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Optimal Recommendation Strategies for AI-Powered E-Commerce Platforms: A Study of Duopoly Manufacturers and Market Competition

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Abstract: Artificial intelligence-powered recommendation systems have gained popularity as a tool to enhance user experience and boost sales. Platforms often need to make decisions about which seller to recommend and the strength of the recommendation when conducting recommendations. Therefore, it is necessary to explore the recommendation strategy of the platform in the case of duopoly competition. We develop a game model where two competing manufacturers sell products through an agency contract on a common platform, and they can decide whether or not to provide recommendations to the manufacturers. Our highlight lies in the endogenous recommendation strength of the platform. The findings suggest that it is optimal for the platform to offer recommendation services when the commission rate is high. The platform also prefers to only recommend one manufacturer in the market with low or high competition, but it prefers to recommend both manufacturers in moderately competitive markets. From the view of manufacturers, they can benefit from the recommendation service as long as the commission rate is not too low. Moreover, recommending only one manufacturer consistently yields stronger recommendations compared to recommending multiple manufacturers. However, the impact of recommendation on prices is influenced by the commission rate and product substitutability. These results have significant implications for platform decision making and provide valuable insights into the trade-offs involved in the development of recommendation systems.

Keywords: recommendation strategy; platform operations; duopoly competition; pricing strategies; product substitutability

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1. Introduction

In recent times, the surge of e-commerce platforms has offered consumers the ability to explore and acquire a broad array of products online, delivering a convenient and diverse shopping experience. According to the China Internet Network Information Center (CNNIC), by December 2021, China's online shopping user base reached 842 million, constituting 81.6% of all Internet users (https://www.askci.com/news/chanye/20220317/1635411746234.shtml accessed on 15 May 2023). Statista reported that the global e-commerce market expanded from $1.3 trillion in 2014 to $4.9 trillion in 2021 and is projected to hit $7.4 trillion by 2025 (https://thesocialshepherd.com/blog/ecommerce-statistics accessed on 15 May 2023). This swift expansion has enticed an increasing number of manufacturers to enter the platform through an agency sales model, leading to intensified market competition. Currently, approximately 2 million third-party sellers operate globally via Amazon, with 57% of Amazon sales in Q2 2022 originating from these sellers, generating revenues of $27.38 billion, which is a 9.12% increase from the previous year (http://jnec.jnbusiness.jinan.gov.cn/content-27-49102-1.html accessed on 11 January 2023).

Given the vast amount of product information available, consumers often struggle to find suitable products. Online platforms have increasingly adopted recommendation systems to enhance user experience and boost sales. Recommendation is a platform-initiated action in order to increase sales. For example, as an e-commerce platform in China, Pinduoduo offers a free recommendation service for sellers. By participating in the platform's specific campaigns and partnership programs, sellers can gain free exposure and recommendation promotion. By providing personalized recommendations, online platforms can increase customer engagement, satisfaction, and loyalty while driving revenue growth. For example, over 35% of Amazon sales and 60% of Netflix rentals are driven by recommendations [1]. Leading e-commerce platforms, such as Amazon, eBay and JD.com, routinely display tags such as "We have recommendations for you", "Hot item", and "Guess you like" to offer recommendations to customers.

As an emerging technology, the recommendation strategy has been explored in the field of platform operations. However, most of the existing literature explores the impact of recommendations on consumers' willingness to pay from an empirical aspect [2,3], and very few provide in-depth analysis at the theoretical level. We fill the gap by developing a game model to investigate the recommendation strategy of an e-commerce platform selling products for duopoly manufacturers under agency contracts and analyze the influence of commission rate and competition intensity on the manufacturers' optimal pricing decisions and the platform's recommendation strategies. Furthermore, we consider recommendation strength as an endogenous decision variable, which is not explored in the studies of [4,5].

The challenge for e-commerce platforms lies in how to utilize recommendations to influence competition between competing upstream manufacturers. Simultaneously, platform recommendations can impact sellers' pricing strategies, necessitating in-depth research to address these concerns. In response to these questions, we introduce a Stackelberg game model in which two competing manufacturers sell substitutable products through an e-commerce platform capable of providing recommendation services. The platform must first decide whether to employ the recommendation system, and subsequently, determine which manufacturer to recommend and the intensity of the recommendation. Upon observing the platform's decision, manufacturers adjust their retail prices accordingly. We specifically examine three platform strategies: (1) no recommendation strategy (mode NN); (2) exclusive recommendation strategy (mode RN); and (3) non-exclusive recommendation strategy (mode RR).

Based on the above analysis, we focus on the optimal recommendation strategy for the platform and investigate the impacts of recommendation on the decisions of supply chain members. Our innovation lies in the endogenous decision of selecting recommendation grades, which is an aspect that is relatively unexplored in the existing literature. We find that providing recommendation services is optimal for the platform when the commission rate is high and vice versa. The platform also prefers to recommend only one manufacturer in low- or high-competition markets but prefers to recommend both in moderately competitive markets. Manufacturers can benefit from the recommendation service when the commission rate is not too low. Additionally, the recommendation strength under a exclusive recommendation scenario is always higher than under a non-exclusive recommendation scenario. However, the effect of recommendations on prices is jointly influenced by the commission rate and product substitutability. Our study contributes to a better understanding of how different recommendation modes impact the interactions among manufacturers and the platform, thus offering valuable insights for business participants and policymakers.

The rest of this paper is organized as follows. Section 2 provides the literature review in three streams: recommendation systems, duopoly competition and platform operations. We propose the model in Section 3 and then derive and analyze equilibrium results under three scenarios in Section 4. Moreover, we compare the three scenarios in Section 5 to obtain the optimal recommendation strategies and conclude the paper in Section 6 at last. All proofs are presented in Appendix A.

2. Literature Review

Our work is related and contributes to the following three research streams: (i) recommendation systems, (ii) duopoly competition, (iii) platform operations.

2.1. Recommendation Systems

Recommendation systems are becoming increasingly crucial for the operation of e-commerce platforms (Dogan, 2023) [6]. The system helps to reduce consumers' search costs and enhance their willingness to pay (Adomavicius et al., 2018 [7]). Yoon and Lee (2021) [8] examined the impact of perceived technology quality and personalization quality on consumers' behavioral intentions and found that AI recommendation service increased perceived technology quality and personalization quality, thus increasing their willingness to pay. A large-scale field experiment conducted by Lee and Hosanagar (2021) [2] verified that recommendation systems increased product views and conversion rates, although this effect was moderated by product attributes and review ratings. Zhou et al. (2022) [4], considering both uniform and differential pricing strategies, analyzed the impact of recommendation systems on the competition between store and national brands as well as consumers' search behavior when recommendations were provided. Ettl et al. (2020) [3] aimed to offer online shoppers personalized, discounted product bundle recommendations, taking into account factors such as profit maximization, inventory management, and consumer preferences. Balancing these factors ensured that the chosen products were relevant and appealing to consumers. Zhou and Zou (2022) [9] developed a model incorporating recommendation precision using Bayes' rule, finding that as recommendation accuracy improved, the equilibrium price first decreased and then increased, but both platform and seller profits declined.

From the manufacturer's perspective, Ghose et al. (2007) [10] asserted that implementing a recommendation service constituted a strategic decision, leading to a shift in supply chain profits from third-party information media to manufacturers. However, adopting recommendation systems could also negatively impact supply chain members. Li et al. (2018) [11] constructed a model involving two competing manufacturers and a shared retailer, discovering that recommendation systems could harm retailers employing recommendations, as the retailer may prioritize its own interests while neglecting upstream manufacturers' strategies. This issue is exacerbated when the costs of developing recommendation systems are high.

Our work is closely related to the research conducted by Wu et al. (2015) [5], who investigated whether a manufacturer should provide exclusive or non-exclusive recommendations to downstream retailers. Their results indicate that a non-exclusive strategy is optimal for the manufacturer if the target market is sufficiently large; otherwise, an exclusive strategy is preferable. In contrast, our research centers on identifying the optimal recommendation strategies for platforms. Specifically, a platform determines whether to implement recommendation systems and, if implemented, whether to exclusively endorse a single upstream manufacturer or provide non-exclusive recommendations for two manufacturers. Additionally, in the majority of previous studies, the recommendation grade was determined exogenously. In contrast, our study treats the recommendation grade as an endogenous decision variable.

2.2. Duopoly Competition

Research on competition between supply chain members can be traced back to Jeuland and Shugan (1983) [12]. Choi (1991) [13] was the first to conduct a thorough analysis of the channel structure involving two rival manufacturers and a shared retailer carrying both manufacturers' products, examining three non-cooperative games under distinct power structures, namely two Stackelberg games and a Nash game. Based on the works of Choi (1991) [13], Huang et al. (2016) [14] investigated the effects of pricing strategies and power structures on a two-tier supply chain composed of a common manufacturer and duopoly retailers. Yu et al. (2022) [15] developed a game model to study the strategic interactions between two brand manufacturers under an e-commerce platform, and they significantly analysed the impact of brand competition on the business mode choices of the e-commerce platform. Wang et al. (2020) [16] assessed the pricing strategies of competing dual-channel retailers as well as the influence of retailer cost and supplementary cross-selling profit on BOPS (buy online and pick up in store) approaches.

Jena and Sarmah (2014) [17] and Zhu et al. (2016) [18] examined recycling, remanufacturing, and trade-in approaches in a duopoly closed-loop supply chain, concluding that employing trade-in strategies can yield a competitive edge in terms of market share and profits. Tang et al. (2023) [19] extended this analysis to a duopoly context in which companies can choose either exclusive or non-exclusive trade-in strategies. Acknowledging that duopoly manufacturers' power may not be equal, Wei et al. (2020) [20] investigated the best pricing and selling formats for leader and follower manufacturers. Additionally, service competition is a significant issue to explore within duopoly supply chains. Ding et al. (2018) [21] delved into the relationship between pricing competition and service competition among duopoly retailers and assessed the impact of service time on optimal choices. Li et al. (2019) [22] analyzed a retailer's best strategy, either pure batch ordering or hybrid shipping, when adopting a drop-shipping strategy with two competing manufacturers.

Existing literature on duopoly competition mainly focuses on pricing strategies and power structures, such as Stackelberg and Nash games. In practice, the service is an essential factor in competition. However, most studies in the literature focus on logistics or recycling services within the duopoly competition. In this study, we examine the recommendation strategy of an e-commerce platform selling products for duopoly manufacturers under agency contracts and analyze the influence of commission rate and competition intensity on the manufacturers' optimal pricing decisions and the platform's recommendation strategies.

2.3. Platform Operations

The digital economy's growth in recent years has elevated the importance of platforms in e-commerce, drawing significant attention from researchers. Manufacturers pay a slotting fee and a share of sales revenue to platforms for the chance to sell their products, making pricing decisions crucial for platform operations (Shen et al., 2019 [23]; Zhen and Xu, 2022 [24]). Mantin et al. (2014) [25] suggested that the existence of multiple third-party sellers could cause price disparities for some products. Lu et al. (2018) [26] employed game theory models to explore e-book pricing strategies under wholesale and agency sales models, taking into account decentralized and centralized channels' effects. The findings indicate that both publishers and e-bookstores can achieve Pareto improvements with the agency model in a decentralized channel. Similarly, Chen et al. (2020) [27] assessed the impact of inventory levels and market competition on optimal pricing and business models. Considering fairness concern, Zhou et al. (2023) [28] investigated the pricing strategies of third-party sellers on ecommerce-platforms and found that different power structures have different influences on fairness concerns.

