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Can the Conditional Rebate Strategy Work? Signaling Quality via Induced Online Reviews

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Abstract: Online reviews are an important part of product information and have important effects on consumers' purchasing decisions. Some sellers try to manipulate the market by inducing online reviews. In this study, a signal game model based on Bayesian conditional probability is constructed to analyze the preconditions, decision-making process, and effect on market demand and profit of this behavior. The results show that first, when consumer sensitivity to rebates reaches a certain threshold, low-quality sellers will adopt a conditional rebate strategy to induce consumers to give positive reviews. Second, the optimal rebate cost (β^*) is obtained, where β^* increases with the product price (p), but it is not necessarily monotonic in consumers' sensitivity to rebates (ρ) or the proportion of high-quality products (α). Third, the conditional rebate strategy can only work in a market dominated by low-quality goods. Using the conditional rebate strategy in a market dominated by high-quality goods will not bring benefits to low-quality sellers but will harm their profits. This study proposes that some developing online markets have collusive behaviors owing to a lack of regulations and laws, as well as consumers' concern for small interests. Ensuring the orderly development of online markets will require joint efforts by platform enterprises, government agencies, and consumers.

Keywords: induced online review; conditional rebate strategy; online market; quality signaling

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1. Introduction

In 2021, retail e-commerce sales amounted to approximately USD 5.2 trillion worldwide. This figure is forecasted to grow by 56 percent over the next years, reaching about USD 8.1 trillion by 2026 [1]. Against the backdrop of e-commerce retail progressively encompassing a larger share of the overall retail landscape, electronic word of mouth (eWOM) has emerged as a pivotal domain for merchants. Electronic word of mouth is incredibly important when shopping online [2]. Consumers refer to online reviews of goods and services to reduce risk and uncertainty in their purchasing decisions [3]. Park et al. (2007) [4] suggested that online reviews are credible because the opinions about and descriptions of products posted on Internet forums come from other consumers who are considered to have no desire to manipulate readers. Therefore, when making decisions, online shoppers are more likely to trust information provided by independent, credible Internet users than by merchants [5]. Free and easy access to such information has weakened the power of marketing communication, as information provided by online peers influences customer perceptions, preferences, and decisions much more than information provided by companies [6].

Given consumers' considerable reliance on online reviews, a growing number of online sellers are making efforts to influence buyers' review behaviors, motivating them to submit positive feedback. For instance, on certain online platforms, sellers have recently adopted a conditional rebate strategy to manipulate the tone of online reviews. The conditional-rebate strategy revolves around offering customers a rebate or discount on a product, contingent on them leaving a positive online review after their purchases. These induced reviews can enhance both the quality and quantity of reviews, which can improve store rating rankings and brand reputations [7]. Several Chinese e-commerce platforms, including Alibaba, have witnessed a proliferation of sellers employing the conditional rebate strategy to elicit consumer reviews. This phenomenon is particularly pronounced in the realm of experienced goods, in stark contrast to search goods. Drawing from our earlier comprehensive survey of China's e-commerce landscape, a discernible pattern emerges: sellers embracing the conditional rebate approach often gravitate toward vending low-quality products. Conversely, purveyors of high-quality merchandise frequently manage to cultivate a positive reputation without resorting to such manipulative tactics. Although specific regulations have been implemented, their impact has been limited due to the strategy's nature of covert collusion between sellers and buyers. Insufficient attention has been devoted to this phenomenon in research, leaving significant gaps in understanding key aspects such as the mechanics behind sellers' adoption of conditional rebate strategies and the appropriate regulatory measures that platforms should undertake.

Targeted regulations have been introduced, but they have had little effect because the strategy is a type of hidden collusion between sellers and buyers. Generally, sellers who adopt the conditional rebate strategy sell low-quality goods. Sellers of high-quality goods, meanwhile, are generally able to gain a good reputation without using such strategies. Thus, the key concern with conditional rebates is that they can easily induce fake reviews, which can have negative effects on the overall e-commerce market. However, research has paid insufficient attention to this phenomenon, and problems such as the mechanism of sellers' behavior in adopting conditional rebate strategies and how platforms should regulate them remain to be studied. This study, therefore, aims to answer the following research questions:

1. Under what conditions will online sellers adopt a conditional rebate strategy to obtain positive online reviews?
2. When using such rebates, what is the optimal cost to sellers to maximize profit?
3. How does the conditional rebate strategy affect different kinds of sellers' profits?

To answer these questions, we created a two-stage model with sellers who sell experience goods. Product quality is the seller's private information, and it may be high or low. An h-type (or l-type) seller sells a product of high (or low) quality. Consumers are uninformed about the product quality in stage 1 but can obtain a signal about the product quality in stage 2 from early consumers through online reviews. Our research outcomes reveal several key insights. Firstly, the manipulation of comments through the conditional rebate strategy disseminates erroneous signals about product quality throughout the market, inducing disorder. However, this strategy only proves effective when consumers exhibit a certain level of sensitivity toward the rebate. Secondly, the conditional rebate strategy's effectiveness is confined to scenarios where low-quality goods hold a dominant position within the market. Our study stands among the earliest examinations of conditional rebate behavior, exploring it through the lens of interference in quality signals. This enriches the existing body of research on quality signals in e-commerce transactions. By employing a two-stage mathematical model built on Bayesian conditional probability, this paper achieves a precise characterization of conditional rebate behavior, incorporating innovative ideas.

The rest of this paper is organized as follows. Section 2 reviews the literature. Section 3 introduces our benchmarking model in the case of no rebate and in the case of receiving positive reviews through rebate strategies. In Section 4, we calculate and analyze the seller's rebate decisions and profits, and the factors that affect the seller's profit are discussed. A numerical study is presented in Section 5. The conclusions and managerial implications are presented in Section 6.

2. Literature Review

This work examines a new rebate mechanism used to encourage online product reviews. This study is closely related to three research streams: rebates, online reviews, and quality signaling.

Consumer rebates have been explored in the economics and marketing literature [8]. Promotional tools such as rebates and coupons present different ways to price discriminate against consumers [9]. Chen et al. (2007) [10] examined the effect of manufacturer rebates on the attraction of end customers and found that unless all customers claim the rebate, the rebate always benefits the manufacturer. Ho et al. (2017) [11], however, studied the online cash-back mechanism, which reimburses the consumer for part of the transaction amount in the form of cash back, a form of rebate that does not always profit the consumer. Cash-back offers increase the likelihood that consumers will make additional purchases and increase the size of those purchases [12]. Qiu and Rao (2020) [13] studied the effect of cash-back sites on retailers and found that cash-back sites can serve as strategic partners for retailers, making them profitable and stickier. Surprisingly, in some cases, consumers who use cash-back sites pay higher prices. Unlike the existing literature on rebates, which mostly focuses on unconditional rebates, we focus on rebates received by consumers on e-commerce platforms who are induced by sellers to give positive reviews. In other words, people who leave positive reviews receive rebates. This is a way for sellers on e-commerce platforms to control online word-of-mouth.

