

Article

Rotordynamic System-Level Effects of Three-Lobe Journal Bearings Including Manufacturing Variations

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Abstract

This paper examines the impact of pad-to-pad manufacturing variations in three-lobe journal bearings on system-level rotordynamics. Two sources of non-uniform clearance were studied: dissimilar pad clearance and preload. Both were varied independently within standard manufacturing tolerances. The results show that the conventional assumption that all pads having equal clearances at tolerance extremes does not capture worst-case conditions. Instead, specific non-uniform pad combinations caused the most significant amplification factors and the lowest stability margin. By applying a Surface Response Design of Experiments (SRDOE) method, surrogate models were developed to represent the nonlinear influence of the pads' dissimilarity. The models identified the most critical combinations of pad and journal variables, revealing that industry-standard practice does not provide the most adverse system behavior. Worst-case conditions arise from non-uniform pad geometry: SRDOE models predict critical combinations, while uniform assumptions of industry-based standards underestimate risk. Incorporating realistic manufacturing variability in rotordynamic models provides a more reliable basis for turbomachinery design.

Keywords: stability; robust rotordynamics; three-lobe journal bearings; manufacturing variation

1. Introduction

Three-lobe journal bearings have been used for many decades in turbomachinery. They are still often encountered in many applications, like centrifugal compressors, vertical centrifugal pumps, integrally geared compressors, steam turbines, and electric motors. The customary design of three-lobe bearings, following Pinkus [1], Allaire [2], and Malik [3], varies key parameters such as L/D ratio, clearance, preload, and offset to optimize performance. These methods, however, often assume uniform pad geometry. Some examples are given in Saruhan et al. [4], Urbiola-Soto et al. [5], and Moroz et al. [6], where the stability and imbalance response are optimized using surrogate models varying all pads evenly. The American Petroleum Institute (API) [7,8] sets the leading globally recognized standard for the natural gas and oil industry, also adopted by [9]. However, the standard practice is focused on idealized configurations, overlooking pad-to-pad variability that naturally arises in manufacturing. Such variations alter stiffness, damping, and cross-coupling, significantly affecting imbalance response and stability thresholds. While tilting-pad bearings have been extensively studied under tolerance effects, at component-level [10–18], and at rotor system-level [19], the literature on fixed-pad three-lobe bearings is limited. The



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impact of manufacturing tolerances of three-lobe journal bearings is explained by Martin and Ruddy [20], including varying tilt angle, preload, and bearing clearance. Generally, the threshold speed of instability increased with preload. However, the study focused on homogeneous preload in all pads. The groove location of multi-lobe journal bearings, and for instance, the pad arc extent, was optimized by Roy and Kakoty [21]. Ma et al. [22] researched several standard tolerancing grades for plain journal bearings. Wider variation was seen in the rotordynamic coefficients for larger tolerance grades. A recent component-level work by Roy et al. [23] deals with the design of a three-lobe fixed-pad journal bearing with uncertainties in the input design variables, namely the pad clearance, preload, and offset factor under independent 5% variation in each. The probabilistic response of mass parameter, load bearing capacity, and minimum film thickness was performed using Monte Carlo stochastic simulations. The present work focuses on investigating realistic dissimilar bearing pad geometry for robust system-level rotordynamic design. The remainder of this paper is organized as follows: Section 2 introduces the bearing factors and defines the key pad-to-pad independent variables studied, including clearance and preload, along with their tolerances. Section 3 describes the Surface Response Design of Experiments (SRDOE) methodology applied to arrive at surrogate models. Section 4 presents the machine configuration and imbalance response and stability output variables considered in the analysis. Section 5 provides a component-level assessment of bearing force coefficients, while Section 6 expands to system-level rotordynamics to predict critical speeds, amplification factors, and stability margins. Finally, Section 7 discusses maximization and minimization results of the responses. Afterwards, the tolerance of the journal is also introduced in combination with those bearing cases corresponding to the best and worst cases. Finally, Section 8 summarizes the main conclusions and implications for design practice.

2. Bearing Factors

The two bearing factors to study are the bearing clearance C_b and the preload m per pad. Each factor can vary independently of the others. The levels of factorial experimentation were within the bounds of the manufacturing tolerances around the nominals, as shown in Table 1. In the SRDOE argot, coded units are scaled values of -1 and 1 for the lower specification limit (or minimum) and upper specification limit (or maximum). The star points, also known as axial points, are located at a distance α from the center or mean value, calculated according to $k^{0.25}$, where $k = 32$ factorial runs [24]. The use of the star points is explained in the next section.

Table 1. Experimentation levels.

	Star	Min	Mean	Max	Star
Factor Levels	-2.378	-1	0	1	2.378
Brg Clearance (C_b) [μm]	41.3	68.9	88.9	108.9	136.5
Preload (m)	0.26	0.40	0.50	0.60	0.74
Resulting parameters	Calculated from Equation (1)				
Pad Clearance (C_p) [μm]	158	172	178	182	185
$f = C_p/D$ [$\mu\text{m}/\text{mm}$]	1.55	1.70	1.75	1.79	1.82

The pad clearance (C_p) depends on the bearing clearance (C_b) and preload (m). Equation (1) shows this relationship, where R_p , R_b and R_j are the as-machined pad radius, the assembled or bored pad radius, and journal radius. Extreme values of C_b and m yield the maximum and minimum C_p . For instance, the pair of minimum C_b and minimum m renders the minimum C_p . Conversely, the pair of maximum C_b and maximum

m delivers the maximum C_p . Similar logic applies to the star levels. The well-known rule-of-thumb for designing the ratio of the diametral pad clearance to the journal diameter is 1.5 to 2 $\mu\text{m}/\text{mm}$. Thus, the associated factor f in Table 1 reasonably falls within such conventional design practice, which confirms a good exploration range for C_b . At the same time, the star points of m lie within the extreme values of the customary design practice (0.2 to 0.75) [25,26]. On the other hand, the diametral tolerance range of C_b is 40 μm , which is fairly close to the ISO H7 tolerance grade for a sliding contact type. The latter has a $+35/-0$ μm tolerance for a 101.6 mm diameter shaft. Note that the minimum C_b is at its nominal value. Finally, the preload tolerance of 0.1 around the mean or nominal, in this case, also reflects customary manufacturing practice and the possible preload loss in service [27]. Either the pad clearance C_p or the bearing clearance C_b may be chosen to vary. However, modeling a dissimilar C_p individually per pad is not customary; rather, the C_b is locally modified using pressure dams or pockets in some pads, particularly the top pad [26]. The procedure to model a dissimilar C_b per pad is explained, along with its correlation to the C_p variation. Figure 1 exemplifies when (a) the bearing clearance varies by a C_{bi} value, and (b) the preload varies by a m_i magnitude. In both cases, only one pad is shown for clarity.

$$C_p = \frac{C_b}{1-m}; \text{ where } C_p = R_p - R_j, \text{ and } C_b = R_b - R_j \quad (1)$$

