Abstract: Efficient machinery operation relies on the performance of high-quality lubricants. Currently, mineral oils of different grades are widely employed for lubricating machine components, but their environmental impact is a concern. Biolubricants are potential alternatives to mineral oils due to environmental factors. The present study focuses on assessing the rheological characteristics of SiO\textsubscript{2} nanoparticle (NP)-enhanced ecofriendly biolubricants for near zero and high-temperature conditions. Pure neem oil, pure castor oil and a 50:50 blend of both oils were considered as the base oils. Nanobiolubricants with enhanced dispersion stability were prepared for varied concentrations of NPs using an ultrasonification method. Viscosity analysis was conducted using an MCR-92 rheometer, employing the Herschel Bulkley model to precisely characterize the viscosity behavior of bio-oils. Due to the fluid–solid interaction between SiO\textsubscript{2} NPs and bio-oils, a crossover trend was observed in the flow curves generated for different base oils enriched with SiO\textsubscript{2} NPs. For neem oil, a significant increase in viscosity was noted for 0.2 wt% of NPs. Using the multilayer perceptron (MLP) algorithm, an artificial neural network (ANN) model was developed to accurately predict the viscosity variations in nanobiolubricants. The accuracy of the predicted values was affirmed through experimental investigations at the considered nanoSiO\textsubscript{2} weight concentrations.

Keywords: rheological studies; machine learning; viscosity; bio-lubricant; SiO\textsubscript{2} nanoparticles

1. Introduction

Currently, there is a growing trend towards the adoption of eco-friendly lubricants designed for total loss applications consisting of biodegradable base oil blended with appropriate additives. A higher fraction of biodegradable components in lubricants makes it more attractive for diverse tribological applications [1,2]. Soybean oil, rapeseed oil, and palm oil are among the bio-oils being investigated for this purpose. Due to their natural biodegradability and lack of toxicity, such bio-oils are particularly enticing for these applications [3–5]. Many researchers have used sunflower oil, castor oil, and soybean oil as an alternative fuel for diesel engines; however, these deteriorated engine components [5–7]. These studies, still in their early stages, face inherent challenges and limitations that necessitate further research.

The primary performance issues encountered while employing vegetable oils as lubricants stem from their vulnerability to oxidative degradation and limited functionality in low-temperature conditions. The addition of lubricant additives such as organic phosphates, organic sulfides, and organic metallic compounds, exhibits an improvement in the dispersing stability and tribological properties of lubricant. However, certain additives have toxicity and may lead to the release of phosphorus, sulfur, and sulphated ash [8].
concentration level of nanoparticles (NPs) significantly affects the tribological characteristics of nanolubricants (NL). Incorporating nanoparticles (NPs) into lubricants generally offers benefits by reducing friction and wear. However, identifying the optimal concentration presents significant challenges. Adding NPs below 1 wt% or exceeding 2 wt% does not necessarily result in a proportional reduction in friction and wear. Therefore, an optimal concentration needs to be achieved to attain the maximum reduction in friction and wear [9,10].

Yi and Zhang [11] observed that molybdenum disulfide (MoS$_2$) with a lamellar structure demonstrated notably superior tribological capabilities compared to a petal structure. This highlights the potential for heightened chemical activity and the development of interlayer sliding effects with lamellar MoS$_2$. Hu et al. [12,13] proved that spherical MoS$_2$ nanoparticles exhibited higher antioxidant activity than stacked MoS$_2$ nanoparticles, contributing to increased stability in lubrication. Moreover, Chen et al. [14] found out that ultrathin MoS$_2$ nanosheets exhibited superior performance compared to other metal sulfides, particularly in high-pressure conditions. This superiority was attributed to their enhanced accessibility to friction contact regions. Metal hydroxides, particularly layered double hydroxides (LDHs), have demonstrated remarkable lubricating properties due to their layered structure, small particle size, and robust chemical reactivity. LDH powders have proven to be particularly effective, especially in situations involving boundary lubrication [15,16]. Further, Zhao et al. [17] synthesized La(OH)$_3$ nanoparticles with granular morphologies through the sol-gel process. Multiple base oils blended with La(OH)$_3$ NPs resulted in improved friction reduction and wear resistance qualities.