Platforms generally provide manufacturers with two standard contract options: revenue-sharing contracts, where the platform claims a percentage of the manufacturer's revenue, and fixed-fee contracts, where the platform charges a set fee per sale. Zhang et al. (2019) [29] discovered that revenue-sharing contracts may lead to lower manufacturer prices, while fixed-fee contracts could result in higher quality. Additionally, numerous aspects of platform operations warrant further investigation. For instance, Choi et al. (2020) [30] and Zhang et al. (2021) [31] studied the application of blockchain technology in platform operations. Taking into account the platform's bilateral network externalities characteristic, Anderson et al. (2014) [32] devised a model to explore the platform's optimal investment strategy under three distinct scenarios: monopoly, price-setting duopoly, and price-taking duopoly. Taylor (2018) [33] examined the effect of delay sensitivity and agent independence on on-demand service platforms' optimal service price and wage, finding that delay sensitivity reduces customers' and agents' expected utility, while the influence of agent independence on price remains uncertain. Furthermore, information asymmetry issues may arise in platform operations. Zha et al. (2022) [34] considered a conventional reselling model and the agency/marketplace model, and they found that the platform prefers to share imformation with at least one seller. Tsunoda and Zennyo (2021)[35] investigated how demand information sharing between platforms and suppliers can alter suppliers' multi-channel management. Additionally, suppliers' inventory information-sharing strategies impact pricing decisions on the platform (Martinez-de et al., 2022 [36]).

Our research expands on the existing literature by examining a supply chain in which duopoly manufacturers sell products on the same platform via an agency contract. Moreover, the platform can choose to develop its own recommendation system to boost sales and decide which manufacturer(s) to recommend. Specifically, the platform can choose no recommendation strategy, an exclusive recommendation strategy, or a non-exclusive recommendation strategy.

3. The Model

3.1. The Manufacturers and Platform

We consider an online supply chain involving two competing manufacturers (M_1 and M_2) that sell substitutable products through an e-commerce platform who can provide recommendation services to the manufacturers under an agency-selling agreement. The platform has the option to decide whether to offer recommendation services to one or more manufacturers and determine the grade of recommendation service $\hat{\alpha}$, which signifies the strength of the recommendation. Specifically, there are three strategies for the platform: (1) no recommendation strategy (mode NN); (2) exclusive recommendation strategy, where the platform recommends only one manufacturer (mode RN); and (3) non-exclusive recommendation strategy, where the platform recommends both manufacturers (mode RR). The retail prices for the products of the two manufacturers are represented by p_1 and p_2, respectively. To simplify, we assume a zero marginal cost of production.

3.2. Market Demand

In line with the works of Ghose et al. (2007) [10] and Wu et al. (2015) [5], we assume that the market is segmented into two parts: the traditional market and the market with a recommendation system. In the traditional market, there is no recommendation system, meaning consumers' intention to purchase is not affected by the grade of recommendation. On the other hand, in the market with a recommendation system, consumers' buying choices are influenced by the platform's recommendations, meaning they browse the platform and may purchase the products recommended by the platform.

Consumer can buy products from either of the two manufacturers, indicating that the manufacturers' products are substitutable. Intuitively, the greater the substitutability of products, the more competitive the market is. We use θ to represent the degree of substitutability between the two products. Specifically, the products are considered independent when $\theta = 0$, while they are perfect substitutes when $\theta = 1$.

We first focus on consumers in the traditional market who buy products from the manufacturers without the assistance of the recommendation system. Following the studies of Spence (1976) [37], Cai et al. (2012) [38], Wu et al. (2015) [5] and Zhou and Zou (2022) [9], the consumer utility function in the traditional market can be expressed as:

$$U = \sum_{i=1,2} (\alpha D_i - p_i D_i - \frac{1}{2} D_i^2) - \theta D_1 D_2, \tag{1}$$

where α denotes the potential demand of the traditional market, and D_1 and D_2 represent the demands of the two manufacturers in the traditional market. By maximizing U in Equation (1), we obtain the aggregate demands of the manufacturers as:

$$D_i = \frac{(1-\theta)\alpha - p_i + \theta p_j}{1-\theta^2}, j = 3-i; i = 1, 2. \tag{2}$$

Next, we examine consumers' utilities in the market with recommendation system, where the platform offers recommendation services for manufacturers. There are consumers who favor buying products with the assistance of recommendations. In practical terms, as the strength of the recommendation increases, a larger number of consumers make purchases based on these recommendations. Using $\hat{\alpha}$ to represent the recommendation strength and drawing a parallel to (1), when both manufacturers receive recommendations, we can formulate the consumer utility function in a quadratic form as follows:

$$\hat{U} = \sum_{i=1,2} (\hat{\alpha}\hat{D}_i - p_i\hat{D}_i - \frac{1}{2}\hat{D}_i{}^2) - \theta\hat{D}_1\hat{D}_2, \tag{3}$$

Moreover, when the platform recommends only one manufacturer, i.e., only manufacturer i exists in the market with a recommendation system, thus, by letting $\hat{D}_j = 0$, the consumer utility function can be rewritten as

$$\hat{U} = \hat{\alpha}\hat{D}_i - p_i\hat{D}_i - \frac{1}{2}\hat{D}_i{}^2. \tag{4}$$

Maximizing U in Equations (3) and (4) yields the aggregate demands of the manufacturers in the recommended market.

$$\hat{D}_i = \begin{cases} \frac{(1-\theta)\hat{\alpha}-p_i+\theta p_j}{1-\theta^2}, & \text{if both manufacturers are recommended;} \\ \hat{\alpha} - p_i, & \text{if only manufacturer } i \text{ is recommended;} \\ 0, & \text{if neither manufacturer is recommended.} \end{cases} \quad j = 3 - i; i = 1, 2. \tag{5}$$

3.3. Timing of the Game

The sequence of events in a Stackelberg game unfolds as follows: In the initial stage, the platform declares its recommendation strategy, specifying if it should recommend one manufacturer, both, or neither. If the platform uses recommendation systems, simultaneously, it determines the recommendation strength as a leader while taking the recommendation cost into account. In the subsequent stage, the manufacturers set their respective retail prices as followers. The game's resolution is achieved through backward induction in the following section. If the platform does not use recommendation systems, the game turns to a Nash game and two manufacturers simultaneously determine their prices.

3.4. Assumptions and Limitations

For simplicity, we assume a zero marginal cost of production, which is commonly used in the existing literature such as [9,22] etc. In addition, $\rho\epsilon[\rho_l^{RN}, \rho_h^{RN}]$ in Mode RN, which ensures positive equilibrium results in Mode RN, where $\rho_l^{RN} = \frac{k(2+\theta)(8-5\theta^2)}{8+7\theta-3\theta^2-3\theta^3}$, $\rho_h^{RN} = \frac{k(2+\theta)(3-2\theta^2)(8-5\theta^2)}{(1-\theta^2)(8+\theta(1-\theta)(5+2\theta))}$. In addition, $\rho\epsilon[\rho_l^{RR}\rho_h^{RR}]$ in Mode RR, which ensures positive equilibrium results in Mode RR, where $\rho_l^{RR} = k(2-\theta)(1+\theta)$ and $\rho_h^{RR} = \frac{k(2-\theta)(1+\theta)(3-\theta)}{1-\theta}$.

4. Equilibrium Analysis

In this section, we explore the context and equilibrium outcomes for three distinct scenarios. First, we assess a benchmark scenario where the platform opts not to employ a recommendation system (mode NN). Next, we investigate a scenario in which the platform decides to recommend only one manufacturer (mode RN). Lastly, we concentrate on a scenario where the platform recommends both manufacturers (mode RR). We employ superscripts "NN*", "RN*" and "RR*" to denote equilibrium outcomes in these varying situations.

4.1. Mode NN

In mode NN, serving as a benchmark, the platform does not utilize a recommendation system, meaning all consumers are on the traditional market. Manufacturers simultaneously establish their retail prices. The profit functions can be expressed as follows:

$$\pi_1 = (1-\rho)p_1 D_1, \tag{6}$$

$$\pi_2 = (1-\rho)p_2 D_2, \tag{7}$$

$$\pi_e = \rho(p_1 D_1 + p_2 D_2). \tag{8}$$

where ρ represents the flat commission rate imposed by the platform. Typically, the commission rate is associated with the product category and constitutes a stable, long-term policy for the platform, persisting consistently over an extended duration. Consequently, we assume that the commission rate is exogenous while examining its influence on the interaction between the platform and manufacturers.

By the backward approach, we obtain the equilibrium in Lemma 1.

Lemma 1. *In model NN, the equilibrium prices and profits of the supply chain members are* $p_1^{NN*} = p_2^{NN*} = \frac{\alpha(1-\theta)}{2-\theta}$; $\pi_1^{NN*} = \pi_2^{NN*} = \frac{\alpha^2(1-\theta)(1-\rho)}{(2-\theta)^2(1+\theta)}$; $\pi_e^{NN*} = \frac{2\alpha^2\rho(1-\theta)}{(2-\theta)^2(1+\theta)}$.

In the subsequent corollary, we investigate the effects of channel competition and commission rates on retail prices and the profits of firms.

Corollary 1. *When the platform does not adopt a recommendation system, we obtain the following results regarding the impacts of θ and ρ on the prices and profits.*

(i) The equilibrium prices p_1^{NN*} and p_2^{NN*} decrease with product substitutability θ. The profits of the manufacturers and platform π_1^{NN*}, π_2^{NN*} and π_e^{NN*} decrease with θ.

(ii) The profits of the manufacturers π_1^{NN*} and π_2^{NN*} decrease with ρ, whereas π_e^{NN*} increases with ρ.

Corollary 1 and Figure 1 collectively demonstrate that as product substitutability intensifies, manufacturers need to lower their prices to secure a larger market share. Nevertheless, although market demand may increase due to reduced prices as product substitutability escalates, the positive effect of increased demand on profits is outweighed by the negative impact of lower prices, leading to a decline in profits as competition becomes fiercer. Consequently, the platform's profit also decreases since it derives a share from the manufacturers. Furthermore, as illustrated in Figure 2, a higher commission rate benefits the platform but adversely affects manufacturers, which is an intuitive outcome.

4.2. Mode RN

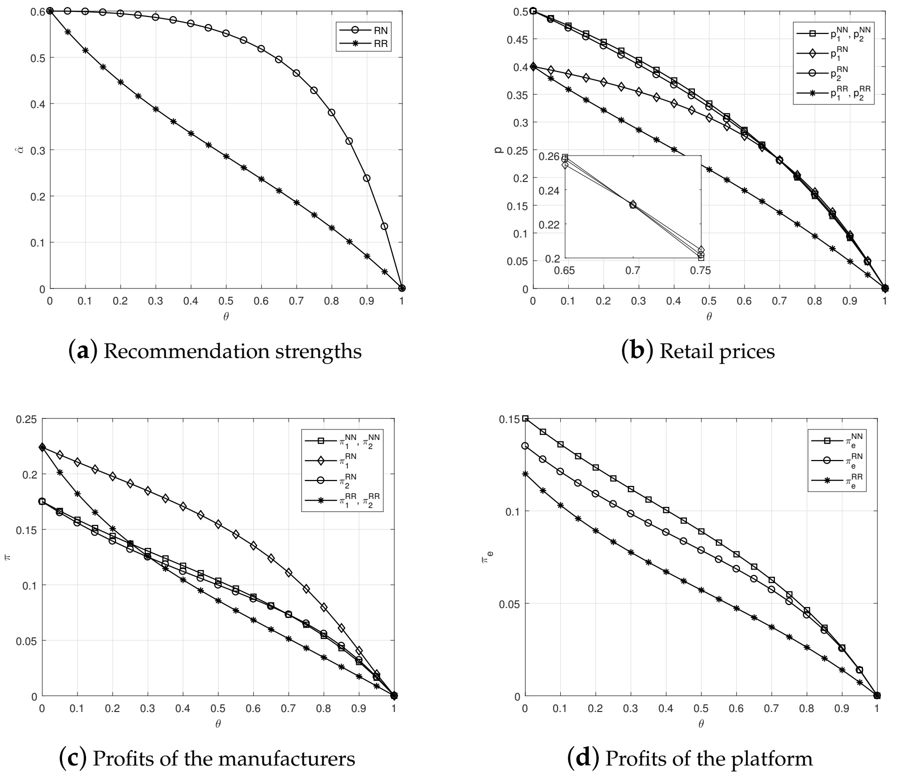
In mode RN, the platform deploys a recommendation system and provides the service to just one manufacturer. Assuming, without loss of generality, that Manufacturer 1's product is recommended, the manufacturers set their retail prices after the platform determines the recommendation strength. The profit functions can be expressed as follows:

$$\pi_1 = (1-\rho)p_1(D_1 + \hat{D}_1), \tag{9}$$

$$\pi_2 = (1-\rho)p_2 D_2, \tag{10}$$

$$\pi_e = \rho(p_1 D_1 + p_1 \hat{D}_1 + p_2 D_2) - k\hat{\alpha}^2. \tag{11}$$

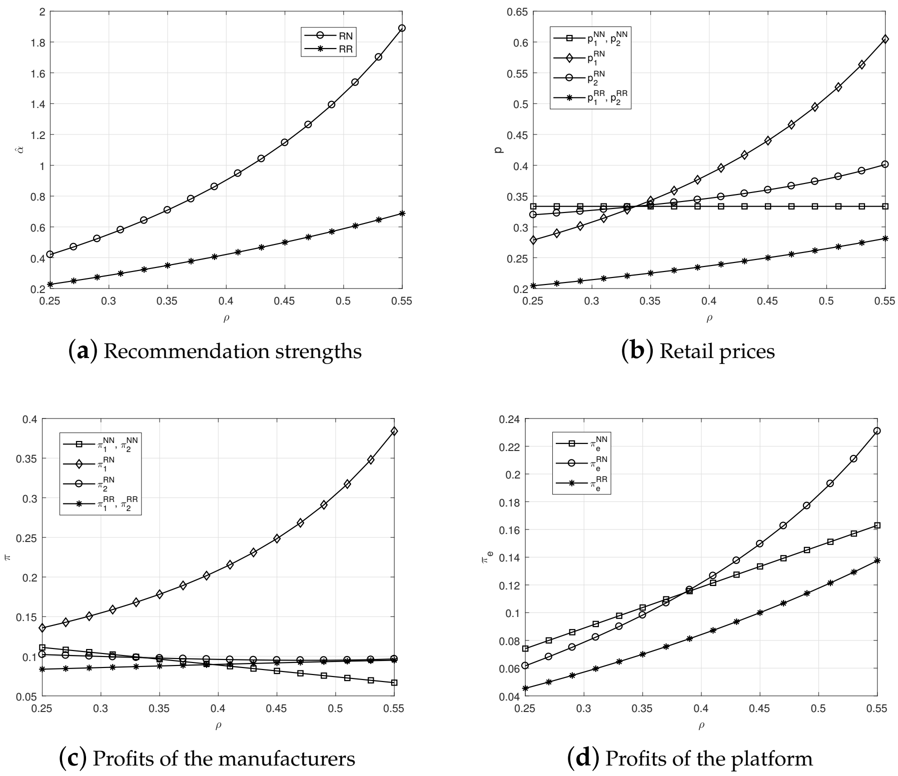
where $k > 0$ is the coefficient representing the cost of the recommendation.



(**a**) Recommendation strengths (**b**) Retail prices

(**c**) Profits of the manufacturers (**d**) Profits of the platform

Figure 1. The impacts of channel competition on equilibrium outcomes ($\alpha = 1, k = 0.1, \rho = 0.3$).



(**a**) Recommendation strengths (**b**) Retail prices

(**c**) Profits of the manufacturers (**d**) Profits of the platform

Figure 2. The impacts of commission rate on equilibrium outcomes ($\alpha = 1, k = 0.1, \theta = 0.5$).

By the backward approach, we obtain the equilibrium in Lemma 2.

Lemma 2. *In model RN, the equilibrium prices and recommendation strength are*

$$p_1^{RN*} = \frac{\alpha(1-\theta)[\theta\rho(1-\theta^2) + k(\theta+2)(8-5\theta^2)]}{k(8-5\theta^2)^2 - \rho(3\theta^4 - 11\theta^2 + 8)}, \tag{12}$$

$$p_2^{RN*} = \frac{\alpha(1-\theta)[k(8-5\theta^2)(4+\theta-2\theta^2) - 2\rho(2-3\theta^2+\theta^4)]}{k(8-5\theta^2)^2 - \rho(3\theta^4 - 11\theta^2 + 8)}, \tag{13}$$

$$\hat{\alpha}^{RN*} = \frac{\alpha\rho[8-\theta^2(-4\theta^2+\theta+11)]}{k(8-5\theta^2)^2 - \rho(3\theta^4 - 11\theta^2 + 8)}, \tag{14}$$

The corresponding equilibrium profits are

$$\pi_1^{RN*} = \frac{(1-\theta)(2-\theta^2)(1-\rho)[k\alpha(\theta+2)(5\theta^2-8) + \alpha\theta\rho(\theta^2-1)]^2}{(\theta+1)[k(8-5\theta^2)^2 - \rho(3\theta^4 - 11\theta^2 + 8)]^2}, \tag{15}$$

$$\pi_2^{RN*} = \frac{(1-\theta)(1-\rho)[k\alpha(5\theta^2-8)(2\theta^2-\theta-4) - 2\alpha\rho(2-3\theta^2+\theta^4)]^2}{(\theta+1)[k(8-5\theta^2)^2 - \rho(3\theta^4 - 11\theta^2 + 8)]^2}, \tag{16}$$

$$\pi_e^{RN*} = \frac{\alpha^2\rho(1-\theta)[k(3\theta^4 - 8\theta^3 - 17\theta^2 + 16\theta + 24) - \rho(2-3\theta^2+\theta^4)]}{(\theta+1)[k(8-5\theta^2)^2 - \rho(3\theta^4 - 11\theta^2 + 8)]}. \tag{17}$$

where $\rho \epsilon [\rho_l^{RN}, \rho_h^{RN}]$, $\rho_l^{RN} = \frac{k(2+\theta)(8-5\theta^2)}{8+7\theta-3\theta^2-3\theta^3}$, $\rho_h^{RN} = \frac{k(2+\theta)(3-2\theta^2)(8-5\theta^2)}{(1-\theta^2)(8+\theta(1-\theta)(5+2\theta))}$.

In Corollary 2, we investigate the effects of product substitutability and commission rate on retail prices and the profits of firms.

Corollary 2. *When the platform recommends only one manufacturer, we obtain the following results regarding the impacts of* θ *and* ρ *on the prices, recommendation strength and profits.*

(i) *The equilibrium prices* p_1^{RN*}, p_2^{RN*} *and recommendation strength* $\hat{\alpha}^{RN*}$ *decrease with product substitutability* θ.

(ii) p_1^{RN*}, p_2^{RN*} *and* $\hat{\alpha}^{RN*}$ *increase with commission rate* ρ. *When* $0 < k < k_1^{RN}$, *or* $k_1^{RN} < k < k_2^{RN}$ *and* $\rho_l^{RN} < \rho < \rho_1^{RN}$, *the profit of manufacturer who is recommended* π_1^{RN*} *increases with* ρ; *when* $k_2^{RN} < k < 1$, *or* $k_1^{RN} < k < k_2^{RN}$ *and* $\rho_1^{RN} < \rho < \rho_h^{RN}$, π_1^{RN*} *decreases with* ρ. *The profit of platform* π_e^{RN*} *increases with* ρ.

Similar to our observations in Corollary 1, Corollary 2 (i) suggests that equilibrium prices decrease with the level of product substitutability. Furthermore, as the product substitutability increases, the platform correspondingly lowers the recommendation strength for the manufacturer's products. This occurs because as the market becomes more competitive, the product similarity and substitutability increase, leading to the platform being less inclined to recommend them and resulting in a reduced recommendation strength. As indicated in Corollary 2 (ii), the commission rate positively impacts equilibrium prices and recommendation grades. This is because an increase in the platform's commission rate leads to higher costs for manufacturers, who may then raise their product prices to maintain their profits.

Additionally, since the platform's profit comes from commissions, the higher the commission rate, the more inclined the platform is to enhance its recommendation strength to sell more products and generate higher profits. The effect of the commission rate on the profit of the manufacturer utilizing the recommendation service depends on the recommendation cost. Specifically, this manufacturer's profit increases with the commission rate when the recommendation cost is low, whereas it decreases when the recommendation cost is high. This can be attributed to the fact that when the recommendation cost is low, the platform can easily implement recommendations to boost sales, thereby increasing the manufacturer's profit. Meanwhile, an increase in the commission rate results in higher prices for manufacturers. However, if the recommendation cost is high, the negative impact on sales due to higher prices outweighs the positive effect on sales resulting from a higher recommendation grade, which is detrimental to the recommended manufacturer.

Moreover, when the recommendation cost is moderate, the profit of the recommended manufacturer initially increases and then decreases as the commission rate rises. This occurs because when the commission rate is low, the positive effect of a higher recommendation grade dominates the negative effect of the increased commission rate and vice versa.

Now that one manufacturer's product is recommended while the other is not, it is worth comparing the prices and profits of these two manufacturers.

Proposition 1. *When the platform recommends only one manufacturer's product, we obtain the following results regarding the prices and profits of the two manufacturers.*

(i) *If* $\rho_l^{RN} < \rho < \frac{2k(8-5\theta^2)}{4+\theta-2\theta^2}$, $p_1^{RN*} < p_2^{RN*}$; *if* $\frac{2k(8-5\theta^2)}{4+\theta-2\theta^2} < \rho < \rho_h^{RN}$, $p_1^{RN*} > p_2^{RN*}$.

(ii) *When* $0 < \theta \le \frac{\sqrt{3}-1}{2}$, *if* $\rho_l^{RN} < \rho < \tilde{\rho}^{RN}$, $\pi_1^{RN*} < \pi_2^{RN*}$; *if* $\tilde{\rho}^{RN} < \rho < \rho_h^{RN}$, $\pi_1^{RN*} > \pi_2^{RN*}$. *When* $\frac{\sqrt{3}-1}{2} < \theta < 1$, $\pi_1^{RN*} > \pi_2^{RN*}$.

where $\tilde{\rho}^{RN} = \frac{(-3\theta^2+4\theta+8)k}{1-\theta^2} - \frac{(-4\theta^3-3\theta^2+8\theta+8)k}{(1-\theta^2)\sqrt{2-\theta^2}}$.

Proposition 1 highlights the relationship between the two manufacturers' prices and profits under mode RN, which is affected by the commission rate and product substitutability. Specifically, when the commission rate is low, the manufacturer whose product is recommended sets a lower retail price than the manufacturer whose product is not recommended. As the recommended manufacturer gains more exposure to consumers, they are willing to set a lower retail price than the non-recommended manufacturer to secure a larger market share and profit when the platform commission is low. On the other hand, as seen in Corollary 2, when the commission rate is high, both manufacturers increase the prices of their products to compensate for the higher cost. However, the recommended manufacturer possesses more market power due to the platform's recommendation. It is more likely to transfer the burden of the increased commission rate to customers by raising prices more aggressively. As a result, the recommended manufacturer charges a higher price than the manufacturer whose product is not recommended.

Furthermore, when the product substitutability and commission rate are both low, the non-recommended manufacturer actually earns more profit than the recommended manufacturer. This indicates that a low-grade commission rate could potentially harm the manufacturer whose product is recommended when competition is not intensive. Nevertheless, as the commission rate increases, the platform becomes more willing to provide a higher grade of recommendation service, which benefits the recommended manufacturer to such an extent that its profit surpasses the non-recommended one. However, when product substitutability is relatively large, the recommended manufacturer consistently obtains greater profit than the non-recommended one, regardless of the commission rate. This suggests that the recommendation service indeed strengthens the competitive position of the manufacturer who receives it when competition is fierce.

