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The Dual-Channel Retailer's Channel Synergy Strategy Decision

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Abstract: The main research question asked in this paper is whether and when a dual-channel retailer (retailer in short) should adopt the "buy online and pick up in store" (BOPS) strategy. To answer this question, we first derive the optimal price decision using the non-BOPS and BOPS strategies. Subsequently, we compare the performance of retailers under non-BOPS and BOPS scenarios. Our main findings are that under the monopoly scenario, retailers may not always benefit from the BOPS strategy. Retailers will benefit only if the offline operational costs are low and the degree of customer acceptance of the online channel is high. However, the BOPS strategy cannot improve dual-channel retailers' market share. Furthermore, under a Stackelberg game scenario with e-retailers as leaders, when the value of a product is medium and the transaction costs of the offline channel are high, retailers can use the BOPS strategy to enhance their market share. If the degree of customer acceptance of the online channel is also high, retailers can further improve their profits by using the BOPS strategy. Overall, these findings not only provide decision support for retailers, but also enrich the theories on dual-channel retailing in operations management.

Keywords: retailing; dual channel; synergy strategy; Stackelberg game

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1. Introduction

In 2020, China's online retail sales exceeded USD 1800 billion, an increase of 10.9% over 2019. With the development of e-commerce, more and more brick-and-mortar retailers are turning into dual-channel retailers, i.e., operating both offline and online channels simultaneously. However, in the e-commerce market, many pure e-retailers (e.g., Amazon, JD, Tmall) entered earlier than dual-channel retailers. Hence, at present in the e-commerce market, dual-channel retailers are still at a disadvantage in competition with pure e-retailers. For example, in 2018, the market share of dual-channel retailers (e.g., Suning, Gome) in the B2C e-commerce of China was just 5.7%.

To improve their competitiveness, many dual-channel retailers have begun to adopt certain channel synergy strategies. At present, "buy online and pick up in store" (BOPS) is a common channel synergy strategy that is adopted by many dual-channel retailers, e.g., Suning, Uniqlo, 7 Eleven, and many more. In practice, some dual-channel retailers achieve good results by using the BOPS strategy. For example, 50% of Circuit City's online orders come from the BOPS strategy. However, some retailers do not achieve the desired results. Uniqlo adopted the BOPS strategy in 2014, but it not only failed to enhance sales, it also increased operational costs. Meanwhile, some empirical studies have showed that the BOPS strategy is not always beneficial to dual-channel retailers [1].

Therefore, the following question arises—when should a dual-channel retailer adopt the BOPS strategy? To this end, we first establish a monopoly model with a "same product, different price" condition. Then, we investigate the optimal price decision under non-BOPS and BOPS strategies. By comparing the performance of dual-channel retailers using these two strategies, we study the scope of application of the BOPS strategy. Subsequently, we extend the monopoly model to competitive environments. In practice, dual-channel retailers often use the BOPS strategy as a means to compete with pure e-retailer. Therefore, under competition conditions, we consider a pure e-retailer and a dual-channel retailer who compete with each other using an e-retailer leadership Stackelberg game scenario. We then study the equilibrium price using non-BOPS and BOPS strategies. After this, we investigate the scope of application of the BOPS strategy in competitive environments.

Gao and Su [2] and Zhang et al. [3] studied the application of the BOPS strategy. In comparison, this paper makes at least two important contributions. First, we study the BOPS strategy under a "same product, same price" scenario. "Same product, same price" means that the same produce should be set at the same prices in the offline and online channels. However, at present, differential prices in the offline and online channels are still very common (e.g., Gome and Wal-Mart). To reflect this business practice, this paper considers a "same product, different price" scenario. By comparing the results of this paper with the relevant literature, we also find that the application of the BOPS strategy is greatly affected by the price strategy ("same product, same price" and "same product, different price"). Second, although Zhang et al. [3] extended Gao and Su's model [2] from non-competitive to competitive environments, they assumed that pure e-retailers and dual-channel retailers have comparable market power in the e-commerce market (i.e., a Nash game approach). However, in some countries, pure e-retailers have more power than dual-channel retailers. Meanwhile, customers often go to the pure e-retailers, such as Tmall.com, to track prices. Hence, pure e-retailers have more power to influence product pricing. To fill this research gap, this paper considers that the dual-channel retailers are at a disadvantage in the competition with pure e-retailers, i.e., the e-retailer leader Stackelberg game approach. In summary, this paper not only provides decision support for a retailer to adopt the BOPS strategy under the "same product, different price" with e-retailer as the leaders, but also further enriches the theories on dual-channel retailing in operations management.

This paper is structured as follows. Section 2 reviews the literature related to our study. Section 3 describes the model and initial analysis. In Sections 4 and 5, we study whether and when the retailer adopts the BOPS strategy. In the final section, we conclude the findings and offer future research perspectives.

2. Literature Review

As more retailers begin to use the dual-channel model, synergies between channels become more important. Many scholars have begun to pay attention to such issues. Steinfield et al. [4] found that despite case study evidence showing the benefits of channel integration, a high degree of integration across channels is relatively uncommon. They proposed that the required level of investment and IT sophistication is the main reason for this phenomenon. Bendoly et al. [5] mainly investigated the impacts of channel integration on purchasing decisions. They found that the degree of channel integration can be critical to increasing consumer retention rates. Oh and Teo [6] studied an information-sharing strategy between online and offline channels, finding that the strategy offers a promising opportunity for retail firms to enhance their relationships, with their customers as well as their firm performance. Herhausen et al. [7] defined the channel synergy strategy as integrating access to and knowledge about the offline channel into an online channel. They examined the impacts of online–offline channel integration on purchase decisions and found that the integrated dual-channel approach can improve competitive advantage. Li et al. [8] studied one monopolistic publisher with both physical and electronic channels. They found that by coordinating the sale times and prices for the different channels, the profits of publishers could be effectively enhanced. Lee [9] studied the impacts of omni-channel characteristics on customer satisfaction and found that integrated promotion and integrated information can significantly improve customer satisfaction.

Although the above-mentioned literature studied the channel synergy, it did not involve the BOPS strategy. Steinfield [10] used transaction cost economics, inter-organizational systems, competitive strategy, and economic sociology to develop a framework that can outline the potential synergies arising from the integration of e-commerce with traditional channels. As one of the synergistic strategies, he referred to the BOPS strategy. Chopra [11] proposed a framework for helping managers to design the distribution channel according to a variety of customer and product characteristics. In this paper, he mentioned the BOPS strategy and believed that it could reduce the cost of distribution. Berman and Thelen [12] proposed many channel synergy strategies: BOPS strategy, synergy promotion, synergy information system, and synergy pricing. They believed that such synergy strategies can result in an increased customer base, improved revenue, and higher market share. Zhang et al. [13] studied the motivations and constraints of going multichannel, the challenges of crafting multichannel retailing strategies, and opportunities for creating synergies across channels. In the multichannel retailing strategies, they mainly discussed the BOPS strategy, "buy-online-and-delivery-in-store", and "buy-online-and-return-in-store".

