

Article

Hyperbolic-Secant-Function-Based Fast Sliding Mode Control for Pantograph Robots

Muhammad Ali Hassan, Zhenwei Cao *  and Zhihong Man 

Department of Telecommunications, Electrical, Robotics and Biomedical Engineering, Swinburne University of Technology, Hawthorn 3122, Australia; muhammadalihassan@swin.edu.au (M.A.H.); zman@swin.edu.au (Z.M.)

* Correspondence: zcao@swin.edu.au

Abstract: Technological advancements in control systems are indispensable and require a robust controller with fast convergence in numerous sectors like robotics, space engineering, etc. A sliding mode controller (SMC) is a robust controller when subjected to nonlinearities, external disturbances, and uncertainties in system dynamics. However, SMC is prone to chattering, which can damage the plant in long term and results in low tracking accuracy. In this research paper, a novel hyperbolic secant function (HSF)-based SMC is proposed that achieves fast convergence, reduces chattering, and improves tracking performance. Stability analysis is conducted based on Lyapunov stability criteria. Comparative simulations and experiments on a pantograph robot (PR) show the effectiveness of the proposed algorithm.

Keywords: sliding mode controller; hyperbolic secant function; end-effector; position control; pantograph robot



Citation: Hassan, M.A.; Cao, Z.; Man, Z. Hyperbolic-Secant-Function-Based Fast Sliding Mode Control for Pantograph Robots. *Machines* **2023**, *11*, 941. <https://doi.org/10.3390/machines11100941>

Academic Editor: Zheng Chen

Received: 28 August 2023

Revised: 26 September 2023

Accepted: 26 September 2023

Published: 3 October 2023



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1. Introduction

Robotic manipulators are considered a much-needed tool in the current era of industrial revolution. These manipulators are used for various purposes like cutting, mounting, transporting, and welding in numerous sectors where automatic precise machine operations are required. Pantograph robots are common for these type of industrial applications. A mechanical structure of four links connected in the form of a parallelogram is called a pantograph, where the motion of joint angles control the end-effector's desired position, as shown in Figure 1. The end-effector's accurate position control is required for a desired trajectory tracking task [1]. Forward, as well as inverse kinematics, along with the dynamics of a servo motor need to be derived for the accurate control of the end-effector of a pantograph robot (PR). Therefore, the end-effector position control of robotic manipulators has gained much attention from researchers [2].

Linear-quadratic regulators (LQRs) as well as proportional-integral-derivative (PID) controllers are commonly utilized by a combination of various intelligent control algorithms for these manipulators. These intelligent algorithms are specifically used for tuning the gains of LQR and PID [3,4], but the disadvantages of these basic algorithms lie in the key fact that these are unable to handle modeling errors or plant uncertainties. In [4], a genetic algorithm (GA) is combined with a PID controller to achieve the desired tracking, but it does not cope with the unmodeled system dynamics and parameters uncertainties. Moreover, the tuning of gains in PID controllers needs to be performed optimally. LQRs and PID controllers have been used for controlling the position of end-effectors of PRs, and particle swarm optimization (PSO) is also used for tuning purposes [5], but it lacks in tracking performance in the case of uncertainties. However, the introduction of optimization tools like GA, PSO, and Quantum PSO always enhances the complexity of a system, which eventually results in more overshoot with large steady-state error and even slow settling time with more iterations [5–7]. To address these issues and the disadvantages of

the aforementioned algorithms, the SMC is considered to be a robust controller in such a scenario.

Sliding mode was designed in the era of the Soviet Union in the early 1950s [8]. In the case of model uncertainties, the robustness of SMC is a prominent feature [9]. SMC importance was neglected at that time because of chattering problems. However, the main benefits were considered, such as, for instance, the dynamic response of a plant in a closed-loop domain with a switching function makes the system robust against uncertainties. Therefore, researchers had a keen interest to investigate the pros and cons of SMCs. Currently, SMCs have been utilized as robust controllers in different fields pertaining to motion control, especially for precise tracking like in robotics, agriculture, automation and control manufacturing units, energy and mining industry, power electronics, and aerospace engineering [10].

The main reason behind the selection of SMCs is due to their robust properties against nonlinearities, external disturbances, and unknown model dynamics, as well as their reliability for multi-input multioutput (MIMO) systems. But, in spite of their many advantages, one of the prominent drawbacks of SMCs is chattering, which could damage the hardware during long runtimes. Another drawback of SMCs is slow convergence, as there is always a trade-off between faster convergence and chattering phenomena alongside better tracking performance, which eventually introduces large oscillations into systems. Chattering induces heat loss in electronic circuits, wear and tear of mechanical components, and results in low accuracy [11]. Although a fast SMC for a PR is proposed in [12], it is prone to chattering and not feasible for real-time applications. To address the chattering issues, a second-order SMC was developed, and then the higher-order SMC received much attention [13–15]. Then, disturbance-observer-based techniques were designed alongside SMCs with a feed-forward control in [16] to cope with chattering. Similarly, in [17], a disturbance-observer-based SMC was designed. Moreover, the boundary-layer strategy was also introduced in order to address discontinuity in the layer, but its drawback included that the ideal sliding mode was lost and eventually resulted in the cost of robustness [18]. To control the reaching process directly, as nonideal reaching in the reaching phase could also induce chattering, [19] proposed the reaching law strategy. Firstly, the constant reaching law was proposed, then the constant as well as proportional reaching law alongside the power reaching law were presented and extended to discrete-time systems [20,21]. Further modifications of these laws were also introduced [22–24]. Likewise, in [25], a continuous terminal SMC was proposed, but initial condition sensitivities led to large control inputs. Furthermore, in [26], an adaptive SMC based on barrier function was proposed for a linear motor. An integral-based fast terminal SMC was proposed in [27] for a permanent magnet synchronous motor.

An adaptive SMC based on proportional integral observer was proposed for a synchronous motor [28]. Recently, a pantograph control for high-speed trains based on rapid adaptation with deep meta-reinforcement learning was proposed. For transmitting electrical energy, a pantograph–catenary system is the vital component of the train, mounted at the top, which requires the robust control of a pantograph such that by the speed variation it optimally maintains contact and improves the current collection quality [29]. As discussed earlier, the introduction of intelligent optimization tools increases system complexity, needs more computational resources, and requires more iterations for optimal learning, which eventually makes the system slower. A robust control which requires less computational power, is simple in design, and feasible for real-time systems is much-needed. For nonlinear chaotic systems, another adaptive fuzzy fractional control based on a supervised SMC is proposed to deal with uncertainties, but it is prone to slower convergence and overall lack in robustness for trajectory tracking [30].

Therefore, issues in regards to chattering, tracking performance, and faster convergence are still a huge challenge for researchers. Thus, this article contributes with an emphasis on the reduction in chattering, such that it should be feasible for real-time systems, along with faster convergence, stabilization, and improved tracking performance.

Firstly, the novelty of the proposed algorithm includes a hyperbolic secant function, which is introduced in the power of a sliding surface such that it significantly reduces chattering. Secondly, a thorough stability analysis is carried out which guarantees the stability of the system. Lastly, the validation on hardware demonstrates the superiority of the proposed algorithm, and simulation studies verify the robustness of the proposed algorithm over the current existing state-of-the-art algorithms.

This article is organized such that Section 1 discuss the introduction. Section 2 consists of the mathematical modeling of the PR in which the servo motor transfer function is derived along with robot links kinematics, i.e., forward and inverse kinematics. In Section 3, the proposed control structure along with the stability analysis is explained briefly. Next, Section 4 includes the comparative simulation studies that are carried out in MATLAB/Simulink 2021a by considering a PR model. In Section 5, experimental results are outlined by considering three different cases for the PR with thorough discussion and implications, which show the effectiveness of the proposed algorithm. Finally, the conclusion is provided at the end.

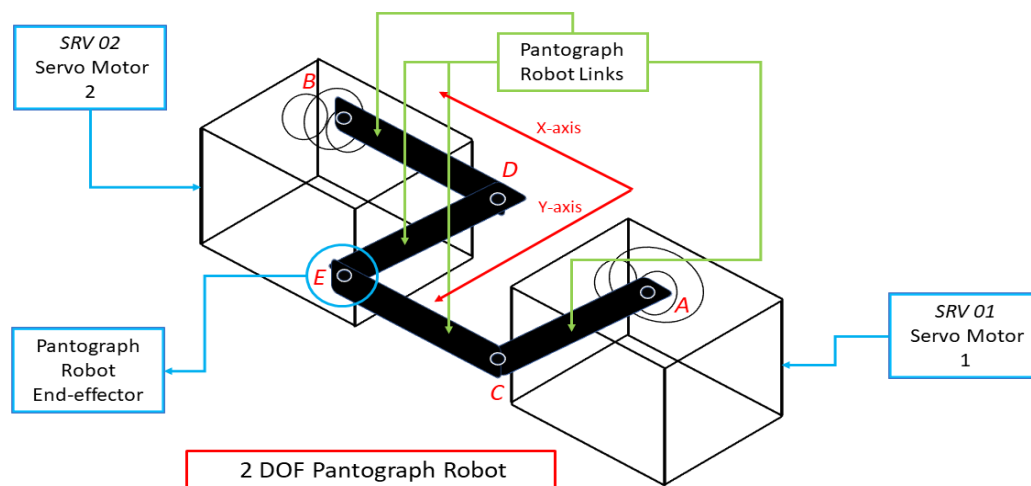


Figure 1. Pantograph robot.

2. Mathematical Modeling of the Pantograph Robot

The PR consists of two servo motors on which four links are attached, with each link being 5 inches in length. Two joints are actuated and three joints of the links are not actuated. As shown in Figure 1, two of the links are mounted on SRV01 and SRV02 on A and B points, respectively, and the control objective is to manipulate the end-effector position of the PR by controlling the coordinates of point E. In this section, mathematical modeling of the PR is carried out. Firstly, the transfer function and the state equation of the servo motor is given, followed by the inverse and forward kinematics of the PR links. Although the kinematics of the PR links are derived in [12], this paper presents much more details and thorough explanations.

2.1. Servo Motor SRV01

The servo motor consists of a linear or rotary actuator which is coupled with a motor and an encoder. It works in such a way that helps to accurately control the position or speed of a plant, and it is commonly used in robotics. The angular rate of the servo motor load shaft to input voltage ratio is represented by the following transfer function:

$$\frac{\Omega_m(s)}{U_m(s)} = \left(\frac{K_m}{T_ms + 1} \right), \quad (1)$$

where Ω_m is the Laplace transform of ω_m , which is the angular rate of the servo motor, and U_m is the Laplace transform of control input u_m , respectively. T_m is the time constant, K_m is the gain, and s is the Laplace operator. T_m and K_m are the nominal model parameters. The value of $T_m = 0.0254$ (s) and $K_m = 1.53$ (rad/sV). These nominal model parameters can be experimentally found by the bump test or by using frequency response. Equation (1) can be illustrated as follows:

$$\Omega_m(s)(T_ms + 1) = K_m U_m(s), \quad (2)$$

now, by the inverse Laplace transform, Equation (2) is simplified as follows:

$$\dot{\omega}_m(t)T_m + \omega_m(t) = K_mu_m(t), \quad (3)$$

where $\omega_m(t)$ is the angular rate of the servo motor, i.e., $\omega_m(t) = \dot{\theta}_m(t)$, substituting $\dot{\theta}_m(t)$ into (3) yields

$$\ddot{\theta}_m(t) = -\frac{1}{T_m}\dot{\theta}_m(t) + \frac{K_m}{T_m}u_m(t). \quad (4)$$

The state vector is $\bar{x} = [x_1 = \theta_m, x_2 = \dot{\theta}_m]^T$, and the state equation is expressed as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = r(x, t) + gu_m(t) + d(t), \end{cases} \quad (5)$$

In Equation (5), $r(x, t) = -\frac{1}{T_m}\dot{\theta}_m$, $g = \frac{1}{T_m}K_m$, and $d(t)$ are considered as external disturbances.