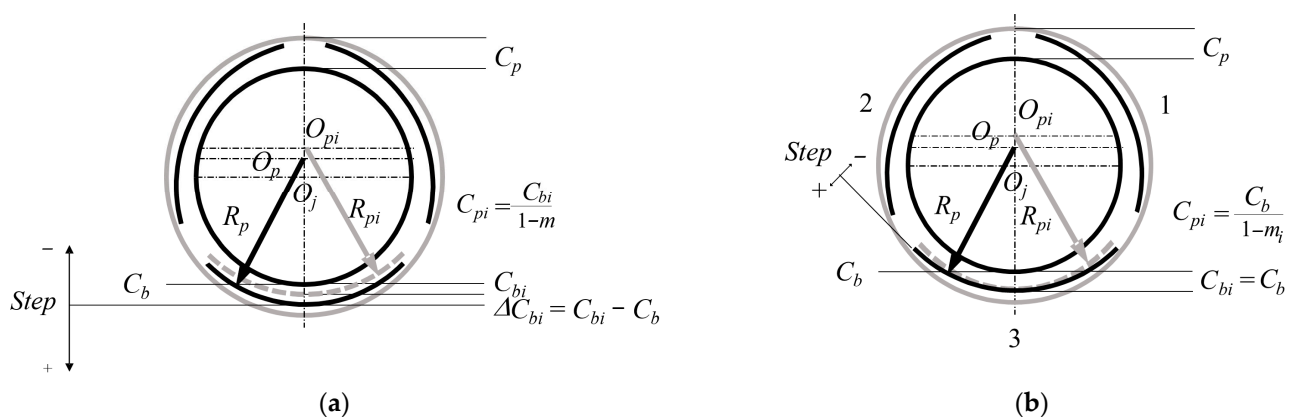


Figure 1. Three-lobe bearing factors: (a) bearing clearance C_b variation; (b) preload m variation.

Figure 1a illustrates that for a given change in the bearing clearance ΔC_{bi} , the pad set bore center O_p is adjusted while varying the pad machined R_p radius clearance to maintain the same preload m . Then $R_{pi} = R_p + \Delta C_{pi}$, which is calculated using Equation (1). Therefore, varying C_{bi} is equivalent to varying R_{pi} , and in turn equivalent to varying C_{pi} for a given preload per Equation (1).

On the other hand, Figure 1b shows that for varying the preload m , the pad machined radius R_{pi} is modified while maintaining the set bore clearance C_b . In this other scenario, only the preload changes from a nominal or mean preload to a non-nominal preload m_i . Equation (2) provides the total change in C_b ; the first term is the experimentation level of the pad clearance, whereas the second term is the actual pad clearance conditioned by the experimentation level of m . When C_{bi} equals its nominal (minimum) value, and m_i equals its nominal (mean) value, then $\Delta C_{bi} = 0$.

$$\Delta C_{bi} = C_{bi} - C_p(1 - m_i) \quad (2)$$

Finite element-based thermo-elasto-hydrodynamic (TEHD) code [28] was used in this study. A full-film solution of the Reynolds equation, including variable viscosity and heat transfer effects, is executed. Thermal deformations of the journal and bearing are considered. Turbulence correction factors are incorporated and applied independently in the axial and circumferential directions. Thermal effects are added through solutions to the energy equation. A thermal solution to determine lubricant film temperature from inlet conditions is performed. Sump temperature is applied for convective heat transfer from the ends of each pad. The heat conduction equation of the pads and the thermal loading of the pads are solved simultaneously. The variation of C_b in Equation (2) may be modeled as a positive step (recess) or negative step (layer), which completely extends from the leading to the trailing edge of the pad and fully across the pad axial length. On the other hand, the pad preload m may be declared as an independent variable per pad.

3. Surface Response Design of Experiments

A Surface Response Design of Experiments (SRDOE) is a regression-based approach for nonlinear modeling, typically fully quadratic per Equation (3). A plan of experiments is defined as such that the cloud of datapoints complies with certain array properties, like rotatability to avoid bias in directional variance, orthogonality to separate main effects from interactions, and circumscription to fully cover the range of tolerances [24], where the experiments could also be numerical simulations, as is the case in this work. Table 2 displays the SRDOE used in this study, where level -1 , 0 , and 1 are the lower specification limit, mean, and upper specification limit, respectively. On the other hand, levels -2.37841 and 2.37841 are the star points lying outside the specification limits and useful for increasing fitting accuracy near the boundaries. Note that the center point, i.e., all factors in their means, needs to be replicated nine times to fulfill the key matrix properties described beforehand. Finally, the Max C_b and Min C_b runs belong to the homogeneous maximum and minimum pad clearance in all pads, respectively. The journal radius remained constant at its nominal value for all the runs. This other important parameter will be integrated at a later stage of this work for three reasons: first, to maintain a short number of combinations and keep computational runs to a minimum; second, to understand the isolated effect of the bearing manufacturing variation per se on the imbalance response and stability of the example rotor for a nominal journal; and third, to further expand the study to understand the responsivity of specific bearing-geometry selected cases, which will be combined with extreme values of the journal diameter and then make the comparison vs. the API approach [7,8].

$$Y = \sum_{i=1}^n b_{ii}x_{ii}^2 + \sum_{i=1}^n b_i x_i + \sum_{i < j} b_{ij} x_i x_j + b_0 + error \quad (3)$$

Table 2. Surface Response Design of Experiments.

Run Order	Point Type	C_{b1}	m_1	C_{b2}	m_2	C_{b3}	m_3
1	Factorial	-1	-1	-1	-1	-1	-1
2	Factorial	1	-1	-1	-1	-1	1
3	Factorial	-1	1	-1	-1	-1	1
4	Factorial	1	1	-1	-1	-1	-1
5	Factorial	-1	-1	1	-1	-1	1
6	Factorial	1	-1	1	-1	-1	-1
7	Factorial	-1	1	1	-1	-1	-1

Table 2. Cont.

Run Order	Point Type	C_{b1}	m_1	C_{b2}	m_2	C_{b3}	m_3
8	Factorial	1	1	1	−1	−1	1
9	Factorial	−1	−1	−1	1	−1	1
10	Factorial	1	−1	−1	1	−1	−1
11	Factorial	−1	1	−1	1	−1	−1
12	Factorial	1	1	−1	1	−1	1
13	Factorial	−1	−1	1	1	−1	−1
14	Factorial	1	−1	1	1	−1	1
15	Factorial	−1	1	1	1	−1	1
16	Factorial	1	1	1	1	−1	−1
17	Factorial	−1	−1	−1	−1	1	1
18	Factorial	1	−1	−1	−1	1	−1
19	Factorial	−1	1	−1	−1	1	−1
20	Factorial	1	1	−1	−1	1	1
21	Factorial	−1	−1	1	−1	1	−1
22	Factorial	1	−1	1	−1	1	1
23	Factorial	−1	1	1	−1	1	1
24	Factorial	1	1	1	−1	1	−1
25	Factorial	−1	−1	−1	1	1	−1
26	Factorial	1	−1	−1	1	1	1
27	Factorial	−1	1	−1	1	1	1
28	Factorial	1	1	−1	1	1	−1
29	Factorial	−1	−1	1	1	1	1
30	Factorial	1	−1	1	1	1	−1
31	Factorial	−1	1	1	1	1	−1
32	Factorial	1	1	1	1	1	1
33	Star	−2.37841	0	0	0	0	0
34	Star	2.37841	0	0	0	0	0
35	Star	0	−2.37841	0	0	0	0
36	Star	0	2.37841	0	0	0	0
37	Star	0	0	−2.37841	0	0	0
38	Star	0	0	2.37841	0	0	0
39	Star	0	0	0	−2.37841	0	0
40	Star	0	0	0	2.37841	0	0
41	Star	0	0	0	0	−2.37841	0
42	Star	0	0	0	0	2.37841	0
43	Star	0	0	0	0	0	−2.37841
44	Star	0	0	0	0	0	2.37841
45	Center	0	0	0	0	0	0
46	Max C_b	1	−1	1	−1	1	−1
47	Min C_b	−1	1	−1	1	−1	1

