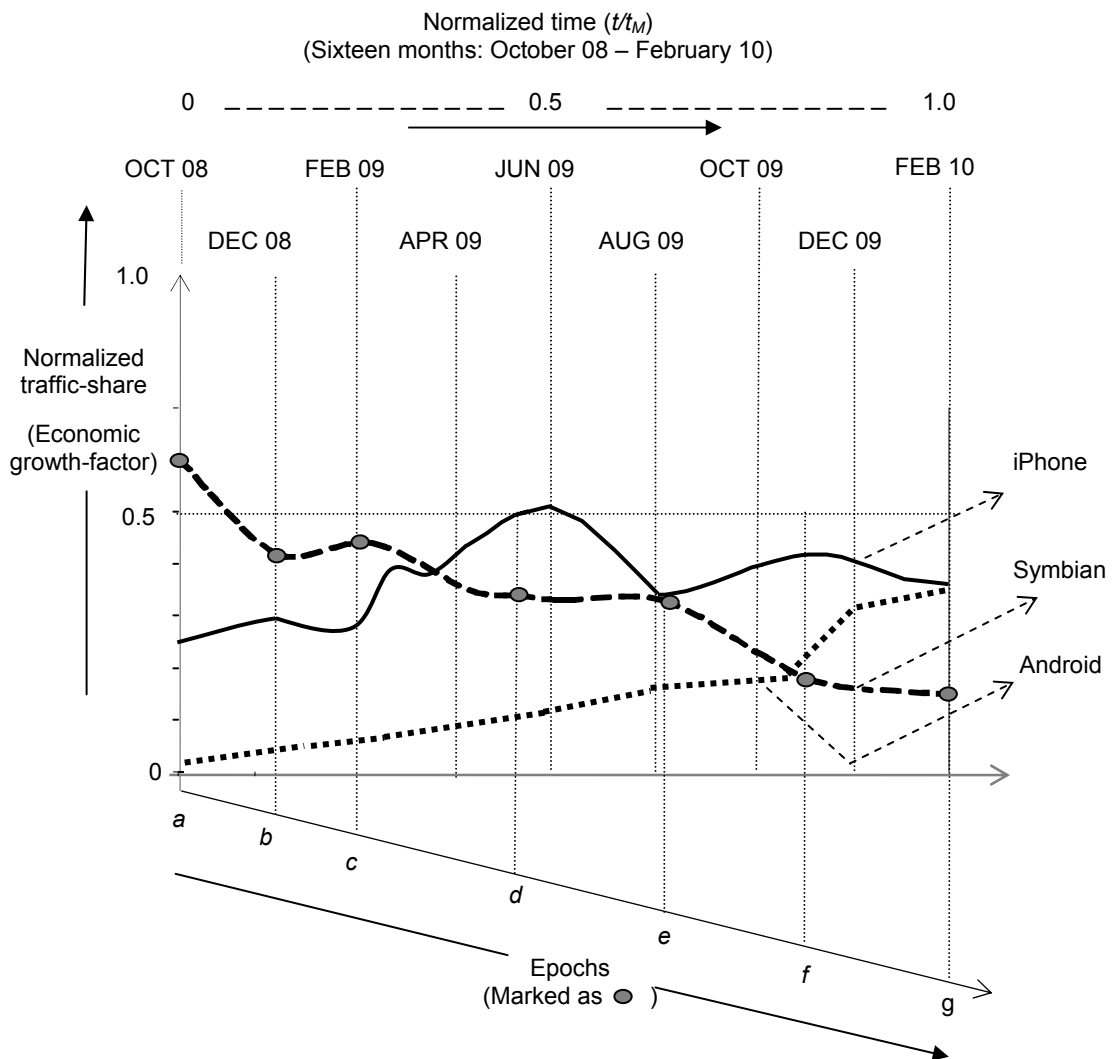


Figure 3: Normalized profiles of mobile OS traffic-shares in the USA

Table 1: Competing mobile OS platforms: A chronological summary of marketed Items across the period October 2008 to February 2010

Mobile OS company	Epoch: Introduction in the market of the Device/Application	Date	Epoch designation: As in Figs. 3 and 4
iPhone	Beta version OS 3.0	March 2009	<i>C</i>
iPhone	3GS	June 2009	<i>de</i>
Android	Android 2.0 OS Update	October 2009	<i>f</i>
Motorola	Droid	November 2009	<i>fg</i>
Android	Google Nexes One	January 2010	<i>g</i>

In Figure 2, the profiles indicate actual traffic-shares in percentage by Symbian, iPhone and Android from October 2008 to February 2010 [3]-[4], [14]-[15], [17]. Figure 3 is the normalized version of Figure 2. The total period of  $t_m$  equal to 16 months (from October 2008 to February 2010) is normalized on the x-axis with a time-scale,  $t$  from 0 to 1. Salient epochs are indicated by circles. With reference to the profiles of traffic-shares indicated in Figures 2 and 3, relevant logic behind the flip-flops (depicting growth/decay dichotomy) can be explained in terms of business ploys and/or new devices/applications introduced by the competing species. For this purpose, presented in Table 1 is a

summary *vis-à-vis* the observations in Figures 2 and 3. The model developed in this paper and presented *via* Equation (3) can be validated against the details of Figures 2 and 3 consistent with the business epochs described in Table 1. Relevant considerations can be identified in terms of the following temporal history:

- In the growth/decay profile dictated by Equation (3), the dichotomy in switching a state is signaled by the growth (or decay) function showing a saturation (flat temporal trend), namely,  $dG(t)/dt \rightarrow 0$  where  $G(t)$  depicts explicitly,  $G_A$  or  $G_B(t)|_{Growth/Decay}$  of Equation (3) for two competitors  $A$  and  $B$
- The epoch(s) designated as above *via*  $dG(t)/dt \rightarrow 0$  may remain sustained approximately over a (short) time-duration shown as overlapping regimes (OL) in Figure 1 at which either or both competitors would introduce new promotion ploys or withdraw any existing incentives
- Suppose the initial slope of  $G(t)$  at the commencement of growth or decay is written as,  $m_o = [dG(t)/dt]_{t \rightarrow start-time}$ . It is (randomly) decided by the endogenous and exogenous technoeconomic inputs. Further, the corresponding regulation of evolution (or devolution) may face an intense (or significantly observable), jagged variations in the dynamic activity, mostly during the nascent stages of commencement of the processes involved. As such, the entities, namely,  $Q_A$  and  $Q_B$  of the Bernoulli-Langevin function in Equation (3) that control the initial slope of  $G(t)$  are considered as non-deterministic random variables in the simulations, but within the specified range of  $(1/2 \text{ to } \infty)$
- Suppose at  $t = 0$ , the competition between  $A$  and  $B$  commences such that  $G_A(t)$  is evolving (growing) and  $G_B(t)$  is devolving (decaying). This dynamics places  $A$  in predator status and  $B$  in prey status; and, the corresponding models due to Equation (3) are specified by:  $G_A(t)|_{Growth} = \{\alpha_i L_{QA}(t)\}$  and  $G_B(t)|_{Decay} = \{\beta_i [1 - L_{QB}(t)]\}$ , ( $t \geq 0$ ) respectively. At a certain instant ( $t + \tau$ ) of epoch in the stretch of OLR regime suppose a state-transition is invoked. This makes  $A$  and  $B$  to flip their roles with  $G_A|_{Decay} = \{\gamma_i [1 - L_{QA}(t)]\}$  and  $G_B(t)|_{Growth} = \{\delta_i L_{QB}(t)\}$ , ( $t + \tau \geq 0$ ). Such dichotomous dynamics would progress ahead with the flip-flops of evolutionary/devolutionary states of  $A$  and  $B$  along the time-scale of interest

In the studies presented in [11] and [16], it is indicated that at each reversal of state indicated above, “the evolving (devolving) species will try to assume and follow the gradient-trend of its opponent in the episode prior to the change of the state”. This statement can be illustrated as shown in Figures 4(a) and 4(b).

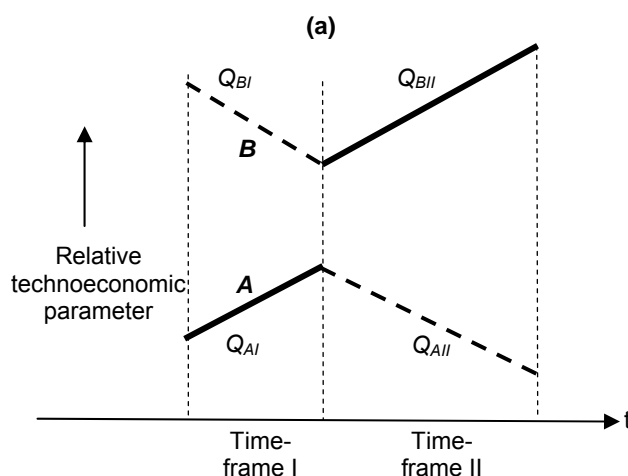


Figure 4 (a): Illustration of evolving (devolving) species: Zero-sum payoff condition

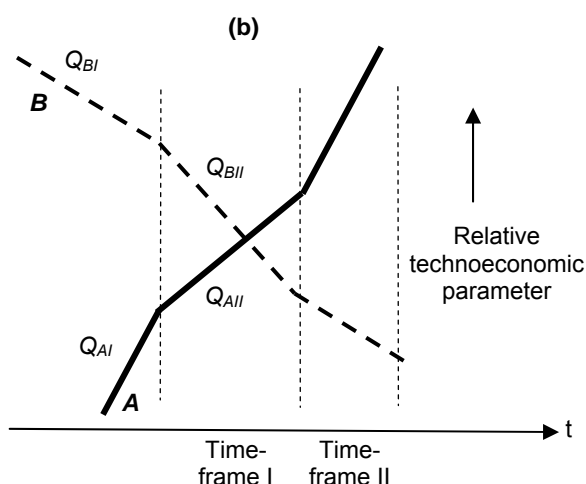


Figure 4(b): Illustration of evolving (devolving) species: *Non-zero sum payoff* condition

