Article
Competitive Policy for Online Retailers’ Intrusion in E-Commerce
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Abstract: In recent years, online retail has developed rapidly. However, as consumer demands become increasingly sophisticated, the traditional online retail model has encountered difficulties with respect to meeting consumers’ needs. As a result, numerous retailers with offline physical stores have emerged in the online retail industry. This paper constructs a game model for the invasion of the market by e-commerce retailers with offline physical stores in a context in which a traditional, online-only incumbent retailer is already in the market. Compared with the new entrant, the incumbent has the advantage of an established good reputation, and consumers prefer the products of the incumbent. This research shows that as consumers’ product valuations increase, the following three situations may occur: a partially covered market, a multiple-equilibrium market or a fully covered market. In a partially covered market, the incumbency advantage does not affect the entrant. In a multiple-equilibrium market or fully covered market, the incumbency advantage impacts the profits of the entrant. However, in a fully covered market, if the incumbency advantage is too large, the profits of both retailers are damaged. Finally, this paper finds that offline physical stores can provide positive benefits to entrants. When consumers are highly sensitive to services, opening offline physical stores is an effective intrusion strategy that entrants can use to overcome the incumbency advantage.

Keywords: retailer invasion; market type; offline physical stores; incumbency advantage

1. Introduction

The profound development of the Internet has provided new opportunities to various retail industries. Many retailers have opened online stores to sell products, and online retail has developed rapidly. The changes in retail resulting from the COVID-19 pandemic have also promoted the development of online sales. According to a report by e-Marketer, global e-commerce sales accounted for 18% of total global retail sales in 2020, and this proportion is expected to increase to 21.8% by 2024. Global online retail sales are expected to exceed five trillion dollars in 2022 and may reach six trillion dollars in 2024. These data indicate that online retail is becoming a very large trading market, and an increasing number of consumers choose to purchase goods directly online. Under such circumstances, an increasing number of retail companies have begun to enter the market by engaging in online retail, and there is fierce competition among online retailers [1]. However, due to the gradual increase in consumers’ demands regarding online shopping, the traditional online retail model can no longer satisfy consumers’ needs easily [2]. Therefore, many retail e-commerce companies have begun to develop offline businesses, and a new retail model integrating both online and offline sales has gradually emerged in the retail market [3]. In June 2017, Amazon acquired Whole Foods for 13.7 billion dollars, which features more than 400 stores in prime locations across the United States and which is associated with high-value resources, such as locations in malls in high-income areas, thereby providing strong support for Amazon’s offline development strategy. This new retail model, which combines the dual advantages of low-cost online operations and high-quality offline services, can inspire new ideas regarding the development of retail companies [4].
Therefore, some companies have begun to open offline stores to meet the expanding needs of consumers. For example, in 2016, Lincherie, a Dutch clothing brand, opened an offline store in Amsterdam under the slogan “you can only try and not buy”; this store allows customers to try on clothes and place orders on digital devices. Northstorm, a high-end department store in the United States, also opened a “no-sales” physical store, which mainly provides services such as personal styling, clothing modification, in-store pickup, and order modification. In recent years, many retail e-commerce companies have opened physical stores in an offline environment in China, such as Super Species, MISSFRESH, and 7 FRESH. These new types of retail companies rely on a combination of online retail platforms with offline physical stores to improve consumers’ shopping experiences.

Although the new retail model of online–offline integration has developed rapidly in recent years, the traditional e-commerce model remains an option used by many enterprises when entering the retail market due to the low operating costs and entry barriers associated with this approach. Pinduoduo, one of the largest e-commerce platforms in China, is a successful example of online retail. Pinduoduo has developed rapidly since it entered the online market. It went online in 2015 and was listed in the United States in July 2018 with a market value of 24 billion dollars [https://www.sohu.com/a/243548176_430392 (accessed on 1 October 2022)], indicating its success in the online retail field. Similar businesses that rely on pure online retail are also doing well, indicating that online retail still has a role to play in e-commerce. At present, consumers are concerned about the price and quality of products as well as the services provided by sellers [5,6]. Consumers may face search costs or experience costs when purchasing goods both online and offline [7,8].

However, due to the convenience and accessibility of the internet, consumers can quickly compare products from different stores online and communicate with other consumers via social networks [9]. In addition, consumers may have different preferences for retailers, which is consistent with the literature indicating that many consumers form habits and demonstrate a willingness to repeatedly purchase products from the same store [10]. When faced with different retailers, consumers tend to choose incumbent companies, i.e., those that entered the market first [11]. Compared with new entrants, incumbents have certain advantages because they have established good reputations in the market or have many loyal consumers [12]. This incumbent advantage may affect an entrant’s intrusion strategy.

Retail companies that are new to the market can consider adopting either the online–offline combined retail model or the pure online retailing model. The online–offline integrated model includes both online and offline sales channels. While selling products online, this model can also rely on offline stores to provide consumers with more comprehensive experience services to meet their increasingly high-level shopping needs [13]. However, pure online retailing can provide a broad market for new entrants that rely on large amounts of network traffic, so this approach remains an option for them [14]. Therefore, the ways in which new entrants can choose a better market intrusion strategy are worth studying.

Against this backdrop, this paper conducts research on the intrusion strategies used by retail enterprises entering the market. The main research questions are as follows:

1. When a pure online retailer is the incumbent, how should a new entrant choose between the pure online retail intrusion strategy and the new retail model of online–offline integration?
2. How does the incumbency advantage affect consumers’ sensitivity to services, and how do different types of market coverage affect entrants’ choice of intrusion strategies?

Against the backdrop of retail model innovation, this paper studies a duopoly game model in which a new entrant invades the market. Since this problem has not been explored fully in the extant literature, this paper uses the Hotelling model to study the market invasion scenario under the new retail model in further detail. On the one hand, the conclusions of this paper expand the research content of the duopoly game model; on the other hand, they provide decision-making suggestions for market invasion under the new retail model. The research methods used in, and the conclusions obtained by, this
paper can inspire new ideas in the context of e-commerce research and enrich the research content of the duopoly game scenario of retail model innovation.

Therefore, this paper produced certain related contributions. First, when studying competition among retail enterprises, most studies have focused on factors such as product quality and services [15,16] but they have not considered the impact of market coverage types on entrants’ strategic choices. This paper used the Hotelling model to obtain the following three types of market coverage: partially covered markets; multiple equilibrium markets; and fully covered markets. The invasion results were divided into scenarios for regional analysis, and the influence of market coverage type on the choice of intrusion strategy was studied in depth. Second, extant research on the manner in which online retail enterprises invade the market has been relatively simple, focusing mainly on the pure online retail strategy [17,18]. In the context of contemporary e-commerce, this paper considered two feasible strategies that retail companies can employ to enter the market, namely, the pure online retail model and the integrated online–offline model. In addition, this paper analyzed the influence of the incumbency advantage and offline services on entrants’ choice of strategy.

2. Literature Review

The retail market constantly faces the threat of new retail e-commerce entrants, and the issue of intrusion has always attracted academic attention [19,20]. This section reviews the literature related to this topic, which can be divided into the following two streams, with one stream focusing on the entrant’s invasion mode and the other stream focusing on the incumbency advantage. For this study, relevant research on the invasion mode can be divided specifically into the pure online retail and the integrated online–offline models.

2.1. Traditional Online Retail Model

Due to the emergence of the online retail mode, a new scenario has emerged for the intrusion of retail enterprises. Liu et al. (2006) studied market invasion in the context of a retailer using the pure online model [21]. The incumbent is a physical retailer and can consider whether to add an online retail channel to deal with potential online retailer entrants. Some scholars have studied the factors that affect online retail. Kim and Krishnan (2019) developed a Markov model to study the impact of price promotion on customer loyalty [22]. These authors found that, unlike offline marketplaces, online retailers focused more on recent changes in consumer buying behaviour than on long-term trends. Feng et al. (2019) found that online reviews can influence other consumers’ purchasing decisions and that online retailers can influence online product reviews via pricing, thus affecting sales [23]. Sun et al. (2020) analyzed the effectiveness of recommendation searches and social media endorsements in the context of online retail [24]. These authors found that both methods significantly increased traffic for online retailers but that recommendation searches were more effective than social media endorsements. Many scholars have proposed that the available information and shopping experience can affect online retailers’ sales [25,26]. Kumar et al. (2019) studied the impact of online stores opened by large clothing retailers on existing customers [20]. The results showed that after these retailers opened online stores, customers’ online shopping increased. They believed that customers make more online purchases due to the high level of engagement facilitated by store interactions [17]. Wu and Cosguner (2020) empirically examined and quantified the sales of a leading online retailer [27]. These authors developed a risk framework for online retail and further simulated the impact of different dominant scenarios on retail profits.