4. Machine Used for the Study and Response Variables

The machine selected for this analysis is shown in Figure 2; it is a seven-stage centrifugal compressor handling hydrogen. The compressor power is 2613 kW, the bearing span is 1.69 m, and the rotor span to mean shaft diameter (L_b/D_{ms}) is 10.5. The rotor mass is 480 kg, with an I_t , and I_p about its center of gravity, and centerline of 3440 and 6450 kg·m², respectively. The rotor is supported by identical bearings of three-lobe type. Due to the closeness of bearing loads, an average load per bearing was employed to simplify the analysis with no loss of generality. Table 3 displays the bearing specifications and operating conditions. The journal surface temperature listed in Table 3 corresponds to the imposed thermal boundary condition used for the thermo-elasto-hydrodynamic

(TEHD) bearing analysis. It represents the bulk journal temperature at the lubricant–journal interface and is used to determine lubricant viscosity and solve for the temperature field.

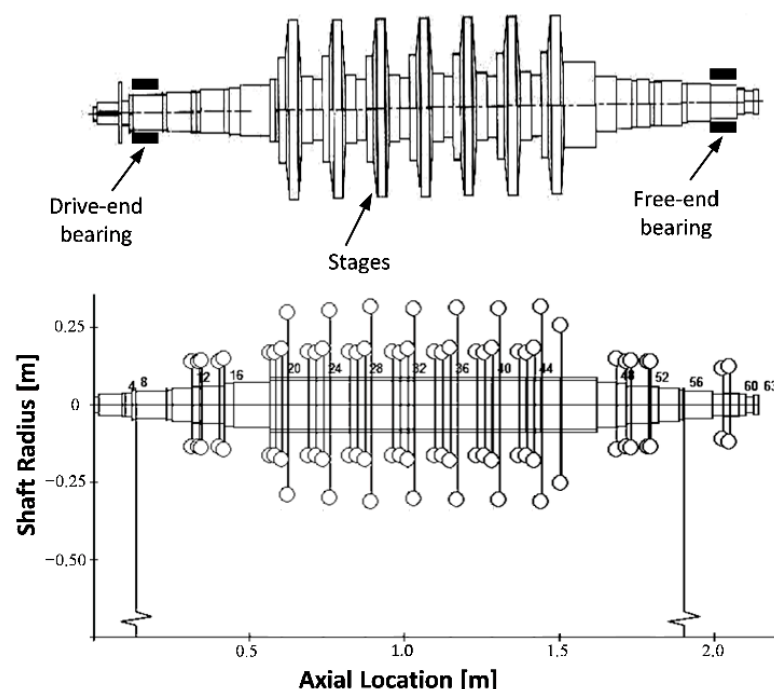


Figure 2. 2D model of compressor rotor selected for this study: geometry (**top**), and rotordynamic model (**bottom**).

Table 3. Bearing specifications and operating conditions.

Characteristic	Specification
Bearing diameter [mm], and L/D ratio	101.6, and 0.5
Pad arc [degrees], and pad offset	100, 0.5
Selected lubricant	ISO VG 32
Oil supply temperature [°C]	50
Journal surface temperature [°C]	53
Oil inlet flow [lt/min]	26.5
Drive-end bearing load [N]	2505
Free-end bearing load [N]	2210
Average load per bearing [N]	2358

The response variables are depicted in Figure 3, where Pk and SS indicate the peak and steady-state imbalance response, respectively. These apply to the *Midspan*, *FE Brg*, or *DE Brg* at their major axis of vibration. On the other hand, Ncm and AFm are the critical speed and the amplification factor at the midspan plane at its major axis of vibration, respectively. AFm is computed according to the Half-Power Bandwidth method described in [7,8]. $Ncreg$ is the peak-to-peak speed range spanning all critical speeds, confirmed by showing an $AF > 2.5$ at the rotor midspan. Nso is introduced as the safe operating speed, resulting from computing the separation margins for all true critical speeds at the rotor midspan, and the minimum operating speed, as shown in Equations (4)–(6). Finally, Nt is the threshold speed of instability when the $\logdec = 0$. These outputs were chosen to represent both dynamic amplification and stability margins. The rotordynamic model does not consider interstage and end seals. However, while the absolute values of the stability margin may vary across experiments, the relative trends are expected to remain consistent if

seals were added. A resolution of 25 rpm was used in the imbalance response and stability to obtain high-accuracy calculations.

$$SM_{ax,k} = 17 \left(1 - \left(\frac{1}{AF_{ax,k} - 1.5} \right) \right) \quad (4)$$

$$Nos_{ax,k} = Nc_{ax,k} \left(1 + \frac{SM_{ax,k}}{100} \right) \quad (5)$$

$$Nso = \max \{ Nos_{ax,k} \} \quad (6)$$

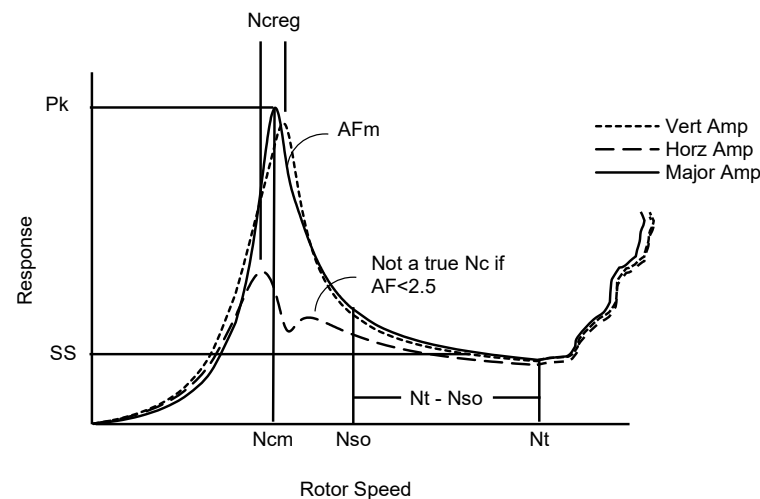


Figure 3. Generic Bode plot illustrating the response variables.

5. Preliminary Assessment at Component Level

The rotodynamic coefficients were numerically computed using the TEHD code, as described in Section 2. Figures 4–6 show the bearing force coefficients as a function of speed for each of the runs. The center point or run 45 corresponds to the optimized bearing design as reported in [5]. Although the SRDOE includes only a subset of the full factorial design, it allows for a preliminary assessment of where the uniform pad cases (46 and 47) lie within the design space. Figure 4a shows that the main stiffness k_{xx} is higher for case 17 at low speed, higher for case 47 at mid speeds, and higher for case 25 at high speed. Therefore, it is difficult to ascertain which case is the stiffest in actuality. On the contrary, case 22 appears as the softest across the entire speed range. Figure 4b shows that cases 1 and 18 are consistently the stiffest and softest across speed for the direct coefficient k_{yy} . The direct damping c_{xx} coefficient in Figure 5a depicts the consistently highest curve across speed for case 17. Meanwhile, cases 1, 18, 32, and 46 overlap at the lowest level. On the other hand, the direct damping c_{yy} is highest and lowest for cases 1 and 22, respectively, as shown in Figure 5b. The cross-coupled k_{xy} coefficient is shown in Figure 6a, where the highest level belongs to case 17, and the lowest level to cases 18 and 46. At both levels, there are some other curves that closely overlap. Finally, Figure 6b outlines cases 22 and 11 as the highest and lowest for the k_{xy} coefficient, where the uniform cases 46 and 47 lie in an intermediate region across the variation. It can be conclusively stated that there is neither a highest nor a lowest consistent predominance of the uniform C_b cases in any of the rotodynamic coefficients.