Online reviews play an important role in consumers' online shopping decision processes and have been widely studied. Chatterjee (2001) [14] first proposed a definition of online review usefulness, namely the degree of impact of online review information. For consumers, searching for reviews before shopping online helps them understand the risks associated with a purchase, thus offering a powerful source of information for consumers [15,16]. Adopting a behavioral and psychological perspective on reviewers, Racherla et al. (2013) [17] found that consumers uphold the principle of not hurting others and are more likely to write positive reviews. The effect of aggregate measures of reviews (e.g., average product ratings and number of reviews) on consumer behavior has been previously investigated. It has been shown that after considering average product ratings, individual reviews also have a strong influence on consumers' purchase decisions [16]. For firms, reviews have a significant effect on product sales [18]. Empirical studies usually find that negative reviews affect sales to a greater extent than positive ones [19]. With the rapid growth of online media, the potential impact of negative reviews is increasing, as anonymous reviews by individuals allow for more authentic sharing of negative experiences that does not lead to social consequences. In this context, companies are beginning to use marketing tools such as promotions to reduce the impact of negative word of mouth on their sales. Previous research has found that manipulating consumer reviews always benefits the company but only when the manipulation is weak [20]. As a result, some platforms try to offer financial incentives to encourage users to contribute [21], where sellers offer rebates to buyers to cover the cost of feedback reports from buyers, regardless of whether the feedback is positive or negative. Studies suggest that buyers will engage in favorable returns with sellers if they offer feedback rebates [22]. Therefore, sellers can "buy" feedback, but such feedback is likely to be biased [23]. It can be seen that the phenomenon of conditional rebates has attracted the attention of scholars. Nonetheless, research pertaining to conditional rebate behavior and its consequential effects on the market remains relatively limited and necessitates further exploration. Overall, eWOM fundamentally serves as quality signals, while merchants engaging in review manipulation essentially disrupt this quality signaling process. Therefore, this paper intends to carry out research based on signaling theory.

In general, product quality is the firm's private information, which creates information asymmetry. Signaling theory came into being to help investigate this problem. Signaling theory is useful for describing behavior when two parties have access to different information. Typically, one party, the sender, must choose whether and how to communicate (or signal) that information, and the other party, the receiver, must choose how to interpret the signal [24]. Recent research has focused on signals as mechanisms to solve problems that arise under asymmetric information. A firm or individual credibly communicates the level of some unobservable element in a transaction by providing an observable signal (e.g., the quality of goods). The most important signal for product quality is the price. Bagwell and Riordan (1991) [25] showed that firms with private information about quality can indicate quality through the price, but frequent price reductions can have a significantly negative effect on brand equity [26]. Zhao (2000) [27] showed that advertising serves as an effective signal of quality and that only high-quality firms can afford to advertise. Therefore, high-quality firms can use advertising expenditures to avoid imitation by low-quality firms and gain greater profits [28,29], and consumers can infer the quality of products through advertising expenditures [27]. In online shopping, return insurance can also be used as a quality signal, with buyers inferring quality from the retailer's price and insurance adoption and insurance companies strategically choosing premiums [30]. Shen et al. (2011) [31] investigated the effect of seller reputation, product condition, and review quality on consumer purchase decisions by classifying the information on online auction pages based on signaling theory. Moreover, Wells et al. (2011) [32] investigated website quality as a potential signal of product quality and considered the moderating effects of product information asymmetry and signal credibility. They found that website quality influences consumers' perceptions of product quality and, in turn, their willingness to purchase online.

The work that is most closely related to ours is the literature examining online platform quality management [33,34]. In particular, Chen et al. (2022) [35] developed a microbehavioral model capturing consumers' review-sharing benefits, review-posting costs, and moral cost of lying to examine the seller's optimal pricing and rebate decisions.The results indicated that it is not always profitable for strategic sellers to employ conditional rebate strategies. Even when a conditional rebate strategy is adopted, it does not always result in fake reviews.

Thus, although some studies have analyzed the role of online reviews in reliably singling product quality, little attention has been paid to the recent phenomenon of induced reviews. The conditions for sellers to implement conditional rebate strategies and their effects on the market are also in need of investigation. In this study, we use a signaling game model with Bayesian conditional probability to describe consumers' purchasing behavior in response to positive review incentives. Then, through a two-stage purchase behavior analysis, the market demand of high- and low-quality sellers is obtained, and the optimal rebate cost is obtained by optimizing the objective function. We compare changes in the profits of the two types of sellers under the conditions of the implementation and non-implementation of a conditional rebate strategy. In this way, our work helps to explore the mechanism, effect, and regulation of the review-inducing behavior of online sellers and contributes to enriching the literature on signaling theory.

3. Models

First, we briefly describe the platform for online sellers and buyers and then specify how quality information about the product is updated. We then analyze the two-stage purchase decision and the game between sellers and buyers.

3.1. The Online Sellers

The existence of differing quality for the same new product on an e-commerce platform is expressed in terms of h-type products with a probability α and l-type products with a probability $1-\alpha$. We use $j \in \{h, l\}$ to label the types of products in the market, and $q \in (0,1)$, where $q_l < q_h$. The true value of q is the seller's private information and is not initially known by the buyers [30,36]. The l-type online sellers may offer a rebate ($\beta(\beta > 0)$) to each customer who leaves a positive review online to encourage more buyers to leave positive reviews.

In practice, the seller does not publicly publish the rebate information (including its existence and quantity) in advance, and the incentive is provided to the customer privately. We therefore assumed this as a general rule.

In undertaking this conditional rebate, the seller's cost includes not only the rebate (discount) itself given to the customer in exchange for a positive review but also the seller's operating and management costs in communicating with each buyer who purchases the product and has the potential to give a positive review. Without any loss of generality, the seller's unit production cost was normalized to zero.

3.2. Information Updates about Quality

Consumers in stage 1 can only access a prior probability of the product's quality [37], but consumers in stage 2 can obtain a posterior quality probability because they can receive a signal about the quality through user-generated reviews by first-stage consumers.

The signal s has two possible values—s_h and s_l—which refer to positive and negative reviews, respectively.

According to practice, consumers are more likely to give positive reviews when buying high-quality goods, and lower quality producers will spend more resources to manipulate reviews [38]. $P(s_h|q_h) = 1$ was assumed. Let m be the accuracy of the signal of l-type products (i.e., $P(s_l|q_l) = m$). Similar information structures are commonly used to model imperfect information [39–41]. The Bayesian-updated probabilities of quality conditional on the signals s_h and s_l are

$$
\begin{aligned}
&P(s_l|q_h) = 0, P(s_h|q_l) = 1 - m,\\
&P(s_h) = P(q_h) \cdot P(s_h|q_h) + P(q_l) \cdot P(s_h|q_l) = \alpha + (1-\alpha)(1-m),\\
&P(q_h|s_h) = \frac{P(q_h) \cdot P(s_h|q_h)}{P(s_h)} = \frac{\alpha}{\alpha + (1-\alpha)(1-m)},\\
&P(q_l|s_h) = \frac{P(q_l) \cdot P(s_h|q_l)}{P(s_h)} = \frac{1-\alpha}{\alpha + (1-\alpha)(1-m)}.
\end{aligned}
\tag{1}
$$

3.3. Consumers' Decisions

We considered a two-stage model and assumed an independent consumer group of a size of one in each stage. In the first stage, consumers make purchase decisions based only on the price and expected product valuation without online reviews. At the end of the first stage, the customers who made purchases in the first stage will give or not give comments about the product, and they will give positive or negative comments [42]. In the second stage, customers read online reviews to update their expectations about the product's value and make purchase decisions. The expected consumer utility is

$$U = v + E[q] - p. \tag{2}$$

Prior to consumption, a consumer's utility is specified as having three components. The first component, v, is a baseline utility that captures benefits from consumption in this product category. Assume that the consumers are heterogeneous in terms of this baseline utility v such that v is uniformly distributed over a range [0, 1]. The second component is the product quality expected by consumers $E[q]$, which is dependent on the overall level of product quality in the market, where $E[q] = \alpha q_h + (1-\alpha)q_l = \mu$. The third component of consumer utility is an exogenously specified fixed price p paid by the consumer to purchase the good.