This stream of literature has focused on the intrusion of online retail enterprises, and these studies have shown that in the online market, consumers pay more attention to their perceptions of products and the services that they can receive. This paper takes this factor into account and examines its effect on the outcome of the game played between retail enterprises.
2.2. Online–Offline Integrated Retail Model

Online–offline integrated retail is a new retail model that has emerged in recent years, which has injected new vitality into the retail market; however, research on this new retail model remains in its infancy. Arya and Mittendorf (2018) proposed a physical store selection model for online retailers [28]. These authors believed that offline channels allow online retailers to reach new customers, thereby giving them an edge over their competitors. Harsha et al. (2019) studied price optimization in the context of the cross-channel interaction of supply and demand, found that when the demand of offline stores is lower than expected but the online demand is higher than forecasted, omnichannel retailers can meet excess online demands effectively by utilizing the unused inventory of physical stores [29]. Similarly, Gao and Su (2018) analyzed a situation in which retailers operate in both online and offline channels and found that online and offline channels can complement each other and enhance the shopping experiences of consumers [30]. These authors found that offline stores and online channels can work together to achieve a win–win situation.

When consumers purchase goods online, they often care about the quality of the goods or their degree of fit. Gao and Su (2017) studied the ways in which retailers can deliver online and offline information effectively to omnichannel consumers who strategically choose whether to collect information online or offline and whether to buy products online or offline [31]. These authors found that offline showrooms may prompt retailers to reduce their inventory, thus increasing availability risk while potentially also increasing online retail profits. Li et al. (2021) studied the influence of online consumer reviews on pricing and new product design strategies in the online-to-offline (O2O) supply chain and found that if the consumer valuation of new products is not sufficiently high, platform sellers are more inclined to limit this influence but also that manufacturers can benefit from this situation [1]. Sardar and Sarkar (2021) discussed the use of advanced radio frequency identification technology (RFID) by retailers to obtain accurate information regarding customer needs and solved the problem of online sales unreliability through the use of different game strategies and advanced technologies [32]. Sarkar et al. (2021) established a price-dependent demand model and found when a product in this model is sold online and offline simultaneously, the offline price is slightly higher than the online price; however due to the quality of the service, some customers prefer to purchase specific products in physical stores [3]. Online retailers provide various services via offline channels, such as commodity displays, offline experiences and receipts, which can mitigate the defects of pure online retail information transmission and improve consumers’ purchase intentions and loyalty [33,34].

Meanwhile, in the online retail market, consumers’ evaluations affect retailers’ sales and profits [35]. Dzyabura and Jagabathula (2019) found that companies are increasingly inclined to sell products both offline and online, thus allowing customers to experience the product before purchasing it [36]. Bell et al. (2018) introduced the concept of offline showrooms, which are physical spaces in which customers can view and try products [37]. They found that such a showroom increases overall demand and helps increase brand awareness and channel awareness. Dzyabura et al. (2018) compared consumer reviews of products online and offline and found that the most valuable parameters vary significantly between online and offline studies [8].

Some scholars have noted that online retailers who open experience-oriented brick-and-mortar stores offline can experience increased sales [38–40]. Offline stores can improve the competitiveness of online retailers in the market [41–43]. Bell et al. (2020) proposed that for online retailers, the experience-centered offline store model can not only expand market coverage but also significantly enhance future positive customer behavior both online and offline [13]. By reference to an analysis of an actual store, these authors found that after encountering the zero-inventory store experience, consumers spent more money, made purchases more quickly and were less likely to return goods. Simultaneously, the opening of offline experience stores also entails certain costs, and online retailers must consider whether they should open offline experience stores in full detail [44,45]. This
A stream of literature has shown that the retail model that integrates both online and offline retail satisfies the need of consumers to perceive or experience products, which cannot be met by pure online retail. The literature has also discussed the various services that offline physical stores can provide, as well as their influence [31]. However, these studies have not taken the retail model of online–offline integration into account as an alternative intrusion strategy. Based on these studies, this paper conducted a comparative study of the pure online retail and the online–offline retail models and considered the influence of market coverage type on the enterprise’s choice of an intrusion strategy.

2.3. Incumbent Advantage

In intrusion research, many scholars have proposed that the incumbent has certain advantages over the entrant [46]. Carpenter and Nakamoto (2005) proposed that consumers exhibit different responses to similar products that are provided by dominant brands and those that are offered by later entrants, which is known as the asymmetric preference effect [11]. The analysis by these authors showed that preference asymmetry can promote the sustained competitive advantage of dominant brands. Chakravarthi and Zhang (2017) established a game theory model to analyze the most basic factors affecting an enterprise’s decision to enter the market, including the effect of the entry sequence [47]. These authors believed that consumers prefer products that enter first. Chen and Grewal (2013) considered the problem of a manufacturer that supplies products to two competing retailers that entered the market successively [12]. One retailer has established a good reputation in the market, while the other retailer is new to the market. Selove (2014) considered the influence of the incumbent’s advantage and incorporated consumers’ preference for the incumbent’s products as an assumption [48]. These authors believed that incumbents usually have many advantages, such as established retail stores and expertise in the use of up-to-date technology.

This paper discusses the advantages of incumbents in terms of consumer differences at the product level [47]. Different consumers have differentiated preferences; that is, based on whether consumers buy products from incumbents or entrants, they pay mismatched costs. The advantage of the incumbent is that, at the same level of mismatch, the mismatched cost that consumers pay for the incumbent’s products is lower than the cost that they pay for the entrant’s products.

The review indicates that the previous literature on market invasion by online retail companies has not taken the impact of market coverage types on entrants’ strategic choices into consideration [49,50]. Based on the research concerning the two entry strategies, this paper analyzes the influence of three types of market coverage, namely, partially covered markets, multiple equilibrium markets and fully covered markets, on the decision-making of invading enterprises. The research on online retailer intrusion has mostly focused on one intrusion mode and has not considered the situation in which intrusion enterprises can choose from multiple intrusion strategies [28].

3. Problem Description and Basic Assumptions

This paper considers a situation in which an online retailer R₁ (known as the incumbent R₁) in the market sells a product to consumers, while another new retailer R₂ (known as the new entrant R₂) invades the market and competes with the incumbent by offering a homogeneous product. To compete with the incumbent R₁, the new entrant R₂ opens not only an online retail store using the internet but also a physical store that features the same prices as the online store to provide positive utility to consumers by displaying products or providing various services.

In this paper, the Hotelling model is used to establish the analytical framework of the game. It is assumed that two retailers R₁ and R₂ are located at the two endpoints of a line segment with a length of 1 in the Hotelling model, that is, R₁ and R₂ are located at positions 0 and 1, respectively. Based on their preferences regarding the difference in product level between R₁ and R₂, consumers in the market are evenly distributed between
0 and 1, and the position of any consumer is assumed to be \( x, x \in [0,1] \). Since \( R_1 \) is an incumbent and has established a good reputation, consumers are more familiar with \( R_1 \)'s products. Although \( R_2 \) is an entrant to the market, it is assumed that consumers who purchase products from \( R_1 \) will have a greater mismatch than those who purchase products from \( R_2 \). This article defines a sensitivity advantage for level differences \( \alpha, \alpha \in [0,1] \). \( \alpha \) is used to indicate the advantage of \( R_1 \) with regard to the preferences of consumers with product level differences. That is, when the consumer at \( x \) purchases a product from \( R_1 \), the cost is \( \alpha x \), and the cost of purchasing the product from \( R_2 \) is \( t(1-x) \), where \( t \) is the consumer’s sensitivity to product level differences [51,52].

Assuming that consumers’ valuations of the products offered by the two retailers are both \( v \), then the utility that the consumer obtains by purchasing the products of incumbent \( R_1 \) is \( U_1 = v - \alpha x - p_1 \). The new entrant \( R_2 \) can provide a certain level of offline service \( s \) by opening a physical store, which improves consumers’ product awareness and shopping experience [53]. Assuming that the sensitivity of consumers to the offline service level is \( \beta, \beta \in [0,1] \), then the utility obtained by consumers purchasing \( R_2 \)'s products is \( U_2 = v - t(1-x) - p_2 + \beta s \). Table 1 provides a set of definitions for the symbols, parameters, and decision variables used in this paper.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>( R_i )</td>
<td>Retailer ( i = 1,2; R_1 ) is the incumbent and ( R_2 ) is the new entrant</td>
</tr>
<tr>
<td>( U_i )</td>
<td>Utility that a consumer obtains by buying a product from retailer ( i = 1,2 )</td>
</tr>
<tr>
<td>( D_i )</td>
<td>Demand of retailer ( i, i = 1,2 )</td>
</tr>
<tr>
<td>( p_i )</td>
<td>Price of retailer ( i, i = 1,2 )</td>
</tr>
<tr>
<td>( c_i )</td>
<td>Unit production cost of retailer ( i, i = 1,2 )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Consumer’s valuation of the product</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Sensitivity of consumers to the level of offline service</td>
</tr>
<tr>
<td>( t )</td>
<td>Sensitivity of consumers to differences in product level</td>
</tr>
<tr>
<td>( s )</td>
<td>Level of offline service provided by ( R_2 )</td>
</tr>
<tr>
<td>( \pi_i )</td>
<td>Profit of retailer ( i, i = 1,2 )</td>
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</table>

It is assumed that consumers choose to purchase one unit of a product or to not to purchase any product from either the incumbent \( R_1 \) or the new entrant \( R_2 \) [19]. When \( R_2 \) invades the market, since its level of offline service in physical stores is \( s \), referring to Wu (2012) and Ma et al. (2016), the cost of providing offline services is \( C(s) = s^2/2 \) [54,55]. Meanwhile, to simplify the calculation, it is assumed that the operating costs of both retailers are \( c_i = 0 \) and that the sensitivity of consumers to product level differences is \( t = 1 \) [52]. In addition, assuming that the rights of the two retailers are equal, the two parties use the Nash game to make decisions. Subsequently, the equilibrium result of the game between the two retailers is solved.