In Figure 4, the evolving (devolving) species that try to assume and follow the gradient-trend of its opponent in the episode prior to the change of the state is illustrated. Figure 4(a) refers to the case of zero-sum payoff condition, for example  $Q_{BII}$  tracking  $Q_{AI}$  and Figure 4 (b) refers to condition, for example *non-zero sum payoff* condition with, for example the dominance of A over B. (Details as in the text). Further, in Figure 4, suppose in an arbitrary time-frame I, A (predator) is evolving and its  $Q_A$ -value is designated as  $Q_{AI}$ ; correspondingly, B (prey) is devolving with its  $Q_B$ -value designated as  $Q_{BI}$ . These Q-values decide the initial slope ( $m_o$ ) of the growth/decay curves [7], [10]. That is, the gradient-trends of A and B can be implicitly specified by the Langevin function logistics expressed in terms of  $Q_A$  and  $Q_B$  values respectively; and, the parameter Q is related to  $m_o$  as follows [7], [10]:  $m_o = (1/3)(1 + 1/Q)$ .

Considering the dichotomy of state-transitions from prey-to-predator or *vice versa* across the temporal dynamics, the progress of evolution of a participant species currently assuming, say the role of a predator in the context of competitive prey-to-predator bio-ecology [16], [18] would track the progress of evolution (in the prior of time-frame) of the competing species currently playing the role of a prey. This track history is decided by two conditions of game-theoretic payoff. *Payoff* is what results from each combination of strategies/decisions (alternatives) adopted or courses of action taken by the participants in the market) [8]: *Condition 1*: A *zero-sum payoff* status implies that the sum of payoffs for any choice of alternatives of decisions by the competitors is zero. For example, in the case of two participants A and B, if one gains profit, the other meets a loss of equal extent for each outcome; and, *condition 2*: The status of *non-zero sum payoff* implies an assured dominance of one competitor against the other. It depends on the proportion of efforts of a dominating species so as to get advantageous edge on its disposition (such as monotonically growing revenue) by invoking appropriate alternatives. Relevant *payoff dominance* would imply that a higher-paying alternative is said to dominate the lower one. Hence, based on the observed initial slopes of  $G(t)$  pertinent to A or B choice of  $Q_A$  and  $Q_B$  is made (in the range  $1/2$  to  $\infty$ ) consistent with game-theoretic considerations indicated above and described in [16]. Relevant theorem based on competitive bio-ecology principles is also enunciated and proved in [16].

In summary, as per *Condition 1*, the *ex ante* profile of growth (or decay) trend of an incumbent species posterior to a state-transition would track the *ex post* profile of growth (or decay) trend of the competitor species prior to the state-transition (and *vice versa*). Hence, the outcome of this game is that asymptotically, the progress of dichotomy of states will continue until each state assuming a stability of its own. With reference to *Condition 2*, the *ex ante* profile of growth (or decay) trend of an incumbent species posterior to a state-transition will be autonomous and independent of the *ex post* profile of growth (or decay) trend of the competitor species, prior to the state-transition (and *vice versa*); and, the said autonomous track is decided by the underlying dominance criterion.

## 5 Computations, Results and Discussions

Pursuant to the considerations discussed above, Equation (3) is simulated and validated with reference to growth/decay details presented in Figure 3. First, using the data of Figure 3, initial slopes ( $m_o$ ) of growth/decay curves in each time-frame are ascertained and listed as in Table 2(a). Next, the corresponding Q-values of Bernoulli-Langevin functions are determined pertinent to Equation (3) for each of the Mobile OS system (Symbian, iPhone or Android) under consideration. The indices/suffices, AN, iP and Sy used in Table 2(a) explicitly denote the respective services of Android, iPhone and Symbian). Further, as regard to the associated nonlinear temporal growth/decay profiles having  $m_o$  and Q-values as listed in Table 2(a), corresponding explanatory remarks are presented in Table 2(b).

Table 2(a): Determination of  $Q_{Sy}$ ,  $Q_{iP}$  and  $Q_{AN}$  values in terms of the initial slope ( $m_o$ ) of growth/decay curves [7], [10] of each time-frame indicated in Figure 3 (Note:  $Q_{Sy} = [1/(3m_o - 1)_{Sy}]$ ,  $Q_{iP} = [1/(3m_o - 1)_{iP}]$  and  $Q_{AN} = [1/(3m_o - 1)_{AN}]$ )

