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Elastoplastic Multi-Physics Modeling of Sliding Electrical Contact in Slip Rings

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Abstract

Electrical slip rings are key components for power and signal transmission in rotating equipment, and degradation of sliding electrical contact is a major factor limiting their reliability. To analyze the sliding electrical contact behavior of slip rings, an elastoplastic contact framework incorporating thermal–mechanical–electrical coupling is developed. The semi-analytical method combined with discrete convolution-fast Fourier transform is employed to efficiently solve the coupled contact problem, while J_2 flow theory and radial return algorithm are adopted to determine plastic deformation. Based on the proposed model, the elastoplastic sliding electrical contact behaviors of smooth and sinusoidal surfaces are systematically investigated. The results show that plastic deformation increases the contact area, thereby reducing the electrical contact resistance, current density at the contact edge, and maximum temperature rise, although it may induce the residual stress. Reducing the asperity height of sinusoidal surfaces while maintaining multiple discrete micro contact spots can effectively lower the electrical contact resistance and interfacial temperature rise. The proposed model provides a useful theoretical tool for evaluating the thermal–mechanical–electrical performance of sliding electrical contact in slip rings.

Keywords: slip ring; electrical contact; multi-physics coupling; elastoplastic contact; electrical contact resistance; residual stress

1. Introduction

Electrical slip rings are critical components for the continuous transmission of power and signal in rotating equipment and are widely used in satellites, offshore installations, and wind turbines [1–3]. Failure of sliding electrical contact is one of the main factors limiting the operational reliability of slip rings. Electrical contact resistance (ECR), temperature rise, and stress are key indicators of sliding electrical contact performance and are strongly influenced by thermal–mechanical–electrical coupling [4,5]. Consequently, accurate prediction of these parameters under coupled thermal–mechanical–electrical conditions is important for evaluating the contact performance of slip rings and understanding the underlying contact behavior.

Among these parameters, ECR has received considerable attention because it directly reflects the current-carrying capability of the contact interface. Greenwood [6] derived an expression for the ECR of multiple contact spots that accounts for both the parallel contribution of all spots and the interaction resistance between them. However, the contact spots were assumed to be circular, and the contact problem for rough surfaces was not fully addressed. To estimate the ECR of rough surfaces, many studies adopted the parallel method, in which the total resistance is calculated by treating the resistances of all spots



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as parallel resistors [7–10]. Because the parallel method neglects the interactions between contact spots, it may lead to significant errors when the spots are numerous and closely spaced. In our previous work, a semi-analytical model was developed to calculate the ECR of rough surfaces, accounting for the irregular shapes of contact spots and the interactions between them [11]. Recently, Xu et al. [12] incorporated the effect of dielectric breakdown across narrow gaps into ECR prediction, enabling a more accurate evaluation of electrical contact performance.

The aforementioned studies primarily focused on the relationship between ECR and contact state while neglecting the effect of Joule-heating-induced temperature rise on both ECR and contact state. To address this limitation, several studies investigated electrical contact under thermal–mechanical–electrical coupling. Hennessy et al. [13] employed the finite element method (FEM) to solve the electrical contact problem between a hemisphere and a half-space. Michopoulos et al. [14] proposed a multiscale model for rough surface electrical contact, in which the macroscale was solved using FEM, whereas the mesoscale and microscale were treated analytically. Li et al. analyzed the thermal–mechanical–electrical contact behavior of a single asperity [15], and later extended the study to rough surfaces using the parallel method [16]. Recently, Li et al. [17] adopted the boundary element method (BEM) to model the rough surface electrical contact while accounting for spot interactions, achieving higher accuracy than the parallel method.

The above studies focused on static electrical contact problems, whereas in sliding electrical contact, the effects of frictional heating cannot be ignored. Komvopoulos [18] developed an analytical model for sliding electrical contact of fractal surfaces, but its applicability is limited to low-current conditions because Joule heating was neglected. The semi-analytical method (SAM) and BEM are both efficient for solving sliding electrical contact problems involving both frictional and Joule heating. In our previous work [19], SAM was employed to analyze sliding electrical contact behavior, enabling efficient prediction of ECR, temperature rise, and stress. Similarly, Luo et al. [20] applied SAM to systematically investigate the interplay between frictional heating and Joule heating. Dai et al. utilized BEM to solve the two-dimensional sliding electrical contact problem [21], and subsequently extended the model to three dimensions with a profile-dependent friction coefficient [22]. However, these sliding electrical contact models were generally developed within an elastic framework and therefore cannot capture plastic deformation, residual displacement, and residual stress.

Elastoplastic deformation is another critical factor in sliding electrical contact because it governs the real contact area and the distribution of contact spots, which directly influence ECR and contact performance. Existing elastoplastic electrical contact models are mainly confined to static scenarios [16] or neglect Joule heating [18]. Elastoplastic sliding electrical contact is difficult to solve because plastic deformation is path-dependent and requires incremental loading. At each loading increment, the coupled thermal–mechanical–electrical contact problem must be solved while the plastic strain, residual displacement, and residual stress are updated. Therefore, an efficient framework that simultaneously accounts for elastoplastic deformation, frictional heating, Joule heating, and current constriction is still lacking. The SAM has proven efficient for solving coupled multi-physics problems [23–26] and elastoplastic contact problems [27–29]. In this paper, a semi-analytical elastoplastic multi-physical model is developed for sliding electrical contact. The thermal–mechanical–electrical coupling problem is solved using SAM combined with DC-FFT, while J_2 flow theory and the radial return algorithm are adopted to determine plastic strain increments. The proposed framework is then used to investigate the effects of plastic deformation on ECR, temperature rise, von Mises stress, and residual stress for both smooth and sinusoidal surfaces.

2. Theory of Sliding Electrical Contact

2.1. Sliding Electrical Contact Model

Figure 1a shows a schematic of a disk-type electrical slip ring. Its reliability depends on the sliding electrical contact performance of brush-slip ring friction pair, and the corresponding model is illustrated in Figure 1b. The brush, subjected to normal force P_z , slides along the x -axis over the slip ring surface at constant velocity V , while the current I flows through the contact interface. The brush has a spherical tip of radius R , and the slip ring is modeled as a half-space; both bodies are elastoplastic. Since the current can only flow through the contact region, current constriction occurs at the contact interface (Figure 1c), giving rise to the ECR R_c . Under the applied mechanical and electrical loads, the contact pressure $p_z(x, y)$, friction $p_x(x, y)$, and current density $J(x, y)$ are distributed over the contact region Γ . The friction p_x satisfies Coulomb's law, i.e., $p_x = f \cdot p_z$, where f is the friction coefficient. In this study, f is assumed to be constant as an idealized condition. In practical sliding electrical contacts, the friction coefficient may depend on contact pressure, temperature, current, surface films, oxidation, transfer layers, and wear state. These effects are not considered in the present model.

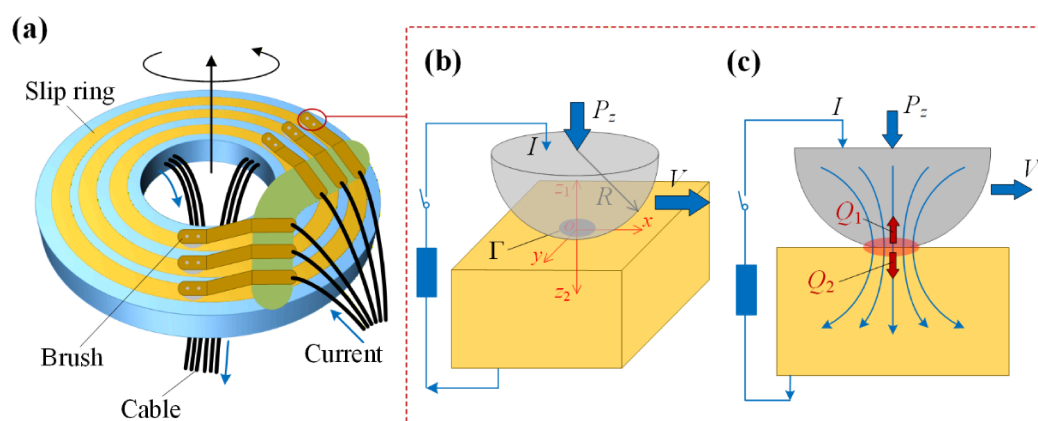


Figure 1. Schematic of (a) disk-type electrical slip ring and (b,c) sliding electrical contact model for the brush-slip ring friction pair.

Sliding friction and current generate frictional heating and Joule heating. Since Joule heating generated by ECR contributes much more to the temperature rise than that by bulk resistance and is concentrated near the contact interface [11], it is assumed to originate entirely from the ECR and is modeled as a surface heat source. The heat flux density caused by frictional heating and Joule heating at the contact interface, $Q(x, y)$, is partitioned into two parts: $Q_1(x, y)$ entering the brush and $Q_2(x, y)$ entering the slip ring with $Q_1 + Q_2 = Q$. The resulting thermal expansion alters the contact configuration and current conduction. Consequently, analyzing the present sliding electrical contact problem requires the simultaneous consideration of thermal, mechanical, and electrical fields.

2.2. Governing Equations and Boundary Conditions

2.2.1. Thermal Field

The thermal field in sliding electrical contact is governed by the steady-state heat conduction equation. In the absence of volumetric heat generation within the contacting bodies, the governing equation can be written as [19,30]:

$$\nabla^2 T + \frac{V}{\gamma} \cdot \frac{\partial T}{\partial x} = 0 \quad (1)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, T and γ denote the temperature rise and thermal diffusivity, respectively. The interfacial heat flux is generated by frictional heating and Joule heating, whereas the temperature rise tends to zero far from the contact region. Accordingly, the thermal boundary conditions can be written as:

$$\begin{cases} -\kappa \frac{\partial T}{\partial z} \big|_{z=0} = Q_a(x, y) \\ T \big|_{z=\infty} = 0 \end{cases} \quad (2)$$

where κ denotes the thermal conductivity, Q_a ($a = 1, 2$) denotes the interfacial heat flux entering the brush (Q_1) and the slip ring (Q_2), respectively.

