Technical Note

Preliminary Verification of the PHITS Code Applicability to Conversion Efficiency Calculation of Direct Charge Nuclear Battery

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Abstract: A direct charge nuclear battery, or DCNB, is one of the nuclear batteries based on direct energy conversion and is characterized by exceptional high voltage generation and conversion efficiency higher than other nuclear batteries. For studying potential applications of DCNB, a preliminary estimation of DCNB electrical power and performance is required; hence, conversion efficiency analysis is crucial. For preliminary verification purposes, an ideal DCNB conversion efficiency was calculated under the simplified electron transport model by using the general-purpose Monte Carlo particle transport calculation code PHITS. The result was compared with a reference experimental efficiency for a T-loaded parallel plate DCNB, and the resulting relative error was approximately 12%. Considering the relative error of 20% or less in DCNB conversion efficiency shown by preceding studies, the resulting error was comparable, and it was concluded that the PHITS code is sufficiently applicable to DCNB conversion efficiency analysis.

Keywords: nuclear battery; DCNB; direct energy conversion; conversion efficiency; Monte Carlo method; electron transport; PHITS; EGS5

1. Introduction

Nuclear batteries can utilize radioactive isotopes included in spent nuclear fuels for electricity generation. In the spent fuel, Strontium and Cesium are known as major heat source elements. If they are removed together with Pu and MA recovery, the size of the geological disposal repository is expected to be significantly decreased [1]. Although the use of a fast-spectrum reactor has been proposed for the transmutation of long-lived fission products such as Se-79, Tc-99, Pd-107, and I-129 [2], the utilization of radioactive isotopes in spent nuclear fuels needs to be further investigated to reduce the environmental and social impact inherent to the geological disposal repository.

Nuclear batteries can be categorized as thermoelectric generators (also called RTG) or direct-conversion-based batteries. The latter ones mainly are β- or γ-voltaics [3,4], which are similar to photo-voltaics in terms of using semiconductors and direct charge nuclear batteries (DCNB). General principles and characteristics of these nuclear batteries are summarized in a review article [5] and in a textbook [6]. This study focuses on DCNB for its inherent simplicity in design. The principle of DCNB was developed back in 1913 [7] and summarized in a textbook [8]. DCNB essentially consists of two electrodes and thin α or β sources, as shown in Figure 1. The kinetic energy of the charged particles is directly converted into electrical energy by an electric field. Equation (1) is the example for a β particle:

$$K_e + (-eV_e) = K_C + (-eV_C) \rightarrow K_e - K_C = e(V_e - V_C)$$ (1)
where $e$ is elementary charge, $K_e$ and $K_c$ ($K_e > K_c$) are kinetic energy of a $\beta$ particle at the emitter and collector electrodes, and $V_e$ and $V_c$ ($V_e > V_c$) are electrostatic potentials at the emitter and collector electrodes. For example, keV-order particles generate kV-class high voltage. If all charged particles have identical energy and are emitted perpendicularly to the collector electrode, the efficiency of conversion from charged particle kinetic energy to electrostatic energy is 100% based on Equation (1), which is different from the Carnot cycle. Under steady-state conditions, the supply of $\beta$ particles to the collector electrode needs to be balanced with the consumption of $\beta$ particles at the electrode, and the voltage is generated and maintained by charges accumulated on the collector electrode. Considering insulation, lower energy $\beta$ particles are preferable for stable battery operation rather than $\alpha$ particles with higher energy. Table 1 lists representative $\alpha$ and $\beta$ emitters. On the other hand, the use of a thin source and the insulation gap between electrodes are unfavorable for designing batteries with high power density and small volume.

