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Reliability Prediction Model for Ball Screws Considering Full-Life Fatigue Damage

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Abstract

This paper addresses the challenges of life prediction and reliability assessment for ball screws under complex operating conditions by proposing a reliability prediction model that incorporates full-life fatigue damage. First, a full-life fatigue life prediction model encompassing the three stages of crack initiation, propagation, and fatigue cumulative spalling is developed. This model comprehensively considers the effects of material properties, geometric parameters, and loading history, enabling a systematic description of the fatigue process of ball screws from initial use to final failure. Based on this life prediction model, an enhanced adaptive Kriging–Monte Carlo simulation (E-AK-MCS) method is introduced to construct a surrogate model, which efficiently solves the high-dimensional nonlinear limit state function, thereby enabling accurate reliability assessment and parameter sensitivity analysis. Experimental results demonstrate that the proposed model achieves an average life prediction accuracy of 94.15% for the 8020 and 5005 specification ball screws, indicating its preliminary engineering applicability under the tested conditions. Reliability analysis indicates that ball diameter fracture toughness, and initial crack size are key factors influencing service reliability. This research provides systematic theoretical methods and technical support for the accurate life prediction, reliability design, and process optimization of ball screws.

Keywords: ball screws; full-life fatigue damage; life prediction; reliability analysis; E-AK-MCS

1. Introduction

As a core transmission component in high-end equipment such as CNC machine tools, aerospace equipment, and precision instruments, the performance and reliability of ball screws directly affect the accuracy, dynamic response characteristics, and service life of the overall system [1]. In practice, ball screws are often subjected to complex stress states involving high speeds, high load, and variable load, making them prone to various failure modes including fatigue pitting, wear, and plastic deformation [2], which severely impacts their service life and operational reliability. Therefore, conducting research on full-life fatigue life prediction and reliability analysis of ball screws is of great significance for improving their design level, optimizing usage strategies, and achieving condition-based maintenance [3]. However, existing studies often neglect the life during the fatigue spalling stage. Although this stage is relatively brief, its omission introduces significant deviations into traditional life prediction models, which may lead to either overestimation or underestimation depending on the specific specimen conditions.



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Classical rolling contact fatigue life prediction is closely related to the work of Palmgren and Lundberg–Palmgren. Palmgren proposed an early fatigue life concept for rolling bearings [4], and the Lundberg–Palmgren theory further established a probabilistic relationship between dynamic load rating and rolling bearing fatigue life [5]. These theories laid the foundation for the rated-life calculation of rolling contact components and have been widely extended to ball screws and other rolling transmission systems. However, traditional rated-life methods mainly describe the statistical life before macroscopic fatigue failure and generally do not explicitly distinguish crack initiation, stable crack propagation, and cumulative spalling. Therefore, when the fatigue damage process after crack initiation is of concern, further full-life modeling is still required.

Building on these classical rolling contact fatigue theories, current research on the life and reliability of ball screws has further focused on specific failure modes, operating conditions, and reliability modeling methods. Ye et al. [6] discovered that severe surface wear, pronounced spalling, and numerous localized cracks on the nut raceway were the primary causes of ball screw unit failure. Liu Lu et al. [7] established a competing failure model considering performance degradation and fatigue pitting, providing a framework for reliability design. Zaid Mael et al. [8] demonstrated that biaxial compressive stress inhibits branch cracks and promotes Mode II fatigue crack growth, obtaining friction-modified crack propagation data. Lai et al. [9] employed a fluid–structure interaction finite element method to reveal the penetration mechanism of lubricant into surface cracks, analyzing the effects of viscosity and load speed on crack growth. Barradas-Berglind et al. [10] combined the rainflow counting method with a hysteresis operator to propose a fatigue damage model based on dissipated energy, embedding it within model predictive control to mitigate drive shaft damage. Wang et al. [11] established a finite element model of a ball screw considering complex loads, analyzed the relationship between axial deformation and position error, and proposed an error compensation method based on deformation prediction. Kim et al. [12] integrated an improved Archard wear model with a damage model to propose a life prediction method for linear guides that comprehensively considers wear and rolling contact fatigue. Zhou et al. [13] further introduced fractal theory and geometric correction factors to establish a more realistic preload degradation model, revealing the influence mechanism of micro-contact behavior on macroscopic performance.

In the realm of reliability modeling, Zhou et al. [14] proposed a reliability model based on performance degradation for extreme operating conditions, utilizing the expectation-maximization algorithm for parameter estimation, offering a new approach for reliability assessment of ball screws with high reliability requirements and short service durations. Zhao et al. [15] proposed a reliability-based design optimization method for ball screw feed systems to address the limitations of traditional deterministic design methods in handling machining errors and material uncertainties. Wu et al. [16] developed a ball load distribution model comprehensively considering geometric errors and composite loads, revealing the significant influence of lead error on fatigue life. Liu Jun et al. [17] introduced the concept of fatigue elastic life, constructed a reliability model considering epistemic uncertainty based on uncertainty theory, and identified contact angle uncertainty as the most influential factor on reliability. Huang Kuan et al. [18] employed a quasi-Monte Carlo method combined with Halton sequences for reliability analysis, improving computational efficiency and accuracy. Ge Yuhong et al. [19] verified the comprehensive performance advantages of domestic 55CrMo steel through fatigue testing and failure analysis.

However, existing research largely concentrates on specific stages or single failure mechanisms. There is a lack of systematic modeling for the full-life fatigue process of ball screws from initial use to final failure. Furthermore, unlike traditional rated-life methods that mainly provide a deterministic life estimate, this study establishes an integrated

full-life fatigue prediction framework for ball screws. The novelty lies in coupling crack initiation, stable crack propagation, and cumulative spalling into a stage-equivalent life model, and further embedding this model into an E-AK-MCS reliability assessment procedure. Therefore, the proposed method provides not only a deterministic life prediction but also failure probability and parameter sensitivity information for ball screws fatigue reliability design.

It should be noted that the present model is intended for heavy-duty ball screw applications operating under high contact pressure and relatively low sliding speed, where boundary or mixed lubrication is likely to occur. Under these conditions, the lubricant film may not fully separate the contacting asperities, and fatigue damage is strongly affected by the local contact stress field. Therefore, lubricant viscosity, EHL film thickness, and lubricant degradation are not explicitly included as independent variables in the current model. This treatment defines the applicable scope of the proposed framework rather than excluding the influence of lubrication. Further extension is required to incorporate EHL behavior and lubricant degradation when the model is applied to high-speed or full-film lubrication conditions.

2. Establishment of a Full-Life Fatigue Life Model for Ball Screws

As shown in Figure 1, ball screws of different specifications are influenced by factors such as lead, material, helix angle, ball recirculation method, operating environment, and raceway surface treatment. Consequently, their life under the rated dynamic load will deviate from 10^6 revolutions.

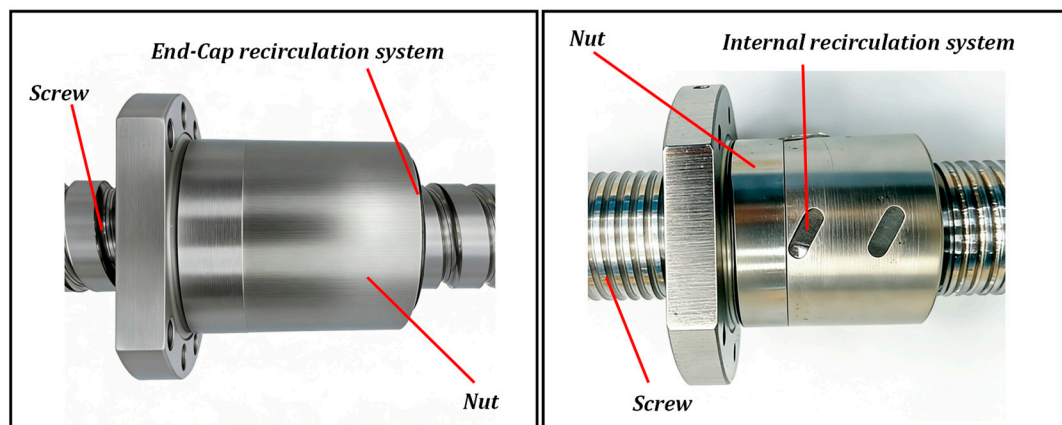


Figure 1. Schematic diagram of a ball screws.