Time-frames (See Fig. 3)	Symbian		iPhone		Android		Remarks
	Initial slope ( $m_o$ ) <sub>Sy</sub>	$Q_{Sy}$ and $G_{Sy}(t)$	Initial slope ( $m_o$ ) <sub>iP</sub>	$Q_{iP}$ and $G_{iP}(t)$	Initial slope ( $m_o$ ) <sub>AN</sub>	$Q_{AN}$ and $G_{AN}(t)$	
<i>ab</i>	1.0 (Decay)	0.50 $1 - L_{Sy}(t)$	1.0 (Growth)	0.50 $L_{iP}(t)$	$\rightarrow 0$	Not specified  (Steady or minor growth or decay)	I
<i>bc</i>	0.5 (Growth)	2.0 $L_{Sy}(t)$	$\rightarrow 0$	Not specified  (Steady or minor decay)	$\rightarrow 0$	Not specified  (Steady or minor growth or decay)	II
<i>cd</i>	0.5 (Decay)	2.0 $1 - L_{Sy}(t)$	1.05 (Growth)	0.47 $L_{iP}(t)$	$\rightarrow 0$	Not specified  (Steady or minor growth or decay)	III
<i>de</i>	$\rightarrow 0$	Not specified  (Steady or minor growth)	0.55 (Decay)	1.54 $1 - L_{iP}(t)$	$\rightarrow 0.5$ (Growth)	1.0 $L_{AN}(t)$	IV
<i>ef</i>	0.33 (Dominant decay)	$\rightarrow \infty$ $1 - L_{Sy}(t)$	0.39 (Growth)	6.45 $L_{iP}(t)$	0.55 (Growth)	1.54 $L_{AN}(t)$	V
<i>fg</i>	$\rightarrow 0$	Not specified  (Steady or minor growth)	Across <i>ef-fg</i> ... 0.58      1.36  Growth: $L_{iP}(t)$		0.67 (Growth)	0.98 $L_{AN}(t)$	VI

Table 2(b): Explanations on the remarks (I through VI) of Table 2(a) *vis-à-vis* Bernoulli-Langevin nonlinear growth/decay profiles of traffic shared by iPhone, Symbian and Android systems

Remarks	Explanatory details	
I (ab)	Symbian	Devolution (decay) tendency toward upper-bound ( $Q_{Sy} = 0.5$ ) of the sigmoid implying a closing-in toward an isotropically unsettled random interaction of technoeconomic entities
	iPhone	Evolution (growth) tendency toward upper-bound ( $Q_{iP} = 0.5$ ) of the sigmoid implying a closing-in toward an isotropically unsettled random interaction of technoeconomic entities
	Android	Insignificant evolution (growth) tendency (with $Q_{AN} \rightarrow$ negative value) towards an almost steady-state condition. So the Bernoulli-Langevin function is not applicable
II (bc)	Symbian	Evolution (growth) tendency toward lower-bound of the sigmoid ( $Q_{Sy} = 2$ ) implying a closing-in toward a settled (anisotropically random) interaction of technoeconomic entities
	iPhone	Minor evolution (growth/decay) tendency (with $Q_{iP} \rightarrow$ negative value) towards an almost steady-state condition. So the Bernoulli-Langevin function is not applicable
	Android	Minor evolution (growth) tendency (with $Q_{AN} \rightarrow$ negative value) towards an almost steady-state condition. So the Bernoulli-Langevin function is not applicable
III (cd)	Symbian	Devolution (decay) tendency toward lower-bound ( $Q_{Sy} = 2.0$ ) of the sigmoid implying a closing-in toward an anisotropically settled random interaction of technoeconomic entities
	iPhone	Evolution (growth) tendency toward upper-bound of the sigmoid ( $Q_{iP} = 0.47$ ) implying a closing-in toward a unsettled (isotropically random) interaction of technoeconomic entities
	Android	Minor evolution (growth) tendency (with $Q_{AN} \rightarrow$ negative value) towards an almost steady-state condition. So the Bernoulli-Langevin function is not applicable
IV (de)	Symbian	Not significant evolution (growth/decay) tendency (with $Q_{Sy} \rightarrow$ negative value) towards an almost steady-state condition. So the Bernoulli-Langevin function is not applicable
	iPhone	Devolution (decay) tendency toward lower-bound of the sigmoid ( $Q_{iP} = 1.54$ ) implying a closing-in toward a settled (isotropically random) interaction of technoeconomic entities
	Android	Evolution (growth) tendency toward upper-bound with $Q_{AN} \rightarrow 1.0$ implying a closing-in toward a settled (anisotropically random) interaction of technoeconomic entities
V (ef)	Symbian	Devolution (decay) tendency degenerates toward signum function ( $Q_{Sy} \rightarrow \infty$ ) implying a closing-in toward <u>totally</u> anisotropic random interaction of technoeconomic entities meaning that variability in economic parameters pose insignificant influence on the decay process --- <b>So, Symbian is loosing the game and a stage is now set for the dominance of the competitors (iPhone and Android)</b>
	iPhone	Evolution (growth) tendency toward lower-bound of the sigmoid ( $Q_{iP} = 6.45$ ) implying a closing-in extensively toward a settled (anisotropically random) interaction of technoeconomic entities --- <b>With growth (winning) trend, iPhone is getting the dominance over the competitor, Symbian</b>
	Android	Evolution (growth) tendency toward lower-bound of the sigmoid ( $Q_{AN} = 1.54$ ) implying a closing-in toward a settled (anisotropically random) interaction of technoeconomic entities --- <b>Android is also getting the dominance over the competitor, Symbian</b>
VI (fg)	Symbian	Minor evolution (almost no-growth) tendency (with $Q_{Sy} \rightarrow$ negative value) towards a steady-state condition. So the Bernoulli-Langevin function is not applicable
	iPhone	Evolution (growth) tendency toward lower-bound of the sigmoid ( $Q_{iP} = 1.36$ ) implying a closing-in toward a settled (anisotropically random) interaction of technoeconomic entities --- <b>iPhone is creeping into the dominance over the competitor, Symbian and competes almost one-to-one with Android</b>
	Android	Evolution (growth) tendency toward lower-bound of the sigmoid ( $Q_{AN} = 0.98$ ) implying a closing-in toward an unsettled (isotropically random) interaction of technoeconomic entities --- <b>Android is also creeping into the dominance over the competitor, Symbian and competes one-to-one with iPhone</b>