2.2.2. Mechanical Field

The elastic deformation and stress in sliding electrical contact are governed by the thermoelastic equations, and the plastic response is derived from the resulting elastic stress field as described in Section 3.3. In the absence of body forces, the governing equations for the mechanical field can be written as [22,26]:

$$\begin{cases} \nabla^2 u_x + \frac{1}{1-2\nu} \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_z}{\partial x \partial z} \right) = \frac{2(1+\nu)\alpha}{1-2\nu} \frac{\partial T}{\partial x} \\ \nabla^2 u_y + \frac{1}{1-2\nu} \left(\frac{\partial^2 u_x}{\partial x \partial y} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_z}{\partial y \partial z} \right) = \frac{2(1+\nu)\alpha}{1-2\nu} \frac{\partial T}{\partial y} \\ \nabla^2 u_z + \frac{1}{1-2\nu} \left(\frac{\partial^2 u_x}{\partial x \partial z} + \frac{\partial^2 u_y}{\partial y \partial z} + \frac{\partial^2 u_z}{\partial z^2} \right) = \frac{2(1+\nu)\alpha}{1-2\nu} \frac{\partial T}{\partial z} \end{cases} \quad (3)$$

where u_i ($i = x, y, z$) denotes the thermoelastic displacement, ν and α are Poisson's ratio and thermal expansion coefficient. The contact pressure $p_z(x, y)$ and friction $p_x(x, y)$ are applied at the contact interface, while the displacement vanishes at infinity. Accordingly, the mechanical boundary conditions can be written as:

$$\begin{cases} \sigma_{xz}(x, y, 0) = -f \cdot p_z(x, y) \\ \sigma_{yz}(x, y, 0) = 0 \\ \sigma_{zz}(x, y, 0) = -p_z(x, y) \\ u_x \big|_{z=\infty} = u_y \big|_{z=\infty} = u_z \big|_{z=\infty} = 0 \end{cases} \quad (4)$$

where σ_{ij} ($i, j = x, y, z$) denotes the thermoelastic stress.

2.2.3. Electrical Field

The electrical potential distribution in sliding electrical contact is governed by the potential equilibrium equation. In the absence of volumetric charge density, the governing equation can be written as [19,31]:

$$\nabla^2 \varphi = 0 \quad (5)$$

where φ denotes the electrical potential. In sliding electrical contact, the current distribution at the contact interface is nonuniform and is characterized by the current density $J(x, y)$. Since the electrical circuit forms a closed path, the current density becomes uniform far from the contact interface. Thus, the electrical boundary conditions can be written as:

$$\begin{cases} \frac{\partial \varphi}{\partial z} \big|_{z=0} = \rho J(x, y) \\ \frac{\partial \varphi}{\partial z} \big|_{z=\infty} = \rho J_u \end{cases} \quad (6)$$

where ρ denotes the electrical resistivity and J_u is the uniform current density at infinity. Because the characteristic dimensions of the contacting bodies are much larger than those of the contact region, the far-field uniform current density J_u is negligible compared with

the unknown interfacial current density $J(x, y)$. Hence, J_u can be taken as zero, and the electrical boundary conditions reduce to:

$$\begin{cases} \frac{\partial \varphi}{\partial z}|_{z=0} = \rho J(x, y) \\ \frac{\partial \varphi}{\partial z}|_{z=\infty} = 0 \end{cases} \quad (7)$$

3. Semi-Analytical Method

3.1. Frequency Response Functions and Influence Coefficients

The semi-analytical method (SAM) is employed to solve the multi-physics problem described in Section 2. The basic idea of SAM is to analytically derive the unit excitation responses of each field, i.e., the influence coefficients (ICs), and then numerically compute the coupled responses under arbitrary loads based on the superposition principle [32]. To obtain the ICs for the coupled thermal–mechanical–electrical fields, their frequency domain counterparts, i.e., the frequency response functions (FRFs), are derived from the governing Equations (1), (3) and (5), as detailed in Appendix A. The boundary conditions (2), (4) and (7) are then applied to determine the ten unknown coefficients in the FRFs. Because the variables in thermal–mechanical–electrical fields vanish at infinity, the coefficients A_2 , B_3 , B_4 , M_2 , and N_2 in the FRFs are zero. The remaining coefficients can be determined from the following matrix equations (parameters in matrix are given in Appendix A):

$$\begin{bmatrix} iG\omega_y/\omega & 2iG\omega_x & -2ig_3\omega_x/\omega & 2iGKr\omega_x & 0 \\ -iG\omega_x/\omega & 2iG\omega_y & -2ig_3\omega_y/\omega & 2iGKr\omega_y & 0 \\ 0 & -2G\omega & -2[\mu/d - G] & g_7 & 0 \\ 0 & 0 & 0 & 0 & \kappa r \\ 0 & 0 & 0 & 0 & -\omega/\rho \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ B_1 \\ B_2 \\ M_1 \\ N_1 \end{bmatrix} = \begin{bmatrix} -f \cdot \tilde{p}_z \\ 0 \\ -\tilde{p}_z \\ \tilde{Q}_a \\ -\tilde{f} \end{bmatrix} \quad (8)$$

The unknown coefficients in the FRFs for unit mechanical, thermal, and electrical loading are obtained by setting $(\tilde{P}_z, \tilde{Q}_a, \tilde{f}) = (1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, respectively. Once the FRFs are obtained, the ICs in continuous Fourier transform form $\tilde{C}$ are expressed by multiplying the FRFs $\tilde{F}$ by the shape function $\tilde{Y}$ [33]:

$$\begin{cases} \tilde{C}(\omega_x, \omega_y, z) = \tilde{F}(\omega_x, \omega_y, z) \cdot \tilde{Y}(\omega_x, \omega_y) \\ \tilde{Y}(\omega_x, \omega_y) = \frac{4 \sin(\omega_x \Delta_x / 2) \sin(\omega_y \Delta_y / 2)}{\omega_x \omega_y} \end{cases} \quad (9)$$

where Δ_x and Δ_y are the grid spacings in the x and y directions, respectively. For a square computational domain with side length L , $\Delta_x = L/M$ and $\Delta_y = L/N$, where M and N are the numbers of grids in the x and y directions. For numerical implementation, the ICs in discrete Fourier transform form are then derived with aliasing processing to reduce discretization errors [33]:

$$\hat{C} = \frac{1}{\Delta_x \Delta_y} \sum_{r_x=-AL}^{r_x=AL} \sum_{r_y=-AL}^{r_y=AL} \tilde{C}\left(\frac{2\pi}{M_e \Delta_x} m - \frac{2\pi}{\Delta_x} r_x, \frac{2\pi}{N_e \Delta_y} n - \frac{2\pi}{\Delta_y} r_y\right) \quad (-M_e/2 < m \leq M_e/2, -N_e/2 < n \leq N_e/2) \quad (10)$$

where “ $\hat{\cdot}$ ” denotes the discrete Fourier transform; AL is aliasing control level; $M_e = 2^X M$ and $N_e = 2^X N$ are the numbers of refined grids based on the numbers of original grids; m ,

n and χ are integers. The coupled field variables in sliding electrical contact can then be determined using the discrete convolution-fast Fourier transform (DC-FFT) [34]:

$$\begin{cases} \sigma_{ij} = IFFT(\hat{C}_{\sigma_{ij}}^{p_z} \hat{p}_z + \hat{C}_{\sigma_{ij}}^Q \hat{Q}_a) \\ u_i = IFFT(\hat{C}_{u_i}^{p_z} \hat{p}_z + \hat{C}_{u_i}^Q \hat{Q}_a) \\ T = IFFT(\hat{C}_T^Q \hat{Q}_a) \\ \varphi = IFFT(\hat{C}_\varphi^J \hat{J}) \end{cases} \quad (11)$$

where $\hat{C}_{X_1}^{X_2}$ denotes the IC of variable X_1 induced by load X_2 , and $\hat{p}_z, \hat{Q}_a, \hat{J}$ are the discrete Fourier transform of the input loads. $IFFT$ denotes the inverse fast Fourier transform. The detailed procedure for solving Equation (11) is summarized in Table 1.

Table 1. The procedure for solving $\sigma_{ij}, u_i, T, \varphi$ using DC-FFT.

Step	Operation
1	Input the multi-physics loads p_z, Q_a, J ; the ICs $\hat{C}_{X_1}^{X_2}$; and the numbers of grids M and N .
2	Double the domain of the ICs to obtain $\{\hat{C}\}_{2M \times 2N}$ over the region $[-M: M-1, -N: N-1]$.
3	Mirror $\{\hat{C}\}_{2M \times 2N}$ to obtain new matrix $\{\hat{C}\}_{2M \times 2N}$ with a range of $[1:2M, 1:2N]$.
4	Discretize the multi-physics loads numerically to obtain $\{p_z\}_{M \times N}, \{Q_a\}_{M \times N}, \{J\}_{M \times N}$.
5	Double the domain of the multi-physics loads and perform zero-padding in the x and y directions to obtain new matrix $\{\underline{p}_z\}_{2M \times 2N}, \{\underline{Q}_a\}_{2M \times 2N}, \{\underline{J}\}_{2M \times 2N}$.
6	Perform double discrete Fourier transform on the loads in the x and y directions to obtain $\{\hat{p}_z\}_{2M \times 2N}, \{\hat{Q}_a\}_{2M \times 2N}, \{\hat{J}\}_{2M \times 2N}$.
7	Multiply the corresponding element of the ICs matrix and loads matrix to obtain the temporary frequency-domain sequence, such as $\{\hat{C}_{\sigma_{ij}}^{p_z} \circ \hat{p}_z\}_{2M \times 2N}$.
8	Perform $IFFT$ on the frequency-domain sequence in the x and y directions, and retain the original computational domain to obtain $\sigma_{ij}, u_i, T, \varphi$ at any depth z .