Although the studies mentioned above involve the BOPS strategy, they did not conduct in-depth research on the BOPS strategy. In business practice, many retailers believe that the BOPS strategy is the most important channel synergy strategy [14], thus some scholars have begun to pay attention to it. Chatterjee [15] studied the influence of channel synergy strategy on consumer purchase decisions, using the BOPS strategy to make an empirical analysis. The result illustrated that dual-channel retailers who used the BOPS strategy can obtain greater profitability than those who operated dual channels independently. Kim et al. [16] studied the impact of BOPS strategy on consumer buying behavior by using an empirical research method. They found that the consumer perceptions of relative advantage, complexity, compatibility, and risks involved in online shopping played an important role in the use of BOPS strategy. Gallino and Moreno [1] found that the offline channels can benefit from the BOPS strategy, but online channels suffered losses. By using an empirical analysis, though Kim et al. [16] and Gallino and Moreno [1] proposed that the BOPS strategy was not always beneficial to dual-channel retailers, they did not further analyze when the retailers should use the BOPS strategy. Gao and Su [2] and Zhang et al. [3] studied this question by using a mathematical model, and found that the BOPS strategy was not appropriate for products that sell well in stores. However, they only considered a "same product same price" condition. However, in practice, differential prices in dual channel are still very common. In additional, though Zhang et al. [3] studied this question in a competitive environment, they only considered the retailer and pure e-retailer had the comparable market power in the e-commerce market. However, it is a more common phenomenon that the pure e-retailer has more power than the retailer in the e-commerce market. Therefore, this paper will study this question under a "same product different price" condition and a e-retailer leader's Stackelberg game.

In addition, compared with the dual-channel retailers, manufacturers adopt the dual-channel model earlier in practice. The dual-channel manufacturer means that the manufacturer distributes his product through an online direct channel and an offline retailer simultaneously. Previous studies on channel synergy mainly focused on this field. Scholars propose many channel synergy strategies, such as cooperative advertising strategy [17,18], lateral transshipment strategy [19–21], and information sharing strategy [22–24]. They all found that in some cases, these channel synergy strategies can improve the performance of manufacturer and retailer simultaneously.

3. The Model

Following Gao and Su [2] and Zhang et al. [3], we will use the methodology of mathematical modeling to study the application of the BOPS strategy. The basic procedure of mathematical modeling is as follows. The first step is to establish a mathematical model according to the actual situation. Next, the model is solved by a mathematical method. Finally, the result of the solution is used to solve the practical problem.

This section mainly introduces the basic model of a monopolist dual-channel retailer. The retailer sells the product at price p_r and p_e through the offline channel and the online channel, respectively. The operational costs per unit of different sales channels are generally different, so we use c_r and c_e to denote the operational cost in the offline channel and in the online channel, respectively. Some empirical studies showed that compared with offline channels, online channels could effectively reduce operating costs [25,26], thus we assume that $c_r > c_e$. By using empirical research, Kacen et al. [27] found that the degree of customer acceptance of the online channel is generally lower than the offline channel. The main reason is that customers cannot possess it immediately, as well as the lack of detailed physical inspection when they buy the product from online channel. Therefore, we use v and θv denote the value of purchasing the product from offline channels and from online channels, respectively, where $0 < \theta < 1$. The value of purchasing product refers to the value that the product can bring to consumers. For example, the product can generally meet the needs of consumers in terms of functions and/or emotions.

Many related studies also have the same assumption, such as Chiang et al. [28] and Zhang et al. [29]. If customers buy the product from online channels, they generally need to pay for delivery cost b_e and transaction cost l_e. We then can determine that the customer utility of online channel U_e is

$$U_e = \theta v - p_e - b_e - l_e \tag{1}$$

Consumer utility refers to the difference between the value obtained by consumers for owning the product and the cost of obtaining the product.

Compared with online channel, customers need to go to physical stores when they buy from the offline channel. Therefore, customers need to pay shopping trip costs $b_r s$, where b_r is the unit shopping trip cost and s is the distance from customer to store. Following Larralde et al. [30] and Osborne and Pitchik [31], we assume that s is uniformly distributed within the customer population from 0 to 1, with a density of 1, and the physical store is located at $s = 0$. In the physical store, customers need to spend time locating the desired items and standing in line to pay, etc., thus we use l_r to denote the offline transaction cost. In reality, because online transactions are just "a click away", we assume that the offline transaction cost is greater than online transaction cost, i.e., $l_r > l_e$. We then can determine that the customer utility of online channel U_r is

$$U_r = v - p_r - b_r s - l_r \tag{2}$$

In practice, due to distance, it is difficult for the offline channel to cover the entire market, i.e., even if the transaction cost and offline channel's price are both zero, customers who are far away from the offline channel will not buy. Therefore, we assume $v - b_r < 0$. If customers choose BOPS, they do not need to pay the delivery fee b_e, but they need to bear shopping trip cost $b_r s$. We use p_{ei} to denote sale price under BOPS. Therefore, the customer utility of BOPS strategy U_{ei} is

$$U_{ei} = \theta v - p_{ei} - b_r s - l_e \tag{3}$$

4. Monopoly Case

In practice, some retailers exclusively sell certain types of products from manufacturers. Under this monopoly condition, one must ask whether and when the retailer should adopt BOPS strategy. To answer this question, we investigate the performance of retailers under the non-BOPS strategy and BOPS strategy. For clarity, we use superscript *mo* and *mn* to denote the BOPS case and non-BOPS case, respectively.

4.1. Non-BOPS Strategy

We first derive the retailer's demand functions for online and offline channels. Under the non-BOPS strategy, customers cannot pick up the online purchase from the store, and will purchase from the channel with more surplus. A customer will prefer the offline channel if $v - p_r^{mn} - b_r s - l_r > \theta v - p_e^{mn} - b_e - l_e$. When $s^a = \frac{(1-\theta)v + b_e + p_e^{mn} + l_e - p_r^{mn} - l_r}{b_r}$, the customer is indifferent between the channels, as shown in Figure 1.

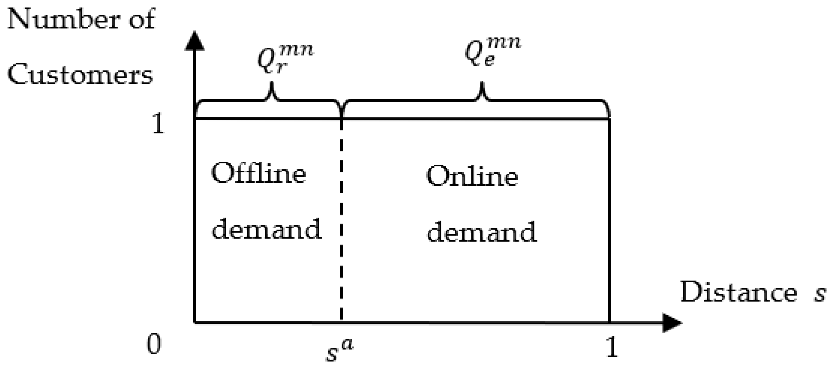

Figure 1. Retailer's demand under non-BOPS and monopoly case.

We then can determine that the offline demand Q_r^{mn} and the online demand Q_e^{mn} are

$$Q_r^{mn} = \frac{(1-\theta)v + b_e + p_e^{mn} + l_e - p_r^{mn} - l_r}{b_r} \tag{4}$$

$$Q_e^{mn} = \frac{b_r - (1-\theta)v - b_e - p_e^{mn} - l_e + p_r^{mn} + l_r}{b_r} \tag{5}$$

Based on the demand function, we can determine the retailer's profit (Π_R^{mn}):

$$\Pi_R^{mn} = (p_r^{mn} - c_r)\frac{(1-\theta)v + b_e + p_e^{mn} + l_e - p_r^{mn} - l_r}{b_r} + (p_e^{mn} - c_e)\frac{b_r - (1-\theta)v - b_e - p_e^{mn} - l_e + p_r^{mn} + l_r}{b_r} \tag{6}$$

From (6), we can derive that optimal price decisions (non-uniqueness) exist that maximize the retailer's profits.