Therefore, based on the material parameters and geometric parameters of each ball screws, this study calculates the fatigue crack initiation life, fatigue crack propagation life, and fatigue cumulative spalling life to establish a full-life model for ball screws. Before establishing the full-life model for ball screws, the following assumptions are made:

- (1) It is assumed that all ball screws operate under the same environmental conditions and are all subjected to the rated dynamic load.
- (2) The tested ball screws are assumed to have no detectable macroscopic manufacturing defects or abnormal damage before testing. Microscopic inclusions, surface imperfections, and equivalent initial crack sizes are considered in the reliability analysis through the random variable a_0 .
- (3) Assembly errors are within a reasonable range and are identical for all ball screws.
- (4) The lubrication condition is assumed to be boundary or mixed lubrication, where the contact surfaces are partially separated by a thin lubricant film and asperity interactions dominate the stress distribution. Consequently, the effects of EHL film

thickness and viscosity variation are neglected. This assumption is justified by the high contact pressures and relatively low sliding velocities typical of the rated-load operating conditions examined in this study.

2.1. Load Distribution and Contact Stress Analysis of Ball Screws

To establish a full-life fatigue life prediction model for ball screws, it is first necessary to clarify the stress state, load distribution, and the resulting critical stress and strain fields under rated dynamic load. This subsection, based on Hertzian contact theory, systematically derives the maximum contact stress and deformation between the ball and raceway, providing input parameters for the subsequent fatigue crack initiation life model.

The ball screws transmits axial force F_a between the screw and nut raceways via balls. At any working position, the load distribution on the ball screws subjected to axial force F_a exhibits periodic variation. The equilibrium equations, constitutive equations, and geometric equations are formulated and solved. The nut and screw are segmented at the ball-raceway contact points, and the balls, nut segments, and screw segments are numbered. The load-carrying ball closest to the nut flange face is designated as ball No. 1. The screw segment between ball i and ball $i + 1$ is denoted as $S^{(i)}$, and the nut segment as $N^{(i)}$. The contact force of ball i acts only on $S^{(i)}$ and $N^{(i)}$.

Force analysis of the nut yields:

$$F_a = \sum_{j=1}^N Q_j \sin \alpha_j \cos \varphi \quad (1)$$

where F_a is the axial load, Q_j is the normal contact force between the j -th ball and the raceway, α_j is the contact angle of the j -th ball, and N is the number of load-carrying balls.

Force analysis of the nut segment $N^{(i)}$ yields:

$$F_{n,i-1} = F_{n,i} + Q_i \sin \alpha_i \cos \varphi \quad (2)$$

where $F_{n,i}$ is the internal force of nut segment $N^{(i)}$. Simplifying gives:

$$F_{n,i} = F_a - \sum_{j=1}^i Q_j \sin \alpha_j \cos \varphi \quad (3)$$

Similarly, the internal force of screw segment $S^{(i)}$ can be expressed as:

$$F_{s,i} = F_a - \sum_{j=1}^i Q_j \sin \alpha_j \cos \varphi \quad (4)$$

According to Hertzian contact theory, the relationship between the normal contact force and the normal contact deformation for the i -th ball is:

$$Q_i = K_h \delta_i^{\frac{3}{2}} \quad (5)$$

where K_h is the Hertzian contact stiffness of the contact zone, and δ_i is the axial deformation of the i -th ball. The axial deformations $N^{(i)}$ and $S^{(i)}$ of nut segment Δn_i and screw segment Δs_i can be expressed as:

$$\Delta n_i = \frac{F_{n,i} \Delta L}{E_n A_n} \quad (6)$$

$$\Delta s_i = \frac{F_{s,i} \Delta L}{E_s A_s} \quad (7)$$

where E_n and E_s are the elastic moduli of the nut and screw materials, respectively, A_n and A_s are the effective cross-sectional areas of the nut and screw, respectively, and ΔL is the length of the nut or screw segment, given by:

$$\Delta L = \frac{nP_h}{N} \quad (8)$$

where n is the number of load-carrying ball turns and P_h is the lead.

It is stipulated that the axial deformations Δn_i and Δs_i of the screw and nut are positive in tension and negative in compression. A deformation compatibility equation can then be formulated based on the distance from the contact point of ball i with the nut to the contact point of ball $i + 1$ with the screw:

$$\Delta n_i - \delta'_{i+1} = \Delta s_i - \delta'_i \quad (9)$$

where δ'_i is the axial deformation of the i -th ball, given by:

$$\delta'_i = \frac{\delta_i}{\sin \alpha_i \cos \varphi} \quad (10)$$

After deformation occurs, the contact angle also changes. This can be determined based on the distance between the curvature centers of the screw and nut raceways. In the initial state, the distance between curvature centers can be expressed as:

$$\overline{O_n O_s} = (f_{rn} + f_{rs} - 1)D_w \quad (11)$$

where f_{rn} and f_{rs} are the conformity ratios of the nut and screw raceways, respectively. After deformation, the distance between curvature centers in the contact zone of the i -th ball can be expressed as:

$$\overline{O_n O_s}' = (f_{rn} + f_{rs} - 1)D_w + \delta_i \quad (12)$$

Then, based on geometric relationships, the change in contact angle can be expressed as:

$$\alpha_j = \cos^{-1} \left(\frac{\overline{O_n O_s} \cos \alpha_0}{\overline{O_n O_s}'} \right) \quad (13)$$

Combining Equations (1) and (3)–(12), the system of equations for solving the load distribution can be derived:

$$\left\{ \begin{array}{l} F_a - \sum_{j=1}^i K_h \delta_j^{\frac{3}{2}} \sin \alpha_j \cos \varphi - \frac{Z}{nP_h \left(\frac{1}{E_s A_s} + \frac{1}{E_n A_n} \right) \cos \varphi} \cdot \left(\frac{\delta_{i+1}}{\sin \alpha_{i+1}} - \frac{\delta_i}{\sin \alpha_i} \right) = 0 \quad (i = 1, \dots, Z-1; \delta_j > 0) \\ F_a - \sum_{j=1}^i K_h \delta_j^{\frac{3}{2}} \sin \alpha_j \cos \varphi = 0 \quad (\delta_j > 0) \\ \alpha_i - \cos^{-1} \left[\frac{(f_{rn} + f_{rs} - 1)D_w \cos \alpha_0}{(f_{rn} + f_{rs} - 1)D_w + \delta_i} \right] = 0 \end{array} \right. \quad (14)$$

Here, n is the number of load-carrying ball turns. Equation (14) constitutes a system of $2Z$ equations, which can be solved for the $2Z$ unknowns: the contact deformations δ_i and contact angles α_i .

The contact between the ball and raceway can be simplified as a point contact problem between two curved bodies, suitable for analysis using Hertzian contact theory. In the cross-

section passing through the ball center and perpendicular to the screw axis, the curvature radii of the screw and nut raceways determine the contact geometry. The contact of the ball with the screw raceway and the nut raceway is treated as a contact problem within two principal curvature planes. For the point contact between a ball and a raceway, the contact area is an ellipse. According to Hertzian theory, the semi-major axis a_0 , semi-minor axis b_0 of the contact ellipse, and the maximum Hertzian contact stress σ_1 can be calculated by the following equations:

$$a_0 = a^* \left[\frac{3Q}{2\sum\rho} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right) \right]^{\frac{1}{3}} \quad (15)$$

$$b_0 = b^* \left[\frac{3Q}{2\sum\rho} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right) \right]^{\frac{1}{3}} \quad (16)$$

$$\sigma_1 = \frac{3K_h\delta^{\frac{3}{2}}}{2\pi a_0 b_0} \quad (17)$$

where a^*, b^* are Hertzian contact coefficients related to the contact geometry, functions of the principal curvature function $F(\rho)$. $\sum\rho$ is the sum of principal curvatures of the two bodies at the contact point. E_1, E_2 are the elastic moduli of the ball and screw/nut materials, respectively. ν_1, ν_2 are the Poisson's ratios of the ball and screw/nut materials, respectively.

