Article

Application of the Taguchi Method and Grey Relational Analysis for Multi-Objective Optimization of a Two-Stage Bevel Helical Gearbox

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Abstract: This paper introduces a novel approach to deal with the multi-objective optimization of a two-stage bevel helical gearbox by applying the Taguchi method and Grey Relation Analysis (GRA). The goal of the study is to find optimal main design factors that minimize the gearbox volume and maximize the gearbox efficiency. To accomplish this, five main design parameters were selected: the coefficients of wheel face width (CWFW) of the bevel and the helical gear sets, the allowable contact stresses (ACS) of the first and the second stages, and the gear ratio of the first stage. Furthermore, two single targets were investigated: minimum gearbox volumes, and maximum gearbox efficiency. Also, the multi-objective optimization problem is solved through two steps: Step 1 for closing the gap between variable levels and Step 2 for determining the optimal main design factors. The study’s findings were used to introduce the optimum values of five major design parameters for designing a two-stage helical gearbox.

Keywords: gearbox; two-stage bevel helical gearbox; gear ratio; multi-objective optimization; Taguchi method; grey relational analysis

1. Introduction

Bevel gearboxes are an essential component of vehicle and machine transmission systems. They are used to change the direction of motion (transmit motion to a perpendicular axis) as well as to reduce speed and increase torque from the motor to the working machine. For that reason, the optimal design of a bevel gearbox is a matter of interest to many scientists.

Until now, there have been several studies on the optimization of bevel gearboxes. J. Astol et al. [1] proposed a simulation model to reduce the transmission error of spiral bevel gears. Z. Zhang et al. [2] introduced optimal design parameters for a three-stage bevel helical gearbox to obtain a minimal gearbox volume. Optimal partial gear ratios were found for bevel helical gearboxes by different methods. F. Mendi et al. [3] have determined the optimal parameters of bevel gearboxes such as the module, shaft diameter and bearings, by using a genetic algorithm (GA). P. Eremeev et al. [4] introduced a work on both single- and multi-objective optimization of a bevel helical gearbox. In this study, three single targets including the total mass, the assemblability and the performance were investigated. Moreover, the multi-objective optimization problem was conducted with two single objectives: the performance and assemblability. The optimization problem to determine optimal partial gear ratios for bevel gearboxes has been conducted by many
scientists. Kudreavtev V.N. et al. [5] and T. Chat and L. V. Uyen [6] introduced the graph method for finding the gear ratio of the bevel gear set. Milou et al. [7] proposed a practical method in which the optimal partial gear ratios can be determined from practical data. The most useful method for determination of optimal gear ratios is the model method. This method was applied in many studies to find directly optimal gear ratios for different single targets, such as the minimal mass of gearbox [8], minimal gearbox volume [9], minimal gearbox length [10], minimal gearbox cross section area [11,12], minimal gearbox size [13] and minimal gearbox cost [14].

Single-objective optimization, which is an absolute optimization, determines the best or optimal level of the selected criterion. A multi-objective optimization problem has two or more simple objectives. As a consequence, the optimal solution to the multi-objective optimization problem cannot satisfy all criteria simultaneously. For example, it is not possible to meet the gearbox’s efficiency and cost demands. In simple terms, it is impossible to find a solution to a problem that is both “white” and “black”; only a “gray” solution can be found. The gray solution is the solution that comes in between the best and worst solutions, or “white” and “black” in the multi-objective optimization problem [15]. As a consequence, it is known as GRA optimization. To solve the single-objective problem, the original Taguchi method is used, and the Taguchi method and GRA are necessary to solve the multi-objective optimization problem.

According to the results of the preceding analysis, there have been numerous studies on the optimization of bevel helical gearboxes. The majority of these studies, however, are single-objective optimization. Furthermore, several methods for solving multi-objective optimization problems, such as the Pareto optimization method [4] and the discrete version of the Non-Dominated Sorting Genetical Algorithm II (NSGA-II) [7,16,17], have been used. However, no studies have been carried out to date that use the Taguchi method and GRA for multi-objective optimization of a gearbox. Furthermore, determining the optimal main design factors for these types of gearboxes has received insufficient attention. Additionally, previous research on multi-objective optimization for bevel helical gearboxes has failed to demonstrate a link between optimal input factors and total gearbox ratio. This is an important consideration when designing a new gearbox.

In this paper, we present a multi-objective optimization study for a two-stage bevel helical gearbox, with two single objectives in mind: maximizing gearbox efficiency and minimizing gearbox volume. The proposal of five optimal main design factors for the two-stage bevel helical gearbox is a key indication of this research. When the total gearbox ratio is known, these design factors are the key variables to consider when designing a two-stage helical gearbox. The CWFW and the ACS for both stages, and the gear ratio of the bevel gear set are among these elements. Furthermore, a novel approach to addressing the multi-objective optimization problem in gearbox design by combining the Taguchi method and the GRA in a previously unseen two-stage process to solve the problem was proposed. A relationship between optimal input factors and total gearbox ratio was also suggested.