Over the last few years significant research has been carried out to explore the impact of NP additions on mineral oil-based lubricants and their impact on friction, wear and other tribological parameters. Using a four-ball machine tribometer, Zulkifli et al. [18,19] evaluated the tribological properties of paraffin oil and palm oil biolubricant containing TiO$_2$ NPs as additives. The incorporation of TiO$_2$ NPs into a trimethylolpropane (TMP) ester resulted in a 14% reduction in the friction coefficient and a 10% decrease in the wear scar width compared to the TMP ester without TiO$_2$ NPs. Gulzar et al. [20] examined MoS$_2$ NPs on modified palm oil and found that incorporating a 1 wt% concentration of the NPs enhanced the extreme pressure (EP) properties by 1.5 times. Katpatal et al. [21,22] further investigated the dispersion stability, viscosity, and thermal conductivity variations in a copper oxide (CuO)-blended oil-based nanolubricant. Nanoparticle blended lubricants were prepared by ISO VG46 oil and Jatropha oil in ratios of 90:10 and 80:20. The dispersion process involved introducing surface-modified CuO nanoparticles into the blend in two stages, ranging from concentrations of 0.5–3 wt.%. Notably, the NP blended lubricant at a 90:10 ratio, containing 1.5 wt.% CuO, exhibited increased sensitivity at lower nanoparticle concentrations compared to the NP blended lubricant at a 80:20 ratio. Dispersion of 1 wt% CuO NPs and multiwalled carbon nanotubes (MWCNTs) in SAE 40 engine oil generated an increase in viscosity of hybrid NLs. Pure oil and hybrid NLs exhibited the Ostwald-de Waele relationship, indicating their non-Newtonian behavior characterized by shear thinning tendencies [23,24].

Dhanola and Garg [25] further analyzed the dispersion stability and behaviour of the size aggregation of TiO$_2$ nanoadditives in canola oil. The dynamic light scattering (DLS) method was utilized to evaluate the impact of different surfactants and various TiO$_2$-to-surfactant mass ratios on dispersion stability. Additionally, rheological assessments were conducted on canola oil with varying quantities of TiO$_2$ nanoadditives at various temperature and shear rate ranges. All nanofluids exhibited Newtonian behavior, indicating an increase in canola oil viscosity with higher concentrations of nanoadditives [26]. Cortes and Ortega [27] investigated the rheological and tribological properties of coconut oil blended with SiO$_2$ and CuO nanoparticles. These nanoparticle additions were found to effectively reduce wear volume loss by 37% and 33% respectively.

Singh et al. [28] investigated the tribological properties of epoxidized Madhuca indica oil with SiO$_2$ nanoparticles as additions. To improve the oil’s acceptability, the unsaturated
fatty acid content was chemically enhanced using the epoxidation process. The inclusion of nanoparticles at 100 °C increased the kinematic viscosity of the chemically treated oil by 1.2 wt%. Moreover, the addition of SiO$_2$ nanoparticles led to an elevation in the flash point while causing a reduction in the pour point. Additionally, the incorporation of SiO$_2$ in the epoxidized oil resulted in an increased level of distribution uniformity [29,30]. Further research studies focused on investigating the tribological potential of neem oil and sesame oil as a biolubricant, by integrating hybrid additives such as SiO$_2$ NPs and an imidazolium-based ionic liquid. Improved friction and wear characteristics were noted for an optimal concentration of NP addition [31,32]. Rheological analysis of hazelnut oil blended with 1.5 wt% of zirconium dioxide (ZrO$_2$) NPs and juliflora oil with TiO$_2$ NPs, effectively reduced the COF and pin wear, and improved the flash point and viscosity properties of the base fluid [33,34]. Studies conducted into nanobiolubricants have indicated that the addition of NPs in optimal quantities can enhance the dispersion stability and antiwear characteristics of bio-oils.