3.2. Solution of Elastic Contact

To determine the coupled thermal–mechanical–electrical fields from Equation (11), the distributions of p_z, J , and Q_a must first be obtained from the contact problem. This contact problem can be transformed into an equivalent problem satisfying the Kuhn-Tucker complementary conditions [35]:

$$\begin{cases} p_z(x, y) > 0 \ \& \ w(x, y) = 0, \ (x, y) \in \Gamma \\ p_z(x, y) = 0 \ \& \ w(x, y) > 0, \ (x, y) \notin \Gamma \end{cases} \quad (12)$$

where w denotes the gap between the two contacting surfaces. In the absence of plastic deformation, it is given by:

$$w(x, y) = c_g(x, y) - d_0 + u_z(x, y, 0) \quad (13)$$

where c_g is the initial gap, including the initial separation, geometric profile and surface roughness; d_0 is the rigid body approach of the two contacting bodies; thus, $c_g(x, y) - d_0$ represents the geometric interference. u_z is the sum of the surface thermoelastic deformations of the two contacting bodies and is obtained from Equation (11).

Discretizing Equations (12) and (13) yields the following discrete equation for the surface gap:

$$\begin{cases} w(m, n) = c_g(m, n) - d_0 + IFFT[(\hat{C}_{u_{z1}}^{p_z} + \hat{C}_{u_{z2}}^{p_z})\hat{p}_z + \hat{C}_{u_{z1}}^Q \hat{Q}_1 + \hat{C}_{u_{z2}}^Q \hat{Q}_2] \Big|_{z=0} \\ p_z(m, n) \geq 0, w(m, n) \geq 0, \quad p_z(m, n) \cdot w(m, n) = 0 \end{cases} \quad (14)$$

where $X(m, n)$ denotes the nodal values at node (m, n) , with $m = 1, \dots, M$ and $n = 1, \dots, N$. Solving Equation (14) is equivalent to minimizing the total complementary potential energy, which can be formulated as a quadratic programming problem.

In Equation (14), the heat flux density Q_a ($a = 1, 2$) can be expressed as:

$$Q_a(m, n) = \eta_a(m, n) \cdot [fvp_z(m, n) + IR_c J(m, n)] \quad (15)$$

where η_1 and η_2 denote the fractions of heat flux entering the brush and slip ring. R_c can be determined from Ohm's law, in which the current is I and the voltage drop is determined from the electrical potential distribution calculated using Equation (11). In sliding electrical contact problem, the current density is nonuniform within the contact region, whereas outside the contact region, $J = 0$. Because the current passes vertically through the contact region, φ is uniform within the contact region. Therefore, J and φ satisfy the following relationship:

$$\begin{cases} J(m, n) > 0 \ \& \ \bar{\varphi} - \varphi(m, n, 0) = 0, \ (m, n) \in \Gamma \\ J(m, n) = 0 \ \& \ \bar{\varphi} - \varphi(m, n, 0) \neq 0, \ (m, n) \notin \Gamma \end{cases} \quad (16)$$

where $\bar{\varphi}$ indicates the average electrical potential within the contact region. Solving Equation (16) is likewise equivalent to minimizing the total complementary potential energy, which can be formulated as a quadratic programming problem.

Q_1 and Q_2 are determined such that the temperature rise they induced on the contact surface, T_1 and T_2 , are equal within the contact region. On this basis, η satisfies:

$$\begin{cases} Q_1(m, n) = \eta_1(m, n) \cdot [fvp_z(m, n) + IR_c J(m, n)] \\ Q_2(m, n) = [1 - \eta_1(m, n)] \cdot [fvp_z(m, n) + IR_c J(m, n)] \\ IFFT(\hat{C}_{T_1}^Q \hat{Q}_1) \Big|_{z=0} = IFFT(\hat{C}_{T_2}^Q \hat{Q}_2) \Big|_{z=0}, \quad (m, n) \in \Gamma \end{cases} \quad (17)$$

where $\eta_1(m, n)$ can be determined by relaxation iteration. The algorithm for determining $p_z(m, n)$, $J(m, n)$, and $Q_a(m, n)$ is summarized in Appendix B.

3.3. Calculation of Plastic Deformation

During elastoplastic contact, plastic strain generates residual displacement on the contact surface, which persists after unloading. Accordingly, Equation (14) is modified for the elastoplastic contact problem as follows:

$$\begin{cases} w(m, n) = c_g(m, n) - d_0 + u_z^p(m, n, 0) + IFFT[(\hat{C}_{u_{z1}}^{p_z} + \hat{C}_{u_{z2}}^{p_z})\hat{p}_z + \hat{C}_{u_{z1}}^Q \hat{Q}_1 + \hat{C}_{u_{z2}}^Q \hat{Q}_2] \Big|_{z=0} \\ p_z(m, n) \geq 0, w(m, n) \geq 0, \quad p_z(m, n) \cdot w(m, n) = 0 \end{cases} \quad (18)$$

where u_i^p ($i = x, y, z$) denotes the plastic deformation, also referred to as residual displacement. For two elastoplastic bodies, the plastic deformation in Equation (18) represents the sum of residual displacements for two contacting bodies. The plastic strain, residual displacement, and residual stress of each body are calculated separately using the same solution procedure and then superposed in the contact formulation. If one body is rigid, its plastic displacement is zero, and the formulation reduces to the rigid-elastoplastic contact case. Following Liu's method [29,36], plastic deformation can be expressed as:

$$u_i^p(m, n, h) = -\frac{1}{2\pi} \sum_{m'=1}^M \sum_{n'=1}^N \sum_{h'=0}^H \left[U^*((m-m')\Delta x, (n-n')\Delta y, (h-h')\Delta z) \cdot e_{ij}^p(m', n', h') \right] \quad (19)$$

where $e_{ij}^p = \varepsilon_{ij}^p$ ($i = j$), $e_{ij}^p = 2\varepsilon_{ij}^p$ ($i \neq j$), and ε_{ij}^p denotes the plastic strain; $h = 0, \dots, H$ and H is the numbers of grids in the z direction. U^* is the ICs for plastic deformation induced by unit plastic strain, as given in Ref. [36].

The determination of u_i^p requires the plastic strain ε_{ij}^p . In the present study, the elastoplastic model is based on the assumptions of small deformation and plastic incompressibility. The total strain ε'_{ij} can be expressed as:

$$\varepsilon'_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^t + \varepsilon_{ij}^p \quad (20)$$

where ε_{ij}^e and ε_{ij}^t denotes the elastic strain and thermal strain, respectively. Based on Hooke's law, the stress σ'_{ij} can be expressed as:

$$\sigma'_{ij} = C_{ijkl}^e \varepsilon_{ij}^e = C_{ijkl}^e (\varepsilon'_{ij} - \varepsilon_{ij}^t - \varepsilon_{ij}^p) = C_{ijkl}^e (\varepsilon'_{ij} - \varepsilon_{ij}^p) - (3\mu + 2G)\alpha T \delta_{ij} \quad (21)$$

where C_{ijkl}^e denotes the elastic stiffness tensor, $G = E/[2(1 + \nu)]$, $\mu = \nu E/[(1 + \nu)(1 - 2\nu)]$, and δ_{ij} is the Kronecker delta. The stress σ'_{ij} can also be divided into the stress caused by contact pressure $\sigma_{ij}^{\text{pressure}}$, thermal stress σ_{ij}^t and residual stress σ_{ij}^r :

$$\sigma'_{ij} = \sigma_{ij}^{\text{pressure}} + \sigma_{ij}^t + \sigma_{ij}^r = \sigma_{ij} + \sigma_{ij}^r \quad (22)$$

where the sum of the stress induced by contact pressure and the thermal stress σ_{ij} is obtained by Equation (11) and the residual stress σ_{ij}^r can be solved based on Liu's method (see details in Ref. [36]).

In the present study, the plastic flow follows the J_2 flow theory and isotropic strengthening criterion. The associated flow rule is considered to be:

$$\dot{\varepsilon}_{ij}^p = \dot{\lambda} \cdot \partial f_Y / \partial \sigma'_{ij} \quad (23)$$

where $\dot{\varepsilon}_{ij}^p$ denotes the plastic strain rate, $\dot{\lambda}$ is a scale variable representing the plastic flow rate. f_Y represents the yield function, which can be expressed as:

$$f_Y = \bar{\sigma}' - \sigma_Y(\bar{\varepsilon}^p) \quad (24)$$

where $\bar{\sigma}'$ denotes the equivalent stress, $\bar{\varepsilon}^p$ represents the equivalent plastic strain and $\sigma_Y(\bar{\varepsilon}^p)$ is the isotropic hardening function. In the present study, the linear hardening law is applied, and it can be written as:

$$\begin{cases} \sigma_Y(\bar{\varepsilon}^p) = \sigma_{Y0} + E_p \cdot \bar{\varepsilon}^p \\ E_p = \frac{E_T}{1 - E_T/E} \end{cases} \quad (25)$$

where σ_{Y0} denotes the initial yield stress, E_p , E_T , and E are the plastic modules, tangential modules and elastic modules, respectively. The return mapping algorithm for rate-independent plasticity [27] is used to solve the above elastoplastic problem, as given in Table 2.

Because plastic deformation depends on the loading path, the load should be applied incrementally. The residual stress and plastic strain are mutually coupled and need to be solved iteratively. The numerical solution procedure for this elastoplastic contact problem is shown in Figure 2. The final residual stress is obtained by incrementally unloading the normal force to zero after the prescribed load is reached. During unloading, the electrical and thermal loads are also incrementally reduced to zero, and the temperature field is

assumed to recover to the reference temperature. Therefore, no residual thermal stress is retained, and the reported residual von Mises stress represents the mechanical residual stress caused by plastic deformation after unloading.

Table 2. The radial return algorithm for solving the plastic strain ε_{ij}^p .