In practice, the retailer can use a single online channel, a single offline channel, or dual channels to distribute products. We first compare the single offline channel and dual-channel. Because we assume that it is difficult for the offline channel to cover the entire market, some demands are not satisfied under the single offline channel mode. In this case, because the consumers have homogenous values when purchasing through the online channel (with delivery), either all consumers are willing to buy online, or none are. Thus, when the costs do not exceed this common valuation, the retailer can utilize the online channel to reach consumers who do not shop offline, which will lead to more profit. Therefore, the dual-channel mode is always better than the single offline channel mode.

We then compare the single online channel and dual-channel. Under the single online channel structure, the retailer will set $p_e = \theta v - b_e - l_e$ to maximize his profits. We derive that the retailer's margin equals $\theta v - b_e - l_e - c_e$ under this condition. Under the dual-channel structure, to ensure $U_r = v - p_r - b_r s - l_r \geq 0$, the maximum offline price equals $v - l_r$. If $v - l_r - c_r < \theta v - b_e - l_e - c_e$, the retailer's margin will decrease when he moves from a single online channel to a dual channel. We then can obtain the following proposition:

Proposition 1 . *Under a monopoly case and a non-BOPS strategy, if* $\theta \geq \frac{v - c_r - l_r + b_e + c_e + l_e}{v}$*, the retailer should adopt a single online channel, i.e.,* $s^a \leq 0$ *; otherwise, the retailer should adopt dual channels, i.e.,*$s^a > 0$.

From Proposition 1, we know that when the degree of customer acceptance of the online channel is not very high, the dual-channel mode can lead to more profit for the retailer.

4.2. BOPS Strategy

Under the BOPS strategy, customers can choose between online channel with delivery (online channel in short), BOPS mode and offline channel. We first compare the BOPS mode with the online channel. A customer will prefer the BOPS mode if $\theta v - p_{ei}^{mo} - b_r s - l_e > \theta v - p_e^{mo} - b_e - l_e$. When $s^\beta = \frac{p_e^{mo}+b_e-p_{ei}^{mo}}{b_r}$, the customer is indifferent between the channels. Next, we compare the offline channel with the BOPS model. A customer will prefer the BOPS model if $v - p_r^{mo} - l_r < \theta v - p_{ei}^{mo} - l_e$. Because we also can achieve $s^\alpha \geq s^\beta$ under $v - p_r^{mo} - l_r > \theta v - p_{ei}^{mo} - l_e$, we can derive that no customers will adopt the BOPS mode. Under this condition, we will reach the same results to the non-BOPS strategy condition. Therefore, to study whether the BOPS strategy can help the retailer, we just consider $v - p_r^{mo} - l_r < \theta v - p_{ei}^{mo} - l_e$, as shown in Figure 2.

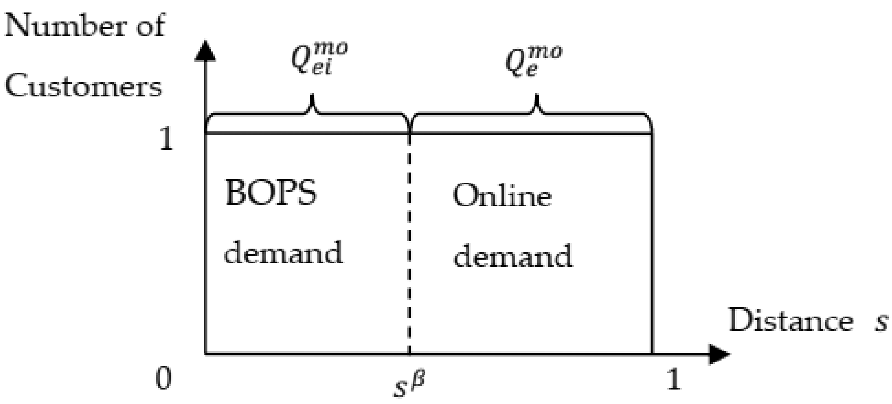

Figure 2. Retailer's demand under BOPS and monopoly case.

Under this condition, the BOPS mode's demand Q_{ei}^{mo} and the online channel's demand Q_e^{mo} are

$$Q_{ei}^{mo} = \frac{p_e^{mo} + b_e - p_{ei}^{mo}}{b_r} \tag{7}$$

$$Q_e^{mo} = \frac{b_r - p_e^{mo} - b_e + p_{ei}^{mo}}{b_r} \tag{8}$$

Based on the demand function, we can determine the retailer's profit (Π_R^{mo}):

$$\Pi_R^{mo} = (p_{ei}^{mo} - c_r)\frac{p_e^{mo} + b_e - p_{ei}^{mo}}{b_r} + (p_e^{mo} - c_e)\frac{b_r - p_e^{mo} - b_e + p_{ei}^{mo}}{b_r} \tag{9}$$

Based on (9), we derive that the of optimal price decisions exist (non-uniqueness) that maximize the retailer's profits.

4.3. Strategy Decision: Non-BOPS or BOPS

Based on the above analysis, the retailer under both non-BOPS and BOPS strategy can obtain full market coverage, thus the BOPS strategy cannot improve the market share. Next, we discuss the impact of BOPS strategy on the retailer's profits. From Proposition 1, we know if $\theta \geq \frac{v-c_r-l_r+b_e+c_e+l_e}{v}$, the single online channel mode is better than the dual-channel mode and the retailer will set $p_e = \theta v - b_e - l_e$ to maximize his profits. However, under the dual-channel mode, if the retailer adopts the BOPS strategy, he can set the BOPS price $p_{ei} = \theta v - l_e$ and $p_e = \theta v - b_e - l_e$. If the BOPS's margin $\theta v - l_e - c_r$ is higher than the margin of the online channel $\theta v - b_e - l_e - c_e$, the BOPS strategy will improve the retailer's profits. Therefore, we can obtain the following proposition.

Proposition 2. *Under a monopoly case, when* $\theta \geq \frac{v-c_r-l_r+b_e+c_e+l_e}{v}$ *and* $b_e > c_r - c_e$*, the retailer's profit under the BOPS strategy is more than that under the non-BOPS strategy.*

Customers who use the BOPS strategy could see a decrease in their delivery cost and a corresponding increase in their shopping trip cost. Those who are close to a store can use the BOPS strategy to increase their utility. The BOPS strategy offers a way for a retailer to implement price discrimination, i.e., the setting a higher price for customers who use the BOPS strategy. Meanwhile, when the offline operational cost is lower, the retailer will obtain more margin from the BOPS strategy, which is the main source of his profit increase.

From Proposition 1, we know that if $\theta < \frac{v-c_r-l_r+b_e+c_e+l_e}{v}$, both online and offline channels have demand under the non-BOPS strategy. Based on demand analysis under the BOPS strategy, we know that only if $s^\beta > s^\alpha$, BOPS strategy will be adopted by some customer. Therefore, under this condition, when the retailer moves from the non-BOPS strategy to the BOPS strategy, there will be two types of consumers who change their purchase method to BOPS. One type is all offline customers under non-BOPS. The reason for customers' change is that they can enjoy both free delivery cost and lower transaction cost simultaneously. Another type is a section of online customers under non-BOPS who are closer to the store. Relative to the delivery cost, they can have a lower shopping trip cost under BOPS which is main the main reason for the change. From above, we know that the optimal price decisions under non-BOPS and BOPS strategies are non-uniqueness, thus it is hard to study how these prices change when the BOPS option is added and why. Therefore, we then compare the profits by using the numerical example. The base parameter values are as follows: $v = 1$, $c_r = 0.3$, $c_e = 0.2$, $b_r = 1.2$, $b_e = 0.3$, $l_r = 0.2$, $l_e = 0.1$ and $\theta = 0.9$. From Figures 3 and 4, we acquire the observation:

Observation 1. *Under a monopoly case, if* $\theta < \frac{v-c_r-l_r+b_e+c_e+l_e}{v}$*, when the delivery cost or degree of customer acceptance of the online channel is higher, the retailer can adopt the BOPS strategy to improving the profits.*