Within the Hertzian contact stress field, the maximum principal stress σ_1 and maximum principal strain ε_1 typically occur in the subsurface region at a certain depth directly below the contact area. For high-cycle fatigue, this region is the primary site for crack initiation. Under pure rolling or rolling-with-sliding contact conditions, the stress state at any point inside the contacting bodies can be obtained through integration of the Hertzian stress field or by numerical methods. The maximum principal strain ε_1 can be determined from the principal stresses using the generalized Hooke's law:

$$\varepsilon_1 = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] \quad (18)$$

where $\sigma_x, \sigma_y, \sigma_z$ are the normal stress components along the directions of the contact ellipse's major axis, minor axis, and depth, respectively.

Therefore, through the basic mechanical model described above, starting from the external axial load F_a , the key parameters required for the SWT multiaxial fatigue criterion—specifically, the maximum principal strain amplitude σ_1 and the maximum normal stress ε_1 on the plane where this maximum principal strain occurs—can be progressively derived. The subsequent full-life fatigue prediction model for ball screws in this paper will be based on the stress-strain field derived in this section.

2.2. Crack Initiation Life Prediction Model for Ball Screws

The stress state during the fatigue process of ball screw raceways is typically multiaxial, meaning the raceway is subjected to multi-directional alternating stresses. Relevant studies indicate that fatigued ball screws exhibit Mode I cracks [20]. The Smith–Watson–Topper (SWT) multiaxial fatigue life criterion is applicable to materials failing in the Mode I crack growth mode. The SWT parameter can predict crack initiation and growth on the plane of maximum principal stress and strain. Therefore, the multiaxial fatigue life criterion is adopted to predict the fatigue crack initiation life of ball screws, as shown in Equation (19):

$$\frac{\Delta\varepsilon_1}{2}\sigma_1 = \frac{\sigma_f^2}{E}(2N_i)^{2n} + \varepsilon'_f\sigma'_f(2N_i)^{n+g} \quad (19)$$

where $\Delta\epsilon_1$ is the maximum principal strain amplitude; σ_1 is the maximum normal stress on the plane of maximum principal strain amplitude, which can be calculated using Hertzian contact theory or finite element analysis; σ'_f is the fatigue strength coefficient; E is the material's elastic modulus; N_i is the fatigue crack initiation life; ϵ'_f is the fatigue ductility coefficient; n is the fatigue strength exponent; g is the fatigue ductility exponent. For the material fatigue parameters (σ'_f , n , ϵ'_f , g) in Equation (19), as shown in Table 1, scholars have proposed several different estimation methods based on extensive experimental data fitting [21]. In the table, σ_n is the material's ultimate tensile strength, R_A is the reduction in area, and ϵ_f is the true fracture strain.

Table 1. Estimation methods for material crack initiation life model parameters.

Parameter	Universal Slopes Method	Modified Universal Slopes Method	Homogeneous Material Criterion	Median Method
σ'_f	$1.9\sigma_n$	$0.623\sigma_n^{0.832}E^{0.168}$	$1.5\sigma_n$	$1.5\sigma_n$
n	-0.12	-0.09	-0.087	-0.09
ϵ'_f	$0.76\epsilon_f^{0.6}$	$0.0196\epsilon_f^{0.6}(\sigma_n/E)^{-0.53}$	$0.59R_A$	0.45
g	-0.6	-0.56	-0.58	-0.59

Neglecting plastic deformation during contact, we have:

$$\frac{\Delta\epsilon_1}{2}\sigma_1 = \frac{\sigma_f'^2}{E}(2N_i)^{2n} \quad (20)$$

Compared to the methods for obtaining material fatigue parameters mentioned above, combining the simplified SWT parameter method with the S-N curve provides a simpler and more accurate way to determine the values of each material parameter. Taking GCr15, a common material for ball screws, as an example, based on the GCr15 fatigue life S-N curve from the literature [22], an appropriate fitting function $y = ax^b$ is selected. The relationship curves for the fatigue strength coefficient and the fatigue strength exponent can be obtained by fitting. The fitting results are shown in Table 2:

Table 2. Fitting parameters for GCr15 fatigue life S-N curve.

Parameter	Value
σ'_f (MPa)	2204.6162
n	-0.0409
E (MPa)	208,000

Therefore, the crack initiation life of a ball screws can be expressed as:

$$N_i = \frac{1}{2} \left(\frac{\frac{\Delta\epsilon_1}{2}\sigma_1 \cdot E}{\sigma_f'^2} \right)^{\frac{1}{2n}} \quad (21)$$

2.3. Crack Propagation Life Prediction Model for Ball Screws

According to previous research, fatigue cracks in ball screws are generally elliptical in shape [23]. Assuming the semi-major axis of the crack is a and the semi-minor axis is b , the general formula for calculating the stress intensity factor is:

$$K_I = A\sigma\sqrt{\pi a} \quad (22)$$

where σ is the contact stress; a is the length of the semi-major axis of the crack; A is a geometric shape factor.

As shown in Figure 2, for a semi-elliptical surface crack embedded in a three-dimensional curved surface, the stress intensity factor varies along the crack front. The Newman–Raju formula provides expressions for K_I at the deepest point of the crack (point A) and at the surface (point C):

$$K_A = \sigma_1 \sqrt{\frac{\pi a}{Q_0}} \cdot F_\alpha \quad (23)$$

$$K_C = \sigma_1 \sqrt{\frac{\pi a}{Q_0}} \cdot F_\beta \quad (24)$$

where F_α and F_β are boundary correction factors, which are complex functions of the semi-major axis length a , semi-minor axis length b , and the elliptic integral function $\emptyset$, etc. They need to be obtained from literature or specialized manuals. Q_0 is the crack shape factor, which can be calculated from the crack depth t and surface crack width c :

$$Q_0 = 1 + 1.146 \left(\frac{t}{c} \right)^{1.65} \quad (25)$$

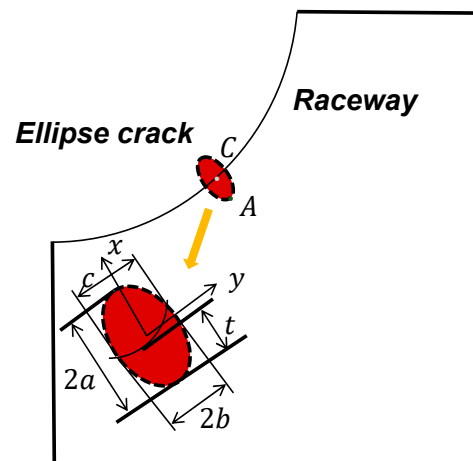


Figure 2. Raceway crack in a ball screws.

The Paris formula is used to describe the crack propagation rate in ball screws:

$$\frac{da}{dN_k} = C \cdot \text{Max}\{\Delta K_C, \Delta K_A\}^m \quad (26)$$

where N_k is the crack propagation life; C and m are fatigue crack propagation parameters, which depend on material properties such as temperature, hardness, and toughness. They can be obtained through experiments or by consulting relevant material handbooks. The values used in the present model are taken from Ref. [24]. These values were adopted as engineering reference parameters because independent standard crack growth tests were not included in the present ball screws life test program. This treatment may introduce uncertainty into the crack propagation life, and direct measurement of C and m for the tested material will be carried out in future work. ΔK_C , ΔK_A are the stress intensity factor ranges.

The number of cycles for an elliptical crack to propagate from its initial size to a critical area is the fatigue crack propagation life N_k :

$$N_k = \frac{1}{C_k} \int_{a_0}^{a_c} \frac{1}{\text{Max}\{\Delta K_C, \Delta K_A\}^m} da; (a_0 \leq a \leq a_c) \quad (27)$$

where K'_c is the ball recirculation method factor; C_k is the fatigue crack propagation factor; a_0 is the initial semi-major axis length of the crack, defined by material defects or the crack size at the end of the initiation stage, which can be determined by non-destructive testing or empirical values. Stable crack propagation occurs when $K_{th} < \text{Max}\{\Delta K_C, \Delta K_A\} < K_{IC}$. a_c is the critical semi-major axis length of the fatigue crack before the onset of cumulative fatigue spalling, determined by the material's fracture toughness K_{IC} . Unstable propagation occurs when the stress intensity factor range $\text{Max}\{\Delta K_C, \Delta K_A\} > K_{IC}$.

$$a_c = \frac{1}{\pi} \left(\frac{K_{IC}}{Y \cdot \Delta \sigma_1} \right)^2 \quad (28)$$

where Y is a geometric correction factor.