Step	Operation
1	Input initial values $\varepsilon_{ij}^p, \bar{\varepsilon}^p, \Delta\lambda = 0, \sigma'_{ij} = \sigma_{ij} + \sigma_{ij}^r, \bar{\sigma}'$
2	Calculate the yield function $f_Y = (\bar{\sigma}' - 3G\Delta\lambda) - \sigma_Y(\bar{\varepsilon}^p)$, and check if $f_Y < 0$. If so, stop the iteration, otherwise go to Step 3.
3	Calculate $\delta\lambda = (\bar{\sigma}' - 3G\Delta\lambda) - \sigma_Y(\bar{\varepsilon}^p) / (3G + E_p)$.
4	Update the middle variables $\hat{\mathbf{n}} = \boldsymbol{\sigma}'_{\text{dev}} / \ \boldsymbol{\sigma}'_{\text{dev}}\ $, where subscript “dev” denotes the deviatoric operator; $\Delta\varepsilon^p = \delta\lambda\sqrt{3/2}\hat{\mathbf{n}}$; $\bar{\varepsilon}^p = \bar{\varepsilon}^p + \delta\lambda$; $\Delta\lambda = \Delta\lambda + \delta\lambda$; $\Delta\boldsymbol{\sigma}' = 2G\delta\lambda\sqrt{3/2}\hat{\mathbf{n}}$
5	Update the plastic strain $\varepsilon_{ij}^p = \varepsilon_{ij}^p + \Delta\varepsilon_{ij}^p$ and stress $\sigma'_{ij} = \sigma'_{ij} - \Delta\sigma'_{ij}$, go to step 2.

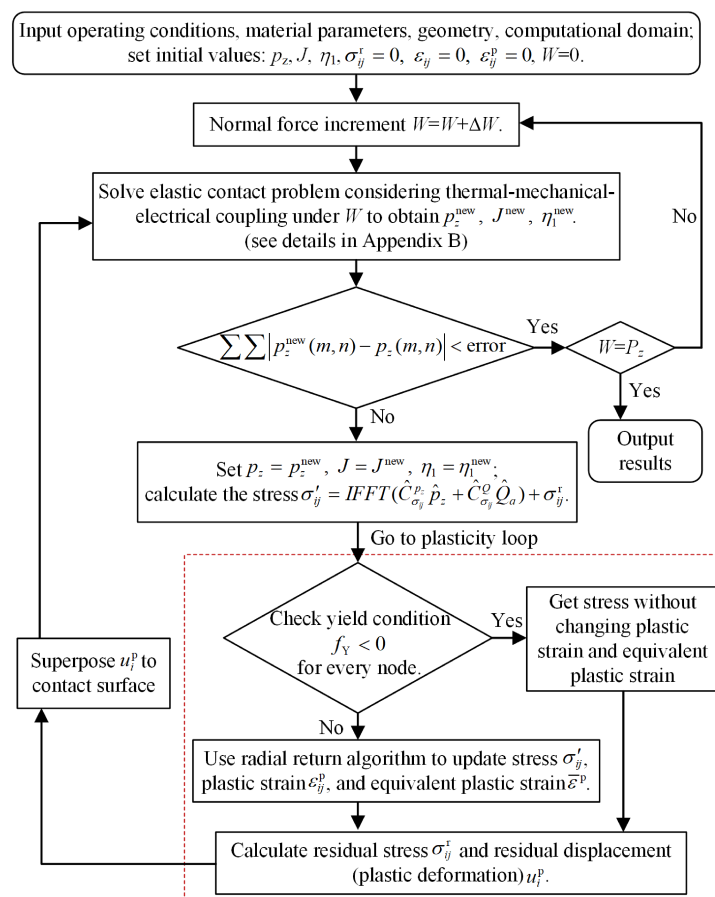


Figure 2. The numerical solution procedure for the contact problem considering both thermal-mechanical-electrical coupling and plastic deformation.

4. Results and Discussion

4.1. Model Verification

In this section, the key components of the proposed framework are verified by comparison with published results and analytical solutions. Since direct validation of the complete elastoplastic sliding electrical contact problem involving frictional heating, Joule heating, thermal expansion, and plasticity is challenging, the present model is verified through several reduced limiting cases. First, the present model is reduced to a purely

elastoplastic contact model to verify the plastic deformation solution by comparison with the independent FEM-based elastoplastic contact results reported in Ref. [27] under the same conditions. Specifically, the brush is rigid with a spherical tip of radius $R = 20$ mm; the slip ring is elastoplastic with $E = 100$ GPa, $\nu = 0.3$, $\sigma_{Y0} = 600$ MPa, $E_T = 0.5 E$; and the normal load is $P_z = 800$ N. Comparisons of the von Mises stress along the z -axis, the contact pressure and residual displacement along the x -axis are shown in Figure 3. The results of present model agree well with those reported in Ref. [27]. The rigid-elastoplastic case can be regarded as a limiting case of the two-elastoplastic-body formulation. The elastoplastic deformation is solved using the same procedure, with the only difference being that the deformation of the rigid body is zero rather than being superposed with that of the other body. Therefore, this comparison verifies the core elastoplastic solution procedure adopted in the present two-body model.

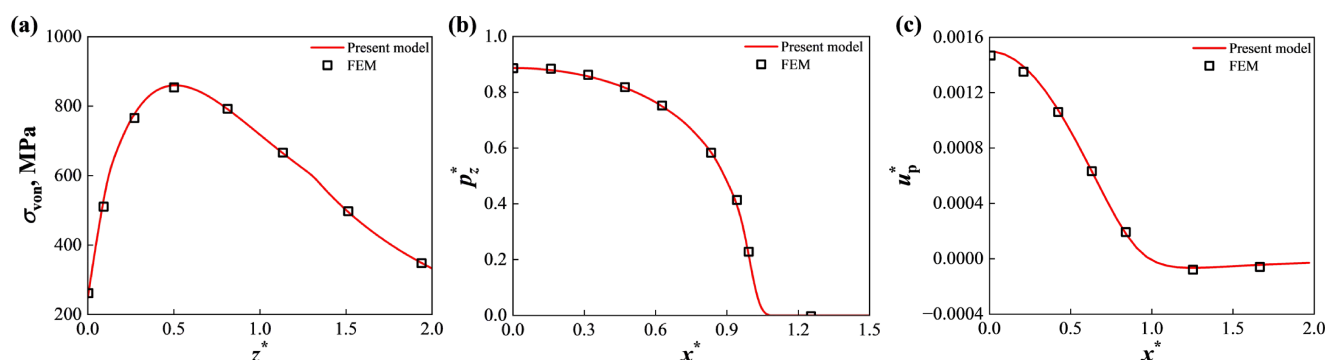


Figure 3. The comparisons of (a) von Mises stress, (b) contact pressure, and (c) residual displacement of present model with Ref. [27] (“*” denotes the nondimensional form of the variables).

Next, the present model is reduced to a thermal–mechanical coupling contact model to verify the prediction of frictional heating effects by comparison with the results in Ref. [37]. The verification case considers two elastic bodies in contact, with a spherical tip of radius $R = 15$ mm, $E = 210$ GPa, $\nu = 0.3$, $\kappa = 50.2$ W/m·K, $\alpha = 11.7$ $\mu\text{m}/\text{m}\cdot\text{K}$, $\eta_1 = 0$, under $P_z = 7.5$ kN and $f \cdot v = 0.05$ m/s. The comparison of contact pressure is shown in Figure 4a, and good agreement is observed. Finally, the electrical field solution and Joule heating effects are verified by comparison with the analytical solution [15]. For a rigid brush ($R = 10$ mm) in contact with an elastoplastic slip ring ($E = 100$ GPa, $\nu = 0.3$, $\kappa = 401$ W/m·K, $\rho = 1.75 \times 10^{-8}$ $\Omega\cdot\text{m}$, $\eta_1 = 0$) under $P_z = 100$ N, $I = 100$ A, $f \cdot v = 0$, the contact radius and ECR are $a = 0.239$ mm and $R_c = 3.63 \times 10^{-2}$ m Ω , respectively. Comparisons of the current density and the Joule-heating-induced temperature rise are shown in Figure 4b,c and good agreement is also observed.

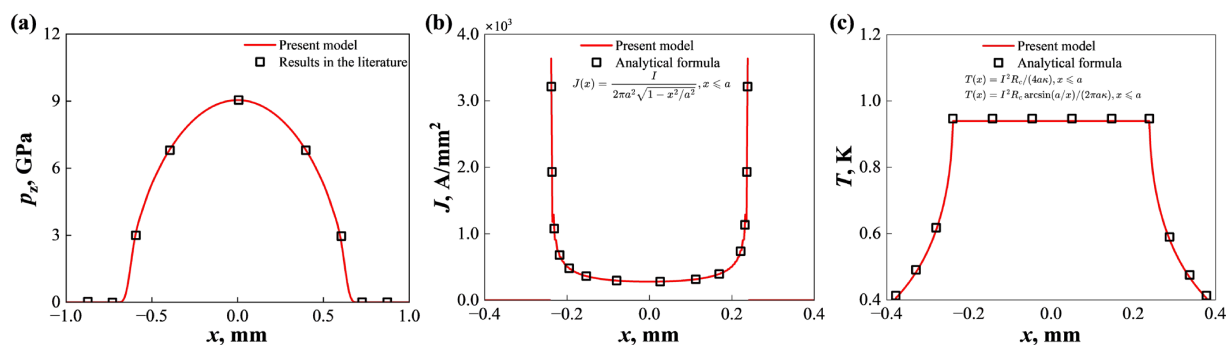


Figure 4. The comparisons of (a) contact pressure with Ref. [37], (b) current density and (c) Joule-heating-induced temperature rise with the analytical formula in Ref. [15].

4.2. Smooth Surface Contact

In this section, an idealized reference contact between a brush with a spherical tip of radius $R = 10$ mm and a smooth slip ring is investigated to focus on the influence of elastoplastic deformation. The brush and slip ring are both assumed to be elastoplastic and have identical material properties: $E = 119$ GPa, $\nu = 0.326$, $\kappa = 401$ W/m·K, $\rho = 1.75 \times 10^{-8}$ $\Omega \cdot \text{m}$, $\alpha = 17.7$ $\mu\text{m}/\text{m} \cdot \text{K}$, $\gamma = 116$ mm^2/s , $\sigma_{Y0} = 240$ MPa. Because the two contacting bodies have identical material properties, the interfacial heat flux is equally partitioned between them, i.e., $\eta_1 = \eta_2 = 0.5$. This identical material setting is used as a reference case to isolate the effects of plastic deformation and surface morphology on ECR, temperature rise, and stress. It should be noted that this setting is not intended to represent all practical brush-ring material pairs, and the proposed model can be further applied to dissimilar material pairs. Moreover, the smooth surface considered in this section represents an idealized reference morphology. Therefore, the following results should be interpreted as revealing the fundamental influence of elastoplastic deformation on the coupled thermal–mechanical–electrical contact responses, rather than as direct predictions for all practical slip ring systems.