![Image of a steady-state DCNB using parallel plate electrodes with a $2\pi$ $\beta$ source.](image)

**Figure 1.** Concept of a steady-state DCNB using parallel plate electrodes with a $2\pi$ $\beta$ source.

**Table 1.** Representative $\alpha/\beta$ emitters.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Decay</th>
<th>Average Energy</th>
<th>Half-Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>$\beta$</td>
<td>5.7 keV</td>
<td>12.3 years</td>
</tr>
<tr>
<td>Ni-63</td>
<td>$\beta$</td>
<td>17.4 keV</td>
<td>100.1 years</td>
</tr>
<tr>
<td>Pm-147</td>
<td>$\beta$</td>
<td>62 keV</td>
<td>2.62 years</td>
</tr>
<tr>
<td>Sr-90</td>
<td>$\beta$</td>
<td>198 keV</td>
<td>28.9 years</td>
</tr>
<tr>
<td>Y-90</td>
<td>$\beta$</td>
<td>930 keV</td>
<td>64.6 hours</td>
</tr>
<tr>
<td>Cs-137</td>
<td>$\beta$</td>
<td>157 keV</td>
<td>30.2 years</td>
</tr>
<tr>
<td>Pu-238</td>
<td>$\alpha$</td>
<td>5.59 MeV</td>
<td>87.7 years</td>
</tr>
</tbody>
</table>

Recent studies on DCNB date back to the 2010s. A research group of the University of Illinois Urbana-Champaign (UIUC) performed theoretical, experimental, and numerical studies for H-3 (T)-loaded DCNB using parallel plate electrodes with a $2\pi$ source [9] and $^{147}$Pm-loaded DCNBs using parallel plate electrodes with a $2\pi$ source and cylindrical electrodes with a $4\pi$ source [9,10]. UIUC used Geant4 code [11], a toolkit developed by CERN for Monte Carlo simulations of the passage of particles or radiation through matter, for conversion efficiency calculation and showed a 20% difference between experimental 3.5% and numerical 4.2% efficiencies for the $^{147}$Pm-loaded DCNB using parallel plate electrodes. In the Geant4 code simulation, dominant loss effects on conversion efficiency, such as electron transport in the $^{147}$Pm source, geometrical electron leakage, Coulomb repulsion, electron backscattering, and secondary electron production at the collector electrode, were considered. A numerical optimization of the conversion efficiency of DCNB using a $4\pi$ $^{63}$Ni source and parallel plate electrodes was performed using SuperMC code [12], a CAD-based Monte Carlo program developed by the Fusion Design Study team in China for integrated simulation of nuclear systems. The calculation showed an ideal efficiency of over 20% [13]. Another approach to estimating ideal conversion efficiency was
made by using only theoretical and empirical models of electron transport for $^{63}$Ni-loaded DCNB using parallel plate electrodes with a $2\pi$ source [14–16]. The result showed the largest difference of 18.8% between experimental 6.4% and numerical 8.2% efficiencies. All the preceding studies reviewed here focused only on the scientific investigation of DCNB conversion efficiency, which is required before proposing a promising application of DCNB. However, DCNB is characterized by its inherent simplicity in design compared to RTG and $\beta$- or $\gamma$-voltaics, which require a thermoelectric transducer and semiconductor, respectively. For designing batteries with a long-life and low power density, DCNB is still worth studying.

For studying potential applications of DCNB, a preliminary estimation of the electrical power and performance of DCNB is required; hence, conversion efficiency analysis is crucial. This work presents preliminary verification of a methodology for calculating DCNB steady-state conversion efficiency $\eta_{ss}$ by using Particle and Heavy Ion Transport code System PHITS [17], a general-purpose Monte Carlo particle transport calculation code developed under collaboration between JAEA, RIST, KEK, and several other institutes. MC-based methodology is useful due to its applicability to any geometrical configurations, i.e., planer, cylindrical, and spherical DCNBs, so it is beneficial to have another MC-code that is applicable to DCNB conversion efficiency analysis in addition to the Geant4 code for benchmark studies. Since there have been a few examples of DCNB conversion efficiency calculation by using the PHITS code [18] and no verification research has been reported, this work preliminarily confirmed the applicability of the PHITS code to DCNB analysis by reproducing an ideal conversion efficiency $\eta_{ss}$ using the PHITS code and comparing it with an experimental one. Experimental result of UIUCs T-loaded in parallel plates DCNB [9] was used as the reference because most of experimental conditions are available.

