

Article

Application of a Multi-Criterion Decision-Making Method for Solving the Multi-Objective Optimization of a Two-Stage Helical Gearbox

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Abstract: This paper provides a novel application of a multi-criterion decision-making (MCDM) method to the multi-objective optimization problem of designing a two-stage helical gearbox. This study's goal is to identify the ideal primary design elements that increase gearbox efficiency while reducing the gearbox cross-section area. In this work, three primary design parameters were selected for investigation: the gear ratio of the first stage and the coefficients of wheel face width (CWFV) of the first and second stages. The multi-objective optimization problem was further split into two phases: phase 1 solved the single-objective optimization problem of minimizing the gap between the variable levels, and phase 2 solved the multi-objective optimization issue of identifying the ideal key design factors. Moreover, the multi-objective optimization problem was handled by the SAW method as an MCDM approach, and the weight criteria were computed using the entropy approach. This study's significant characteristics are as follows: First, a multi-objective optimization problem was successfully solved using the MCDM approach (SAW technique) for the first time. Second, the power losses in idle motion were investigated in this work in order to determine the efficiency of a two-stage helical gearbox. From this study's findings, the ideal values for three major design parameters can be determined for the design of a two-stage helical gearbox.

Keywords: helical gearbox; gear ratio; gearbox efficiency; gearbox cross-section area; multi-objective optimization; SAW method



Citation: Dinh, V.-T.; Tran, H.-D.; Vu, D.-B.; Vu, D.; Vu, N.-P.; Luu, A.-T. Application of a Multi-Criterion Decision-Making Method for Solving the Multi-Objective Optimization of a Two-Stage Helical Gearbox. *Machines* **2024**, *12*, 365. <https://doi.org/10.3390/machines12060365>

Academic Editor: Gianni Campatelli

Received: 15 April 2024

Revised: 17 May 2024

Accepted: 21 May 2024

Published: 24 May 2024



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1. Introduction

The most crucial component of a mechanical drive system is the gearbox. It facilitates the lowering of the torque and speed transfer from the motor shaft to the working shaft. Because of this, numerous experts are working on optimization of the gearbox. In practice, multi-objective optimization refers to optimization issues in practice that involve the simultaneous optimization of two or more objective functions. Maximizing multiple performance parameters at once, including efficiency, size, mass, and load-carrying capability, can be difficult and complex when building a gearbox utilizing multi-objective optimization. In order to solve these issues, numerous optimization approaches have been created.

Using the response surface method, H. Wang et al. conducted a multi-objective optimization research for the helical gear in a centrifugal compressor [1]. Two specific goals were chosen for this project: the minimal gear mass and the highest gear stress. According to reports, the helical gear's maximum stress is within permissible bounds and its mass

has decreased by 27.4%. The Non-Dominated Sorting Genetic Algorithm II methodology (NSGA-II) was employed by D. Miller et al. [2] in a multi-objective spur gear pair optimization investigation. Enhancement of gear efficiency and decrease in gear volume were this study's goals. It was observed that a trade-off between efficiency and volume was necessary and that a lower gear module, a lower face width, greater profile shift coefficients, and a higher pinion tooth count all worked well together to achieve these goals. A mono-objective self-adaptive algorithm approach was used in [3] to solve the optimization of tooth changes for spur and helical gears. The PSO (particle swarm optimization) method was the foundation of this strategy. Multi-objective optimization enhanced the transmission error signal's maximal contact pressures and root mean square values. Edmund S. Maputi and Rajesh Arora [4] used the NSGA-II and the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) approach to solve multi-objective optimization by concurrently taking three objectives into consideration: volume, power output, and center distance. Their study's findings can provide insight into the design of small gearboxes. In [5], a planetary gearbox was optimized using the NSGA-II method while taking into account crucial tribological and regular mechanical restrictions. Comparing the results of single-objective optimization with and without tribological restrictions, the multi-objective optimization result demonstrated a considerable reduction in weight and power loss. In order to minimize gear weight, contact stress throughout the contact path, and optimal film thickness at the contact point, Jawaz Alam and Sumanta Panda [6] established a strategy for spur gear set design optimization. In order to guarantee a notable decrease in weight and contact stress of a profile-modified spur gear set with sufficient film thickness at the site of contact, this work used particle swarm optimization, particle swarm optimization-based teaching–learning optimization, and Jaya algorithms. The gear design was considerably better with optimum addendum coefficient values within the design space than with conventional designs, according to the study's findings. A unique multi-objective optimization of a two-stage spur gearbox utilizing the NSGA-II method was introduced by M. Patil et al. [7]. Two goal functions—minimal gearbox volume and minimum gearbox power losses—were involved in this effort. Their study's findings suggest that solutions derived from single-objective minimization had a significant likelihood of wear failure. Additionally, when utilizing multi-goal optimization as opposed to single-objective optimization, the overall power loss was cut in half. Chrystopher V.T. et al. [8] used the MCDM method to find the ideal gear material for a gearbox in order to enhance surface fatigue and boost its wear resistance. Their study aimed to enhance surface fatigue resistance and maximize its efficiency when applied to a gearbox. In [9], the multi-objective design of helical gear pair transmission was taken into consideration. The objective functions included gear volumes and opposing number of overlap ratio. In this study, the Parameter-Adaptive Harmony Search Algorithm (PAHS) was used to address the optimization problem. Recently, X.H. Le and N.P. Vu [10] explored the multi-objective optimization problem of designing a two-stage helical gearbox using the Taguchi technique and grey relation analysis (GRA). Finding the optimal primary design parameters that maximize gearbox efficiency while minimizing gearbox mass was the goal of their study. This method was also used in [11] to identify the best major design parameters for a two-stage bevel helical gearbox with the goal of maximizing gearbox efficiency and reducing gearbox volume. A multi-target optimization research for the two-stage gearbox of xEV-axle drives was carried out by M. Hofstetter et al. [12]. Three specific goals were chosen for this work's investigation: gearbox efficiency, packaging metric, and total expenses. Additionally, a closed loop of choosing the gearbox design parameters and then performing a gearbox analysis was included in the optimization process. The above study's findings suggested that the benchmark solution could still be improved, particularly in light of the cost and packaging trade-off. In order to build a drive system for electric vehicles, Istenes G. and J. Polák [13] carried out research to jointly optimize an electric motor and a gearbox. The goal of their effort was to minimize both the driving system's weight and the overall energy waste. To highlight the additional potential of cooperative optimization, the optimization