The state-space representation is given as follows,

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu_m(t) \\ y(t) = Cx(t), \end{cases} \quad (6)$$

where the state vector $x(t) = [\theta_m, \dot{\theta}_m]^T$, $A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T_m} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ K_m \frac{1}{T_m} \end{bmatrix}$, and $C = [1 \ 0]$.

2.2. Forward Kinematics

In forward kinematics, the end-effector coordinates along the X–Y axis need to be calculated, while θ_1 and θ_2 can be directly measured by the encoders of both servo motors, i.e., SRV01 and SRV02, respectively, as shown in Figure 2.

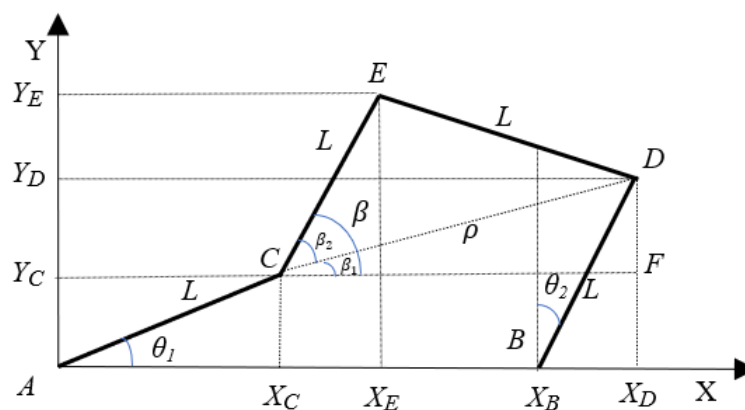


Figure 2. Forward kinematics for the four links of the pantograph robot [12].

Firstly, point C coordinates along the X–Y axis, i.e., X_C and Y_C are calculated and represented as follows:

$$X_C = L \cos \theta_1, \cos \theta_1 = \frac{X_C}{L}, \quad (7)$$

$$Y_C = L \sin \theta_1, \sin \theta_1 = \frac{Y_C}{L}, \quad (8)$$

where $L = 5$ (inch) represents the length of each link of the PR. Angular positions θ_1 and θ_2 along the axis of rotation are given in anticlockwise and clockwise, respectively. X_B is known, i.e., $X_B = 2L$. Thus, point D components along the X–Y axis are written as follows:

$$X_D = X_B - L \cos(90 - \theta_2) = X_B - L \sin \theta_2, \quad (9)$$

$$Y_D = L \cos \theta_2, \quad (10)$$

now, ρ needs to be calculated, which is the distance between point C and D. By applying the Pythagorean theorem on the triangle CDE, ρ is derived as follows:

$$\rho = \sqrt{(Y_D - Y_C)^2 + (X_D - X_C)^2}. \quad (11)$$

To find the end-effector, entitled coordinate E; the β angle shown in Figure 2 needs to be computed by the following equation:

$$\beta = \beta_2 + \beta_1. \quad (12)$$

Next, by considering a bisector from point E on line CD such that it divides the length ρ into two equal lengths, i.e., $\rho/2$ for each part, then β_2 and β_1 are calculated as follows:

$$\cos \beta_2 = \frac{\frac{\rho}{2}}{L}, \beta_2 = \arccos\left(\frac{\rho}{2L}\right), \quad (13)$$

$$\cos \beta_1 = \frac{X_D - X_C}{\rho}, \beta_1 = \arccos \frac{X_D - X_C}{\rho}. \quad (14)$$

Substituting β_2 and β_1 from (13) and (14) into (12) yields β . Now, to calculate X_E and Y_E , β in terms of X_E and Y_E is written as follows:

$$\cos \beta = \frac{X_E - X_C}{L}, \beta = \arccos \frac{X_E - X_C}{L}. \quad (15)$$

$$\sin \beta = \frac{Y_E - Y_C}{L}, \beta = \arcsin \frac{Y_E - Y_C}{L}. \quad (16)$$

Based on (15) and (16), the PR end-effector coordinates along the X–Y axis, i.e., X_E and Y_E , are given as

$$X_E = X_C + L \cos \beta. \quad (17)$$

$$Y_E = L \sin \beta + Y_C. \quad (18)$$

Now, from (7) and (8), by substituting the values of X_C and Y_C into (17) and (18), the final expression of X_E and Y_E can be written as

$$\begin{cases} X_E = L \cos \theta_1 + L \cos \beta. \\ Y_E = L \sin \beta + L \sin \theta_1. \end{cases} \quad (19)$$

In this section, X_E and Y_E are calculated as mentioned above in (19) when the positions of SRV01 and SRV02 are known, i.e., θ_1 and θ_2 , respectively.

2.3. Inverse Kinematics

For the inverse kinematics, θ_1 and θ_2 need to be computed for the given end-effector E coordinates along the X–Y axis, i.e., X_E and Y_E , as shown in Figure 3.

At first, φ_1 and φ_2 , the angles across points C and D, need to be calculated by the following equations, where ρ_1 and ρ_2 are the distances between the AE and BE points, respectively, and L is the length of each link, i.e., $L = 5$ (inch).

$$\varphi_1 = \arccos\left(1 - \frac{\rho_1^2}{2L^2}\right), \quad (20)$$

$$\varphi_2 = \arccos\left(1 - \frac{\rho_2^2}{2L^2}\right), \quad (21)$$

where the values of ρ_1 and ρ_2 can be calculated as follows:

$$\rho_1 = \sqrt{(X_E)^2 + (Y_E)^2}, \quad (22)$$

$$\rho_2 = \sqrt{(B - X_E)^2 + (Y_E)^2}. \quad (23)$$

Now, the equations to derive the desired position angles of both servo motors of the PR, while the end-effector X–Y axis coordinates are known, i.e., X_E and Y_E , γ_1 , γ_2 , as shown in Figure 3, can be given as

$$\gamma_1 = \frac{\pi - \varphi_2}{2}, \quad (24)$$

$$\gamma_2 = \frac{\pi - \varphi_1}{2}, \quad (25)$$

where γ_1 and γ_2 are known as per (24) and (25), and α_1 and α_2 are calculated as follows:

$$\alpha_1 = \arctan\left(\frac{Y_E}{X_E}\right), \quad (26)$$

$$\alpha_2 = \arctan\left(\frac{B - X_E}{Y_E}\right). \quad (27)$$

The servo motors SRV01 and SRV02 position angles have the following expressions:

$$\theta_1 = \alpha_1 - \gamma_2. \quad (28)$$

$$\theta_2 = \alpha_2 - \gamma_1. \quad (29)$$

Now, substituting the values of γ_1 , γ_2 from (24), (25), α_1 , α_2 from (26), and (27) into (28) and (29), respectively, yields the final expression of the desired θ_1 and θ_2 in terms of X_E and Y_E , as per following equations:

$$\begin{cases} \theta_1 = \arctan\left(\frac{Y_E}{X_E}\right) - \frac{\pi - \varphi_1}{2} \\ \theta_2 = \arctan\left(\frac{B - X_E}{Y_E}\right) - \frac{\pi - \varphi_2}{2} \end{cases} \quad (30)$$

In this section, the desired θ_1 and θ_2 are derived as per (30) for the given end-effector position along the X–Y axis, i.e., X_E and Y_E .

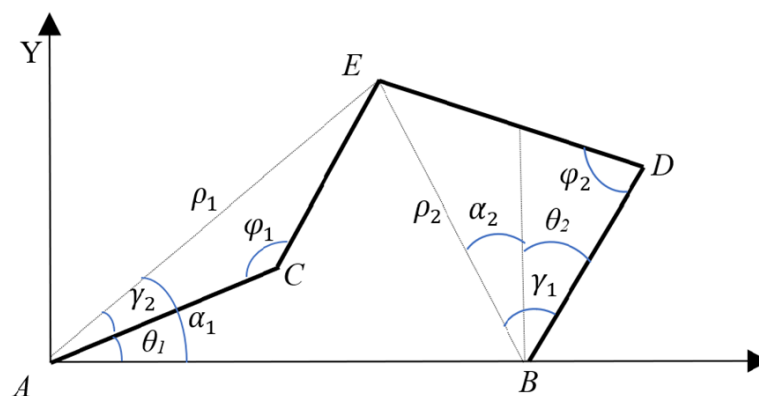


Figure 3. Inverse kinematics for the four links of the pantograph robot [12].

3. Design of the Proposed Control and Stability Analysis

In this section, firstly, the proposed control structure is explained, and then the proposed algorithm is derived, followed by the stability analysis.

3.1. The Proposed Control Structure of the PR

The proposed control structure of the PR is shown in the schematic block diagram in Figure 4.

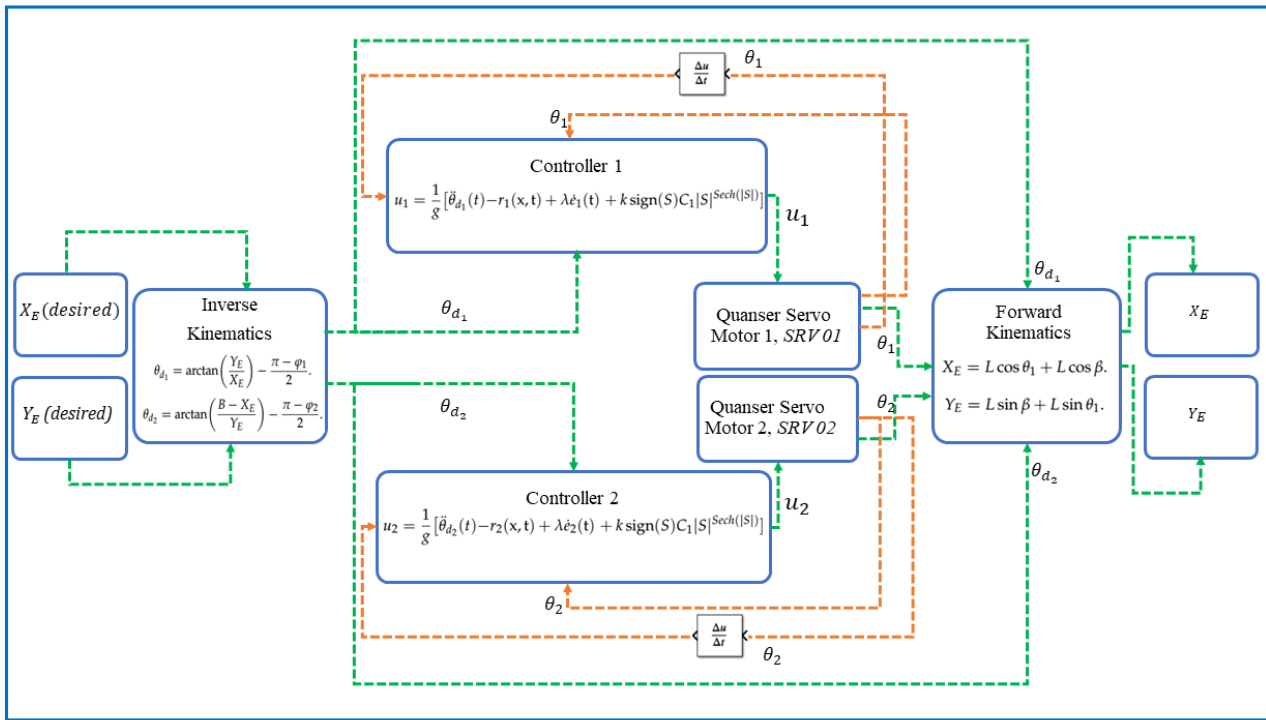


Figure 4. The proposed PR control structure.