All notations are given in Table 1, where β is used as the decision variable.

3.4. Gaming

The game consisted of two stages (Figure 1 summarizes the sequence of events). Before the game started, it was assumed that all consumers were aware of the existence of the new product and the distribution of its ex ante quality probabilities but not of the true quality type. At the beginning of the first stage, the natural choice of type is made, but only the seller on the platform is aware of the product quality. Consumers have alternative credible avenues to assess product quality, notably through online reviews. As such, the online reviews served as the quality signal. Consumers in the first period make purchase decisions based on ex ante quality probabilities. Sellers send products to consumers, who purchase them with a positive review conditional rebate card, and the consumers receive the product as well as the information about the conditional rebate. Then, they decide whether to put a positive review online. The seller and buyer communicate privately regarding the rebate. In the second period, there are some comments on the products, and buyers enter the marketplace, update product quality information based on quality signals, and make purchase decisions.

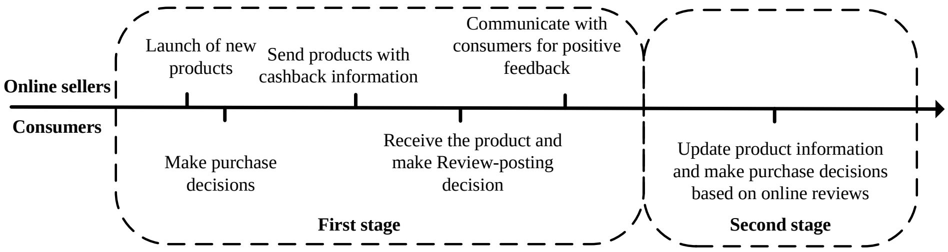

Figure 1. Timing of the game.

Table 1 summarizes the notations for the variables and parameters.

Table 1. Notations for variables and Parameters.

Notation	Description
j	Product type, $j \in \{h, l\}$
q_j	j-type product quality
α	Probability that the product is type h
$1-\alpha$	Probability that the product is type l
s_j	Signal type
ρ	Sensitivity factor of consumers to rebate for positive reviews
λ	Coefficient of improvement in information accuracy brought by sales of the l-type product in the first stage
β	Cost of rebate for positive reviews
v	Consumers' baseline utility
$E[q]$	Quality of products expected by consumers
p	Price of product
U	Consumer utility
d_{ij} (d_{ij}^n)	Demand for the j-type product at period i with (without) the rebate strategy
Π_j (Π_j^n)	j-type seller's total profit with (without) rebate strategy

4. Equilibrium Analysis

In this section, we first consider a benchmark case in which the sellers do not adopt the conditional rebate strategy. Then, we characterize the case with the conditional rebate strategy. Finally, we investigate the effect of conditional rebates on platform sellers' equilibrium profit through a comparison of the two scenarios.

4.1. Benchmark Case without the Rebate Strategy

We considered a platform seller who did not adopt the rebate strategy. We assumed each consumer who purchased a product in the first period posted a review online. The amount of such information was assumed to be able to improve the informativeness of the signal of l-type products; that is, $m = \lambda d_{1l}^n$, where λ is the improvement to the signal accuracy from a unit sale of the first-stage l-type product and d_{1l}^n denotes the demand for l-type products in the first period without the rebate (i.e., the negative reviews of l-type products become more prevalent as consumers' purchase volume increases).

Thus, we assumed $P^n(s_h|q_h) = 1$, $P^n(s_l|q_l) = \lambda d_{1l}^n$. The Bayesian-updated probabilities of quality conditional on the signals s_h and s_l are

$$\begin{aligned}
&P^n(s_l|q_h) = 0, P^n(s_h|q_l) = 1 - \lambda d_{1l}^n,\\
&P^n(s_h) = P(q_h) \cdot P^n(s_h|q_h) + P(q_l) \cdot P^n(s_h|q_l) = \alpha + (1-\alpha)(1-\lambda d_{1l}^n),\\
&P^n(q_h|s_h) = \frac{P(q_h) \cdot P^n(s_h|q_h)}{P(s_h)} = \frac{\alpha}{\alpha + (1-\alpha)(1-\lambda d_{1l}^n)},\\
&P^n(q_l|s_h) = \frac{P(q_l) \cdot P^n(s_h|q_l)}{P(s_h)} = \frac{(1-\alpha)(1-\lambda d_{1l}^n)}{\alpha + (1-\alpha)(1-\lambda d_{1l}^n)}.
\end{aligned} \tag{3}$$

The consumer's utility $U = v + E[q] - p$, and it follows that the aggregate demand function is $d = Prob\{U(v) > 0\} = \int_{p-E[q]}^{1} 1dv = 1 - p + \mu$. Therefore, the demand for h-type products in the first stage is $d_{1h}^n = P(q_h)d = \alpha(1-p+\mu)$, and the demand for l-type products is $d_{1l}^n = P(q_l)d = (1-\alpha)(1-p+\mu)$. In the second stage, consumers update their product quality expectations based on online reviews, where the demand for h-type products in the second stage is $d_{2h}^n = P^n(q_h|s_h)d = \frac{\alpha}{\alpha+(1-\alpha)(1-\lambda d_{1l}^n)}(1-p+\mu)$ and the demand for l-type products is $d_{2l}^n = P^n(q_l|s_h)d = \frac{(1-\alpha)(1-\lambda d_{1l}^n)}{\alpha+(1-\alpha)(1-\lambda d_{1l}^n)}(1-p+\mu)$.

The seller's expected profit across the two stages is as follows:

$$\Pi_j^n = pd_{1j}^n + pd_{2j}^n, \tag{4}$$

where the first and second terms on the right-hand side are the revenues from purchasing consumers in the first and second periods.

Proposition 1. *Without the rebate strategy, the profit of l-type merchants decreases with the increase in the coefficient of improvement in information accuracy brought by sales of the l-type product in the first stage. The profit of high-quality merchants increases with the increase in information accuracy; that is,* $\frac{\partial \Pi_l^n}{\partial \lambda} < 0$, $\frac{\partial \Pi_h^n}{\partial \lambda} > 0$.

Proof: All proofs are given in Appendix A.

Proposition 1 shows that without the conditional rebate strategy, consumers entering the market in the first stage posted real reviews based on their consumption experience, and buyers entering the market in the second stage made purchase decisions based on real and credible product comments and reviews. The more accurate the information given by the first-stage customers, the lower the probability that an l-type seller would receive positive reviews in the first period, which in turn reduced the sales of l-type products in the second stage. As the accuracy of information increased, the probability of h-type sellers receiving positive reviews in the first period increased, which in turn increased the sales volume of h-type products in the second stage.

