



Article

Study on Heat Partition in Sliding Contact Pairs Considering Conduction Heat Flux

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Abstract

Regarding heat conduction in sliding contact pairs, this paper investigates the interfacial heat partition problem with conduction heat flux taken into account to address the issue of the heat partition coefficient falling outside its physically reasonable range. The main contributions of this study are as follows. First, conduction heat flux is explicitly introduced, and the governing equation for the heat partition coefficient incorporating conduction heat flux is derived via Green's function method. Subsequently, to tackle the nonlinearity caused by the time-varying velocity and heat source of the contact pair, least-squares estimation is adopted to solve for the heat partition coefficient and conduction heat flux. The results indicate that under extreme operating conditions with drastic variations in heat source and velocity, traditional heat partition models yield unphysical results where the heat partition coefficient is less than 0 or greater than 1, whereas the modified model effectively resolves this issue. Furthermore, this paper analyzes the effects of material parameters, motion characteristics, and thermal loads on heat partition. The findings of this work provide a reference for interfacial thermal design and thermal management of various sliding contact pairs.

Keywords: heat partition; conduction heat flux; least-squares estimation; armature–rail system; sliding contact

1. Introduction

Sliding contact pairs are frictional systems composed of two components that remain in contact and undergo relative sliding under applied loads; they are widely used in mechanical equipment such as rail transit systems, brakes, bearings, and gears [1–3]. During the operation of sliding contact pairs, frictional heat induces a significant temperature rise in the components, which affects the operating condition and performance of the contact pairs [4]. Particularly in current-carrying friction, the temperature rise in the friction pair is even more severe and may even lead to melting and phase transitions. The interfacial heat partition coefficient is defined as the ratio of heat flowing into one contact body to the total heat generated at the contact interface [5–7]. Accurate calculation of heat partition is a prerequisite and foundation for elucidating interfacial thermal damage mechanisms, predicting material wear behavior, and optimizing thermal management strategies, and it is of great significance for both theoretical research and engineering applications.

Jaeger were the first to conduct research in the heat partition, proposing principles based on either the average interface temperature or the maximum interface temperature for determining the heat partition coefficient [8]. Bogdanovich et.al [9] reviewed the literature on early heat partition studies, analyzed the friction heat models developed by



Received: 25 June 2026

Revised: 28 July 2026

Accepted: 3 August 2026

Published: 5 August 2026

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Blok, Jaeger, Archard, and Kuhlmann, and examined the effects of various factors on heat partition. Sardar [10] used five classical heat partition models to calculate the temperature evolution and found that the maximum difference in flash temperature predictions among the different models can reach 1.65 times, and there is no universally optimal model for all operating conditions. Based on Jaeger's classical heat source method, Hou et. al [11] proposed a general solution for the temperature rise at any point caused by stationary or moving planar heat sources of various shapes (uniform, parabolic, and normal), thereby expanding the scope of application of traditional methods. Reference [12] compared the performance of different models in sliding contact under high Peclet numbers. Bauzin et. al [13] studied the inverse problem of heat conduction in one-dimensional dry-friction sliding contact, simultaneously calculating the frictional heat flux, the contact thermal conductivity, and the heat partition coefficient based on temperature measurements. The above research provides theoretical foundations for the heat partition problem.

Regarding the problem of heat partition under non-ideal contact conditions, Balakin [14] proposed a series of thermophysical models that account for dynamic factors such as volumetric heat sources, plastic deformation of the surface layer, material transfer, and oxide films. Kennedy et. al [15] studied thermal partition in sliding friction pairs while taking surface roughness into account and proposed an experimental observation model based on contact temperature. Gecim et. al [16] studied the thermal behavior of a single roughness peak within the apparent contact area and obtained an analytical solution for the heat partition coefficient in low speed where the Peclet number is less than 5. Building on this work, Yongwoo et. al [17] derived expressions for the heat partition coefficient as a function of surface statistical parameters, nominal contact pressure, and sliding velocity and provided simple curve-fitting formulas, enabling the computational results for a single roughness peak to be applied as macroscopic heat conduction conditions in numerical simulations. Liu [18] investigated heat conduction in sliding contact between two rough surfaces based on the Greenwood–Williamson contact model, an established analytical model for deriving the macroscopic equivalent of thermal contact from microscopic roughness peaks, and obtained expressions for parameters such as the nominal frictional heat flux through statistical integration. Oleksii [19] overcame the limitation of traditional models that treat frictional heat as a boundary heat source by approximating the frictional heat source as a heat source at the adhesive interface and a heat source in the deformed volume. The above studies implicitly addressed the issue of heat conduction in contact and provided a relatively comprehensive understanding of non-ideal thermal contact.

Regarding methods for solving heat partition problems, Komanduri [20] derived an analytical solution using Green's function method for classic sliding systems. Reference [21] proposed a generalized integral transform method, which overcomes the problem of non-separable variables caused by time-varying coefficients in traditional methods of separation of variables. Belyakow [22] compared classical solutions to the heat partition problem and concludes that combining Green's function method with the FFT method can provide a new approach to solving the problems on real rough surfaces. Yao [23] proposed a computational method based on least-squares regression to determine the heat partition at the armature–rail interface. Fang [24] used the FEM to analyze friction heat accumulation under high-speed rolling–sliding contact. Avevor [25] investigated heat distribution in dry-friction machinery and used a combined FE–analytical approach to determine the time-dependent variation in the heat partition coefficient during the cutting process. Zoltan et. al [26] studied heat partition under three-dimensional surface wear conditions, using an incremental FEM to calculate the evolution of the actual contact area and heat partition during sliding.

Addressing specific issues related to heat partition in engineering applications, Reference [27] provides a review of heat partition during metal cutting processes. Chen [28] studied the heat partition in a milling machine using a combination of theoretical analysis, numerical calculations, and experimental validation and calculated the instantaneous temperature changes during the cutting process. Goswami et al. [29,30] studied the temperature field in disk–pin contact, deriving analytical and approximate solutions for the heat partition coefficient under one-dimensional steady-state and three-dimensional transient conditions, respectively, and validated the computational results through experiments. Reference [31] investigated the influence of thermally inhomogeneous and anisotropic heat-conducting shells on the heat partition process. To address the issue of temperature evolution of wheel–rail, Alizadeh [32] proposed a one-dimensional analytical heat conduction model that accounts for contact thermal conductivity. Kennedy [33] established a two-dimensional thermal model of wheel–rail sliding, simulated the heat partition process using an artificial contact layer, and compared the simulation results with those of classical analytical heat partition models to analyze the applicability of the Iwano and Bock models [8]. Yevtushenko [34] used FEM to study frictional heat generation in disk brakes and discussed changes in the maximum temperature of the contact surface under single and repeated contact conditions. Abdullah [35] developed a fluid–structure-coupled convective heat transfer model and established equations to characterize the effective energy fluxes for convective heat transfer and heat generation at the sliding interface. Gkinis [36] investigated the thermodynamic–kinetic coupling behavior under dry-friction conditions in a clutch and analyzed the interaction mechanisms between temperature rise and friction and wear. The above studies include a large number of contact pairs involving heat partition but lack research on thermal effects in electrical contacts.

Given that heat partition during friction is closely related to material properties, Nosko [37] studied friction problems involving coatings based on the assumption of a linear distribution of surface temperature. Akoussan [38] investigated sliding friction in polycrystalline materials under extreme contact loads and discussed the effects of frictional heat on this process. Wang [39] investigated the heat partition during three-dimensional thermoelastic contact between coated solids, solving the contact and heat partition problems using the DC-FFT and CGM methods. YANG [40] established a thermoelastic contact model between a sliding sphere and a rail with a transversely isotropic coating and calculated the heat partition coefficient using a semi-analytical method. Yevtushenko [41] proposed a theoretical approach for determining the heat partition coefficient in a friction pair composed of functionally graded materials.