At first, the desired position of the end-effector of the PR along the X–Y axis is given and, with the help of inverse kinematics as per (30), the desired θ_{d1} and θ_{d2} are calculated, which are required for the design of the proposed HSF-based SMC. As per Section 3.2, the proposed SMCs are designed, which can control the actual positions of both motors, i.e., θ_1 and θ_2 , by generating two control inputs u_1 and u_2 , respectively. Now, θ_1 and θ_2 are measured across the encoders of the servo motors SRV01 and SRV02, and end-effector coordinates along the X–Y axis (X_E, Y_E) are derived accordingly as per the forward kinematics in Equation (19). In this way, the PR end-effector position is successfully controlled, as shown in Figure 4.

Simulations and hardware results verify the effectiveness of the proposed control, with less chattering, fast convergence, and better tracking performance.

3.2. The Proposed Hyperbolic-Secant-Function-Based SMC

In the proposed SMC, an appropriate switching function is first designed such that the system states can reach the sliding surface. Once, the states reach the sliding surface, a robust control law is necessary that enforces the states to slide across the designed sliding manifold. The following process outlines the steps to design the proposed control law.

First, the sliding surface can be designed as follows:

$$S = \dot{e}(t) + \lambda e(t), \quad \lambda > 0 \quad (31)$$

where the tracking error is represented as $e(t)$ and can be chosen as follows:

$$e(t) = \theta_d(t) - \theta_m(t), \quad (32)$$

$$\dot{e}(t) = \dot{\theta}_d(t) - \dot{\theta}_m(t). \quad (33)$$

Taking the first derivative of (31) yields

$$\dot{S} = \ddot{e}(t) + \lambda \dot{e}(t). \quad (34)$$

Substituting $\ddot{e}(t)$ from (33) into (34) produces

$$\dot{S} = (\ddot{\theta}_d(t) - \ddot{\theta}_m(t)) + \lambda \dot{e}(t). \quad (35)$$

Based on (4), substituting $\ddot{\theta}_m(t)$ into (35), we obtain

$$\dot{S} = (-r(x, t) - gu(t) - d(t) + \ddot{\theta}_d(t)) + \lambda \dot{e}(t). \quad (36)$$

Substituting $\dot{S} = 0$ in (36), if $d(t) = 0$, results in

$$u_{eq} = \frac{1}{g} [-r(x, t) + \ddot{\theta}_d(t) + \lambda \dot{e}(t)]. \quad (37)$$

A novel improved switching law is proposed as follows:

$$u_{sw} = -\frac{1}{g} k \operatorname{sign}(S) C_1 |S|^{\operatorname{Sech}(|S|)},$$

$$u = \frac{1}{g} [-r(x, t) + \ddot{\theta}_d(t) + \lambda \dot{e}(t) + k \operatorname{sign}(S) C_1 |S|^{\operatorname{Sech}(|S|)}], \quad (38)$$

where $\lambda > 0$, $C_1, k > 0$, $r(x, t) = -\frac{1}{T_m} \dot{\theta}_m$, $g = \frac{1}{T_m} k_m$, and $\operatorname{Sech}(|S|) \in (0, 1]$, as shown graphically in Figure 5.

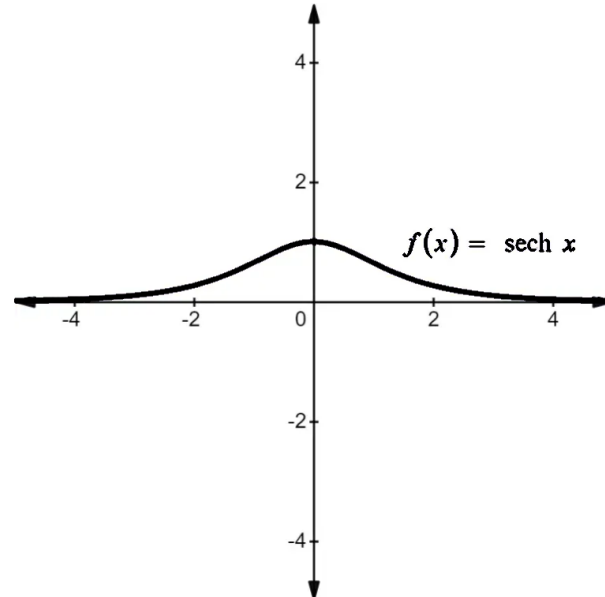


Figure 5. Hyperbolic secant function graphical representation.

3.3. Stability Analysis

The sliding surface as per Equation (31) is chosen as follows:

$$S = \dot{e}(t) + \lambda e(t). \quad \lambda > 0 \quad (39)$$

The first derivative of (39) yields

$$\dot{S} = \ddot{e}(t) + \lambda \dot{e}(t). \quad (40)$$

Define a Lyapunov function as per [31],

$$V = \frac{1}{2}S^2. \quad (41)$$

If the claim of the system's energy decreasing with time is valid, then its derivative has to be strictly negative, i.e., $S\dot{S} < 0$. By substituting the value of $\dot{S}$ in the reachability condition, i.e., $S\dot{S} < 0$, we obtain

$$S\dot{S} = S(\ddot{e}(t) + \lambda\dot{e}(t)). \quad (42)$$

Substituting $\ddot{e}(t)$ from (33) into (42) gives

$$S\dot{S} = S(\ddot{\theta}_d(t) - \ddot{\theta}_m(t) + \lambda\dot{e}(t)). \quad (43)$$

Substituting $\ddot{\theta}_m(t)$ from (4) into (43) yields

$$S\dot{S} = S(-r(x, t) - gu(t) - d(t) + \ddot{\theta}_d(t) + \lambda\dot{e}(t)). \quad (44)$$

Substituting $r(x, t)$ and g from (5) into (44) results in

$$S\dot{S} = S\left[\left(\frac{1}{T_m}\dot{\theta}_m\right) - \left(\frac{1}{T_m}k_m\right)u(t) - d(t) + \ddot{\theta}_d(t) + \lambda\dot{e}(t)\right]. \quad (45)$$

Substituting u from (38) into (45), we obtain

$$S\dot{S} = S\left[\left(\frac{1}{T_m}\dot{\theta}_m\right) - \left(\frac{1}{T_m}k_m\right)\left[\frac{1}{g}\left[\ddot{\theta}_d(t) - r(x, t) + \lambda\dot{e}(t) + C_1 \text{sign}(S)k|S|^{\text{Sech}(|S|)}\right] - d(t) + \ddot{\theta}_d(t) + \lambda\dot{e}(t)\right]\right]. \quad (46)$$

Substituting $r(x, t)$ and g into (46) as $d(t) = 0$ produces

$$S\dot{S} = S\left[\left(\frac{1}{T_m}\dot{\theta}_m\right) - \left(\frac{1}{T_m}k_m\right)\frac{1}{k_m}T_m\ddot{\theta}_d(t) - \frac{1}{T_m}\dot{\theta}_m - \lambda\dot{e}(t) - C_1 \text{sign}(S)k|S|^{\text{Sech}(|S|)} + \ddot{\theta}_d(t) + \lambda\dot{e}(t)\right]. \quad (47)$$

Further simplification of (47) yields

$$S\dot{S} = S(-C_1 \text{sign}(S)k|S|^{\text{Sech}(|S|)}). \quad (48)$$

Case 1: In (48), if $S > 0$,

$$S\dot{S} = S(-C_1 \text{sign}(S)k|S|^{\text{Sech}(|S|)}) < 0$$

$$S\dot{S} = -S(C_1 \text{sign}(S)k|S|^{\text{Sech}(|S|)}) < 0; \text{ iff } C_1, k > 0, \text{ Sech}(|S|) \in (0, 1] \quad (49)$$

Case 2: In (48), if $S < 0$,

$$S\dot{S} = -S(-C_1 \text{sign}(-S)k|S|^{\text{Sech}(|S|)}) < 0$$

$$S\dot{S} = -S(C_1 \text{sign}(S)k|S|^{\text{Sech}(|S|)}) < 0; \text{ iff } C_1, k > 0, \text{ Sech}(|S|) \in (0, 1] \quad (50)$$

It is pertinent to mention here that if $|s| \rightarrow 0, |s| = 0, |s| < 0$, or $|s| > 0$, $\text{Sech}(|S|)$ is always bounded within the stable range, i.e., $\text{Sech}(|S|) \in (0, 1]$, as shown above in Figure 5. Hence, stability is guaranteed.

4. Comparative Simulation Analysis

This section outlines the simulation results carried out in MATLAB/Simulink (R2021a) on a PR model. The control structure is chosen as per Section 3.1. The desired reference signals given to the PR along the X–Y axis for tracking are $X_{Ed} = \sin(\pi t) + 0.5\sin(\pi/2t) + 5$ and $Y_{Ed} = \sin(\pi t) - 0.5\sin(\pi/2t - 0.5\pi) + 5$, respectively. The comparison is performed with an SMC and a recent adaptive sliding mode control (ASMC) [28]. Firstly, comparative simulations with same control parameters are carried out, as per Section 4.1. For this purpose, the control parameters of the proposed SMC are selected as per Table 1. For the ASMC, gains

are taken as $c = 14$, $k = 140$, and the rest of the parameters are $n = 50$, $\lambda = 0.20$. In Section 4.2, the optimal performance comparison of each controller is also conducted by individual tuning of each controller based on minimum chattering and better tracking performance.

Table 1. Control parameters for experimental analysis.

Controller	Controller Parameters
SMC	$K_m = 1.53$ (rad/sV), $K = 14$, $T_m = 0.0254$ (s), $\lambda = 14$, $L = 5$ (inch)
Proposed	$K_m = 1.53$ (rad/sV), $K = 14$, $T_m = 0.0254$ (s), $\lambda = 14$, $L = 5$ (inch), $C_1 = 6.5$

4.1. Comparative Simulation with Same Parameters

Comparative simulation results with the same parameters are shown in Figures 6–10.

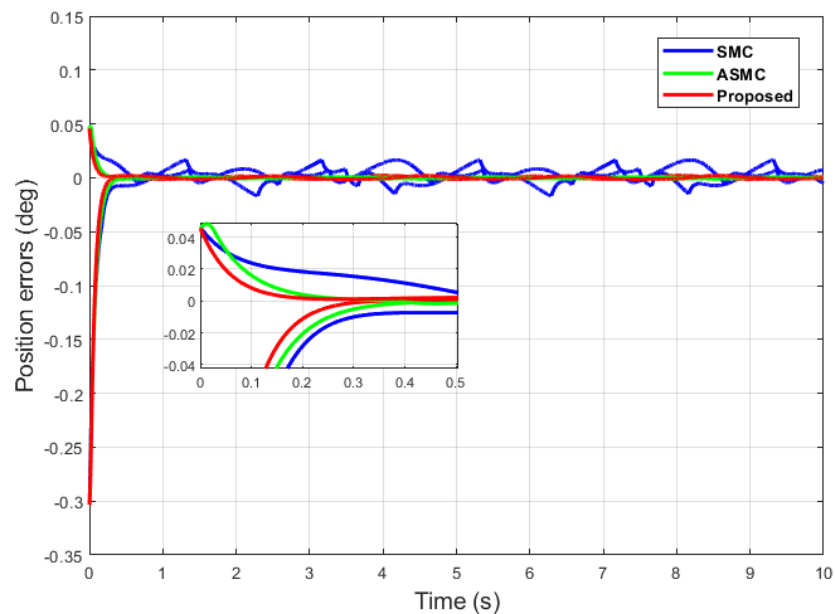


Figure 6. Position error comparison of dual sine wave.

It can be seen from Figure 6 that in the case of dual-sine-wave tracking, the proposed algorithm outperforms the others with faster convergence, which can be seen in the zoomed section of Figure 6. In Figures 7 and 8, the sliding surfaces comparison and control inputs comparison for dual-sine-wave tracking are conducted, respectively. However, the SMC input can be adjusted to a similar bandwidth for fair comparison in simulations, which further deteriorates its performance.