results were contrasted with earlier findings. If a drive system is optimized overall, it has been stated that increasing the gear ratio increases the system's overall efficiency. The most effective power flow solutions for a power system, taking into account the electric market and renewable energy, more particularly, nodal prices and wind turbine placement, were presented in [14]. The placement of wind turbines was optimized in this study to increase transmission power systems' profits. Nevertheless, the study did not take into account the possibility of reducing power loss in the wind turbine system by improving the gearbox's efficiency. To enhance the hypoid gears' operational features, a multi-objective optimization technique was developed [15]. The maximum tooth contact pressure, the minimum transmission error, the minimum gear mesh temperature, and the maximum gear pair efficiency were the four goals to be studied. The model was also solved using the NSGA-II approach. In [16], a study on the mechanical efficiency forecasting model and energy loss for aero-engine bevel gear power transmission was presented. Based on the research findings, a novel model for predicting energy loss and mechanical efficiency was put forth, utilizing enhanced thermal elastohydrodynamic lubrication analysis and taking into account the non-Newtonian fluid effect and entrainment angle. Higher and higher degrees of aero-engine bevel gear production efficiency and design sustainability were made possible by this approach. It can offer essential access to geometric and physical evaluations that can enhance the aero-engine power transmission system's overall energy generation, conversion, transfer, and usage.

It can be found from previous studies that the best primary design parameters for helical gearboxes have not been found using the MCDM technique, despite a great deal of research on multi-objective optimization for helical gearboxes. The present study reports on a multi-objective optimization study carried out for a two-stage helical gearbox, with two particular goals in mind: increasing gearbox efficiency and minimizing the gearbox cross-section area. The three optimum main design features for the two-stage helical gearbox were examined in this study: the gear ratio of the first stage and the CWWF of both stages. Additionally, the SAW method was selected to solve the multi-objective optimization issue, and the entropy methodology was used to establish the weights of the criteria. One of the primary research conclusions recommends using an MCDM technique in conjunction with two-step problem solving to address multi-objective optimization problems as well as single- and multi-objective problems. Furthermore, the solutions to the challenge out-perform those from previous research.

2. Optimization Problem

In order to construct the optimization problem, the gearbox cross-section areas and gearbox efficiency are first calculated in this section. The stated objective functions and limitations are then provided. Table 1 lists the nomenclatures used in the optimization problem to make calculations easier.

Table 1. The nomenclatures used in the optimization work.

Parameters	Nomenclature	Units
Allowable contact stress of stage 1	AS_1	MPa
Allowable contact stress of stage 2	AS_2	MPa
Allowable shear stress of shaft material	$[\tau]$	MPa
Arc of approach on i stage	β_{ai}	
Arc of recess on i stage	β_{ri}	
Base-circle radius of the pinion	R_{01i}	mm
Base-circle radius of the gear	R_{02i}	mm
Contacting load ratio for pitting resistance	$k_{H\beta}$	-
Center distance of stage 1	a_{w1}	mm
Center distance of stage 2	a_{w2}	mm
Diameter of shaft i	d_{si}	mm
Efficiency of a helical gearbox	η_{hb}	-

Table 1. Cont.

Parameters	Nomenclature	Units
Efficiency of the i stage of the gearbox	η_{gi}	-
Efficiency of a helical gear unit	η_{hg}	-
Efficiency of a rolling bearing pair	η_b	-
Friction coefficient	f	-
Friction coefficient of bearing	f_b	-
Gearbox cross-section area	A_{gb}	dm ²
Gearbox length	L	dm
Gearbox height	H	dm
Gear ratio of stage 1	u_1	-
Gear ratio of stage 2	u_2	-
Gearbox ratio	u_{gb}	-
Gear width of stage 1	b_{w1}	mm
Gear width of stage 2	b_{w2}	mm
Hydraulic moment of power losses	T_H	Nm
ISO viscosity grade number	VG_{40}	
Length of shaft i	l_{si}	mm
Load of bearing i	F_i	N
Material coefficient	k_a	MPa ^{1/3}
Output torque	T_{out}	Nmm
Outside radius of the pinion	R_{e1i}	mm
Outside radius of the gear	R_{e2i}	mm
Outside diameter of the gear of stage i	d_{a2i}	mm
Peripheral speed of bearing	v_b	m/s
Pitch diameter of the pinion of stage 1	d_{w11}	mm
Pitch diameter of the gear of stage 2	d_{w21}	mm
Pitch diameter of the pinion of stage 2	d_{w12}	mm
Pitch diameter of the gear of stage 2	d_{w22}	mm
Power loss in the gears	Pl_g	kW
Power loss in the bearings	Pl_b	kW
Power loss in the seals	Pl_s	kW
Power loss in the idle motion	P_{zo}	kW
Pressure angle	α	rad.
Sliding velocity of gear	v	m/s
Total power loss in the gearbox	Pl	
Torque on the pinion of stage i	T_{1i}	Nmm
Wheel face width coefficient of stage 1	X_{ba1}	-
Wheel face width coefficient of stage 2	X_{ba2}	-