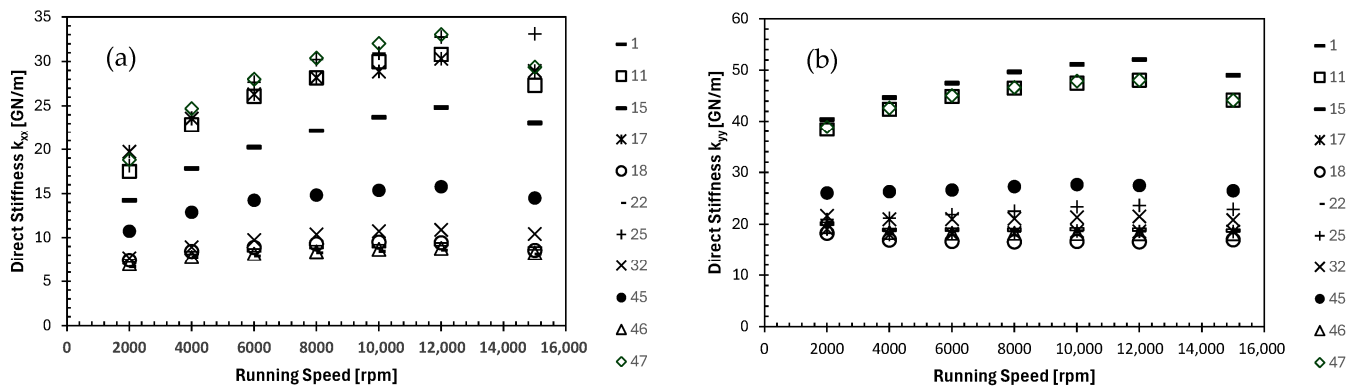


Figure 4. Bearing direct stiffness: (a) k_{xx} ; (b) k_{yy} for selected cases.

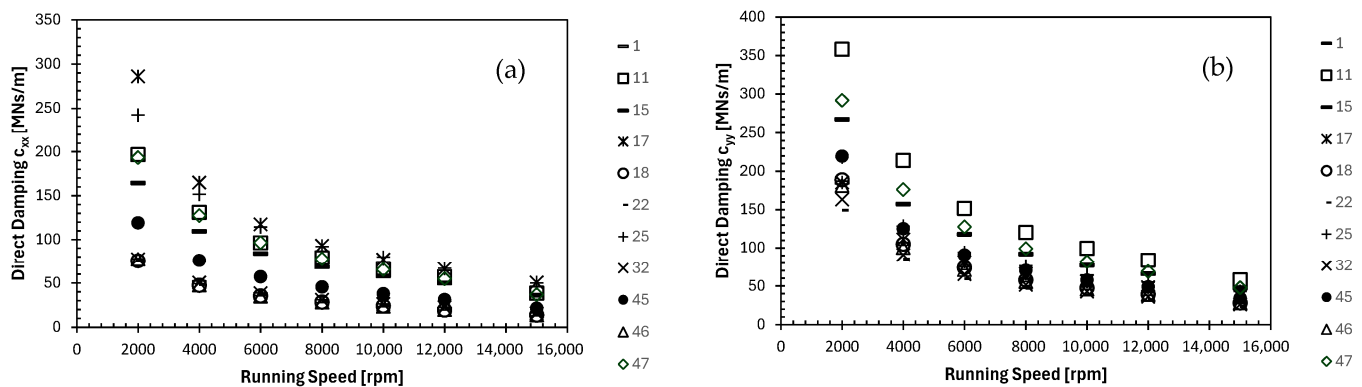


Figure 5. Bearing direct damping: (a) c_{xx} ; (b) c_{yy} for selected cases.

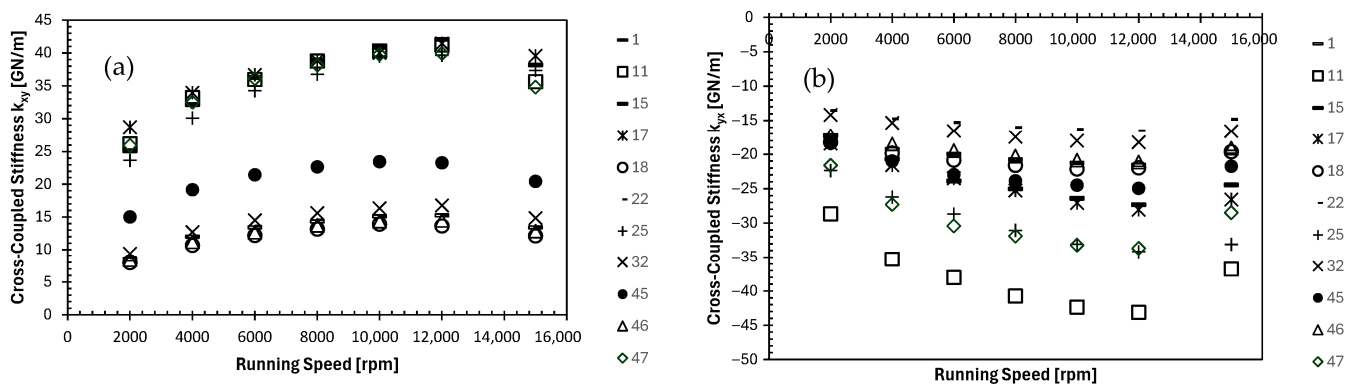


Figure 6. Bearing cross-coupled stiffness: (a) k_{xy} ; (b) k_{yx} for selected cases.

As general trends, as C_b tends to be minimum across pads, and m approaches its maximum value, both the main stiffness and damping tend to be maximum, while the absolute values of the cross-coupled stiffness increase (cases 11, 15, 25, and 47). High damping is desirable from an imbalance response standpoint, while high stiffness is preferred for stability, but it is counteracted by high cross-coupled stiffness. As the preload across pads is more dissimilar (cases 11, 17, and 25), the absolute magnitude of the cross-coupled stiffness coefficients increases, which is detrimental for stability. As C_b and m approach their maximum values across pads, the absolute cross-coupled stiffness values tend to decrease (cases 25 and 32), but also damping tends to decrease. The first can improve stability, but the second is not favorable for either stability or imbalance response. Figure 7 illustrates the whirl frequency ratio (WFR) as defined in Ref. [29]. This parameter accounts for the combined influence of all force coefficients on bearing stability. The lower the WFR,

the higher the stability margin and the safe operating speed, calculated by dividing the first rotor-bearing natural frequency by the WFR. The results indicate that case 15 offers the greatest stability, whereas case 18 exhibits the poorest (highest) WFR. Although these findings provide valuable insight into bearing stability, a system-level analysis is required to assess the imbalance response and the impact of rotor flexibility.