The distribution of von Mises stress σ_{von} in plane $y = 0$ for different E_T under $P_z = 20$ N, $I = 100$ A, $f = 0.2$, and $v = 0.1$ m/s is shown in Figure 5a. Due to friction, σ_{von} is asymmetrically distributed and biased towards the direction opposite to sliding. For the ideal elastoplastic case ($E_T = 0$), the maximum σ_{von} appears both on and beneath the contact surface, and does not exceed the material initial yield strength σ_{Y0} . As E_T increase, the maximum σ_{von} increases, and is located only beneath the surface near the contact center. The distributions of residual von Mises stress σ_{von}^r in the planes $y = 0$ and $z = 0$ for the same case are shown in Figure 5b,c. The distribution of σ_{von}^r is symmetric along the x -axis but asymmetric along the y -axis because friction acts along the x -axis. For the ideal elastoplastic case ($E_T = 0$), residual stress appears both beneath the surface near the contact center and at the trailing edge of the contact region on the surface, indicating that these two locations are more susceptible to failure initiation. As E_T increases, the maximum σ_{von}^r decreases and becomes concentrated mainly beneath the surface.

The distributions of contact pressure p_z , current density J , and temperature rise T along the x -axis are shown in Figure 6. It can be found that p_z , J , and T are all asymmetrically distributed due to friction. Figure 6a shows that plastic deformation significantly reduces the maximum contact pressure, and this reduction becomes more pronounced as E_T decreases. Meanwhile, the maximum p_z shifts further towards the trailing edge of the contact region as E_T decreases. Figure 6b shows that plastic deformation significantly reduces the current density at the front edge of the contact region, while having little effect on the current density at the contact center. Furthermore, although plastic deformation has a negligible effect on the temperature rise outside the contact region, it significantly reduces the temperature rise within the contact region, and maximum T decreases as E_T decreases, as shown in Figure 6c. The above results indicate that plastic deformation can reduce the contact pressure, the current density peak at the contact edge, and the temperature rise, thereby alleviating current density concentration and local overheating at the contact interface.

The variations of sliding electrical contact behavior with normal force P_z for different E_T are further investigated. Figure 7 shows the variations of the maximum von Mises stress σ_{vonmax} during loading and the residual von Mises stress σ_{vonmax}^r after unloading. For elastoplastic contact, both σ_{vonmax} during loading and σ_{vonmax}^r after unloading increase with P_z . Furthermore, it is worth noting that, for the ideal elastoplastic case ($E_T = 0$), both σ_{vonmax} and σ_{vonmax}^r do not exceed the initial yield strength σ_{Y0} . When strain hardening is considered ($E_T = 0.4 E$), both σ_{vonmax} and σ_{vonmax}^r continue to increase with P_z , and their values may exceed σ_{Y0} because the yield stress evolves with plastic deformation.

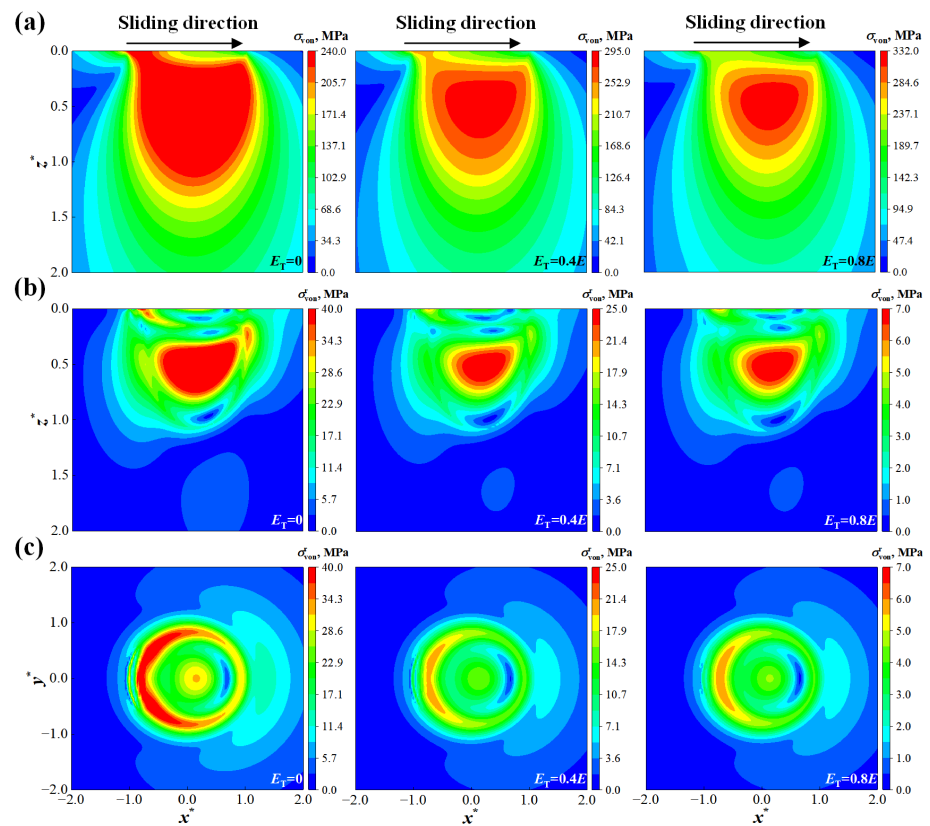


Figure 5. Distributions of (a) von Mises stress, residual von Mises stress in planes (b) $y = 0$ and (c) $z = 0$ ($x^* = x/a_0$, $y^* = y/a_0$, $z^* = z/a_0$, $a_0 = 0.131$ mm is the Hertz contact radius).

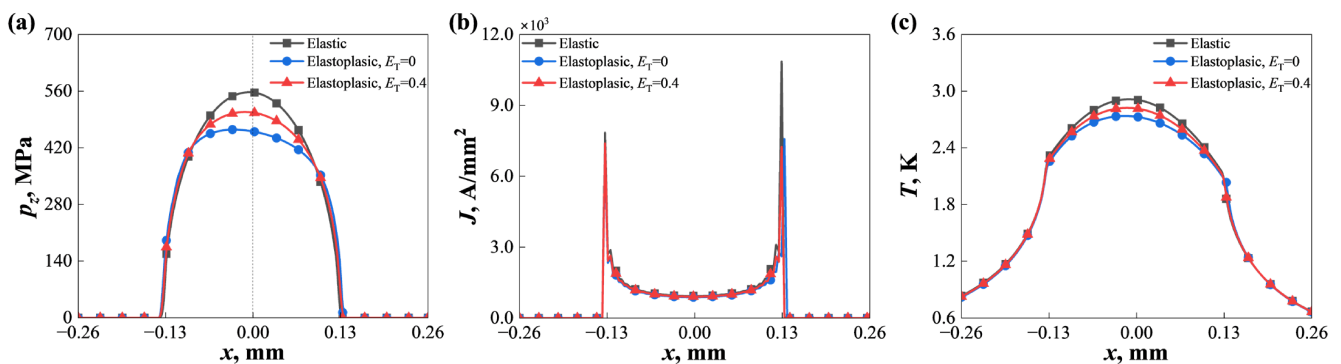


Figure 6. Distributions of (a) contact pressure, (b) current density, and (c) temperature rise along the x -axis for different E_T .

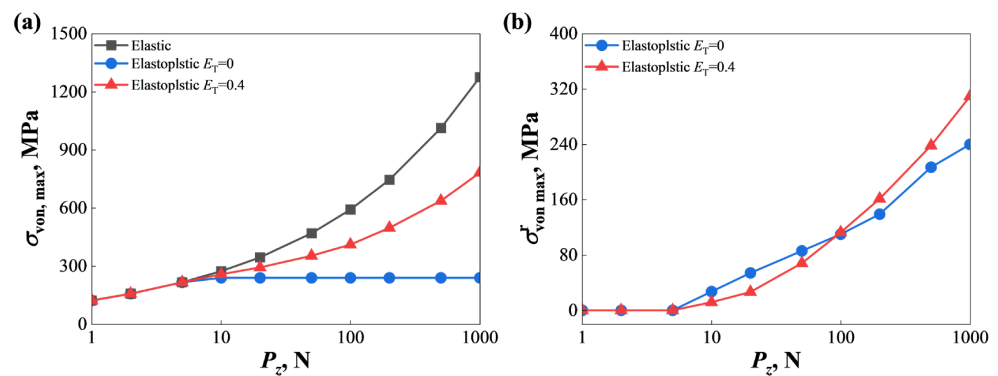


Figure 7. Variations of maximum (a) von Mises stress during loading and (b) residual von Mises stress after unloading with normal force.

The variations of contact area S , ECR R_c , and maximum temperature rise $T_{\max}$ with P_z are shown in Figure 8. S increases with P_z increasing, and once plastic deformation occurs, the increase of S becomes greater than that in elastic case, and this effect becomes more pronounced as E_T decreases. The increase of S means enlarging the conductive region, leading to a decrease in R_c with increasing P_z , as shown in Figure 8b. Under relatively high normal loads, plastic deformation becomes more pronounced, and a smaller E_T gives rise to a lower ECR. Figure 8c shows that $T_{\max}$ varies non-monotonically with increasing P_z , first increasing and then decreasing. At relatively low P_z , R_c is high, and Joule heating is the dominant contributor to the temperature rise. Since plastic deformation is negligible in this regime, the results for elastic and elastoplastic contact are nearly identical. At relatively high P_z , frictional heating becomes the dominant source of temperature rise, while plastic deformation can effectively reduce $T_{\max}$.