Previous studies have made notable progress in investigating heat partition under diverse operating conditions; however, they have not explicitly considered the conduction heat flux that may exist at the contact interface. Particularly for sliding electrical contacts or scenarios with time-varying contact pressure, variations in the interfacial heat source are not synchronized with changes in sliding velocity. Explicit calculation of the conduction heat flux contributes to a better understanding of the thermal behavior of contact pairs and eliminates unphysical values of the heat partition coefficient.

This paper investigates the heat partition problem in sliding contact pairs with explicit consideration of conduction heat flux, focusing on the current-carrying sliding issue in armature–rail systems. First, based on Green’s function method, a composite interfacial heat source model incorporating both frictional heat and Joule heat generated by contact resistance is established, and a conduction heat flux term across the contact interface is introduced to derive the modified heat partition governing equation. Second, the Volterra integral governing equation is discretized and solved using the least-squares estimation method. The accuracy and validity of the modified model are verified by comparing the

obtained results with the analytical solution under uniform motion conditions. Finally, the influences of key parameters, such as material thermal properties and sliding velocity, on the heat partition coefficient are analyzed, revealing the physical mechanism underlying interfacial heat partition. The findings of this study provide a theoretical foundation for predicting interfacial thermal damage and developing thermal management strategies for current-carrying sliding contacts and other sliding contact systems.

2. Experimental Setup and Test Results

Figure 1a depicts the experimental setup in which the following measurement instruments are indicated: a Rogowski coil for measuring the excitation current amplitude, a high-voltage differential probe for measuring the muzzle voltage (from which the contact resistance can be derived), B-dot probes, and a laser velocimetry target for determining the armature velocity.

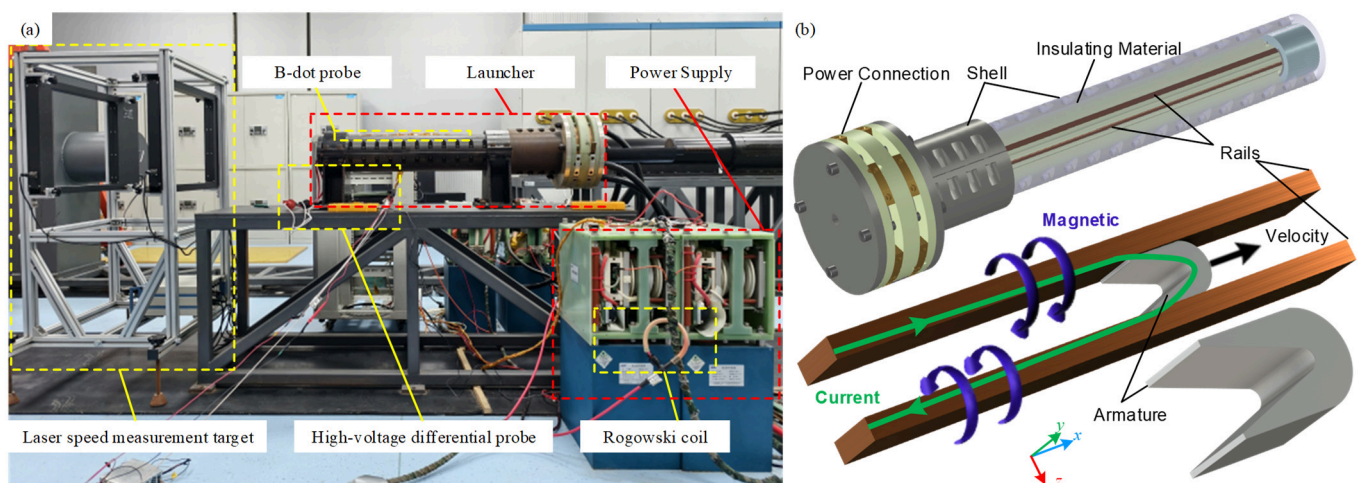


Figure 1. (a) Railgun test platform. (b) Structure of the armature–rail system.

The structure of the railgun employed in this study is illustrated in Figure 1b. For clarity, only a partial segment of the rails is presented in the figure. A pulsed power supply is connected to the breech end of the rails: current flows into rail 1 and returns through rail 2 via the armature. The y-direction magnetic field generated by the rail currents inside the barrel interacts with the z-direction current in the armature, producing an x-direction Lorentz force that drives the armature forward. In addition, the interaction between the current in the armature arms and the magnetic field induces a z-direction electromagnetic force that presses the armature arms against the rail surfaces. To counteract the tendency of the armature to separate from the rails and maintain stable contact, an interference fit is adopted to provide an initial contact pressure of approximately 3800 N at the armature–rail interface, as determined via static testing.

Driven by electromagnetic forces, the armature travels along the rails and is accelerated to an ultra-high velocity exceeding 1500 m/s within 2 ms, resulting in substantial frictional heat generation at the armature–rail contact pair within a short duration. Meanwhile, with a peak excitation current of 550 kA applied to the railgun, considerable Joule heat is also generated at the contact pair.

The full nomenclature of parameters employed in this study is summarized in Table 1.

In this paper, a subscript of 1 refers to the armature, and a subscript of 2 refers to the rails; the meanings of other subscripts will be explained as needed.

Table 1. Parameter nomenclature.

Symbol	Description	Symbol	Description
T	Temperature	μ	Coefficient of Friction
t	Time	k	Thermal Conductivity
I	Current Amplitude	c	Heat Capacity
q	Heat Flux	ρ	Density
α	Heat Partition Coefficient	η	Resistivity
F	Force	H_c	Hardness of the Armature
P	Contact Pressure	θ	Armature Tail Angle
$\mathbf{r}$	Position Vector	C_d	Aerodynamic Drag Coefficient
v	Velocity	N	Degree of Polynomial
A	Area	B	Coefficient of Polynomial
G	Green's Function	ε	Fitting Error
$\mathbf{n}$	the normal to the surface	L'	Inductive Gradient
m	Mass	R_c	Contact Resistance

The thrust and friction forces acting on the armature are calculated using Equations (1) and (2):

$$F_t = \frac{1}{2} L' I^2 \quad (1)$$

$$F_f = 2\mu(F_{n0} + F_{ne}) + \frac{1}{2} C_d \rho_0 A_f v^2 \quad (2)$$

Here, F_{n0} represents the initial contact force; F_{ne} represents the contact force provided by the electromagnetic force; the term $0.5C_d\rho_0 A_f v^2$ represents the air resistance acting on the armature; ρ_0 is the density of the air.

Based on Equations (1) and (2), the armature velocity can be calculated in accordance with kinematic laws. The excitation current I can be directly measured via a Rogowski coil (as shown in Figure 1), and the measurement results are presented in Figure 2a. The abrupt change in the current waveform indicates that the armature has exited the barrel.