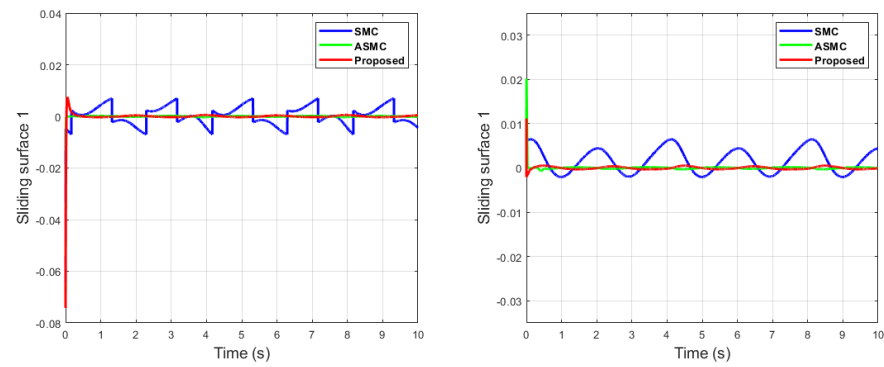


Figure 7. Sliding surfaces comparison of dual sine wave.

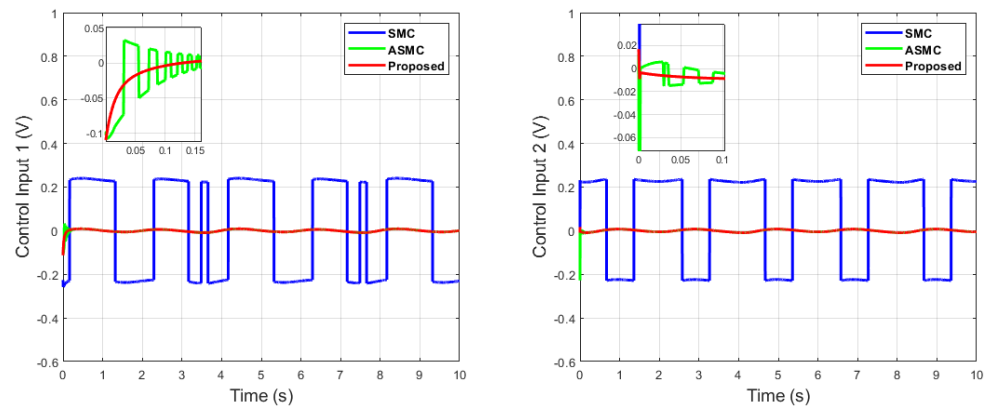


Figure 8. Control inputs comparison of dual sine wave.

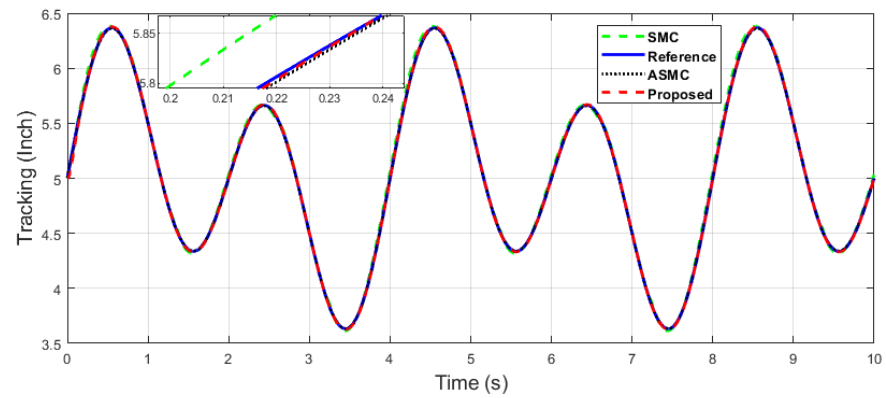


Figure 9. The x-axis end-effector tracking of dual sine wave.

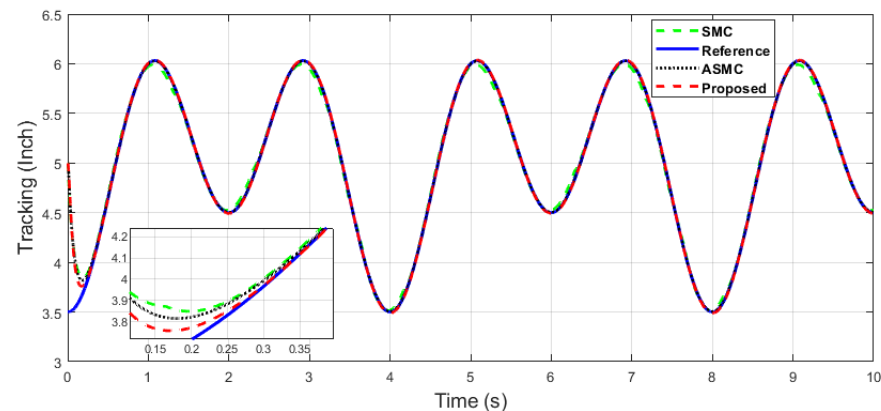


Figure 10. The y-axis end-effector tracking of dual sine wave.

It can be seen in Figures 9 and 10 that the proposed controller has overall fast convergence, as the PR control is the collaborative control along the X–Y axis in contrast to the SMC and ASMC, which depicts the superiority of the proposed algorithm.

Next, an impulse disturbance is introduced with a magnitude of 0.2 V at 4 (s) for a short duration in the dual-sine-wave tracking to replicate in a real-world scenario if the robot is touched by a human or if any other sudden obstacle comes into contact with it during operation in order to consider uncertainty in the system. It can be seen from Figures 11–14, which depict the comparative analysis with the ASMC [28], that when the system is subjected to impulse disturbance, the proposed control is robust with smooth control input when the disturbance is introduced at 4 (s), while the ASMC is prone to chattering in the initial phase, as well as during disturbance.

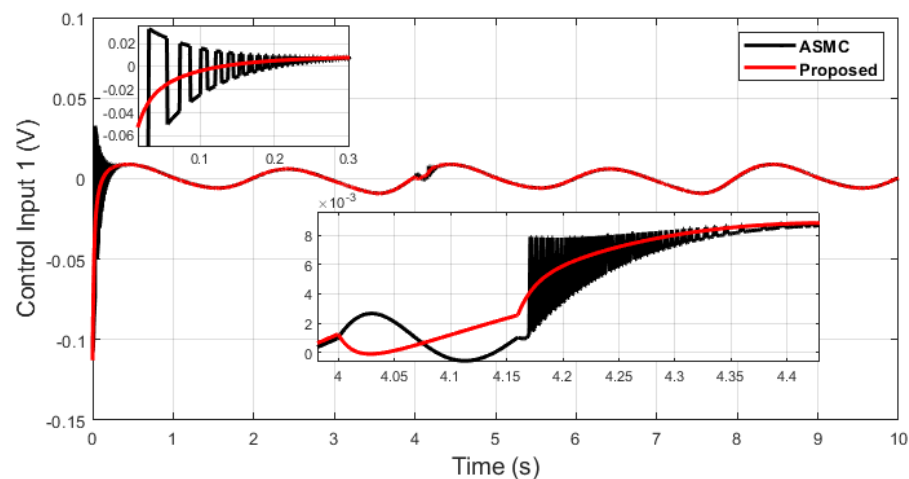


Figure 11. Control input 1 comparison of dual sine wave under impulse disturbance introduced at 4 (s).

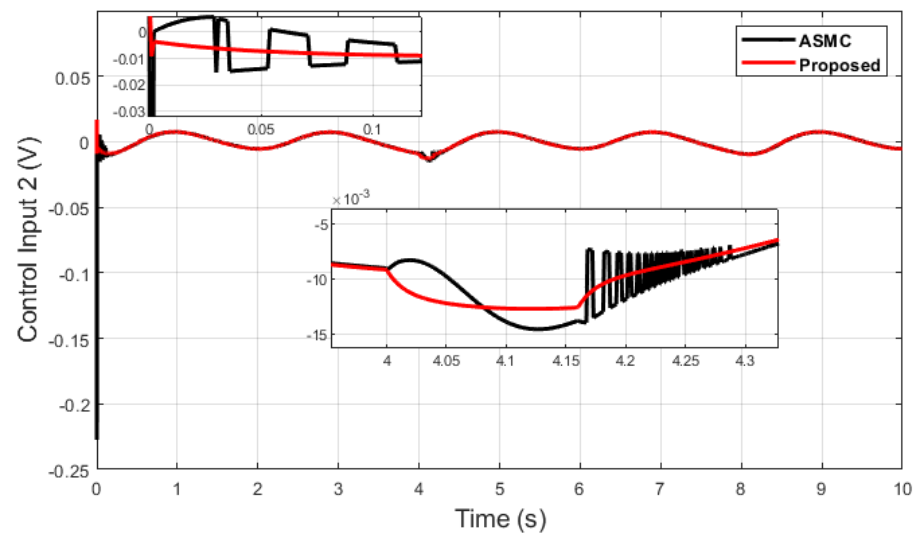


Figure 12. Control input 2 comparison of dual sine wave under impulse disturbance introduced at 4 (s).

It can be seen in Figures 13 and 14 that when an impulse disturbance is introduced at 4 (s), the proposed algorithm is robust in contrast to the ASMC, with better tracking and ability of disturbance rejection, which can be seen in the zoomed section of these figures.

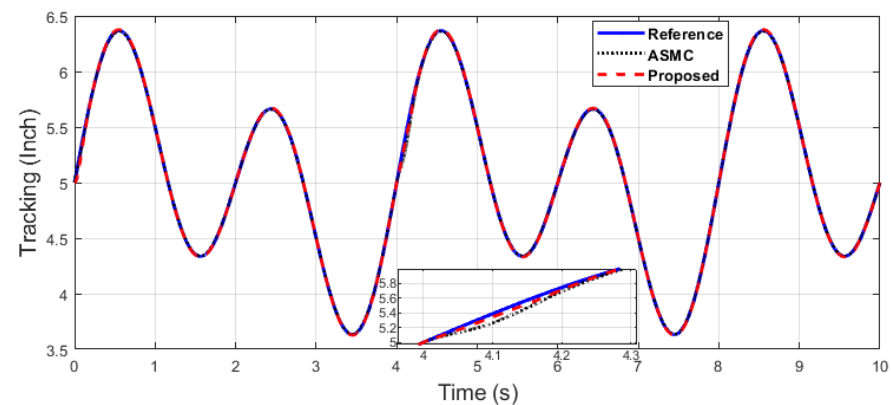


Figure 13. The x-axis end-effector tracking of dual sine wave under impulse disturbance introduced at 4 (s).

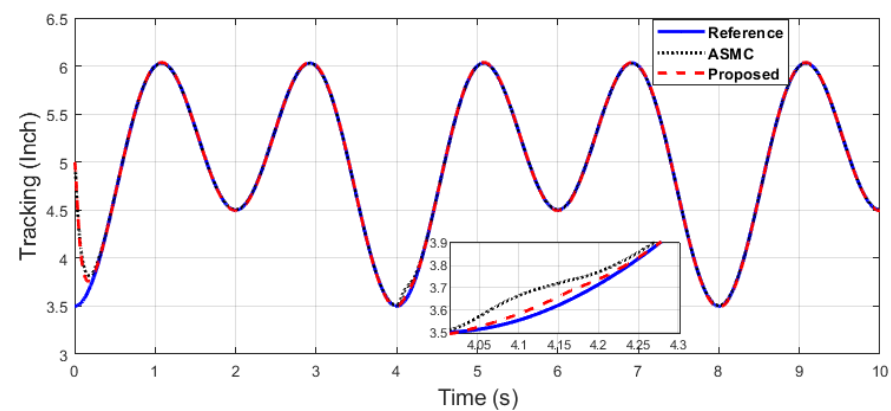


Figure 14. The y-axis end-effector tracking of dual sine wave under impulse disturbance introduced at 4 (s).