4.2. Benchmark Case with Rebate Strategy

Here, we consider the adoption of a positive review conditional rebate strategy by platform sellers. We assumed that for l-type products, consumers gave positive reviews influenced by the sensitivity of the rebate, and we therefore assumed that $m = 1 - \rho\beta$; that is, $P(s_l|q_l) = 1 - \rho\beta$, where $\rho(0 < \rho \leq 1)$ indicates the consumer's rebate sensitivity and β $(0 < \beta < p)$ indicates the rebate cost. The Bayesian-updated probabilities of quality conditional on signals s_h and s_l are

$$
\begin{aligned}
&P(s_l|q_h) = 0, P(s_h|q_l) = \rho\beta, \\
&P(s_h) = Pr(q_h) \cdot P(s_h|q_h) + P(q_l) \cdot P(s_h|q_l) = \alpha + (1-\alpha)\rho\beta, \\
&P(q_h|s_h) = \frac{P(q_h) \cdot P(s_h|q_h)}{P(s_h)} = \frac{\alpha}{\alpha + (1-\alpha)\rho\beta}, \\
&P(q_l|s_h) = \frac{P(q_l) \cdot P(s_h|q_l)}{P(s_h)} = \frac{(1-\alpha)\rho\beta}{\alpha + (1-\alpha)\rho\beta}.
\end{aligned} \tag{5}
$$

Similar to Equation (3), the demand for h-type products in the first stage is $d_{1h} = P(q_h)d = \alpha(1-p+\mu)$, and the demand for l-type products is $d_{1l} = P(q_l)d = (1-\alpha)(1-p+\mu)$. The demand for h-type products in the second stage is $d_{2h} = P(q_h|s_h)d = \frac{\alpha}{\alpha+(1-\alpha)\rho\beta}(1-p+\mu)$, and the demand for l-type products is $d_{2l} = P(q_l|s_h)d = \frac{(1-\alpha)\rho\beta}{\alpha+(1-\alpha)\rho\beta}(1-p+\mu)$, where $P(q_h|s_h)d(P(q_l|s_h)d)$ indicates the number of buyers in the second stage who updated their quality expectations based on the positive reviews and purchased the real quality in the form of h (l)-type products.

The sellers' expected profits across the two stages are as follows:

$$
\begin{aligned}
\Pi_h &= pd_{1h} + pd_{2h}, \\
\Pi_l &= pd_{1l} + pd_{2l} - \beta d_{1l},
\end{aligned} \tag{6}
$$

where pd_{ij} is the revenue from sales in the first and second stages and βd_{1l} represents the cost of the rebate to the buyers induced to post positive reviews. The l-type seller controls its rebate costs to maximize profits. The profit of l-type sellers on the platform is equal to the sales revenue in both periods minus the cost of the conditional rebate, and its decision model is

$$
\max_{\beta} \Pi_l = pd_{1l} + pd_{2l} - \beta d_{1l}, \tag{7}
$$

where $\Pi_l^* \geq \Pi_l^n$ is the incentive compatibility condition; that is, the l-type seller can make more profit by adopting the conditional rebate strategy.

Proposition 2. *In equilibrium, l-type sellers adopt the rebate strategy for positive reviews if* $\rho \geq \bar{\rho}$, *where* $\bar{\rho} = \frac{\alpha}{(\sqrt{A}-1)^2 p}$, $A = \frac{(1-\alpha)(1-\lambda(1-\alpha)(1-p+\mu))}{\alpha+(1-\alpha)(1-\lambda(1-\alpha)(1-p+\mu))}$, *and* $0 < \alpha < p$. *The l-type sellers' optimal decision and profit are as follows:*

$$
\begin{cases}
\beta^* = \frac{\sqrt{p\rho\alpha}-\alpha}{(1-\alpha)\rho}, \\
d_{2l}^* = \frac{\sqrt{p\rho\alpha}-\alpha}{\sqrt{p\rho\alpha}}(1-p+\mu), \\
\Pi_l^* = p((1-\alpha)(1-p+\mu) + \frac{\sqrt{p\rho\alpha}-\alpha}{\sqrt{p\rho\alpha}}(1-p+\mu)) - \beta(1-\alpha)(1-p+\mu).
\end{cases} \tag{8}
$$

Thus, we obtained the optimal rebate costs for l-type sellers who adopted the conditional rebate strategy, the second-stage demand under the effect of the rebate strategy, and the optimal profit under the rebate strategy.

Corollary 1. *When online sellers adopt the rebate strategy,* β^* *increases in p but is not necessarily monotonic in* ρ; *that is,* $\frac{\partial\beta^*}{\partial p} > 0$, $\frac{\partial\beta^*}{\partial\rho} = \frac{-2\alpha+\sqrt{p\rho\alpha}}{2(\alpha-1)\rho^2}$.

When $P^n(q_l|s_h) \geq \frac{1}{4}$, *then* $\rho \in (\frac{\alpha}{(\sqrt{A}-1)^2 p}, 1)$, $\frac{\partial\beta^*}{\partial\rho} < 0$.

When $P^n(q_l|s_h) < \frac{1}{4}$, *then* $\rho \in (\frac{4\alpha}{p}, 1)$, $\frac{\partial\beta^*}{\partial\rho} < 0$, $\rho \in (\frac{\alpha}{(\sqrt{A}-1)^2 p}, \frac{4\alpha}{p})$, $\frac{\partial\beta^*}{\partial\rho} > 0$.

Corollary 1 shows that the higher the product pricing in the market, the higher the rebate costs paid by l-type sellers to obtain positive reviews. This suggests that the higher the value of a product, the higher the cost of a positive review. Buyers do not give unjustified positive feedback because of a small rebate, and incentives are positively correlated with the product value.

The relationship between the rebate cost and consumers' sensitivity to the rebate depends on the probability of l-type sellers receiving positive reviews without the conditional rebate strategy. Why might l-type sellers receive positive reviews? This happens in such a market where online buyers are keen on comments and tend to give friendly and positive comments after each purchase. Even if the product quality is not as good as expected, buyers might still voluntarily post positive comments. Therefore, when $P^n(q_l|s_h) \geq \frac{1}{4}$ (i.e., the rebate costs paid by l-type sellers are inversely related to consumers' sensitivity to the rebate), in this scenario, the seller can easily receive an induced positive review. When $P^n(q_l|s_h) < \frac{1}{4}$ (i.e., when l-type sellers do not adopt the rebate strategy), it is quite hard for them to receive positive reviews or good comments. Customs in the online market are more demanding and rigorous. In this case, the relationship between the rebate costs paid by l-type sellers and consumers' sensitivity to the rebate is not necessarily monotonic. When consumers' sensitivity to the rebate is less than a certain threshold ($\hat{\rho} = \frac{4\alpha}{p}$), the rebate costs increase with the increase in consumers' sensitivity to rebates for positive reviews. This means consumers are not sensitive to rebates at this time. Sellers should pay a high price for inducing positive reviews. When the sensitivity of consumers to rebates for positive reviews is greater than a certain threshold ($\hat{\rho} = \frac{4\alpha}{p}$), the rebate costs decrease with the increase in consumer sensitivity. At this time, consumers have an overall higher sensitivity, and it is easier for l-type sellers to take certain measures to "buy" potential consumers' positive reviews.

Corollary 2. *When online l-type sellers adopt the conditional-rebate strategy, Π_l^* increases in ρ or q_l; that is, $\frac{\partial \Pi_l^*}{\partial \rho} > 0$, $\frac{\partial \Pi_l^*}{\partial q_l} > 0$.*

Corollary 2 shows that the optimal profit of l-type sellers under the conditional rebate strategy is positively correlated with consumers' sensitivity to the rebates. This suggests that the more sensitive consumers are to rebates, the more easily they can be induced, and the more profitable l-type sellers are. Under different systems and cultural environments, online markets in different countries and regions have different consumer groups with different characteristics. This is why induced positive reviews work in some markets and not in others. In addition, when the quality level of the l-type product rises, so do the profits of l-type sellers who adopt the conditional rebate strategy. This means consumers are more likely to accept the rebates for positive reviews if the product is of acceptable quality, which in turn brings more profit to l-type sellers.