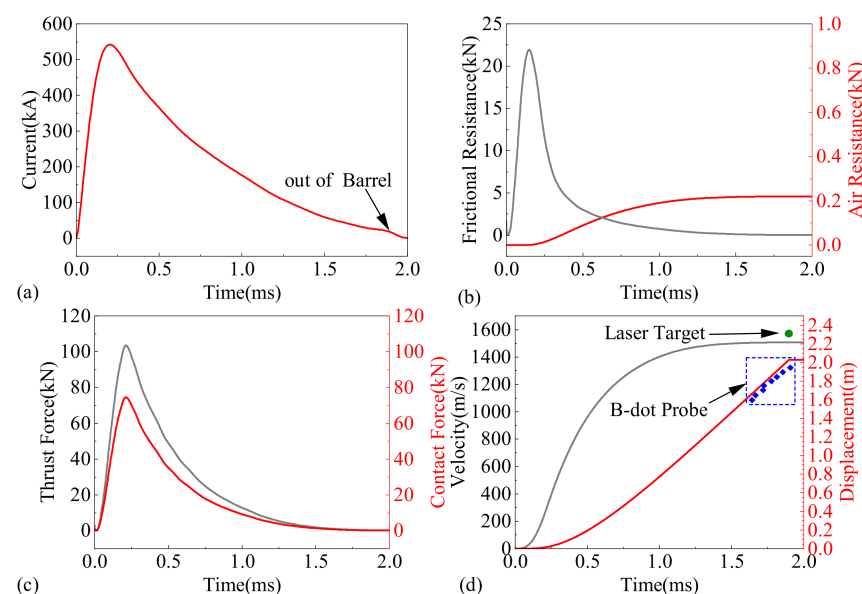


Figure 2. Simulation and experimental Results. (a) Excitation current measured in the experiment, (b) resistance on the armature, (c) thrust and contact force on the armature, (d) velocity and displacement of the armature.

The other two unknown parameters in Equations (1) and (2) are the inductance gradient and the friction coefficient. These parameters are difficult to measure directly but can be calibrated against the measured velocity data. Specifically, the inductance gradient is a constant determined solely by the structural configuration of the experimental setup. By polishing both the armature and the rails, the friction coefficient can be kept consistent across all tests. Therefore, these two parameters can be calibrated by conducting two tests at different current levels, with the results listed in Table 2.

Table 2. Parameters used in the calculation.

Symbol	Value	Unit	Symbol	Value	Unit
L'	0.46	[$\mu\text{H}/\text{m}$]	c_1	900	[J/(kg $\times$ K)]
μ_s	0.45	-	ρ_1	2700	[kg/m ³]
μ_d	0.0625	-	η_1	2.649×10^{-8}	[$\Omega \times \text{m}$]
m	34.72	[g]	k_2	400	[W/(m $\times$ K)]
θ	0.31	[rad]	c_2	385	[J/(kg $\times$ K)]
A	6.9×10^4	[m ²]	ρ_2	8960	[kg/m ³]
T_e	300	[K]	η_2	1.667×10^{-8}	[$\Omega \times \text{m}$]
k_1	238	[W/(m $\times$ K)]	H_c	5.22×10^7	[Pa]

In the experiments, the armature and rails were fabricated from aluminum and copper, respectively. For the simulations, the materials are assumed to be homogeneous and isotropic, and the detailed material parameters are provided in Table 2.

$$\mu = \mu_d + (\mu_s - \mu_d)e^{-t_r v} \quad (3)$$

Based on the forces acting on the armature, the time-dependent variations in the armature's acceleration, velocity, and displacement can be further determined. The calculation results are presented in Figure 2b–d.

In particular, Figure 2d also presents the measurement results obtained from the laser speed measurement target and the B-dot probes, plotted in green and blue, respectively. Good agreement is observed between the experimental and calculated results.

3. Solving of the Heat Partition Problem

3.1. Basic Assumptions

Figure 3a shows the heat sources at the armature–rail contact pair. The contact area is approximately rectangular, and the heat sources include frictional heat and Joule heat. Frictional heat is caused by the motion of the armature, while Joule heat is caused by the shrinkage resistance. Let the contact area of the armature–rail be A . Then, the total boundary heat source q at the contact surface is the sum of the Joule heat q_e and the frictional heat q_f , e.g., $q = q_f + q_e$, where

$$q_f = \mu p v(t) \quad (4)$$

$$q_e = \frac{I^2(t)R_c}{A} \quad (5)$$

As shown in Figure 3a, the contact surfaces are connected via discrete contact points, referred to as A-spots; therefore, current can only flow through the A-spots, which introduces an additional resistance known as the shrinkage resistance. Holm provided the following formula for calculating the contraction resistance:

$$R_c = \frac{\eta_1 + \eta_2}{2} c \left(\frac{H_c}{F_c} \right)^n \quad (6)$$

where F_c is the total contact force. c and n are constants, which are set to $c = 9.45 \times 10^{-4}$ and $n = 0.63$, respectively, based on the experimental results.

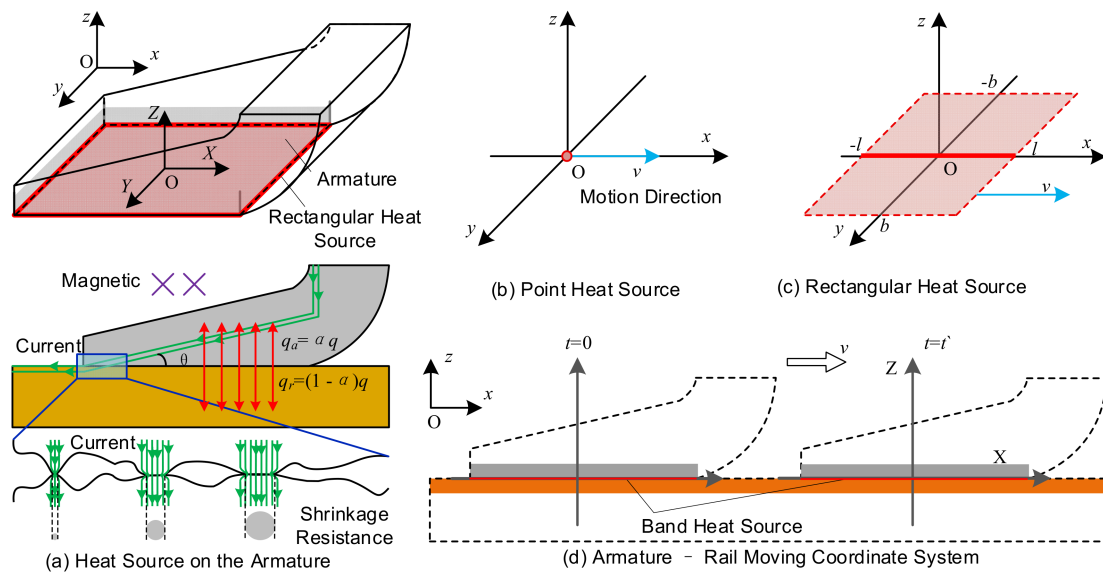


Figure 3. Schematic diagram of the armature–rail boundary heat source. (a) Heat source on the armature, (b) point heat source, (c) rectangular heat source, (d) armature–rail moving coordinate system.

3.2. Basic Solution for Temperature Field

As shown in Figure 3a, the heat generated by the boundary heat source q is partially transferred to the armature and partially to the rails. The heat partition coefficient α is defined as the proportion of the total boundary heat source that is transferred to the sliding component (armature); that is,

$$\alpha = \frac{q_a}{q_a + q_r} \quad (7)$$

Here, q_a and q_r represent the heat entering the armature and the rails, respectively.

When the boundary heat source is known, Green's function method can be used to solve for the temperature rise at the contact surface between the armature and rails. Consider the heat conduction governing equations under isotropic material conditions:

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T \quad (8)$$

Here, $\kappa = k/(\rho c)$ is the thermal diffusion coefficient.

For an infinite medium with an initial temperature of 0, where a heat source Q is present at point $\mathbf{r}'$ at time t' , the temperature generated at point $\mathbf{r}$ by this heat source at any time $t > 0$ is given by Equation (10), which is the solution to the inhomogeneous equation in Equation (9).

$$\frac{\partial G}{\partial t} - \kappa \nabla^2 G = Q \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \quad (9)$$

$$G(\mathbf{r}, t; \mathbf{r}', t') = \frac{Q}{8\rho c(\pi\kappa(t - t'))^{3/2}} e^{-|\mathbf{r} - \mathbf{r}'|^2 / 4\kappa(t - t')} \quad (10)$$

Here, δ is the Dirichlet function; Equation (10) is also known as Green's function of the heat conduction equation.