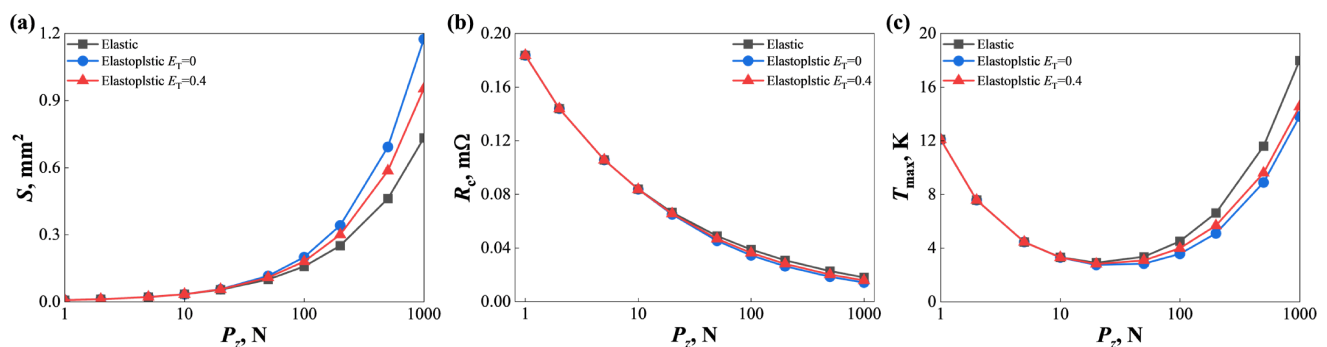


Figure 8. Variations of (a) contact area, (b) ECR, and (c) maximum temperature rise with normal force.

4.3. Sinusoidal Surface Contact

After the smooth surface case, a sinusoidal surface is considered as a representative regular non-smooth morphology to further investigate the influence of surface morphology, and the same solution procedure as in Section 4.2 is applied. This prescribed sinusoidal surface is also an idealized reference morphology and is used to clarify the effect of non-smooth morphology on the coupled contact responses. By modifying the surface geometry in Equation (18), the sliding electrical contact for arbitrary surfaces can be investigated. In this section, the sinusoidal surface is taken as an example, and the effects of asperity height on the sliding electrical contact behavior are investigated. The sinusoidal surface can be expressed as:

$$z(x, y) = A \cos\left(\frac{2\pi x}{Ba_0}\right) \cos\left(\frac{2\pi y}{Ba_0}\right) \quad (26)$$

where A and B determine the height and spacing of the surface peak. Other conditions are the same as those in Section 4.2 and $E_T = 0.4 E$.

Figure 9 shows the distribution of contact pressure p_z along the x -axis, as well as the distributions of von Mises stress σ_{von} and residual von Mises stress σ_{von}^r in the plane $y = 0$ for the contact of sinusoidal surface with different A . It can be found that the high value regions of von Mises stress and residual stress are mainly concentrated in the subsurface regions below the asperity peaks. Comparison of the results for different sinusoidal surface morphologies shows that increasing A , which corresponds to a larger asperity height, leads to higher p_z , σ_{von} , and σ_{von}^r . Meanwhile, the distribution of σ_{von}^r extends to deeper positions in the z direction.

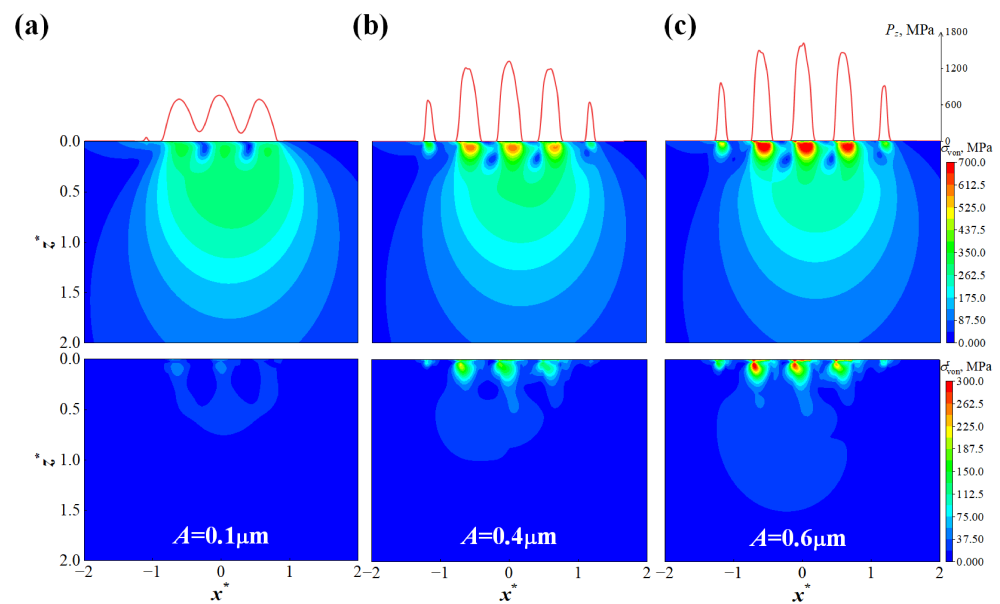


Figure 9. Distributions of contact pressure along the x -axis, von Mises stress and residual von Mises stress in the plane $y = 0$ for different sinusoidal surface ($B = 0.6$): (a) $A = 0.1 \mu\text{m}$, (b) $A = 0.4 \mu\text{m}$ (c) $A = 0.6 \mu\text{m}$ ($x^* = x/a_0$, $y^* = y/a_0$, $z^* = z/a_0$, $a_0 = 0.131 \text{ mm}$ is the Hertz contact radius).

Figure 10a shows the variation of maximum von Mises stress σ_{vonmax} with the asperity height A of the sinusoidal surface over a wide range. As A increases, σ_{vonmax} increases monotonically. For elastoplastic contact, both the magnitude and slope of this increase are markedly lower than those for purely elastic contact. Nevertheless, the maximum residual von Mises stress σ_{vonmax}^r increases significantly with increasing A , as shown in Figure 10b. It is also worth noting that, for both elastic and elastoplastic contact, the effect of A on σ_{vonmax} is negligible at small A . This indicates that reducing the asperity height can alleviate stress only within a limited range.

Figure 11 shows the variations in contact area S , ECR R_c , and maximum temperature rise T_{max} with the asperity height A of sinusoidal surface. As A increases, S decreases monotonically, and the contact morphology evolves through three distinct stages. In Stage 1, the asperity height is low, and the contact region remains nearly circular and continuous. In Stage 2, the contact boundary gradually breaks into multiple contact spots, while several circular noncontact holes appear in the central region as the asperity height increases. In Stage 3, the contact region becomes fully dispersed into many discrete micro contact spots.

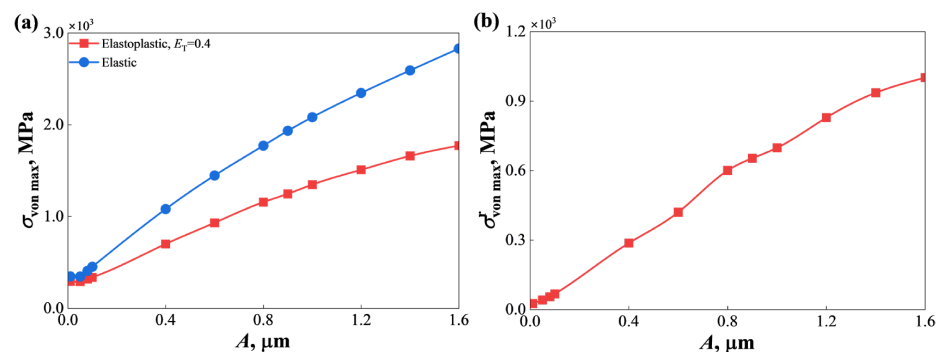


Figure 10. Variations of maximum (a) von Mises stress and (b) residual von Mises stress with the height A of the sinusoidal surface peak ($B = 0.6$).

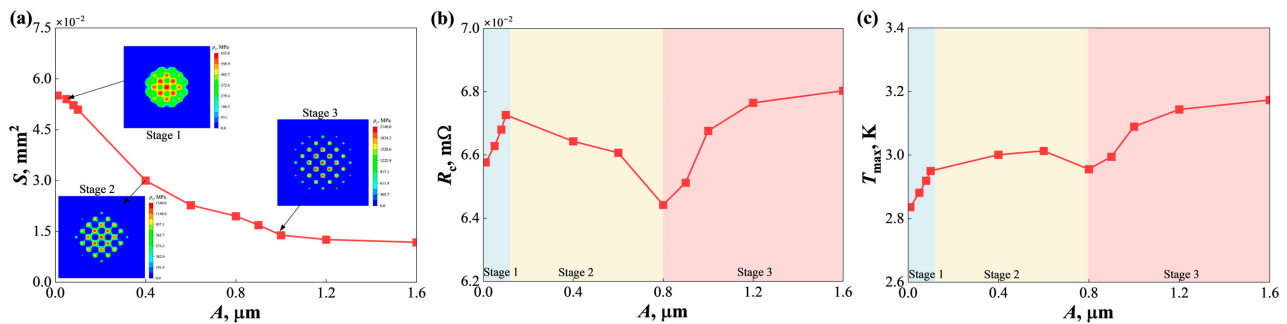


Figure 11. Variations of (a) contact area, (b) ECR, and (c) maximum temperature rise with the height A of the sinusoidal surface peak ($B = 0.6$).

The evolutions of R_c and T_{max} are governed by the contact morphology, as shown in Figure 11b,c. In Stage 1, the conductive area decreases with increasing A , which leads to the increase in R_c and T_{max} . In Stage 2, although the conductive area continues to decrease, the number of conductive spots increases, and the influence of the increased spot number becomes dominant; as a result, R_c decreases, as explained in our previous work [11]. During this stage, however, T_{max} does not vary synchronously with R_c , because the contact pressure continues to increase, leading to a competition between increasing frictional heating and decreasing Joule heating, and hence a trend of first increasing and then decreasing temperature rise. In Stage 3, the number of conductive spots no longer increases, whereas the contact area continues to decrease; therefore, both R_c and T_{max} increase again.

5. Conclusions

In this paper, an elastoplastic multi-physics model is developed to investigate the sliding electrical contact behavior of slip rings. The semi-analytical method is employed to solve the thermal–mechanical–electrical coupled contact problem, while the radial return algorithm and J_2 flow theory are adopted to determine plastic deformation. Based on the presented model, the sliding electrical contact behavior considering both multi-field coupling and plastic deformation is systematically investigated for reference material pairs with smooth and sinusoidal surface morphologies, respectively.