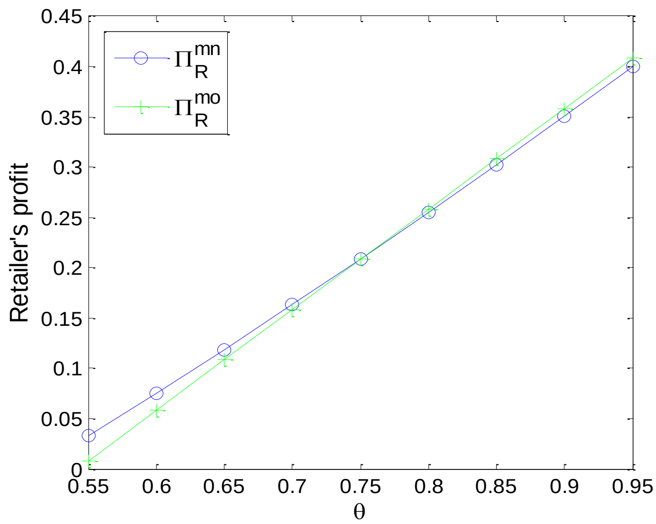

Figure 3. Impact of θ on retailer's profits under monopoly case.

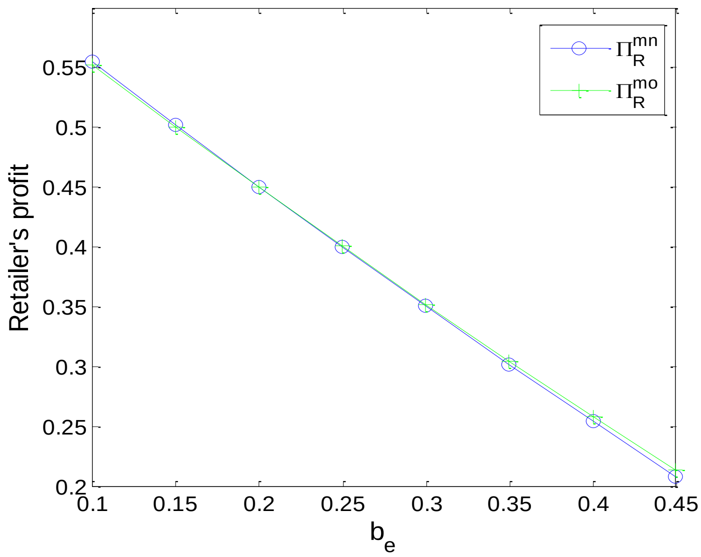

Figure 4. Impact of b_e on retailer's profits under monopoly case.

According to the above analysis, all offline customers and a section of online customers will change their purchase method to BOPS when the retailer adopts the BOPS strategy. When the offline customers adopt the BOPS mode, they will face lower product value (from v to θv) and lower transaction cost (from l_r to l_e), thus the higher degree of customer acceptance of the online channel may lead to more surplus, which can provide a chance for the retailer to set higher prices under the BOPS strategy. Therefore, the higher degree of customer acceptance of the online channel can be benefit for the retailer to gain more profit from the original offline customer. When the online customers adopt the BOPS mode, they will face lower shopping trip costs (from b_e to $b_r s$), the higher delivery cost means the online customer can achieve more benefit from BOPS. Therefore, the retailer can set a higher price for the original online customer and acquire more profit.

Based on the above analysis, we can determine that if a retailer has more offline customers, he should pay more attention to the degree of customer acceptance of the online channel when he decides whether to adopt the BOPS strategy. On the contrary, if a retailer has more online customers, he should pay more attention to delivery cost when he makes the decision for the BOPS strategy.

Gao and Su [2] and Zhang et al. [3] also studied whether the retailer should adopt the BOPS strategy under a monopoly case. The findings of Gao and Su [2] showed that the BOPS strategy is beneficial to the improvement of the retailer's profit. The main reason for this is the cross-selling opportunities, i.e., the customers purchase additional products when they pick up the purchase in store. However, in commercial practice, in order to facilitate the pick-up of consumers who use BOPS, retailers generally set the pick-up station outside the store. Some retailers (e.g., Wal-Mart) even provide "delivery to car" service for consumers. Therefore, in practice, the revenue/profit from cross-selling may be limited. The results of our paper (Proposition 2 and Observation 1) show that the BOPS strategy is not good choice for retailers in some condition, which is consistent with some commercial practices. For example, Uniqlo's revenue was reduced when it used the BOPS strategy in 2014. Zhang et al. [3] illustrated that under a monopoly case, if the dual-channel retailer moves from the non-BOPS strategy to the BOPS strategy, his profit will decrease. The main reason is that the BOPS strategy increases the retailer's operating costs, but because of the "same product same price", the retailer cannot set a higher price for consumers who use BOPS. In our paper, we consider that the retailer can use the "same product different price" strategy, i.e., the same produce can be set differential prices in the offline and online channels. This strategy is currently widely adopted in the retail industry (e.g., Gome and Wal-Mart). Therefore, the results of our paper show that retailers have the opportunity to use the BOPS strategy to obtain higher revenue in some cases. In reality, many retailers (e.g., Suning, 7 Eleven and Circuit City) choose to adopt the BOPS strategy, which also confirms our result.

5. Competition Case

In practice, the BOPS is often used as a competitive strategy between the dual-channel retailer and pure e-retailer. Therefore, we consider that a pure e-retailer (e-retailer in short) competes with a dual-channel retailer. For clarity, we use superscript d to denote this scenario. The e-retailer sells the product to customers at price p_t^d. Because the e-retailer has richer operating experience in the e-commerce market, we assume that the operational cost of the e-retailer c_t is lower that the operational cost of the retailer's online channel.

Our goal is to identify whether and when the retailer should adopt the BOPS strategy under competition case. To answer question, we first investigate game equilibrium price. In many countries, the pure e-retailer has more power than the dual-channel retailer. We assume here, therefore, that the e-retailer is the Stackelberg leader and the retailer is the follower. The game sequence is as follows. First, the e-retailer acts as the Stackelberg leader and decides her price. Second, the retailer acts as the Stackelberg follower and decides on his online and offline channel's prices. We use backward deduction to solve this game.

5.1. Non-BOPS Strategy

We first study the retailer and e-retailer's demand functions. Under the non-BOPS strategy, customers can choose between the retailer's offline channel, the retailer's online channel, and the e-retailer. We first compare the e-retailer and the retailer's offline channel. A customer will prefer the e-retailer if $\theta v - p_t^{dn} - b_e - l_e > v - p_r^{dn} - b_r s - l_r$. When $s^{\gamma} = \frac{(1-\theta)v+b_e+p_t^{dn}+l_e-p_r^{dn}-l_r}{b_r}$, the customer is indifferent between the e-retailer and offline channel. Next, we compare the e-retailer with the retailer's online channel. A customer will prefer the e-retailer if $\theta v - p_e^{dn} - b_e - l_e > \theta v - p_t^{dn} - b_e - l_e$. When $p_e^{dn} = p_t^{dn}$, the customer is indifferent between the retailer's online channel and e-retailer. Because $c_t < c_e$, the e-retailer can always set the price p_t^{dn} lower than the retailer's online channel price p_e^{dn} to ensure her sales. For example, for the retailer, the lowest online channel's price is $p_e^{dn} = c_e$. Under this condition, the e-retailer also can set $p_t^{dn} = c_e - \varepsilon$ (where $0 < \varepsilon < c_e - c_t$) to ensure sales and obtain profit simultaneously. Therefore, the retailer just uses his online channel to limit the e-retailer's price ceiling, i.e., $p_t^{dn} < p_e^{dn}$. To sum up, under this condition, the retailer's demand for the offline channel Q_r^{dn} and the e-retailer's demand Q_t^{dn} are

$$Q_r^{dn} = \frac{(1-\theta)v + b_e + p_t^{dn} + l_e - p_r^{dn} - l_r}{b_r} \tag{10}$$

$$Q_t^{dn} = \frac{b_r - (1-\theta)v - b_e - p_t^{dn} - l_e + p_r^{dn} + l_r}{b_r} \tag{11}$$