Proposition 3. *The demand and profit of h-type sellers will be affected when there is induced review behavior by l-type sellers in the online market. In equilibrium, we have*

$$\begin{cases} d_{2h}^* = \frac{\alpha}{\sqrt{p\rho\alpha}}(1-p+\mu), \\ \Pi_h^* = p(\alpha(1-p+\mu) + \frac{\alpha}{\sqrt{p\rho\alpha}}(1-p+\mu)). \end{cases} \tag{9}$$

In this case, the profit of the h-type seller is smaller than that in the case of no rebate strategy (i.e., $\Pi_h^* < \Pi_h^n$).

Corollary 3. *When l-type sellers adopt the rebate strategy, Π_h^* increases as q_l increases and decreases as ρ; that is, $\frac{\partial \Pi_h^*}{\partial \rho} < 0$, $\frac{\partial \Pi_h^*}{\partial q_l} > 0$.*

Proposition 3 suggests that when market consumers are sensitive to rebates, they can be easily induced to post positive comments about l-type products, resulting in inaccurate quality signals. This can cause h-type goods to become unrecognizable, leading to less profits for h-type sellers. Thus, conditional rebate behavior disrupts the market order.

In addition, the higher the l-type quality level, the smaller the difference between h-type and l-type products. In this case, the profits of h-type sellers will decrease. Therefore, quality is key for h-type sellers. They have to make high-quality products that are differentiated from other low-quality products.

5. Numerical Study

In this section, we investigate how each parameter (α, p, and ρ) affects the decision variable β and other equilibrium results, such as demand and profit. For α, we divided it into two cases. In one, α was relatively small, which means that a large number of low-quality products flooded the online market. In the other, α was relatively large, indicating that high-quality goods dominated the online market. In two different markets, the same parameter values were used for comparative analysis. Table 2 shows the specific parameter values.

Table 2. Parameter settings.

	q_h	q_l	p	ρ
Case 1 $\alpha = 0.2$	0.8	0.4	1	0.8
Case 2 $\alpha = 0.6$	0.8	0.4	1	0.8

5.1. *Effect of ρ on the Equilibrium Outcomes*

We first consider the online market in which low-quality products dominated ($\alpha = 0.2$), which is also one of the characteristics of immature e-commerce markets. We let ρ vary to see how it affected the rebate cost, demand, and profit of two different types of sellers. Figure 2 shows the results.

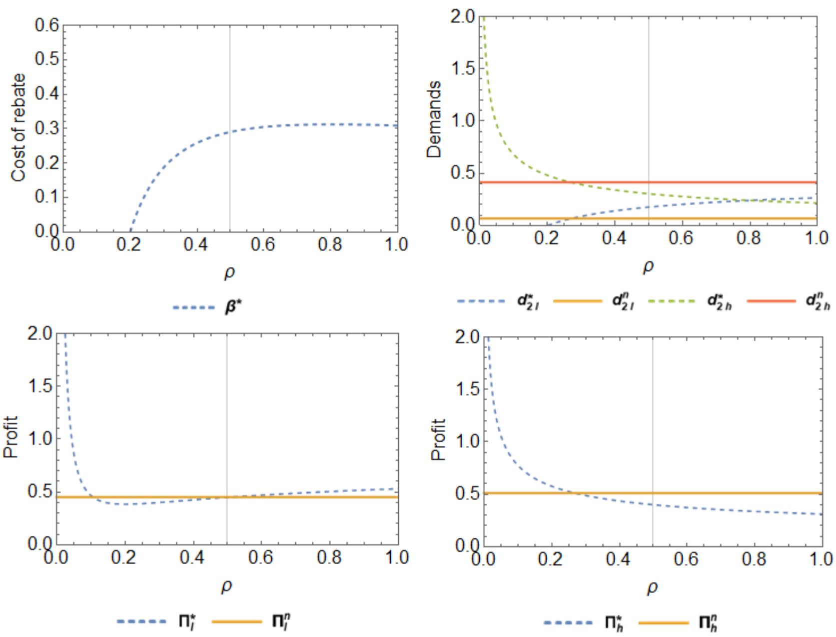

Figure 2. Effect of ρ on equilibrium outcomes ($\alpha = 0.2$).

Figure 2 shows that we had a dividing line of $\rho = 0.5$, which is the condition of l-type sellers adopting the conditional rebate strategy under certain parameter values in Table 2. This is what we learned from proposition 2. The feasible area for discussion is on the right side of the dividing line. From Figure 2, we first know that when consumers' sensitivity to the rebate reached its critical value, the rebate cost decreased with the increase in consumers' sensitivity, but the change was small. Second, considering the demands of two types of sellers before and after the conditional rebate strategy, we can see that before adopting the rebate strategy, the market demand for h-type products was obviously higher than that for l-type goods. The market shares changed after the rebate strategy was adopted by the l-type sellers. The demand for l-type products increased, and the demand for h-type products decreased until the former overtook the latter, taking a greater market share. Third, the difference in profits between the two types of sellers was small before the conditional rebate strategy. However, the adoption of the strategy by l-type sellers increased their profits and reduced the profits of h-type sellers, ultimately leading to a loss of profits for high-quality products in the market. It is possible for h-type sellers to exit the market because it is not profitable, ultimately leading to the survival of the inferior rather than the superior goods.

Next, we consider the other case in which high-quality products dominated ($\alpha = 0.6$). We also let ρ vary to see how it affected the rebate cost, demand, and profit of the two different types of sellers. Figure 3 shows the results.

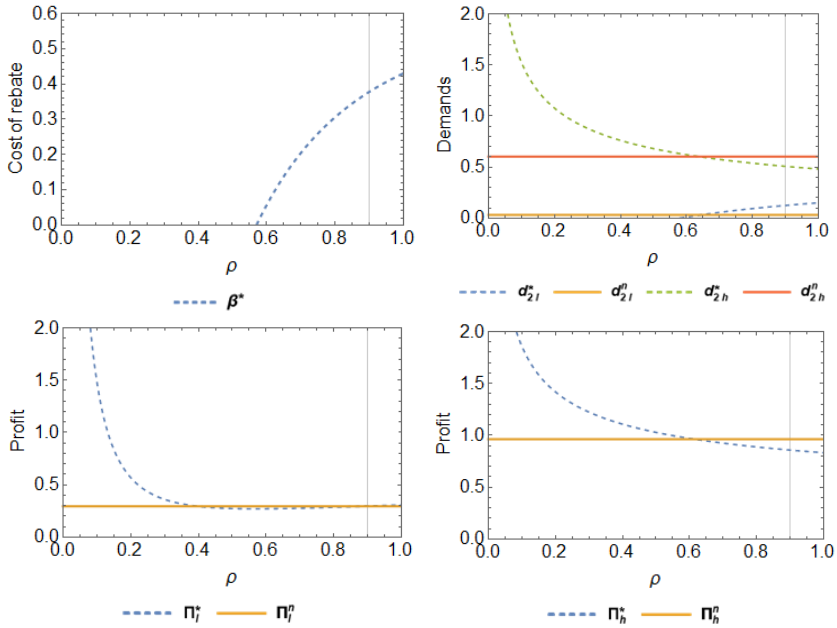

Figure 3. Effect of ρ on equilibrium outcomes ($\alpha = 0.6$).