The regions of high stress and residual stress are mainly located at the trailing edge of the contact region on the surface and in the subsurface beneath the asperity, indicating that these locations are more susceptible to failure. Plastic deformation increases the contact area, thereby reducing ECR, the temperature rise in the contact region, and the current density peak at the contact edge. However, plastic deformation also induces residual stress after unloading, and the residual stress increases with normal force. Therefore, plastic deformation can improve the electrical and thermal contact performance of the interface, but the accompanying residual stress may increase the risk of mechanical damage.

Reducing the asperity height A of the contact surface effectively lowers the stress and residual stress. However, once A becomes sufficiently small, its influence is negligible, indicating that further reducing A is ineffective in improving the stress state. Decreasing A increases the contact area, but does not necessarily reduce ECR and temperature rise, because both are jointly determined by the contact area and contact spot distribution. Lower ECR and temperature rise are achieved when a large contact area is accompanied by a greater number of discrete micro contact spots.

Overall, the developed framework enables efficient analysis of the coupled thermal–mechanical–electrical contact responses of slip rings, including ECR, temperature rise, von Mises stress, and residual stress. It provides a theoretical tool for evaluating the sliding electrical contact performance of slip rings. Further work is needed to incorpo-

rate temperature-dependent material properties, degradation mechanisms such as wear, oxidation, and transfer-film evolution, and dielectric breakdown across intercontact gaps. These extensions would support a more comprehensive understanding of sliding electrical contact behavior in practical slip ring systems.

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Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Applying the Fourier transform to Equations (1), (3) and (5) with respect to x and y , and introducing two intermediate variables, $\tilde{\Pi} = i\omega_y \tilde{u}_x - i\omega_x \tilde{u}_y$ and $\tilde{H} = i\omega_x \tilde{u}_x + i\omega_y \tilde{u}_y$, yields the following governing equations:

$$\begin{aligned} \partial^2 \tilde{T} / \partial z^2 - (\omega^2 - i\omega_x V / \gamma) \tilde{T} &= 0 \\ \partial^2 \tilde{\Pi} / \partial z^2 - \omega^2 \tilde{\Pi} &= 0 \\ \partial^2 \tilde{H} / \partial z^2 - \omega^2 (1 + d) \tilde{H} - \omega^2 d \cdot \partial \tilde{u}_z / \partial z &= -\zeta \omega^2 \tilde{T} \\ (1 + d) \cdot \partial^2 \tilde{u}_z / \partial z^2 - \omega^2 \tilde{u}_z + d \cdot \partial \tilde{H} / \partial z &= \zeta \cdot \partial \tilde{T} / \partial z \\ \partial^2 \tilde{\varphi} / \partial z^2 - \omega^2 \tilde{\varphi} &= 0 \end{aligned} \quad (A1)$$

where “ $\sim$ ” denotes the continuous Fourier transform, ω_x and ω_y are the frequency variables with respect to the x and y directions, $\omega = \sqrt{\omega_x^2 + \omega_y^2}$, $i = \sqrt{-1}$, $d = 1/(1 - 2\nu)$, $\zeta = 2\alpha(1 + \nu)/(1 - 2\nu)$. For the above ordinary differential equations, the general solutions can be expressed as:

$$\begin{aligned} \tilde{T} &= M_1 e^{-rz} + M_2 e^{rz} \\ \tilde{\Pi} &= A_1 e^{-\omega z} + A_2 e^{\omega z} \\ \tilde{u}_z &= B_1 e^{-\omega z} + B_2 z e^{-\omega z} + B_3 e^{\omega z} + B_4 z e^{\omega z} - K \cdot \partial \tilde{T} / \partial z \\ \tilde{H} &= \beta_1 e^{-\omega z} + \beta_2 z e^{-\omega z} + \beta_3 e^{\omega z} + \beta_4 z e^{\omega z} + (K/d) \tilde{T} \\ \tilde{\varphi} &= N_1 e^{-\omega z} + N_2 e^{\omega z} \end{aligned} \quad (A2)$$

where $r = \sqrt{\omega^2 - i\omega_x V / \gamma}$, $K = \zeta \gamma / [iV\omega_x(1 + d)]$. A, B, M, N are unknown coefficients to be solved and b is the intermediate variable defined by B :

$$\begin{cases} \beta_1 = \omega B_1 - (2/d + 1)B_2 \\ \beta_2 = \omega B_2 \\ \beta_3 = -\omega B_3 - (2/d + 1)B_4 \\ \beta_4 = -\omega B_4 \end{cases} \quad (A3)$$

The general solutions of $\tilde{u}_x$ and $\tilde{u}_y$ can be represented by $\tilde{\Pi}$ and $\tilde{H}$:

$$\begin{cases} \tilde{u}_x = -i(\omega_x \tilde{H} + \omega_y \tilde{\Pi})/\omega^2 \\ \tilde{u}_y = -i(\omega_y \tilde{H} - \omega_x \tilde{\Pi})/\omega^2 \end{cases} \quad (\text{A4})$$

Based on the elastic displacement, the elastic stress can be derived by thermoelastic constitutive equations:

$$\begin{cases} \sigma_{ij} = 2G\varepsilon_{ij} + \mu\varepsilon_{kk}\delta_{ij} - (3\mu + 2G)\alpha T\delta_{ij} \\ \varepsilon_{ij} = (\partial u_i/\partial j + \partial u_j/\partial i)/2 \end{cases} \quad (\text{A5})$$

where $i, j, k = x, y, z$; $G = E/[2(1 + \nu)]$; $\mu = \nu E/[(1 + \nu)(1 - 2\nu)]$; E and ν are the elastic modulus and Poisson's ratio; δ_{ij} is the Kronecker delta.

Therefore, the FRFs can be expressed as:

$$\begin{aligned} \tilde{\sigma}_{xx} = & 2G\frac{\omega_x\omega_y}{\omega^2}A_1e^{-\omega z} + 2G\frac{\omega_x\omega_y}{\omega^2}A_2e^{\omega z} + 2G\frac{\omega_x^2}{\omega}B_1e^{-\omega z} - 2\left(\frac{\mu}{d} + \frac{g_1\omega_x^2}{\omega^2}\right)B_2e^{-\omega z} \\ & - 2G\frac{\omega_x^2}{\omega}B_3e^{\omega z} - 2\left(\frac{\mu}{d} + \frac{g_2}{\omega^2}\omega_x^2\right)B_4e^{\omega z} + g_5M_1e^{-rz} + g_5M_2e^{rz} \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} \tilde{\sigma}_{yy} = & -2G\frac{\omega_x\omega_y}{\omega^2}A_1e^{-\omega z} - 2G\frac{\omega_x\omega_y}{\omega^2}A_2e^{\omega z} + 2G\frac{\omega_y^2}{\omega}B_1e^{-\omega z} - 2\left(\frac{\mu}{d} + \frac{g_1\omega_y^2}{\omega^2}\right)B_2e^{-\omega z} \\ & - 2G\frac{\omega_y^2}{\omega}B_3e^{\omega z} - 2\left(\frac{\mu}{d} + \frac{g_2}{\omega^2}\omega_y^2\right)B_4e^{\omega z} + g_6M_1e^{-rz} + g_6M_2e^{rz} \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \tilde{\sigma}_{zz} = & -2G\omega B_1e^{-\omega z} - 2\left[\frac{\mu}{d} - G(1 - \omega z)\right]B_2e^{-\omega z} + 2G\omega B_3e^{\omega z} \\ & - 2\left[\frac{\mu}{d} - G(1 + \omega z)\right]B_4e^{\omega z} + g_7M_1e^{-rz} + g_7M_2e^{rz} \end{aligned} \quad (\text{A8})$$

$$\begin{aligned} \tilde{\sigma}_{xy} = & -G\frac{\omega_x^2 - \omega_y^2}{\omega^2}A_1e^{-\omega z} - G\frac{\omega_x^2 - \omega_y^2}{\omega^2}A_2e^{\omega z} + 2G\frac{\omega_x\omega_y}{\omega}B_1e^{-\omega z} \\ & - 2g_1\frac{\omega_x\omega_y}{\omega^2}B_2e^{-\omega z} - 2G\frac{\omega_x\omega_y}{\omega}B_3e^{\omega z} - 2g_2\frac{\omega_x\omega_y}{\omega^2}B_4e^{\omega z} \\ & + 2GK\omega_x\omega_yM_1e^{-rz} + 2GK\omega_x\omega_yM_2e^{rz} \end{aligned} \quad (\text{A9})$$

$$\begin{aligned} \tilde{\sigma}_{xz} = & iG\frac{\omega_y}{\omega}A_1e^{-\omega z} - iG\frac{\omega_y}{\omega}A_2e^{\omega z} + 2iG\omega_xB_1e^{-\omega z} - 2ig_3\frac{\omega_x}{\omega}B_2e^{-\omega z} \\ & + 2iG\omega_xB_3e^{\omega z} + 2ig_4\frac{\omega_x}{\omega}B_4e^{\omega z} + 2iGKr\omega_xM_1e^{-rz} - 2iGKr\omega_xM_2e^{rz} \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} \tilde{\sigma}_{yz} = & -iG\frac{\omega_x}{\omega}A_1e^{-\omega z} + iG\frac{\omega_x}{\omega}A_2e^{\omega z} + 2iG\omega_yB_1e^{-\omega z} - 2ig_3\frac{\omega_y}{\omega}B_2e^{-\omega z} \\ & + 2iG\omega_yB_3e^{\omega z} + 2ig_4\frac{\omega_y}{\omega}B_4e^{\omega z} + 2iGKr\omega_yM_1e^{-rz} - 2iGKr\omega_yM_2e^{rz} \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} \tilde{u}_x = & -i\frac{\omega_y}{\omega^2}A_1e^{-\omega z} - i\frac{\omega_y}{\omega^2}A_2e^{\omega z} - i\frac{\omega_x}{\omega}B_1e^{-\omega z} + i\frac{g_1}{G}\frac{\omega_x}{\omega^2}B_2e^{-\omega z} \\ & + i\frac{\omega_x}{\omega}B_3e^{\omega z} + i\frac{g_2}{G}\frac{\omega_x}{\omega^2}B_4e^{\omega z} - iKw_xM_1e^{-rz} - iKw_xM_2e^{rz} \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} \tilde{u}_y = & i\frac{\omega_x}{\omega^2}A_1e^{-\omega z} + i\frac{\omega_x}{\omega^2}A_2e^{\omega z} - i\frac{\omega_y}{\omega}B_1e^{-\omega z} + i\frac{g_1}{G}\frac{\omega_y}{\omega^2}B_2e^{-\omega z} \\ & + i\frac{\omega_y}{\omega}B_3e^{\omega z} + i\frac{g_2}{G}\frac{\omega_y}{\omega^2}B_4e^{\omega z} - iKw_yM_1e^{-rz} - iKw_yM_2e^{rz} \end{aligned} \quad (\text{A13})$$