In the present study, detailed rheological studies were carried out on a MCR 92 rheometer to evaluate the influence of nanoSiO$_2$ addition on the viscosity of multiple base oils. The rheological properties of biobased nanofluids were studied to further utilize them as an alternate lubricant in journal bearing applications, replacing the conventional mineral oil lubricants. This would help to enhance sustainability due to the eco-friendly nature of biobased oils. The base oils considered for this study included pure neem oil, pure castor oil, and a blend of neem and castor oil in a 50:50 proportion. The volume fractions of nanoSiO$_2$ added in the lubricant mixture varied from 0 wt% to 0.2 wt%. The viscosity variations were experimentally measured for varying operating temperatures from −10 °C to 90 °C. An ANN model was developed to accurately predict the viscosity and shear stress variation for different in input parameter combinations. Use of such predictive models will help to reduce the number of experimental runs to be conducted for generating intermediate data points. By using predictive models, researchers can optimize their experimental design, save time and resources, and still obtain reliable results.

2. Materials and Methods

2.1. SiO$_2$ Nanoparticles Selection

In this study, SiO$_2$ nanoparticles procured from local vendors in Udupi, Karnataka were utilized to develop multiple nanobiolubricants, effectively enhancing the rheological characteristics of neem and castor oil. SiO$_2$ NPs possess specific physical, chemical, and optical properties, rendering them highly versatile and applicable across various tribological applications [30]. Figure 1a represents the SiO$_2$ NPs utilized at varying concentrations for the rheological experimental studies. Table 1 presents a detailed overview of the physical properties of SiO$_2$ NPs.

Table 1. Properties of SiO$_2$ nanoparticles.

<table>
<thead>
<tr>
<th>Nanoparticle</th>
<th>SiO$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purity</td>
<td>99.5%</td>
</tr>
<tr>
<td>Average particle size</td>
<td>15 nm</td>
</tr>
<tr>
<td>Specific surface area</td>
<td>650 m$^2$/g</td>
</tr>
<tr>
<td>Molecular weight</td>
<td>60.08 g/mol</td>
</tr>
<tr>
<td>Shelf life</td>
<td>60 months</td>
</tr>
</tbody>
</table>

2.2. Selection of Biolubricants

Neem oil (NO) and castor oil (CO) were utilized as base oils for developing nanobiolubricants with enhanced properties. The characteristic properties of neem oil and castor oil is detailed in Table 2. Both oils are hydrophobic in nature, having a distinctive odor. Castor oil has high ricinoleic acid content, accounting for approximately 90% of its total fatty acids. Ricinoleic acid contains a hydroxyl group in its structure, imparting exceptional
stability to its viscosity at various temperatures, a characteristic not commonly found in other vegetable oils [35].

Table 2. Characteristics of Neem and Castor Oil [35–37].

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Neem Oil</th>
<th>Castor Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic viscosity at 40 °C (Cst)</td>
<td>51.93</td>
<td>292</td>
</tr>
<tr>
<td>Viscosity Index (VI)</td>
<td>134</td>
<td>321</td>
</tr>
<tr>
<td>Flash point (°C)</td>
<td>284</td>
<td>145</td>
</tr>
<tr>
<td>Pour point (°C)</td>
<td>−9</td>
<td>2.7</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>860</td>
<td>959</td>
</tr>
<tr>
<td>Refractive Index</td>
<td>1.477</td>
<td>1.480</td>
</tr>
</tbody>
</table>

Figure 1. Nanoparticle selection and nanobiolubricant preparation: (a) SiO₂ nanoparticles, (b) ultrabath sonicator, (c) biolubricants, (d) process flowchart for experimental study.