Based on the linearity of the heat conduction equation, integrating Equation (10)—the fundamental solution for an instantaneous point heat source—over time and space yields the temperature field generated by a heat source with an arbitrary distribution.

As shown in Figure 3b, when a point heat source located at the origin moves along the x -axis at a velocity v , its position can be expressed as

$$\mathbf{r}_0(\tau) = (\int_0^\tau v(s)ds, 0, 0) \quad (11)$$

If the point heat source continues to release heat during the time interval $0-t$, integrating Equation (10) with respect to time yields the temperature field generated by the moving point heat source:

$$T(\mathbf{r}, t) = \int_0^t \frac{q}{8\rho c[\pi\kappa(t-\tau)]^{3/2}} \exp\left[-\frac{(x - \int_0^\tau v(s)ds)^2 + y^2 + z^2}{4\kappa(t-\tau)}\right] d\tau \quad (12)$$

Furthermore, by integrating Equation (12) over the regions $x \in (-l, l)$ and $y \in (-b, b)$, we obtain the temperature rise caused by the rectangular heat source at the armature-track contact surface, as shown in Figure 3c. Considering only the temperature at the contact surface where $z = 0$, it can be expressed as:

$$T(x, y, t) = \frac{1}{4k\sqrt{\pi\kappa}} \int_0^t \frac{q(\tau)}{\sqrt{t-\tau}} \cdot I_x \cdot I_y d\tau \quad (13)$$

$$I_x = \int_{-l}^l \exp\left(-\frac{(x-x' - \int_\tau^t v(s)ds)^2}{4\kappa(t-\tau)}\right) dx' \quad (14)$$

$$I_y = \frac{1}{2\sqrt{\pi\kappa(t-\tau)}} \int_{-b}^b \exp\left(-\frac{(y-y')^2}{4\kappa(t-\tau)}\right) dy' \quad (15)$$

For Equation (13), setting $y = 0$ and considering only the temperature rise at the center of the heat source further simplifies the problem to a one-dimensional problem—specifically, the problem of heat diffusion and heat partition under one-dimensional conditions for railguns, as shown in Figure 2d. The initial and boundary conditions for this problem are set as follows:

$$T_1(X, Y, 0) = T_2(x, y, 0) = T_e \quad (16)$$

where T_1 is the temperature of the armature, and T_2 is the temperature of the rails. (X, Y) represents the moving coordinate with the armature, and (x, y) represents the fixed coordinate.

At infinity, far from the armature and rails, the temperature approaches ambient temperature; that is,

$$T_1 \rightarrow T_e \quad Z \rightarrow \infty \quad (17)$$

$$T_2 \rightarrow T_e \quad z \rightarrow -\infty \text{ or } |x| \rightarrow \infty \quad (18)$$

The thermal insulation boundary condition is applied to the region where the armature does not come into contact with the rails:

$$\frac{\partial T_1}{\partial \mathbf{n}} = \frac{\partial T_2}{\partial \mathbf{n}} = 0 \quad (19)$$

Since the dimensions of the armature and the rails are much larger than the thermal diffusion depth, they can be approximated as semi-infinite bodies. In this case, in a fixed coordinate system, the surface temperature of the rails can be expressed as:

$$T_2(x, t) = \frac{1}{2k_2\sqrt{\pi\kappa_2}} \int_0^t \frac{q(\tau)}{\sqrt{t-\tau}} \int_{-l}^l (1 - \alpha(x', \tau)) \exp\left(-\frac{(x - x' - \int_\tau^t v(s)ds)^2}{4\kappa_2(t-\tau)}\right) dx' d\tau \quad (20)$$

In the moving coordinate system (X, Y, Z) , the temperature on the armature surface is:

$$T_1(X, t) = \frac{1}{2k_1\sqrt{\pi\kappa_1}} \int_0^t \frac{q(\tau)}{\sqrt{t-\tau}} \int_{-l}^l \alpha(X', \tau) \exp\left(-\frac{(X - X')^2}{4\kappa_1(t-\tau)}\right) dX' d\tau \quad (21)$$

3.3. Control Equation for the Heat Partition Coefficient

Under ideal conditions, a contact pair should maintain temperature continuity across the interface during heat conduction. However, under actual operating conditions, rough surfaces only make contact via discrete asperity peaks, and heat is conducted through both the contacting asperities and the air trapped in the interfacial gaps. Accordingly, when rough surfaces are simplified as smooth surfaces, point-wise temperature continuity at the interface no longer corresponds to the real physical scenario. The simplification approach based on average temperature matching inherently satisfies the requirement of energy conservation at the macroscopic scale, ensures the continuity of total heat flux, and reduces the complex distributed heat partition coefficient to a scalar value. Under this assumption, the heat partition coefficient depends solely on time; that is, $\alpha(x, t) = \alpha(t)$. This method effectively reduces computational complexity, making it a reasonable approximation for engineering applications.

Let the average temperature at the contact surface be $\bar{T}_i(t) = \frac{1}{2l} \int_{-l}^l T_i(x, t) dx$; taking the spatial average of Equations (20) and (21) along the contact zone and setting $\bar{T}_1(t) = \bar{T}_2(t)$, we obtain

$$\int_0^t \frac{1}{\sqrt{t-\tau}} [K_1(t-\tau) + \beta K_2(\Delta x(\tau, t), t-\tau)] \alpha(\tau) d\tau = \beta \int_0^t \frac{1}{\sqrt{t-\tau}} K_2(\Delta x(\tau, t), t-\tau) d\tau \quad (22)$$

Here, $\beta = k_1\sqrt{\kappa_1}/k_2\sqrt{\kappa_2}$, and

$$K_1(\theta) = 2\sqrt{\pi\kappa_1\theta} \operatorname{erf}\left(\frac{l}{\sqrt{\kappa_1\theta}}\right) - \frac{2\kappa_1\theta}{l} \left[1 - \exp\left(-\frac{l^2}{\sqrt{\kappa_1\theta}}\right)\right] \quad (23)$$

$$K_2(\Delta, \theta) = \frac{\sqrt{\pi\kappa_2\theta}}{2l} \int_{-l}^l \left[\operatorname{erf}\left(\frac{x+l-\Delta}{\sqrt{4\kappa_2\theta}}\right) - \operatorname{erf}\left(\frac{x-l-\Delta}{\sqrt{4\kappa_2\theta}}\right) \right] dx \quad (24)$$

In this context, erf is the error function, and erfi is imaginary the error function.

Equation (21) is a first-kind Volterra integral equation. Reference [42] provides the analytical solution under uniform motion conditions:

$$\alpha(t) = \frac{1}{\pi} \int_0^\infty e^{-\frac{v_0 t}{2} r} \frac{\beta \sqrt{r} \tilde{F}(-r)}{\sqrt{\pi} r [1 + \beta^2 r \tilde{F}(-r)^2 / \pi]} dr \quad (25)$$

where $\tilde{F}(-r) = \sqrt{\pi}(1 + \frac{3}{2r})\operatorname{erfi}(\sqrt{r}) - \frac{3e^r}{\sqrt{r}}$.