Based on the game sequence, the retailer will first determine its best response to the e-retailer's price. From (10), we can determine the retailer's profit (Π_R^{dn}):

$$\Pi_R^{dn} = (p_r^{dn} - c_r)\frac{(1-\theta)v + b_e + p_t^{dn} + l_e - p_r^{dn} - l_r}{b_r} \tag{12}$$

From (12), we can determine that the best response $\widetilde{p}_r^{dn}$ is unique and the value is

$$\widetilde{p}_r^{dn} = \frac{(1-\theta)v + b_e + p_t^{dn} + l_e + c_r - l_r}{2} \tag{13}$$

We then study how the retailer will decide their online channel's price. From above, we know that although there is no demand on his online channel, he can use the online channel to limit the e-retailer's price, i.e., the e-retailer must set price p_t^{dn} less than p_e^{dn}.

Substituting (13) into (12), we can easily check that Π_R^{dn} increases in p_t^{dn}, thus the retailer's optimal online channel's price is

$$p_t^{dn*} = \theta v - b_e - l_e \tag{14}$$

From (13) and (11), we obtain the profit function of the e-retailer (Π_T^{dn}):

$$\Pi_T^{dn} = \left(p_t^{dn} - c_t\right)\frac{2b_r - (1-\theta)v - b_e - l_e - p_t^{dn} + c_r + l_r}{2b_r} \tag{15}$$

From (15) and (12), we can gather the proposition:

Proposition 3. *Under the competition case, if the retailer adopts the non-BOPS strategy, Stackelberg equilibrium prices are*

$$p_r^{dn*} = \frac{v + c_r - l_r}{2} \tag{16}$$

$$p_t^{dn*} = \theta v - b_e - l_e \tag{17}$$

From Proposition 3, we find that although the retailer competes with the e-retailer, the retailer's offline price is not related to parameters of the online channel, such as customer acceptance of the online channel and delivery cost. The retailer just needs to consider the operational cost and transaction cost of the offline channel when he sets the offline price.

5.2. BOPS Strategy

Again, we first study the retailer and e-retailer's demand functions. Under the BOPS strategy, customers can choose between BOPS mode, retailer's online channel by delivery (online channel), offline channel, and e-retailer. Next, we compare the e-retailer with the retailer's BOPS mode. A customer will prefer the BOPS mode if $\theta v - p_{ei}^{do} - b_r s - l_e > \theta v - p_t^{do} - b_e - l_e$. When $s^\chi = \frac{p_t^{do} + b_e - p_{ei}^{do}}{b_r}$, the customer is indifferent between the e-retailer and the BOPS mode. Similar to the monopoly case under BOPS strategy, we need to consider $v - p_r^{do} - l_r < \theta v - p_{ei}^{do} - l_e$. In addition, to ensure her sales, the e-retailer needs to set the price p_t^{dn} less than the online channel price of the retailer. To sum up, we can derive that customers will only choose between the e-retailer and the BOPS mode. Therefore, the BOPS mode's demand Q_{ei}^{do} and the e-retailer's demand Q_t^{do} are

$$Q_{ei}^{do} = \frac{p_t^{do} + b_e - p_{ei}^{do}}{b_r} \tag{18}$$

$$Q_t^{do} = \frac{b_r - p_t^{do} - b_e + p_{ei}^{do}}{b_r} \tag{19}$$

The retailer will first determine its best response to the e-retailer's price. From (18), we can determine the retailer's profit (Π_R^{do}):

$$\Pi_R^{do} = \left(p_{ei}^{do} - c_r\right)\frac{p_t^{do} + b_e - p_{ei}^{do}}{b_r} \tag{20}$$

From (20), we can determine that the best response $\widetilde{p}_{ei}^{do}$ is

$$\widetilde{p}_{ei}^{do} = \begin{cases} \theta v - b_e - l_e & if\ \ p_t^{do} > 2\theta v - 3b_e - c_r \\ \frac{p_t^{do} + b_e + c_r}{2} & if\ \ p_t^{do} \le 2\theta v - 3b_e - c_r \end{cases} \tag{21}$$

From (21), we can determine the e-retailer's profit (Π_T^{do}):

$$\Pi_T^{do} = \begin{cases} \left(p_t^{do} - c_t\right)\frac{b_r - p_t^{do} - 2b_e + \theta v - l_e}{b_r} & if \;\; p_t^{do} > 2\theta v - 3b_e - c_r \\ \left(p_t^{do} - c_t\right)\frac{2b_r - p_t^{do} - b_e + c_r}{b_r} & if \;\; p_t^{do} \le 2\theta v - 3b_e - c_r \end{cases} \quad (22)$$

From (21) and (22), we can determine the proposition. All proofs, if not provided in the main text, are in the Appendix A.

Proposition 4. *Under the competition case, if the retailer adopts the BOPS strategy, Stackelberg equilibrium solutions are*

$$p_{ei}^{do*} = p_t^{do*} = \begin{cases} b_e + c_r & if \;\; 0 < b_e < \frac{v - c_r}{2} \; and \; \frac{c_r + 2b_e}{v} < \theta < 1 \\ \theta v - b_e - l_e & otherwise \end{cases} \quad (23)$$

From Proposition 4, we can find two groups of equilibrium prices, as shown in Figure 5. When the delivery cost is low and customer acceptance of the online channel is high, i.e., part I, both the retailer and e-retailer will set the BOPS price and online price lower than under the other conditions. The delivery cost and the customer acceptance of the online channel can measure the degree of e-commerce maturity. As the degree of e-commerce maturity improves, the competitiveness of the e-retailer strengthens, and the online channel becomes more and more important for the dual-channel retailer. Subsequently, the competition between online channels increases, providing the main reason for declining BOPS price and online prices.

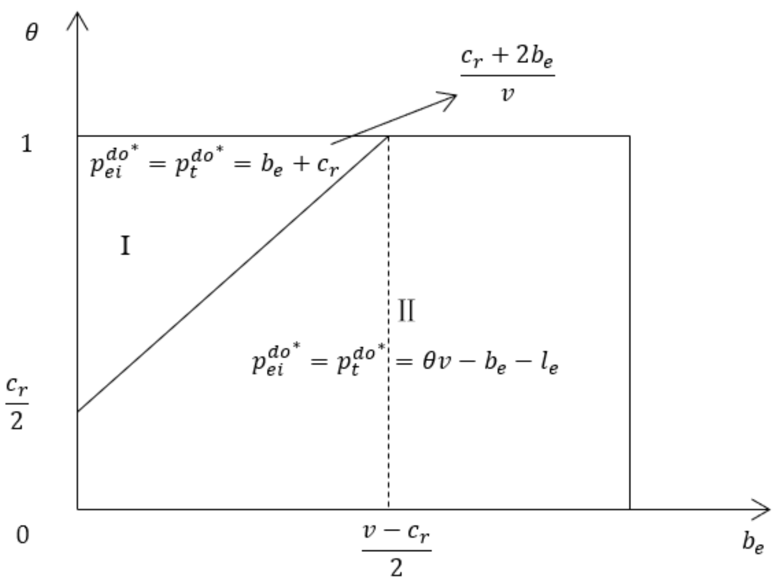

Figure 5. Stackelberg equilibrium prices under the BOPS strategy.