2.1. Determination of Gearbox Cross-Section Area

The gearbox cross-section area, A_{gb} , is determined by

$$A_{gb} = L \cdot H \quad (1)$$

where L and H (Figure 1) are calculated by the following [17]:

$$L = (d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22}/2 + 22.5)/0.975 \quad (2)$$

$$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot S_G \quad (3)$$

$$S_G = 0.005 \cdot L + 4.5 \quad (4)$$

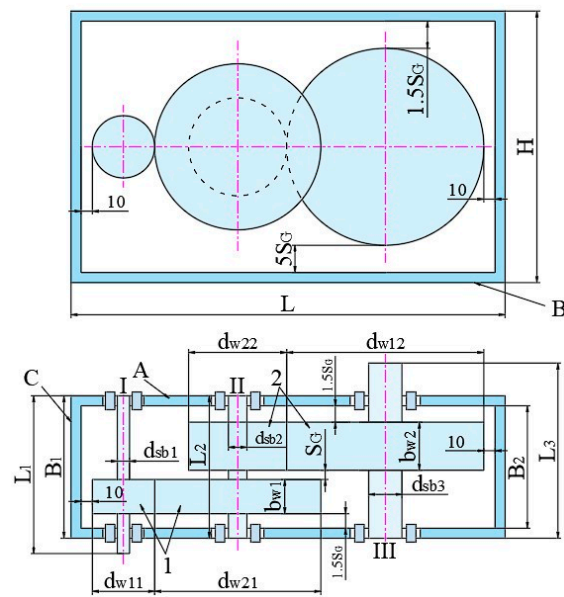


Figure 1. Calculated schema.

In the above equations,

$$d_{w1i} = 2 \cdot a_{wi} / (u_i + 1) \quad (5)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i / (u_i + 1) \quad (6)$$

$$b_{w1} = X_{ba1} \cdot a_{w1} \quad (7)$$

$$b_{w2} = X_{ba2} \cdot a_{w2} \quad (8)$$

In them, a_{wi} is calculated by the following equation based on contact strength condition (for the involute gear profile with a pressure angle of 20 degrees) [18]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta} / ([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (9)$$

with

$$T_{1i} = \frac{T_r}{\prod_{j=i}^3 (u_i \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})} \quad (10)$$

2.2. Determination of Gearbox Efficiency

The efficiency of a two-stage helical gearbox (%) can be found by

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (11)$$

wherein P_l is calculated by [19]

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{zo} \quad (12)$$

where P_{lg} , P_{lb} , P_{ls} , and P_{zo} are determined by the following:

+) Calculation of power loss in the gears, P_{lg} :

$$P_{lg} = \sum_{i=1}^2 P_{lgi} \quad (13)$$

where

$$P_{lgi} = P_{gi} \cdot (1 - \eta_{gi}) \quad (14)$$

In this, η_{gi} is determined by [20]

$$\eta_{gi} = 1 - \left(\frac{1 + 1/u_i}{\beta_{ai} + \beta_{ri}} \right) \cdot \frac{f_i}{2} \cdot (\beta_{ai}^2 + \beta_{ri}^2) \quad (15)$$

with β_{ai} and β_{ri} calculated by [20]

$$\beta_{ai} = \frac{\left(R_{e2i}^2 - R_{02i}^2 \right)^{1/2} - R_{2i} \cdot \sin \alpha}{R_{01i}} \quad (16)$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2 - R_{01i}^2 \right)^{1/2} - R_{1i} \cdot \sin \alpha}{R_{01i}} \quad (17)$$

From the data in [20], f in (15) can be determined by the following regression equations [10]:

- If $v \leq 0.424$ (m/s):

$$f = -0.0877 \cdot v + 0.0525 \quad (18)$$

- If $v > 0.424$ (m/s):

$$f = 0.0028 \cdot v + 0.0104 \quad (19)$$

+) Calculation of power loss in bearings P_{lb} [19]:

$$P_{lb} = \sum_{i=1}^6 f_b \cdot F_i \cdot v_i \quad (20)$$

where $i = 1 \div 6$ and $f_b = 0.0011$ because the radical ball bearings with angular contact were used [19].

+) Calculation of power loss in seals, P_s [19]:

$$P_s = \sum_{i=1}^2 P_{si} \quad (21)$$

where i is the ordinal number of seal ($i = 1 \div 2$), and P_{si} can be determined by

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot \log \log (VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \quad (22)$$

+) Calculation of power loss in idle-motion P_{zo} [19]:

$$P_{zo} = \sum_{i=1}^k T_{Hi} \cdot \frac{\pi \cdot n_i}{30} \quad (23)$$

where k is the total number of gear pairs ($k = 2$), n is the number of revolution of driven gear, and T_{Hi} is calculated by [19]

$$T_{Hi} = C_{Spi} \cdot C_1 \cdot e^{\frac{C_2 \cdot v}{v_{i0}}} \quad (24)$$

where $C_{Spi} = 1$ for stage 1 when the involved oil must pass till the mesh, and, for stage, 2 C_{Spi} can be found by (Figure 2)

$$C_{Spi} = \left(\frac{4 \cdot e_{max}}{3 \cdot h_{Ci}} \right)^{1.5} \cdot \frac{2 \cdot h_{Ci}}{l_{hi}} \quad (25)$$

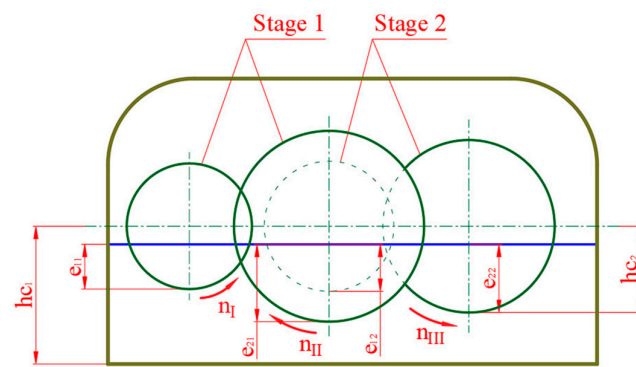


Figure 2. Schema for the determination of the lubrication parameters.