Now, using the details as in Tables 2(a) and 2(b), simulation of underlying Bernoulli-Langevin function for each time-frame under consideration can be done with the corresponding  $Q$ -value. The simulations performed in each time-frame refer to an ensemble of evaluations on  $G(t)$  with the set  $\{Q \pm \Delta Q_R\}$  where  $\{\Delta Q_R\}$  denotes an ensemble set of random values in the range of say,  $\pm 10\%$  of  $Q$  so as to provide the statistical error-bar on simulated curves. (For the specific cases of  $Q \rightarrow 0$  and  $Q \rightarrow \infty$  respectively,  $10^{-4}$  and  $10^{+4}$  are used in the simulations).

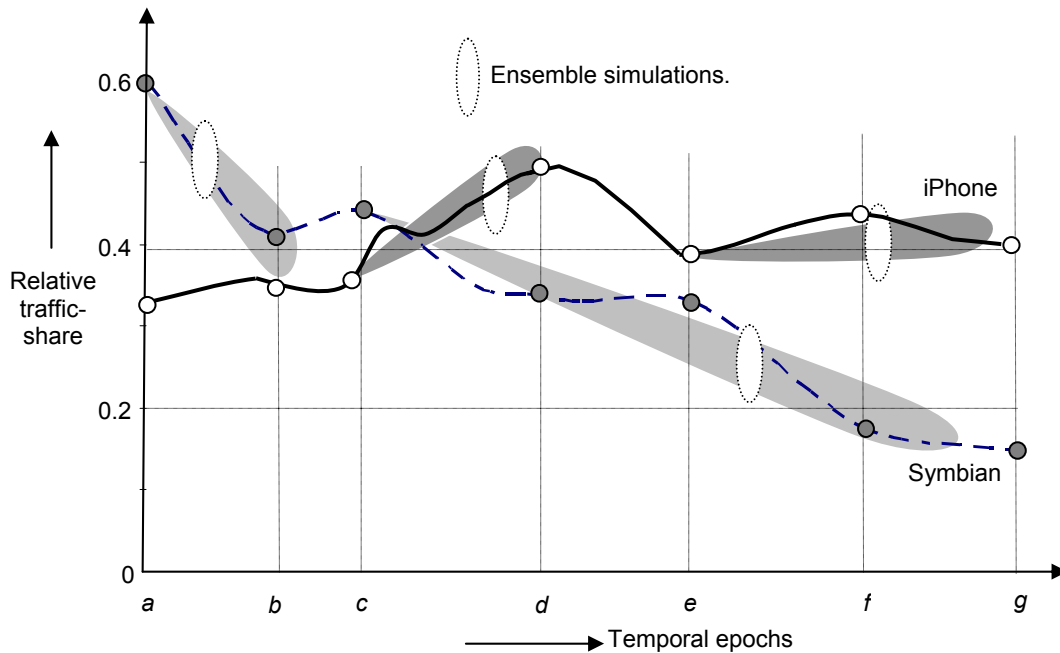


Figure 5: Symbian versus iPhone: Progress of traffic-shares from October 2008 to February 2010