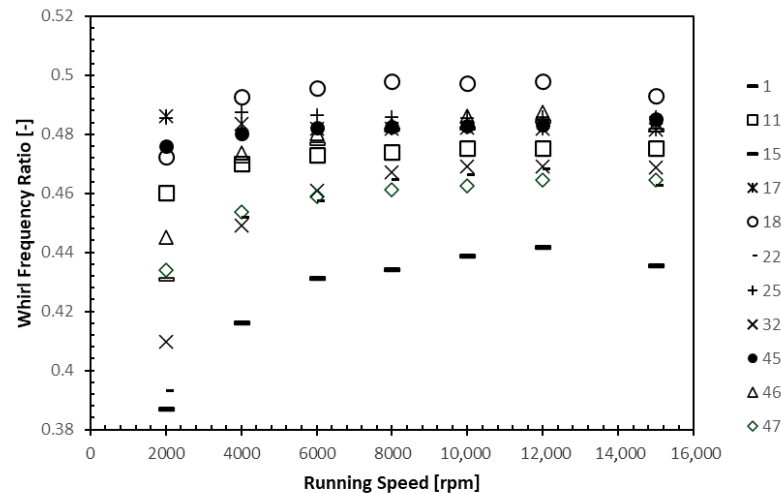


Figure 7. Bearing whirl frequency ratio (WFR) for selected cases.

Table 4 shows a screening of tribological parameters of the most relevant cases extracted. A constant supply pressure boundary condition was used in all bearing calculations. The supply flow rate shown in Table 3 represents the nominal inlet condition, while the flow rates reported in Table 4 are resulting quantities determined by the TEHD solution as bearing geometry and operating conditions vary. The largest and smallest magnitudes of the tribological parameters are shown in bold numbers. It can be conclusively stated that there is neither a highest nor a lowest consistent predominance of the uniform C_b cases in any of the tribological parameters.

Table 4. Screening of tribological parameters.

Case	Power [W]	$P_{\max}$ [kPa]	Film $T_{\max}$ [°C]	$H_{\min}$ [microns]	Bearing Flow [Lt/min]
1	2076	1468	88.1	38	22
11	2326	2409	95.4	27	16
15	2445	2555	89.4	30	19
17	2578	1840	92.0	22	18
18	1972	1445	91.2	37	20
22	2118	1551	87.1	37	23
25	2608	1810	92.1	21	18
32	2124	1696	85.5	40	24
45	2148	1817	90.9	34	20
46	2075	1454	87.6	38	23
47	2285	2549	95.0	28	17

6. System-Level Analysis

This section evaluates the response variables shown in Figure 3, namely imbalance response, critical speeds, and stability margins at the rotor system level. A critical speed map of the selected rotor is shown in Figure 8. Only the stiffness for a few of the cases is shown for the sake of clarity. These results illustrate the bounds for the locations of the undamped critical speeds at the crossing of the stiffness with the natural frequency curves.

The rotor is sufficiently stiff such that variations in bearing stiffness influence the natural frequencies within the transitional region, before they become asymptotic. As damping is introduced, some variation in the critical speed location is also expected.

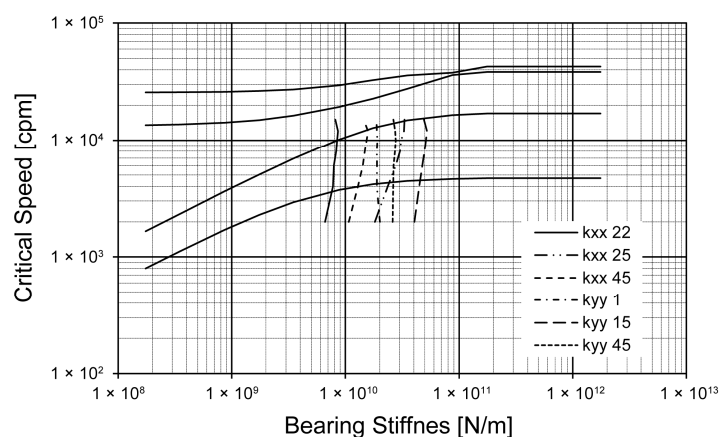


Figure 8. Critical speed map for selected cases.

Equations (7)–(10) display some of the SRDOE surrogate models retaining only significant terms. All models are presented in uncoded units, i.e., dimensional units when applicable. The models have been reduced to significant terms. A small p -value (typically <0.05) suggests that the term is likely a significant term influencing the response. Therefore, the models can be simplified or reduced by identifying which terms in the model are statistically significant and contribute most to the variability of the response [24]. Simplification also involves analyzing the models' R^2 , R^2 (adj) and R (pred) correlation coefficients so they are as close as possible to each other. This ensures a given model adequately fits the data while having good predictive power. Table 5 shows a summary of the models' correlation coefficients. At their largest values, all coefficients are above 0.9 for Nt , while at their lowest values they are 0.82, 0.79 and 0.71 for R^2 , R^2 (adj) and R (pred), respectively for Ncm . Apart from the latter, all correlation coefficients for the rest of the responses look remarkably good. These surrogate models provide an efficient way to predict worst-case combinations without requiring exhaustive simulations.

$$Nt \text{ [rpm]} = -5013880 C_{b1} + 4870 m_1 + 37082706 C_{b2} + 6910 m_2 + 34081023 C_{b3} + 5416 m_3 + 121803788819 C_{b1} \\ \times C_{b1} + 105230973635 C_{b3} \times C_{b1} - 37693698 C_{b1} \times m_1 - 178719010760 C_{b1} \times C_{b3} - 33006198 C_{b2} \times m_2 \\ - 186531510760 C_{b2} \times C_{b3} - 26756198 m_2 \times C_{b3} - 42381198 C_{b3} \times m_3 + 548 \quad (7)$$

$$AFm = -84292 C_{b1} + 14.00 m_1 + 50855 C_{b2} + 12.51 m_2 - 140148 C_{b3} - 2.065 m_3 + 601565169 C_{b1} \times C_{b1} \\ + 1061084135 C_{b3} \times C_{b3} - 118169 C_{b1} \times m_2 - 116717 m_1 \times C_{b3} - 586178491 C_{b2} \times C_{b3} + 11.01 \quad (8)$$