2. Optimization Problem

2.1. Determination of Gearbox Volume

The calculation scheme for a two-stage bevel helical gearbox is shown in Figure 1. For purposes of simplicity, consider the gearbox to be a rectangular box with the dimensions $L$, $H$ and $B$. The gearbox volume ($m^3$)$V_{\text{gb}}$ is determined by (Figure 1):

$$V_{\text{gb}} = L \cdot B \cdot H$$  

(1)

in which $L$, $H$, $B$ are the length, height and width of the gearbox which can be calculated by:

$$L = 2 \cdot \frac{l_0}{3} + \frac{d_{e21}}{2} + \frac{d_{w12}}{2} + d_{w22} + 2 \cdot k$$  

(2)

$$H = \max (d_{e21}; d_{w22}) + 8.5 \cdot S_G$$  

(3)
where

\[ B = b \cdot \cos \delta_2 + b_w + 4 \cdot S_G \]  

(4)

\[ (+) k = 8 - 12 \text{ (mm)} [6] \text{ (mm)}; S_G \text{ is a design parameter which can be determined by [18]}:\]

\[ S_G = 0.005 \cdot L + 4.5 \]  

(5)

\[ L = 2 \cdot l_0/3 + d_{e21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot k \]  

(2)

\[ H = \max (d_{e21}; d_{w22}) + 8.5 \cdot S_G \]  

(3)

\[ B = b \cdot \cos \delta_2 + b_w + 4 \cdot S_G \]  

(4)

Figure 1. Calculated schema.

In (2), \( \delta_2 \) is gear pitch cone angle; \( \tan \delta_2 = u_1 \). \( l_0 = 3 \cdot d_{s1} \) is the first shaft diameter which is found by [6]:

\[ d_{s1} = \left[ T_{11} / (0.2 \cdot [\tau]) \right]^{1/3} \]  

(6)

(+) \( d_{e21} \) is the outer pitch diameter of the bevel gear (mm) which is found by [6]:

\[ d_{e21} = 2 \cdot u_1 \cdot R_c / \left(1 + u_1^2\right)^{1/2} \]  

(7)

in which \( R_c \) is the cone distance (mm) which can be calculated by [6]:

\[ R_c = k_R \cdot \sqrt{u_1^2 + 1} \cdot \sqrt{T_{11} \cdot k_{hp1} / \left[ (1 - k_{hp}) \cdot k_{hp1} \cdot u_1 \cdot [\sigma_H]^2 \right]} \]  

(8)
where \( k_R \) is coefficient; \( k_R = 50 \) (MPa) [6]; \( k_{HP} \) is the contacting load ratio for pitting resistance; \( k_{HP} = 1.04 - 1.18 \) [6] and it can be chosen as \( k_{HP} = 1.11 \); \( k_{be} = \frac{k}{R} = 0.25 - 0.3 \) is the coefficient of face width and it was chosen as \( k_{be} = 0.27 \); \( T_{12} \) is the torque on pinion of the first stage (N.mm); \( d_{e1} \) and \( d_{e2} \) are the outer pitch diameters of the pinion and the gear:

\( d_{w1} \) and \( d_{w2} \) are the pitch diameters of pinion and gear of the second stage which can be determined by [6]:

\[
d_{w12} = 2 \cdot a_w / (u_1 + 1)
\]

\[
d_{w22} = 2 \cdot a_w \cdot u_j / (u_j + 1)
\]

wherein \( a_w \) is the centre distance of the second stage which can be determined by [2]:

\[
a_w = k_a \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{HP} / (\sigma_{H2}^2 \cdot u_2 \cdot X_{ba})}
\]

where \( T_{12} \) is the torque on the pinion of the second stage (N.mm); \( X_{ba} = 0.25 - 0.4 \) is the wheel face width coefficient of the helical gear stage.

2.2. Determination of Gearbox Efficiency

The gearbox efficiency can be calculated by:

\[
\eta_{gb} = \frac{100 \cdot P_l}{P_m}
\]

in which \( P_l \) is the total gearbox power loss [19]:

\[
P_l = P_{lg} + P_{lb} + P_{ls}
\]

where \( P_{lg} \) is the gear power losses in all gears; \( P_{lb} \) is the bearing power loss; and \( P_{ls} \) is the seal power loss. These elements can be found as follows:

(+) The power losses in the gears:

\[
P_{lg} = \sum_{i=1}^{2} P_{lg_i}
\]

where \( P_{lg_i} \) is the gear power losses of \( i \) stage which is found by:

\[
P_{lg_i} = P_{gi} \cdot (1 - \eta_{gi})
\]

in which \( \eta_{gi} \) is the efficiency of the \( i \) stage of the gearbox which is calculated by [19]:

\[
\eta_{gi} = 1 - \left( \frac{1 + 1 / u_i}{\beta_{ai} + \beta_{ri}} \right) \frac{f_i}{2} \left( \beta_{ai}^2 + \beta_{ri}^2 \right)
\]

where \( u_i \) is the gear ratio of \( i \) stage; \( f \) is the friction coefficient; \( \beta_{ai} \) and \( \beta_{ri} \) are arc of approach and recess on \( i \) stage which are calculated by [20]:

(+) For the bevel gear stage:

\[
\beta_{ai} = \frac{(R_{e1i}^2 - R_{e2i}^2)^{1/2} - R_{2i} \cdot \sin \alpha}{R_{01i}}
\]

\[
\beta_{ri} = \frac{(R_{e1i}^2 - R_{02i}^2)^{1/2} - R_{1i} \cdot \sin \alpha}{R_{01i}}
\]
in which \( R_{aev1} \) and \( R_{aev2} \) are outside radii of equivalent pinion and gear, respectively; \( R_{v1} \) and \( R_{v2} \) are pitch radii of equivalent pinion and gear, respectively; \( R_{0v1} \) and \( R_{0v2} \) are base radii of equivalent pinion and gear, respectively; \( \alpha \) is pressure angle.