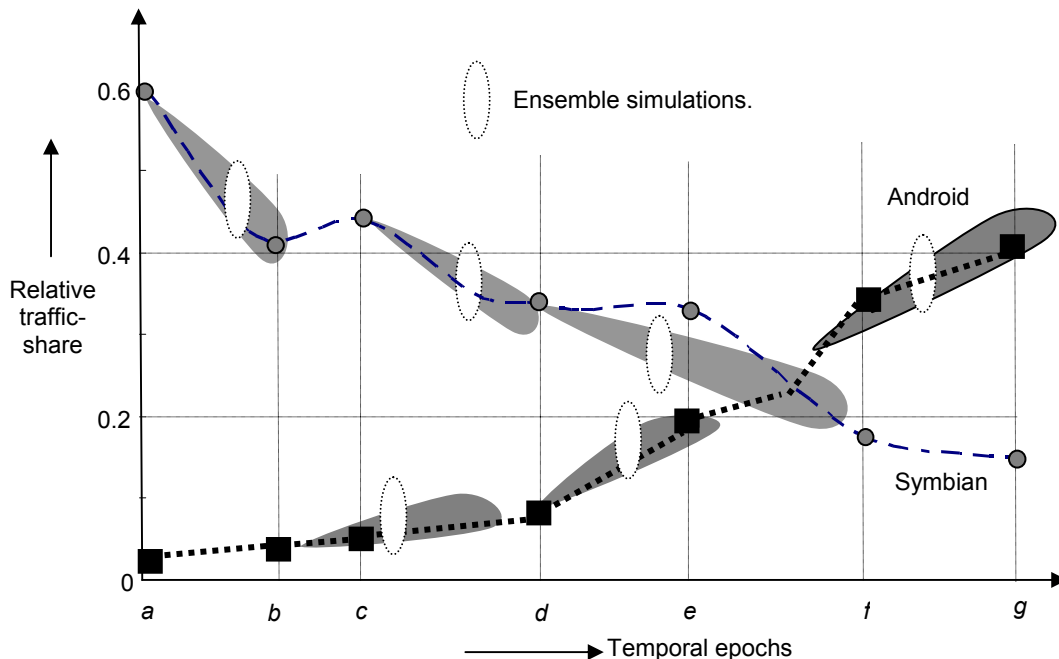


Figure 6: Symbian versus Android: Progress of traffic-shares from October 2008 to February 2010

Figure 5 is the Illustration of growth pursuit by a participant (predator) in a given time-frame versus decay suffered by the competitor (prey) in the previous time-frame. Likewise, Figure 6 depicts the Illustration of growth pursuit by a participant (predator) in a given time-frame versus decay suffered by the competitor (prey) in the previous time-frame. In essence, Figures 5 and 6 depict respectively, the simulated curves superimposed on data curves of (Symbian versus iPhone) and (Symbian versus Android) as observed during each time-frames,  $ab, bc, \dots, fg$ . In terms of the model developed in this work, the results depicted in Figures 5 and 6 are explained in Tables 3 and 4.

Table 3: Symbian versus iPhone: Prey/predator status of Figure 5 – Details

Time-frame	Mobile OS	Growth or decay	Status	Q-value	Comment
ab	Symbian	Decay	Prey	0.50	Commencement of the game-theoretic competition. $Q = 0.5$ implies the economic variability would strongly influence the growth/decay making $G(t)$ dynamics jagged
	iPhone	Growth	Predator	0.50	
bc	Symbian	Growth	Predator	2.00	Symbian with $Q = 2.0$ shows less variability in getting affected by economic factors, but has a growing trend relative to iPhone
	iPhone	Steady or minor growth	Prey	Not specified in $L_Q(t)$ context	
cd	Symbian	Decay	Prey	2.00	The profile of growth trend of iPhone (posterior to the state-transition at c) appears to gracefully track the profile of growth trend of the competitor (Symbian) prior to the state-transition. Hence, the suite of Condition 1, seem to fit in implying a zero sum payoff situation.
	iPhone	Growth	Predator	0.47	
de	Symbian	Steady or minor growth	Predator	Unspecified	Symbian tries to maintain a steady profile with no event-specific moves
	iPhone	Decay	Prey	de: 1.54	
ef-fg	iPhone	Growth	Predator	6.45 Across ef-fg: 1.36	The growth regime of iPhone (predator) in the time-frame, ef tracks more or less the competitor's (prey's) decay profile all the way from the previous time-frame, cd. Further, across ef-fg, Symbian loses its dependence on any variability (with $Q \rightarrow \infty$ ) in the technoeconomic inputs. As such, its decay tendency continues. Here, Condition 2 tendency appears to prevail. That is, with Symbian continuously losing grip on the game, the growth-trend of the competing iPhone (posterior to its $e^{\text{th}}$ epochal state-transition) becomes more autonomous; that is, an autonomous track sets in due to the underlying dominance criterion, which in this case takes the iPhone to be the eventual winner as a dominant Mobile OS system. Thus, the Condition 2, implying a non-zero sum payoff situation creeps in.
	Symbian	Decay	Prey	ef: $\rightarrow \infty$ fg: Unspecified	

Table 4: Symbian versus Android: Prey/predator status of Figure 6 – Details

Time-frame	Mobile OS	Growth or decay	Status	Q-value	Comment
ab	Symbian	Decay	Prey	0.50	ab: Commencement of the game with Android posing no extensive competitive trend and Symbian significant decay trend.  Condition 1, namely, the zero-sum payoff situation can be expressed with respect to competing enterprises in the initial time-frames. Towards the end (ef-fg) Condition 2 tendency prevails. That is, the growth-trend of competing Android becomes more autonomous and independent of the profile of decay trend of the incumbent Symbian. The autonomous track sets in due to the underlying worsening state of the Symbian and dominance of the Android. Hence, at the end of the observed period, Android becomes the eventual winner.
	Android	Steady or minor growth	Predator (?)	Unspecified	
cd	Android	Minor growth	Predator	Unspecified	
	Symbian	Decay	Prey	2.00	
de	Android	Growth	Predator	1.00	
	Symbian	Steady or minor decay	Prey	de: $\rightarrow \infty$ ef: Not specified	
ef-fg	Android	Growth	Predator	0.98-1.54	
	Symbian	Decay	Prey	Not specified	