2.3. Preparation of SiO₂-Enriched Nanobiolubricants

Biolubricants blended with nanoSiO₂ were prepared using both ultrasonification and magnetic stirring approaches. Ultrasonication employs high-frequency ultrasound waves to disperse and disintegrate nanoparticle agglomerates. The associated shear forces and shock waves play a crucial role in breaking down these agglomerates, facilitating a uniform dispersion of nanoparticles and formulating a stable nanofluid. Further, the nanofluid is subjected to a magnetic stirring technique to attain an enhanced dispersion of nanoparticles in the liquid. The magnetic stirrer bar utilized is coated with a chemically inert material to prevent chemical reactions with the nanofluid. Adjusting the stirring speed allows for control over the level of agitation, with higher speeds potentially aiding in the breakdown of agglomerates and achieving uniform dispersion. As shown in Figure 1c, three distinct base oils were identified: pure neem oil, pure castor oil, and a blend comprising equal proportions of neem and castor oils. For each base oil case, different weight concentrations of SiO₂ NPs were added. The process included dispersing SiO₂ NPs in neem and castor oil through an ultrasonication process (Figure 1b). Pure base oil, and base oil blended with 0.05, 0.1, 0.15 and 0.2 wt% of SiO₂ NPs were prepared after sonification. The dispersion stability of NPs in the base oil is substantially influenced by the ultrasonication method. Both
agglomeration and sedimentation need to be limited as it results in an uneven distribution of NPs in the lubricant, affecting the lubricant performance characteristics.

2.4. Rheological Measurements

The rheological behavior of the nanobiolubricants prepared was assessed using an Anton Par MCR-92 rheometer, as illustrated in Figure 2. Experimental tests were conducted on neem and castor oil blended with different concentrations of nanoSiO$_2$ at various operating temperatures. The temperature range was varied from $-10 \, ^\circ C$ to $90 \, ^\circ C$. This study clearly focused on understanding the viscosity-temperature relationship at near-zero and subzero temperatures. This aspect holds substantial importance for potential applications in cold environments, where conventional lubricants may encounter difficulties concerning their flow and effectiveness. The rheological characteristics of nanobiolubricant samples, characterized by their non-Newtonian behavior, were assessed using the Herschel–Bulkley model (Equation (1)) on the MCR-92 rheometer [38,39]. Each test case was performed twice to minimize the errors and generate robust data for analysis and comparison.

\[
\tau = \tau_0 + k\gamma^n
\]  

(1)

Figure 2. MCR 92 rheometer for viscosity measurement.

3. ANN Development and Optimization

Prediction Model Using ANN

In the present study, 1575 input data points comprising varying bio-oil types, shear rates, volume fractions and temperatures were considered. Both viscosity and shear stress were defined as the output variables, influencing the rheological behavior of the lubricant. The designed ANN model was developed using a feed-forward multilayer perceptron (MLP) algorithm. This algorithm functions by receiving input data, training neurons using a portion of the inputs to determine weights and biases, and subsequently generating outputs through testing and error validation. In case of disparity between the computed error and the user’s predicted error, the weights and biases undergo adjustments, and the training is reiterated until the network achieves the specified error level. Typically, this algorithm comprises three layers: the input layer, a hidden layer, and the output layer. The performance of the neural network is directly impacted by the hidden layer, which is composed of multiple layers, each having varying numbers of neurons. Figure 3 represents the architecture of the ANN model employed in this study. After training, testing and
validation, the best ANN model was selected based on the value of R-square and mean square error (MSE). Equations (2) and (3) detail the correlations of these parameters.

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (\mu_{nf, \text{expt}} - \mu_{nf, \text{pred}})^2$$

(2)

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (\mu_{nf, \text{expt}} - \mu_{nf, \text{pred}})_i^2}{\sum_{i=1}^{N} (\mu_{nf, \text{expt}})_i^2}$$

(3)

Figure 3. Architecture of ANN model.

Using the developed ANN model, variation in biolubricant viscosity can be accurately predicted for different operating temperatures and weight concentrations that lie in between the initially considered values. The predicted values can be validated by conducting experimental rheological study on different nanobiolubricants for such in-between weight concentrations. This could help researchers to utilize the developed ANN model to precisely predict the viscosity variations for weight concentrations and temperatures other than those defined in this study. The neural network achieved its optimal and best structure through the method depicted in Figure 4.