4.2. Comparative Simulation with Optimal Tuning

In this section, the performance of all controllers with optimal parameter tuning is considered, in which the parameters of each controller are individually tuned to achieve less chattering, better tracking, and faster convergence. There is always a trade-off among chattering, fast convergence, and better tracking performance; thus, the control parameters are chosen as per Table 2 for the optimal tuning of all comparative algorithms.

Table 2. Control parameters for optimal tuning.

Controller	Controller Parameters
SMC	$K_m = 1.53$ (rad/sV), $K = 0.14$, $T_m = 0.0254$ (s), $\lambda = 10$, $L = 5$ (inch).
ASMC	$K_m = 1.53$ (rad/sV), $T_m = 0.0254$ (s), $n = 30$, $L = 5$ (inch), $c = 10$, $k = 172$, $\lambda = 0.095$.
Proposed	$K_m = 1.53$ (rad/sV), $K = 14$, $T_m = 0.0254$ (s), $\lambda = 14$, $L = 5$ (inch), $C_1 = 27$.

The optimal tuning comparison is shown in Figures 15–23.

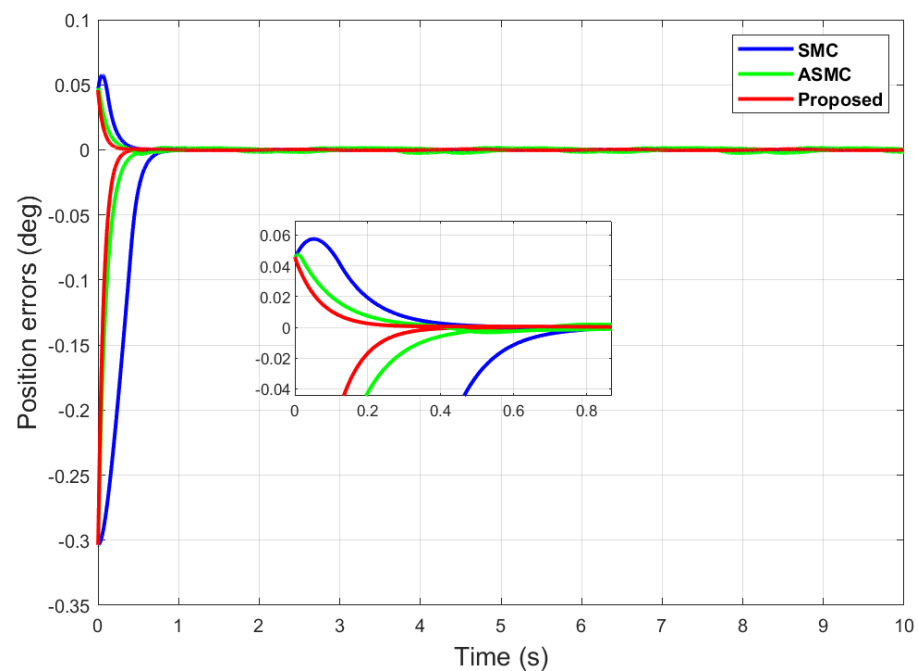


Figure 15. Position error comparison of dual sine wave with optimal tuning.

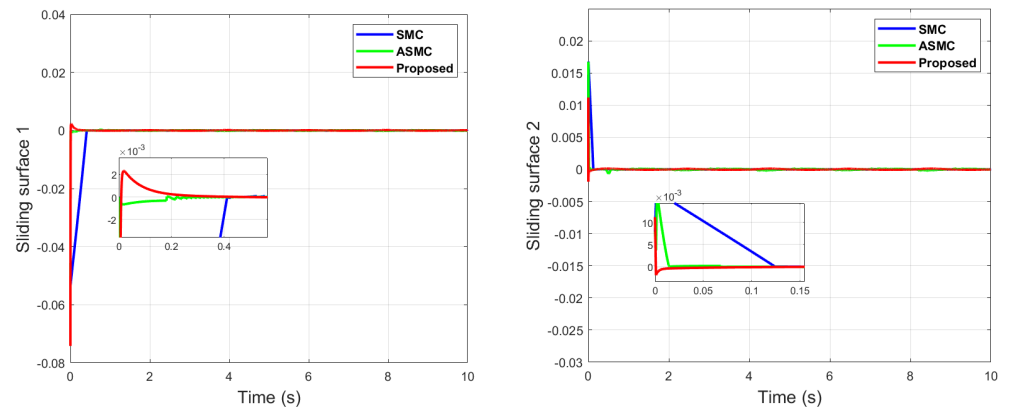


Figure 16. Sliding surface comparison of dual sine wave with optimal tuning.

It can be seen in Figures 15 and 16 that the proposed controller has overall fast convergence in contrast to the SMC and ASMC, which depicts the superiority of the proposed algorithm. In Figure 17, the proposed algorithm outperforms in terms of chattering reduction in contrast to the SMC and ASMC, which are prone to chattering, even in the case with individual optimal tuning.

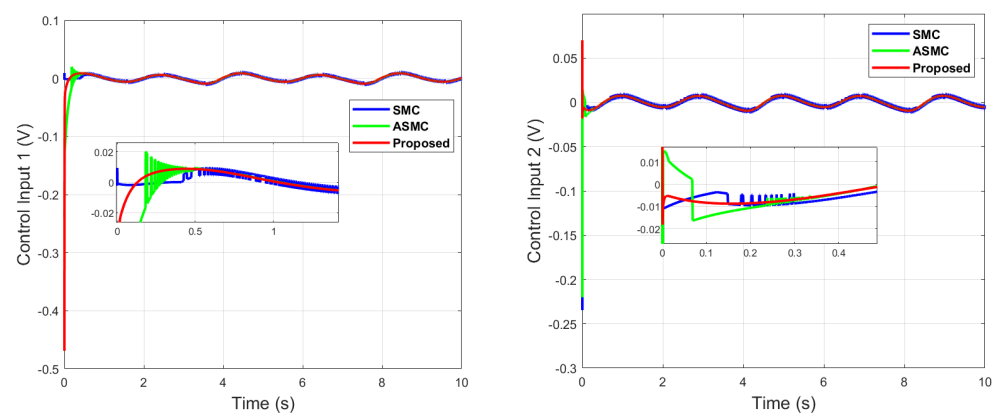


Figure 17. Control inputs comparison of dual sine wave with optimal tuning.

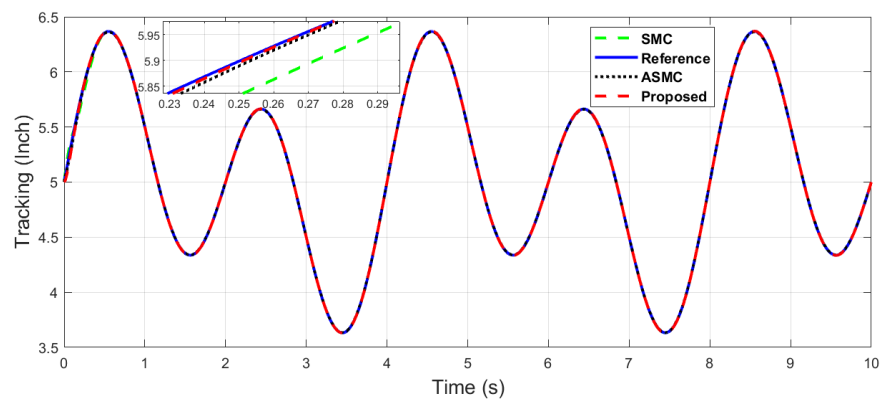


Figure 18. The x-axis end-effector tracking of dual sine wave with optimal tuning.

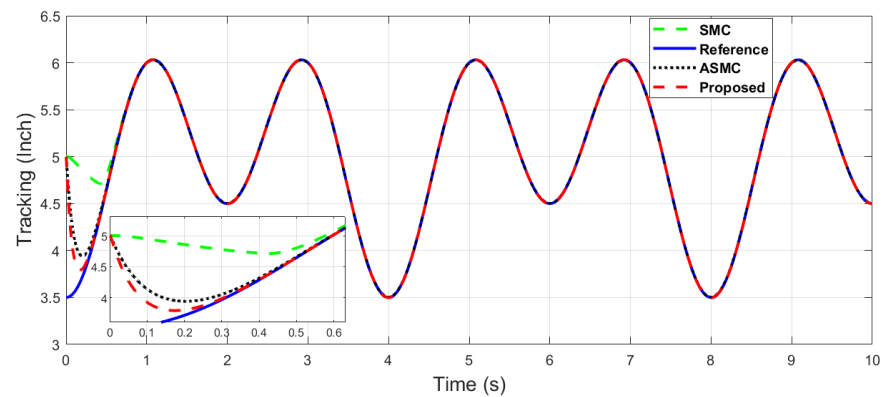


Figure 19. The y-axis end-effector tracking of dual sine wave with optimal tuning.

Figures 18 and 19 show the robustness of the proposed control, with overall fast convergence and better tracking performance compared with the SMC and ASMC, which can be seen in the zoomed sections of these figures.

Furthermore, an impulse disturbance is introduced with a magnitude of 0.2 V at 4 (s) for a short duration in dual-sine-wave tracking, similar to Section 4.1. Figure 20 shows the superiority of the proposed algorithm when subjected to an impulse disturbance.

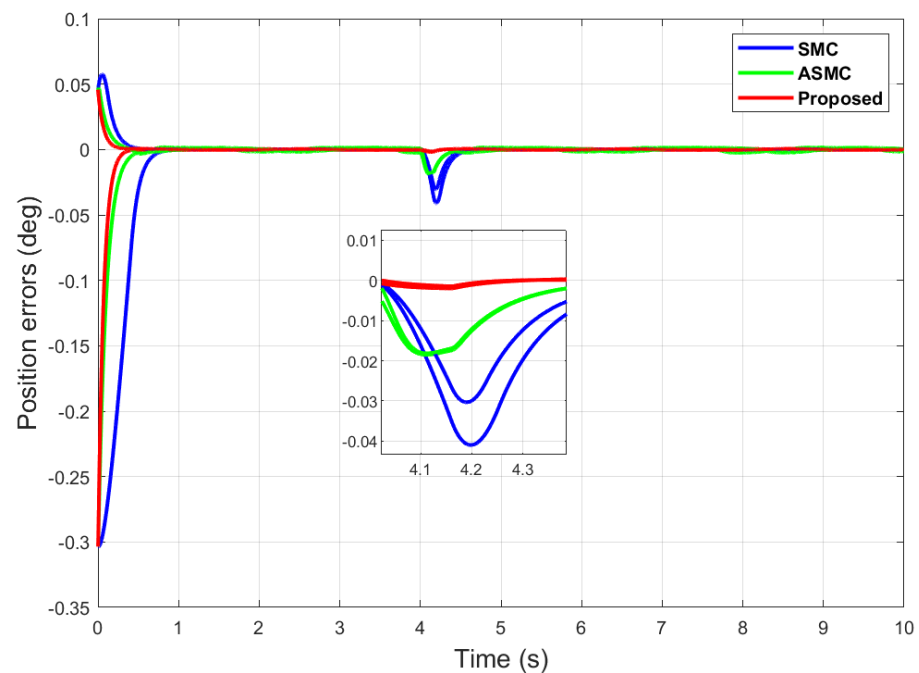


Figure 20. Position error comparison of dual sine wave with optimal tuning when subjected to an impulse disturbance at 4 (s).