3.4. Least-Squares Solution for the Heat Partition Coefficient

Reference [42] employed a high-Peclet-number simplification in deriving Equation (25). However, this simplification applies exclusively to steady-state conditions and fails to meet the requirements for calculating the heat partition coefficient in general sliding friction processes. When the

velocity $v(t)$ varies with time, Equation (22) cannot be reduced to an algebraic form, and no analytical solution can be obtained. Therefore, a least-squares fitting method is adopted to derive an approximate solution for the heat partition coefficient under time-varying velocity conditions.

Express the heat partition coefficient as a N th-degree polynomial in time:

$$\alpha(t) = \sum_{n=0}^N B_n t^n + \varepsilon(t) \quad (26)$$

Here, N is the degree of the polynomial; B_n is the coefficient of the polynomial; $\varepsilon(t)$ is the fitting error.

Substituting into Equation (22), we have

$$\begin{aligned} & \sum_{n=0}^N B_n \int_0^t \frac{\tau^n q(\tau)}{\sqrt{t-\tau}} [K_1(t-\tau) + \beta K_2(\Delta x(\tau, t), t-\tau)] d\tau + \\ & \int_0^t \frac{\varepsilon(\tau) q(\tau)}{\sqrt{t-\tau}} [K_1(t-\tau) + \beta K_2(\Delta x(\tau, t), t-\tau)] d\tau \\ & = \beta \int_0^t \frac{q(\tau)}{\sqrt{t-\tau}} K_2(\Delta x(\tau, t), t-\tau) d\tau \end{aligned} \quad (27)$$

Discretize the calculation range into M time points, and define the least-squares error as:

$$E = \sum_{i=0}^M \varepsilon^2(t_i) = \|\mathbf{LB} - \mathbf{R}\|^2 \quad (28)$$

where $\mathbf{B} = [B_0, B_1, \dots, B_N]^T$ is the vector of coefficients to be solved for, and $\mathbf{L}$ and $\mathbf{R}$ are the matrices on the left and right sides of the control equation, respectively.

Define the normalized least-squares error as a measure of goodness-of-fit:

$$\mathbf{E}^* = \sqrt{\frac{E}{\mathbf{R}^T \mathbf{R}}} = \sqrt{\frac{\varepsilon^T \varepsilon}{\mathbf{R}^T \mathbf{R}}} \quad (29)$$

Let the coefficient matrix $\mathbf{B}$ obtained from the solution be expressed as $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$, where each entry is denoted as B_{1n} and B_{2n} , respectively:

$$B_{1n} = \int_0^t \frac{\tau^n q(\tau)}{\sqrt{t-\tau}} K_1(t-\tau) d\tau \quad (30)$$

$$B_{2n} = \int_0^t \frac{\tau^n q(\tau)}{\sqrt{t-\tau}} \beta K_2(\Delta x(\tau, t), t-\tau) d\tau \quad (31)$$

The fitting results for the average temperatures at the armature–rail contact interface can be denoted as $\mathbf{T}_{1f} = \mathbf{B}_1 \mathbf{B}$ and $\mathbf{T}_{2f} = \mathbf{R} - \mathbf{B}_2 \mathbf{B}$, respectively. Each component of $\mathbf{T}_{1f}$ and $\mathbf{T}_{2f}$ represents the fitted value of the average temperature at the contact surface at a different point in time. Under ideal conditions, the true values of the average temperatures at the contact interface between the armature and the track are equal at all times, i.e., $T_1(t) = T_2(t)$. However, fitting errors may cause $T_{1f}(t)$ and $T_{2f}(t)$ to differ at certain times. The temperature matching error is defined as:

$$\Delta T_f(t) = \frac{|T_{1f}(t) - T_{2f}(t)|}{1/2 |\bar{T}_{1f} - \bar{T}_{2f}|} \quad (32)$$

where $\bar{T}_{1f} = (1/M) \sum_{k=1}^M T_{1f}(t)$ and $\bar{T}_{2f} = (1/M) \sum_{k=1}^M T_{2f}(t)$.

Figure 4 presents the normalized least-squares error and temperature matching error, as defined by Equations (29) and (32), calculated for various polynomial orders and constant velocities. To simplify the factors affecting the heat partition coefficient and provide a more intuitive physical interpretation, the following settings were adopted for the calculations: (1) Both the armature and the rails are made of aluminum. (2) The boundary heat source consists solely of frictional heat, and the friction coefficient is taken as a constant static friction coefficient; therefore, the heat source does not vary with time. (3) Material properties are assumed constant and are not affected by temperature changes.

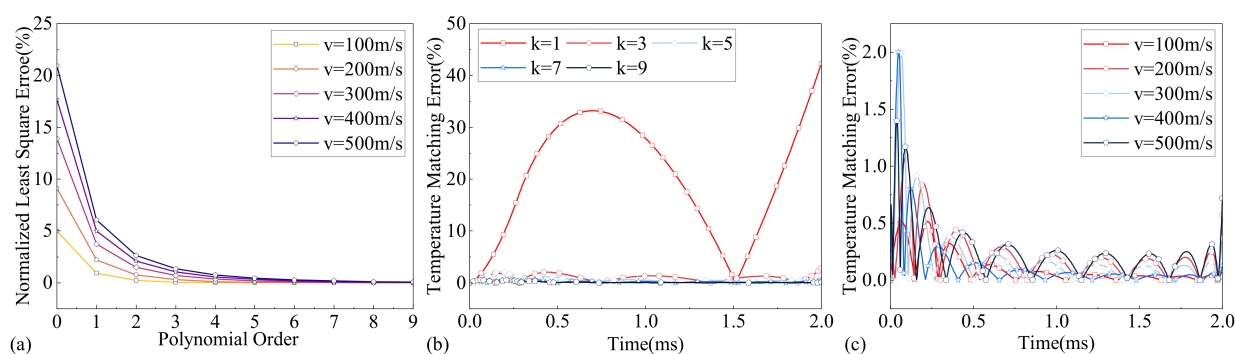


Figure 4. Error caused by the least-squares method. (a) Normalized least-squares error, (b) temperature matching error at different polynomial order, (c) temperature matching error at different velocity.

Specifically, the calculations for Figure 4b are performed at a constant velocity of $v = 100$ m/s, while those for Figure 4c adopt a polynomial order of $k = 9$. As illustrated in Figure 4a,b, both the normalized least-squares error and the temperature matching error decrease gradually with increasing fitting order. At a polynomial order of $k = 0$, the normalized least-squares error reaches up to 21%, and the maximum temperature matching error approaches 50%. When the polynomial order exceeds 6, the fitting error drops to nearly zero, indicating that the estimated heat partition coefficient is close to its true value. A comparison of Figure 4a,c also reveals that the accuracy of the least-squares fit is velocity dependent: higher velocities correspond to lower fitting accuracy. Furthermore, the temperature matching error peaks at the initial moment and then follows a fluctuating downward trend, which is consistent with the inherent nature of the fitting method.

Figure 5 presents the time evolution of the analytical solution and the least-squares solution for the heat partition coefficient at a constant velocity of $v = 100$ m/s, with the least-squares polynomial order set to 9. In these calculations, both the armature and the rails are made of aluminum; therefore, the initial value of the heat partition coefficient is 0.5.

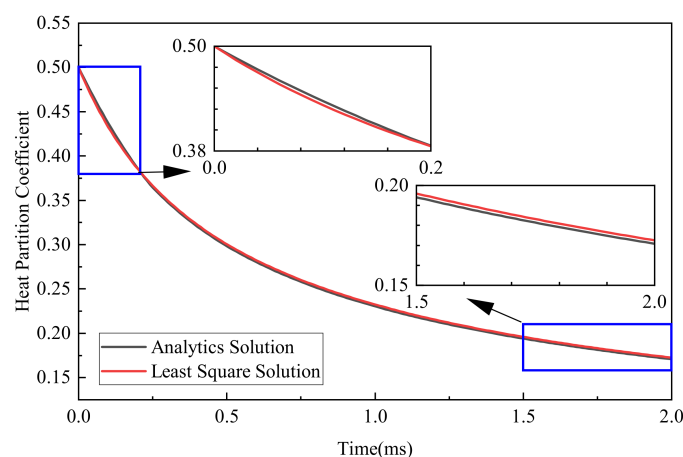


Figure 5. Comparison of analytics and least-squares solutions for the heat partition coefficient.