Figure 4. Methodology for designing ANN model and optimization.
4. Results

4.1. Rheological Analysis of Lubricant

The rheological behavior of neem oil (NO) and castor oil (CO) as biolubricants blended with different volume fractions of nano-SiO$_2$ was determined from this study. Three different base oils were considered—pure NO, pure CO, and neem and castor oil blended in a 50:50 ratio (NOCO). The volume fraction of SiO$_2$ NP was varied from 0 wt% to 0.2 wt% with an increment of 0.05 wt%. Figures 5 and 6 represent the flow curves generated by testing the prepared nanobiolubricants in the MCR 92 rheometer for varying temperatures. The operating temperature was varied over a broad range from 90 °C to −10 °C. Rheological behavior of NLs was assessed at higher, near zero, and subzero temperatures in this study.

![Experimental (10 °C)](image1)

![Experimental (50 °C)](image2)

![Experimental (70 °C)](image3)

![Experimental (90 °C)](image4)

**Figure 5.** Variation in shear stress and shear rate relationship for different operating temperatures: (a) temperature 10 °C, (b) temperature 50 °C, (c) temperature 70 °C, (d) temperature 90 °C.
In Figure 5a–d, the variation in shear stress vs. shear rate for varying operating temperatures and different weight concentration NLs are illustrated in detail. For pure neem oil at a temperature of 10 °C, a reduction in yield stress was observed indicating a fluid-like quality. This trend persisted as the temperature of the neem oil was further raised to 30 °C. With the addition of 0.05 wt% of SiO$_2$ NPs, the nano-neem oil demonstrated non-Newtonian behavior, exhibiting the characteristics of Herschel–Bulkley flow. In this study, the Herschel–Bulkley model was employed to fit the viscosity curve, and the slope of curve generated provides the viscosity value for the lubricant test samples. The model assists in capturing the NP blended bio-oils undergoing shear-thinning or shear-thickening
behavior in response to varying conditions. This model was valuable in describing the flow properties of nano-bio fluids that deviate from Newtonian behavior [40]. As the weight concentration of NPs was increased to 0.15 wt% in neem oil, a reduction in viscosity and shear thinning behavior was observed. Nano-neem oil was found to be less resistant to flow under increased shear rate conditions. Such behavior is typical of non-Newtonian fluids, deviating from the linear correlation between shear stress and shear rate that is characteristic of Newtonian fluids [41]. Meanwhile for neem oil infused with 0.2 wt% of SiO₂ NPs, a reduction in fluid viscosity was noted at 10 °C, and Newtonian behavior was observed for the 30 °C operating temperature. From the experimental tests, Newtonian flow behavior was observed at the higher temperatures of 70 °C to 90 °C. Elevated temperatures disturb the cohesive forces that bind molecules, facilitating a smoother flow. In non-Newtonian fluids demonstrating shear-thinning behavior, a rise in temperature can result in decreased viscosity due to alterations in the fluid’s internal structure. This phenomenon was especially noteworthy in fluids such as polymer solutions or specific colloidal suspensions [41]. For pure castor oil at 30 °C, the viscosity almost remained constant, with only a negligible variation, reaffirming its Newtonian fluid nature. For 50% blends of base oils, a similar variation in flow curve exhibiting Newtonian behavior was noted for higher temperature conditions. As the temperature was reduced to −10 °C, a significant increase in the yield stress of neem oil and 50% blend oils (NOCO) was observed. By incorporating 0.05 wt% of SiO₂ NPs into the neem oil, the yield stress value surpassed that of pure neem oil. An increase in yield stress was noted for the increased weight concentration of NPs in biolubricants. Whereas, for pure castor oil at a volume of 250 mL and temperature of −10 °C, its viscosity remained nearly constant with increasing shear rate, indicating a Newtonian fluid behavior.