$$\tilde{u}_z = B_1e^{-\omega z} + zB_2e^{-\omega z} + B_3e^{\omega z} + zB_4e^{\omega z} + KrM_1e^{-rz} - KrM_2e^{rz} \quad (\text{A14})$$

$$\tilde{T} = M_1e^{-rz} + M_2e^{rz} \quad (\text{A15})$$

$$\tilde{\varphi} = N_1e^{-\omega z} + N_2e^{\omega z} \quad (\text{A16})$$

where the key parameters in FRFs are given as follows:

$$\begin{aligned} g_1 &= G(2/d + 1 - \omega z) & g_2 &= G(2/d + 1 + \omega z) \\ g_3 &= G(1/d + 1 - \omega z) & g_4 &= G(1/d + 1 + \omega z) \\ g_5 &= -E\alpha/(1 - 2\nu) + K(\omega^2\mu + 2\omega_x^2G - r^2\mu) \\ g_6 &= -E\alpha/(1 - 2\nu) + K(\omega^2\mu + 2\omega_y^2G - r^2\mu) \\ g_7 &= -E\alpha/(1 - 2\nu) + K(\omega^2\mu - 2r^2G - r^2\mu) \end{aligned} \quad (\text{A17})$$

Appendix B

The detailed procedure for solving p_z , J , and Q_a using the conjugate gradient methods (CGMs) and relaxation iteration is given below:

Input: Operating conditions P_z, f, l, V ; the ICs; initial gap $w^0(m, n)$; initial parameters estimations $p_z^0(m, n), J^0(m, n), \eta_1^0(m, n)$. Set $p' = p_z^0, J' = J^0, \eta' = \eta_1^0$.

Step 1: Enter the pressure-solution loop to determine p_z . Set initial conjugate gradient direction $t^0 = 0$ and iteration step $\tau = 1$.

Step 2: Apply DC-FFT to obtain the thermoelastic deformation $u_z(m, n, 0)$.

Step 3: Calculate the average gap over the contact region $\bar{w}$ and update the surface gap as $w^\tau(m, n) = w^{\tau-1}(m, n) - \bar{w}$.

Step 4: Calculate the conjugate gradient direction $t^\tau(m, n)$:

$$t^\tau(m, n) = -w^\tau(m, n) + \alpha t^{\tau-1}(m, n), \quad \alpha = \frac{\sum_{m,n \in \Gamma} [w^\tau(m, n)]^2}{\sum_{m,n \in \Gamma} [w^{\tau-1}(m, n)]^2} \quad (\text{A18})$$

Step 5: Calculate the step size β :

$$\beta = \frac{\sum_{m=1}^M \sum_{n=1}^N (-w^\tau(m, n) \cdot t^\tau(m, n))}{\sum_{m=1}^M \sum_{n=1}^N (\zeta^\tau(m, n) \cdot t^\tau(m, n))} \quad (\text{A19})$$

where ζ^τ is determined from $\zeta_1^\tau(m, n) = \text{IFFT}(\hat{C}_{u_z}^{p_z} \hat{t}^\tau)$ and its average value $\bar{\zeta}_1^\tau$, which is given by $\zeta^\tau(m, n) = \zeta_1^\tau(m, n) - \bar{\zeta}_1^\tau$.

Step 6: Update contact pressure in contact region as $p_z^\tau(m, n) = p_z^{\tau-1}(m, n) - \beta t^\tau(m, n)$.

Step 7: Calculate the iteration errors e_1 and e_2 :

$$e_1 = \frac{\sum_{m=1}^M \sum_{n=1}^N |p_z^\tau(m, n) - p_z^{\tau-1}(m, n)|}{P_z}; \quad e_2 = \frac{\left| P_z - \sum_{m=1}^M \sum_{n=1}^N p_z^\tau(m, n) \Delta_x \Delta_y \right|}{P_z} \quad (\text{A20})$$

Step 8: If $e_1 > \text{error}$ or $e_2 > \text{error}$, set $p_{\text{old}}(m, n) = p_z^\tau(m, n)$, update the contact pressure:

$$p_z^\tau(m, n) = \frac{P_z}{\sum_{m=1}^M \sum_{n=1}^N p_{\text{old}} \Delta_x \Delta_y} p_{\text{old}} \quad (\text{A21})$$

Set $\tau = \tau + 1$ and return to step (2). Otherwise, terminate the pressure iteration and output the contact pressure $p_z(m, n)$.

Step 9: Enter the current-solution loop to determine $J(m, n)$. Set $t^0 = 0, \phi^0 = 1, \tau = 1$.

Step 10: Apply DC-FFT to obtain electrical potential $\phi(m, n, 0)$.

Step 11: Calculate the average electrical potential over the contact region $\bar{\phi}$ and set $\phi^\tau(m, n) = \bar{\phi} - \phi(m, n, 0)$.

Step 12: Calculate the conjugate gradient direction $t^\tau(m, n)$:

$$t^\tau(m, n) = \phi^\tau(m, n) + \alpha t^{\tau-1}(m, n), \quad \alpha = \frac{\sum_{m,n \in \Gamma} [\phi^\tau(m, n)]^2}{\sum_{m,n \in \Gamma} [\phi^{\tau-1}(m, n)]^2} \quad (\text{A22})$$

Step 13: Calculate the step size β :

$$\beta = \frac{\sum_{m=0}^M \sum_{n=0}^N (\phi^\tau(m, n) \cdot t^\tau(m, n))}{\sum_{m=0}^M \sum_{n=0}^N (r^\tau(m, n) \cdot t^\tau(m, n))} \quad (\text{A23})$$

where r^τ is determined from $r^\tau(m, n) = \text{IFFT}(\hat{C}_\phi^J \hat{t}^\tau)$.

Step 14: Update current density in contact region as $J^\tau(m, n) = J^{\tau-1}(m, n) + \beta t^\tau(m, n)$.

Step 15: Calculate the iteration errors e_1 and e_2 :

$$e_1 = \frac{\sum_{m=1}^M \sum_{n=1}^N |J^\tau(m, n) - J^{\tau-1}(m, n)|}{I}; \quad e_2 = \frac{\left| I - \sum_{m=1}^M \sum_{n=1}^N J^\tau(m, n) \Delta_x \Delta_y \right|}{I} \quad (\text{A24})$$

Step 16: If $e_1 > \text{error}$ or $e_2 > \text{error}$, set $J_{\text{old}}(m, n) = J^\tau(m, n)$, update the current density:

$$J^\tau(m, n) = \frac{I}{\sum_{m=1}^M \sum_{n=1}^N J_{\text{old}} \Delta_x \Delta_y} p_{\text{old}} \quad (\text{A25})$$

Set $\tau = \tau + 1$ and return to step (10). Otherwise, terminate the current-solution loop, output $J(m, n)$ and $R_c = (\bar{\varphi}_1 + \bar{\varphi}_2)/I$, where subscript “1” and “2” denote the brush and slip ring, respectively.

Step 17: Enter the heat-solution loop to determine $Q_a(m, n)$. Set $\tau = 1$.

Step 18: Apply DC-FFT to obtain the temperature rise $T_1(m, n, 0)$ and $T_2(m, n, 0)$.

Step 19: Apply relaxation iteration to update $\eta_1(m, n)$:

$$\eta_1^\tau(m, n) = \eta_1^{\tau-1}(m, n) + \vartheta \frac{T_2(m, n) - T_1(m, n)}{T_1(m, n) + T_2(m, n)} \quad (\text{A26})$$

where ϑ denotes the iteration coefficient.

Step 20: Set $T_{1\text{old}} = T_1$, $T_{2\text{old}} = T_2$, update T_1 , and T_2 based on Equation (11).

Step 21: Calculate the iteration errors e_1 and e_2 :

$$e_1 = \sum_{m=1}^M \sum_{n=1}^N \left| \frac{\eta_1^\tau(m, n) - \eta_1^{\tau-1}(m, n)}{\eta_1^{\tau-1}(m, n)} \right|$$

$$e_2 = \sum_{m=1}^M \sum_{n=1}^N \left| \frac{T_1(m, n) + T_2(m, n) - T_{1\text{old}}(m, n) - T_{2\text{old}}(m, n)}{T_{1\text{old}}(m, n) + T_{2\text{old}}(m, n)} \right| \quad (\text{A27})$$

Step 22: If $e_1 > \text{error}$ or $e_2 > \text{error}$, set $\tau = \tau + 1$ and return to step (18). Otherwise, terminate the current-solution loop, output $\eta_1(m, n)$ and $Q_a(m, n)$.

Step 23: Calculate the total iteration errors e_p , e_j , and e_q :

$$e_p = \sum_{m=1}^M \sum_{n=1}^N \left| \frac{p'(m, n) - p_z(m, n)}{p'(m, n)} \right|$$

$$e_j = \sum_{m=1}^M \sum_{n=1}^N \left| \frac{J'(m, n) - J(m, n)}{J'(m, n)} \right|$$

$$e_q = \sum_{m=1}^M \sum_{n=1}^N \left| \frac{\eta'(m, n) - \eta_1(m, n)}{\eta'(m, n)} \right| \quad (\text{A28})$$

Step 24: If $e_p > \text{error}$ or $e_j > \text{error}$ or $e_q > \text{error}$, set $p' = p_z^0 = p_z$, $J' = J^0 = J$, $\eta' = \eta_1^0 = \eta_1$ and return to step (1). Otherwise, terminate the total loop, output p_z , J , R_c , and Q_a .

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