4. Model

When the new entrant \( R_2 \) invades the market, if \( U_1 = v - \alpha x - p_1 > 0 \), that is, when the distance between consumers and the product level of \( R_1 \) satisfies \( x < (v-p_1)/\alpha \), consumers can obtain positive utility from the product offered by \( R_1 \). If \( U_2 = v - (1-x) - p_2 + \beta s > 0 \), namely, when the distance between consumers and the product level of \( R_2 \) meets \( 1-x < v - p_2 + \beta s \), consumers can obtain positive utility from the product offered by \( R_2 \). When the two retailers’ products provide equal utility to consumers, \( v - \alpha x - p_1 = v - (1-x) - p_2 + \beta s \), and so the consumer position is \( x = (1-p_1 + p_2 - \beta s)/(1+\alpha) \).

It can be proven that there is a threshold \( \delta = (p_1 + \alpha (p_2 - \alpha \beta s))/(1+\alpha) \). When \( v < \delta \), the consumers’ valuation of the product is too low, and the market is partially covered. This paper defines this type of market as a partially covered market, which is indicated by \( Pa \). This situation indicates that although the products offered by the two retailers in the market are identical, they have corresponding
consumer groups. In other words, the incumbent $R_1$ retains his old customers, while
the new entrant $R_2$ attracts new customers, thus producing a situation of differentiated
competition. In this case, the demand functions of the two retailers are as follows:

$$D_{Pa}^1 = (v - p_{Pa}^1) / \alpha. \quad (1)$$

$$D_{Pa}^2 = v - p_{Pa}^2 + \beta s_{Pa}. \quad (2)$$

However, when $v > \hat{v}$, consumers’ valuations of products can cover the market
completely, and this type of market is defined as a fully covered market, which is indicated
by $Fu$. This situation suggests that the customers of the incumbent $R_1$ and those of the new
entrant $R_2$ can shift allegiances, so the competition between the two retailers is fierce. In
this case, the demand functions of the two retailers are as follows:

$$D_{Fu}^1 = \frac{1 - p_{Fu}^1 + p_{Fu}^2 - \beta s_{Fu}}{1 + \alpha}. \quad (3)$$

$$D_{Fu}^2 = \frac{p_{Fu}^1 - p_{Fu}^2 + \alpha + \beta s_{Fu}}{1 + \alpha}. \quad (4)$$

After obtaining the demand functions of the two retailers in the two situations dis-
cussed above, their decision results are analyzed below.

4.1. The Partially Covered Market

In the game sequence used in this study, a Nash game takes place between the in-
cumbent retailer and the entrant retailer; that is, both competitors decide their retail prices
simultaneously, and consumers make purchase decisions after becoming aware of the
product prices of both parties.

When $v < \hat{v}$, the profit functions of the two retailers are as follows:

$$\pi_{Pa}^1 = \frac{(v - p_{Pa}^1)p_{Pa}^1}{\alpha}. \quad (5)$$

$$\pi_{Pa}^2 = (v - p_{Pa}^2 + \beta s_{Pa})p_{Pa}^2 - C(s_{Pa}). \quad (6)$$

In this situation, the equilibrium prices of $R_1$ and $R_2$ can be calculated as follows:

$$p_{Pa}^{1*} = v / 2$$

$$p_{Pa}^{2*} = v / (2 - \beta^2)$$

The equilibrium service level of $R_2$ is as follows:

$$s_{Pa}^{*} = v \beta / (2 - \beta^2)$$

Accordingly, the equilibrium demand and profit of the two retailers can be further
obtained as follows:

$$D_{Pa}^{1*} = v / 2\alpha$$

$$D_{Pa}^{2*} = v / (2 - \beta^2)$$

$$\pi_{Pa}^{1*} = v^2 / 4\alpha$$

$$\pi_{Pa}^{2*} = v^2 / (2 - \beta^2)$$

By substituting the equilibrium result into $\hat{v}$, it can be concluded that the threshold
value is $\hat{v} = 2\alpha (2 - \beta^2) / (2 + 2\alpha - \beta^2)$, which is denoted as $\hat{v}_1$.

Lemma 1 can be obtained by comparing the equilibrium results.
Lemma 1. The incumbent $R_1$ and the new entrant $R_2$ compete by offering homogeneous products. When $v < \hat{v}_1$ is satisfied, i.e., when the market is partially covered, the following results can be obtained:

1. $p_1^{Pa*} \leq p_2^{Pa*}$;
2. There exists a threshold $\beta^{Pa*} = \sqrt{2(1 - \alpha)}$; when the sensitivity of consumers to the offline service level of new entrant $R_2$ is $\beta \leq \beta^{Pa*}$, then $D_1^{Pa*} \geq D_2^{Pa*}$ and $\pi_1^{Pa*} \geq \pi_2^{Pa*}$; otherwise, $D_1^{Pa*} < D_2^{Pa*}$ and $\pi_1^{Pa*} < \pi_2^{Pa*}$.

Proof. Solve for $\pi_1^{Pa*} - \pi_2^{Pa*} = \frac{v^2}{4\alpha} - \frac{\beta^2}{8(1-\beta^2)} = \frac{v}{2} (D_1^{Pa*} - D_2^{Pa*}) = \frac{\alpha}{2} \left( 1 - \frac{1}{2-\beta^2} \right)$ with the condition $\beta > 0$, the threshold $\beta^{Pa*} = \sqrt{2(1 - \alpha)}$ is obtained.

Lemma 1 shows that when consumer utility satisfies $v < \hat{v}_1$, the product price of the new entrant $R_2$ is higher than that of the incumbent $R_1$. Obviously, in the partially covered market, no direct competition takes place between the two retailers, and the two retailers each set prices based on their own consumer groups. That is, as existing customers prefer incumbent $R_1$, the incumbent maintains this customer group due to this advantage, while the new entrant $R_2$ attracts new customer groups by using its newly established offline services. The new entrant $R_2$ improves the consumer shopping experience by opening physical stores offline and thus gains the ability to set higher prices. However, $R_2$ must also pay some service costs to provide such services to its consumers. Therefore, when consumers are less sensitive to the offline service level, that is, when $\beta \leq \beta^{Pa*}$, incumbent $R_1$ has an advantage in terms of both demand and profit. Otherwise, $R_2$ has this type of advantage. In this scenario, consumers’ sensitivity to the offline service level is a key factor.

4.2. The Fully Covered Market

As mentioned above, when the consumers’ utility meets $v > \hat{v}_2$, the two retailers’ markets have been completely covered; that is, consumer shifting and competition have begun to emerge. In this case, the profit functions of the two retailers are as follows:

$$\pi_1^{Fu} = \frac{(1 - p_1^{Fu} + p_2^{Fu} - \beta s^{Fu}) p_1^{Fu}}{1 + \alpha}.$$ (7)

$$\pi_2^{Fu} = \frac{(p_1^{Fu} - p_2^{Fu} + \alpha + \beta s^{Fu}) p_2^{Fu}}{1 + \alpha} - C(s^{Fu}).$$ (8)

Taking the first derivative of (7) with respect to $p_1^{Fu}$, taking the first derivative of (8) with respect to $p_2^{Fu}$ and $s$, and letting the first derivatives shown above equal to zero, the equilibrium pricing of $R_1$ and $R_2$ in a fully covered market is obtained as follows:

$$p_1^{Fu*} = \frac{2 - \beta^2 + \alpha(3 + \alpha - \beta^2)}{3 + 3\alpha - \beta^2}, \quad p_2^{Fu*} = \frac{(1 + \alpha)(1 + 2\alpha)}{3 + 3\alpha - \beta^2}.$$

The equilibrium service level of $R_2$ is $s^{Fu*} = \frac{\beta(1+2\alpha)}{3+3\alpha-\beta^2}$.