As shown in Figure 5, the initial values yielded by the analytical method and the least-squares method are consistent with the theoretical values, verifying the accuracy of the calculation results. Further analysis of Figure 5 indicates that the error between the analytical solution and the least-squares solution is pronounced in the initial stage (before 0.2 ms). This is because all fitting methods inherently exhibit notable errors at data boundaries, which is consistent with the temporal variation in the temperature matching error presented in Figure 4c. After 0.2 ms, the discrepancy between the analytical solution and the least-squares solution remains small. Nevertheless, as time elapses, the least-squares solution gradually exceeds the analytical solution, and the error increases progressively. This can be attributed to the cumulative effect of the temperature field: the temperature error calculated at the previous time step has to be compensated by the heat partition coefficient at the subsequent step, causing it to deviate from the true value. Furthermore, the heat partition coefficient represents the fraction of total heat flowing into the armature. As the armature undergoes continuous heating, the heat partition coefficient is expected to decrease over time to maintain temperature consistency between the armature and the rails, which is in line with the calculation results.

The foregoing analysis demonstrates that the heat partition coefficient results obtained via the least-squares method agree well with the analytical solution and conform to the physical interpretation of the heat partition coefficient, confirming the reliability of the proposed calculation method.

Figure 6 depicts the temporal variations in the heat partition coefficient as well as the surface temperatures of the armature and rails for an aluminum armature in contact with rails made of aluminum, copper, iron, and steel, respectively. For these calculations, the armature velocity is assumed to be constant at $v = 200$ m/s. The material properties adopted in the calculations are listed in Table 3.

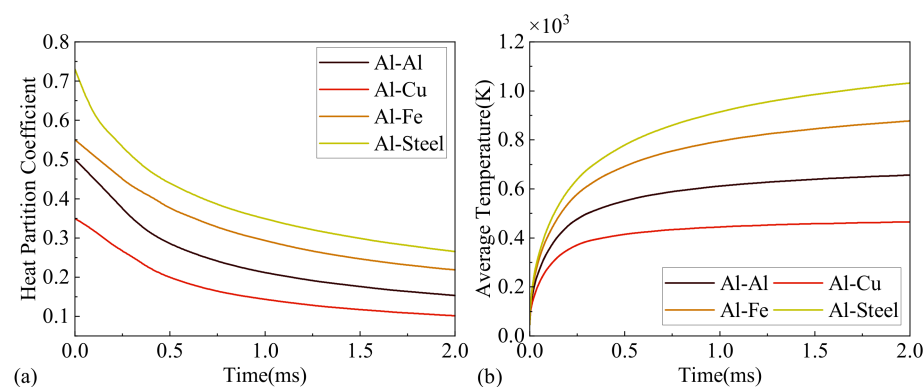


Figure 6. Calculation results for different material contact pairs under uniform motion. (a) Heat partition coefficient, (b) average temperature on the contact surface.

Table 3. Material parameters of the rails.

Material	Aluminum	Copper	Iron	Steel
Density/[kg/m ³]	2700	8960	7870	7900
Thermal Conductivity/[W/m × K]	238	400	76	44.5
Heat Capacity/[J/(kg × K)]	900	385	447	475
Initial Value of Heat Partition Coefficient	0.50	0.35	0.55	0.73

As illustrated in Figure 6, for rails made of steel, iron, aluminum, and copper respectively, both the heat partition coefficient and the rail surface temperature decrease sequentially in this order. The initial value of the heat partition coefficient is essentially consistent with the theoretical value, and it exhibits a decreasing trend over time with a

gradually diminishing rate of decline. The rail surface temperature rises with time at a gradually decelerating rate, which is consistent with the foregoing analysis.

These observations indicate that improving the rail's resistance to temperature rise by increasing the thermal conductivity, density, or specific heat capacity of the rail material helps alleviate severe temperature rise and phase transition during the operation of electromagnetic rail launchers. Meanwhile, when the rails are reused, aluminum from the melted armature deposited on the rail surface reduces the rail's resistance to temperature rise, exacerbating the temperature rise and consequently deteriorating the contact state and operating conditions of the railgun.

Figure 7 depicts the time-dependent variations in the calculated heat partition coefficient and the average contact surface temperature over a 3 s period at different constant velocities, assuming both the armature and the rails are made of aluminum. In the calculations, an identical heat flux density is adopted for all velocity conditions. As shown in Figure 7, when the operating duration is sufficiently long, both the heat partition coefficient and the average contact surface temperature gradually reach a steady state.

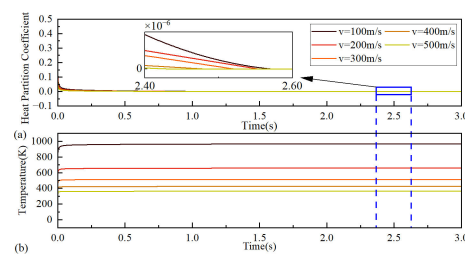


Figure 7. Calculation results within 3 s at different velocities. (a) Heat partition coefficient, (b) average temperature on the contact surface.

As shown in Figure 7a, when the constant velocity is 100 m/s, 200 m/s, 300 m/s, 400 m/s, and 500 m/s, the heat distribution coefficient decays to zero and stabilizes at 2570 ms, 2553 ms, 2520 ms, 2479 ms, and 2431 ms, respectively. This is consistent with the results of theoretical analysis and can be explained in terms of the physical significance of the heat partition coefficient: when the contact pair is in operation, the armature is continuously heated, while the rails in contact with the armature is heated only at the current moment; the heat source must heat the rail to the same temperature as the armature. At higher velocities, the length of rail traversed by the heat source within the same time interval increases, which is equivalent to a decrease in heat flux density; therefore, the heat partition coefficient must be reduced to lower the proportion of heat absorbed by the armature, thereby increasing the heat received by the rails. As the heat partition coefficient decreases to 0, the average temperature of the contact surface also ceases to rise and reaches an equilibrium temperature. Furthermore, the higher the velocity, the shorter the time required to reach equilibrium and the lower the temperature at equilibrium. At this point, the armature surface temperature remains constant, and the combined effect of the total heat flux density is precisely to heat the track to a temperature equal to the armature's average temperature, thereby achieving dynamic equilibrium.

Figure 8 presents the heat sources and friction coefficient at the armature–rail contact surface of an electromagnetic railgun under actual operating conditions. The parameters employed in the calculations are shown in Figure 3 and Table 2.

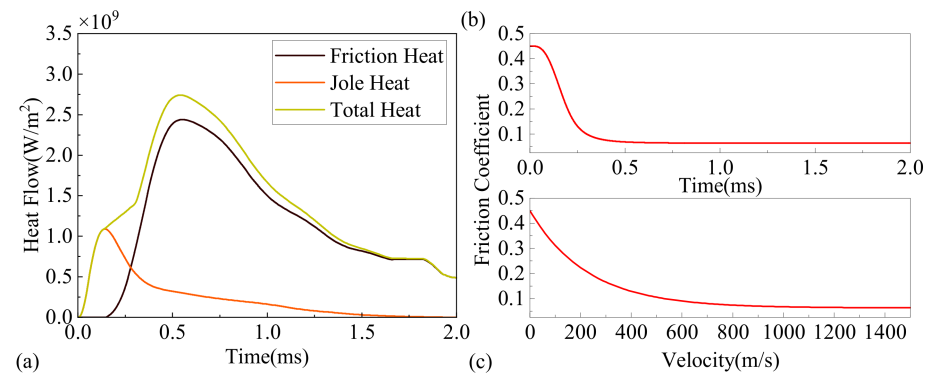


Figure 8. Parameters required for launching condition calculations. (a) Joule heat flow and friction heat flow versus time, (b) friction coefficient versus time, (c) friction coefficient versus velocity.