2. Methods

2.1. The Definition of DCNB Steady State Conversion Efficiency

Steady state conversion efficiency $\eta_{ss}$ of a DCNB is defined by the ratio of converted electrical energy $E_{\text{electrical}}$ to thermal energy generated from a $\beta$ source $E_{\text{thermal}}$. If $N_{\text{source}}$ $\beta$ particles with an average energy of $E_{0,\beta}$ are emitted in a $\beta$ source, $E_{\text{thermal}} = E_{0,\beta} \times N_{\text{source}}$. On the other hand, if $N_{\text{arrival}}$ $\beta$ particles arrive at the collector electrode, the converted electrical energy can be calculated as $E_{\text{electrical}} = e(V_{c} - V_{e}) \times N_{\text{arrival}}$. The conversion efficiency $\eta_{ss}$ can be calculated by taking the ratio:

$$\eta_{ss} \equiv \frac{E_{\text{electrical}}}{E_{\text{thermal}}} = \frac{e(V_{c} - V_{e}) \times N_{\text{arrival}}}{E_{0,\beta} \times N_{\text{source}}} = \frac{eV \times P_{\text{arrival}}(V)}{E_{0,\beta}},$$

where $V$ is steady-state voltage between the electrodes, i.e., $V = V_{c} - V_{e}$, and $P_{\text{arrival}}(V)$ is the probability of a source $\beta$ particle to arrive at the collector at steady state voltage $V$, which is defined by:

$$P_{\text{arrival}}(V) \equiv \frac{N_{\text{arrival}}(V)}{N_{\text{source}}},$$

where $N_{\text{arrival}}$ depends on the steady-state voltage $V$ between the electrodes. The probability $P_{\text{arrival}}(V)$ can be further decomposed to:

$$P_{\text{arrival}}(V) = \frac{N_{\text{escape}}}{N_{\text{source}}} \times \frac{N_{\text{arrival}}(V)}{N_{\text{escape}}} = P_{\text{escape}} \times P_{\text{cross}},$$

where $N_{\text{escape}}$ is the number of $\beta$ particles escaped from the source layer out of the $N_{\text{source}}$ source particles. The $P_{\text{escape}}$ is the probability of a source $\beta$ particle to escape from the source layer in the direction to the collector electrode and it depends on the interaction between $\beta$ particles and source layer material. The $P_{\text{cross}}$ is the probability of a $\beta$ particle which escaped from the source layer to cross the electric field between electrodes and it depends only on steady-state voltage $V$. 

$\beta$- or $\gamma$-voltaics.
2.2. Calculation Model

The calculation model used for the preliminary verification is shown on the right-hand side of Figure 2. In the reference UIUC T-loaded parallel plates DCNB experiment, 24 sources (emitter electrodes) and 13 stainless steel collector electrodes were used. The reproduction calculation modeled only a unit cell consisting of a pair of emitter and collector electrodes. The specifications of the model are the same as in the reference experiment. T was used as a β source in the chemical form of ScT₂. The energy spectrum of β particles emitted from T is shown in Figure 3 [19]. The polyimide coating of 0.3–0.4 µm on the collector electrode, which was used for suppressing secondary and backscattering electrons in the UIUC experiment, was ignored for calculating an ideal conversion efficiency. The gap between electrodes was at a pressure lower than 10⁻⁵ Torr in the UIUC experiment but was changed to an inner void in this study.

Figure 2. Calculation model of a unit cell of the UIUC T-loaded parallel plates DCNB.

Figure 3. An example of energy spectrum of β particles emitted from T [19].

2.3. Calculation Methodology

The PHITS code was used for calculating \( P_{\text{arrival}}(V) \) in Equation (3) and \( P_{\text{escape}} \) in Equation (4) for reproducing the conversion efficiency \( \eta_{ss} \) of the reference UIUC experiment. The PHITS code is a general-purpose Monte Carlo particle transport simulation code developed under collaboration between JAEA, RIST, KEK, and several other institutes. It can deal with the transport of all particles over wide energy ranges, using several nuclear reaction models and nuclear data libraries.