\[
R_{v1} = R_1 / \cos \delta_1
\]

\[
R_{v2} = R_2 / \cos \delta_2
\]

wherein \( R_1 \) and \( R_2 \) are pitch radii of bevel pinion and gear at large end, respectively; \( \delta_1 \) and \( \delta_2 \) are pitch angles of bevel pinion and gears, respectively.

\[
R_{aev1} = R_{v1} + a_p
\]

\[
R_{aev2} = R_{v2} + a_g
\]

(+) For the helical gear stage:

\[
\beta_{ai} = \left( R_{e2i} - R_{02i} \right)^{1/2} - R_{2i} \cdot \sin \alpha \cdot R_{01i}
\]

\[
\beta_{ri} = \left( R_{e1i} - R_{01i} \right)^{1/2} - R_{1i} \cdot \sin \alpha \cdot R_{01i}
\]

where \( R_{e1i} \) and \( R_{e2i} \) are outside radii of pinion and gear, respectively; \( R_{1i} \) and \( R_{2i} \) are pitch radii of pinion and gear, respectively; \( R_{01i} \) and \( R_{01i} \) are base-circle radii of pinion and gear, respectively; \( \alpha \) is pressure angle.

From the data in [20], the following regression Equations were found to find the friction coefficient:

- When the sliding velocity \( v \leq 0.424 \) (m/s) (with \( R^2 = 0.9958 \)):

\[
f = -0.0877 \cdot v + 0.0525
\]

- When the sliding velocity \( v > 0.424 \) (m/s) (with \( R^2 = 0.9796 \)):

\[
f = 0.0028 \cdot v + 0.0104
\]

(+) The bearing power losses [19]:

The power losses in rolling bearings is determined by:

\[
P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i
\]

in which \( f_b \) is the friction coefficient of bearing; as the radical ball bearings with angular contact were used, \( f_b = 0.0011 \) [19]; \( F \) is the bearing load (N); \( v \) is the peripheral speed; and \( i \) is the ordinal number of the bearing (\( i \) equals 1 to 6).

(+) The total seal power losses can be found by [19]:

\[
P_s = \sum_{i=1}^{2} P_{si}
\]

where \( i \) is the ordinal number of seals (\( i = 1 - 2 \)); \( P_{si} \) is the power loss caused by sealing for a single seal (\( w \)), which is determined by:

\[
P_{si} = 7.69 \cdot 10^{-6} \cdot d_{si}^2 \cdot n_i
\]

in which \( d_{si} \) and \( n_i \) are the diameter and the shaft speed \( i \)th.
2.3. Objective Functions and Constraints

2.3.1. Objectives Functions

The multi-objective optimization problem in this study comprises two single objectives:

- Minimizing the gearbox volume:
  \[ \text{min} f_1(X) = V_{gb} \]  
  (30)

- Maximizing the gearbox efficiency:
  \[ \text{max} f_2(X) = \eta_{gb} \]  
  (31)

in which \( X \) is the design variable vector reflecting variables. In this work, five main design factors including \( u_1, X_{ba_1}, X_{ba_2}, AS_1, AS_2 \) were selected as variables and we have:

\[ X = \{ u_1, k_{be}, X_{ba}, AS_1, AS_2 \} \]  
(32)

2.3.2. Constraints

The maximum gear ratio for a bevel gear set is 6 [6]. Furthermore, the CWFW of a bevel gear set ranges from 0.25 to 0.3, while that of a helical gear set ranges from 0.25 to 0.4 [6]. Furthermore, the gear materials used in this work are steel 40, 45, 40X, and 35XM refining, with surface teeth hardness of 350 HB (these are the most commonly used gear materials in gearboxes). The calculated results show that the allowable contact stresses for the first and second stages range from 350 to 420 (MPa). As a result, the following constraints were derived from these comments:

\[ 1 \leq u_1 \leq 6 \quad \text{and} \quad 1 \leq u_2 \leq 9 \]  
(33)

\[ 0.25 \leq k_{be} \leq 0.3 \quad \text{and} \quad 0.25 \leq v \leq 0.4 \]  
(34)

\[ 350 \leq AS_1 \leq 420 \quad \text{and} \quad 350 \leq AS_2 \leq 420 \]  
(35)

3. Methodology

As was mentioned in Section 2.3, the variables for the multi-objective optimization problem are five major design factors. Table 1 lists these factors as well as their minimum and maximum values. The most common input first pinion speed of 1480 (rpm) was chosen. Steel 40, 45, 40X and 35XM refining gear materials were used in this work, with teeth hardness on the surface of 350 HB (these are the most commonly used gear materials). Steel 45 was also chosen as the shaft material because it is a common shaft material. The Taguchi method and GRA are used in this study to deal with a multi-objective optimization problem with five variables. The greater the number of variable levels, the easier it is to determine the solution of the optimization problem. To maximize the number of levels per variable using the Taguchi method, the design L25 (5\(^5\)) was chosen (for the scenario with five variables). However, variable \( u_1 \) has the widest distribution of the selected variables (\( u_1 \) ranges from 1 to 6, as described in Section 2.3.2). As a consequence, even with five orders, the distance between each level of this variable remains quite considerable (in this case, the distance between the levels is \((6 - 1)/4 = 1.5\)). A multi-objective optimization problem overcoming procedure has been proposed to reduce this gap while also saving time and improving solution accuracy (Figure 2). This procedure is divided into two stages: the first stage solves a single-objective optimization problem to close the gap between variable orders, and the second solves an optimization problem with multiple objectives to identify optimal main design elements.
Table 1. Input factors and their ranges.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Notation</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear ratio of first stage</td>
<td>$u_1$</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>CWFW of bevel gear set</td>
<td>$X_{ba1}$</td>
<td>0.25</td>
<td>0.3</td>
</tr>
<tr>
<td>CWFW of helical gear set</td>
<td>$X_{ba2}$</td>
<td>0.25</td>
<td>0.4</td>
</tr>
<tr>
<td>ACS of stage 1 (MPa)</td>
<td>$A_{S1}$</td>
<td>350</td>
<td>420</td>
</tr>
<tr>
<td>ACS of stage 2 (MPa)</td>
<td>$A_{S2}$</td>
<td>350</td>
<td>420</td>
</tr>
</tbody>
</table>