4.3. Mode RR

In mode RR, the platform deploys a recommendation system and recommends products from both manufacturers. The manufacturers establish their retail prices following the platform's determination of the recommendation strength. The profit functions can be given as:

$$\pi_1 = (1-\rho)p_1(D_1 + \hat{D}_1), \tag{18}$$

$$\pi_2 = (1-\rho)p_2(D_2 + \hat{D}_2), \tag{19}$$

$$\pi_e = \rho(p_1 D_1 + p_1 \hat{D_1} + p_2 D_2 + p_2 \hat{D_2}) - 2k\hat{\alpha}^2. \quad (20)$$

By the backward approach, we obtain the equilibrium in Lemma 3.

Lemma 3. *In model RR, the equilibrium prices and recommendation strength are*

$$p_1^{RR*} = p_2^{RR*} = \frac{k\alpha(1-\theta^2)(2-\theta)}{2k(2-\theta^2)(1+\theta) - \rho(1-\theta)}, \quad (21)$$

$$\hat{\alpha}^{RR*} = \frac{\alpha\rho(1-\theta)}{2k(2-\theta^2)(1+\theta) - \rho(1-\theta)}, \quad (22)$$

The corresponding equilibrium profits are

$$\pi_1^{RR*} = \pi_2^{RR*} = \frac{2k^2\alpha^2(1-\rho)(1-\theta^2)(2-\theta)^2}{[2k(2-\theta^2)(1+\theta) - \rho(1-\theta)]^2}, \quad (23)$$

$$\pi_e^{RR*} = \frac{2k\alpha^2\rho(1-\theta)}{2k(2-\theta^2)(1+\theta) - \rho(1-\theta)}. \quad (24)$$

where $\rho\epsilon[\rho_l^{RR}, \rho_h^{RR}]$, $\rho_l^{RR} = k(2-\theta)(1+\theta)$, $\rho_h^{RR} = \frac{k(2-\theta)(1+\theta)(3-\theta)}{1-\theta}$.

Corollary 3. *When the platform recommends both manufacturers' products, we obtain the following results regarding the impacts of θ and ρ on the prices, recommendation strength and profits.*

(i) *The equilibrium prices* p_1^{RR*}, p_2^{RR*} *and recommendation strength* $\hat{\alpha}^{RR*}$ *decrease with product substitutability* θ*. The profits of manufacturers* π_1^{RR*}, π_2^{RR*} *and platform* π_e^{RR*} *decrease with* θ*.*

(ii) p_1^{RR*}, p_2^{RR*} *and* $\hat{\alpha}^{RR*}$ *increase with commission rate* ρ*. When* $0 < k < k_1^{RR}$*, or* $k_1^{RR} < k < k_2^{RR}$ *and* $\rho_l^{RR} < \rho < \rho_1^{RR}$, π_1^{RR*} *and* π_2^{RR*} *increase with* ρ*; when* $k_2^{RR} < k < 1$*, or* $k_1^{RR} < k < k_2^{RR}$ *and* $\rho_1^{RR} < \rho < \rho_h^{RR}$, π_1^{RR*} *and* π_2^{RR*} *decrease with* ρ*. Moreover, the profit of platform* π_e^{RN*} *increases with* ρ*.*

When the platform recommends both manufacturers' products, it creates a situation similar to when the platform exclusively recommends one manufacturer. In such cases, as illustrated in Figure 1, the competition between the manufacturers is mainly reflected in the form of price competition, as both manufacturers lower their prices to gain a larger market share. This strategy is ultimately harmful to their profits. Additionally, since the platform's profit is linearly linked to the manufacturers' profits, the platform's profit also reduces as the substitutability between the products increases. The strength of the platform's recommendations and the equilibrium prices increase with the commission rate, which is perceived as a cost by the manufacturers and a source of revenue for the platform. Furthermore, the impact of the commission rate on the manufacturers' profits depends on the cost of the recommendation. If the recommendation cost is low, the manufacturers' profits increase with the commission rate, but they decrease when the recommendation cost is high. If the recommendation cost is moderate, the manufacturers' profits first increase and then decrease with the increase in commission rate. However, the platform always benefits from a higher commission rate as long as the rate is not so high that the manufacturers decide to exit the market.

5. Equilibrium Comparison

Building on our previous analysis and the equilibrium outcomes, this section is dedicated to comparing the equilibrium outcomes under different recommendation modes. This comparison will allow us to explore the optimal strategies for both the manufacturers and the platform. Note that $\rho\epsilon[\rho_l^{RR}, \rho_h^{RN}]$ in the following analysis.

Proposition 2. *$\hat{\alpha}^{RN*} > \hat{\alpha}^{RR*}$ always holds when $\rho_l^{RR} < \rho < \rho_h^{RN}$.*

Proposition 2 states that the grade of recommendation is consistently higher in scenarios where exclusive recommendations are made compared to those where non-exclusive recommendations are made. This can be attributed to the fact that the cost of recommendation not only increases with the grade of recommendation but also increases with the number of manufacturers being served. Therefore, when the platform provides non-exclusive recommendation services for both manufacturers, the level of recommendation is lower than that in the exclusive recommendation scenario.

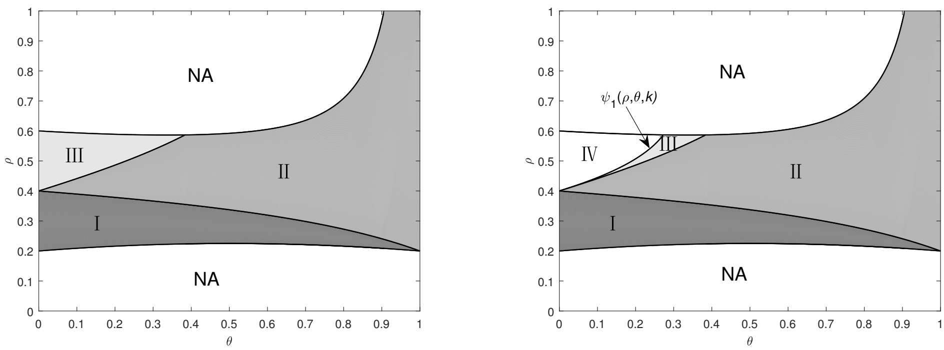
Proposition 3. **(i)** *If $\rho_l^{RR} < \rho < \frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4}$, $p_1^{NN*} > p_1^{RN*}$; if $\frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4} < \rho < \rho_h^{RN}$, $p_1^{NN*} < p_1^{RN*}$.*

(ii) *If $\rho_l^{RR} < \rho < \frac{k(2-\theta^2)(1+\theta)}{1-\theta}$, $p_1^{NN*} > p_1^{RR*}$; if $\frac{k(2-\theta^2)(1+\theta)}{1-\theta}$, $p_1^{NN*} < p_1^{RR*}$.*

(iii) *$p_1^{RN*} > p_1^{RR*}$ always holds.*

Proposition 3 examines the retail price differences between Manufacturer 1 under various scenarios, as depicted in Figure 3a and Table 1. When the platform creates an exclusive recommendation system for Manufacturer 1, the recommended manufacturer sets a higher retail price compared to the non-exclusive recommendation scenario. This occurs because the exclusive recommendation leads to a significant surge in demand, allowing the manufacturer to increase prices and maximize profits. However, in the non-exclusive recommendation case, the recommendation's impact on demand growth is diminished, resulting in a limited effect on price elevation.

The comparison between retail prices with and without recommendations is also influenced by the commission rate. Specifically, when the commission rate is low, the retail price without recommendations is higher than in recommendation cases, whether the recommendation is exclusive or non-exclusive. Conversely, when the commission rate is high, the opposite holds true. This suggests that when the commission rate is low, the recommended manufacturer can set a lower price, and the combined effect of recommendation and low price leads to higher demand. In contrast, a high commission rate necessitates a higher retail price to cover the increased commission expense.



(a) Retail price of the Manufacturer 1 **(b)** Retail price of the Manufacturer 2

Figure 3. Prices comparison ($\alpha = 1, k = 0.1$).

Table 1. Comparison of p_1.

Zone	Comparison
I	$p_1^{NN*} > p_1^{RN*} > p_1^{RR*}$
II	$p_1^{RN*} > p_1^{NN*} > p_1^{RR*}$
III	$p_1^{RN*} > p_1^{RR*} > p_1^{NN*}$

Proposition 4. **(i)** *If* $\rho_l^{RR} < \rho < \frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4}$, $p_2^{NN*} > p_2^{RN*}$; *if* $\frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4} < \rho < \rho_h^{RN}$, $p_2^{NN*} < p_2^{RN*}$.

(ii) *If* $\rho_l^{RR} < \rho < \frac{k(2-\theta^2)(1+\theta)}{1-\theta}$, $p_2^{NN*} > p_2^{RR*}$; *if* $\frac{k(2-\theta^2)(1+\theta)}{1-\theta}$, $p_2^{NN*} < p_2^{RR*}$.

(iii) *If* $\psi_1(\rho,\theta,k) > 0$, $p_2^{RN*} < p_2^{RR*}$; *otherwise,* $p_2^{RN*} > p_2^{RR*}$, *where* $\psi_1(\rho,\theta,k) = \frac{k(2-\theta)(\theta+1)}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k} - \frac{(5\theta^2-8)(2\theta^2-\theta-4)k-2(\theta^4-3\theta^2+2)\rho}{(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k}$.

Proposition 4 compares the retail price of Manufacturer 2 under different scenarios, with Figure 3b and Table 2 providing visual representations for different parameter values. The results demonstrate that Proposition 4 (i) and (ii) share similarities with Proposition 3, indicating that the exclusive recommendation strategy has comparable effects on the retail prices for both recommended and non-recommended manufacturers.

Table 2. Comparison of p_2.

Zone	Comparison
I	$p_2^{NN*} > p_2^{RN*} > p_2^{RR*}$
II	$p_2^{RN*} > p_2^{NN*} > p_2^{RR*}$
III	$p_2^{RN*} > p_2^{RR*} > p_2^{NN*}$
IV	$p_2^{RR*} > p_2^{RN*} > p_2^{NN*}$

However, unlike Proposition 3 (iii), Proposition 4 (iii) reveals that whether a non-recommended manufacturer's retail price will increase after receiving a non-exclusive recommendation depends on the commission rate and the substitutability between products. Specifically, when the commission rate is high and product substitutability is low, the retail price of a manufacturer not recommended under the no recommendation (RN) scenario is lower than when they are non-exclusively recommended under the dual recommendation (RR) scenario. As shown in Corollary 2, prices increase with the commission rate. Nevertheless, the market share of a manufacturer not recommended in the RN scenario is smaller than when it is non-exclusively recommended, allowing it to boost demand by reducing the retail price when products are less substitutable.

In the following step, we analyze the manufacturers' preferences regarding various recommendation scenarios.

Proposition 5. **(i)** *If* $\rho_l^{RR} < \rho < \tilde{\rho}_1$, $\pi_1^{NN*} > \pi_1^{RN*}$; *if* $\tilde{\rho}_1 < \rho < \rho_h^{RN}$, $\pi_1^{NN*} < \pi_1^{RN*}$.

(ii) *If* $\rho_l^{RR} < \rho < \tilde{\rho}_3$, $\pi_1^{NN*} > \pi_1^{RR*}$; *if* $\tilde{\rho}_3 < \rho < \rho_h^{RN}$, $\pi_1^{NN*} < \pi_1^{RR*}$.

(iii) $\pi_1^{RN*} > \pi_1^{RR*}$ *always holds.*

Proposition 5 contrasts Manufacturer 1's equilibrium profits under three distinct recommendation strategies implemented by the platform. Manufacturer 1's profit is higher when no recommendation system is in place and the commission rate is low. As illustrated in Propositions 3 and 4, this occurs because Manufacturer 1 sets a high retail price when the platform does not implement a recommendation system and the commission rate is low. This relationship, however, reverses when the commission rate is high, as can be directly inferred from the results of Propositions 3 and 4.