2.4. Fatigue Cumulative Spalling Life Prediction Model for Ball Screws

Yang Huiming et al. [25] established a fatigue cumulative spalling life model based on the modified critical plane method and the modified Paris model, using the volume difference of the spalled semi-elliptical “pit” as the characteristic quantity, as shown in Figure 3. To maintain analytical tractability in the calculation of crack propagation and spalling volume, an equivalent simplification is adopted based on existing models for rolling contact fatigue and bearing spalling. It is assumed that the ratio of the semi-major axis, semi-minor axis, and depth of the semi-elliptical “pit” is 2:1:1. Compared to the original Paris model and L-P model, this approach can more accurately predict bearing fatigue life. Using a similar method and concept, a fatigue cumulative spalling life prediction model for ball screws is established. It can be expressed as:

$$N_b = \frac{1}{C_b} \int_{V_c}^{V_s} \frac{1}{\text{Max}\{\Delta K_C, \Delta K_A\}^m} dV; (a \geq a_c) \quad (29)$$

where N_b is the fatigue cumulative spalling life; ΔK_s is the spalling factor for fatigue cracks; C_b is the crack fracture fatigue factor; V_c is the initial spalling volume; V_s is the final spalling volume. According to the standard “JB-T13813.2-2020 Rolling functional units—Reliability and life—Part 2: Functional reliability test specification for ball screws” [26], a ball screw is considered failed when the spalling depth on the raceway surface reaches ≥ 0.05 mm or the spalled area reaches ≥ 0.5 mm². Therefore, the volume of the final spalled semi-elliptical “pit” is:

$$V_s = \frac{\pi d^3}{3072} \quad (30)$$

where d is the maximum diameter of the rolling element.

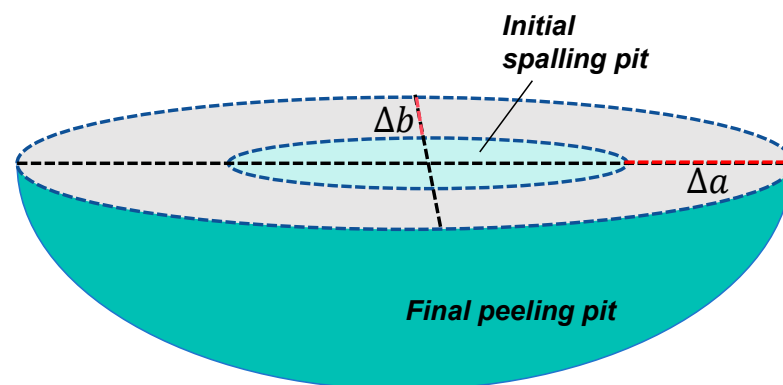


Figure 3. Schematic diagram of the cumulative fatigue spalling model for ball screws.

The fatigue spalling process is considered a continuation of fatigue crack propagation. Therefore:

$$\frac{1}{C_b} \int_0^{V_c} \frac{1}{\text{Max}\{\Delta K_C, \Delta K_A\}^m} dV = \frac{1}{C_k} \int_{a_0}^{a_c} \frac{1}{\text{Max}\{\Delta K_C, \Delta K_A\}^m} da \quad (31)$$

2.5. Full-Life Fatigue Life Prediction Model

In summary, life prediction models for three stages—crack initiation, crack propagation, and crack fatigue spalling—have been established. From Equations (21), (27) and (29), the full-life fatigue life prediction model for a ball screws under rated load can be obtained:

$$N_n = N_i + N_k + N_b \quad (32)$$

It should be noted that crack initiation, crack propagation, micropitting, crack coalescence, and material removal may overlap during rolling contact fatigue. Therefore, the total life expression should be regarded as a dominant-stage equivalent life approximation. Here, N_i , N_k , and N_b represent the equivalent numbers of cycles dominated by crack initiation, stable crack propagation, and cumulative spalling, respectively. This approximation is evaluated by comparing the predicted total life with experimental life data, while direct verification using interrupted tests and microscopic observation remains a topic for future work.

3. Reliability Assessment Method Based on the Full-Life Model

In engineering practice, the fatigue life of ball screws is significantly influenced by various uncertainties, including material properties, geometric parameters, and loading conditions [27]. Although traditional deterministic life models can provide average life predictions, they cannot quantify life dispersion or failure risk [28]. To address this, based on the full-life fatigue life model established in the previous section, this section introduces reliability theory to construct a limit state function for the fatigue life of ball screws. The limit state function derived from the previously established physical model for full-life fatigue life exhibits a high-order nonlinear form with multiple coupled variables and involves complex calculations such as crack propagation integrals. This makes direct reliability analysis using the Monte Carlo Simulation (MCS) method computationally expensive. To solve this problem, an enhanced adaptive Kriging–Monte Carlo Simulation (E-AK-MCS) method is introduced. This method also constructs an adaptive Kriging surrogate model to efficiently approximate the true limit state function, which is then combined with MCS for rapid reliability assessment, thus achieving a balance between accuracy and efficiency. Furthermore, sensitivity analysis is conducted to identify key parameters affecting life reliability.

3.1. Limit State Function

The core of reliability analysis is to establish a limit state function to determine whether a structure is in a safe state. Under axial loading, the cumulative spalling volume V_b increases with the number of screw revolutions N . Therefore, the limit state function can be defined as the difference between the critical spalling volume and the fatigue cumulative spalling volume at N revolutions:

$$G(X) = V_s - V_b(X) \quad (33)$$

where X is the vector of uncertain parameter variables.

When $G(X) \leq 0$, it indicates that the structure has failed due to fatigue before reaching the number of cycles N . The corresponding failure probability is:

$$P_f = P_f(G(X) \leq 0) = P_f(V_b(X) \geq V_s) \quad (34)$$

According to Equations (29) and (31), the fatigue cumulative spalling life L_b is linked to the crack propagation life L_k , with the spalling process considered a continuation of crack growth. Therefore, the cumulative spalling volume $V_b(X)$ is a function of the number of cycles N , and its development depends on the crack propagation history.

The spalling process is simplified as the expansion of a semi-elliptical “pit”. Its volume V is related to the crack size a (ellipse semi-major axis). Assuming the ratio of the pit’s semi-major axis, semi-minor axis, and depth is 2:1:1, the volume V can be expressed as:

$$V(a) = k_v a^3 \quad (35)$$

where k_v is a volume shape factor.

According to the Newman–Raju formula, generally $K_A > K_C$. Therefore, combining Equations (10), (16) and (19), performing the integration and rearranging yields the crack size $a(X)$ after N revolutions:

$$a(X) = \left[\left(1 - \frac{m}{2}\right) C_b \left(\sigma_1 \sqrt{\frac{\pi}{Q}}\right)^m (N - N_i - N_k) + a_0^{1-\frac{m}{2}} \right]^{\frac{2}{2-m}} \quad (36)$$

Subsequently, the fatigue cumulative spalling volume at N revolutions is obtained:

$$V_b(X) = k_v \cdot \left(\left[\left(1 - \frac{m}{2}\right) C_b \left(\sigma_1 \sqrt{\frac{\pi}{Q}}\right)^m (N - N_i - N_k) + a_0^{1-\frac{m}{2}} \right]^{\frac{2}{2-m}} \right)^3 \quad (37)$$

Substituting $V_b(X)$ into the limit state function yields an explicit expression:

$$G(X) = V_s - k_v \cdot \left(\left[\left(1 - \frac{m}{2}\right) C_b \left(\sigma_1 \sqrt{\frac{\pi}{Q}}\right)^m (N - N_i - N_k) + a_0^{1-\frac{m}{2}} \right]^{\frac{2}{2-m}} \right)^3 \quad (38)$$

3.2. E-AK-MCS Method

Given the complexity of the limit state function derived from the physical model, a Kriging surrogate model is employed to approximate it. The enhanced AK-MCS method is also a combination of an adaptive Kriging surrogate model and Monte Carlo simulation. It requires fewer model evaluations and less computation, can yield more accurate prediction results, and effectively enhances the efficiency of reliability analysis [29]. Its flowchart is shown in Figure 4:

The E-AK-MCS procedure includes: (1) generating S_{MC} by Monte Carlo or Latin Hypercube Sampling; (2) selecting an initial training set from S_{MC} using the Maxmin strategy; (3) calculating the true limit-state function values using the physical full-life model; (4) constructing and adaptively updating the Kriging surrogate model; and (5) estimating the failure probability and coefficient of variation using the converged surrogate model. The specific reliability analysis calculation process is as follows:

- (1) Generate MCS sample pool: Generate a large-scale sample pool S_{MC} in the random variable space using Latin Hypercube Sampling, with a size N_{MC} , typically $10^5 \sim 10^6$.