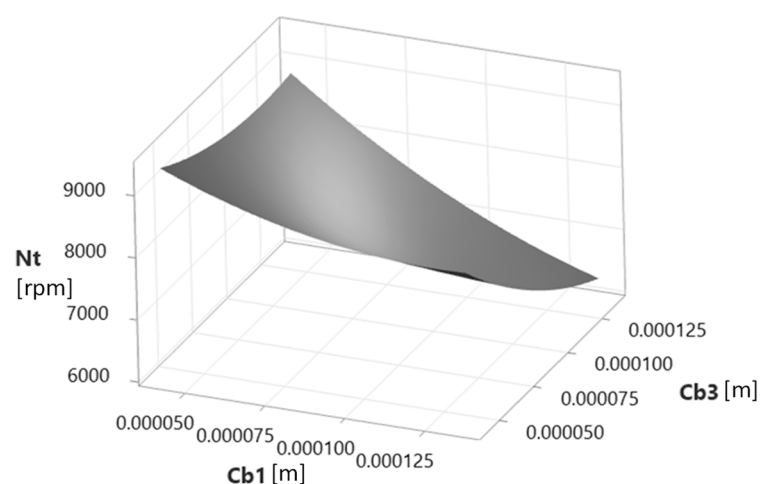
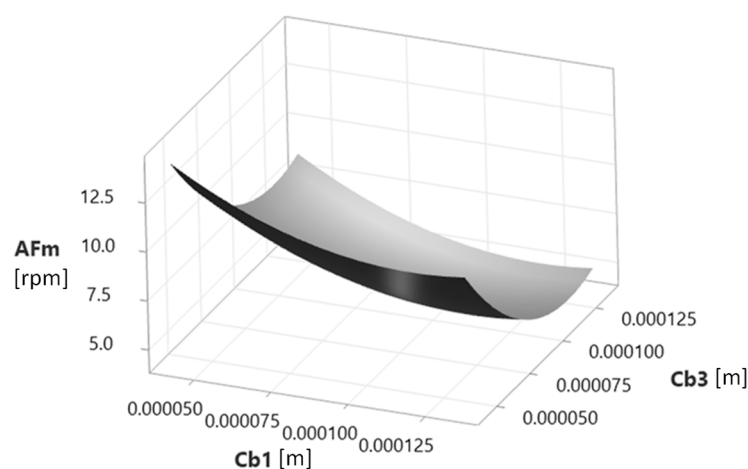
$$Ncreg \text{ [rpm]} = -48255165 C_{b1} - 3517 m_1 - 20708153 C_{b2} - 11432 m_2 - 39186357 C_{b3} - 4178 m_3 + \\ 137172702222 C_{b1} \times C_{b1} + 2172 m_2 \times m_2 + 308344529205 C_{b1} \times C_{b3} + 16456094 C_{b1} \times m_3 + 5928 m_1 \times m_2 + \\ 19581094 C_{b2} \times m_2 + 85688279205 C_{b2} \times C_{b3} + 18018594 m_2 \times C_{b3} + 4209 m_2 \times m_3 + 9874 \quad (9)$$

$$DE \text{ Brg SS } [\mu\text{m}] = [40.0 C_{b1} - 0.1435 m_1 - 503 C_{b2} - 0.0943 m_2 - 261.9 C_{b3} - 0.0521 m_3 + 3576757 C_{b1} \times C_{b3} + \\ 579 m_1 \times C_{b2} + 0.0707 m_1 \times m_2 + 0.0877 m_1 \times m_3 + 478 C_{b2} \times m_2 + 0.1351] \times 25.4 \quad (10)$$

Table 5. SRDOE model fitting.

Response	R ²	R ² (adj)	R ² (pred)
<i>Nt</i>	0.96	0.94	0.91
<i>Ncm</i>	0.82	0.79	0.71
<i>Ncreg</i>	0.93	0.91	0.85
<i>Afm</i>	0.89	0.87	0.77
<i>Nt-Nso</i>	0.94	0.93	0.91
<i>Midspace Pk</i>	0.91	0.89	0.82
<i>FE Brg Pk</i>	0.91	0.88	0.85
<i>DE Brg Pk</i>	0.88	0.86	0.82
<i>Midspace SS</i>	0.88	0.85	0.76
<i>FE Brg SS</i>	0.92	0.90	0.85
<i>DE Brg SS</i>	0.92	0.90	0.84

Figures 9–11 depict the surface response for some key variables. The rest of the input variables are held at their means. Note that the shapes of the surfaces correspond to upward-opening tilted paraboloids.

**Figure 9.** Surface response plot displaying the speed of instability vs. C_{b1} and C_{b3} .**Figure 10.** Surface response plot displaying amplification factor vs. C_{b1} and C_{b3} .

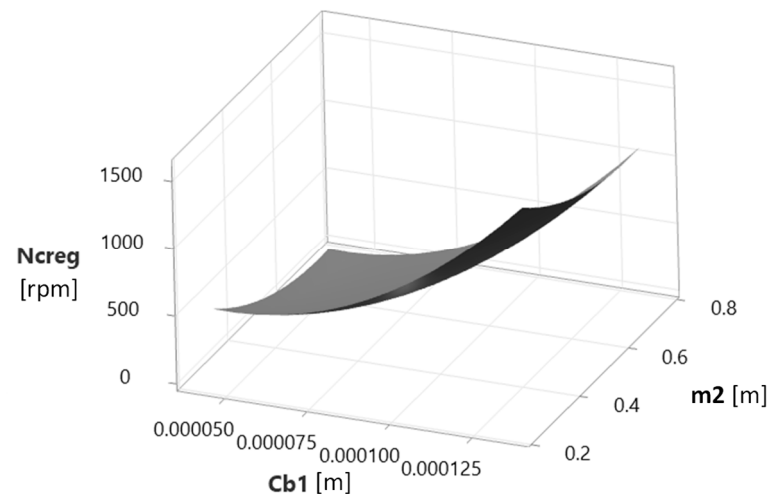


Figure 11. Surface response plot displaying critical speed region vs. C_{b1} and m_2 .

7. Maxima and Minima Using SRDOE Surrogate Models

All the SRDOE surrogate models were analyzed for the specific bearing geometry that delivered the maximum and minimum magnitudes. The following desirability method was used:

1. Establish constraints: The boundaries for the bearing geometry are the Min and Max levels in Table 1.
2. Define individual desirability d functions for each response: The method involves the transformation of each predicted response to a dimensionless partial desirability function, d_i . One- or two-sided functions are used. L_i , T_i and U_i are the lowest, target and highest values obtained for the response i , respectively.

If a response is to be maximized, the individual desirability is defined as

$$d = \begin{cases} 0 & \text{if } \hat{Y}_i(x) < L_i \\ \frac{\hat{Y}_i(x) - L_i}{T_i - L_i} & \text{if } L_i \leq \hat{Y}_i(x) \leq T_i \\ 1 & \text{if } \hat{Y}_i(x) > T_i \end{cases} \quad (11)$$

On the other hand, the desirability function to minimize a response is given by

$$d = \begin{cases} 0 & \text{if } \hat{Y}_i(x) > U_i \\ \frac{U_i - \hat{Y}_i(x)}{U_i - T_i} & \text{if } T_i \leq \hat{Y}_i(x) \leq U_i \\ 1 & \text{if } \hat{Y}_i(x) < T_i \end{cases} \quad (12)$$

3. Maximize the overall desirability D in Equation (13) with respect to the input variables: This is calculated using the geometric mean. Customarily, an acceptable overall desirability is $D > 0.5$. A weighting strategy of 1 for all individual d_i was used. In other words, all responses have the same importance in the optimization process.

$$D^Y(x) = (d_1^Y(x) \cdot d_2^Y(x) \cdot \dots \cdot d_i^Y(x))^{1/y}, y = 11 \quad (13)$$

If D is close to 1, the combination of the individual desirabilities is globally optimum. In this formulation of D , possible correlations between the responses are not taken into account, and hence, it is assumed that the responses are independent of each other. That is why this method is well-suited when used in combination with an SRDOE. To solve the optimization problem of maximizing the continuous but nondifferentiable D , a univariate

search technique that does not employ derivative information was used to solve the overall optimization problem.