It can be seen from Figure 21 that the comparative algorithms of the SMC and ASMC are prone to chattering when subjected to an impulse disturbance in contrast to the proposed control, which shows its superiority with much smoother control input.

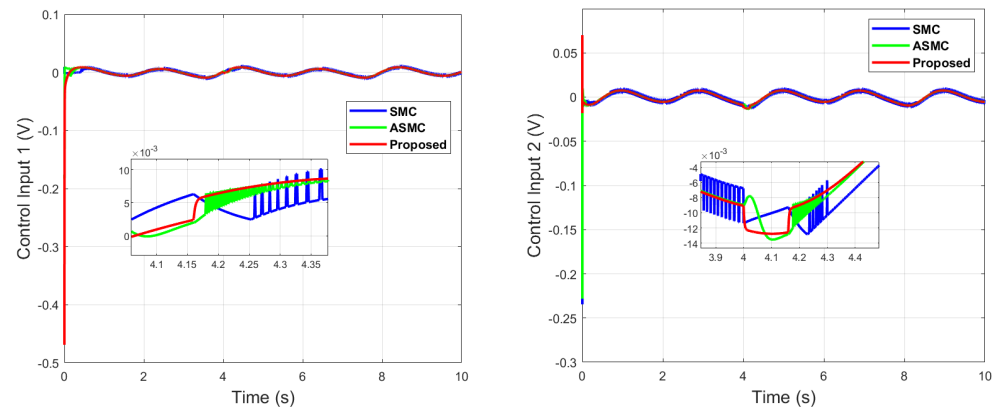


Figure 21. Control input comparison of dual sine wave with optimal tuning when subjected to an impulse disturbance at 4 (s).

It can be seen in Figures 22 and 23, which show the optimal tuning comparison when an impulse disturbance is introduced at 4 (s), that the proposed algorithm is robust in contrast to the SMC and ASMC, with better tracking and ability of disturbance rejection, which can be seen in the zoomed section of these figures. The proposed control outperforms in the optimal tuning scenario.

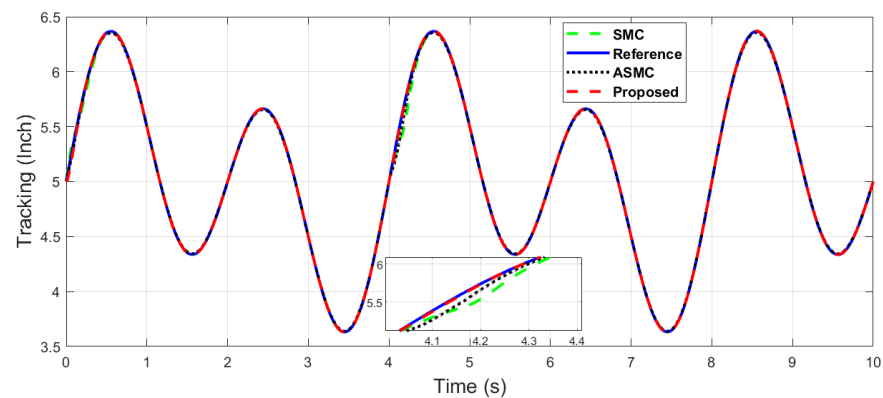


Figure 22. The x-axis end-effector tracking of dual sine wave with optimal tuning when subjected to an impulse disturbance at 4 (s).

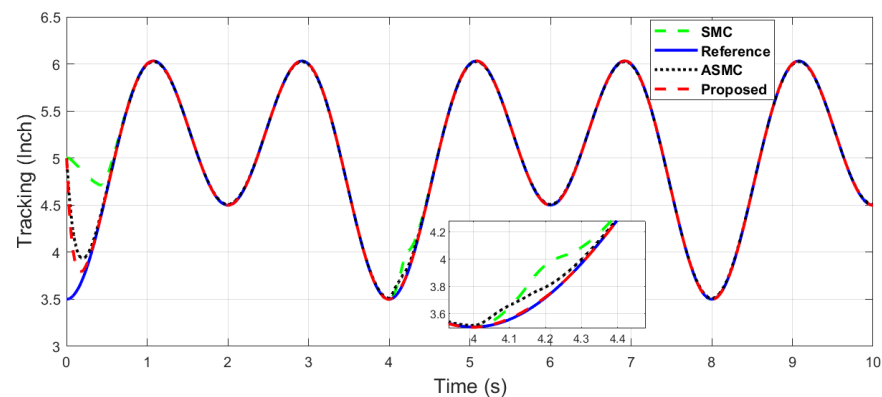


Figure 23. The y-axis end-effector tracking of dual sine wave with optimal tuning when subjected to an impulse disturbance at 4 (s).

5. Comparative Experimental Validation

Hardware implementation was carried out on the 2 DOF pantograph robot. The experimental block diagram is shown below in Figure 24.

Real-time hardware experiments were carried out on the 2 DOF PR, manufactured by Quanser, with MATLAB and Quarc software. A detailed explanation is provided for ease to replicate the hardware results. From Figure 24, it can be seen that it consists of two servo motors, entitled SRV01 and SRV02, connected with each other through four links such that it has three nonactuated and two actuated joints. Actuated joints with two links are mounted on SRV01 and SRV02, on points A and B, respectively, at a fixed distance such that each link is 5 inches in length. From the PC, a USB cable is connected to a Q8 USB data acquisition device, which consists of two ports for Encoder 1 and 2, which are connected with both motors position measuring points. If the position of the motors is measured by the encoders of each servo motor and speed is not a known state of the system, then two state observers can be designed to estimate the speed and achieve better performance accordingly. The data acquisition device also comprises two ports for analog output 1 and 2, which are connected to two amplifiers, Volt FAQ-X1 Amplifier 1 and Volt FAQ-X1 Amplifier 2, respectively. By appropriately controlling the positions of both motors and link kinematics, the end-effector E position is controlled.

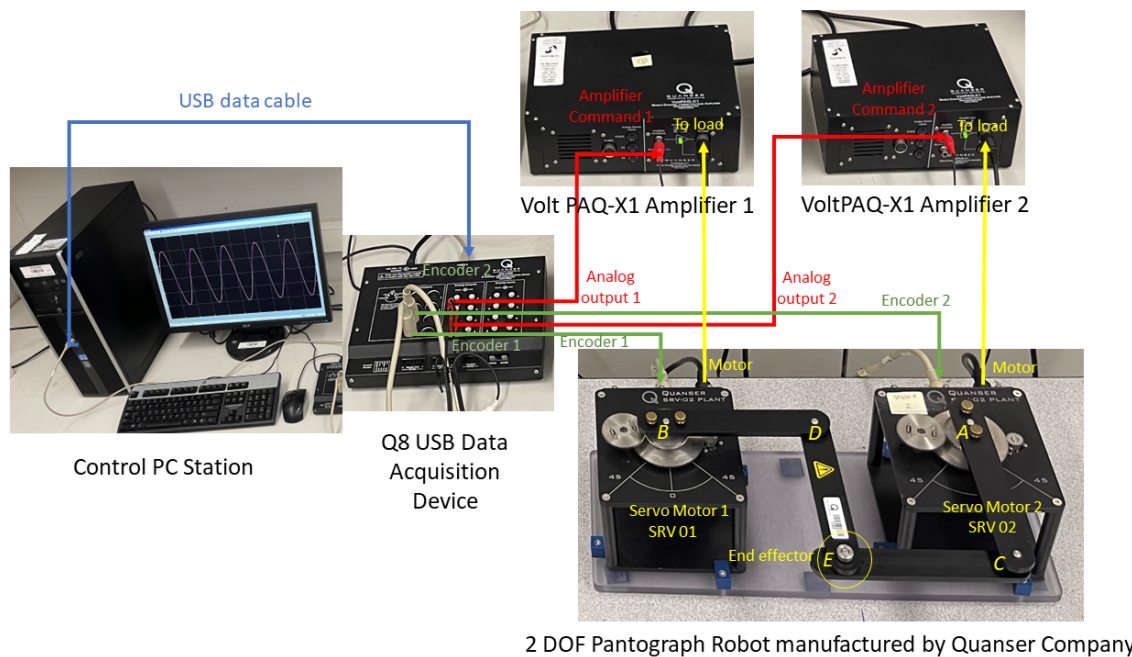


Figure 24. Quanser 2 DOF PR hardware implementation setup in the robotics lab.

The control parameters for the proposed and comparative algorithms are selected as per Table 1. For the sake of fair comparison, common gains are taken as equal; moreover, the control input bandwidth is adjusted the same way for the proposed control and SMC.

Three different cases for the experimental validation of the proposed control are carried out in Figures 25–39. The first case includes the single-sine-wave tracking, and the second case deals with dual-sine-wave tracking, followed by dual-sine-wave tracking with an input disturbance introduced at 4 (s) for a short duration in the third case of the experimental validation.

Case 1: Single-sine-wave tracking performance comparison and analysis for the PR.

In the first experiment, the PR tracks the simple sine wave along the x-axis as $X_{E_d} = \sin(\pi t) + 5$ and the y-axis as $Y_{E_d} = \sin((\pi t) - 0.5\pi) + 5$.

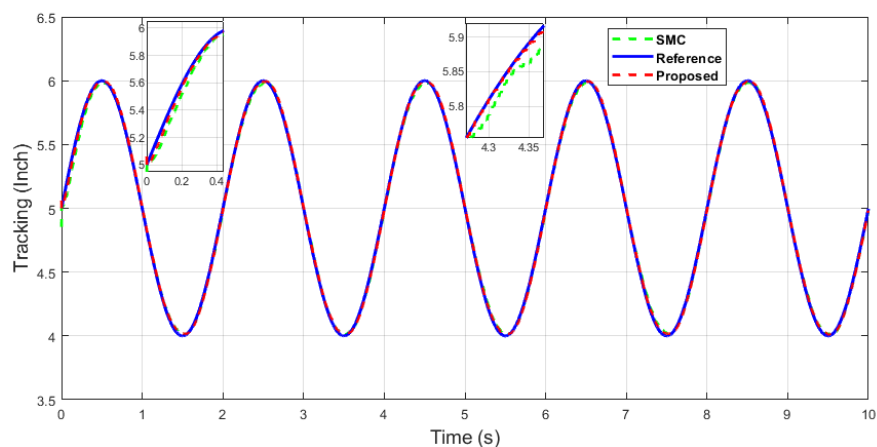


Figure 25. The x-axis end-effector tracking of single sine wave.

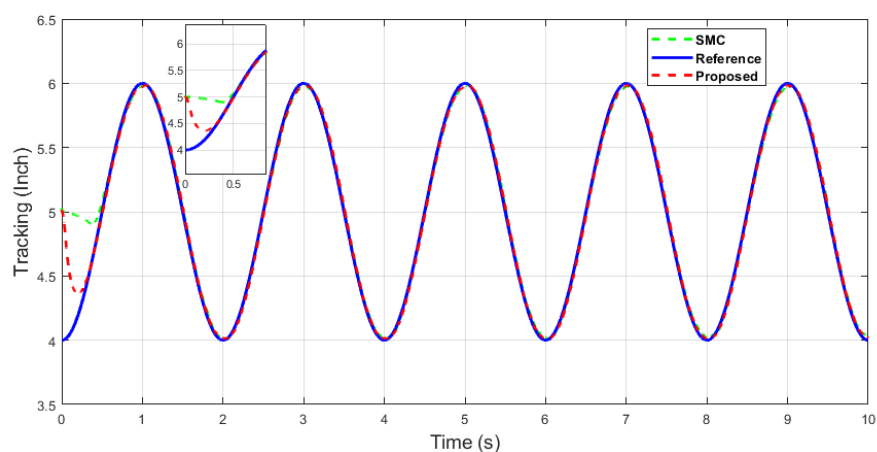


Figure 26. The y-axis end-effector tracking of single sine wave.