Furthermore, as shown in Figure 4a, the no recommendation scenario becomes suboptimal for Manufacturer 1 as products become more substitutable. This implies that the manufacturer requires the support of a recommendation system to boost sales when product substitutability is relatively large. Additionally, if the platform develops a recommendation system, Manufacturer 1 consistently earns more profit when exclusively recommended rather than non-exclusively recommended. Exclusive access to the recommendation system's benefits aids the recommended manufacturer by securing a larger market share in the competitive landscape.

Proposition 6. **(i)** *If* $\rho_l^{RR} < \rho < \frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4}$, $\pi_2^{NN*} > \pi_2^{RN*}$; *if* $\frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4} < \rho < \rho_h^{RN}$, $\pi_2^{NN*} < \pi_2^{RN*}$.

(ii) *If* $\rho_l^{RR} < \rho < \tilde{\rho}_3$, $\pi_2^{NN*} > \pi_2^{RR*}$; *if* $\tilde{\rho}_3 < \rho < \rho_h^{RN}$, $\pi_2^{NN*} < \pi_2^{RR*}$.

(iii) *If* $\psi_4(\rho,\theta,\alpha,k) > 0$, $\pi_2^{RN*} > \pi_2^{RR*}$; *otherwise,* $\pi_2^{RN*} < \pi_2^{RR*}$, *where* $\psi_4(\rho,\theta,\alpha,k) = \frac{2k^2\alpha^2(\theta^2-1)(\theta-2)^2}{[(\theta-1)\rho+2(\theta+1)(\theta-2)^2k]^2} + \frac{(1-\theta)[\alpha(5\theta^2-8)(\theta(2\theta-1)-4)k-2\alpha(\theta^4-3\theta^2+2)\rho]^2}{(\theta+1)[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2}$.

Similar to Proposition 5, Proposition 6 shows that when the commission rate is low, Manufacturer 2 achieves higher profits without any recommendation strategies from the platform, and the opposite is true when the commission rate is high. However, unlike Proposition 5, the no recommendation (RN) scenario is not always preferable to the dual recommendation (RR) scenario for Manufacturer 2. Figure 4b visually presents Manufacturer 2's preferences. Specifically, if the platform employs a recommendation system, Manufacturer 2 prefers to be recommended alongside Manufacturer 1 only when products are not substitutable and opts not to be recommended otherwise. This occurs because when the products are less substitutable, consumers may be more inclined to try new products, allowing the recommendation system to increase Manufacturer 2's exposure and attract new customers. However, when the products are substitutable, which means competition is intense, being recommended alongside a competitor may not be advantageous for Manufacturer 2. As demonstrated in Corollary 2, Manufacturer 2's prices and profits decline with increasing product substitutability. Moreover, being recommended together with Manufacturer 1 could result in heightened price competition, negatively impacting both manufacturers' profit margins.

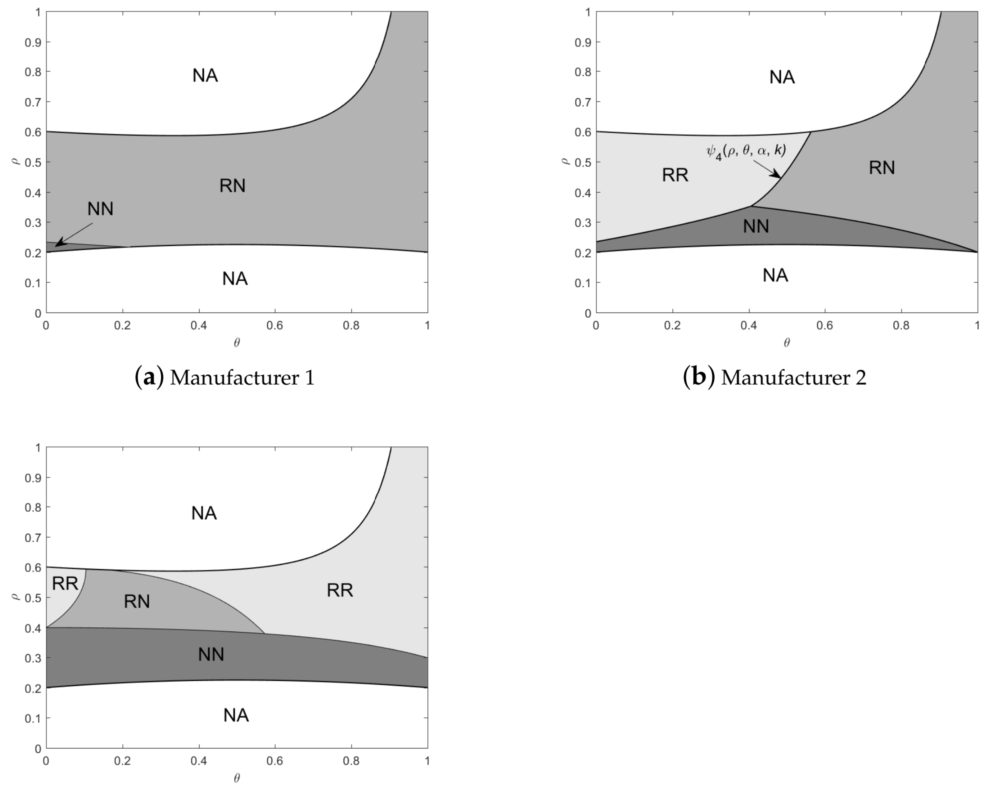
Proposition 7. **(i)** *If* $\rho_l^{RR} < \rho < \frac{k(3\theta^4-20\theta^3-20\theta^2+32\theta+32)}{\theta^4-4\theta^3-4\theta^2+8\theta+8}$, $\pi_e^{NN*} > \pi_e^{RN*}$; *if* $\frac{k(3\theta^4-20\theta^3-20\theta^2+32\theta+32)}{\theta^4-4\theta^3-4\theta^2+8\theta+8} < \rho < \rho_h^{RN}$, $\pi_e^{NN*} < \pi_e^{RN*}$.

(ii) *If* $\rho_l^{RR} < \rho < \frac{k(\theta-2)^2(1+\theta)}{(1-\theta)}$, $\pi_e^{NN*} > \pi_e^{RR*}$; *if* $\frac{k(\theta-2)^2(1+\theta)}{(1-\theta)} < \rho < \rho_h^{RN}$, $\pi_e^{NN*} < \pi_e^{RR*}$.

(iii) *If* $\rho_l^{RR} < \rho < \tilde{\rho}_5$, *we have* $\pi_e^{RN*} > \pi_e^{RR*}$; *if* $\tilde{\rho}_5 < \rho < \rho_h^{RN}$, $\pi_e^{RN*} < \pi_e^{RR*}$.

Proposition 7 uncovers the platform's preferences regarding different recommendation scenarios. When the commission rate is low, the platform benefits more from a no recommendation scenario. This is because a low commission rate means the platform earns minimal revenue per transaction, which may result in less incentive to actively promote manufacturers' products. However, as the commission rate rises, the platform's optimal strategy is to adopt recommendation systems. The comparison then lies between choosing an exclusive or non-exclusive recommendation strategy. When product substitutability is either low or high, the platform prefers a non-exclusive recommendation strategy, while it favors an exclusive recommendation strategy when product substitutability is moderate.

In a highly competitive market, consumers may be more willing to explore a variety of products, rendering an exclusive recommendation strategy less effective. In contrast, a non-exclusive recommendation strategy can expose a broader range of products to customers, offering more purchasing options. If the competition level is low, the substitutability between the two products is minimal, and the platform would want to recommend both manufacturers to consumers. When product substitutability is moderate, the platform may prefer an exclusive recommendation strategy. This is because in a moderately competitive market, consumers may be overwhelmed by the number of products, making it difficult for them to decide and potentially reducing sales. In such a case, an exclusive recommendation strategy can help simplify the purchasing process by limiting the number of options and highlighting a few specific products.



(**a**) Manufacturer 1

(**b**) Manufacturer 2

(**c**) Platform

Figure 4. Supply chain members' strategy preference ($\alpha = 1, k = 0.1$).

6. Conclusions and Discussion

6.1. Conclusions

In the highly competitive market of today, platforms are progressively embracing recommendation strategies to increase sales and improve customer engagement. As a result, pricing strategies under various recommendation scenarios have emerged as a critical consideration for manufacturers. In this study, we examine an online supply chain comprising duopoly manufacturers and an e-commerce platform, which permits manufacturers to sell products in exchange for commission payments. The platform must determine whether to provide recommendation services to manufacturers, either exclusively or non-exclusively, and also decide on the grade of recommendations. After observing the platform's decisions, the duopoly manufacturers set their retail prices according.

In particular, we derive the Stackelberg equilibrium results and examine the influence of commission rate and competition on these results under three scenarios: "no recommendation", where the platform does not implement recommendation systems; "exclusive recommendation", where the platform recommends one of the manufacturers; and "non-exclusive recommendation", where the platform supports both manufacturers. The findings indicate that competition between duopoly manufacturers can heighten price competition and decrease the recommendation strength provided by the platform. Conversely, an increase in commission rate leads to a rise in prices and recommendation strength. Furthermore, when the commission rate is low, the platform may opt not to recommend manufacturers' products, as the low commission income does not offset the cost of offering the recommendation service. When the commission rate is high, adopting a recommendation strategy is the optimal choice for the platform. The selection between exclusive and non-exclusive recommendation strategies depends on specific market conditions and the level of product substitutability. A non-exclusive recommendation strategy may be more effective in markets with low or high product substitutability, while an exclusive recommendation strategy may be more advantageous in a moderately competitive market. Additionally, as long as the commission rate is not too low, recommendations are beneficial to manufacturers. In competitive markets, a manufacturer is better off being exclusively recommended rather than non-exclusively recommended, and the opposite holds true in less competitive markets.

The effects of a recommendation system on the decisions of supply chain members can be summarized as follows. First, regardless of whether a manufacturer is exclusively or non-exclusively recommended, the recommendation system lowers retail prices when the commission rate is low and raises them when the commission rate is high. Second, when exclusively recommended, the manufacturer consistently sets a higher price than when it is non-exclusively recommended alongside another manufacturer. Moreover, under an exclusive recommendation scenario, the non-recommended manufacturer sets a lower price than without any recommendation system when the commission rate is low, and the opposite occurs when the commission rate is high. Interestingly, when the commission rate is high and competition is weak, the retail price of a manufacturer not recommended in an exclusive recommendation scenario is lower than that of a manufacturer recommended under a non-exclusive recommendation scenario. Finally, the platform consistently offers stronger recommendation services when recommending the product of only one manufacturer compared to recommending products from both manufacturers.

6.2. Discussion

In this paper, we focus on the optimal recommendation strategy for the platform and investigate the impacts of recommendation on the decisions of supply chain members. Our innovation lies in the endogenous decision of selecting recommendation grades, which is an aspect that is relatively unexplored in the existing literature about recommendations, such as Zhou et al. (2022) [4] and Wu et al. (2015) [5]. Our findings suggest that duopoly competition can amplify price competition and decrease the intensity of recommendation services provided by the platform. A higher commission rate leads to increased selling prices and a more robust recommendation. When the commission rate is relatively low, the platform may opt not to offer recommendations, as the limited commission income fails to offset the cost of providing such a service. In contrast, when the commission rate is high, recommendations become profitable for the platform, and the choice between an exclusive and non-exclusive recommendation strategy depends on market conditions and competition intensity. A non-exclusive recommendation strategy may be preferred in markets with low or high levels of product substitutability, whereas an exclusive recommendation strategy may be more profitable when product substitutability is moderate. As long as the commission rate is not excessively low, manufacturers can benefit from recommendation services. Furthermore, when product substitutability is relatively high, manufacturers are better off being exclusively recommended than being non-exclusively recommended and vice versa.