The minimized response of Nt is shown in Figure 12, where C_{b1} contributes most to Nt , followed by m_2 and C_{b3} . Tables 6 and 7 summarize the one-at-a-time maximization and minimization of the responses. This enables visualization of the most adverse bearing-geometry conditions for each individual response. Note again that none of the bearing geometries correspond to the homogeneous cases, except the maximum *Midspan SS* in Table 6; that is, 1 out of the 22 responses (11 max, and 11 min) agrees with a homogeneous bearing pad geometry. A few additional runs (48–53) were executed, as such bearing geometries were not inside the original SRDOE subset. The responses sharing the same bearing geometry are shaded with a gray background. The individual desirabilities are very good, while the correlation of the SRDOE surrogate model to the system-level rotodynamic model is reasonably good. Note that in terms of stability, the one-at-a-time maximization and minimization of Nt correspond to the same cases obtained while evaluating the WFR, i.e., case 15 and 18, respectively.

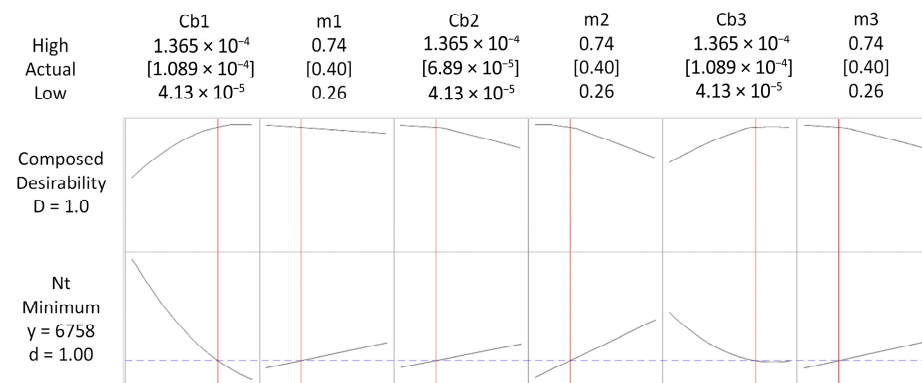


Figure 12. Minimized response for the threshold speed of instability.

Table 6. Maximization of individual (one-at-a-time) responses.

Max Response	C_{b1}	m_1	C_{b2}	m_2	C_{b3}	m_3	D	SRDOE	System-Level	Diff. [%]	Case
Nt [rpm]	−1	1	1	1	−1	1	0.91	9164	9425	2.8	15
Ncm [rpm]	1	1	−1	−1	−1	−1	0.99	4647	4625	−0.5	4
$Ncreg$ [rpm]	1	−1	−1	−1	1	1	1.00	1211	1100	−10.1	18
AFm	−1	1	1	1	−1	−1	0.70	9.9	8.4	−18.2	48
$Nt - Nso$ [rpm]	−1	1	1	1	−1	1	0.93	3882	3621	−7.2	15
$Midspan Pk$ [μm]	−1	1	1	1	−1	−1	0.84	18.5	17.6	−4.9	53
$FE Brg Pk$ [μm]	1	1	−1	1	1	1	0.95	2.3	2.2	−6.0	51
$DE Brg Pk$ [μm]	1	1	−1	1	1	1	0.96	2.5	2.3	−7.2	51
$Midspan SS$ [μm]	−1	1	−1	1	−1	1	0.90	4.1	4.1	0.2	47
$FE Brg SS$ [μm]	1	−1	−1	−1	1	−1	1.00	1.5	1.5	−4.6	18
$DE Brg SS$ [μm]	1	−1	−1	−1	1	−1	1.00	1.5	1.4	−4.3	18

The worst and best scenarios were searched for all responses simultaneously per Equation (13). Figure 13 shows the worst scenario, where all responses have the same importance in the optimization process. Some responses are set to be maximum, while others are set to be minimum, depending on their adversity to the imbalance response and stability. For instance, to obtain the worst case $Ncreg$, AFm , $Midspan Pk$, $FE Brg Pk$, $DE Brg Pk$, $Midspan SS$, $FE Brg SS$, and $DE Brg SS$ are simultaneously maximized, while Nt , and Ncm are minimized. The opposed scenario occurred when obtaining the best case. Inspection of Figure 13 allows C_{b1} and C_{b3} to be highlighted as the most significant factors

for all responses, while also showing a relevant change of direction of the responses owing to their curvature behavior.

Table 7. Minimization of individual (one-at-a-time) responses.

Max Response	C_{b1}	m_1	C_{b2}	m_2	C_{b3}	m_3	D	SRDOE	System-Level	Diff. [%]	Case
<i>Nt</i> [rpm]	1	−1	−1	−1	1	−1	0.95	6758	6800	0.6	18
<i>Ncm</i> [rpm]	−1	1	1	1	1	1	0.87	4453	4450	−0.1	52
<i>Ncreg</i> [rpm]	−1	1	1	−1	1	1	1.00	50	100	50.0	23
<i>AFm</i>	1	−1	1	1	1	1	1.00	3.9	4.4	10.0	49
<i>Nt−Nso</i> [rpm]	1	−1	−1	−1	1	−1	1.00	1276	1449	12.0	18
<i>Midspan Pk</i> [μm]	1	−1	1	−1	1	1	1.00	7.7	8.5	8.9	22
<i>FE Brg Pk</i> [μm]	−1	−1	1	−1	1	−1	1.00	2.0	2.0	0.0	21
<i>DE Brg Pk</i> [μm]	−1	−1	1	−1	1	−1	1.00	2.1	2.1	0.5	21
<i>Midspan SS</i> [μm]	1	−1	1	−1	1	1	1.00	3.2	3.3	1.2	22
<i>FE Brg SS</i> [μm]	−1	1	−1	1	1	1	0.85	0.8	0.8	3.2	27
<i>DE Brg SS</i> [μm]	−1	1	−1	1	1	−1	0.85	0.7	0.7	2.2	50

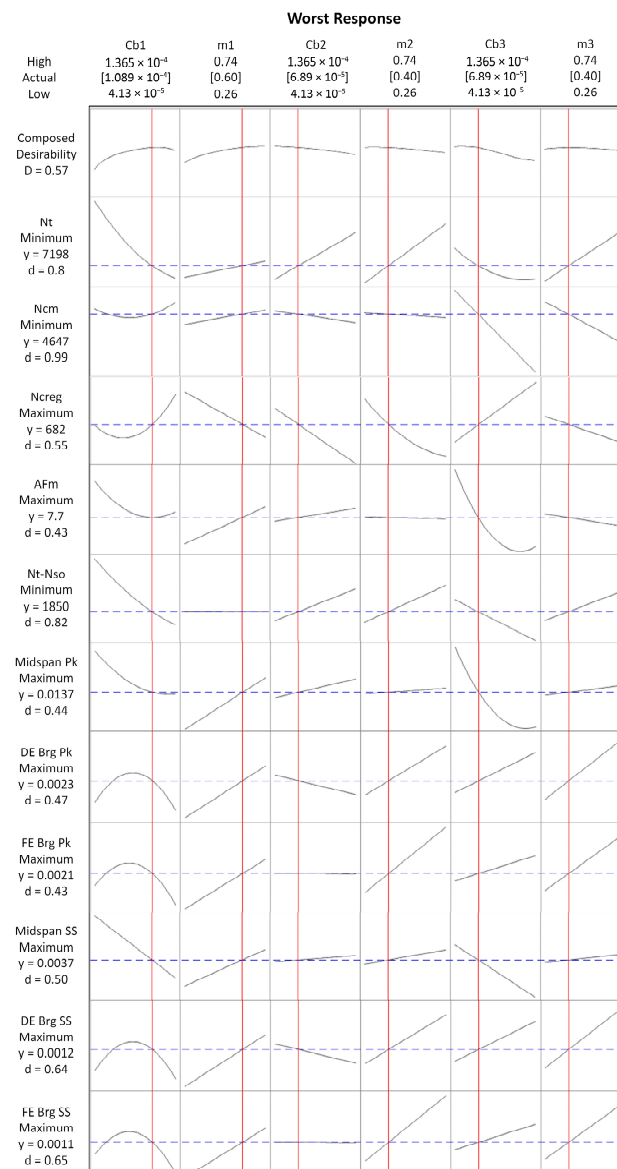


Figure 13. Desirability and response optimization plot for the worst-case scenario and all responses.