Figure 3 shows that in a mature online market, it is not easy for l-type sellers to use a conditional rebate strategy to control online reviews. Under the same parameter setting, the critical value of adopting the strategy increased, and the conditional rebate strategy could be adopted only when ρ was greater than 0.9; that is, consumers in this market are highly sensitive to rebates, which is rare in the real market. When looking at the region to the right of the dividing line, we can see that the effect of ρ on the decision variable and other equilibrium results in this market situation differed significantly from that in the previous market situation. First, the rebate costs were positively correlated with consumer sensitivity, and the rebate costs were relatively large. Second, the market demand for l-type products was rather small without the conditional rebate strategy, and it increased after the strategy was adopted but still only accounted for a small part of the market share. Conditional rebate strategies also reduced the market share of h-type products. Generally speaking, however, the market share of h-type products was always higher than that of l-type products, indicating that the conditional rebate strategy would not change the market positions of the two types of sellers. Third, in mature online markets, the profit of h-type sellers was significantly higher than that of l-type sellers, and l-type sellers' profits increased to some extent after adopting the rebate strategy, but the increase was quite limited. On the contrary, this brought about a significant decline in the profits of h-type sellers. This shows that conditional rebate behavior harms others, does not bring significant benefits, and disturbs the competitive order of the market.

5.2. Effect of p on Equilibrium Outcomes

As in Section 5.1, we first considered an online market in which low-quality products dominated ($\alpha = 0.2$). We let p vary to see how it affected the rebate cost, demand, and profit of the two different types of sellers. Figure 4 shows the results.

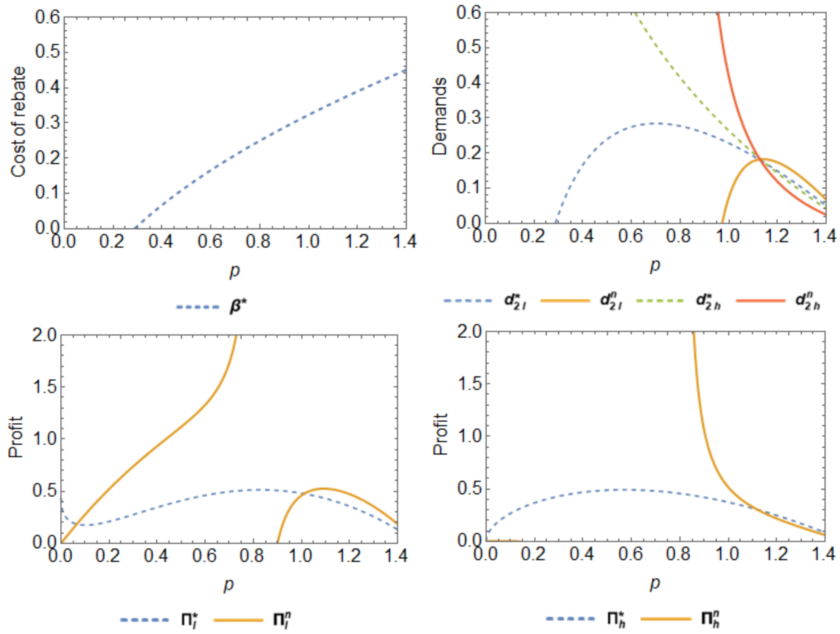

Figure 4. Effect of p on equilibrium outcomes ($\alpha = 0.2$).

Figure 4 shows that first of all, the rebate cost was positively correlated with the price, which is consistent with Corollary 1. This indicates that if the product price is high, the seller will benefit buyers more in inducing positive comments and might need to pay more in management costs (e.g., communicating and coordinating with different customers), which is consistent with the actual market situation. Second, before the rebate strategy was adopted, the market demand for h-type products decreased with the rise in prices, and the market demand for l-type products increased first and then decreased with the rise in prices. When the product's price was equal to 1.1, the demand for the two kinds of products was equal. Generally speaking, after the rebate strategy was adopted, the demand for l-type products increased, and the demand for h-type products decreased. However, when the price increased to a certain extent ($p = 1.1$), the rebate strategy would reduce the demand for l-type products and increase the demand for h-type products. This shows that within a certain range of product prices, a conditional rebate strategy is feasible. We will discuss in detail the price range within which this strategy can be adopted in Section 5.3. Third, we can see in the figure that the profits of the two types of sellers changed due to changes in price and demand as well as changes in rebate cost. We will compare them with a market dominated by high-quality products ($\alpha = 0.6$).

When $\alpha = 0.6$, we also let p vary to see how it affected the rebate cost, demand, and profit of the two different types of sellers. Figure 5 shows the results.

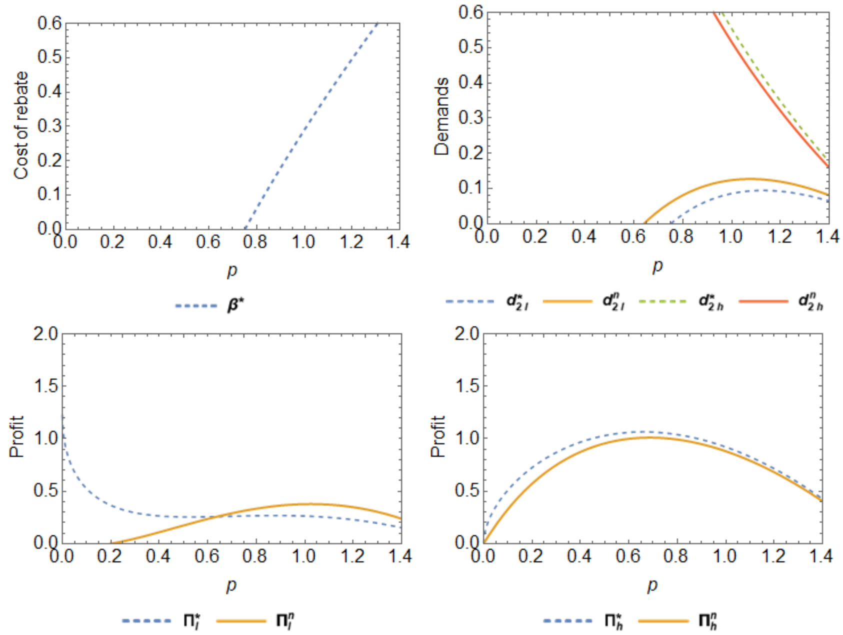

Figure 5. Effect of p on equilibrium outcomes ($\alpha = 0.6$).

Compared with Figure 4, Figure 5 shows that the mature online markets were more orderly. First, as when $\alpha = 0.2$, the cost of the rebate was positively correlated with the price. Second, in a market dominated by h-type goods, a conditional-rebate strategy did not result in an increase in l-type sellers' own demand but rather a decrease in their own demand and an increase in the demand of the other party. We can see a slight increase in demand for h-type products. Third, the rebate strategy increased the profits of l-type sellers in the case of relatively low product prices, but when the price rose to a certain extent, the adoption of the conditional-rebate strategy would cause a decline in the profits of l-type sellers. For h-type sellers, profits could increase by a small margin in this market.

5.3. Comparison under Parameter Combinations

Above, we analyzed the effect of a single parameter variation on the equilibrium result. Here, we consider the optimal strategy selection of l-type sellers under different parameter variation combinations (Figure 6). First, when ρ and p varied within a certain range, the blue area indicates that the conditional-rebate strategy could be adopted. Similarly, when ρ and α changed within a certain range, the blue area also indicates that in this region, the profit for l-type sellers adopting the conditional rebate strategy was greater than that without the strategy. This provides a reference for the decision making of l-type sellers, Here, h-type sellers were the recipients of the strategy. We assumed that no counteraction was taken against the behavior of inducing consumer reviews by l-type sellers.