In addition, following the works of Ettl et al. (2020) [3] and Wu et al. (2015) [5], this paper investigates the impact of recommendation on pricing strategies. Regardless of whether the manufacturer is exclusively recommended or non-exclusively recommended, the recommendation reduces prices when the commission rate is low and increases prices otherwise. Furthermore, similar to Zhou and Zou (2022) [9], we also explore prices under different recommendation scenarios. Specifically, if the platform develops a recommendation system, the manufacturer always sets a higher price when he is exclusively recommended compared to non-exclusively recommended. The manufacturer not recommended under the exclusive recommendation scenario would set a lower price when the commission rate is low and set a higher price when it is high compared to the no recommendation scenario. In the case of high commission rate and weak competition, the manufacturer not recommended under the exclusive recommendation scenario sets a lower retail price than when recommended under the non-exclusive recommendation scenario. However, the platform always determines a higher exclusive recommendation strength compared to non-exclusive recommendation strength. Our study significantly advances the understanding of the profound impact that different recommendation modes have on the intricate interactions between manufacturers and the platform. As a result, it provides invaluable insights for both business participants and policymakers, empowering them with strategic knowledge and actionable recommendations.

6.3. Future Research Directions

In future research, our study can be expanded in several ways. First, we only considered the competition of manufacturers. In fact, there may be reference price-dependent behavior among manufacturers, and joint pricing between them may improve their bargaining power in the face of the platform, which is a collaboration to some extent. Additionally, the impact of recommendations on consumers might not always be positive; thus, examining the uncertain effects of recommendation strategies on consumer behaviour could provide valuable insights.

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Appendix A

Proof of Corollary 1. $\frac{\partial p_1^{NN*}}{\partial \theta} = \frac{\partial p_2^{NN*}}{\partial \theta} = -\frac{\alpha}{(\theta-2)^2} < 0$, $\frac{\partial \pi_1^{NN*}}{\partial \theta} = \frac{\partial \pi_2^{NN*}}{\partial \theta} = \frac{2\alpha^2(\theta^2-\theta+1)(1-\rho)}{(\theta-2)^3(\theta+1)^2} < 0$, $\frac{\partial \pi_1^{NN*}}{\partial \rho} = \frac{\partial \pi_2^{NN*}}{\partial \rho} = \frac{\alpha^2(\theta-1)}{(\theta-2)^2(\theta+1)} < 0$, $\frac{\partial \pi_e^{NN*}}{\partial \theta} = \frac{4\alpha^2(\theta^2-\theta+1)\rho}{(\theta-2)^3(\theta+1)^2} < 0$, $\frac{\partial \pi_e^{NN*}}{\partial \rho} = \frac{2\alpha^2(1-\theta)}{(\theta-2)^2(\theta+1)} > 0$. □

Proof of Corollary 2. $\frac{\partial \hat{\alpha}^{RN*}}{\partial \theta} = -\frac{\alpha\theta\rho[\theta(3\theta^2-16\theta-24)(\theta-1)^2\rho+(-25\theta^5+90\theta^4-80\theta^3-224\theta^2+192\theta+128)k]}{[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2} < 0$, $\frac{\partial \hat{\alpha}^{RN*}}{\partial \rho} = \frac{k\alpha(8-5\theta^2)^2(4\theta^4-\theta^3-11\theta^2+8)}{[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2} > 0$,
$\frac{\partial p_1^{RN*}}{\partial \theta} = -\frac{\alpha[(\theta^2-1)^2(3\theta^2-16\theta+8)\rho^2+(5\theta^2-4\theta+8)(8-5\theta^2)^2k^2-4(10\theta^6-27\theta^5-3\theta^4+72\theta^3-36\theta^2-48\theta+32)k\rho]}{[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2} < 0$,
$\frac{\partial p_1^{RN*}}{\partial \rho} = \frac{2\alpha(\theta-1)^2[20\theta^6+35\theta^5-57\theta^4-136\theta^3+128\theta+64]k}{[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2} > 0$,
$\frac{\partial p_2^{RN*}}{\partial \rho} = \frac{\alpha(\theta-1)^2\theta[20\theta^6+35\theta^5-57\theta^4-136\theta^3+128\theta+64]k}{[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2} > 0$,
$\frac{\partial p_2^{RN*}}{\partial \theta} = -\frac{\alpha[2(\theta^2-1)^2(3\theta^4-18\theta^2+4\theta+16)\rho^2+(10\theta^4-33\theta^2+8\theta+24)(8-5\theta^2)^2k^2-(80\theta^8-567\theta^6+86\theta^5+1345\theta^4-208\theta^3-1312\theta^2+128\theta+448)k\rho]}{[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2} < 0$,
$\frac{\partial \pi_1^{RN*}}{\partial \rho} = -\frac{\alpha^2(\theta-1)(\theta^2-2)A_1A_2}{(\theta+1)[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^3}$, $A_1 = [\theta(\theta^2-1)\rho + (5\theta^3+10\theta^2-8\theta-16)k]$, $A_2 = [-\theta(\theta^2-1)^2(3\theta^2-8)\rho^2 + (\theta+2)(5\theta^2-8)^3k^2 + 2(5\theta^4-13\theta^2+8)k(\theta^3(9\rho-8) + 3\theta^2(\rho-2) - 16\theta(\rho-1) - 8(\rho-2))]$. When $0 < k < k_1^{RN}$, or $k_1^{RN} < k < k_2^{RN}$ and $\rho_l^{RN} < \rho < \rho_1^{RN}$, $\frac{\partial \pi_1^{RN*}}{\partial \rho} > 0$; when $k_2^{RN} < k < 1$, or $k_1^{RN} < k < k_2^{RN}$ and $\rho_1^{RN} < \rho < \rho_h^{RN}$, $\frac{\partial \pi_1^{RN*}}{\partial \rho} < 0$. Where $k_1^{RN} = \frac{4(\theta^2-1)(-2\theta^3-3\theta^2+5\theta+8)^2}{100\theta^8+335\theta^7-290\theta^6-1906\theta^5-552\theta^4+3512\theta^3+2336\theta^2-2112\theta-1792}$,

$k_2^{RN} = \frac{4(\theta^2-1)(-3\theta^3-3\theta^2+7\theta+8)^2}{120\theta^8+345\theta^7-422\theta^6-1862\theta^5-112\theta^4+3256\theta^3+1568\theta^2-1856\theta-1280}$, $\rho_1^{RN} = \frac{(45\theta^5+15\theta^4-152\theta^3-64\theta^2+128\theta+64)k+\theta(-3\theta^4+11\theta^2-8)\sqrt{H}}{\theta(\theta^2-1)(3\theta^2-8)}$, $H = \frac{k(20\theta^5+15\theta^4-72\theta^3-64\theta^2+64\theta+64)[(120\theta^5+15\theta^4-392\theta^3-64\theta^2+320\theta+64)k-4\theta(3\theta^4-11\theta^2+8)]}{\theta^2(3\theta^4-11\theta^2+8)^2}$. $\frac{\partial\pi_e^{RN*}}{\partial\rho} = \frac{\alpha^2(1-\theta)[(\theta^2-1)^2(3\theta^4-14\theta^2+16)\rho^2+(3\theta^4-8\theta^3-17\theta^2+16\theta+24)(8-5\theta^2)^2k^2-2(\theta^4-3\theta^2+2)(8-5\theta^2)^2k\rho]}{(\theta+1)[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2} > 0$. □

Proof of Corollary 3. $\frac{\partial\hat{\alpha}^{RR*}}{\partial\theta} = \frac{4k\rho\alpha(\theta^3-3\theta^2+3\theta-2)}{[(\theta-1)\rho+2(\theta+1)(\theta-2)^2k]^2} < 0$, $\frac{\partial\hat{\alpha}^{RR*}}{\partial\rho} = \frac{2k\alpha(\theta-2)^2(1-\theta^2)}{[(\theta-1)\rho+2(\theta+1)(\theta-2)^2k]^2} > 0$, $\frac{\partial p_1^{RR*}}{\partial\theta} = \frac{\partial p_2^{RR*}}{\partial\theta} = \frac{k\alpha[(\theta-1)^2(2\theta-1)\rho-2k(-\theta^2+\theta+2]^2]}{[(\theta-1)\rho+2k(\theta+1)(\theta-2)^2]^2} < 0$, $\frac{\partial p_1^{RR*}}{\partial\rho} = \frac{\partial p_2^{RR*}}{\partial\rho} = \frac{k\alpha(2-\theta)(\theta-1)^2(\theta+1)}{[(\theta-1)\rho+2k(\theta+1)(\theta-2)^2]^2} > 0$, $\frac{\partial\pi_1^{RR*}}{\partial\theta} = \frac{\partial\pi_2^{RR*}}{\partial\theta} = -\frac{4k^2\alpha^2(2-\theta)(\theta^2-\theta+1)(1-\rho)[(1-\theta)\rho+2k(\theta+1)(\theta-2)^2]}{[(\theta-1)\rho+2k(\theta+1)(\theta-2)^2]^3} < 0$, $\frac{\partial\pi_1^{RR*}}{\partial\rho} = \frac{\partial\pi_2^{RR*}}{\partial\rho} = \frac{2k^2\alpha^2(\theta-2)^2(\theta^2-1)[2k(\theta-2)^2(\theta+1)-(\theta-1)(\rho-2)]}{[(\theta-1)\rho+2k(\theta+1)(\theta-2)^2]^3}$. When $0 < k < k_1^{RR}$, or $k_1^{RR} < k < k_2^{RR}$ and $\rho_l^{RR} < \rho < \rho_1^{RR}$, $\frac{\partial\pi_1^{RR*}}{\partial\rho} > 0$; when $k_2^{RR} < k < 1$, or $k_1^{RR} < k < k_2^{RR}$ and $\rho_1^{RR} < \rho < \rho_h^{RR}$, $\frac{\partial\pi_1^{RR*}}{\partial\rho} < 0$. Where $k_1^{RR} = \frac{2-2\theta}{3\theta^3-10\theta^2+\theta+14}$, $k_2^{RR} = \frac{2-2\theta}{3\theta^3-8\theta^2-\theta+10}$, $\rho_1^{RR} = 2-\frac{2k(\theta-2)^2(\theta+1)}{1-\theta}$. $\frac{\partial\pi_e^{RR*}}{\partial\theta} = \frac{8k^2\alpha^2\rho(\theta^3-3\theta^2+3\theta-2)}{[(\theta-1)\rho+2k(\theta+1)(\theta-2)^2]^2} < 0$, $\frac{\partial\pi_e^{RR*}}{\partial\rho} = \frac{4k^2\alpha^2(\theta-2)^2(1-\theta^2)}{[(\theta-1)\rho+2k(\theta+1)(\theta-2)^2]^2} > 0$. □

Proof of Proposition 1. Let $p_1^{RN*} - p_2^{RN*} = \frac{\alpha(\theta-1)^2(\theta+1)[(-2\theta^2+\theta+4)\rho+2(5\theta^2-8)k]}{(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k} = 0$, we have $\rho = \frac{2k(8-5\theta^2)}{4+\theta-2\theta^2}$, thus if $\frac{2k(8-5\theta^2)}{4+\theta-2\theta^2} < \rho < \rho_h^{RN}$, $p_1^{RN*} > p_2^{RN*}$; if $\rho_l^{RN} < \rho < \frac{2k(8-5\theta^2)}{4+\theta-2\theta^2}$, $p_1^{RN*} < p_2^{RN*}$.