### 5.1 Sample Calculation and Sensitivity Considerations

With reference to the compiled results, indicated below is an example calculation and sensitivity aspects of the observed performance. Considering Symbian versus iPhone competition, relevant prey/predator status shown in Figure 5 is explained in Table 3. It refers to, for example: (i) in time-frame *bc* Symbian shows a growth trend with an incumbent role of being a predator; and, (ii) in the next time-frame *cd*, the competitor namely, iPhone assumes a growth trend playing a predator role. This growth trend of iPhone, which happens posterior to a state-transition (in *cd*) appears to track the profile of the growth trend of the competitor prior to the state-transition (in *bc*). Hence, the suite of Condition 1, seem to fit in implying a zero sum payoff situation. This can be justified using numerical details on the growth/decay functions. From the traffic-share data (in normalized form) fitted to the time-frames *bc* and *cd*, it can be inferred from Table (2a) that:

**bc time-frame:**

Computed initial slope (magnitude) of Symbian's growth profile ( $m_{oS_Y}$ )  $\approx 0.5$   
 The corresponding Q-value =  $1/(3m_{oS_Y} - 1) \approx 2.0$   
 $G(t)|_{ab} = L_{S_Y}(t)$  as indicated in Table 2(a)

**cd time-frame:**

Computed initial slope (magnitude) of iPhone's growth profile ( $m_{oIP}$ )  $\approx 1.05$   
 The corresponding Q-value =  $1/(3m_{oIP} - 1) \approx 0.47$   
 $G(t)|_{cd} = L_{IP}(t)$  as indicated in Table 2(a)

Thus, the growth trend in *cd* of iPhone tracks the growth trend of the competitor (Symbian) in the prior time-frame, *bc*. The tracking as above is in conformance with the suite of Condition 1 namely, zero sum payoff situation. The efficacy of tracking trend can be specified in terms of the sensitivity set by the Q-values. In both cases as above, the Q-values (2.0 and 0.47) are near to the upper-bound of the statistical variability or disorder set by the function  $G(t) \equiv L_Q(t)$  on the growth features introduced by the exogenous and endogenous factors of the underlying technoeconomics. This is true inasmuch as the time-frames in question (*bc* and *cd*) conform to the initial business stages where such high variability or isotropic randomness can be anticipated.

However, the sensitivity of growth/decay to such variations will be felt to a lesser extent at later time-frames wherein the businesses stabilize for better or worse. This can be illustrated with the following exemplification: Again, consider the traffic-share data (in normalized form) fitted to the time-frames *ef* through *fg* relevant to Symbian versus iPhone competition. From Table 2(a):

**ef time-frame:**

Computed slope (magnitude) of Symbian's  $G(t)$  profile ( $m_{oS_Y}$ )  $\approx 0.39$   
 The corresponding Q-value =  $1/(3m_{oS_Y} - 1): \rightarrow \infty$



$G(t)|_{ef} = 1 - L_{SY}(t)$  as indicated in Table 2(a)

Computed slope (magnitude) of iPhone's  $G(t)$  profile ( $m_{oIP}$ )  $\approx 0.33$

The corresponding Q-value =  $1/(3m_{oIP} - 1)$ : 6.45

$G(t)|_{ef} = L_{iP}(t)$  as indicated in Table 2(a)

*fg* time-frame:

Computed slope (magnitude) of Symbian's  $G(t)$  profile ( $m_{oSY}$ )  $\rightarrow 0$

The corresponding Q-value =  $1/(3m_{oSY} - 1)$ : Unspecified in using  $L_Q(t)$  function

$G(t)|_{ef} \rightarrow$  Steady-state as indicated in Table 2(a)

Computed slope (magnitude) of iPhone's  $G(t)$  profile ( $m_{oIP}$ )  $\approx 0.58$

The corresponding Q-value =  $1/(3m_{oIP} - 1)$ : 1.36

$G(t)|_{ef} = L_{iP}(t)$  as indicated in Table 2(a)

At these later time-frames (*ef-fg*), the businesses stabilize for better or worse. That is, the sensitivity of growth/decay to technoeconomic variability is less felt. With iPhone, the Q-value (= 6.45) is much offset from the limiting value of  $Q = 0.5$ ; and hence, the statistical variability on the growth features introduced by the exogenous and endogenous factors of the underlying technoeconomics becomes *steadier* rendering the associated disorder closer to being more *anisotropic* or invariant. As such, iPhone becomes negatively sensitive to decay in its growing trends.