- (2) Construct initial training set: Select n_0 sample points from S_{MC} using the *Maxmin* strategy to form the initial training set S_T :

$$S_T^* = \operatorname{argmax} \operatorname{mind}(S_{MC}^*) \quad (39)$$

where $d(S_{MC}^*)$ is the Euclidean distance of samples in S_{MC} . Calculate their true limit state function values $G(X)$ using the physical model from Section 2.

- (3) Build/update Kriging model: Build a Kriging surrogate model $\hat{G}(X)$ based on the current S_T .
- (4) Assess convergence: Calculate the U learning function value and the expected improvement function $E(X)$ for all samples in S_{MC} . The U learning function and $E(X)$ function are defined respectively as:

$$U(X) = \left| \frac{\hat{G}(X)}{\sigma_{\hat{G}}(X)} \right| \quad (40)$$

$$E(X) = \begin{cases} (G_{min} - \hat{G}(X)) \cdot \Phi\left(\frac{G_{min} - \hat{G}(X)}{\sigma_{\hat{G}}(X)}\right) + \sigma_{\hat{G}}(X) \cdot \phi\left(\frac{G_{min} - \hat{G}(X)}{\sigma_{\hat{G}}(X)}\right), & \sigma_{\hat{G}}(X) > 0 \\ 0, & \sigma_{\hat{G}}(X) = 0 \end{cases} \quad (41)$$

where $\sigma_{\hat{G}}(X)$ is the prediction standard deviation. A smaller $U(X)$ value indicates that the point is closer to the estimated limit state surface and has higher uncertainty, making it the most valuable point for updating. G_{min} is the minimum absolute value of the limit state function in the current training set. Φ and ϕ are the cumulative distribution function and probability density function of the standard normal distribution, respectively.

If $\min_{X \in N_{MC}} U(X) \geq 2$ and the relative change R in failure probability for three consecutive iterations is less than 1%, proceed to step (6); otherwise, continue to step (5).

$$R = \frac{|\hat{p}_f^{(k)} - \hat{p}_f^{(k-1)}|}{\hat{p}_f^{(k-1)}} < 0.01 \quad (42)$$

- (5) Adaptive update: Update sample points $X_1^H = \arg \min_{X \in N_{MC}} U(X)$, $X_2^H = \arg \max_{X \in N_{MC}} E(X)$, and $X_3^H = \arg \max_{X \in N_{MC}} \sigma_{\hat{G}}(X)$. Add X_1^H , X_2^H , and X_3^H to S_T and return to step 3.
- (6) Calculate failure probability: Use the converged Kriging model to predict the state of all samples in S_{MC} . Count the number of failed samples N_f . The estimated failure probability and its coefficient of variation are respectively:

$$\hat{p}_f = \frac{N_f}{N_{MC}} \quad (43)$$

$$Cov(\hat{p}_f) = \sqrt{\frac{1 - \hat{p}_f}{(N_{MC} - 1)\hat{p}_f}} \quad (44)$$

Here, the number of failed samples N_f represents the count of samples for which $\hat{G}(X^{(i)})$ ($i = 1, 2, \dots, N_{MC}$) is less than or equal to 0. If $Cov(\hat{p}_f) < 5\%$ is considered satisfactory; otherwise, expand the sample pool N_{MC} and proceed to step (4).

- (7) Calculate the failure domain indicator function value for each sample $X^{(i)}$ in S_{MC} based on the current Kriging surrogate model $\hat{G}(X)$:

$$I_F(X^{(i)}) = \begin{cases} 0, \hat{G}(X^{(i)}) > 0 \\ 1, \hat{G}(X^{(i)}) < 0 \end{cases} (X^{(i)} \in S_{MC}) \quad (45)$$

- (8) When $Cov(\hat{P}_f) < 5\%$, terminate the E-AK-MCS solution process; otherwise, expand the sample pool S_{MC} and proceed to step (4).

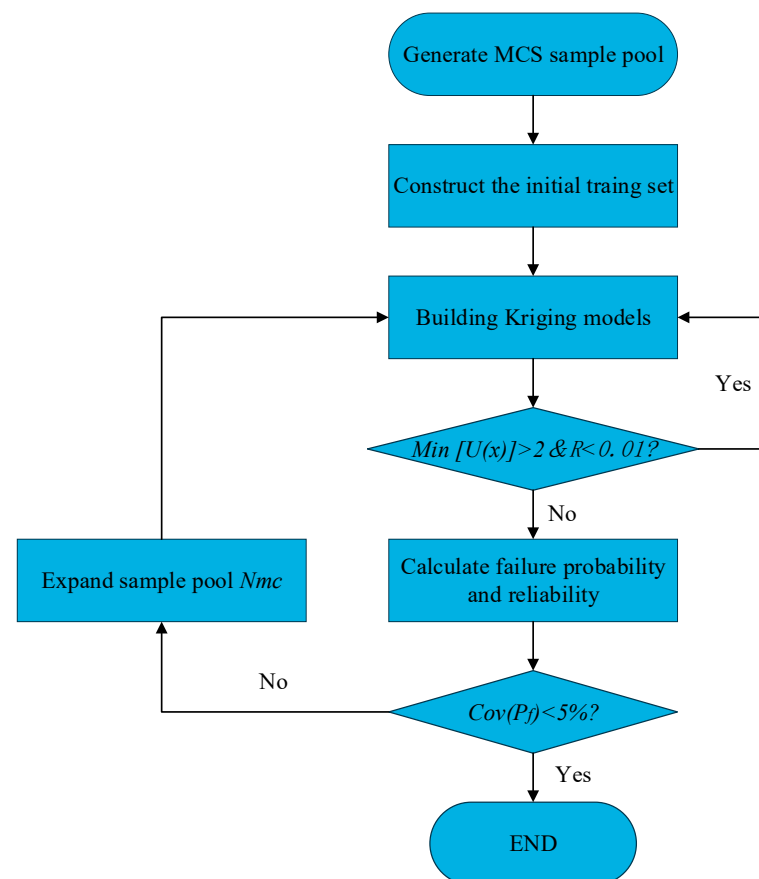


Figure 4. Steps of the enhanced AK-MCS method.

4. Experimental Verification and Analysis of Reliability Results

4.1. Verification of the Full-Life Fatigue Life Prediction Model for Ball Screws

This paper analyzes the full-life data measured under constant temperature and humidity conditions using a heavy-load ball screws life test rig, as shown in Figure 5. The test rig mainly consists of two loaded ball screws, two linear guide pairs, a worktable, a loading motor, and a drive motor.