By analyzing the viscosity variations recorded in Figure 6, SiO₂ NP addition was found to effectively influence the rheological flow behavior of neem oil, castor oil and 50% blend oils. The variations recorded for a temperature of −10 °C was not reported due to the exponential increase in viscosity at subzero temperatures. The variation in viscosity for different nano-biolubricants in the temperature range 10 °C to 90 °C is illustrated in Figure 7 and viscosity variation at −10 °C is represented in detail in Figure 8. For a clear understanding of the flow viscosity, Figure 6a–d represents the flow curves generated from 10 °C to 90 °C. At a near zero temperature of 10 °C, the viscosity of pure neem oil was 171.68 mPa-s, which was 11.45 times and 3.43 times lower than that of base castor oil and the 50% blend mixture. Further at 90 °C, neem oil viscosity was 12.058 mPa-s, which was only 2.47 times lower than pure castor oil. At higher temperatures, the variation in viscosity was found to be minimal for different base oils without NP additives. With the addition of nanoSiO₂, an increase in flow viscosity was noted for neem oil. At 0.2 wt% of SiO₂ NPs, around 10.37% increase in viscosity was noted in nano-neem oil at 70 °C. A gradual increase in viscosity was observed for different NP concentrations in Neem oil. With the increase in temperature, a reduction in viscosity can be noted for all tested cases of NLs. This phenomenon occurs because, at lower temperatures, the rise in viscosity is primarily attributed to the resistance posed by the nanoparticles against the motion of fluid particles. The lower viscosity is advantageous for reducing friction and decreasing power consumption. This phenomenon primarily occurs because, at higher temperatures, the nanoparticles absorb heat and extend the decay of cohesive forces. Elevated temperatures typically contribute to enhanced fluidity by providing additional energy to the fluid molecules. This increased kinetic energy helps overcome intermolecular forces, resulting in a reduction in viscosity [42].
Figure 7. Variation in viscosity of different nano-biolubricants (a) neem oil (b) castor oil (c) neem and castor Oil blend.

Figure 8. Variation in viscosity of different nano-biolubricants at $-10^\circ C$. 
For castor oil with NP additives, significant variation in viscosity was noted at near zero temperatures. At 10 °C, the viscosity recorded for castor oil with 0.05 wt% nano-SiO$_2$ was 2380.5 mPa·S, which was 21.05% higher than that of pure castor oil. A small variation in the viscosity range was noted for the increased quantity of nano-SiO$_2$ in castor oil lubricant. At 70 °C, a reduction in viscosity was noted with least value (53.31 mPa·S) generated for 0.1 wt% of nano-SiO$_2$. At operating temperature of 10 °C, a maximum viscosity of 636.23 mPa·S was noted for the blend mixture with 0.2 wt% of nano-SiO$_2$. For Neem and castor oil blend, higher viscosities were obtained for increased concentration of nano-SiO$_2$. A crossover trend was noted in the flow curves generated for different base oils with nano-SiO$_2$ additions. With the rise in operating temperature, the crossover point also increases indicating a transition in the rheological behavior of biolubricants. The crossover phenomenon noted was attributed to the interaction between the base oil and the SiO$_2$ nanoparticles, indicating fluid–solid interaction and temperature dependent characteristics. The decrease in adhesive forces between the SiO$_2$ NPs was more significant compared to the reduction in adhesive forces between bio-oils. This has led to the observation of a crossover trend at elevated temperatures. Such crossover variation was mainly influenced by the reduction in adhesive force magnitude observed within SiO$_2$ NPs and base oils. Adhesive forces present between base oil molecules and NPs was proven to affect the rheological behavior of NLs [43].