4. Single-Objective Optimization

The total gearbox ratio of a two-stage bevel helical gearbox can be calculated using $u_t = u_1 \cdot u_2$. As a result, for a given total gearbox ratio (for example, $u_t = 20$), only one optimal partial gear ratio ($u_1$ or $u_2$) must be determined. The bevel gear set’s gear ratio is chosen as a variable in this work, and the task of this problem is to find its optimal value for a given $u_t$ value. The direct search method is used to solve the single-objective optimization problem in this work. In addition, based on the objective functions (30) and (31) and the constraints (33)–(35), a computer program written in Matlab was created to solve two single-objective problems: minimizing gearbox volume and maximizing gearbox efficiency. Figure 3 depicts the relationship between the optimal gear ratio of the first stage $u_1$ and the total gearbox ratio $u_t$ based on the outcomes of this program. Moreover, new constraints for the variable $u_1$ have been identified as shown in Table 2.

Table 2. New constraints of $u_1$.

<table>
<thead>
<tr>
<th>$u_t$</th>
<th>$u_1$ Lower Bound</th>
<th>$u_1$ Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>−2.24</td>
<td>3.39</td>
</tr>
<tr>
<td>15</td>
<td>2.24</td>
<td>4.44</td>
</tr>
<tr>
<td>20</td>
<td>2.24</td>
<td>5.53</td>
</tr>
<tr>
<td>25</td>
<td>2.77</td>
<td>6</td>
</tr>
<tr>
<td>30</td>
<td>3.32</td>
<td>6</td>
</tr>
<tr>
<td>35</td>
<td>3.88</td>
<td>6</td>
</tr>
</tbody>
</table>
Figure 3. Optimal gear ratio of the first stage versus total gearbox ratio.

5. Multi-Objective Optimization

The goal of this research’s multi-objective optimization problem is to determine the optimal main design parameters that satisfy two single-objective functions: minimizing gearbox volume and maximizing gearbox efficiency in the design of a two-stage helical gearbox with a specific total gearbox ratio. A simulation experiment was carried out to address this issue. The Taguchi method was used to design the experiment, and Minitab R18 software was used to analyze the results. Furthermore, as previously stated, the design L25 (5^5) was selected to obtain the highest levels of variable. To carry out these experiments, a computer program was created. An investigation was carried out to reduce programming complexity by defining the orthogonal matrix of the Taguchi simulation experiment at each specified total gearbox ratio. The total gearbox ratios studied were 10, 15, 20, 25, 30 and 35. A total of 25 simulation experiments were performed using a 5-level Taguchi design (L25) for each total gearbox ratio mentioned above. Table 3 describes the main design factors and their levels, while Table 4 shows the experimental plan and the corresponding output findings, which include gearbox volume and efficiency for a total gearbox ratio of 20.

Table 3. Main design factors and their levels for $u_1 = 20$.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Notation</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Level 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear ratio of first stage</td>
<td>$u_1$</td>
<td>2.24</td>
<td>3.065</td>
<td>3.89</td>
<td>4.715</td>
<td>5.53</td>
</tr>
<tr>
<td>CWFW of stage 1</td>
<td>$X_{ba1}$</td>
<td>0.25</td>
<td>0.2625</td>
<td>0.275</td>
<td>0.2875</td>
<td>0.3</td>
</tr>
<tr>
<td>CWFW of stage 2</td>
<td>$X_{ba2}$</td>
<td>0.25</td>
<td>0.2875</td>
<td>0.325</td>
<td>0.3625</td>
<td>0.4</td>
</tr>
<tr>
<td>ACS of stage 1 (MPa)</td>
<td>$A_{S1}$</td>
<td>350</td>
<td>368</td>
<td>386</td>
<td>404</td>
<td>420</td>
</tr>
<tr>
<td>ACS of stage 2 (MPa)</td>
<td>$A_{S2}$</td>
<td>350</td>
<td>368</td>
<td>386</td>
<td>404</td>
<td>420</td>
</tr>
</tbody>
</table>
Table 4. Experimental matrix and output responses for \( u_t = 20 \).