To verify the accuracy of the full-life fatigue life prediction model for ball screws established in this paper, full-life fatigue life tests were conducted on a group of ball screws specimens under rated dynamic load using the heavy-load ball screws fatigue life test rig shown in Figure 5, under constant temperature (20 °C) and constant humidity (60%) conditions. According to JB/T 13813.2-2020, the specimen was deemed to have failed when the standard failure criterion related to raceway spalling depth or spalled area was met. In this study, failure identification was mainly based on optical microscope observation of pitting and spalling morphology on the raceway surface. Firstly, the accuracy of the model is verified using screw fatigue life data from the literature [30]:

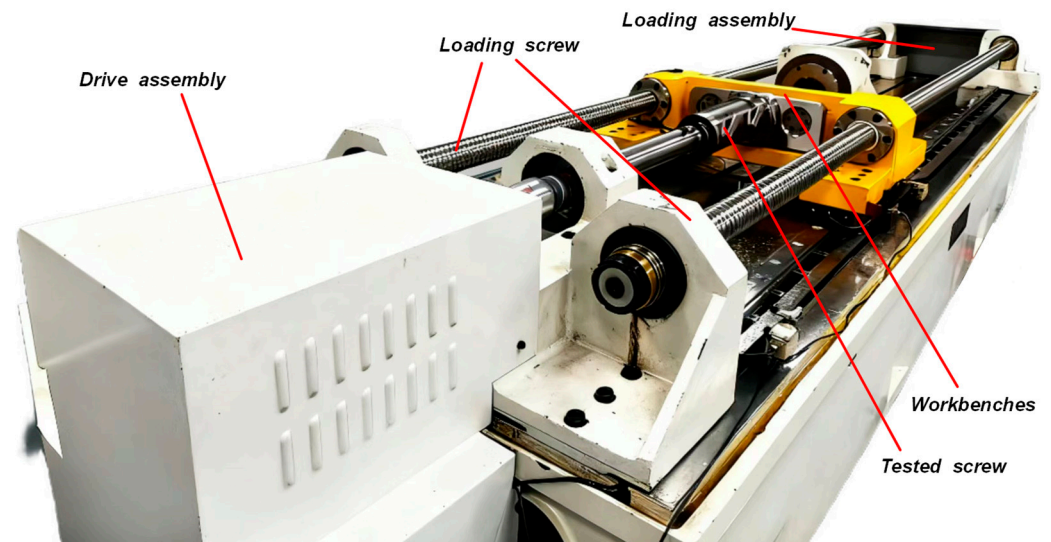


Figure 5. Heavy-load ball screws fatigue life test rig.

As shown in Figure 6, under rated dynamic load, the experimental life of the 8020 specification ball screws shows high consistency with the model predictions. The traditional L-P model, which uses a fixed life value of 10^6 revolutions as the evaluation benchmark, generally exhibits significant deviations in actual tests, indicating its failure to adequately reflect the fatigue damage mechanism of ball screws under real operating conditions. In contrast, the full-life prediction model proposed in this paper shows smaller deviations than the traditional L-P model for the tested samples. This suggests that the model can more accurately characterize the fatigue evolution process of ball screws under dynamic loading.

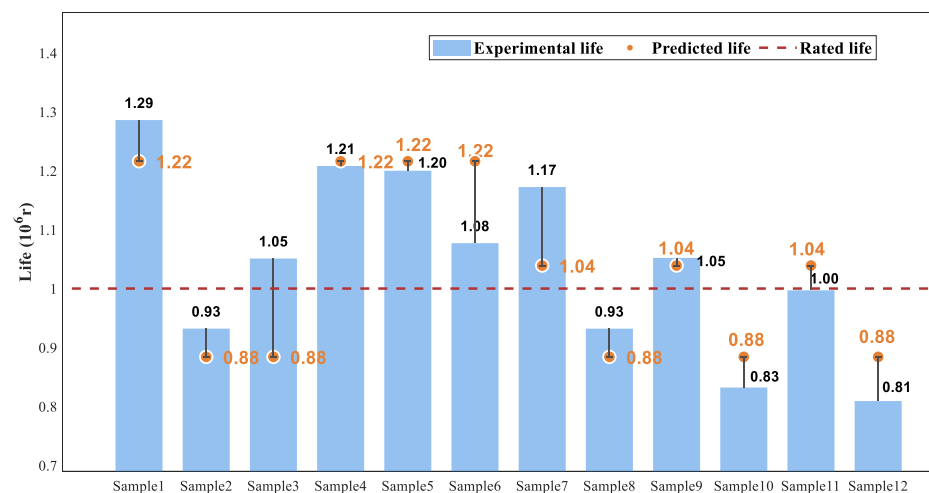


Figure 6. Predicted life diagram.

Further examining the prediction accuracy distribution shown in Figure 7, the model achieves an average prediction accuracy of 93.4% for the tested specimens, suggesting acceptable consistency within the current experimental dataset. The prediction error for the vast majority of samples is controlled within 10%, with only a few individual samples showing slightly larger deviations, which may stem from minor fluctuations in material parameters, assembly errors, or slight variations in the test environment. However, because of the limited number of specimens, the robustness of the model still requires further verification using larger sample populations.

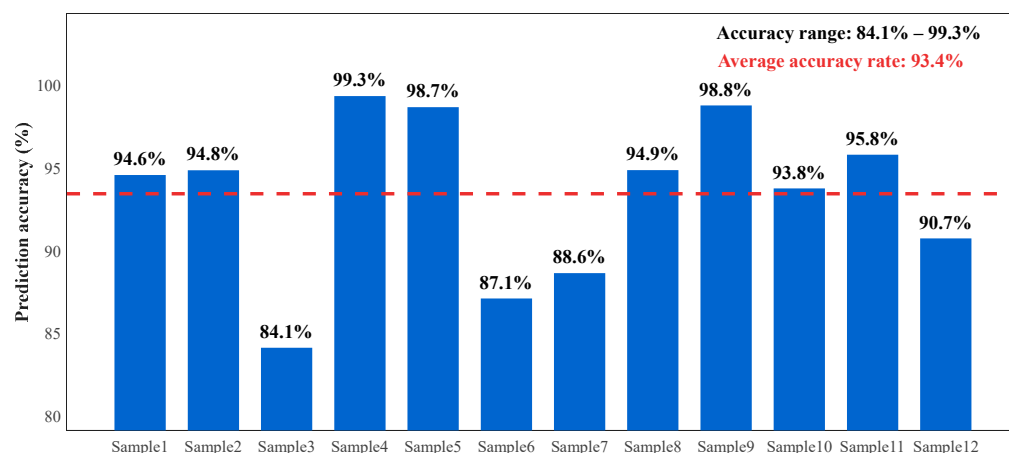


Figure 7. Prediction model accuracy diagram.

Using the ball screws fatigue life test rig shown in Figure 5, full-life fatigue life test data for the 5005 specification screw were obtained. Substituting the actual parameters of the test screws into the established full-life fatigue life model, the results show: for the 5005 specification screws, the model predicts a life of 975,000 revolutions for specimens 5005-1 and 5005-2, and 931,900 revolutions for specimens 5005-3 and 5005-4. The fatigue failure conditions of each specimen are shown in Figure 8. It is the fatigue damage morphology of the 5005 specification ball screw raceway observed under an optical microscope. The areas outlined by the red boxes indicate the primary spalling and pitting defects. The fatigue failure mode for specimens 5005-1 and 5005-2 was large-area fatigue spalling, while for specimens 5005-3 and 5005-4, it was deeper fatigue pitting. The actual experimental lives for specimens 5005-1 and 5005-2 were 912,500 and 1,012,500 revolutions, corresponding to prediction accuracies of 93.59% and 96.30%, respectively. The actual experimental lives for both specimens 5005-3 and 5005-4 were 912,500 revolutions, with a prediction accuracy of 97.92%. These results indicate that the model has preliminary applicability for the tested 5005 samples.

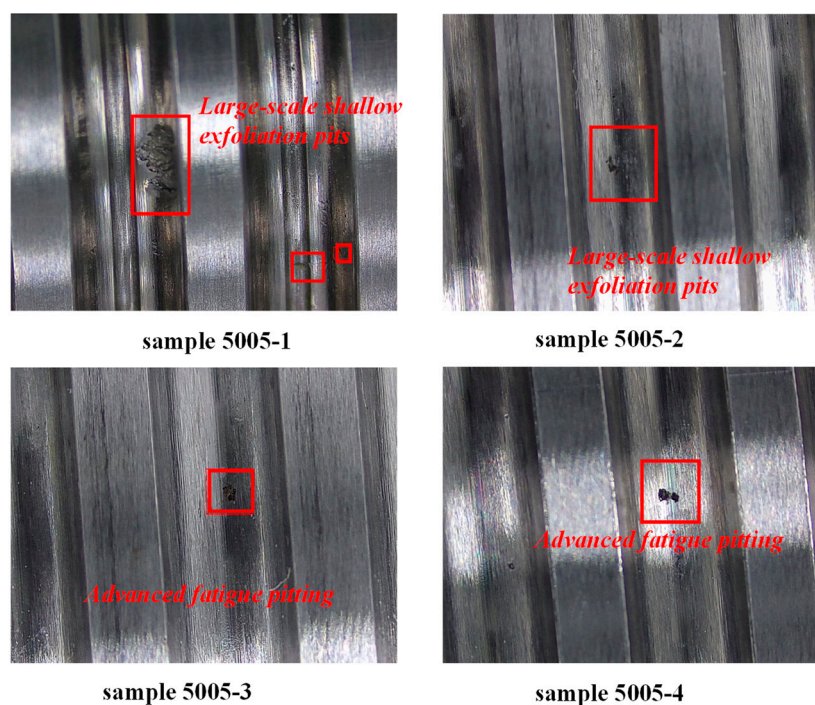


Figure 8. Fatigue spalling of the 5005 specification screw raceway.