Figure 7a–c represents the detailed variation in viscosity measured for neem oil, castor oil and blend mixture. Maximum viscosity was found to be generated at near zero temperatures. At lower temperatures, the increase in viscosity was mainly influenced by the resistance offered by the NPs for the fluid motion. Such an increase in viscosity values was further affected by the increase in NP wt%. The highest variation in viscosity was observed at 0.2 wt% of nano-SiO$_2$, showing a significant 260% increase in viscosity. The resistance offered by the addition of NPs was more than the molecular force resistance in the pure Neem oil. However, the addition of SiO$_2$ NPs was found to have minimal impact on viscosity as the temperature increased to room temperature (30 °C) and above (70–90 °C). Such variation at room temperatures indicates that the volume of NPs blended in biolubricant needs to be increased to modify the rheological characteristics. The increase in the NP concentration in biolubricant further increases the contact area thus providing higher resistance. In Figure 7b, a non-uniform variation trend in viscosity was noted for varying concentrations of NPs in castor oil. In comparison with castor oil, around 113.6% increase in viscosity was noted for castor oil mixed with 0.05 wt% SiO$_2$ nanoparticles compared to pure castor oil. The incorporation of 0.05 wt% SiO$_2$ NPs led to an increase in the viscosity of neem oil at 10 °C. However, the effect of NPs on blend mixture of neem and castor oil was minimal.

In this study, the addition of NPs has primarily affected the rheological behavior of lubricant at near zero and even subzero temperatures as indicated in Figure 8. At subzero temperatures (−10 °C), approximately 185% increase in viscosity was noted for Neem oil blended with 0.05 wt% SiO$_2$ NPs than compared with pure Neem oil. With the addition of 0.1 wt% and 0.15 wt% SiO$_2$ NPs, a further increase of 138% and 115% in viscosity was noted at −10 °C. With the addition of 0.1 wt% and 0.2 wt% SiO$_2$ NPs, the viscosity of the castor oil decreases to 74.28% and 94.74% respectively, at −10 °C. At subzero temperatures, higher viscosities were noted for blend mixture of base oils than compared with pure neem oil and castor oil. From Figure 9, for different weight concentrations of NPs, higher viscosity was generated for castor oil. Further studies need to be conducted on castor oil to accurately assess the variation in rheological dynamics of castor oils as compared to neem oil or a 50% blend mixture. Due to the presence of castor oil in equal proportion in the blend mixture, higher viscosities were noted for varying temperatures when compared with neem oil as the base lubricant.
Figure 9. Variation in viscosity of NLs at different temperatures: (a) temperature 10 °C, (b) Temperature 50 °C, (c) Temperature 70 °C, (d) temperature 90 °C.

Figures 10 and 11 represent the pair plot and correlation matrix generated for the interaction of multiple input parameters with lubricant viscosity. The effect of varying volume fractions of NPs, temperature, and shear rate during testing on the lubricant viscosity is clearly illustrated for a detailed analysis. Temperature variations were found to exert maximum influence on the fluid viscosity. However, with the addition of nanoSiO$_2$ a nominal improvement in viscosity was noted for different base oils. Higher volume fractions of NPs were found to generate an increase in viscosity when compared with base oil with no additives. At 10 °C, the viscosities of different lubricant compositions reach a point of intersection. This convergence takes place in three different cases of base oils and the blend mixture with 0.05 wt% of nano-SiO$_2$. The viscosity trend exhibits a noticeable change at this specific temperature, suggesting potential interactions or synergistic effects between these parameters. Further study into this viscosity crossover phenomenon could provide valuable insights into the underlying molecular dynamics and formulation behavior of these bio-oil–SiO$_2$ mixtures.
4.2. Performance Prediction Model and Output Analysis

In this study, an objective function considering multiple input parameters influencing the shear stress and viscosity of NLs was built using ANN. Figure 12 represents the neural network training, and performance carried out by defining a set number of neurons to input, output and hidden layers. The training was carried out by allotting major data points to train the ANN model, with the rest utilized for testing and validation purposes. The optimal ANN model with the least MSE and high R-Square value was determined by executing continuous iterations of the ANN model. MSE attained its lower value at 221 epochs. A precise predictive artificial neural network (ANN) model was achieved with a hidden layer comprising 20 neurons. The capacity of the developed ANN model to accurately predict the viscosity variations in nanobiolubricant was assessed by comparing its results with the experimental data as shown in Figure 13.
Figure 11. Correlation matrix of various input parameters with lubricant viscosity.