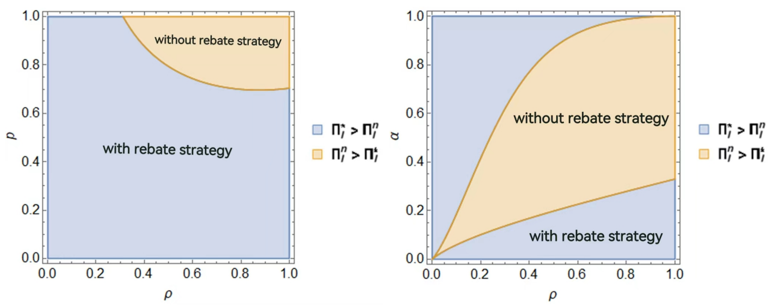

Figure 6. Comparison under parameter combinations.

6. Conclusions and Implications

6.1. Conclusions

Considering the phenomenon in which some l-type sellers in online markets induce consumers to post positive reviews of purchased goods through a conditional rebate strategy, we constructed a signal game model based on Bayesian conditional probability. The basic conclusions are given below.

First, there are preconditions for l-type sellers to adopt a conditional rebate strategy. This strategy will be adopted only when consumers are sensitive to the rebate cost to a certain extent. In this study, we obtained the critical value of consumer sensitivity to the rebate and carried out a specific analysis of the feasible range.

Second, we found the optimal rebate cost by optimizing the l-type seller's profit function. As shown in Equation (8) the optimal rebate cost (β^*) increased with the product price (p) but was not necessarily monotonic in consumers' sensitivity to the rebate (ρ) or the proportion of h-type products (α). At the same time, we found the equilibrium demand for the two types of products and the profit of the two types of sellers.

Third, the market share of high-quality products (α) is an important parameter. A low α value indicates that the market is filled with a large number of low-quality products; otherwise, high-quality products dominate the market. By comparing the equilibrium results of α under high and low values, we found that when α was low, the critical value $\bar{\rho}$ for l-type sellers to adopt the conditional rebate strategy was relatively small. Moreover, the conditional-rebate strategy can increase the demand and profit of l-type sellers. Under this market situation, it is profitable to adopt the conditional rebate strategy. However, when α was high, the critical value $\bar{\rho}$ for l-type sellers to adopt the conditional rebate strategy was relatively large. The behavior of inducing consumer reviews does not seem to work, nor can it affect the demand for h-type products that dominate the market, and it will even lead to an increase in profits for h-type sellers.

In addition, when the quality level of l-type product rose, so did the profits of l-type sellers who adopted the conditional rebate strategy. This means consumers are more likely to accept the rebates for positive reviews if the product is of acceptable quality, which in turn brings more profit to l-type sellers.

6.2. Implications

Our findings underscore that the practice of inducing consumers, as observed among certain l-type sellers, to post positive reviews through conditional rebate strategies disrupts the equilibrium of the market. This manipulation misleads consumers by generating fabricated quality signals. Such conduct not only boosts profits for l-type sellers but also impacts the earnings of h-type sellers and tarnishes the online shopping experience for customers. Effective regulation is imperative to avert the influx of subpar products into online markets at the expense of superior ones.

Addressing this issue requires a multi-pronged approach. Firstly, the primary responsibility lies with platform enterprises. E-commerce platforms should oversee and govern sellers and buyers within their ecosystems, instituting robust rules for responsible transactions and interactions. These platforms should actively thwart collusive actions and impose more stringent penalties on deceitful sellers.

Secondly, governmental intervention through laws and regulations is essential. In many developing nations, the growth of e-commerce markets outpaces legal frameworks. This creates opportunities for dominant online sellers to exploit regulatory gaps. Therefore, the government needs to enact legislation that effectively curbs such malpractices.

Thirdly, the behavior of buyers significantly influences the market. In online marketplaces, buyers function not just as consumers but also as content creators, as their user-generated reviews hold considerable sway. Consequently, responsible review behavior from buyers is pivotal for fostering the orderly progression of online markets.

This concerted effort involving the three aforementioned stakeholders is imperative to accentuating the competitiveness of high-quality products in the market, especially for experience goods.

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Appendix A. Proofs of Propositions

Proof of Proposition 1. From Equation (1), we can obtain that

$$\Pi_l^n = p((1-\alpha)(1-p+\mu) + \frac{(1-\alpha)(1-\lambda d_{1l}^n)}{\alpha+(1-\alpha)(1-\lambda d_{1l}^n)}(1-p+\mu)),$$
$$\Pi_h^n = p(\alpha(1-p+\mu) + \frac{\alpha}{\alpha+(1-\alpha)(1-\lambda d_{1l}^n)}(1-p+\mu)).$$

By using the first-order condition, we obtain

$$\frac{\partial \Pi_l^n}{\partial \lambda} = \frac{-p\alpha(1-\alpha)d_{1l}^n(1-p+\mu)}{(\alpha+(1-\alpha)(1-\lambda d_{1l}^n))^2} < 0, \frac{\partial \Pi_h^n}{\partial \lambda} = \frac{p\alpha(1-\alpha)d_{1l}^n(1-p+\mu)}{(\alpha+(1-\alpha)(1-\lambda d_{1l}^n))^2} > 0.$$

Thus, the proposition is proven. □

Proof of Proposition 2. From Equation (2), we can obtain that

$$\Pi_l = p((1-\alpha)(1-p+\mu) + \frac{(1-\alpha)\rho\beta}{\alpha+(1-\alpha)\rho\beta}(1-p+\mu)) - \beta(1-\alpha)(1-p+\mu),$$
$$\Pi_h = p(\alpha(1-p+\mu) + \frac{\alpha}{\alpha+(1-\alpha)\rho\beta}(1-p+\mu)).$$

By taking the derivatives of Π_l with respect to β, we have

$$\frac{\partial \Pi_l}{\partial \beta} = p(1-p+\mu)\frac{(1-\alpha)\rho(\alpha+(1-\alpha)\rho\beta) - (1-\alpha)^2\rho^2\beta}{(\alpha+(1-\alpha)\rho\beta)^2} - (1-\alpha)(1-p+\mu),$$

and

$$\frac{\partial^2 \Pi_l}{\partial \beta^2} = \frac{2p\alpha(1-\alpha)^2(1-p+\mu)\rho^2}{(-\alpha+(\alpha-1)\beta\rho)^3} < 0.$$

For any given β, this implies that $\Pi_l(\beta)$ is strictly concave in β. Consequently, there is a unique optimal solution (denoted by β^*) to maximize l-type sellers' profit $\Pi_l(\beta)$ for a given β.

Let $\frac{\partial \Pi_l}{\partial \beta} = 0$. Then, we obtain

$$\begin{aligned}\beta^* &= \frac{(1-\alpha)^2\alpha\rho - \dfrac{\sqrt{p(\alpha-1)^4\alpha(1-p+\mu)^2\rho^3}}{1-p+\mu}}{(\alpha-1)^3\rho^2} \\ &= \frac{\sqrt{p\alpha\rho}-\alpha}{(1-\alpha)\rho}, \\ \beta_1 &= \frac{(1-\alpha)^2\alpha\rho + \dfrac{\sqrt{p(\alpha-1)^4\alpha(1-p+\mu)^2\rho^3}}{1-p+\mu}}{(\alpha-1)^3\rho^2} \\ &= \frac{\sqrt{p\alpha\rho}+\alpha}{(\alpha-1)\rho}.\end{aligned}$$

Thus, $0 < \alpha < 1$, $\beta_1 (\beta_1 < 0)$ does not satisfy the condition, and β^* satisfies the optimal solution.

Let $A = \frac{(1-\alpha)(1-\lambda(1-\alpha)(1-p+\mu))}{\alpha+(1-\alpha)(1-\lambda(1-\alpha)(1-p+\mu))}$, $\alpha < p$.