From Figures 25 and 26, it can be seen that the proposed control is faster and has better tracking performance compared with the SMC, which is shown in the zoomed portions of the figures.

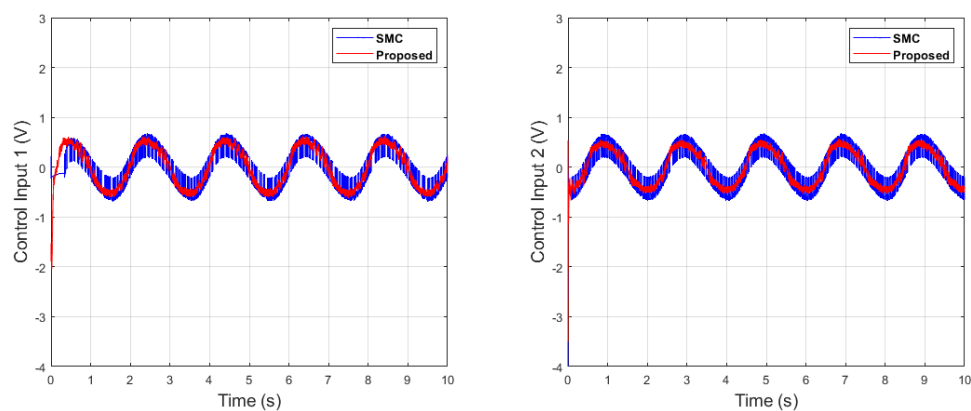


Figure 27. Control inputs comparison of single sine wave.

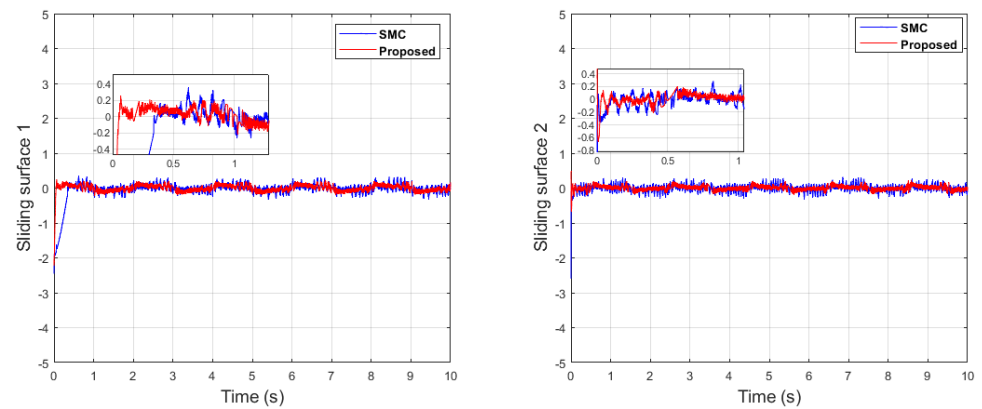


Figure 28. Sliding surfaces comparison of single sine wave.

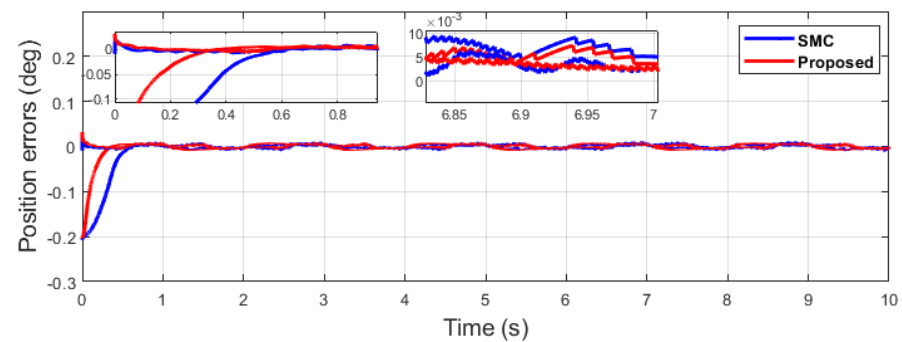


Figure 29. Position errors comparison of single sine wave.

In the following case, dual-sine-wave tracking was also considered for both end-effector positions along the x- and y axes, respectively. The following results show the effectiveness of the proposed algorithm under dual-sine-wave tracking, as per Figures 30–34.

Case 2: Dual-sine-wave tracking comparison and analysis for the PR.

In the case of dual-sine-wave tracking, the following desired signal was given along the x-axis and the y-axis, respectively, i.e., $X_{E_d} = \sin(\pi t) + 0.5\sin(\pi/2t) + 5$ and $Y_{E_d} = \sin(\pi t) - 0.5\pi + 0.5\sin(\pi/2t - 0.5\pi) + 5$.

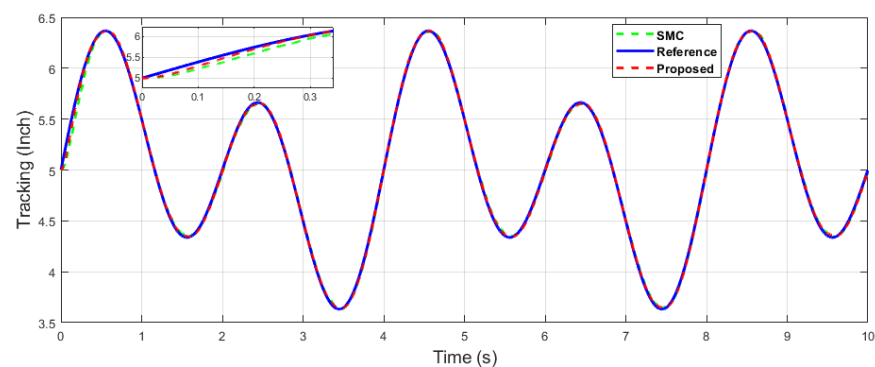


Figure 30. The x-axis end-effector tracking of dual sine wave.

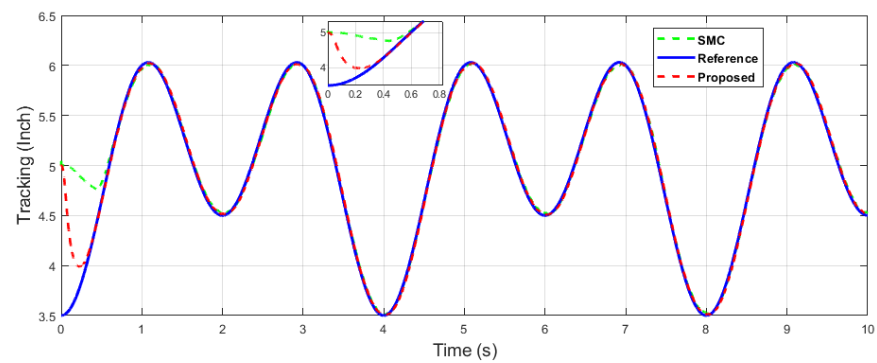


Figure 31. The y-axis end-effector tracking of dual sine wave.

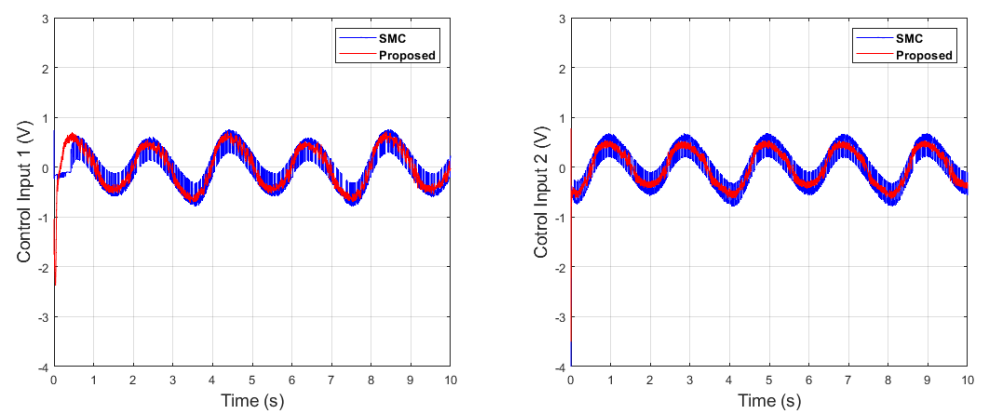


Figure 32. Control input comparison of dual sine wave.

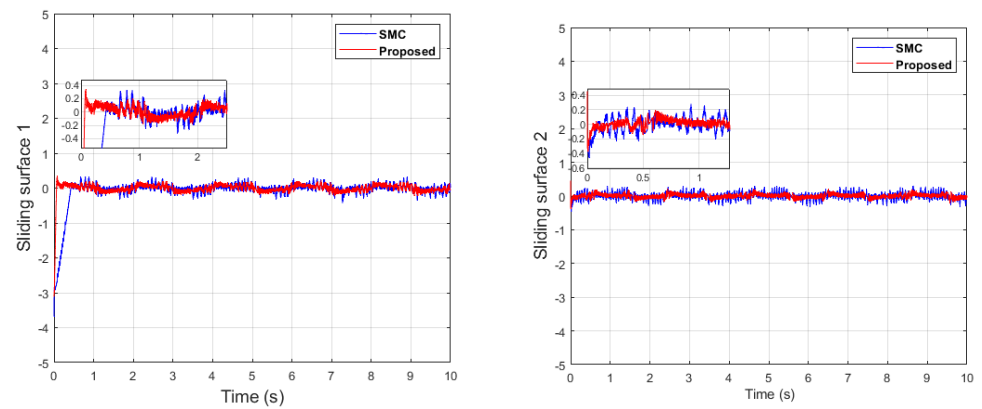


Figure 33. Sliding surface comparison of dual sine wave.

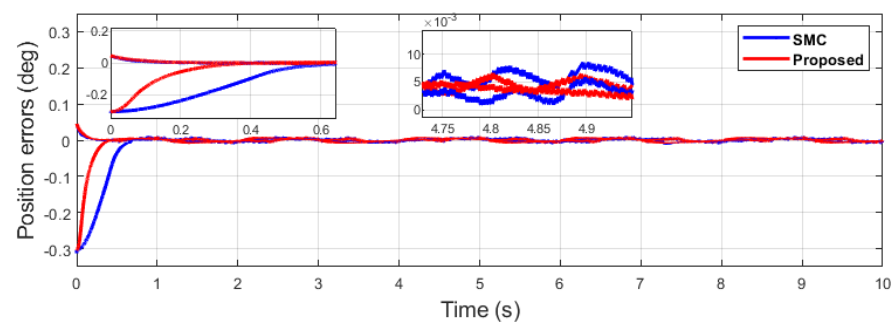


Figure 34. Position error comparison of dual sine wave.

Case 3: Dual-sine-wave tracking under impulse disturbance with comparison and analysis for the PR.

In the case of dual-sine-wave tracking under an impulse disturbance, the following desired signal was given along the x-axis and the y-axis, respectively, i.e., $X_{E_d} = \sin(\pi t) + 0.5\sin(\pi/2t) + 5$ and $Y_{E_d} = \sin(\pi t) - 0.5\sin(\pi/2t - 0.5\pi) + 5$ with an impulse disturbance of 0.2V magnitude at 4 (s) for a short duration to consider the uncertainty in the system.