In the above, l_{hi} can be found by the following [19]:

$$l_{hi} = (1.2 \div 2.0) \cdot d_{a2i} \quad (26)$$

In (24), C_{1i} and C_{2i} ($i = 1 \div 2$) are determined by the following [19]:

$$C_{1i} = 0.063 \cdot \left(\frac{e_{1i} + e_{2i}}{e_0} \right) + 0.0128 \cdot \left(\frac{b_{wi}}{b_0} \right) \quad (27)$$

$$C_{2i} = \frac{e_{1i} + e_{2i}}{80 \cdot e_0} + 0.2 \quad (28)$$

2.3. Objective Functions and Constrains

2.3.1. Objective Functions

In this work, the multi-objective optimization problem consists of two single objectives:

- Minimizing the gearbox cross-section area:

$$\min f_1(X) = V_{gb} \quad (29)$$

- Maximizing the gearbox efficiency:

$$\min f_2(X) = \eta_{gb} \quad (30)$$

where X is the vector that represents the design variables. A two-stage helical gearbox has five main design parameters: u_1 , X_{ba1} , X_{ba2} , AS_1 , and AS_2 [10]. Moreover, it has been demonstrated that the maximum values of AS_1 and AS_2 match their ideal values [10]. Consequently, u_1 , X_{ba1} , and X_{ba2} —the three main design factors—have been selected as variables for the optimization issue in this work. Consequently, we have the following:

$$X = \{u_1, X_{ba1}, X_{ba2}\} \quad (31)$$

2.3.2. Constrains

The multi-objective function needs to be subject to the following constraints:

$$1 \leq u_1 \leq 9 \text{ and } 1 \leq u_2 \leq 9 \quad (32)$$

$$0.25 \leq X_{ba1} \leq 0.4 \text{ and } 0.25 \leq X_{ba2} \leq 0.4 \quad (33)$$

3. Methodology

3.1. Method to Solve Multi-Objective Optimization

In this work, the two objectives of the multi-objective optimization problem are the lowest gearbox cross-section area and the best gearbox efficiency. Furthermore, the opti-

mization problem selects three main design factors to be variables, as noted in Section 2. These variables are listed in Table 2 along with the lowest and maximum values. In practice, an MCDM method is difficult to use to solve the MOO (multi-objective optimization) problem. The reason is that, for solving an MOO problem, the number of possible solutions or possibilities is very large. In this work, with three parameters and their limits as shown in Table 2, and the step between variables being 0.02 (to ensure the accuracy of the parameters and not miss the solution of the optimization problem), the number of options (or number of experimental runs) that need to be determined and compared is $(9 - 1)/0.02 \cdot (0.4 - 0.25)/0.02 \cdot (0.4 - 0.25)/0.02 = 22,500$ (runs). With such a large number of options, it is impossible to directly use the MCDM method to solve the MOO problem. This paper uses the SAW method to solve the MCDM problem and the entropy methodology to establish the criterion weights. To supply the input data for the multi-objective optimization problem for a two-stage helical gearbox in the MCDM problem, a simulation experiment is built. The number of experiments that can be conducted by applying the full-factorial design is unlimited because this is a simulation experiment. The total number of experiments is $5^3 = 125$ because there are three experimental variables (as previously mentioned) and five levels for each variable. A method for resolving multi-objective problems is presented in order to lessen this discrepancy, save time, and enhance the correctness of the results (Figure 2). This process is divided into two stages: phase 1 factors solve the single-objective optimization problem to narrow the gap between levels, and phase 2 factors solve the multi-objective optimization problem to identify the best primary design. Furthermore, if the levels of the variables are not sufficiently close to one another or if the best answer is not appropriate for the requirement, the SAW issue is rerun using the smaller distance between two levels of the u_1 in order to address the multi-objective problem (see Figure 3).

Table 2. Input parameters.

Parameters	Symbol	Lower Bound	Upper Bound
Gearbox ratio of the first stage	u_1	1	9
CWFW of stage 1	X_{ba1}	0.25	0.4
CWFW of stage 2	X_{ba2}	0.25	0.4

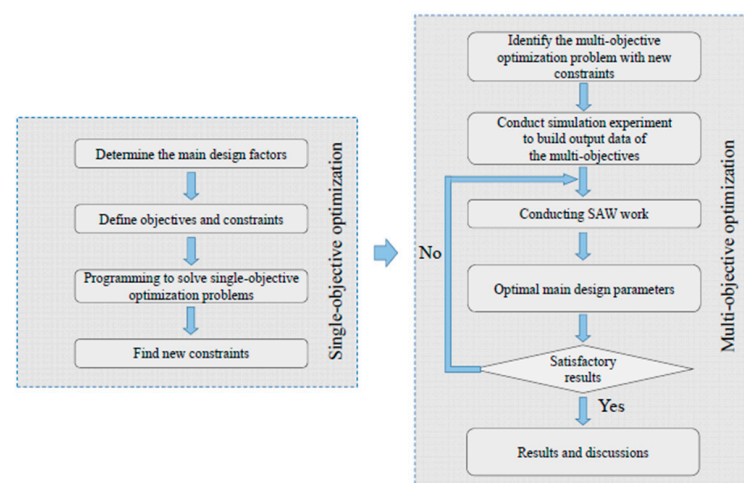


Figure 3. The process for solving the multi-objective problem.

3.2. Method to Solve the MCDM Problem

The SAW approach was first suggested in 2006 [21]. The stages of implementing this method are as follows.

- Creating initial decision-making matrix:

$$X = \begin{matrix} & \begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix} \end{matrix} \quad (34)$$

In which, m and n are alternative and criterion numbers.

- Finding the normalized matrix by

$$n_{ij} = \frac{r_{ij}}{\max r_{ij}} \quad (35)$$

$$n_{ij} = \frac{\min r_{ij}}{r_{ij}} \quad (36)$$

Equation (35) is used for the criterion as gearbox efficiency, while (36) is used for the criterion as the gearbox cross-section area.

- Determine the preference value for each option:

$$V_i = \sum_{j=1}^n w_j \cdot n_{ij} \quad (37)$$

- Rank the options according to the principle that the option with the highest V_i is the best one.

3.3. Method to Find the Weight of Criteria

The weights of the criteria in this research were determined using the entropy technique. This technique can be implemented following the steps outlined below [22].