5.3. Strategy Decision: Non-BOPS or BOPS

This subsection studies the main research questions: whether the BOPS strategy can enhance the performance of the retailer. To this end, we start by comparing the demands, and we can achieve the proposition.

Proposition 5. *Under the competition case, if the retailer moves from the non-BOPS strategy to the BOPS strategy, the retailer's market share will increase when* $\frac{v - c_r - l_r}{2} \le b_e \le \theta v - c_e - l_e$*, and will decrease when*$b_e < \frac{v - c_r - l_r}{2}$.

Because the delivery cost is eliminated under the BOPS strategy, the advantage of BOPS will be more pronounced, compared to the e-retailer, as the delivery cost increases, leading to an increase in market share. Because the demand does not change with the delivery cost under the non-BOPS strategy, the demand under the BOPS strategy will be larger than that under the non-BOPS strategy when the delivery cost exceeds a threshold, i.e., $\frac{v-c_r-l_r}{2}$. Because the delivery cost is usually not very high in practice, generally the delivery cost will be higher than the threshold only when the value of the product is also not too high. In addition, we find also that the value of threshold decreases in the transaction costs of the offline channel l_r. Therefore, when the value of the product is medium and the transaction costs of the offline channel are higher, retailers can use the BOPS strategy to enhance their market share.

Proposition 6. *Under the competition case, if the retailer moves from the non-BOPS strategy to the BOPS strategy, the retailer's profits will increase only when* $\frac{v-c_r-l_r}{2} < b_e < \frac{(v-c_r)+\sqrt{(v-c_r)^2-(v-c_r-l_r)^2}}{2}$ *and* $\frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v} < \theta < 1$*; otherwise, retailer's profit will decrease.*

The product's value θv in the online channel increases with the degree of customer acceptance of the online channel. Hence, if the degree (θ) is higher, the BOPS strategy outperforms the non-BOPS strategy, as shown in Figure 6. Moreover, under the BOPS strategy, the retailer's profits are also affected by the delivery cost. From Proposition 4, we know that under the BOPS strategy, the market share of the retailer increases with the delivery cost. We then also find that the retailer's BOPS price first increases and then decreases with the delivery cost under the BOPS strategy. We can derive then that under the BOPS strategy, the retailer's profits also first increase and then decrease with the delivery cost, as shown in Figure 7. Meanwhile, the retailer's profits do not change with the delivery cost under the non-BOPS strategy, so the retailer's profits under the BOPS strategy will be larger than that under the non-BOPS strategy when the delivery cost is higher than one threshold, i.e., $\frac{v-c_r-l_r}{2}$, and lower than the other threshold, i.e., $\frac{(v-c_r)+\sqrt{(v-c_r)^2-(v-c_r-l_r)^2}}{2}$, simultaneously. As in the analysis of Proposition 5, the delivery cost is not often high in practice; the delivery cost will satisfy the range only when the value of the product is also not too high. In addition, we find that the feasible interval range expands as the transaction costs of offline channel l_r increase. Therefore, when the value of the product is medium and the transaction costs of the offline channel and the degree (θ) are higher, the BOPS strategy will be more effective for improving the profit of the retailer.

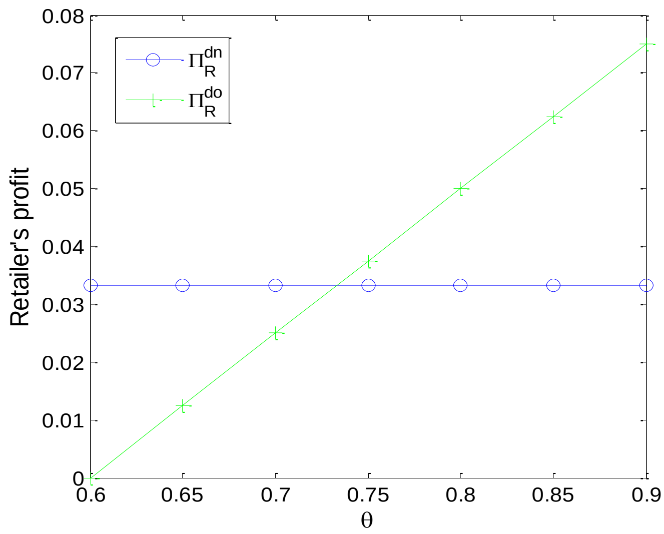

Figure 6. Impact of θ on retailer's profits under competition case.

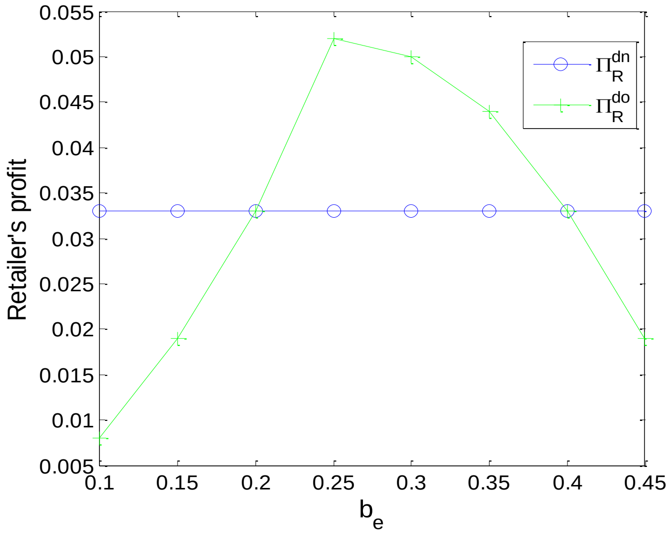

Figure 7. Impact of b_e on retailer's profits under competition case.

Zhang et al. [3] also studied whether the retailer should adopt the BOPS strategy under a competition case. The result of Zhang et al. [3] expressed that when the degree of customer acceptance of the online channel (θ) was lower than a threshold, the BOPS strategy was the best choice for the retailer. On the contrary, the result of our paper shows that only when the degree is higher than a threshold, the BOPS strategy may be benefit to the retailer. The main reason for the opposite result is the different pricing strategies. In the model of Zhang et al. [3], the retailer used "same product same price" strategy, which leaded to lower pricing in offline channels if the degree (θ) is lower. It means that when the retailer adopts non-BOPS strategy, his profit is also lower. Under this condition, when the retailer adopts the BOPS strategy, his profit will increase due to the increase of his market share. In our model, the retailer uses "same product different price" strategy, which means that under the non-BOPS strategy, the offline price is not affected by the degree (θ). Proposition 3 also confirms this conclusion. Hence, under the non-BOPS strategy, when the degree (θ) is lower, the offline profit can still be maintained at a normal level. Under this condition, the BOSP strategy is a double-edged sword for the retailer. On the one hand, the BOSP strategy may improve the retailer' market share and hence profit in the competition case. On the other hand, the lower degree leads to a lower online price, thus consumers will switch from offline to BOPS. Due to the lower online price, the BOSP strategy will hurt the retailer. Therefore, only when the degree is higher than a threshold, the former impacts the retailer's profit more significantly than the latter.

In addition, comparing Figures 3 and 4 with Figures 6 and 7, we can find that compared with monopoly, the BOPS strategy is more effective in competitive cases. It implies that with competition rising, the BOPS strategy can be a more effective competitive method for the retailer.

6. Conclusions

In this paper, we mainly focus on whether and when a dual-channel retailer should adopt the BOPS strategy in the "same product different price" strategy when the e-retailer is the leader in a Stackelberg game. Through the methodology of mathematical modeling, we obtain the following results and managerial insight implications.