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>( u_t )</th>
<th>( k_{bat} )</th>
<th>( X_{bat} )</th>
<th>( AS_1 )</th>
<th>( AS_2 )</th>
<th>( V_{gb} ) (kg)</th>
<th>( \eta_{gb} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.240</td>
<td>0.2500</td>
<td>0.2500</td>
<td>350</td>
<td>350</td>
<td>4.963</td>
<td>94.906</td>
</tr>
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<td>0.2875</td>
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<tr>
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<td>386</td>
<td>386</td>
<td>4.055</td>
<td>94.655</td>
</tr>
<tr>
<td>4</td>
<td>2.240</td>
<td>0.2875</td>
<td>0.3625</td>
<td>404</td>
<td>404</td>
<td>3.725</td>
<td>94.617</td>
</tr>
<tr>
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<td>0.3000</td>
<td>0.4000</td>
<td>420</td>
<td>420</td>
<td>3.468</td>
<td>94.548</td>
</tr>
<tr>
<td>6</td>
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<td>0.2500</td>
<td>0.2875</td>
<td>386</td>
<td>404</td>
<td>3.603</td>
<td>94.657</td>
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<tr>
<td>7</td>
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<td>0.3250</td>
<td>404</td>
<td>420</td>
<td>3.320</td>
<td>94.620</td>
</tr>
<tr>
<td>8</td>
<td>3.065</td>
<td>0.2750</td>
<td>0.3625</td>
<td>420</td>
<td>350</td>
<td>3.850</td>
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</tr>
<tr>
<td>9</td>
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<td>0.3000</td>
<td>0.4000</td>
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</tr>
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<td>386</td>
<td>404</td>
<td>3.511</td>
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</tr>
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<td>0.3250</td>
<td>404</td>
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<td>13</td>
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<td>0.2750</td>
<td>0.3625</td>
<td>420</td>
<td>368</td>
<td>3.085</td>
<td>94.512</td>
</tr>
<tr>
<td>14</td>
<td>3.890</td>
<td>0.2875</td>
<td>0.4000</td>
<td>386</td>
<td>420</td>
<td>3.391</td>
<td>94.561</td>
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<td>0.3000</td>
<td>0.2875</td>
<td>404</td>
<td>350</td>
<td>3.865</td>
<td>94.578</td>
</tr>
<tr>
<td>16</td>
<td>4.715</td>
<td>0.2500</td>
<td>0.3625</td>
<td>350</td>
<td>368</td>
<td>2.930</td>
<td>94.514</td>
</tr>
<tr>
<td>17</td>
<td>4.715</td>
<td>0.2625</td>
<td>0.4000</td>
<td>386</td>
<td>350</td>
<td>3.364</td>
<td>94.560</td>
</tr>
<tr>
<td>18</td>
<td>4.715</td>
<td>0.2750</td>
<td>0.2500</td>
<td>404</td>
<td>368</td>
<td>3.653</td>
<td>94.515</td>
</tr>
<tr>
<td>19</td>
<td>4.715</td>
<td>0.2875</td>
<td>0.2875</td>
<td>420</td>
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<td>94.426</td>
</tr>
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</tr>
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</tr>
<tr>
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<td>420</td>
<td>404</td>
<td>3.206</td>
<td>94.461</td>
</tr>
<tr>
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<td>0.2750</td>
<td>0.2875</td>
<td>350</td>
<td>420</td>
<td>3.351</td>
<td>94.554</td>
</tr>
<tr>
<td>24</td>
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<td>0.2875</td>
<td>0.3250</td>
<td>368</td>
<td>350</td>
<td>3.502</td>
<td>94.474</td>
</tr>
<tr>
<td>25</td>
<td>5.530</td>
<td>0.3000</td>
<td>0.3625</td>
<td>386</td>
<td>368</td>
<td>3.312</td>
<td>94.463</td>
</tr>
</tbody>
</table>

In this work, the Taguchi and GRA methods are used to solve the multi-optimization optimization problem. The following are the primary stages in the procedure:

(+): Finding the signal-to-noise ratio (S/N) using the following Equations, with the goal of reducing gearbox volume and enhancing gearbox efficiency:

- To reduce the gearbox volume, the-smaller-is-the-better S/N:

\[
SN = -10\log_{10}\left(\frac{1}{m}\sum_{i=1}^{m} y_i^2\right)
\]  

(36)

- To increase the gearbox efficiency, the-larger-is-the-better S/N:

\[
SN = -10\log_{10}\left(\frac{1}{m}\sum_{i=1}^{m} \frac{1}{y_i^2}\right)
\]  

(37)

In which \( y_i \) is the outcome response value, and \( m \) is the number of experimental repetitions; in this case, \( m = 1 \) because the experiment is a simulation, and no repetition is required.

Table 5 displays the calculated \( S/N \) for the two mentioned targets of output.
Table 5. Values of S/N of each run of experiment for $u_t = 20$.