It can be observed that during the initial phase of railgun operation, Joule heating induced by contact resistance dominates the evolution of the boundary heat source. As time progresses, frictional heat becomes the primary component of the boundary heat source. Overall, the boundary heat source first increases and then decreases over a 2 ms period.

Figure 9 presents the temporal variations in the heat partition coefficient and the average contact surface temperature under actual operating conditions of the electromagnetic railgun. As shown in Figure 9a, the heat partition coefficient drops to zero within 0.85 ms, indicating that the entire boundary heat source is transferred to the rails and the armature no longer receives heat from the boundary heat source. At this point, the average temperatures of the armature and the rails can no longer remain equal, as illustrated in Figure 9b.

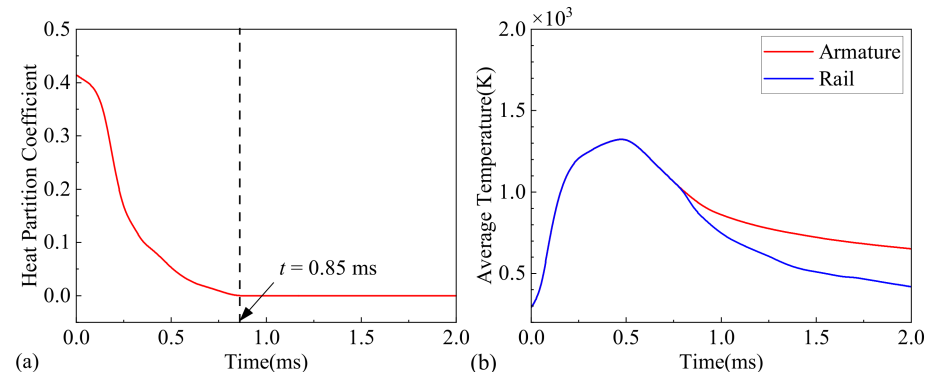


Figure 9. Calculation results for launching condition. (a) Heat partition coefficient, (b) average temperature on the contact surface.

It can be observed that after the heat partition coefficient falls to zero, the armature surface temperature continues to decrease, and the rail temperature in the curve also exhibits a declining trend with a higher cooling rate than that of the armature. Nevertheless, the temperature curves of the armature and rails describe distinct physical quantities. The rail temperature curve consistently represents the average temperature of the rail segment in contact with the armature. Accordingly, the reduction in rail temperature does not signify outward heat dissipation from the rail; instead, it implies a lower initial rail temperature or weaker thermal heating owing to a diminished heat source at the current time step. Once the heat partition coefficient reaches zero, the armature ceases to receive thermal input, and the entire armature is considered adiabatic. Hence, the mechanism behind the temperature drop differs between the armature and the rails. Driven by boundary heat sources, the internal temperature field of the armature is highly nonuniform, featuring

a substantial temperature gradient normal to the contact surface. A larger temperature gradient accelerates the trend toward thermal equilibrium. Therefore, the decline in the armature surface temperature demonstrates that heat gradually diffuses inward from the contact surface into the bulk of the armature.

4. Heat Partition Coefficient Considering the Conduction Heat Flux

For Equation (25), as $t \rightarrow \infty$, $\alpha_\infty := \lim_{t \rightarrow \infty} \alpha(t) = 0$. That is, the heat partition coefficient eventually converges to zero under constant velocity conditions, which indicates that the boundary heat source can always enable the rail to attain the same temperature as the armature. At this stage, the temperature rise induced by the heat source on the armature and rail achieves dynamic equilibrium with the surface temperature reduction arising from thermal diffusion within both components. Nevertheless, under railgun operating conditions or other scenarios featuring time-varying heat sources or velocities, boundary heat sources alone cannot continuously satisfy the thermal equilibrium requirement between the armature (slider) and the rail; conduction heat flux thus becomes a vital factor for sustaining thermal equilibrium. This arises because the operating characteristics of the armature–rail contact pair differ from those of conventional sliding contact pairs, featuring high relative sliding velocities, short operation durations, and intense heat fluxes. Additionally, Joule heating and dynamically varying contact pressure lead to distinctly different evolution patterns of heat source intensity and velocity.

Under the actual operating conditions of the sliding contact pairs, maintaining a continuous temperature between the slider and the rails is an inevitable requirement of Fourier's law of heat conduction and the second law of thermodynamics. When boundary heat sources cannot maintain equal temperatures at the contact surface, it is necessary to introduce a heat flux caused by the potential temperature difference at the contact surface. Let the conductive heat flux q_c represent the net heat flux flowing from the armature to the rails; then, the heat gained by the armature and rails contact surfaces is $\alpha q - q_c$ and $(1 - \alpha)q + q_c$, respectively. At this point, the average surface temperatures of the armature and rails are:

$$\bar{T}_1(t) = \frac{1}{2k_1\sqrt{\pi\kappa_1}} \int_0^t \frac{\alpha(\tau)q(\tau) - q_c}{\sqrt{t-\tau}} K_1(t-\tau) d\tau \quad (33)$$

$$\bar{T}_2(t) = \frac{1}{2k_2\sqrt{\pi\kappa_2}} \int_0^t \frac{(1-\alpha(\tau))q(\tau) + q_c}{\sqrt{t-\tau}} K_2(\Delta x(\tau, t), t-\tau) d\tau \quad (34)$$

Therefore, the control equation for the heat partition coefficient α is:

$$\begin{aligned} & \int_0^t \frac{q(\tau)}{\sqrt{t-\tau}} [K_1(t-\tau) + \beta K_2(\Delta x(\tau, t), t-\tau)] \alpha(\tau) d\tau \\ &= \int_0^t \frac{q_c(\tau)}{\sqrt{t-\tau}} [K_1(t-\tau) + \beta K_2(\Delta x(\tau, t), t-\tau)] d\tau + \int_0^t \frac{q(\tau)}{\sqrt{t-\tau}} \beta K_2(\Delta x(\tau, t), t-\tau) d\tau \end{aligned} \quad (35)$$

At this point, the control equation for the heat partition coefficient includes the boundary heat source q and the unknown conduction heat flux q_c , making it impossible to solve directly. Given that the relationship between q_c and α must be satisfied:

$$\begin{cases} \text{if } q_c > 0, \text{ then } \alpha = 0 \\ \text{if } q_c < 0, \text{ then } \alpha = 1 \\ \text{if } q_c = 0, \text{ then } 0 \leq \alpha \leq 1 \end{cases} \quad (36)$$

That is, when the heat partition coefficient falls within the interval $[0, 1]$, q_c is forced to be 0. In this case, the control equation for the heat distribution coefficient reduces to the control equation without conductive heat flux and can be solved directly using the method described in Section 3.3. When the heat distribution coefficient $\alpha = 0$, there exists $q_c > 0$.