In the beginning of the calculation, β particles were generated uniformly and isotropically in the β source layer in Figure 2. The initial energy of each source β particle was decided based on an energy spectrum of the β source element. Transport calculation of the source β particles in the β source layer and in the uniform electric field between electrodes was performed by changing the voltage between electrodes from 0.0 to 20.0 kV. The electron transport model used in the PHITS code is continuous-slowing-down approximation.
(CSDA), including the electron density effect on stopping power, which is the same with the EGS5 code [20], a general-purpose package for the Monte Carlo simulation of the coupled transport of electrons and photons in an arbitrary geometry for particles with energies ranging from a few keV up to several TeV. CSDA is based on the Bethe-Bloch formula and takes the atomic properties of the source layer material (atomic number, atomic mass, mass density) into account in the $P_{\text{escape}}$ calculation. The effect of the electron density of the source layer material on stopping power, i.e., the polarization effect made by source $\beta$ particles in the source layer, is also considered. Secondary and backscattering electrons on the surface of the collector electrode are included in the PHITS calculation. However, for estimating an ideal conversion efficiency $\eta_{\text{ss}}$, the secondary electrons and backscattered $\beta$ particles were not taken into account in the $P_{\text{arrival}}(V)$ calculation. In the reproduction calculation, an ideal conversion efficiency $\eta_{\text{ss}}$ and the probability $P_{\text{escape}}$ were compared with the reference experiment result and condition. Reproduction calculation conditions are summarized in Table 2 and are compared to conditions used in the reference UIUC experiment.

Table 2. Reproduction calculation conditions compared to conditions used in the UIUC reference experiment.

<table>
<thead>
<tr>
<th>UIUC Condition</th>
<th>Reproduction Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ source isotope</td>
<td>$T$</td>
</tr>
<tr>
<td>Material of the source</td>
<td>$\text{ScT}_2$</td>
</tr>
<tr>
<td>Dimensions of the source</td>
<td>10 cm in diameter</td>
</tr>
<tr>
<td>Mass density of $\text{ScT}_2$</td>
<td>2.9 g/cm$^3$</td>
</tr>
<tr>
<td>Material of the electrodes</td>
<td>Stainless steel</td>
</tr>
<tr>
<td>Mass density of the electrodes</td>
<td>N/A</td>
</tr>
<tr>
<td>Dimensions of the electrodes</td>
<td>10 cm in diameter</td>
</tr>
<tr>
<td>Coating on collector electrode</td>
<td>Polyimide</td>
</tr>
<tr>
<td>Electrode gap</td>
<td>0.5 mm in thickness</td>
</tr>
<tr>
<td>Gap pressure</td>
<td>Less than $10^{-5}$ Torr</td>
</tr>
<tr>
<td>Voltage between electrodes</td>
<td>N/A</td>
</tr>
<tr>
<td>PHITS version</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of particles</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of batches</td>
<td>N/A</td>
</tr>
<tr>
<td>Direction of source $\beta$ particles</td>
<td>N/A</td>
</tr>
<tr>
<td>Energy of source $\beta$ particles</td>
<td>N/A</td>
</tr>
<tr>
<td>Distribution of source $\beta$ particles</td>
<td>N/A</td>
</tr>
<tr>
<td>Maximum energy for tallying $P_{\text{escape}}$ and $P_{\text{arrival}}(V)$</td>
<td>N/A</td>
</tr>
</tbody>
</table>

2.4. The Reference Experiment Methodology

As described in Section 2.2, the reference experiment used a multilayer T-loaded parallel plates DCNB, as shown on the left-hand side of Figure 2. Detailed experimental conditions and results are opened to the public through a reference [9], and important specifications of the experimental DCNB are summarized in Table 2. In the reference experiment by UIUC, the $P_{\text{escape}}$ probability in Equation (4) was calculated, and then short circuit current $I_{\text{SC}}$, saturation voltage $V_{\text{sat}}$, and leakage resistance $R_{\text{leak}}$ were measured for estimating the optimum conversion efficiency. In the following subsections, the calculation and measurement processes are summarized.
2.4.1. Calculation of the \( P_{\text{escape}} \) Probability