- Determining indicator-normalized values:

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^m x_{ij}^2} \quad (38)$$

- Finding the entropy for each indicator:

$$me_j = -\sum_{i=1}^m [p_{ij} \times \ln(p_{ij})] - \left(1 - \sum_{i=1}^m p_{ij}\right) \times \ln\left(1 - \sum_{i=1}^m p_{ij}\right) \quad (39)$$

- Calculating the weight of each indicator:

$$w_j = \frac{1 - me_j}{\sum_{j=1}^m (1 - me_j)} \quad (40)$$

4. Single-Objective Optimization

In this work, the single-objective optimization problem is solved using the direct-search approach. Furthermore, two single-objective problems—maximizing gearbox efficiency and minimizing gearbox cross-section area—have been investigated using a Excel computer program (Device ID: D4FEAEE3-C716-5EEE-850A-8F37FAFD3C23). Several figures and observations from the program's findings are listed below: The relationship between u_1 and A_{gb} is depicted in Figure 4. A_{gb} reaches its lowest value when u_1 is at its ideal value (Figure 4). The relation between η_{gb} and u_1 is described in Figure 5. From the figure, it is clear that there is an ideal value of u_1 at which η_{gb} reaches its maximum. The relationships between X_{ba1} and X_{ba2} and A_{gb} and η_{gb} , respectively, are depicted in

Figures 6 and 7. These findings (Figures 6 and 7) show that a rise in X_{ba1} and X_{ba2} will cause A_{gb} and η_{gb} to fall.

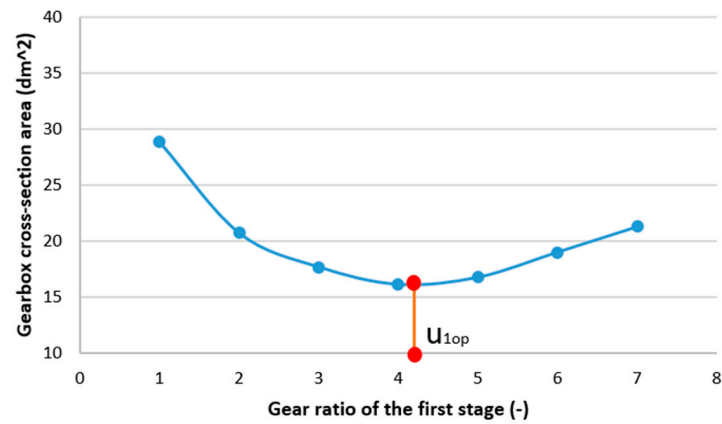


Figure 4. Gearbox cross-section area versus first-stage gear ratio.

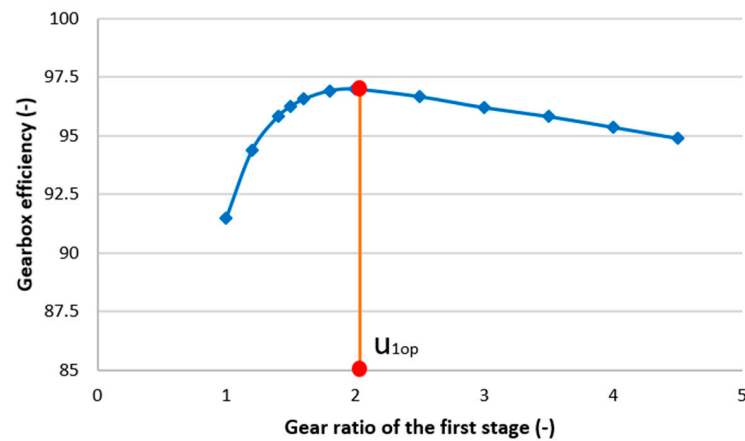


Figure 5. Gearbox efficiency versus first-stage gear ratio.

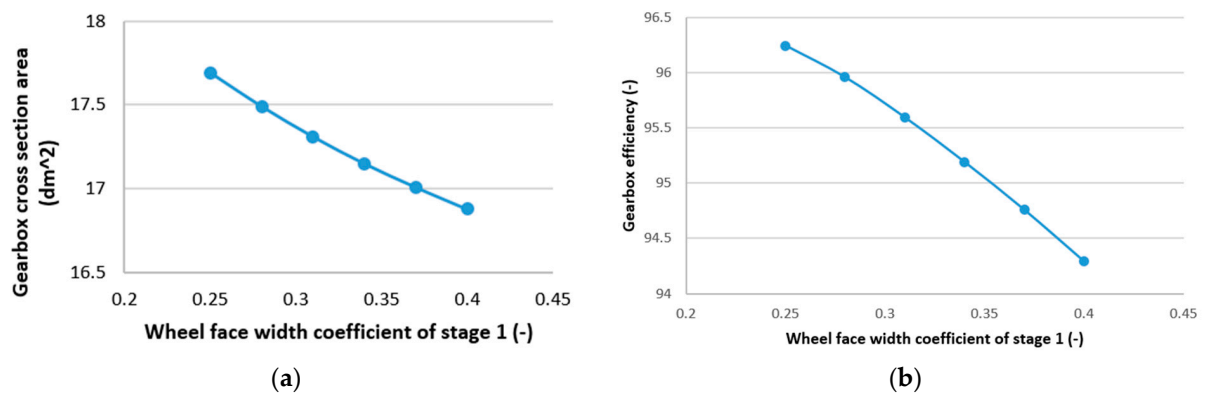


Figure 6. X_{ba1} versus gearbox cross-section area (a) and gearbox efficiency (b).

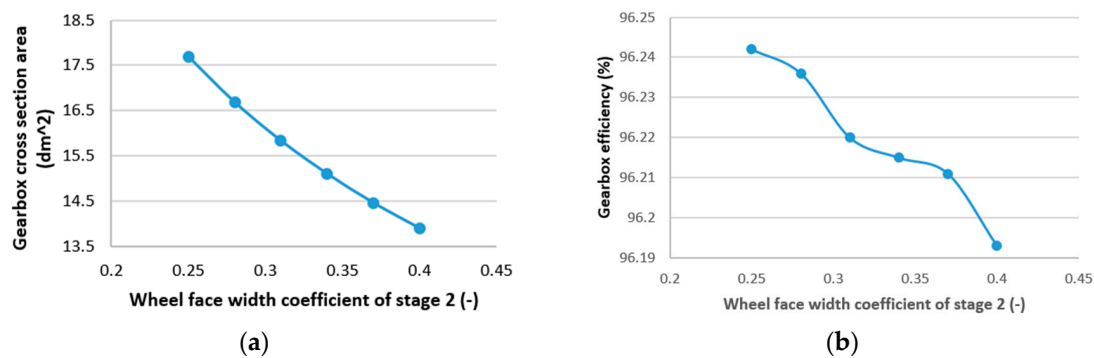


Figure 7. X_{ba2} versus gearbox cross-section area (a) and gearbox efficiency (b).