Proof. Clearly, the profit function $\pi_1^{Fu}$ is a concave function of $p_1^{Fu}$.

The Hessian matrix of $\pi_1^{Fu}$ is as follows:

$$H = \begin{pmatrix}
\frac{\partial^2 \pi_1^{Fu}}{\partial p_1^{Fu}^2} & \frac{\partial^2 \pi_1^{Fu}}{\partial p_1^{Fu} \partial s^{Fu}} \\
\frac{\partial^2 \pi_1^{Fu}}{\partial p_2^{Fu} \partial s^{Fu}} & \frac{\partial^2 \pi_1^{Fu}}{\partial s^{Fu}^2}
\end{pmatrix} = \begin{pmatrix}
-\frac{2}{1+\alpha} & \frac{\beta}{1+\alpha} \\
\frac{\beta}{1+\alpha} & -1
\end{pmatrix}.$$
The first-order principal subexpression of $H$ is $H_1 = -\frac{2}{1+\alpha} < 0$, and the second-order principal subexpression is $H_2 = \frac{2+\beta^2}{1+\alpha} > 0$, so $H$ is a negative definite matrix. Thus, the solution indicated above is optimal.

In this case, the demand and profit of the two retailers are as follows:

\[
D_{1u}^k = \frac{2+\alpha-\beta^2}{3+3\alpha-\beta^2}, \\
D_{2u}^k = \frac{1+2\alpha}{3+3\alpha-\beta^2}, \\
\pi_{1u}^k = \frac{(1+\alpha)(2+\alpha-\beta^2)^2}{(3+3\alpha-\beta^2)^2}, \\
\pi_{2u}^k = \frac{(1+2\alpha)^2(2+2\alpha-\beta^2)}{2(3+3\alpha-\beta^2)^2}.
\]

By substituting the equilibrium results shown above into threshold $\hat{\alpha}$, the following can be obtained: $\hat{\alpha} = \frac{(1+\alpha)(2+\alpha-\beta^2)}{3+3\alpha-\beta^2}$. Lemma 2 can be obtained by comparing the equilibrium results.

**Lemma 2.** The incumbent $R_1$ and the new entrant $R_2$ compete by offering homogeneous products. When consumer utility meets $\nu > \hat{\nu}_2$, that is, when the market is fully covered, there are two thresholds for the sensitivity of consumers to $R_2$'s level of offline service, and the following conclusions can be drawn.

1. When $\beta \leq \beta_{1u}^k$, $D_{1u}^k \geq D_{2u}^k$ and $p_{1u}^k \geq p_{2u}^k$; otherwise, $D_{1u}^k < D_{2u}^k$ and $p_{1u}^k < p_{2u}^k$.

2. When $\beta \leq \beta_{2u}^k$, $\pi_{1u}^k \geq \pi_{2u}^k$; otherwise, $\pi_{1u}^k < \pi_{2u}^k$.

Note that $\beta_{1u}^k = \sqrt{1-\alpha}$, $\beta_{2u}^k = \sqrt{\frac{7+8\alpha-(1+2\alpha)\sqrt{1+12\alpha+12\alpha^2}}{4(1+\alpha)}}$, and $\beta_{1u}^k < \beta_{2u}^k$.

**Proof.** Since $p_{1u}^k - p_{2u}^k = \frac{2-\beta^2+\alpha(3+\alpha-\beta^2)}{3+3\alpha-\beta^2} - \frac{(1+\alpha)(1+2\alpha)}{3+3\alpha-\beta^2} = (1+\alpha)(D_{1u}^k - D_{2u}^k)$ and $\beta > 0$, the threshold in lemma 2(1) is $\beta_{1u}^k = \sqrt{1-\alpha}$. Similarly, it can be concluded from $\pi_{1u}^k - \pi_{2u}^k = \frac{(1+\alpha)(2+\alpha-\beta^2)^2}{(3+3\alpha-\beta^2)^2} - \frac{(1+2\alpha)^2(2+2\alpha-\beta^2)}{2(3+3\alpha-\beta^2)^2} = \frac{2(1+\alpha)^2(7+8\alpha)^2+6(1-\alpha)(1+\alpha)^2}{2(3+3\alpha-\beta^2)^2}$ and $0 < \beta < 1$ that the threshold in lemma 2(2) is $\beta_{2u}^k = \sqrt{\frac{7+8\alpha-(1+2\alpha)\sqrt{1+12\alpha+12\alpha^2}}{4(1+\alpha)}}$.

Lemma 2 shows that in a fully covered market, when consumers are less sensitive to offline services (i.e., $\beta \leq \beta_{1u}^k$), $R_1$ can set a higher price and obtain higher demand due to its long-term dominance. Otherwise, the demand and price of $R_1$ are lower than those of $R_2$. Regarding the revenue of the two retailers, there is a threshold $\beta_{2u}^k$ regarding consumers' sensitivity to offline services. When $\beta > \beta_{2u}^k$, namely, when consumers are more sensitive to offline services, $R_2$ can use the service experience of offline physical stores to obtain higher profits.

Figure 1 shows the trend curves of $\beta_{1u}^k$ and $\beta_{2u}^k$. By reference to Lemma 2, it can be concluded that below the curve $\beta_{1u}^k$, the product price, demand, and profit of $R_1$ are all higher than those of $R_2$. As Figure 1 shows, below the curve, $\beta_{1u}^k$ indicates the area where $\alpha$ and $\beta$ are smaller, so $R_1$ can set a higher price and obtain higher demand and profit. In contrast, above curve $\beta_{2u}^k$, both $\alpha$ and $\beta$ are large, the advantage of $R_1$ is weak in this case, and consumers are more sensitive to the service, so $R_2$ can obtain higher profits. $\hat{\beta}$ is set at a medium level in the area between curves $\beta_{1u}^k$ and $\beta_{2u}^k$. $R_2$ can set a higher price and obtain higher demand, but $R_2$ faces a certain cost due to the need to provide services, so the profit of $R_2$ in this area remains lower than that of $R_1$. In addition, when the advantage of $R_1$ is sufficiently large, i.e., when $\alpha < \hat{\alpha}$, the profit of $R_1$ in a competitive market is always be greater than that of $R_2$. 


are as follows: 

\[ \beta_1 \] and \( \beta_2 \) is located at \( x \) in this case has multi-

\( x \) is the indifference point that is indicated by \( M_u \) resulting from the outcome of this multiple game as a multiple equilibrium market, which is indicated by \( M_u \) in this paper refers to the market situation resulting from the outcome of this multiple game as a multiple equilibrium market, which is indicated by \( M_u \).

Therefore, for any indifference point \( x \in [1 - v/(2 - \beta^2), v/2\alpha] \), the equilibrium price of \( R_1 \) is \( p_{M_u}^{R_1} = v - \alpha x \), and the equilibrium price of \( R_2 \) is \( p_{M_u}^{R_2} = v - (1 - x) \). In this context, the equilibrium demands of the two retailers are as follows:

\[
\begin{align*}
D_{M_u}^{R_1} &= x, \\
D_{M_u}^{R_2} &= 1 - x.
\end{align*}
\]

In this model setting, it is known that \( R_1 \) has a preference advantage \( \alpha \) over \( R_2 \). It is assumed that this asymmetric problem setting affects multiple equilibrium results, i.e., \( x \) is located at \( 1/(1 + \alpha) \) of \( [1 - v/(2 - \beta^2), v/2\alpha] \) and \( x = \frac{2\alpha v}{2\alpha v + (2 - \beta^2)} \). Therefore, this paper refers to the market situation resulting from the outcome of this multiple game as a multiple equilibrium market, which is indicated by \( M_u \).