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Main Design Factors</th>
<th>$V_{gb}$</th>
<th>$\eta_{gb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_1$</td>
<td>$k_{be}$</td>
<td>$X_{ka}$</td>
</tr>
<tr>
<td>1</td>
<td>2.3400</td>
<td>0.2500</td>
<td>0.2500</td>
</tr>
<tr>
<td>2</td>
<td>2.3400</td>
<td>0.2625</td>
<td>0.2875</td>
</tr>
<tr>
<td>3</td>
<td>2.3400</td>
<td>0.2750</td>
<td>0.3250</td>
</tr>
<tr>
<td>4</td>
<td>2.3400</td>
<td>0.2875</td>
<td>0.3625</td>
</tr>
<tr>
<td>5</td>
<td>2.3400</td>
<td>0.3000</td>
<td>0.4000</td>
</tr>
<tr>
<td>6</td>
<td>3.1375</td>
<td>0.2500</td>
<td>0.2875</td>
</tr>
<tr>
<td>7</td>
<td>3.1375</td>
<td>0.2625</td>
<td>0.3250</td>
</tr>
<tr>
<td>8</td>
<td>3.1375</td>
<td>0.2750</td>
<td>0.3625</td>
</tr>
<tr>
<td>9</td>
<td>3.1375</td>
<td>0.2875</td>
<td>0.4000</td>
</tr>
<tr>
<td>10</td>
<td>3.1375</td>
<td>0.3000</td>
<td>0.2500</td>
</tr>
<tr>
<td>11</td>
<td>3.9350</td>
<td>0.2500</td>
<td>0.3250</td>
</tr>
<tr>
<td>12</td>
<td>3.9350</td>
<td>0.2625</td>
<td>0.3625</td>
</tr>
<tr>
<td>13</td>
<td>3.9350</td>
<td>0.2750</td>
<td>0.4000</td>
</tr>
<tr>
<td>14</td>
<td>3.9350</td>
<td>0.2875</td>
<td>0.4000</td>
</tr>
<tr>
<td>15</td>
<td>3.9350</td>
<td>0.3000</td>
<td>0.2500</td>
</tr>
<tr>
<td>16</td>
<td>4.7325</td>
<td>0.2500</td>
<td>0.3250</td>
</tr>
<tr>
<td>17</td>
<td>4.7325</td>
<td>0.2625</td>
<td>0.3625</td>
</tr>
<tr>
<td>18</td>
<td>4.7325</td>
<td>0.2750</td>
<td>0.4000</td>
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<tr>
<td>19</td>
<td>4.7325</td>
<td>0.2875</td>
<td>0.4000</td>
</tr>
<tr>
<td>20</td>
<td>4.7325</td>
<td>0.3000</td>
<td>0.3250</td>
</tr>
<tr>
<td>21</td>
<td>5.5300</td>
<td>0.2500</td>
<td>0.4000</td>
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<tr>
<td>22</td>
<td>5.5300</td>
<td>0.2625</td>
<td>0.3250</td>
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<tr>
<td>23</td>
<td>5.5300</td>
<td>0.2750</td>
<td>0.2500</td>
</tr>
<tr>
<td>24</td>
<td>5.5300</td>
<td>0.2875</td>
<td>0.2875</td>
</tr>
<tr>
<td>25</td>
<td>5.5300</td>
<td>0.3000</td>
<td>0.3625</td>
</tr>
</tbody>
</table>

In reality, the data of the two single objective functions under consideration have different dimensions. To guarantee that they are comparable, the data need to be normalized, or brought to a standardized scale. The data are normalized using the normalization value $Z_{ij}$, which ranges from 0 to 1. The following formula is used to calculate this value:

$$Z_i = \frac{SN_i - \min(SN_i, j = 1, 2, \ldots, n)}{\max(SN_i, j = 1, 2, \ldots, n) - \min(SN_i, j = 1, 2, \ldots, n)}$$  \hspace{1cm} (38)

where $n = 25$ is the experimental number.

(*) Determining the grey relational coefficient:

The grey relational coefficient can be found by the following Equation:

$$y_i(k) = \frac{\Delta_{\min} + \zeta \Delta_{\max}(k)}{\Delta_i(k) + \zeta \Delta_{\max}(k)}$$  \hspace{1cm} (39)

with $i = 1, 2, \ldots, n$.

In (37), $k = 2$ is the objective number; $\Delta_i(k)$ is the absolute value; $\Delta_i(k) = ||Z_0(k) - Z_i(k)||$, where $Z_0(k)$ and $Z_i(k)$ are the reference and the specific comparison sequences, respectively;
\( \Delta_{\text{min}} \) and \( \Delta_{\text{max}} \) are the min and max values of \( \Delta_i(k) \); \( \zeta \) is the characteristic coefficient, \( 0 \leq \zeta \leq 1 \); in this work \( \zeta = 0.5 \).

(+1) Determining the mean of the grey relational coefficient:

The mean of the grey relation coefficients associated with the output objectives is used to calculate the degree of grey relation.

\[
\bar{y}_i = \frac{1}{k} \sum_{j=0}^{k-1} y_{ij}(k) \tag{40}
\]

in which \( y_{ij} \) is the grey relation value of the \( j \)th output targets in the \( i \)th experiment.

Table 6 shows the calculated grey relation value \( (y_i) \) and average grey relation value of all tests.

<table>
<thead>
<tr>
<th>Exp. No</th>
<th>S/N</th>
<th>( Z_i )</th>
<th>( \Delta_i(k) )</th>
<th>Grey Relation Value ( y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( V_{gb} )</td>
<td>( \eta_{gb} )</td>
<td>Reference Value</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-13.9149</td>
<td>39.5459</td>
<td>0.0000 1.0000</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
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<td>39.5352</td>
<td>0.2036 0.7785</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-12.1598</td>
<td>39.5229</td>
<td>0.3834 0.5244</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
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<td>39.5194</td>
<td>0.5445 0.4523</td>
</tr>
<tr>
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<td>5</td>
<td>-10.8016</td>
<td>39.5130</td>
<td>0.6801 0.3213</td>
</tr>
<tr>
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<td>6</td>
<td>-11.1333</td>
<td>39.5231</td>
<td>0.6077 0.5282</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>-10.4228</td>
<td>39.5197</td>
<td>0.7629 0.4580</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>-11.7092</td>
<td>39.5168</td>
<td>0.4818 0.3991</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>-11.2029</td>
<td>39.5266</td>
<td>0.5925 0.6022</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>-12.0086</td>
<td>39.5305</td>
<td>0.4165 0.6818</td>
</tr>
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<td>12</td>
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<td>0.7583 0.4447</td>
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<tr>
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<td>13</td>
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<td>0.9022 0.2529</td>
</tr>
<tr>
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<td>14</td>
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<td>0.7227 0.3460</td>
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<td>15</td>
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<td>0.4745 0.3783</td>
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<td>1.0000 0.2567</td>
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<td>18</td>
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<td>19</td>
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<td>39.5018</td>
<td>0.7617 0.0894</td>
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<td>0.8560 0.3612</td>
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<td>39.4975</td>
<td>0.8802 0.0000</td>
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<td>0.8292 0.1560</td>
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<td>0.7452 0.3327</td>
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<td>24</td>
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<td>39.5062</td>
<td>0.6616 0.1807</td>
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<tr>
<td>25</td>
<td>25</td>
<td>-10.4018</td>
<td>39.5052</td>
<td>0.7675 0.1598</td>
</tr>
</tbody>
</table>