The values of the optimal major design factors of two single-objective functions, A_{gb} and η_{gb} , are shown in Table 3. This table makes it clear that the A_{gb} function takes the maximum values of X_{ba1} and X_{ba2} ($X_{ba1} = 0.4$ and $X_{ba2} = 0.4$) as the ideal values for X_{ba1} and X_{ba2} . This is because, in order for A_{gb} to be the smallest, d_{w21} and d_{w22} —which are determined using Equation (6)—must also be the smallest. In order to decrease a_{w1} and a_{w2} , X_{ba1} and X_{ba2} must now be at their highest values (Equation (9)). As opposed to the A_{gb} function, the η_{gb} function has the biggest (ideal) value when X_{ba1} and X_{ba2} ($X_{ba1} = 0.25$ and $X_{ba2} = 0.25$) have the smallest values. This is because the smallest values of P_{lg} , P_{lb} , P_{ls} , and P_{zo} correspond to the maximum value of the η_{gb} function. X_{ba1} and X_{ba2} have a significant impact on power loss in the idle-motion P_{zo} among these performance factors. In order for P_{zo} to be small, C_{li} must also be small (Equation (27)). In this case, in order for b_{w1} and b_{w2} to be tiny, X_{ba1} and X_{ba2} must have small values (Equations (7) and (8)).

Table 3. Optimum main design factors of two single objectives.

Objective	Factor	u_t					
		5	10	15	20	25	30
A_{gb}	u_1	2.82	4.49	5.88	7.12	8.26	9.33
	X_{ba1}	0.4	0.4	0.4	0.4	0.4	0.4
	X_{ba2}	0.4	0.4	0.4	0.4	0.4	0.4
η_{gb}	u_1	1.51	2.49	2.98	3.49	3.98	4.42
	X_{ba1}	0.25	0.25	0.25	0.25	0.25	0.25
	X_{ba2}	0.25	0.25	0.25	0.25	0.25	0.25

From Table 3, the relationship between the total gearbox ratio (u_t) and the ideal gear ratio for the first stage u_1 for both single objectives (A_{gb} and η_{gb}) is depicted in Figure 8. Furthermore, freshly computed constraints for the variable u_1 are shown in Table 4.

Table 4. New constraints of u_1 .

u_t	u_1	
	Lower Limit	Upper Limit
5	1.41	2.92
10	2.39	4.59
15	2.88	5.98
20	3.39	7.22
25	3.88	8.36
30	4.32	9.43

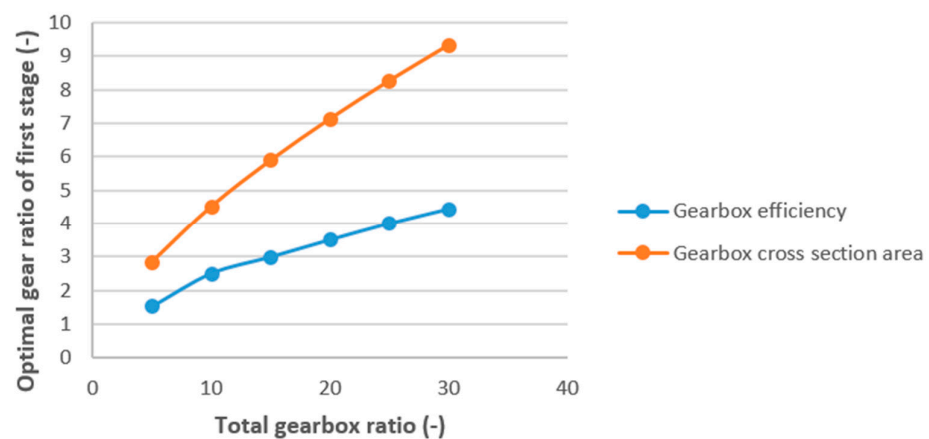


Figure 8. Optimum gear ratio of the first stage versus the total gearbox ratio for single objective functions.

5. Multi-Objective Optimization

In this study, for solving the OMO problem, a simulation experiment was designed and conducted. In this experiment, different variations of the main design factors of a two-stage helical gearbox were established, and the gearbox cross-section area and efficiency (or the output factors) were calculated for use as the input data for the MCDM problem. To conduct simulation experiments, a computer program was developed. For the analysis, the gearbox ratios 5, 10, 15, 20, 25, and 30 were all taken into account. The solutions to this $u_t = 20$ problem are shown below. For the first 125 testing cycles, this gearbox ratio was utilized overall (as indicated in Section 3). To solve the multi-objective optimization issue, SAW received the gearbox cross-section area and the efficiency, which were the experiment's output values. Until there was less than 0.02 between two levels of u_1 , this process was repeated. Table 5 displays the main design parameters and output responses for $u_t = 20$ in the SAW experiment's fourth and final run. The entropy technique (see Section 3.3) was used to determine the weights of the criterion in the following manner: First, we obtained the normalized values of p_{ij} utilizing Equation (38). The entropy value of each indicator m_{ej} was found using Equation (39). Lastly, we determined the weight of the condition w_j using Equation (40). For the latest SAW work run, the weights of A_{gb} and η_{gb} were found to be 0.5676 and 0.4324, respectively. How to use the SAW technique for the MCDM process is explained in Section 3.2. First, we computed the decision-making matrices using Equation (31). The first matrix was then normalized using Equations (35) and (36). After that, V_i was computed using Equation (37). In the end, we sorted the options so that the optimal option had the highest V_i . The options' ranking and number of computed results for $u_t = 20$ (the most recent SAW work run) are shown in Table 6. The table shows that option 95 is the best option out of all the options provided. As a result, Table 5 reveals the ideal values for the three main design features: $u_1 = 6.39$, $X_{ba1} = 0.36$, and $X_{ba2} = 0.4$.