In this scenario, the equilibrium pricing of \( R_1 \) is as follows:

\[
p_{M_u}^{R_1} = \frac{v(2 + 2\alpha^2 - \beta^2 + 2\alpha(2 - \beta^2)) - 2\alpha^2(2 - \beta^2)}{2(2 - \beta^2)(1 + \alpha)},
\]

The equilibrium pricing and service level of \( R_2 \) are as follows:

\[
p_{M_u}^{R_2} = \frac{v(2 + 2\alpha^2 - 3\beta^2 + 4\alpha(2 - \beta^2)) - 2\alpha(2 - 3\beta^2 + \beta^4)}{2\alpha(1 + \alpha)(2 - \beta^2)}, \\
\]

\[
m_{M_u}^{R_2} = \frac{\beta(2(2 - \beta^2) - v(2 - 2\alpha^2 - \beta^2))}{2\alpha(1 + \alpha)(2 - \beta^2)}.
\]

Meanwhile, the demand levels and profits of the two retailers can be obtained as follows:

\[
\begin{align*}
D_{M_u}^{R_1} &= \frac{2\alpha^2(2 - \beta^2) + v(2 - 2\alpha^2 - \beta^2)}{2\alpha(1 + \alpha)(2 - \beta^2)}, \\
D_{M_u}^{R_2} &= \frac{2\alpha(2 - \beta^2) - v(2 - 2\alpha^2 - \beta^2)}{2\alpha(1 + \alpha)(2 - \beta^2)}, \\
\pi_{M_u}^{R_1} &= \frac{v(\beta^4(2 - \beta^2)(2 - 2\alpha^2 - \beta^2) - 4\alpha(2 - \beta^2)^2 - F)}{4\alpha(1 + \alpha)^2(2 - \beta^2)^2} + F, \\
\pi_{M_u}^{R_2} &= \frac{4\alpha(2 - \beta^2)(2 - \beta^2 + 2\alpha) - 4\alpha^2(2 - \beta^2)^2 + F}{8\alpha^2(1 + \alpha)^2(2 - \beta^2)}.
\end{align*}
\]
Note that \( E = \frac{v^2}{\alpha^2} \left( 4\alpha^4 + 4\alpha^3(2 - \beta^2) - (1 + 2\alpha)(2 - \beta^2)^2 \right) \), \( F = \frac{v^2(8\alpha^3 + 4\alpha^4 - 4\alpha(2 - \beta^2) - (2 - \beta^2)^2)}{\alpha} \). In addition, Lemma 3 is obtained by comparing the equilibrium results shown above.

**Lemma 3.** The incumbent \( R_1 \) and the new entrant \( R_2 \) compete by offering homogeneous products. When consumer utility satisfies \( v_1 \leq v < v_2 \), that is, when the market is in a situation of multiple equilibrium, there are consumer sensitivity thresholds related to the level of offline service, i.e., \( \beta_{1n} \), \( \beta_{2n} \) and \( \beta_{3n} \).

1. When \( \beta \leq \beta_{1n} \), the incumbent’s advantage increases, that is, when \( \beta \leq \beta_{1n} \), the two retailers. In the partially covered market and the multiple equilibrium market, as the sensitivity of consumers to the service increases, the profit of consumers is more sensitive to the service, \( \beta_{1n} \), \( \beta_{2n} \) and \( \beta_{3n} \).

2. When \( \beta > \beta_{1n} \), the two retailers. In the partially covered market and the multiple equilibrium market are similar to those obtained in the fully covered market. If \( \beta \) is sufficiently large, i.e., \( \beta > \beta_{1n} \), the optimal demand, price and profit of \( R_2 \) exceed those of \( R_1 \). Otherwise, the results are the opposite. This situation indicates that even though \( R_1 \) has a certain advantage in the competition for the market over \( R_2 \), when consumers are more sensitive to the service, \( R_2 \) can obtain higher profits in the competition with \( R_1 \).

5. Model Analysis

According to the game equilibrium results shown, the influence of \( R_1 \)’s advantage in terms of sensitivity to product level differences, consumers’ sensitivity \( \beta \) to \( R_2 \)’s offline services and the offline service level of \( R_2 \) are analyzed in further detail below.

5.1. The Influence of \( \alpha \) and \( \beta \) on the Intrusion Results

First, Proposition 1 can be obtained by conducting sensitivity analysis on \( \alpha \), the incumbent’s advantage, and \( \beta \), the consumer’s sensitivity to service.

**Proposition 1.** When \( R_2 \) enters the market:

1. \( \frac{\partial \pi_1^{R_2}}{\partial \alpha} < 0 \) and \( \frac{\partial \pi_2^{R_2}}{\partial \beta} > 0 \) when \( v < v_1^B \).
2. \( \frac{\partial \pi_1^{R_2}}{\partial \alpha} < 0 \) and \( \frac{\partial \pi_2^{R_2}}{\partial \beta} > 0 \) when \( v > v_1^B \).
3. \( \frac{\partial \pi_1^{R_2}}{\partial \alpha} > 0 \) and \( \frac{\partial \pi_2^{R_2}}{\partial \beta} < 0 \) when \( v_1^B < v < v_2^B \).

**Proof.** When \( v < v_1^B \), since \( \frac{\partial \pi_1^{R_2}}{\partial \alpha} = -\frac{2(2-\alpha)(\beta^2-2)+(2+4\alpha+2\alpha^2-\beta^2)}{2(1+\alpha)^2(2-\beta^2)} > 0 \), \( \frac{\partial \pi_1^{R_2}}{\partial \alpha} < 0 \) and \( \frac{\partial \pi_2^{R_2}}{\partial \beta} > 0 \) when \( v_1^B < v < v_2^B \).

Similarly, \( \frac{\partial \pi_1^{R_2}}{\partial \beta} > 0 \). Let \( t = \beta^2 \), since \( \frac{\partial \pi_1^{R_2}}{\partial \alpha} = \frac{3\alpha^3+3\alpha^2(3-\beta^2)+a(6-4\beta^2+\beta^4)-\beta^2(2-3\beta^2+4\beta^4)}{(3-3\alpha-\beta^2)^2} > 0 \), and there exists a threshold \( \hat{t} = \frac{1}{2} \left( 3 - \sqrt{2(3 + 4\beta^2 - 4\beta^4)} \right) \). When \( \alpha < \hat{t}, \frac{\partial \pi_2^{R_2}}{\partial \alpha} < 0 \); otherwise, \( \frac{\partial \pi_2^{R_2}}{\partial \beta} < 0 \).

Proposition 1 provides the results concerning the influence of \( \alpha \) and \( \beta \) on the profits of the two retailers. In the partially covered market and the multiple equilibrium market, as the incumbent’s advantage increases, that is, when \( \alpha \) decreases, the profit of \( R_1 \) increases and the profit of \( R_2 \) decreases. In the multiple equilibrium market and the fully covered market, as the sensitivity of consumers to the service increases, the profit of \( R_1 \) decreases, and the profit of \( R_2 \) increases. In the fully covered market, when \( \alpha < \hat{t} \), the profit of \( R_1 \)
decreases as the incumbent’s advantage $\alpha$ increases. Accordingly, there exists a situation in which when $R_1$’s advantage $\alpha$ increases, both retailers’ profits decrease.

To explore the conclusions associated with Proposition 1, the influence of $\alpha$ and $\beta$ on price and demand are analyzed below. Set $v = 0.7$, $\beta = 0.8$, and the influence of $\alpha$ on the price and demand is obtained in Figure 2a,b.

In Figure 2, “I” denotes the partially covered market, and “II” denotes the multiple equilibrium market. There is no competition between the two retailers in area “I”, and the incumbent’s advantage reduces the unit mismatch cost associated with consumers purchasing $R_1$’s product. In a monopolistic market, a reduction in $\alpha$ entails greater product demand for $R_1$, but it does not affect the price of the product. Therefore, the prices of the two retailers in area “I” are not affected by $\alpha$, but the demand for $R_1$’s product is affected. In region II, with the increase in $\alpha$, the prices offered by both retailers decrease. This decrease is due to the fact that the two retailers compete and cooperate with each other in region II. With an increase in $\alpha$, $R_1$’s advantage decreases and the price offered by $R_2$ decreases to obtain greater demand. In region III, $R_1$ and $R_2$ compete with each other. With an increase in $\alpha$, the advantage of $R_1$ weakens, so the demand for $R_1$’s product decreases and the demand for $R_2$’s product increases; thus, the levels of demand obtained by the two retailers tend to be the same, thus easing the competition between them and increasing prices on both sides. The conclusion contained in Proposition 1 can be explained by analyzing the impact of $\alpha$ and $\beta$ on the price and demand associated with the two retailers.

Figure 3 takes $v = 0.7$, $\beta = 0.8$ to show the impact of $\alpha$ on the profits of both retailers. In region I, the two retailers have no influence on each other, so $R_2$’s profit is not influenced by $\alpha$. As $\alpha$ increases, the price of $R_1$ remains unchanged while the demand decreases, so the profit of $R_1$ decreases. In region II, with an increase in $\alpha$, both the price and demand of $R_1$ decrease, and so the profit of $R_1$ also gradually decreases. Although the price of $R_2$ decreases with an increase in $\alpha$, the increase in demand is greater, and so the profit of $R_2$ increases. In area III, with an increase in $\alpha$, the price and demand of $R_1$ also change in the opposite direction, which causes the profit of $R_1$ first to decrease and subsequently to increase. With an increase in $\alpha$, the price and demand of $R_2$ also increase, so the profit of $R_2$ gradually increases.