A higher average grey relation value is preferable to ensure harmony among the output parameters. As a consequence, the multi-objective problem’s objective function is
able to be transformed into a single-objective optimization problem, with the mean grey relation value as the outcomes.

The ANOVA method was used to examine the impact of the main design variables on the average grey relation value ($\bar{y}$), and the results are shown in Table 7. According to the results in Table 7, $AS_1$ has the greatest influence on $y$ (34.64%), followed by $k_{be}$ (24.33%), $u_1$ (15.54%), $AS_2$ (9.11%) and $X_{ba}$ (3.27%). Table 8 describes the order of influence of main design factors on $y$ using ANOVA analysis.

Table 7. Factor effect on $\bar{y}$.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
<th>C (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>4</td>
<td>0.003675</td>
<td>0.003675</td>
<td>0.000919</td>
<td>0.29</td>
<td>0.870</td>
<td>4.50</td>
</tr>
<tr>
<td>$k_{be}$</td>
<td>4</td>
<td>0.021359</td>
<td>0.021359</td>
<td>0.005340</td>
<td>1.70</td>
<td>0.311</td>
<td>26.18</td>
</tr>
<tr>
<td>$X_{ba}$</td>
<td>4</td>
<td>0.006127</td>
<td>0.006127</td>
<td>0.001532</td>
<td>0.49</td>
<td>0.749</td>
<td>7.51</td>
</tr>
<tr>
<td>$AS_1$</td>
<td>4</td>
<td>0.025666</td>
<td>0.025666</td>
<td>0.006416</td>
<td>2.04</td>
<td>0.254</td>
<td>31.45</td>
</tr>
<tr>
<td>$AS_2$</td>
<td>4</td>
<td>0.012182</td>
<td>0.012182</td>
<td>0.003046</td>
<td>0.97</td>
<td>0.512</td>
<td>14.93</td>
</tr>
<tr>
<td>Residual Error</td>
<td>4</td>
<td>0.012587</td>
<td>0.012587</td>
<td>0.003147</td>
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<tr>
<td>Total</td>
<td>24</td>
<td>0.081596</td>
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</table>

Model Summary

<table>
<thead>
<tr>
<th>S</th>
<th>R-Sq</th>
<th>R-Sq(adj)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0561</td>
<td>84.57%</td>
<td>7.45%</td>
</tr>
</tbody>
</table>

Table 8. Order of main design factor effect on $\bar{y}$.

<table>
<thead>
<tr>
<th>Level</th>
<th>$u_1$</th>
<th>$k_{be}$</th>
<th>$X_{ba}$</th>
<th>$AS_1$</th>
<th>$AS_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5407</td>
<td>0.5949</td>
<td>0.5547</td>
<td>0.5895</td>
<td>0.5276</td>
</tr>
<tr>
<td>2</td>
<td>0.5359</td>
<td>0.5591</td>
<td>0.5209</td>
<td>0.5766</td>
<td>0.5188</td>
</tr>
<tr>
<td>3</td>
<td>0.5394</td>
<td>0.5181</td>
<td>0.5308</td>
<td>0.5256</td>
<td>0.5335</td>
</tr>
<tr>
<td>4</td>
<td>0.5685</td>
<td>0.5192</td>
<td>0.5552</td>
<td>0.5179</td>
<td>0.5645</td>
</tr>
<tr>
<td>5</td>
<td>0.5379</td>
<td>0.5311</td>
<td>0.5608</td>
<td>0.5127</td>
<td>0.5762</td>
</tr>
<tr>
<td>Delta</td>
<td>0.0326</td>
<td>0.0768</td>
<td>0.0399</td>
<td>0.0768</td>
<td>0.0574</td>
</tr>
<tr>
<td>Rank</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Average of grey analysis value: 0.544

(+ ) Finding optimal main design factors:

The rational (or optimal) parameter set would theoretically be the set of main design factors with the highest $S/N$ values. As a result, the effect of the main design elements on the $S/N$ ratio was determined (Figure 4). The optimal levels and values of the main design factors for the multi-objective function were also described in Figure 4 (the red dogs) and Table 9.

Table 9. Optimal levels and main design factor values.