First, we find that in the monopoly case, the dual-channel retailer can benefit from the BOPS strategy only if the offline operational cost is lower and the degree of customer acceptance of the online channel is high. It implies that a monopoly dual-channel retailer needs to strive to reduce the offline operational cost and the degree of customer acceptance of the online channel before adopting BOPS strategy. In practice, offline operational cost mainly includes store rent, employee cost, and marketing cost. The store rent is an exogenous variable and is not controlled by the retailer. Hence, the retailer mainly reduces offline operational costs by reducing employee cost and marketing cost. From the perspective of reducing employee cost, on the one hand, the retailer can improve employees' work efficiency through staff training, establishment of reasonable incentive mechanism, reasonable division of labor, and so on. On the other hand, the retailer can endeavor to reduce the number of employees. For example, to reduce the number of cashiers, many large supermarkets have established self-service checkout areas. When consumers buy products from online channels, they generally perceive the products only through pictures and videos. This is one of the main reasons why the degree of customer acceptance of the online channel is generally lower than offline channel. Hence, to improve this degree, the retailer can use technologies such as Virtual Reality to display products more comprehensively to enhance consumer perception. In addition, the retailer also can set a lenient online return policy to reduce shopping risks due to lack of physical inspection. Second, the results of this paper also show that in the competition case, if the value of the product is medium and transaction costs of the offline channel high, the BOPS strategy can enhance the retailer's market share. If the degree of customer acceptance of the online channel is also high simultaneously, the BOPS strategy can further benefit the retailer. It implies that not all retailers should adopt the BOPS strategy to enhance market share. In a physical store, customers need to spend time locating the desired items and standing in line to pay, etc., which is a major part of transaction costs of the offline channel. Hence, the transaction cost of large shopping places is generally high. Meanwhile, it also requires that the value of product sold by the retailer is medium. Therefore, in the competition case, retailers such as supermarkets or bookstores may be more suitable to adopt the BOPS strategy to enhance their market share. If the retailer wants to use the BOPS strategy to further improve profit, he also needs to strive to increase the degree of customer acceptance of the online channel. Lastly, by comparing the results with related earlier research under the monopoly case and competition case, we find that the retailer's price strategy ("same product same price" and "same product different price") has an important influence on the adoption of the BOPS strategy. It implies that the retailer needs to decide whether and when to adopt the BOPS strategy based on his own price strategy.

This paper still has several limitations. First of all, the results of this paper come from the analysis of mathematical models. The validity of the results needs further empirical verification. Second, this paper does not consider the influence of the upstream firms of the supply chain on the retailer's BOPS strategy. Finally, this paper only considers the competition between a dual-channel retailer and a pure e-retailer. To help overcome these limitations, there are a few interesting topics for further research. First, taking this paper as the starting point, empirical analysis is worthy of further research. Second, in real life, more and more manufacturers have established online shops as a direct channel in addition to an existing indirect retail channel. Under this supply chain structure, how will the retailer adopt the BOPS strategy? Last, if two competing dual-channel retailers are considered, how should they choose between the non-BOPS strategy and the BOPS strategy?

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Data Availability Statement: The data presented in this study are available in the article.

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Appendix A

Proof of Proposition 4.

With (22), we construct a Lagrange problem for the e-retailer:

Scenario 1. $p_t^{do} > 2\theta v - 3b_e - c_r$

$$L\left(p_t^{do}, \lambda_1,\ \lambda_2\right) = -\left(p_t^{do} - c_t\right)\frac{b_r - p_t^{do} - 2b_e + \theta v - l_e}{b_r} + \lambda_1\left(p_t^{do} - \theta v + b_e + l_e\right) + \lambda_2\left(2\theta v - 3b_e - c_r - p_t^{do}\right) \quad \text{(A1)}$$

where $\lambda_1\ \geq 0$ and $\lambda_2\ \geq 0$. We now discuss the following three cases:

Case 1. When $\lambda_1\ = 0$ and $\lambda_2\ = 0$

The second-order condition with respect to p_t^{do} is negative. $L\left(p_t^{do}, \lambda_1,\ \lambda_2\right)$ is concave in p_t^{do}. Taking the partial derivative of (A1) with respect to p_t^{do} and setting it to zero:

$$p_t^{do*} = \frac{b_r - 2b_e + \theta v + c_t - l_e}{2} \quad \text{(A2)}$$

$\lambda_1\ = 0$ and $\lambda_2\ = 0$ give that $p_t^{do} - \theta v + b_e + l_e \leq 0$ and $2\theta v - 3b_e - c_r - p_t^{do} \leq 0$. We then can attain $\theta \geq \frac{b_r+c_t+l_e}{v}$ and $\theta \leq \frac{4b_e+2c_r+b_r+c_t-l_e}{3v}$. We can easily check that $\frac{b_r+c_t+l_e}{v} \geq 1$, so there is no feasible solution.

Case 2. When $\lambda_1\ > 0$ and $\lambda_2\ = 0$

$\lambda_1 > 0$ gives $p_t^{do*}\ = \theta v - b_e - l_e$. Substituting p_t^{do*} into (A1), we can acquire $\lambda_1 = \frac{b_r-\theta v+c_t+l_e}{b_r}$. Because $\lambda_1\ > 0$, we can attain $\theta < \frac{b_r+c_t+l_e}{v}$. $\lambda_2\ = 0$ gives $2\theta v - 3b_e - c_r - p_t^{do}\ \leq\ 0$, we then obtain $\theta < \frac{2b_e+c_r-l_e}{v}$. To sum up, when $\frac{v-c_r+l_e}{2} < b_e < \theta v - c_e$ and $0 < \theta < 1$, or when $0 < b_e < \frac{v-c_r+l_e}{2}$ and $0 < \theta < \frac{2b_e+c_r-l_e}{v}$, we can obtain

$$p_t^{do*} = p_{ei}^{do*} = \theta v - b_e - l_e \quad \text{(A3)}$$

Case 3. When $\lambda_1\ = 0$ and $\lambda_2\ > 0$

$\lambda_2\ > 0$ gives $p_t^{do*}\ = 2\theta v - 3b_e - c_r$. Substituting p_t^{do*} into (A1), we can obtain $\lambda_2 = \frac{-b_r+3\theta v-4b_e-2c_r-c_t+l_e}{b_r}$. Because $\lambda_2\ > 0$, we can achieve $\theta > \frac{b_r+4b_e+2c_r+c_t-l_e}{3v}$. $\lambda_1\ = 0$ gives $p_t^{do} - \theta v + b_e + l_e\ \leq\ 0$, we then obtain $\theta < \frac{2b_e+c_r-l_e}{v}$. We can check that if and only if $\frac{2b_e+c_r-l_e}{v} > \frac{b_r+4b_e+2c_r+c_t-l_e}{3v}$. However, when $b_e > \frac{b_r+c_t-c_r+2l_e}{v}$, $\frac{b_r+4b_e+2c_r+c_t-l_e}{3v} > 1$. Therefore, there is no feasible solution.

Scenario 2. $p_t^{do} \leq 2\theta v - 3b_e - c_r$

$$L\left(p_t^{do}, \lambda_1,\ \lambda_2\right) = -\left(p_t^{do} - c_t\right)\frac{2b_r - p_t^{do} - b_e + c_r}{b_r} + \lambda_1\left(p_t^{do} - 2\theta v + 3b_e + c_r\right) + \lambda_2\left(p_t^{do} - b_e - c_r\right) \quad \text{(A4)}$$

Case 1. When $\lambda_1\ = 0$ and $\lambda_2\ = 0$

The second-order condition with respect to p_t^{do} is negative. $L\left(p_t^{do}, \lambda_1,\ \lambda_2\right)$ is concave in p_t^{do}. Taking the partial derivative of (A4) with respect to p_t^{do} and setting it to zero:

$$p_t^{do*} = \frac{2b_r - b_e + c_t + c_r}{2} \quad \text{(A5)}$$

$\lambda_1 = 0$ and $\lambda_2 = 0$ give that $p_t^{do} - 2\theta v + 3b_e + c_r \le 0$ and $p_t^{do} - b_e - c_r \le 0$. We can then obtain $\theta \ge \frac{2b_r+5b_e+c_t+3c_r}{4v}$ and $b_e \ge \frac{2b_r-c_r+c_t}{3}$. We can easily check that when $b_e \ge \frac{2b_r-c_r+c_t}{3}$, $\frac{2b_r+5b_e+c_t+3c_r}{4v} > 1$. Therefore, there is no feasible solution.