In this case, the armature is adiabatic, and the rails must reach the same temperature as the armature under the combined effect of the known boundary heat source q and the conduction heat flux q_c to be solved. The case where the heat distribution coefficient $\alpha = 1$ follows the same logic, but this situation is impossible under the operating conditions of an electromagnetic railgun. Therefore, when the heat distribution coefficient $\alpha = 0$, the control equation for solving the conduction heat flux q_c is simplified to:

$$\begin{aligned} & \int_0^t \frac{1}{\sqrt{t-\tau}} [K_1(t-\tau) + \beta K_2(\Delta x(\tau, t), t-\tau)] q_c(\tau) d\tau \\ &= -\beta \int_0^t \frac{q(\tau)}{\sqrt{t-\tau}} K_2(\Delta x(\tau, t), t-\tau) d\tau \end{aligned} \quad (37)$$

Equation (37) is essentially identical in form to Equation (22) and is also solved using the least-squares method.

Figure 10a,b show, respectively, the heat partition coefficient and the corresponding time-dependent conduction heat flux under uniformly accelerated conditions, calculated using the least-squares method. In the figures, operating conditions 1 through 4 represent, respectively: operating condition 1 (velocity $v = 5 \times 10^5$ t m/s, heat flux density $q = 1 \times 10^9$ W/m²), operating condition 2 (velocity $v = 5 \times 10^5$ t m/s, heat flux density $q = 2 \times 10^9$ W/m²), condition 3 (velocity $v = 1 \times 10^6$ t m/s, heat flux density $q = 1 \times 10^9$ W/m²), and condition 4 (velocity $v = 1 \times 10^6$ t m/s, heat flux density $q = 2 \times 10^9$ W/m²).

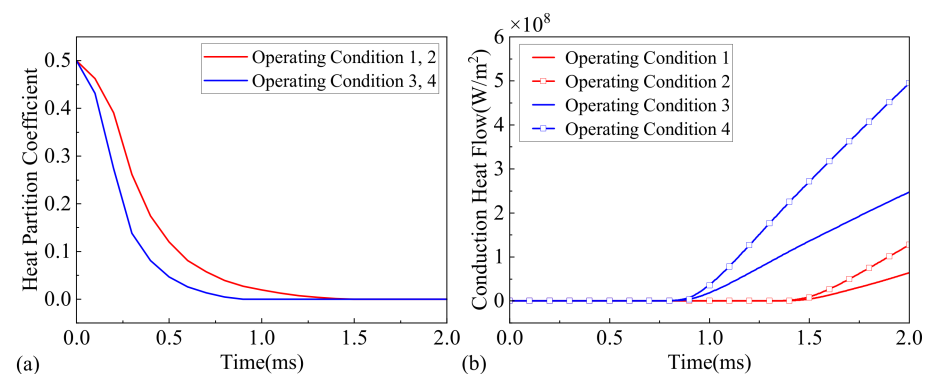


Figure 10. Calculation results including conduction heat flux under different operating conditions. (a) Heat partition coefficient, (b) conduction heat flow.

As illustrated in Figure 10a, the heat partition coefficient decays more rapidly with increasing armature velocity. Nevertheless, the boundary heat flux density exerts no influence on the heat partition coefficient when it remains time-invariant. Compared with Figure 7, the armature velocities in Figure 10 are higher (1000 m/s and 2000 m/s), so the heat partition coefficient decays to zero within 2 ms. By contrast, at lower velocities (below 500 m/s in the operating conditions of Figure 7), approximately ten times longer is required for the heat partition coefficient to drop to zero.

As presented in Figure 10b, a higher applied boundary heat flux density yields a larger conduction heat flux density. Moreover, the growth rate of the conduction heat flux density depends exclusively on the magnitude of the boundary heat source.

Figure 11a depicts the temporal evolution of the heat partition coefficient and conduction heat flux under railgun launch conditions with conduction heat flux considered. Figure 11b shows the average temperatures at the contact interface between the armature and the rails.

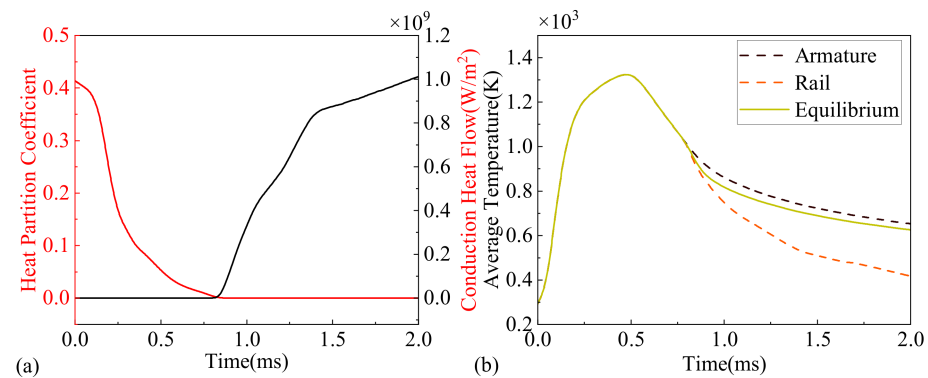


Figure 11. Calculation results including conduction under launching condition. (a) Heat partition coefficient and conduction heat flow, (b) average temperature on the contact surface.

It can be observed that during the operation of the electromagnetic railgun, the heat partition coefficient falls to zero within 0.8 ms, and a conduction heat flux up to 1×10^9 W/m² emerges within 2 ms. The growth rate of the conduction heat flux declines over time, which differs from the time-varying rising rate of the applied boundary heat source.

Figure 11b indicates that under launch conditions, the temperature at the armature–rail contact interface can reach nearly 1400 K. When conduction heat flux is incorporated into the model, the predicted temperature is lower than that obtained without considering this term. Nevertheless, the temporal evolution trend of contact surface temperature remains basically consistent regardless of whether conduction heat flux is included, exhibiting an initial rise followed by a decline, which matches the time-dependent variation in the boundary heat source.

5. Conclusions

In this paper, the heat partition coefficient of sliding contact pairs was calculated using the least-squares method under time-varying heat source and velocity conditions. The authors analyzed the non-physical phenomenon—where the heat partition coefficient is less than 0 or greater than 1—caused by the nonlinear characteristics of the heat source and velocity. They proposed and solved the governing equation for the heat partition coefficient that accounts for the conduction heat flux, thereby effectively resolving the limitation that traditional heat partition models cannot account for nonlinear operating conditions. The main conclusions are as follows:

- 1) Using Green's function method, the governing equation for the heat partition coefficient was derived without considering conduction heat flux, and it was solved using the least-squares method. This effectively addressed the limitation of analytical methods—which are only applicable under constant velocity conditions—and enabled the calculation of the heat partition coefficient under time-varying velocity conditions.
- 2) The causes of non-physical results in traditional heat partition models were analyzed, and a governing equation for the heat partition coefficient that accounts for conduction heat flux was proposed and solved. The results show that the heat partition model incorporating conduction heat flux eliminates non-physical results where the heat partition coefficient is less than 0 or greater than 1 and can accurately describe the heat partition process in an armature–rail system under time-varying heat source and velocity conditions.
- 3) The influence of factors such as motion velocity, material properties, and heat source characteristics on the heat partition coefficient was analyzed, and the variations in the heat partition coefficient and conduction heat flux under electromagnetic railgun launching conditions were calculated.

The findings of this paper provide a comprehensive description of the heat partition process in railguns and serve as a reference for thermal management in armature–rail systems. Additionally, these findings are of value to research on heat partition in other sliding contact pairs, such as brake disks, bearings, and rail transit systems.

Author Contributions: Conceptualization, X.D. and S.L.; methodology, X.D. and X.L.; software, X.D.; validation, X.D. and T.Z.; formal analysis, X.D.; investigation, X.D.; resources, S.L.; writing, X.D.; review, supervision and editing, S.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data that support the findings of this study are available from the author, [Xiangyu Du], upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

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