Let $\pi_1^{RN*} - \pi_2^{RN*} = -\frac{\alpha^2(\theta-1)^2(8-5\theta^2)(1-\rho)[(\theta^4-3\theta^2+2)\rho^2+(8-5\theta^2)^2k^2-2(3\theta^4-4\theta^3-14\theta^2+8\theta+16)k\rho]}{[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2} = 0$, we have $\rho = \tilde{\rho}^{RN} = \frac{(-3\theta^2+4\theta+8)k}{1-\theta^2} - \frac{(-4\theta^3-3\theta^2+8\theta+8)k}{(1-\theta^2)\sqrt{2-\theta^2}}$, thus when $0 < \theta \leq \frac{\sqrt{3}-1}{2}$, if $\rho_l^{RN} < \rho < \tilde{\rho}^{RN}$, $\pi_1^{RN*} < \pi_2^{RN*}$; if $\tilde{\rho}^{RN} < \rho < \rho_h^{RN}$, $\pi_1^{RN*} > \pi_2^{RN*}$. When $\frac{\sqrt{3}-1}{2} < \theta < 1$, $\pi_1^{RN*} > \pi_2^{RN*}$. □

Proof of Proposition 2. $\hat{\alpha}^{RN*} - \hat{\alpha}^{RR*} = \alpha\rho[\frac{\theta^2(-4\theta^2+\theta+11)-8}{(3\theta^4-11\theta^2+8)\rho-(8-5\theta^2)^2k} + \frac{\theta-1}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k}] > 0$ always holds when $\rho_l^{RR} < \rho < \rho_h^{RN}$. □

Proof of Proposition 3. $p_1^{NN*} - p_1^{RN*} = \frac{2\alpha(1-\theta)^2(1+\theta)[2k(-8+5\theta^2)+(4+\theta-2\theta^2)\rho]}{(-2+\theta)[k(8-5\theta^2)^2+(-8+11\theta^2-3\theta^4)\rho]}$, if $\rho_l^{RR} < \rho < \frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4}$, we have $p_1^{NN*} > p_1^{RN*}$; $\frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4} < \rho < \rho_h^{RN}$, we have $p_1^{NN*} < p_1^{RN*}$. $p_1^{NN*} - p_1^{RR*} = \frac{\alpha(1-\theta)}{2-\theta}[1 - \frac{k(\theta-2)^2(\theta+1)}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k}]$, if $\rho_l^{RR} < \rho < \frac{k(2-\theta^2)(1+\theta)}{1-\theta}$, we have $p_1^{NN*} > p_1^{RR*}$; if $\frac{k(2-\theta^2)(1+\theta)}{1-\theta}$, we have $p_1^{NN*} < p_1^{RR*}$. $p_1^{RN*} - p_1^{RR*} = \alpha(\theta-1)[\frac{\theta(\theta^2-1)\rho+(\theta+2)(5\theta^2-8)k}{(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k} - \frac{(\theta-2)(\theta+1)k}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k}] > 0$ always holds when $\rho_l^{RR} < \rho < \rho_h^{RN}$. □

Proof of Proposition 4. $p_2^{NN*} - p_2^{RN*} = \frac{\alpha(\theta-1)^2\theta(\theta+1)[(-2\theta^2+\theta+4)\rho+2k(5\theta^2-8)]}{(\theta-2)[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]}$, if $\rho_l^{RR} < \rho < \frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4}$, we have $p_2^{NN*} > p_2^{RN*}$; if $\frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4} < \rho < \rho_h^{RN}$, we have $p_2^{NN*} < p_2^{RN*}$. $p_2^{NN*} - p_2^{RR*} = p_1^{NN*} - p_1^{RR*} = \frac{\alpha(1-\theta)}{2-\theta}[1 - \frac{k(\theta-2)^2(\theta+1)}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k}]$, so similar to Proposition 3, if $\psi_1(\rho,\theta,k) > 0$, we have $p_2^{NN*} > p_2^{RR*}$; otherwise, $p_2^{NN*} < p_2^{RR*}$. $p_2^{RN*} - p_2^{RR*} = \alpha(\theta-1)[\frac{k(2-\theta)(\theta+1)}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k} - \frac{(5\theta^2-8)(2\theta^2-\theta-4)k-2(\theta^4-3\theta^2+2)\rho}{(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k}]$, let $\psi_1(\rho,\theta,k) = \frac{k(2-\theta)(\theta+1)}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k} - \frac{(5\theta^2-8)(2\theta^2-\theta-4)k-2(\theta^4-3\theta^2+2)\rho}{(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k}$, if $\psi_1(\rho,\theta,k) > 0$, we have $p_2^{RN*} < p_2^{RR*}$; otherwise, $p_2^{RN*} > p_2^{RR*}$. □

Proof of Proposition 5. $\pi_1^{NN*} - \pi_1^{RN*} = \frac{(1-\theta)(1-\rho)}{\theta+1}[\frac{\alpha^2}{(\theta-2)^2} + \frac{(\theta^2-2)(\alpha\theta(\theta^2-1)\rho+\alpha(\theta+2)(5\theta^2-8)k)^2}{((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)^2}]$, let $\psi_2(\rho,\theta,\alpha,k) = \frac{\alpha^2}{(\theta-2)^2} + \frac{(\theta^2-2)(\alpha\theta(\theta^2-1)\rho+\alpha(\theta+2)(5\theta^2-8)k)^2}{((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)^2} = 0$, we can get two thresholds

$\tilde{\rho}_1 = \frac{(8-5\theta^2)(\theta^6-2\theta^5+(8\sqrt{2-\theta^2}-21)\theta^4-2(5\sqrt{2-\theta^2}-6)\theta^3+4(18-7\sqrt{2-\theta^2})\theta^2+16(\sqrt{2-\theta^2}-1)\theta+32(\sqrt{2-\theta^2}-2))k}{(\theta^2-1)(\theta^6-4\theta^5+11\theta^4+8\theta^3-56\theta^2+64)}$ and $\tilde{\rho}_2 = \frac{(8-5\theta^2)(\theta^6-2\theta^5+(8\sqrt{2-\theta^2}-21)\theta^4-2(5\sqrt{2-\theta^2}-6)\theta^3+4(18-7\sqrt{2-\theta^2})\theta^2+16(\sqrt{2-\theta^2}-1)\theta-32(\sqrt{2-\theta^2}+2))k}{(\theta^2-1)(\theta^6-4\theta^5+11\theta^4+8\theta^3-56\theta^2+64)} > \rho_h^{RN}$. Moreover, when $\rho\epsilon[\rho_l^{RR},\rho_h^{RN}]$, $\frac{\partial\psi_2(\rho,\theta,\alpha,k)}{\partial\rho} = \frac{4\alpha^2(\theta^2-2)(20\theta^7+15\theta^6-92\theta^5-79\theta^4+136\theta^3+128\theta^2-64\theta-64)k(\theta(\theta^2-1)\rho+(5\theta^3+10\theta^2-8\theta-16)k)}{((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)^3}$ < 0, therefore, if $\rho_l^{RR} < \rho < \tilde{\rho}_1$, we have $\pi_1^{NN*} > \pi_1^{RN*}$; if $\tilde{\rho}_1 < \rho < \rho_h^{RN}$, $\pi_1^{NN*} < \pi_1^{RN*}$.

$\pi_1^{NN*} - \pi_1^{RR*} = \frac{\alpha^2(1-\rho)}{(\theta-2)^2}[\frac{1-\theta}{\theta+1} + \frac{2k^2(\theta-2)^4(\theta^2-1)}{((\theta-1)\rho+2(\theta+1)(\theta-2)^2k)^2}]$, let $\psi_3(\rho,\theta,k) = \frac{1-\theta}{\theta+1} + \frac{2k^2(\theta-2)^4(\theta^2-1)}{((\theta-1)\rho+2(\theta+1)(\theta-2)^2k)^2} = 0$, we can get two thresholds $\tilde{\rho}_3 = \frac{(2-\sqrt{2})k(1+\theta)(2-3\theta+\theta^2)^2}{(1-\theta)^3}$ and $\tilde{\rho}_4 = \frac{(2+\sqrt{2})k(1+\theta)(2-3\theta+\theta^2)^2}{(1-\theta)^3} > \rho_h^{RN}$. Moreover, when $\rho\epsilon[\rho_l^{RR},\rho_h^{RN}]$, $\frac{\partial\psi_3(\rho,\theta,\alpha,k)}{\partial\rho} = -\frac{4(\theta-2)^4(\theta-1)^2(\theta+1)k^2}{((\theta-1)\rho+2(\theta+1)(\theta-2)^2k)^3} < 0$, therefore, if $\rho_l^{RR} < \rho < \tilde{\rho}_3$, we have $\pi_1^{NN*} > \pi_1^{RR*}$; if $\tilde{\rho}_3 < \rho < \rho_h^{RN}$, $\pi_1^{NN*} < \pi_1^{RR*}$.

$\pi_1^{RN*} - \pi_1^{RR*} = (1-\rho)[\frac{2\alpha^2(\theta^2-1)(\theta-2)^2k^2}{((\theta-1)\rho+2(\theta+1)(\theta-2)^2k)^2} + \frac{(\theta-1)(\theta^2-2)(\alpha\theta(\theta^2-1)\rho+\alpha(\theta+2)(5\theta^2-8)k)^2}{(\theta+1)((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)^2}]$ > 0 always holds, so we have $\pi_1^{RN*} > \pi_1^{RR*}$. □

Proof of Proposition 6. $\pi_2^{NN*} - \pi_2^{RN*} = \frac{(1-\theta)(1-\rho)}{\theta+1}[\frac{\alpha^2}{(\theta-2)^2} - \frac{(\alpha(5\theta^2-8)(\theta(2\theta-1)-4)k-2\alpha(\theta^4-3\theta^2+2)\rho)^2}{((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)^2}]$, if $\rho_l^{RR} < \rho < \frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4}$, we have $\pi_2^{NN*} > \pi_2^{RN*}$; $\frac{2k(8-5\theta^2)}{-2\theta^2+\theta+4} < \rho < \rho_h^{RN}$, we have $\pi_2^{NN*} < \pi_2^{RN*}$.

$\pi_2^{NN*} - \pi_2^{RR*} = \pi_1^{NN*} - \pi_1^{RR*} = \frac{\alpha^2(1-\rho)}{(\theta-2)^2}[\frac{1-\theta}{\theta+1} + \frac{2k^2(\theta-2)^4(\theta^2-1)}{((\theta-1)\rho+2(\theta+1)(\theta-2)^2k)^2}]$, similar to Proposition 5, if $\rho_l^{RR} < \rho < \tilde{\rho}_3$, we have $\pi_2^{NN*} > \pi_2^{RR*}$; if $\tilde{\rho}_3 < \rho < \rho_h^{RN}$, $\pi_2^{NN*} < \pi_2^{RR*}$.

$\pi_2^{RN*} - \pi_2^{RR*} = (1-\rho)[\frac{2k^2\alpha^2(\theta^2-1)(\theta-2)^2}{((\theta-1)\rho+2(\theta+1)(\theta-2)^2k)^2} + \frac{(1-\theta)(\alpha(5\theta^2-8)(\theta(2\theta-1)-4)k-2\alpha(\theta^4-3\theta^2+2)\rho)^2}{(\theta+1)((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)^2}]$, let $\psi_4(\rho,\theta,\alpha,k) = \frac{2k^2\alpha^2(\theta^2-1)(\theta-2)^2}{[(\theta-1)\rho+2(\theta+1)(\theta-2)^2k]^2} + \frac{(1-\theta)[\alpha(5\theta^2-8)(\theta(2\theta-1)-4)k-2\alpha(\theta^4-3\theta^2+2)\rho]^2}{(\theta+1)[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]^2}$, if $\psi_4(\rho,\theta,\alpha,k) > 0$, we have $\pi_2^{RN*} > \pi_2^{RR*}$; otherwise, $\pi_2^{RN*} < \pi_2^{RR*}$. □

Proof of Proposition 7. $\pi_e^{NN*} - \pi_e^{RN*} = -\frac{\alpha^2(\theta-1)^2\rho[(\theta^4-4\theta^3-4\theta^2+8\theta+8)\rho+(-3\theta^4+20\theta^3+20\theta^2-32\theta-32)k]}{(\theta-2)^2[(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k]}$, if $\rho_l^{RR} < \rho < \frac{k(3\theta^4-20\theta^3-20\theta^2+32\theta+32)}{\theta^4-4\theta^3-4\theta^2+8\theta+8}$, we have $\pi_e^{NN*} > \pi_e^{RN*}$; $\frac{k(3\theta^4-20\theta^3-20\theta^2+32\theta+32)}{\theta^4-4\theta^3-4\theta^2+8\theta+8} < \rho < \rho_h^{RN}$, we have $\pi_e^{NN*} < \pi_e^{RN*}$.