<table>
<thead>
<tr>
<th>No.</th>
<th>Input Parameters</th>
<th>Code</th>
<th>Optimum Level</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Total gearbox ratio</td>
<td>$u_1$</td>
<td>4</td>
<td>4.175</td>
</tr>
<tr>
<td>2</td>
<td>CWF of bevel gear set</td>
<td>$k_{be}$</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>CWF of helical gear set</td>
<td>$X_{ba}$</td>
<td>5</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>ACS of stage 1 (MPa)</td>
<td>$AS_1$</td>
<td>1</td>
<td>350</td>
</tr>
<tr>
<td>5</td>
<td>ACS of stage 2 (MPa)</td>
<td>$AS_2$</td>
<td>5</td>
<td>420</td>
</tr>
</tbody>
</table>
(+ ) Evaluation of experimental model:
The Anderson–Darling method is used to assess the satisfactory of the proposed model, and its findings are shown in Figure 5. The data points corresponding to the experimental observations (represented by blue dots) fall within the region bounded by upper and lower limits with a 95% standard deviation, displayed in the graph. Furthermore, the p-value of 0.129 exceeds the significance level of \( \alpha = 0.05 \) significantly. These findings suggest that the proposed model is appropriate for designing a two-stage bevel helical gearbox.

**Figure 4.** Influence of main design factors on S/N ratios of \( \eta \).

**Figure 5.** Probability plot of \( \eta \).

Maintaining from the previous discussion, Table 10 shows the optimal values for the main design factors corresponding to the remaining \( u_t \) values of 10, 20, 25, 30, and 35.
Optimum values of main design factors.

<table>
<thead>
<tr>
<th>No.</th>
<th>$u_1$</th>
<th>$X_{y_{u1}}$</th>
<th>$K_{y_{u}}$</th>
<th>$AS_1$</th>
<th>$AS_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.24</td>
<td>0.25</td>
<td>0.4</td>
<td>350</td>
<td>420</td>
</tr>
<tr>
<td>15</td>
<td>3.77</td>
<td>0.25</td>
<td>0.4</td>
<td>350</td>
<td>420</td>
</tr>
<tr>
<td>20</td>
<td>4.715</td>
<td>0.25</td>
<td>0.4</td>
<td>350</td>
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<td>25</td>
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<td>30</td>
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<td>35</td>
<td>6</td>
<td>0.25</td>
<td>0.4</td>
<td>350</td>
<td>420</td>
</tr>
</tbody>
</table>

Based on the data in Table 10, the following conclusions are given:

The best values for $X_{y_{u1}}$ are the largest (0.4). This can be explained as follows: Because of the slow speed of the helical stage, the torque on the pinion is large. As a result, in order to reduce $dw_{22}$ (Formula (11)), the coefficient $X_{y_{u1}}$ must be maximized (line 119). In contrast to $X_{y_{u1}}$, the values of $K_{y_{u}}$ are at their lowest. Because the bevel stage is high speed, the torque on the pinion is small, and thus $de_{21}$ is small. The cross-section of the gearbox must be small in order to have a small volume. As a result, the diameters $de_{21}$ and $dw_{22}$ should be the same [21]. So $K_{y_{u}}$ is set to the smallest possible value of 0.25 (line 109) in order to increase $Re$ (Formula (8)) and thus increase $de_{21}$ (Formula (7)). Similarly, the optimal values for $AS_1$ are the lowest values, and the optimal values for $AS_2$ are the highest values. It is also explained as above: to reduce the size of $dw_{22}$ and increase $de_{21}$ while keeping them roughly equal to achieve minimal gearbox volume, $AS_2$ must take the maximum value and $AS_1$ must take the minimum value.

Figure 6 shows a clear first-order relationship between the optimal values of $u_1$ and $u_t$. In addition, the following regression equation (with $R^2 = 0.9923$) was discovered to find the optimal values of $u_1$ when $u_t \leq 25$:

$$u_1 = 0.2445 \cdot u_t - 0.0975$$  \hspace{1cm} (41)

Following the discovery of $u_1$, the optimum value of $u_2$ is easily determined by $u_2 = u_t / u_1$. 

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**Figure 6.** Optimal gear ratio of the bevel gear set versus total gearbox ratio.
6. Conclusions

In this paper, the Taguchi method and the GRA are used to solve the multi-objective optimization problem of designing a two-stage bevel helical gearbox. This study’s goal is to identify the best main design factors for maximizing gearbox efficiency while minimizing gearbox volume. Five major design factors were chosen to accomplish this: the CWFW for the bevel and helical gear sets, the ACS for the first and second stages, and the first-stage gear ratio. Furthermore, the multi-objective optimization problem is solved in two steps. The first phase is concerned with resolving the single-objective optimization problem of closing the gap between variable levels, while the second phase is concerned with determining the best main design factors. As a result of this work, the following conclusions were proposed:

- By combining the Taguchi method and the GRA in a two-stage process, a novel approach to dealing with the multi-objective optimization problem in gear-box design was presented. As a result of this approach, the distance between the values of the lower and upper bounds of the constants of \( u_1 \) is reduced, making it easier to determine optimal values.

- Solving the single-objective optimization problem bridges the gap between variable levels, making it easier to solve the multi-objective optimization problem. The proposed solution to the single-objective optimization problem bridges the gap between variable levels, establishing the multi-objective optimization problem solution easier.

- The study’s findings suggested optimal values for the five main design factors in the design of a two-stage bevel helical gear gearbox (Equation (39) and Table 10).

- The ANOVA method was used to examine the effect of the main design parameters on \( y \). \( AS_1 \) has the greatest influence on \( y \) (34.64%), followed by \( k_{be} \) (24.33%), \( u_1 \) (15.54%), \( AS_2 \) (9.11%), and \( X_{ba} \) (3.27%), according to the findings.

- The proposed \( u_1 \) model has a high level of consistency with experimental data, validating its reliability. This model can be used effectively for multi-objective optimization of a two-stage bevel helical gearbox, which is a useful application.

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