The worst and best scenarios are outlined in Table 8. A new case (54) had to be added as it does not fall within the original SRDOE subset. Thus far, the best-case scenario is the only one showing an intermediate value for one of the bearing-geometry parameters (m_2). This is translated from coded units to uncoded units as follows. The center point of m_2 is 0 in coded units, while 0.5 in uncoded units. On the other hand, the corner point +1 of m_2 in coded units corresponds to 0.6 in uncoded units. A regression equation leads to $m_{uncoded} = 1 - 0.8 (m_{coded})$. Then, a 0.48 value of m_2 in coded units translates to 0.55 in uncoded units. Neither the worst nor the best scenario matches the uniform pad cases. Table 9 compares surrogate model predictions with rotordynamic simulations, showing strong agreement except for N_{creg} , where higher-order effects were present.

Table 8. Worst and best scenarios.

Scenario	C_{b1}	m_1	C_{b2}	m_2	C_{b3}	m_3	D	Case
Worst	1	1	−1	−1	−1	−1	0.58	4
Best	−1	0.48	1	−1	1	1	0.72	54

Table 9. Worst (4) and best (54) scenarios vs. system-level predictions.

Response	Worst			Best		
	SRDOE	System-Level	Diff. [%]	SRDOE	System-Level	Diff. [%]
Nt [rpm]	7198	7450	3.4	7965	8025	0.8
N_{cm} [rpm]	4647	4625	−0.5	4473	4538	1.4
N_{creg} [rpm]	682	725	6.0	55	175	68.5
AFm	7.7	7.7	0.5	5.1	5.1	0.0
$Nt - N_{so}$ [rpm]	1850	2105	12.1	2743	2795	1.9
$Midspan Pk$ [μm]	13.7	13.7	0.3	10.1	10.5	3.9
$FE Brg Pk$ [μm]	2.1	2.1	−0.5	2.1	2.1	0.0
$DE Brg Pk$ [μm]	2.3	2.3	−1.2	2.2	2.2	0.0
$Midspan SS$ [μm]	3.7	3.8	1.9	3.6	3.6	−2.1
$FE Brg SS$ [μm]	1.2	1.2	1.5	0.9	1	6.1
$DE Brg SS$ [μm]	1.1	1.1	1.7	0.9	0.8	−6.5

As stated earlier, all simulations thus far have considered the journal at its nominal size, which corresponds to the minimum (R_{jmin}) given its one-sided tolerance. Next, the one-at-a-time minimization of the threshold speed of instability Nt (case 18), and the one-at-a-time maximization of the amplification factor AFm (case 48) were selected to be combined with the journal at its maximum (R_{jmax}) size, as shown in Table 10. The threshold speed of instability increases slightly when the journal size is at its maximum size, thus revealing that the journal tolerance is not as important for stability as the bearing tolerance. On the other hand, the amplification factor significantly increases when the journal is at its maximum size.

Table 10. System-level results varying journal size.

Case	Variable	Dissimilar pads	
		Nom (J_{min})	J_{max}
18	Nt min [rpm]	6800	6825
48	AFm max	8.4	13.4

Finally, Table 11 contains system-level predictions of the threshold speed of instability and amplification factor per API [7,8], where both uniform pads and journal size at its

extreme tolerances are considered. The dissimilar pad cases in Table 10 still show more adverse results than the uniform pads cases when varying journal size. This is consistent with the study of Urbiola-Soto and Pennacchi [19] for tilting-pad journal bearings, where they report that the lowest logdec is found for dissimilar clearance on pads located on the diagonals.

Table 11. API uniform pads varying journal size.

Variable	API [7,8] Uniform pads	
	$C_{b,max}$	$C_{b,min}$
Nt [rpm]	7350	8825
AFm	4.6	9.2

8. Conclusions

This work has demonstrated that the rotordynamic behavior of three-lobe journal bearings is strongly influenced by manufacturing variations in pad clearance and preload. SRDOE surrogate models showed high correlation with system-level rotordynamic calculations, confirming their predictive accuracy. This ensures that SRDOE captures the dominant trends while reducing computational cost. The key findings can be summarized as follows:

1. System-level sensitivity to pad variation

The imbalance response and stability are highly influenced by differences between pads. Critical speeds shifted by several hundred rpm and instability thresholds moved by several thousand rpm. These results highlight the need to include pad variability in system-level models.

2. Opposing trends in stiffness, damping, and cross-coupling

Increased preload across pads raised stiffness and damping, improving imbalance response. However, it also increased cross-coupled stiffness, which undermines stability. The most critical conditions arose from uneven pad configurations. No single case consistently produced the highest or lowest coefficients, underscoring the nonlinear influence of pad variability.

3. Robustness requires considering variability

Surface Response Design of Experiments (SRDOE) methods effectively captured the nonlinear influence of pad tolerances. These models revealed the most critical combinations of variables and showed that the conventional “max–min” assumption does not guarantee robustness.

4. Journal tolerances amplify effects

When journal radius tolerances were included alongside pad variations, amplification factors and stability thresholds shifted further. The combined effects were more severe than those predicted by standard uniform pad assumptions in API guidelines.

5. Implications for design practice

Worst-case conditions did not occur under uniform tolerance extremes. Instead, specific non-uniform pad arrangements produced the largest imbalance response and lowest stability margin. Ignoring pad-to-pad variability can underestimate worst-case behavior. Incorporating bearing dissimilar pads into rotordynamic analysis offers a more reliable approach for assessing turbomachinery performance and safety margins.

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Nomenclature

Bearing parameters

C_b	Radial assembled pad clearance [m]
C_p	Radial as-machined pad clearance [m]
D	Bearing journal diameter [m]
$f = C_p/D$	Diametral clearance per each 1 mm bearing diameter [mm]
J	Journal radius, [m]
L	Bearing axial length [m]
m	Bearing preload [–]

Model Variables

ax	Axis of vibration (horizontal, vertical, maximum)
i, j	Variable subindexes
k	Number of critical speeds per axis
min, max	Minimum and maximum values
x	Model independent variables (design variables)
y	Model dependent variable (responses)

Rotordynamic parameters

$DE\ Brg$	Drive-end bearing
$FE\ Brg$	Free-end bearing
$Midspan$	Rotor midspan
Nos	Minimum operating speed [rpm]
Nso	Safe operating speed [rpm]
Pk	Peak at major axis of vibration [mm]
SM	Separation margin
SS	Steady-state at major axis of vibration [mm]

Tribological parameters

flow	Pad inlet flow [lpm]
H	Film thickness [m]
P	Pressure [kPa]
Power	Power consumption [W]
T	Temperature [°C]

Statistical parameters

d	Individual
D	Overall
p -value	Conditional probability

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