On the contrary, the competitor (Symbian) faces extensive constancy in its dynamics  $G(t) \equiv L_Q(t)$  introduced by the bound  $Q \rightarrow \infty$  (seen in *ef*) on any possible variability in the technoeconomic inputs that would reverse its decaying trend in its performance.

Thus, the growth-trend of competing iPhone (posterior to its  $e^{\text{th}}$  epochal state-transitions through *ef-fg* time-frames) becomes more autonomous and independent of the profile of decay trend of the competitor Symbian; that is, an autonomous track sets in due to the underlying dominance criterion, which in this case takes the iPhone to be the eventual winner as a dominant Mobile OS system. Hence, the Condition 2 implying a non-zero sum payoff situation creeps in.

Similar reasoning can be attributed to rest of the time-frames as well as to the other participant Mobile OS systems analyzed.

## 6 Inferential Remarks and Discussions

Commensurate with the objective of the present study, this paper offers a model to depict the co-evolution of large enterprises such as Mobile OS industry as competitive business structures. The model considers at least two competing corporations with an embodiment of several stochastically interacting subsystems constituting a complex system. Further, considering the underlying dynamics of competition, a novel paradigm of prey-predator model is specified with a dichotomy of (prey-predator) flip-flops along the time-scale. That is, in each state (of predator and prey), the evolution or devolution of the corporate economics is shown to follow a nonlinear growth or decay,  $G(t)$ , which can be expressed in a stochastic differential equation format. Corresponding solution of  $G(t)$  is presented in terms of Bernoulli-Langevin function. Lastly, simulated results on this functional description of  $G(t)$  are compared against real-world Mobile OS data. The computed results lead to the following observations and inferential conclusions toward validation of the model.

- Temporal evolutions/devolutions across each time-frame set:  $\{ab, bc, cd, de, ef, fg\}$ : As presented in Tables 3 and 4
- Considering the incumbent game-players, Android and iPhone show a growth trend. This growth is competitive with respect to each other as could be evinced in the time-frames of *ef* and *fg*. Should iPhone *versus* Android in the post-(February 2010) be an issue of research interest, it is indicated here as an open-question
- In the segments of time-frames considered in this study, relevant simulated data closely follow the actual data within the statistical stretch of computations exercised indicating the efficacy of the model and validating the method of approach
- The computational outcome is exemplified with an illustrative sample calculation and corresponding sensitivity details on the results are furnished
- To the best of the authors' knowledge no parallel efforts in describing the prey-predator dichotomy dynamics of technoeconomics in terms of Bernoulli-Langevin nonlinear perspectives are available in the literature

- The present study offers a model in the analytical form of Equation (3) that describes the growth/decay trends of the businesses in the past (that is, in *ex post* regime). A logical question that arises is how this model could be useful. Once a model such as Equation (3) is available describing the past episode of evolution/devolution, it can be used forecast the details in the futuristic (*ex ante*) time-frame *via* appropriate asymptotic computations that preserve the associated stochastics as well as the stability features of the terminal phases in the technoeconomics scenario under discussion. Relevant forecast computations can be done, first by exercising a *learning phase*, for example in an artificial neural network (ANN) using actual data gathered from the market and modeled *via* Equation (3). Once the ANN is so *trained*, suppose a set of upcoming data on the growth/decay performance is available, it can now be supplied to the ANN as the input set so that the computed output conforms to *predicting* of forecast details. Inasmuch as the ANN training model is based on Equation (3) with the associated stochastic considerations of the growth/decay trends, it can be anticipated in the prediction phase that the growth/decay features will be preserved in the *ex ante* regime following the suite of the *ex post* epochs. Relevant efforts on ANN-based forecasting *via* co-evolution model are in progress
- As indicated before, the use of normalized data in the analysis may apparently restrict the model to describe the evolution/devolution for relative comparison only. However, the normalization performed uses a linear constant weight. Hence the proportionality of performance like traffic share would not change either in the model predictions or in the forecast regimes. Once the relative comparison is elucidated using the normalized data set then, the actual growth/decay performance (in absolute scale) can be decided by denormalization done with the constant linear weight adopted as the normalizer. This proportionality consideration is true for both *ex post* and *ex ante* results.

## 7 Closure

In closure, this paper offers a neoteric approach to view the evolution of competitive species in the framework of prey-predator mind-set projected as a dichotomous dynamics of flip-flopping (prey/predator) roles. Hence the temporal dynamics of competition/co-evolution of known competitors in the mobile-platform market, like Android, Symbian and iPhone is depicted by a novel model posing a dichotomy of prey-predator flip-flops in the market; and, an asymptotic projection of underlying algorithm into the *ex ante* region would correspond to forecast assessments of mobile platforms. That is, the modeled algorithm like Equation (3) can be adopted as a growth/decay profile that feeds an ensemble of input data set to train an ANN in an *ex post* time-frame; and, the trained ANN can be used to deliver projected/forecast data in the *ex ante* regime.

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