However, as shown in Figure 9, during the test on the 4016 specification screw, the test was terminated prematurely due to ball fracture caused by eccentric loading and severe plastic deformation at the edge of the screw raceway after 450,000 revolutions. This failure mode represents an atypical overload damage, falling outside the predictive scope of the fatigue life model established in this paper. It also reflects that in actual operating conditions, sudden damage such as ball fracture can become a critical factor limiting screw life.



Figure 9. Plastic deformation of the 4016 specification screw raceway.

To further evaluate the prediction performance of the proposed full-life fatigue model, several statistical error indices were introduced, including root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), standard deviation of prediction errors, and mean prediction accuracy. The calculation results are listed in Table 3.

Table 3. Statistical error indices of the proposed full-life fatigue life prediction model.

Specimen Type	Number of Specimens	RMSE/ Revolutions	MAE/ Revolutions	MAPE/%	Standard Deviation of Errors/ Revolutions	Mean Prediction Accuracy/%
8020	12	84,193	67,601	6.57	86,998	93.43
5005	4	38,940	34,700	3.70	41,019	96.30
All specimens	16	75,468	59,376	5.85	77,757	94.15

For all tested specimens, the proposed model achieves an RMSE of 75,468 revolutions, an MAE of 59,376 revolutions, a MAPE of 5.85%, and a mean prediction accuracy of 94.15%. These indices provide a more complete quantitative evaluation of the model performance.

In summary, the full-life fatigue life prediction model constructed in this paper shows acceptable prediction accuracy for the tested samples with typical fatigue failure modes, providing a potential theoretical reference for ball screws life assessment. Future research should strengthen the monitoring and diagnosis of early defects in ball screws and further consider the impact of atypical failure mechanisms on full life to enhance the model's robustness under complex operating conditions.

4.2. Reliability Results and Parameter Sensitivity Analysis

Considering the uncertainties inherent in the material properties, manufacturing processes, and operating loads of ball screws, along with the explicit limit state function established above, seven key random variables influencing the full-life fatigue life are selected:

initial crack size a_0 , ball diameter D , helix angle γ , elastic modulus E , Poisson's ratio ν , fracture toughness K_{IC} , and contact angle α . These form the vector $X = [a_0, D, \gamma, E, \nu, K_{IC}, \alpha]$. Fatigue life reliability and sensitivity analyses are conducted. The statistical properties and variation ranges of these seven random variables are summarized in Table 4. The initial crack size a_0 is treated as an equivalent microscopic defect size, with its mean value and range (0.002–0.02 mm) consistent with typical non-metallic inclusion sizes in bearing steels. The ball diameter D and contact angle α are assumed to follow normal distributions with coefficients of variation derived from typical manufacturing tolerances. The elastic modulus $E = 208$ GPa and Poisson's ratio $\nu = 0.30$ are standard material properties for GCr15 bearing steel. The fracture toughness $K_{IC} = 19 \text{ MPa}\cdot\text{m}^{\frac{1}{2}}$ is taken from fracture mechanics studies on GCr15 steel. The helix angle $\gamma = 2.86^\circ$ is calculated from the lead and pitch diameter of the 8020 specification ball screws.

Table 4. Random variables and their statistical properties for reliability analysis.

Parameter	Symbol	Distribution	Mean	Range
Initial crack size	a_0	Lognormal	0.01 mm	0.002–0.02 mm
Ball diameter	D	Normal	9.525 mm	9.43–9.62 mm
Helix angle	γ	Normal	2.86°	2.72 – 3.00°
Elastic modulus	E	Normal	208 GPa	200–216 GPa
Poisson's ratio	ν	Normal	0.30	0.29–0.31
Fracture toughness	K_{IC}	Normal	$19 \text{ MPa}\cdot\text{m}^{\frac{1}{2}}$	17 – $21 \text{ MPa}\cdot\text{m}^{\frac{1}{2}}$
Contact angle	α	Normal	45°	43 – 47°

First, based on the probability density functions of the random parameters, a sample pool of size 5×10^6 is generated using Monte Carlo simulation. Subsequently, using Latin Hypercube Sampling, 30 sets of variable samples are randomly selected, their fatigue life and limit state function values are calculated, and a Kriging surrogate model is constructed and adaptively refined through iterative learning. Additionally, 50 sets of variables are randomly drawn as test samples and evaluated using both the theoretical full-life fatigue life formula for ball screws and the adaptive Kriging surrogate model, as shown in Figure 10. The figure shows that the relative error between the predictions of the adaptive Kriging surrogate model and the theoretical values is within 11%, demonstrating the high fitting accuracy of the established adaptive Kriging surrogate model.

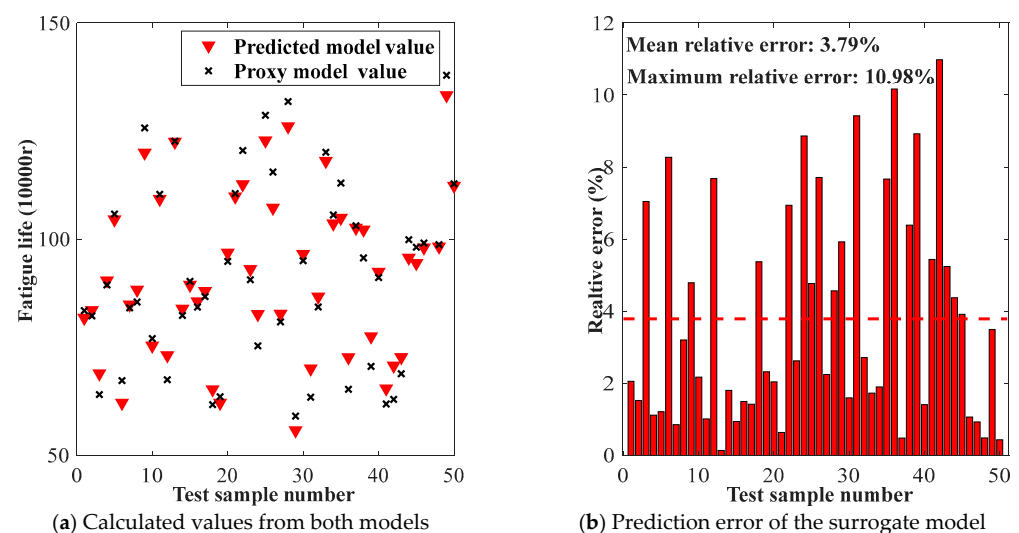


Figure 10. Comparison between the adaptive Kriging surrogate model and theoretical calculation results.

To investigate the influence of the axial force F_x magnitude on the fatigue life reliability of ball screws, the AK-MCS method was used to perform reliability analysis on an 8020 specification ball screw. The resulting life reliability curve is shown in Figure 11.