Figure 4 takes $v = 0.7$, $\alpha = 0.7$ to show the impact of $\beta$ on the levels of price and demand exhibited by both retailers. As Figure 4a,b show, both the price and demand of $R_2$ increase in the three regions with an increase in $\beta$. In region I, $R_1$ is not affected by $\beta$; in region II, with an increase in $\beta$, the price of $R_1$ increases but demand decreases. In region III, both the price and demand of $R_1$ are negatively correlated with $\beta$. Figure 5 takes $v = 0.7$, $\alpha = 0.7$ to show the impact of $\beta$ on the profits of both retailers. In combination with the influence of $\beta$ on the levels of price and demand exhibited by the two retailers analyzed above, $R_2$’s profit always increases with an increase in $\beta$. In contrast, $R_1$’s profit

![Figure 2](image-url)

**Figure 2.** The influence of $\alpha$ on product price and demand. (a) The influence of $\alpha$ on product price. (b) The influence of $\alpha$ on product demand.
is negatively correlated with $\beta$ in all regions with the exception of region I, which is not affected by $\beta$.

![Figure 3](image3.png)

Figure 3. The influence of $\alpha$ on retailers’ profits.

![Figure 4](image4.png)

Figure 4. The influence of $\beta$ on product price and demand: (a) the influence of $\alpha$ on product price; and (b) the influence of $\alpha$ on product demand.

![Figure 5](image5.png)

Figure 5. The influence of $\beta$ on retailers’ profits.

**Corollary 1.** The advantage of the incumbent is not always detrimental to the entrant, nor is it always beneficial to the incumbent. However, the higher the consumer’s sensitivity to the service is, the better the situation is for the entrant.

In a partially covered market, no competition takes place between $R_1$ and $R_2$, so the incumbency advantage does not affect $R_2$. In the multiple equilibrium market and the complete coverage market, competition does take place between $R_1$ and $R_2$, and the incumbency advantage is always disadvantageous to $R_2$. This difference is due to the fact that as the incumbency advantage of $R_1$ increases, the pricing of the two retailers is reduced, and demand also decreases; thus, the profit of $R_2$ decreases. However, for $R_1$, with an increase in its advantage, although its demand increases, when $\alpha < \hat{\alpha}$, the reduction in its price has a greater impact on profits, thus resulting in a decrease in $R_1$’s profit. The effect of $\alpha$ on the prices shows that the incumbent advantage intensifies the competition between
the two retailers. Using its incumbent advantage, $R_1$ competes for the market at the cost of lowering prices to ensure profits. However, in a fully covered market, if the incumbent advantage is too large, this situation can impede $R_1$’s profit. Corollary 1 also shows that the higher the sensitivity of consumers to offline services is, the higher the profits of an entrant that chooses to open a physical store.

5.2. The Influence of $\alpha$ and $\beta$ on $s$ and the Influence of $s$ on Competition

The entrant retailer opens an offline physical store to provide customers with a certain level of service. By analyzing the influence of service $s$ on the game equilibrium results across different market types, Proposition 2 can be obtained.

**Proposition 2.** If consumers are more sensitive to services, the entrant can effectively counter the advantage of the incumbent by opening an offline physical store. When $\beta > \max(\beta^{Pa}, \beta^{Mu})$, $R_2$ can obtain higher profits than $R_1$ by opening an offline physical store.

Although consumers prefer the products of the incumbent, Proposition 2 shows that opening an offline physical store can serve as an effective competitive model for the entrant. Due to the escalation of consumer demand for services in recent years, it has gradually become more difficult to meet the individual needs of consumers using the pure online retail model. By opening offline physical stores, retailers can interact with consumers and provide them with additional services while improving their shopping experience. Therefore, in the context of consumers’ increasingly sophisticated demands for services, to counter the incumbent advantage, offline physical stores can play a more active role for entrants.

The offline services provided by the entrant can provide positive utility to consumers, but the provision of a certain level of service entails corresponding costs. Therefore, the entrant must determine the optimal service level $s$; additionally, $\alpha$ and $\beta$ also impact the entrant’s decision regarding $s$.

**Proposition 3.** When $R_2$ enters the market,

1. $\frac{\partial s^{Pa}}{\partial \beta} > 0 \text{ when } v < v_1$.
2. $\frac{\partial s^{Mu}}{\partial \beta} > 0; \frac{\partial s^{Mu}}{\partial \alpha} > 0 \text{ when } v_1 \leq v < v_2$.
3. $\frac{\partial s^{Fu}}{\partial \beta} > 0; \frac{\partial s^{Fu}}{\partial \alpha} > 0 \text{ when } v \geq v_2$.

Proposition 3 shows that in all market competition situations, the service level is positively correlated with $\beta$. That is, the higher the consumer’s sensitivity to the service is, the more the entrant should do to improve the service level. There is also a positive correlation between the service level and $\alpha$ in all areas with the exception of area I. This finding indicates that as the advantages of the incumbent weaken, the entrant should provide a higher level of service.

Figure 6a takes $v = 0.7$, $\beta = 0.8$ and Figure 6b takes $v = 0.7$, $\alpha = 0.7$ to describe the influence of $\alpha$ and $\beta$ on the service level $s$. Based on Figure 6a, it can be concluded that the service levels provided by the entrant in areas II and III are positively correlated with $\alpha$ but that the service level is not affected by $\alpha$ in area I. Figure 6b reveals a positive correlation between service level $s$ and $\beta$. A comparison of Figure 6a,b suggests that the size of the service level in different market situations is uncertain when different parameters are chosen. For example, in Figure 6a, the service level in the case of partial coverage is higher than the service level in the case of competition, while in Figure 6b, the service level in the case of competition is higher than the service level in the case of partial coverage. This finding demonstrates that the service level provided by the entrant does not depend on the type of market competition in question but is affected by the advantages of incumbency and the sensitivity of consumers to the service.
6. Value of Offline Services

Based on the analysis discussed above, it can be concluded that when consumers are more sensitive to services, opening offline physical stores helps R2 set higher prices, obtain higher demand, and enhance market competitiveness. This section compares two situations, one in which R2 opens offline physical stores and one in which it does not, to analyze the gains that offline services entail for R2 in further detail. By using the previous model solving process, the equilibrium price, demand, and profit of R2 without offline physical stores in the context of different types of market competition can be obtained, and this information is summarized in Table 2.

Table 2. The equilibrium price, demand, and profit of the entrant without opening offline stores in the context of different market types.

<table>
<thead>
<tr>
<th>Market Type</th>
<th>Equilibrium Price</th>
<th>Equilibrium Demand</th>
<th>Equilibrium Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>The partially covered market</td>
<td>$p^\text{NPa*}_2 = \frac{a}{2}$</td>
<td>$D^\text{NPa*}_2 = \frac{a}{2}$</td>
<td>$\pi^\text{NPa*}_2 = \frac{a^2}{4}$</td>
</tr>
<tr>
<td>The multiple equilibrium market</td>
<td>$p^\text{NMus*}_2 = \frac{v(1+a)^2 - 2a}{2a(1+a)}$</td>
<td>$D^\text{NMus*}_2 = \frac{2a - v(1-a^2)}{2a(1+a)}$</td>
<td>$\pi^\text{NMus*}_2 = \frac{G(2a + va^2 - v)}{4a^2(1+a)^2}$</td>
</tr>
<tr>
<td>The fully covered market</td>
<td>$p^\text{NFus*}_2 = \frac{1+2a}{3}$</td>
<td>$D^\text{NFus*}_2 = \frac{1+2a}{3(1+a)}$</td>
<td>$\pi^\text{NFus*}_2 = \frac{(1+2a)^2}{3(1+a)}$</td>
</tr>
</tbody>
</table>

Note: $G = v(1+a)^2 - 2a^2$.

First, the optimal pricing and available demand for R2 with and without offline physical stores are compared. Table 2 shows the equilibrium solution when R2 does not open offline stores.

By comparison, Proposition 4 can be obtained.

**Proposition 4. When R2 enters the market:**

1. $D^\text{NPa*}_2 \leq D^\text{Pa*}_2; p^\text{NPa*}_2 \leq p^\text{Pa*}_2$ when $v < \text{Min}\{v_1, v_1^N\}$.
2. $D^\text{NMus*}_2 \leq D^\text{Mus*}_2$, there exists a threshold $\tilde{v} = \sqrt{\frac{v(2-a^2) - 4a}{a}}$, when $\text{Max}\{v_2, v_2^N\} \leq v < \text{Min}\{v_2, v_2^N\}$, and then $p^\text{NMus*}_2 \leq p^\text{Mus*}_2$ when $\beta \geq \tilde{v}$; otherwise, $p^\text{NMus*}_2 > p^\text{Mus*}_2$.
3. $D^\text{NFus*}_2 \leq D^\text{Fus*}_2; p^\text{NFus*}_2 \leq p^\text{Fus*}_2$ when $v \geq \text{Max}\{v_2, v_2^N\}$.

**Proof.** When $v < \text{Min}\{v_2, v_2^N\}$, $p^\text{NPa*}_2 = \frac{2 - \beta^2}{\beta} p^\text{Pa*}_2$, and since $0 < \frac{2 - \beta^2}{\beta} < 1$, $p^\text{NPa*}_2 \leq p^\text{Pa*}_2$ is obtained. Similarly, $D^\text{NPa*}_2 \leq D^\text{Pa*}_2$.