We assume that

$$f(\beta) = \Pi_l = p((1-\alpha)(1-p+\mu) + \frac{(1-\alpha)\rho\beta}{\alpha+(1-\alpha)\rho\beta}(1-p+\mu)) - \beta(1-\alpha)(1-p+\mu).$$

Let $f(\beta^*) \geq \Pi_l^n$ to obtain $-(1-\alpha)^2\rho(\beta^*)^2 + (\alpha-1)(\alpha+(A-1)p\rho)\beta^* - Ap\alpha \geq 0$. We can find that $\rho \geq \frac{\alpha}{(\sqrt{A}-1)^2 p}$.

Since $0 < \rho \leq 1$, we can obtain

$$\begin{aligned}\frac{(1-\alpha)(1-\lambda d_{1l}^n)}{\alpha+(1-\alpha)(1-\lambda d_{1l}^n)} - (1-\sqrt{\frac{\alpha}{p}})^2 &\leq 0 \\ (1-\alpha)(1-\lambda d_{1l}^n) - (1-\sqrt{\frac{\alpha}{p}})^2(\alpha+(1-\alpha)(1-\lambda d_{1l}^n)) &\leq 0 \\ d_{1l}^n(1-\alpha)((1-\sqrt{\frac{\alpha}{p}})^2-1)\lambda + (1-\sqrt{\frac{\alpha}{p}})^2 + 1 - \alpha &\leq 0 \\ \lambda &\geq \frac{-1+\alpha-(1-\sqrt{\frac{\alpha}{p}})^2}{(1-\alpha)^2((1-\sqrt{\frac{\alpha}{p}})^2-1)(1-p+\mu)}.\end{aligned}$$

Thereby, we can find that $d_{2l}^* = \frac{\sqrt{p\rho\alpha}-\alpha}{\sqrt{p\rho\alpha}}(1-p+\mu)$ and $\Pi_l^* = p((1-\alpha)(1-p+\mu) + \frac{\sqrt{p\rho\alpha}-\alpha}{\sqrt{p\rho\alpha}}(1-p+\mu)) - \frac{\sqrt{p\rho\alpha}-\alpha}{\rho}(1-p+\mu)$.

Thus, the proposition is proven. □

Proof of Corollary 1. By taking the derivatives of β^* with respect to p, we have

$$\frac{\partial \beta^*}{\partial p} = \frac{\alpha}{2(1-\alpha)\sqrt{p\alpha\rho}} > 0.$$

Similarly, by taking the derivatives of β with respect to ρ, we have

$$\frac{\partial \beta^*}{\partial \rho} = \frac{-2\alpha+\sqrt{p\alpha\rho}}{2(\alpha-1)\rho^2}.$$

Let $\frac{\partial \beta^*}{\partial \rho} = 0$. There exists a threshold of the sensitivity factor $\hat{\rho}$ where $\hat{\rho} = \frac{4\alpha}{p}$.

We compare the values of $\frac{\alpha}{(\sqrt{A}-1)^2 p}$ and $\frac{4\alpha}{p}$.

When $A \geq \frac{1}{4}$ and $\max\{\frac{\alpha}{(\sqrt{A}-1)^2 p}, \frac{4\alpha}{p}\} = \frac{\alpha}{(\sqrt{A}-1)^2 p}$, then $\rho \in (\frac{\alpha}{(\sqrt{A}-1)^2 p}, 1)$ and $\frac{\partial \beta^*}{\partial \rho} < 0$.

When $A < \frac{1}{4}$ and $\max\{\frac{\alpha}{(\sqrt{A}-1)^2 p}, \frac{4\alpha}{p}\} = \frac{4\alpha}{p}$, then $\rho \in (\frac{4\alpha}{p}, 1)$, $\frac{\partial \beta^*}{\partial \rho} < 0, \rho \in (\frac{\alpha}{(\sqrt{A}-1)^2 p}, \frac{4\alpha}{p})$, and $\frac{\partial \beta^*}{\partial \rho} > 0$.

Thus, the corollary is proven. □

Proof of Corollary 2. By taking the derivatives of Π_l^* with respect to ρ, we have

$$\frac{\partial \Pi_l^*}{\partial \rho} = \frac{(1 - p + \alpha q_h + (1-\alpha) q_l)(\sqrt{p\alpha\rho} - \alpha)}{\rho^2} > 0.$$

Similarly, by taking the derivatives of Π_l^* with respect to q_l, we have

$$\begin{aligned}
\frac{\partial \Pi_l^*}{\partial q_l} &= \frac{(1-\alpha)(\alpha + 2p\rho - p\rho\alpha - 2\sqrt{p\rho\alpha})}{\rho} \\
&= \frac{(1-\alpha)((\sqrt{\alpha} - \sqrt{p\rho})^2 + (1-\alpha)p\rho)}{\rho} \\
&> 0.
\end{aligned}$$

Thus, the corollary is proven. □

Proof of Proposition 3. From Proposition 2 and Equation (2), we can obtain that

$$\begin{aligned}
\Pi_h^* &= p(\alpha(1 - p + \mu) + \frac{\alpha}{\alpha + (1-\alpha)\rho\beta^*}(1 - p + \mu)) \\
&= p(\alpha(1 - p + \alpha q_h + (1-\alpha) q_l) + \frac{\alpha}{\sqrt{p\rho\alpha}}(\alpha q_h + (1-\alpha) q_l)).
\end{aligned}$$

We assume that $h(\rho) = \Pi_h^n - \Pi_h^*$, and we have

$$\begin{aligned}
h(\rho) &= \frac{p\alpha(1 - p + \mu)}{\alpha + (1-\alpha)(1 - \lambda(1-\alpha)(1 - p + \mu))} - \frac{p\alpha(1 - p + \mu)}{\alpha + (1-\alpha)\rho\beta^*} \\
&= p\alpha(1 - p + \mu)(\frac{1}{\alpha + (1-\alpha)(1 - \lambda(1-\alpha)(1 - P + \mu))} - \frac{1}{\sqrt{p\alpha\rho}}).
\end{aligned}$$

Since $A = 1 - \frac{\alpha}{1 - \lambda(1-\alpha)^2(1-p+\mu)}$ and $\frac{\alpha}{1-A} = 1 - \lambda(1-\alpha)^2(1 - p + \mu)$, we can find that $\frac{\alpha}{(1-A)^2 p} = \frac{(1-\lambda(1-\alpha)^2(1-p+\mu))^2}{p\alpha}$, where $\frac{\alpha}{(1-\sqrt{A})^2 p} < \frac{\alpha}{(1-A)^2 p} < 1$.

Let $h(\rho) \geq 0$. Then, we have

$$\begin{aligned}
\sqrt{\rho\rho\alpha} &\geq 1 - \lambda(1-\alpha)^2(1 - p + \mu) \\
\rho &\geq \frac{(1 - \lambda(1-\alpha)^2(1 - p + \mu))^2}{p\alpha} \\
\rho &\geq \frac{\alpha}{(1-A)^2 p}.
\end{aligned}$$

□

Proof of Corollary 3. By using the first-order condition, we find that

$$\begin{aligned}
\frac{\partial \Pi_h^*}{\partial \rho} &= -\frac{(1 - p + \mu)\sqrt{p\rho\alpha}}{2p^2} < 0, \\
\frac{\partial \Pi_h^*}{\partial q_l} &= p(1-\alpha)\alpha(1 + \frac{\alpha^2}{\sqrt{p\alpha\rho}}) > 0.
\end{aligned}$$

thus, the corollary is proven. □

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