Table 5. Main design parameters and output results for $u_t = 20$ in the fourth run of SAW.

Trial	u_1	X_{ba1}	X_{ba2}	A_{gb} (dm ²)	η_{gb} (%)
1	6.36	0.25	0.25	17.44	93.41
2	6.36	0.25	0.29	16.23	93.39
3	6.36	0.25	0.33	15.25	93.38
4	6.36	0.25	0.36	14.57	93.36
5	6.36	0.25	0.40	14.28	93.39
6	6.36	0.29	0.25	17.18	92.36
...					
23	6.36	0.40	0.33	14.49	88.33
24	6.36	0.40	0.36	13.69	88.31

Table 5. Cont.

Trial	u_1	X_{ba1}	X_{ba2}	A_{gb} (dm ²)	η_{gb} (%)
25	6.36	0.40	0.40	13.02	88.35
...					
50	6.37	0.40	0.40	13.02	88.33
51	6.38	0.25	0.25	17.42	93.39
52	6.38	0.25	0.29	16.21	93.37
...					
71	6.38	0.40	0.25	16.6	88.32
72	6.38	0.40	0.29	15.43	88.3
73	6.38	0.40	0.33	14.47	88.29
...					
94	6.39	0.36	0.36	13.82	89.7
95	6.39	0.36	0.40	13.14	89.74
96	6.39	0.40	0.25	16.59	88.3
...					
123	6.4	0.40	0.33	14.46	88.24
124	6.4	0.40	0.36	13.67	88.22
125	6.4	0.40	0.40	13	88.26

Table 6. Several calculated results and ranking of alternatives by SAW for $u_t = 20$.

Trial	η_{ij}		V_i	Rank
	A_{gb}	η_{gb}		
1	0.7454	1.0000	0.8555	120
2	0.8010	0.9998	0.8870	99
3	0.8525	0.9997	0.9161	75
4	0.8922	0.9995	0.9386	46
5	0.9104	0.9998	0.9490	24
6	0.7567	0.9888	0.8570	109
...				
23	0.8972	0.9456	0.9181	70
24	0.9496	0.9454	0.9478	37
25	0.9985	0.9458	0.9757	14
...				
50	0.9985	0.9456	0.9756	15
51	0.7463	0.9998	0.8559	115
52	0.8020	0.9996	0.8874	92
...				
71	0.7831	0.9455	0.8533	124
72	0.8425	0.9453	0.8870	98
73	0.8984	0.9452	0.9186	67
...				
94	0.9407	0.9603	0.9491	23
95	0.9893	0.9607	0.9770	1
96	0.7836	0.9453	0.8535	122
...				
123	0.8990	0.9447	0.9188	66
124	0.9510	0.9444	0.9482	33
125	1.0000	0.9449	0.9762	9

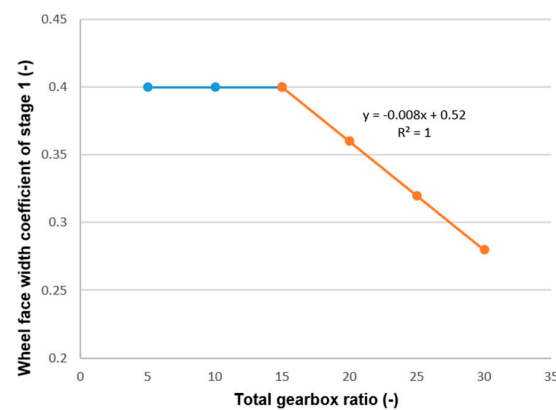
Building on the preceding discussion, Table 7 presents the ideal values for the primary design parameters that correspond to the remaining u_t values of 5, 10, 20, 25, and 30. The data shown in this table allow for the following deductions to be made:

Table 7. Optimum main design parameters.

No.	u_t					
	5	10	15	20	25	30
u_1	2.79	4.37	5.63	6.39	7.72	8.36
X_{ba1}	0.4	0.4	0.4	0.36	0.32	0.28
X_{ba2}	0.4	0.4	0.4	0.4	0.4	0.4

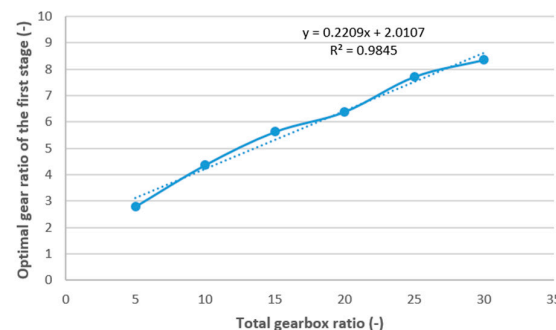
+) X_{ba2} selects the highest value ($X_{ba2} = 0.4$) while X_{ba1} selects the highest value ($X_{ba1} = 0.4$) if $u_t \leq 15$, and it decreases gradually when $15 \leq u_t \leq 30$ (Figure 9). These values are relatively close to the A_{gb} function's ideal values for X_{ba1} and X_{ba2} . This is because it has been discovered that the average weights of A_{gb} and η_{gb} are, respectively, 0.57 and 0.43. Briefly, the A_{gb} function will typically be served by the ideal values of X_{ba1} and X_{ba2} . When $15 \leq u_t \leq 30$, X_{ba2} can be calculated using the following equation (with $R^2 = 1$):

$$X_{ba2} = -0.008 \cdot u_t + 0.52 \quad (41)$$

**Figure 9.** Wheel face width coefficient of the first stage versus the total gearbox ratio.

+) An obvious first-order relationship exists between the ideal values of u_1 and u_t (Figure 10). Additionally, it has been discovered that the ideal values of u_1 may be calculated using the regression equation that follows (with $R^2 = 0.9845$):

$$u_1 = 0.2209 \cdot u_t + 2.0107 \quad (42)$$

**Figure 10.** Optimum gear ratio of the first stage versus the total gearbox ratio for multi-objective function.

The following equation can be used to find the ideal value of u_2 once u_1 has been established:

$$u_2 = u_t / u_1 \quad (43)$$

To assess the model's results for identifying the ideal values when derived using the SAW technique (new method), the findings of this study are compared with those acquired using the Taguchi and grey relational analysis method (old method) in [23]. The ideal values of u_1 corresponding to different u_t determined by the two approaches are shown in Figure 11. Figures 12 and 13, respectively, show the gearbox cross-section area and the efficiency numbers as derived by the old and new methodologies. These statistics demonstrate that the new method generates a substantially smaller gearbox cross-section area (from 28.9 to 45.03%) compared to the calculations obtained using the previous method. However, when $u_t > 13$, the maximum efficiency values determined by the new method are marginally less (less than 4.5%) than those determined by the previous method. For example, when $u_t = 10$, the gearbox cross-section area decreases $(21.43 - 11.78) \cdot 100 / 21.43 = 45.03$ (%), and the gearbox efficiency increases $(91.85 - 89.14) \cdot 100 / 91.85 = 2.95$ (%). When $u_t = 30$, the gearbox cross-section area decreases to $(20.24 - 14.39) \cdot 100 / 20.24 = 28.90$ (%) using new method, but the new method's gearbox efficiency $(94.13 - 90.02) \cdot 100 / 94.13 = 4.57$ (%) is lower than the old method's.

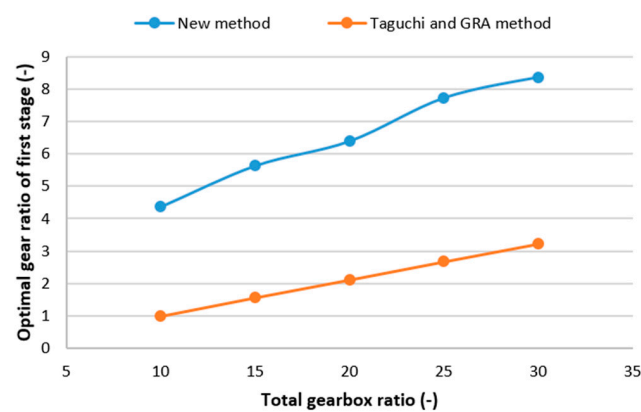


Figure 11. Optimum values of u_1 found by the old and new methods.

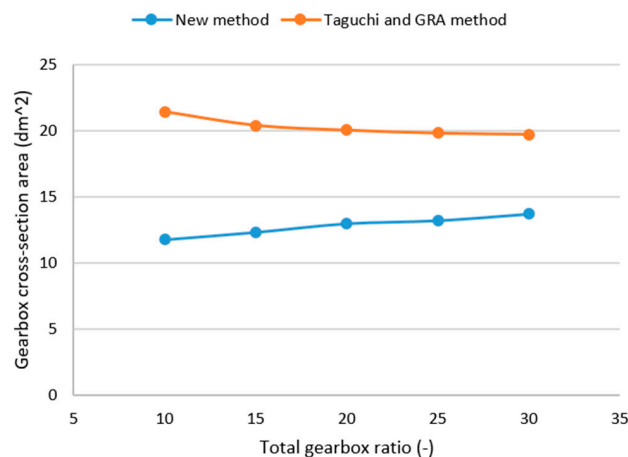


Figure 12. Minimum gearbox cross-section area values found by the old and new methods.

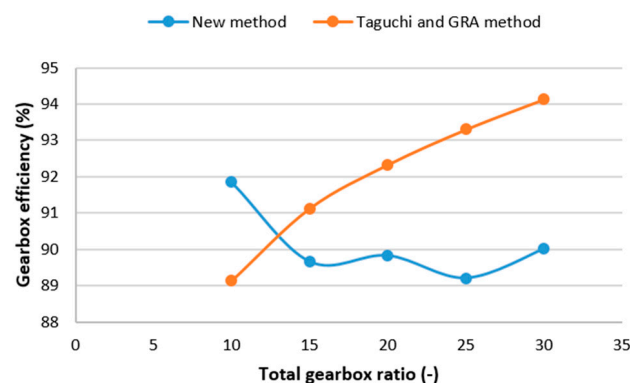


Figure 13. Maximum gearbox efficiency values found by the old and new methods.

6. Conclusions

The SAW technique was utilized in this study to solve the multi-objective optimization problem related to the design of a two-stage helical gearbox. This study's goal was to determine the most critical design parameters that maximized gearbox efficiency while reducing the gearbox cross-section area. To achieve this, three crucial design components were chosen: the CWFV for the first and second stages and the first-stage gear ratio. In addition, there were two phases in the multi-objective optimization problem solution process. Phase 1 was centered on solving the single-objective optimization problem of reducing the difference between variable values, whereas phase 2 was concerned with determining the optimal primary design factors. The following findings were drawn from this work:

- The single-objective optimization problem speeded up and simplified the resolution of the multi-objective optimization problem by bridging the gap between variable levels.
- Two single goals—the maximum gearbox efficiency and the minimum gearbox cross-section area—were assessed in connection with the important design elements.
- By using the SAW technique repeatedly until the required results were reached, the multi-objective optimization problem could be solved more precisely (u_1 had an accuracy of less than 0.02).
- The three main design parameters for a two-stage helical gearbox—Equations (41) and (42) and Table 7—were recommended to have optimal values based on this study's findings.
- Regression Equation (42), with $R^2 = 0.9845$, proved that the experimental data's extraordinary degree of concordance with the proposed model of u_1 verified their reliability.
- The results showed that the novel approach to the multi-objective optimization issue produced better findings when compared with the prior methods (the Taguchi and GRA approaches).

Author Contributions: N.-P.V. came up with the original concept, which all the authors debated. The optimization problem was handled by N.-P.V. with assistance from A.-T.L. The design of the simulation, the examination of the experimental data, and the analysis of the simulation's experimental results were all performed by all authors. The manuscript was written by N.-P.V. with assistance from A.-T.L. and H.-D.T. After reading the manuscript, each author approved it. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data presented in this study are available in this article.

Acknowledgments: The authors express their gratitude to the Thai Nguyen University of Technology for its valuable support in this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest.

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