When $\text{Max}\{v_1, v_1^N\} \leq v < \text{Min}\{v_2, v_2^N\}$, $p^\text{NMus*}_2 - p^\text{Mus*}_2 = \frac{\beta^2(\nu(2-a^2) - 2a(2-\beta^2))}{2a(1+a)(2-\beta^2)}$, so a threshold $\tilde{\beta} = \sqrt{\frac{v(2-a^2) - 4a}{a}}$ is obtained. When $\beta \geq \tilde{v}$, $p^\text{NMus*}_2 \leq p^\text{Mus*}_2$, otherwise, $p^\text{NMus*}_2 > p^\text{Mus*}_2$. $D^\text{NMus*}_2 \leq D^\text{Mus*}_2$ is obtained because $D^\text{NFus*}_2 - D^\text{Fus*}_2 = -\frac{(1+2a)^2}{3(1+a)(3+3a-\beta^2)} < 0$. 

Figure 6. The influence of $a$ and $\beta$ on service level: (a) the influence of $a$ on service level; and (b) the influence of $\beta$ on service level.
The \( v_1^N, v_2^N \) in Proposition 4 are similar to the thresholds \( v_1, v_2 \), which are the thresholds used to distinguish three different types of market competition when \( R_2 \) does not open an offline physical store. According to Proposition 4, in a partially covered market and a fully covered market, \( R_2 \) sets a higher price and obtains higher demand when it opens an offline physical store compared when it does not open an offline physical store. However, in a multiple equilibrium market, although opening offline physical stores can lead to greater demand for \( R_2, R_2 \) can set higher product prices only when consumers are highly sensitive to the services offered, that is, when \( \beta \geq \beta^* \).

Unlike the cases of partially covered markets and fully covered markets, in multiple equilibrium markets, opening offline physical stores does not always allow \( R_2 \) to set higher prices. When consumers are less sensitive to services, \( R_2 \) can set higher prices instead of opening a physical store. This possibility is related to the cooperation between the two retailers involved in this situation. When consumers’ valuation of the product is at a medium level, to determine the equilibrium result, \( R_2 \) and \( R_1 \) are assumed to have a cooperative agreement based on the advantage of \( R_1 \), so that the consumer utility at the point of no difference is 0 and the multiple equilibrium region is divided in a ratio of 1 : \( \alpha \) \( (R_2 \) accounts for \( \alpha/(1+\alpha) \) of the ratio). This situation allows \( R_2 \) to obtain higher demand after opening an offline physical store. In this case, \( R_2 \) may lower its prices to ensure that it is able to satisfy the market demand. Therefore, if consumers are less sensitive to services, in the multiple equilibrium market, \( R_2 \) sets a lower price after opening a physical store than in situation in which it does not open a physical store.

In Table 2, the last column is the equilibrium profit that \( R_2 \) can obtain when it does not open an offline physical store. Proposition 4 suggests that the opening of offline physical stores is beneficial to the pricing and market share of \( R_2 \) as a whole. However, \( R_2 \) must pay a certain fee \( C(s) \) to open an offline store. Therefore, the impact of opening an offline physical store on \( R_2 \)'s profit requires further analysis. In this paper, \( \Delta \) is defined as the difference between \( R_2 \)'s profit when it opens an offline physical store and its profit when it does not open an offline physical store. Proposition 5 can be obtained by comparing the equilibrium profits of \( R_2 \) under each of the three situations shown in Table 2 with the profits obtained by \( R_2 \) when it opens an offline physical store.

**Proposition 5.** When \( R_2 \) enters the market,

1. When \( v < \min \{ v_1, v_1^N \} \), \( \Delta = \pi_2^{Pas} - \pi_2^{NPas} \geq 0 \) and \( \partial \Delta/\partial \beta > 0 \).

2. When \( \partial \Delta/\partial \beta > 0 \); when \( \max \{ v_1, v_1^N \} \leq v < \min \{ v_2, v_2^N \} \), \( \Delta = \pi_2^{Mus} - \pi_2^{NMus} \geq 0 \); and when \( \alpha \leq \bar{\alpha} \), \( \partial \Delta/\partial \alpha \leq 0 \); otherwise \( \partial \Delta/\partial \alpha > 0 \).

3. When \( v < \min \{ v_1, v_1^N \} \), \( \partial \Delta/\partial \alpha > 0 \) and \( \partial \Delta/\partial \beta \geq 0 \).

**Proof.** When \( v < \min \{ v_1, v_1^N \} \), \( \Delta = \pi_2^{Pas} - \pi_2^{NPas} = \frac{v^2\beta^2}{8-4\beta^2} > 0 \) and let \( t = \beta^2 \), then \( \partial \Delta/\partial t = \frac{v^2}{2(2-t)^2} > 0 \) is obtained, and since \( \partial t/\partial \beta = 2\beta > 0 \), \( \partial \Delta/\partial \beta > 0 \) is obtained.

When \( \max \{ v_1, v_1^N \} \leq v < \min \{ v_2, v_2^N \} \), \( \Delta = \pi_2^{Mus} - \pi_2^{NMus} = \frac{\beta^2(4\alpha(2-\beta)+3\beta^2)(2+4\alpha^2+2\beta^2-2\beta^3)}{4(2-t)^4(1+\alpha)^3} > 0 \) and let \( t = \beta^2 \), then \( \partial \Delta/\partial t = \frac{1}{4(2-t)^4(1+\alpha)^3} \left( \frac{\beta^2(4\alpha(2-\beta)+3\beta^2)(2+4\alpha^2+2\beta^2-2\beta^3)}{4(2-t)^4(1+\alpha)^3} \right) > 0 \) is obtained, and since \( \partial t/\partial \beta = 2\beta > 0 \), \( \partial \Delta/\partial \beta > 0 \) is obtained.

When \( v \geq \max \{ v_2, v_2^N \} \), \( \Delta = \pi_2^{Pas} - \pi_2^{NPas} = \frac{(\beta+2\alpha\beta)(3\alpha-2\beta^2)}{18(1+\alpha)(3+3\alpha-\beta^2)} > 0 \) and let \( t = \beta^2 \), then \( \partial \Delta/\partial t = \frac{\beta^2 + 2\alpha\beta^3(3\alpha-2\beta^2)}{18(1+\alpha)(3+3\alpha-\beta^2)} > 0 \) is obtained, and since \( \partial t/\partial \beta = 2\beta > 0 \), \( \partial \Delta/\partial \beta > 0 \) is obtained.

\[
\frac{\partial \Delta/\partial \alpha}{\partial \Delta/\partial \beta} = \frac{1 + 2\alpha}{9(1+\alpha)^2(3+3\alpha-\beta^2)^3} > 0.
\]
Proposition 5 indicates that in the same market type, $R_2$ can always obtain higher profits by choosing to open an offline physical store. This finding indicates that opening offline physical stores can improve $R_2$’s competitiveness and enable $R_2$ to obtain higher profits when engaging in competition with $R_1$. Although $R_2$ chooses whether or not to open a physical store when entering the market, three situations can emerge with respect to the competition between $R_2$ and $R_1$; however, in these three types of market competition, $R_2$ always benefits from opening offline physical stores. This finding indicates that the services offered by offline physical stores have positive value for $R_2$.

Further analysis of the influence of the incumbent’s advantage and the consumer’s sensitivity to the services offered by physical stores on $\Delta$ shows that the influence of the incumbent’s advantage on the value of the service differs across different scenarios. In a multiple equilibrium market, when the incumbent’s advantage is small, namely, $\alpha$ is large; as the incumbent’s advantage weakens, the difference between the profit obtained by $R_2$’s additional offline service and the profit obtained by pure online retail increases, which entails that opening an offline physical store can increase the value of $R_2$. However, when the incumbent’s advantage is small, the value gain effect of offline services for $R_2$ is also weakened as the incumbent’s advantage weakens. These two results indicate that in a multiple equilibrium market, when the incumbent has a greater advantage, the greater the incumbent advantage is, the greater the value that $R_2$ can gain by opening an offline physical store. However, when the incumbent’s advantage is weak, the effect of this advantage is the opposite. Compared with the multiple equilibrium market, in a fully covered market, the weaker the incumbent’s advantage is, the weaker the value gain offered by offline experience stores to $R_2$. These results indicate that the advantage of the incumbent suppresses the value gain of offline physical stores to $R_2$ overall. However, in a multiple equilibrium market, due to the existence of a certain degree of cooperation between $R_1$ and $R_2$, when the advantage of the incumbent is large, an increase in this advantage strengthens the gain effect of offline services. Proposition 5 also indicates that as consumers’ sensitivity to services increases, the profit difference between the two entry strategies also increases. This finding suggests that increased consumer sensitivity to services leads to higher value gains for $R_2$ when it opens offline stores.