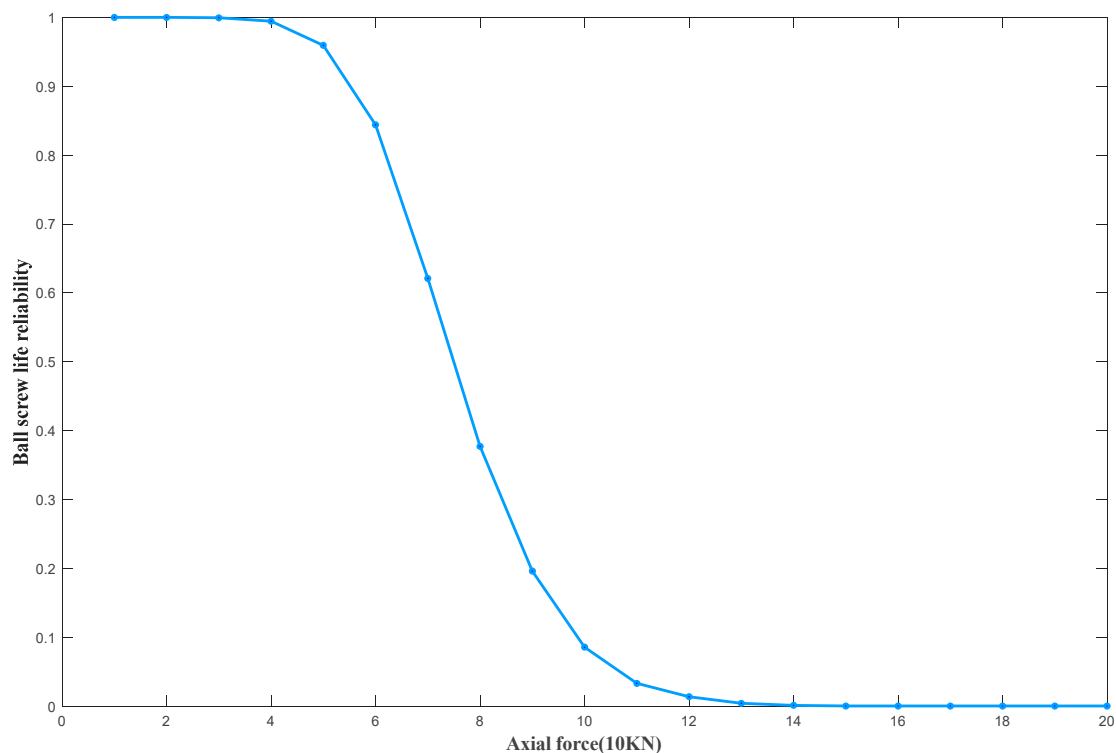


Figure 11. Ball screw life reliability curve varying with axial force.

Figure 11 shows that the axial force experienced by the ball screws reduces its life reliability. When the axial force $F_x = 48$ kN, the life reliability is 0.995, indicating the onset of failure for the ball screws. When F_x exceeds 60 kN, the life reliability decreases sharply with increasing F_x . When F_x increases to 140 kN, the life reliability drops to 0.0012, meaning the ball screws has almost completely failed.

To study the degree of influence of each parameter in the random variable X on the fatigue life failure probability of ball screws, a reliability sensitivity analysis was conducted using AK-MCS, as shown in Figure 12. The analysis reveals that ball diameter D , helix angle γ , fracture toughness K_{IC} , and initial contact angle α are negatively correlated with the fatigue life failure probability. That is, the life reliability of the ball screws increases with larger D , γ , K_{IC} , and α . Conversely, initial crack size a_0 , elastic modulus E , and Poisson's ratio ν are positively correlated with the fatigue life failure probability, meaning life reliability decreases with larger a_0 , E , and ν . The positive correlation between E and failure probability can be explained by the Hertzian contact response in the model. Under the same external load and geometry, a larger elastic modulus reduces elastic compliance and increases contact stiffness, which may increase the local contact stress and the SWT fatigue damage parameter. Consequently, within the selected parameter range, increasing E leads to a higher calculated fatigue failure probability. Among the random variables, ball diameter D has the most significant influence on fatigue life reliability, accounting for 27%. The second most influential factor is fracture toughness K_{IC} , accounting for 19%, followed by initial crack size a_0 , accounting for 16%. Therefore, controlling ball diameter, improving material fracture resistance, and reducing initial defects are important for enhancing fatigue reliability.

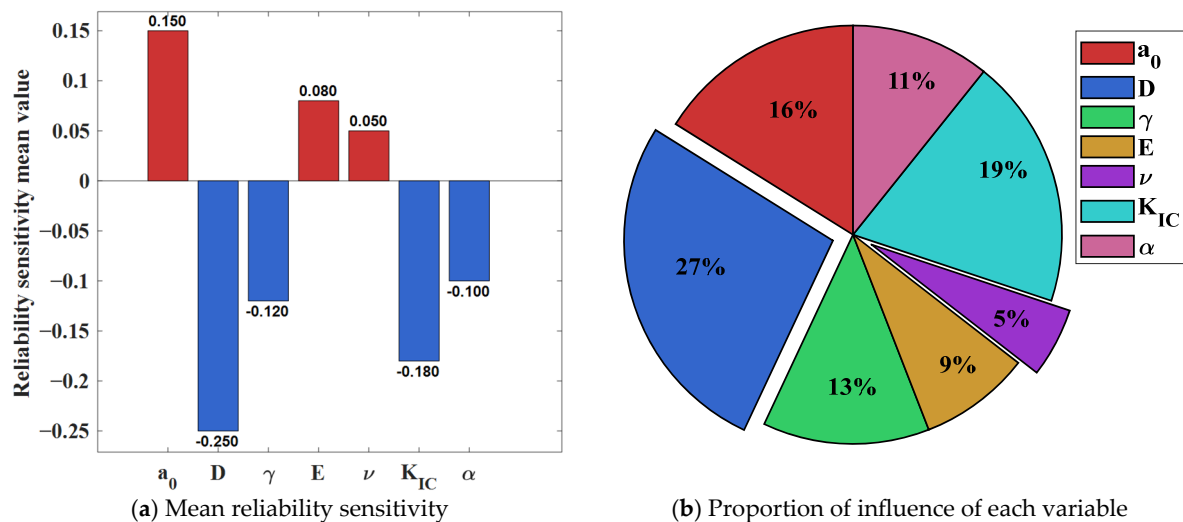


Figure 12. Sensitivity analysis of reliability variables for ball screws fatigue life.

5. Conclusions

This paper focuses on the full-life fatigue life prediction and reliability analysis of ball screws. By combining theoretical modeling, experimental verification, and numerical simulation, a comprehensive framework from the physical mechanism of fatigue damage to the quantitative assessment of service reliability is systematically constructed. The main research achievements can be summarized in the following three aspects:

- (1) A full-life life prediction model encompassing the three stages of fatigue crack initiation, propagation, and fatigue cumulative spalling was developed, integrating material properties, geometric parameters, and loading conditions. This model can more accurately reflect the fatigue evolution process of ball screws under actual dynamic loads, addressing the shortcomings of the traditional L-P model in predicting ball screw fatigue life.
- (2) Experimental validation was conducted on 8020 specification screws using a heavy-load ball screws fatigue life test rig. For the tested specimens, the model-predicted life shows reasonable agreement with the experimental life, the model achieves an average prediction accuracy of 93.4% for the 8020 specimens. The model showed acceptable performance for the current experimental dataset, indicating its potential value for ball screws life assessment. Further validation with larger sample populations is still needed.
- (3) The adaptive Kriging–Monte Carlo method was introduced for reliability assessment, transforming the deterministic life prediction model into an efficient reliability evaluation model, significantly improving computational efficiency and accuracy. Reliability sensitivity analysis indicates that parameters such as ball diameter, initial crack size, and fracture toughness significantly influence life reliability. Notably, ball diameter has the largest impact (27%), followed by fracture toughness K_{IC} (19%) and initial crack size a_0 (16%). These findings provide clear directions for optimizing manufacturing processes.

The proposed full-life prediction model, combined with the E-AK-MCS reliability analysis method, can support life prediction, failure risk quantification, and identification of key influencing factors within the assumptions and tested parameter range of this study. This provides a theoretical reference and methodological basis for further reliability design and life assessment of ball screws.

It is important to emphasize that the proposed model is specifically validated for boundary/mixed lubrication conditions under high contact pressures. The effects of EHL film formation, lubricant viscosity variation, and long-term lubricant degradation are not included in the current framework. Therefore, direct application of the model to full-film lubrication or high-speed operating conditions should be undertaken with caution, and further model extension is required for such scenarios.

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Conflicts of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

E-AK-MCS	Enhanced institute adaptive Kriging–Monte Carlo simulation
AK-MCS	Adaptive Kriging–Monte Carlo simulation
SWT	Smith–Watson–Topper

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