$\pi_e^{NN*} - \pi_e^{RR*} = 2\alpha^2(1-\theta)\rho[\frac{1}{(\theta-2)^2(\theta+1)} - \frac{k}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k}]$, if $\rho_l^{RR} < \rho < \frac{k(\theta-2)^2(1+\theta)}{(1-\theta)}$, we have $\pi_e^{NN*} > \pi_e^{RR*}$; if $\frac{k(\theta-2)^2(1+\theta)}{(1-\theta)} < \rho < \rho_h^{RN}$, $\pi_e^{NN*} < \pi_e^{RR*}$.

$\pi_e^{RN*} - \pi_e^{RR*} = \alpha^2(\theta-1)\rho[\frac{(\theta^4-3\theta^2+2)\rho+(\theta(\theta(\theta(8-3\theta)+17)-16)-24)k}{(\theta+1)((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)} + \frac{2k}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k}]$, let $\psi_5(\rho,\theta,k) = \frac{(\theta^4-3\theta^2+2)\rho+(\theta(\theta(\theta(8-3\theta)+17)-16)-24)k}{(\theta+1)((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)} + \frac{2k}{(\theta-1)\rho+2(\theta+1)(\theta-2)^2k} = 0$, we can get two thresholds $\tilde{\rho}_5 = \frac{(-2\theta^6+4\theta^5+19\theta^4-12\theta^3-47\theta^2-\sqrt{4\theta^{12}-16\theta^{11}-36\theta^{10}+88\theta^9+125\theta^8-320\theta^7-554\theta^6+456\theta^5+1393\theta^4+256\theta^3-1040\theta^2-512\theta+64}+24)k}{2(\theta^4-3\theta^2+2)}$, and $\tilde{\rho}_6 = \frac{(-2\theta^6+4\theta^5+19\theta^4-12\theta^3-47\theta^2+\sqrt{4\theta^{12}-16\theta^{11}-36\theta^{10}+88\theta^9+125\theta^8-320\theta^7-554\theta^6+456\theta^5+1393\theta^4+256\theta^3-1040\theta^2-512\theta+64}+24)k}{2(\theta^4-3\theta^2+2)} > \rho_h^{RN}$. Moveover, $\frac{\partial\psi_5(\rho,\theta,\alpha,k)}{\partial\rho} = \frac{\theta^3-\theta^2-2\theta+2}{(-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k} - \frac{(-3\theta^4+11\theta^2-8)((\theta^4-3\theta^2+2)\rho+(\theta(\theta(\theta(8-3\theta)+17)-16)-24)k)}{(\theta+1)((-3\theta^4+11\theta^2-8)\rho+(8-5\theta^2)^2k)^2} - \frac{2(\theta-1)k}{((\theta-1)\rho+2(\theta+1)(\theta-2)^2k)^2} < 0$, therefore, if $\rho_l^{RR} < \rho < \tilde{\rho}_5$, we have $\pi_e^{RN*} > \pi_e^{RR*}$; if $\tilde{\rho}_5 < \rho < \rho_h^{RN}$, $\pi_e^{RN*} < \pi_e^{RR*}$. □

References

1. Hosanagar, K.; Fleder, D.; Lee, D.; Buja, A. Will the global village fracture into tribes? Recommender systems and their effects on consumer fragmentation. *Manag. Sci.* **2014**, *60*, 805–823. [CrossRef]
2. Lee, D.; Hosanagar, K. How do product attributes and reviews moderate the impact of recommender systems through purchase stages?. *Manag. Sci.* **2021**, *67*, 524–546. [CrossRef]
3. Ettl, M.; Harsha, P.; Papush, A.; Perakis, G. A data-driven approach to personalized bundle pricing and recommendation. *Manuf. Serv. Oper. Manag.* **2020**, *22*, 461–480. [CrossRef]
4. Zhou, C.; Leng, M.; Liu, Z.; Cui, X.; Yu, J. The impact of recommender systems and pricing strategies on brand competition and consumer search. *Electron. Commer. Res. Appl.* **2022**, *53*, 1–15. [CrossRef]

5. Wu, H.; Cai, G.; Chen, J.; Sheu, C. Online manufacturer referral to heterogeneous retailers. *Prod. Oper. Manag.* **2015**, *24*, 1768–1782. [CrossRef]
6. Dogan, O. A recommendation system in e-commerce with profit-support fuzzy association rule mining (p-farm). *J. Theor. Appl. Electron. Commer. Res.* **2023**, *18*, 831–847. [CrossRef]
7. Adomavicius, G.; Bockstedt, G.; Curley, S.; Zhang, J. Effects of online recommendations on consumers willingness to pay. *Inf. Syst. Res.* **2018**, *29*, 84–102. [CrossRef]
8. Yoon, N.; Lee, H.-K. Ai recommendation service acceptance: Assessing the effects of perceived empathy and need for cognition. *J. Theor. Appl. Electron. Commer. Res.* **2023**, *16*, 1912–1928. [CrossRef]
9. Zhou, B.; Zou, T. Competing for recommendations: The strategic impact of personalized product recommendations in online marketplaces. *Mark. Sci.* **2022**, *41*, 1–17.
10. Ghose, A.; Mukhopadhyay, T.; Rajan, U. The impact of internet referral services on a supply chain. *Inf. Syst. Res.* **2007**, *18*, 300–319. [CrossRef]
11. Li, L.; Chen, J.; Raghunathan, S. Recommender system rethink: Implications for an electronic marketplace with competing manufacturers. *Inf. Syst. Res.* **2018**, *29*, 1003–1023. [CrossRef]
12. Jeuland, A.P.; Shugan, S.M. Managing channel profits. *Mark. Sci.* **1983**, *2*, 239–272. [CrossRef]
13. Choi, S.C. Price competition in a channel structure with a common retailer. *Mark. Sci.* **1991**, *10*, 271–296. [CrossRef]
14. Huang, H.; Ke, H.; Wang, L. Equilibrium analysis of pricing competition and cooperation in supply chain with one common manufacturer and duopoly retailers. *Int. J. Prod. Econ.* **2016**, *178*, 12–21. [CrossRef]
15. Yu, J.; Zhao, J.; Zhou, C.; Ren, Y. Strategic business mode choices for e-commerce platforms under brand competition *J. Theor. Appl. Electron. Commer. Res.* **2022**, *17*, 1769–1790. [CrossRef]
16. Wang, R.; Nan, G.; Chen, L.; Li, M. Channel integration choices and pricing strategies for competing dual-channel retailers. *IEEE Trans. Eng. Manag.* **2020**, *69*, 2260–2274. [CrossRef]
17. Jena, S. K.; Sarmah, S. Price competition and co-operation in a duopoly closed-loop supply chain. *Int. J. Prod. Econ.* **2014**, *156*, 346–360. [CrossRef]
18. Zhu, X.; Wang, M.; Chen, G.; Chen, X. The effect of implementing trade-in strategy on duopoly competition. *Eur. J. Oper. Res.* **2016**, *248*, 856–868. [CrossRef]
19. Tang, F.; Dai, Y.; Ma, Z.-J.; Choi, T.-M. Trade-in operations under retail competition: Effects of brand loyalty. *Eur. J. Oper. Res.* **2023**, *310*, 397–414. [CrossRef]
20. Wei, J.; Lu, J.; Zhao, J. Interactions of competing manufacturers leader-follower relationship and sales format on online platforms. *Eur. J. Oper. Res.* **2020**, *280*, 508–522. [CrossRef]
21. Ding, Y.; Gao, X.; Huang, C.; Shu, J.; Yang, D. Service competition in an online duopoly market. *Omega* **2018**, *77*, 58–72. [CrossRef]
22. Li, G.; Zhang, X.; Liu, M. E-tailers procurement strategies for drop-shipping: Simultaneous vs. sequential approach to two manufacturers. *Transport. Res. Part E Logist. Transport. Res.* **2019**, *130*, 108–127. [CrossRef]
23. Shen, Y.; Willems, S. P.; Dai, Y. Channel selection and contracting in the presence of a retail platform. *Prod. Oper. Manag.* **2019**, *28*, 1173–1185. [CrossRef]
24. Zhen, X.; Xu, S. Who should introduce the third-party platform channel under different pricing strategies? *Eur. J. Oper. Res.* **2022**, *299*, 168–182. [CrossRef]
25. Mantin, B.; Krishnan, H.; Dhar, T. The strategic role of third-party marketplaces in retailing. *Prod. Oper. Manag.* **2014**, *23*, 1937–1949. [CrossRef]
26. Lu, Q.; Shi, V.; Huang, J. Who benefit from agency model: A strategic analysis of pricing models in distribution channels of physical books and e-books. *Eur. J. Oper. Res.* **2018**, *264*, 1074–1091. [CrossRef]
27. Chen, P.; Zhao, R.; Yan, Y.; Li, X. Promotional pricing and online business model choice in the presence of retail competition. *Omega* **2020**, *94*, 1–13. [CrossRef]
28. Zhou, C.; Li, X.; Ren, Y.; Yu, J. How do fairness concern and power structure affect competition between e-platforms and third-party sellers? *IEEE Trans. Eng. Manag.* **2023**, *2023*, 1–17. [CrossRef]
29. Zhang, J.; Cao, Q.; He, X. Contract and product quality in platform selling. *Eur. J. Oper. Res.* **2019**, *272*, 928–944. [CrossRef]
30. Choi, T.-M.; Guo, S.; Liu, N.; Shi, X. Optimal pricing in on-demand-service-platform-operations with hired agents and risk-sensitive customers in the blockchain era. *Eur. J. Oper. Res.* **2020**, *284*, 1031–1042. [CrossRef]
31. Zhang, T.; Li, P.; Wang, N. Multi-period price competition of blockchain-technology-supported and traditional platforms under network effect. *Int. J. Prod. Res.* **2021**, *61*, 3829–3843. [CrossRef]
32. Anderson, E.G., Jr.; Parker, G.G.; Tan, B. Platform performance investment in the presence of network externalities. *Inf. Syst. Res.* **2014**, *25*, 152–172. [CrossRef]
33. Taylor, T.A. On-demand service platforms. *Manuf. Serv. Oper. Manag.* **2018**, *20*, 704–720. [CrossRef]
34. Zha, Y.; Li, Q.; Huang, T.; Yu, Y. Strategic information sharing of online platforms as resellers or marketplaces. *Mark. Sci.* **2022**, *2022*, 1–99. [CrossRef]
35. Tsunoda, Y.; Zennyo, Y. Platform information transparency and effects on third-party suppliers and offline retailers. *Prod. Oper. Manag.* **2021**, *30*, 4219–4235. [CrossRef]
36. Martinez-de Albéniz, V.; Pinto, C.; Amorim, P. Driving supply to marketplaces: Optimal platform pricing when suppliers share inventory. *Manuf. Serv. Oper. Manag.* **2022**, *24*, 2367–2386. [CrossRef]

37. Spence, M. Product differentiation and welfare. *Am. Econ. Rev.* **1976**, *66*, 407–414.
38. Cai, G.; Dai, Y.; Zhou, S. Exclusive channels and revenue sharing in a complementary goods market. *Mark. Sci.* **2012**, *31*, 172–187. [CrossRef]