<table>
<thead>
<tr>
<th>Volume fraction</th>
<th>Neem oil (%)</th>
<th>Caster oil (%)</th>
<th>Shear Rate</th>
<th>Temperature</th>
<th>Viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>0.00</td>
<td>1.00</td>
<td>-1.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.23</td>
</tr>
<tr>
<td>-0.00</td>
<td>-1.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.23</td>
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<tr>
<td>-0.00</td>
<td>0.00</td>
<td>-0.00</td>
<td>1.00</td>
<td>-0.00</td>
<td>-0.09</td>
</tr>
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<td>-0.00</td>
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<td>0.00</td>
<td>1.00</td>
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</tr>
<tr>
<td>0.03</td>
<td>-0.23</td>
<td>0.23</td>
<td>-0.09</td>
<td>-0.61</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 12. Neural network training and validation performance.
Using the trained and validated ANN model, viscosity variations obtained for in-between weight concentrations of NPs (0.025 wt%, 0.075 wt%, 0.125 wt% and 0.175 wt%) were predicted and illustrated in Figure 13. The temperatures considered for prediction were 15 °C to 85 °C with an increment of 10 °C. For an increase in operating temperature, a significant variation in lubricant viscosity was noted for a broader range of NP weight concentrations. An optimum nanobiolubricant is such that it experiences minimal variation in viscosity for varied temperature conditions. With the increase in weight concentrations from 0.0125 wt% to 0.0175 wt%, a notable increase in biolubricant viscosity was observed for neem oil, castor oil, and the 50% blend mixture. However, as temperatures increased, the impact of the NP concentration became more pronounced in influencing viscosity variations. The predicted values were further validated through experimental investigations on the NP weight concentrations considered. A near accurate prediction was observed from the results depicted in Figure 13 for different biolubricants enriched with nanoSiO$_2$ of varied concentrations.

![Comparison of predicted and experimentally measured variation in nanobiolubricant viscosities for varied weight concentrations of SiO$_2$ NPs. (a) Neem Oil (b) Castor Oil (c) Neem and Castor Oil Blend.](image-url)
5. Conclusions

The current study exploring the rheological characteristics of nanobiolubricants has yielded essential insights into the viscosity patterns observed among different biolubricant formulations across a wide temperature spectrum. The investigation highlights distinctive viscosity trends within neem oil, castor oil, and their combined blend. Notably, the blend exhibited increased viscosity at subzero temperatures (−10 °C to 0 °C), while neem oil showcased reduced viscosity under the same conditions. The minor viscosity fluctuations noticed across all three biolubricant combinations without nanoparticle additives are primarily influenced by the specific molecular structures of each lubricant. At elevated temperatures, there were slight variations in viscosity observed across the three samples owing to their unique molecular compositions. However, as temperatures increased, the impact of nanoparticle concentration became more pronounced in influencing viscosity changes. Particularly striking was the substantial rise in neem oil viscosity—by 185% with the addition of 0.05 wt% SiO$_2$ nanoparticles at subzero temperatures, escalating further to 260% at a 0.2 wt% nanoparticle concentration. Yet, as temperatures approached and exceeded room temperature, the effect of nanoparticles on viscosity diminished due to reduced adhesive and intermolecular forces. Fluid solid interactions significantly affect the viscosity and flow properties of nanobiolubricants, and a cross-over trend influenced by NL volume fractions was notably observed at different operating temperatures. The ANN model developed in this study was able to precisely predict the variation in lubricant viscosities for an intermediate set of NP weight concentrations. Such machine learning models will be effective in predicting the rheological behavior of bio-oils considered for subzero temperatures and for a broader weight concentration spectrum. The nano-biolubricants formulated need to be further tested in a journal bearing with dimpled profiles to further assess the influence of NPs presence in enhancing the pressure distribution and stability of rotor bearing systems. High speed rotors with improved stability and increased load-bearing capacity are primary requirements in various industrial sectors, including power generation and turbomachinery applications. Usage of currently tested NP blended neem oil and castor oil lubricants holds potential to offer advantages over conventional lubricants and even contribute to improved sustainability.

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