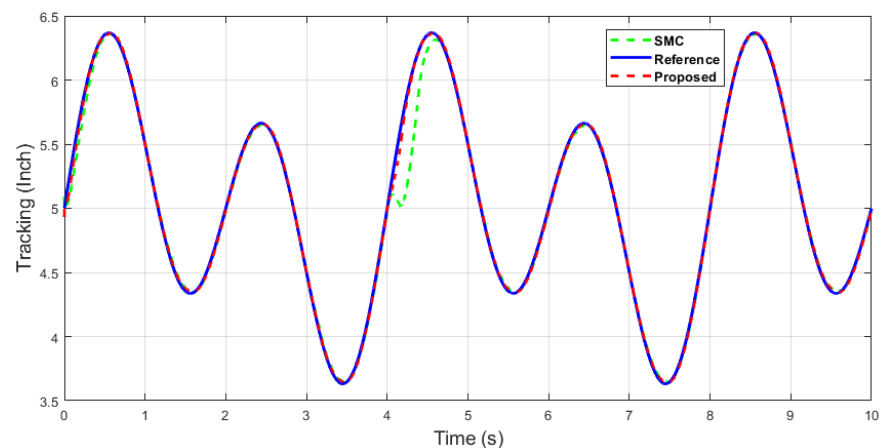


Figure 35. The x-axis end-effector tracking of dual sine wave under impulse disturbance introduced at 4 (s).

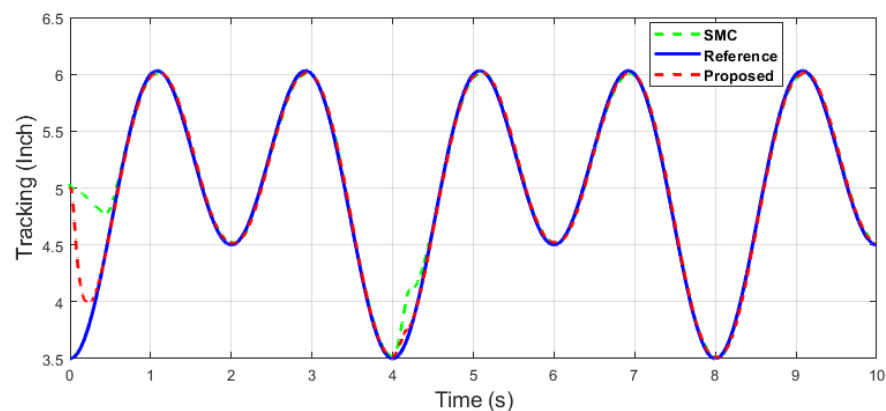


Figure 36. The y-axis end-effector tracking of dual sine wave under impulse disturbance introduced at 4 (s).

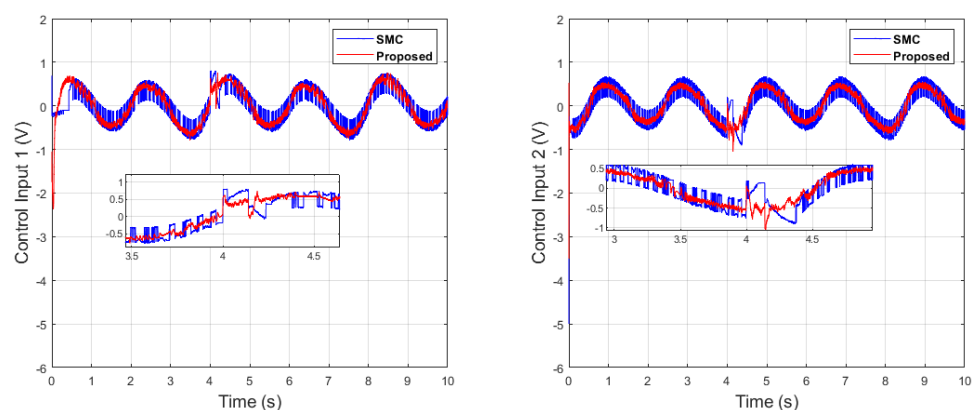


Figure 37. Control input comparison of dual sine wave under impulse disturbance introduced at 4 (s).

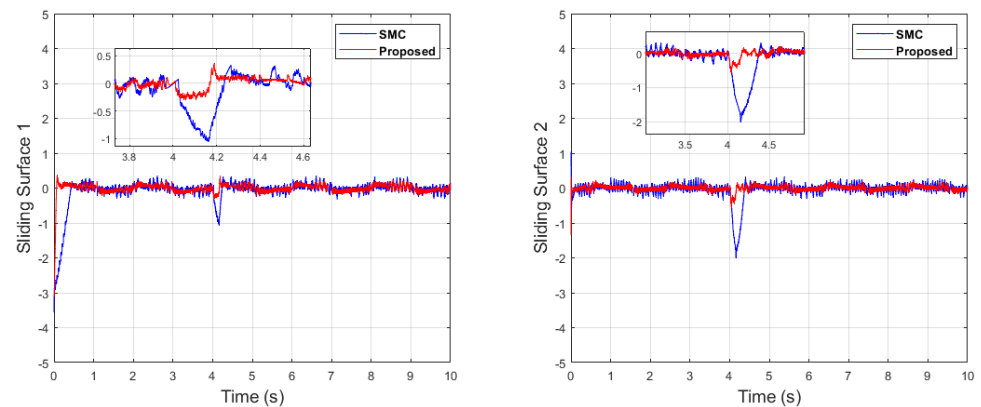


Figure 38. Sliding surface comparison of dual sine wave under impulse disturbance introduced at 4 (s).

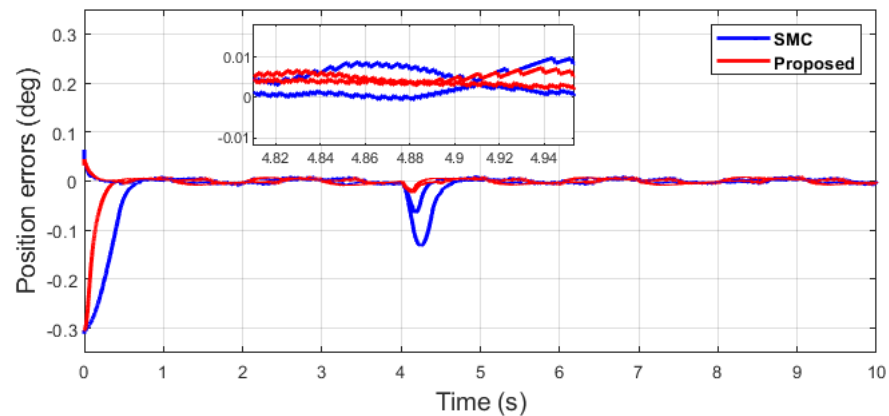


Figure 39. Position error comparison of dual sine wave under impulse disturbance introduced at 4 (s).

6. Discussion

This section outlines the superiority of the proposed algorithm over the comparative algorithm by thorough discussion and in-depth analysis. Firstly, It can be seen from Figures 25–29 that single-sine-wave tracking is achieved, which highlights the effectiveness of the proposed algorithm, as the position error e_1 of the proposed algorithm has significantly less value, i.e., $\text{RMS} = 0.0177$ in contrast to the SMC with an $\text{RMS} = 0.0305$, and position error e_2 has an $\text{RMS} = 0.0032$ compared with the SMC with an $\text{RMS} = 0.0034$, as shown in Table 3. Moreover, it can be seen from Figures 25 and 26, in the zoomed subplot sections, that the proposed algorithm has fast convergence (0.3 s) compared with the comparative algorithm (0.6 s). In Figure 26, which represents the end-effector tracking along the y-axis, the convergence is significantly faster compared with Figure 25, which represents the end-effector tracking along the x-axis. The position control of the end-effector E of the pantograph robot has collaborative controlling coordinates, i.e., X_E and Y_E , as can be seen from Figure 37.

Remark 1. End-effector position control is the collaborative position control along the X–Y axis; thus, the overall convergence of position errors is faster in the proposed algorithm compared with the SMC.

Figure 27 depicts the control input comparison of the proposed algorithm and SMC by single sine wave, and a significant reduction in chattering magnitude can be seen. The sliding surface comparison of single sine wave shows that from Figure 28, the proposed algorithm's sliding surface convergence is faster to origin, with a reduction in chattering magnitude. A similar trend can be seen in Case 2 of dual-sine-wave tracking from Figures 30–34, with the position error e_1 of the proposed algorithm having significantly less

value, i.e., $RMS = 0.0272$ in contrast to the SMC with $RMS = 0.0472$, and position error e_2 has an $RMS = 0.0038$ compared with the SMC with an $RMS = 0.0039$, as shown in Table 3. This is due to the trade-off between convergence and tracking error. Furthermore, it can be seen from Figure 34, in the zoomed subplot sections, that the proposed algorithm has fast convergence (0.3 s) compared with the comparative algorithm (0.6 s). A reduction in chattering magnitude can also be seen in Figure 32. As previously mentioned, the end-effector E has collaborative position control coordinates along the X–Y axis; thus, the overall effect of the proposed algorithm errors convergence is faster. In the case of dual-sine-wave tracking, the proposed algorithm outperforms with faster convergence, less steady-state error, and improved tracking performance.

Similarly, it can be seen in Case 3 of dual-sine-wave tracking from Figures 35–39 that the position error e_1 of the proposed algorithm has less RMS value, i.e., $RMS = 0.0272$ in contrast to the SMC with an $RMS = 0.0478$, and position error e_2 has significantly less $RMS = 0.0047$ compared with the SMC, with an $RMS = 0.0196$, as shown in Table 3. This shows the superiority of the proposed algorithm to handle the external disturbances and uncertainties in the system with better tracking performance and ability of disturbance rejection. In case of impulse disturbance, the SMC has a significantly large error e_2 , i.e., $RMS = 0.0196$, which shows its drawback in this regard.

Remark 2. *The proposed control has broad applications in robotics and can be used for various second-order systems such as pick and place robots, robotic hands, throttle valves, DC motors, etc.*

Table 3. Experimental errors analysis for the proposed and comparative algorithms.

Cases	Index	Errors	Proposed	SMC
Case 1	$RMS(^{\circ})$	e_1	0.0177	0.0305
	$RMS(^{\circ})$	e_2	0.0032	0.0034
Case 2	$RMS(^{\circ})$	e_1	0.0272	0.0472
	$RMS(^{\circ})$	e_2	0.0038	0.0039
Case 3	$RMS(^{\circ})$	e_1	0.0272	0.0478
	$RMS(^{\circ})$	e_2	0.0047	0.0196

7. Conclusions

A robust controller that functions in a closed-loop domain with faster convergence is indispensable in various sectors like robotics, space engineering, mining industry, advanced manufacturing sector, agriculture sector, etc. The sliding mode controller (SMC) has proven to be a robust controller in the case of external disturbances, nonlinearities, uncertainties, and modeled system dynamics. In this article, a novel improved SMC based on the *Sech* function has been proposed that achieves fast convergence, reduced chattering, and has less steady-state error. For the stability analysis, the Lyapunov stability criteria were utilized. Simulations and experimental implementations were carried out on a PR, showing the effectiveness of the proposed algorithm. The work proposed in this paper is focused on asynchronous control, in which the position errors are converging at different time spans. Thus, future work will consider synchronous control, in which both position errors will converge at the same time, which will make the system energy efficient.

Author Contributions: Conceptualization, M.A.H. ; methodology, M.A.H. and Z.C.; software, M.A.H. and Z.C.; validation, M.A.H. ; formal analysis, M.A.H. and Z.C.; investigation, Z.C.; resources, Z.M.; data curation, Z.C. and Z.M.; writing—original draft preparation, M.A.H. and Z.C.; writing—review and editing, M.A.H., Z.C. and Z.M.; visualization, M.A.H.; supervision, Z.C. and Z.M.; project administration, Z.C. and Z.M.; funding acquisition, Z.C. and Z.M.; All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the Australian Research Council under grant number DP190101557.

Data Availability Statement: Not applicable.

Acknowledgments: We would like to acknowledge the support of Jyoti Mishra and other lab mates who helped either at the administrative level or technical side, without whom we would not have been able to complete this work.

Conflicts of Interest: The authors declare no conflicts of interest.

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