Case 2. When $\lambda_1 > 0$ and $\lambda_2 = 0$

$\lambda_1 > 0$ gives $p_t^{do^*} = 2\theta v - 3b_e - c_r$. Substituting $p_t^{do^*}$ into (A4), we can achieve $\lambda_1 = \frac{2b_r-4\theta v+5b_e+c_t+3c_r}{b_r}$. Because $\lambda_1 > 0$, we can $find$ $\theta < \frac{2b_r+5b_e+c_t+3c_r}{4v}$. $\lambda_2 = 0$ gives $p_t^{do} - b_e - c_r \le 0$, we then obtain $\theta < \frac{2b_e+c_r}{v}$. To sum up, when $\frac{v-c_r}{2} < b_e < \theta v - c_e$ and $0 < \theta < 1$, or when $0 < b_e < \frac{v-c_r}{2}$ and $0 < \theta < \frac{2b_e+c_r}{v}$, we can receive

$$p_t^{do^*} = 2\theta v - 3b_e - c_r \tag{A6}$$

$$p_{ei}^{do^*} = \theta v - b_e \tag{A7}$$

Because $p_{ei}^{do^*} \le \theta v - b_e - l_e$, this condition does not exist.

Case 3. When $\lambda_1 = 0$ and $\lambda_2 > 0$

$\lambda_2 > 0$ gives $p_t^{do^*} = b_e + c_r$. Substituting $p_t^{do^*}$ into (A4), we can attain $\lambda_2 = \frac{2b_r-3b_e-c_r+c_t}{b_r}$. Because $\lambda_2 > 0$, we can attain $b_e < \frac{2b_r-c_r+c_t}{3}$. $\lambda_1 = 0$ gives $p_t^{do} - 2\theta v + 3b_e + c_r \le 0$, we then obtain $\theta \ge \frac{2b_r+c_r}{4v}$. To sum up, when $0 < b_e < \frac{v-c_r}{2}$ and $\frac{c_r+2b_e}{v} < \theta < 1$, we can achieve

$$p_t^{do^*} = b_e + c_r \tag{A8}$$

$$p_{ei}^{do^*} = b_e + c_r \tag{A9}$$

Hence, we can reach the proposition.

Proof of Proposition 5.

Under the non-BOPS strategy, the retailer's demand is

$$Q_R^{dn} = Q_r^{dn} = \frac{v - c_r - l_r}{2b_r} \tag{A10}$$

Under the BOPS strategy, the retailer's demand is

$$Q_R^{do} = Q_{ei}^{do} = \frac{b_e}{b_r} \tag{A11}$$

We can attain $Q_R^{do} - Q_R^{dn} = \frac{2b_e-v+c_r+l_r}{2b_r}$. If $Q_R^{do} - Q_R^{dn} \ge 0$, we can obtain $b_e \ge \frac{v-c_r-l_r}{2}$. Therefore, we obtain proposition 5.

Proof of Proposition 6.

Under the non-BOPS strategy, the retailer's profits are

$$\Pi_R^{dn} = \frac{(v - c_r - l_r)^2}{4b_r} \tag{A12}$$

Under the BOPS strategy, the retailer's profits are

$$\Pi_R^{do} = \begin{cases} \frac{b_e^2}{b_r} & if\ 0 < b_e < \frac{v-c_r}{2} \text{ and } \frac{c_r+2b_e}{v} < \theta < 1 \\ (\theta v - b_e - c_r)\frac{b_e}{b_r} & otherwise \end{cases} \tag{A13}$$

Scenario 1. When $\Pi_R^{do} = \frac{b_e^2}{b_r}$

From (A12) and (A13), we can obtain

$$\Pi_R^{do} - \Pi_R^{dn} = \frac{(2b_e + c_r + l_r - v)(2b_e + v - c_r - l_r)}{4b_r} \tag{A14}$$

From (A14), we can find that $\Pi_R^{do} > \Pi_R^{dn}$ if $\frac{v-c_r-l_r}{2} < b_e < \frac{v-c_r}{2}$ and $\frac{c_r+2b_e}{v} < \theta < 1$.

Scenario 2. When $\Pi_R^{do} = (\theta v - b_e - c_r)\frac{b_e}{b_r}$

From (A12) and (A13), we can obtain

$$\Pi_R^{do} - \Pi_R^{dn} = \frac{4b_e(\theta v - b_e - c_r) - (v - c_r - l_r)^2}{4b_r} \quad \text{(A15)}$$

From (A15), we can find that $\Pi_R^{do} > \Pi_R^{dn}$ if and only if $> \frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v}$.

Case 1. When 0< $b_e < \frac{v-c_r}{2}$ and 0< $\theta < \frac{c_r+2b_e}{v}$

Under this case, only when $\frac{c_r+2b_e}{v} > \frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v}$, is there a feasible solution that can lead to $\Pi_R^{do} > \Pi_R^{dn}$. We can check that when 0< $b_e < \frac{v-c_r-l_r}{2}$ and 0< $\theta < \frac{c_r+2b_e}{v}$, $\frac{c_r+2b_e}{v} < \frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v}$, thus the retailer will reduce profits by using BOPS under this condition. We can check that when $b_e > \frac{v-c_r-l_r}{2}$, there exists a feasible solution that can lead to $\Pi_R^{do} > \Pi_R^{dn}$. Under this condition, when $\frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v} < \theta < \frac{c_r+2b_e}{v}$, the BOPS strategy will increase the retailer's profits. When 0< $\theta < \frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v}$, the BOPS strategy will decrease the retailer's profits.

Case 2. When $b_e > \frac{v-c_r}{2}$ and $0 < \theta < 1$

Under this case, only when $\frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v} < 1$, is there a feasible solution that can lead to $\Pi_R^{do} > \Pi_R^{dn}$. We can check that when $\frac{(v-c_r)+\sqrt{(v-c_r)^2-(v-c_r-l_r)^2}}{2} < b_e < \theta v - c_e - l_e$ and $0 < \theta < 1$, $\frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v} > 1$, so the retailer will reduce profits by using BOPS under this condition. We can check that when $b_e < \frac{(v-c_r)+\sqrt{(v-c_r)^2-(v-c_r-l_r)^2}}{2}$, there exists a feasible solution that can lead to $\Pi_R^{do} > \Pi_R^{dn}$. Under this condition, when $\frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v} < \theta < 1$, the BOPS strategy will increase the retailer's profits. When 0 $< \theta < \frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v}$, the BOPS strategy will decrease the retailer's profits.

To sum up, we can find that when $\frac{(v-c_r-l_r)^2+4b_e(b_e+c_r)}{4b_e v} < \theta < 1$ and $\frac{v-c_r-l_r}{2} < b_e < \frac{(v-c_r)+\sqrt{(v-c_r)^2-(v-c_r-l_r)^2}}{2}$, the BOPS strategy will increase the retailer's profits. Otherwise, the retailer will reduce profits by using BOPS.

Hence, we can obtain the proposition.

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