7. Discussion

This paper studies the choice of an intrusion strategy for a new entrant retailer. New entrants often face incumbents that are already in the market. The entrant may choose from two intrusion strategies: one strategy is to adopt the pure online retail mode, which is identical to the approach used by the incumbent, while the other is to adopt a new retail model that integrates online and offline sales. This paper used a game model to analyze the situation in which the entrant chooses one of these two strategies to invade the market as well as the subsequent competition between the entrant and the incumbent. The optimal entry strategy selection and service value of entrants were explored.

The findings show that the offline experiences offered by stores can improve the competitiveness of online retailers in the market; this finding is similar to that reported by Jing (2018) and Caro et al. (2019) [41,43]. Biswajit Sarkar et al. (2021) found that the offline price of a product was slightly higher than the price of that product via the online channel [3]. However, due to the quality of service, some customers prefer to visit the offline store to buy specific products. However, these studies have not considered the retail model featuring online–offline integration as an alternative intrusion strategy. Based on these studies, this paper conducts a comparative study of the pure online retail and online–offline integrated retail models and considers the influence of market coverage type on the choice of intrusion strategy by the invading retail enterprises.

Gao and Su (2017) found that not all products are suitable for in-store pickup and that the online purchase and offline pickup of products that sell well in stores may not increase profits [7]. In addition, while the practice of buying online and picking up offline enables retailers to reach new customers, for existing customers, the shift from online
to offline fulfilment may reduce the benefits of obtaining products. It has been found that consumers’ valuation of a product determines the type of market with which that product is associated. As consumers’ valuation of the product increases, three scenarios can occur, including a partial coverage market, a multiple equilibrium market and a full coverage market. In a partially covered market, no competition takes place between the incumbent retailer and the entrant retailer, so the incumbent advantage cannot affect the entrant. In a multiple equilibrium market or a complete coverage market, the advantage of the incumbent impacts the profits of the entrant negatively. However, the incumbent advantage is always beneficial to the incumbent. In a fully covered market, when the incumbent advantage is too high, the competition between the incumbent and the entrant is fierce, which damages the profits of both retailers.

This paper also examined the impact of quality of service on the intruder’s strategy. The literature has discussed the ways in which online retailers can supplement the defects of retail information transmission that takes place completely online by opening offline channels to provide various services, which can improve consumers’ purchase intentions and loyalty [34]. In addition, Li et al. (2018) found that in the online retail market, consumers’ product evaluations affect retailers’ sales volume and profits [35]. The difference is that the consumer’s sensitivity to the services offered by offline stores affects the outcome of the intrusion. This paper finds that in the three coverage markets, it is always beneficial for entrants to open offline physical stores, indicating that opening offline physical stores provides positive benefits to entrants. Therefore, when consumers are more sensitive to the service, opening offline physical stores is an effective intrusion strategy that allows entrants to cope with the advantages of incumbents. The research contributions of this paper are as follows.

First, the research process and method employed in this paper expanded the research scenario of the duopoly game model by integrating the current business context. Previous literature on market intrusion by online retail firms has not considered the impact of the type of market coverage on the strategic choices of entrants [49,50]. Based on the study of two intrusion strategies, this paper analyzed the impact of three types of market coverage, i.e., the partial coverage market, the multi-equilibrium market and the full coverage market, on the decisions of the invading enterprise. The mode of intrusion involved in online retailer intrusion has mainly been studied by Arya and Mittendorf (2018) [28]. However, these studies did not take the online–offline integrated retail model into account as an alternative intrusion strategy. On the basis of the extant literature, this paper conducted a comparative study of the pure online retail and online–offline integrated retail models and consider ed the impact of market coverage types on the choice of intrusion strategies exhibited by intrusive retail enterprises.

Second, this paper expanded the research scope of the field of e-commerce and discussed the combination of online sales and offline services in detail. Previous studies have explored the intrusion strategy used by online retailers that open offline stores in terms of establishing offline showrooms, introducing advanced technologies, and improving service levels [3,20]. This paper considered the influence of incumbent advantage on the results of intrusion. The results reveal the results when new entrants choose to open offline physical stores and the factors that affect these results when the incumbent retailer has an advantage in online markets.

Third, this paper provides a theoretical foundation for new entrants to make decisions regarding their intrusion business model and discusses the influence of different parameters and scenarios on the strategic decisions made by both sides of the market competition in depth.

8. Conclusions

The development of the Internet has entailed new opportunities for the retail industry, and online retail has developed rapidly. Due to the increasing number of consumers who choose to shop online, the online retail market is attracting an increasing number of retailers.
At present, in addition to the pure online retail model, many retail companies with online stores have also opened offline stores to seize this market. As a new invading enterprise, a retailer can choose simply to offer online sales or to integrate both online and offline retail sales to occupy the market. However, different intrusion strategies affect the retailer’s profit directly. This paper compares the two types of intrusion strategies and discusses the retailer’s choice of intrusion strategy.

Modelling analysis revealed that whether the entrant chooses the pure online retail model or the online–offline integration model, the market is partially covered when the consumer’s product valuation is low. In this case, no actual competition takes place between the entrant and the incumbent. If the entrant chooses a pure online sales strategy, its demand and profit are smaller than those of the incumbent. If the entrant chooses the strategy of online–offline integration, the entrant can set higher prices than the incumbent because the entrant improves the shopping experience of consumers, and the demand obtained by the entrant can also exceed that obtained by the incumbent. In this case, the profit of the entrant may be greater than that of the incumbent. Regardless of the strategy chosen by the entrant, the market is always fully covered if consumers value the product highly. In addition, in this context, regardless of whether the pure online sales strategy or the online–offline integration strategy is chosen, a multiple-equilibrium market situation always emerges. At this point, when the entrant uses the online–offline integration model to invade the market, the equilibrium result becomes more complicated. In a multiple-equilibrium market, the profit of the entrant may be greater than that of the incumbent.

In addition, if the market type is the same when the entrant chooses between the two different strategies, then the entrant should choose an intrusion strategy based on the advantages of the incumbent. If the market type is different when the entrant chooses between the two different strategies, the entrant should avoid direct competition with the incumbent when the incumbent has a large advantage. If consumers have a high valuation of the product, that is, when the entrant must compete with the incumbent, the entrant should choose the online–offline integration strategy to counter the advantages of the incumbent. When consumers are sensitive to the services that are offered by physical stores, offline services can improve the profits of the entrant in the same market type. In addition, in different market types, providing offline services can also benefit the entrant.

These conclusions are of great significance. Due to the emergence of the new retail model, the field of retail is facing a new situation, and many online retail enterprises have begun to develop offline services. Questions concerning whether the retail mode of online–offline integration is the best choice for new entrants and whether the new retail mode can allow new entrants to enter the online retail market more easily than the traditional online retail mode are important to new entrants. Therefore, consumers’ valuation of products, incumbents’ advantages and offline service quality should be considered when making decisions.

Using online fresh product retailers as an example, an increasing number of fresh product retailers, such as Hema and Daily Fresh, are entering the market using a model that features both online and offline sales. For fresh product retailers, opening physical stores and enhancing consumers’ shopping experience can greatly enhance a retailer’s market competitiveness, thereby increasing its profits. On the other hand, consumers have higher requirements regarding the hygiene, quality, and safety of fresh products, which gives incumbents a greater advantage over new entrants. Therefore, entrants should open offline physical stores.

This paper analyzed the entry strategies used by online retailers and studies the influence of the first-mover advantage of incumbents and the last-mover choice of entrants on the results of the ensuing competition. However, this paper faced certain limitations, and further research remains necessary in the future. First, this article did not consider the various costs of products. However, when the cost problem is taken into consideration, the whole model becomes very complicated and difficult to solve via analysis. Second,
according to the model used in this paper, the incumbent has a very limited scope to make decisions and can only seek to ensure maximum profits by changing the price of the product. In the future, the authors of this article may also consider other strategic choices on the part of the incumbent, such as opening a physical store or developing certain defensive strategies. Again, this consideration may cause the model to become too large. Of course, in the future, the possibility that entrants can choose other ways of competing with incumbents or even cooperating with incumbents can be taken into consideration. This topic requires further research in the context of online retailers.

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References
1. Li, Y.; Xiong, Y.; Mariuzzo, F.; Xia, S. The underexplored impacts of online consumer reviews: Pricing and new product design strategies in the O2O supply chain. Int. J. Prod. Econ. 2021, 237, 108148. [CrossRef]
3. Sarkar, B.; Dey, B.; Sarkar, M.; AlArjani, A. A Sustainable Online-to-Offline (O2O) Retailing Strategy for a Supply Chain Management under Controllable Lead Time and Variable Demand. Sustainability 2021, 13, 1756. [CrossRef]