The probability of a source \( \beta \) particle escaping from the source layer in the direction of the collector electrode, \( P_{\text{escape}} \), can be calculated by:

\[
P_{\text{escape}} = \frac{1}{\varepsilon_{\text{sp}}D} \int_{S} dI_{\beta}(A_{\text{sp}}, D),
\]

where \( A_{\text{sp}} \) is the specific activity in the source layer [Bq/mg] and \( D \) is the mass thickness of the layer [mg/cm\(^2\)]. The \( \frac{dI_{\beta}}{dS} \) is the \( \beta \) particles current density on the surface of the source layer [A/cm\(^2\)] with \( A_{\text{sp}} \) and \( D \). For a thin plane source layer, the current density is theoretically given by \([9,21]\):

\[
\frac{dI_{\beta}(A_{\text{sp}}, D)}{dS} = \frac{1}{\varepsilon_{\text{avg}}} \frac{1}{dP(A_{\text{sp}}, D)} = \frac{0.08A_{\text{sp}}}{\varepsilon_{\text{avg}}} \int_{0}^{\pi} \frac{\pi}{2} \frac{D}{\cos \theta} \left( \varepsilon_{\text{avg}} - \int_{0}^{\rho} W(r) dr \right) d\rho d\theta, \tag{6}
\]

where \( \varepsilon_{\text{avg}} \) is the average energy of source \( \beta \) particles, \( \frac{dP}{dS} \) is the corresponding power density of the \( \beta \) particles on the surface of the source layer, \( W(r) dr \) is the energy absorbed in a \( \beta \) particle travel \( dr \), \( \rho \) is a distance parameter and \( \theta \) is an azimuthal parameter in the integration. In Equation (6), the following working expression of \( W(r) \) in the spherical layer with radius \( r \) was used \([9,22]\):

\[
W(r) = 0.25W_{0}e^{-0.10r\nu} + 0.75W_{0}e^{-2r\nu} + (\varepsilon_{\text{avg}}\nu - 0.4W_{0})r\nu e^{-r\nu}, \tag{7}
\]

where \( r \) is the radius in unit of mass thickness [mg/cm\(^2\)]; \( \nu \) is the mass absorption coefficient [cm\(^2\)/mg]; \( W_{0} \) is the stopping power near the source [keV·cm\(^2\)/mg]. It should be noted that the \( P_{\text{escape}} \) of Equation (5) depends only on the mass thickness of source layer \( D \) because the \( \frac{dI_{\beta}}{dS} \) is proportional to the specific activity \( A_{\text{sp}} \) based on Equation (6). In the UIUC experiment, the probability was calculated using Equation (5) through (7) for scandium tritide with the mass density of 2.9 g/cm\(^3\), the thickness of 300 nm, \( \nu \) of 15.1 cm\(^2\)/mg, and \( W_{0} \) of 56.6 keV·cm\(^2\)/mg.

2.4.2. Estimation of the Optimum Conversion Efficiency

Following the measurement of short circuit current \( I_{SC} \), saturation voltage \( V_{\text{sat}} \), and leakage resistance \( R_{\text{leak}} \), the dependencies of \( V_{\text{sat}} \) and \( I_{\text{load}} \) on the resistance of electrical load \( R_{\text{load}} \) connected to the battery were calculated based on the following equation:

\[
V_{\text{sat}} = R_{\text{leak}} \times I_{SC} P_{\text{arrival}}(V_{\text{sat}}), \tag{8}
\]

\[
R_{\text{leak}} = \frac{R_{\text{battery}}R_{\text{load}}}{R_{\text{battery}} + R_{\text{load}}}, \tag{9}
\]

\[
I_{\text{load}} = \frac{V_{\text{sat}}}{R_{\text{load}}} = I_{SC} \left[ P_{\text{arrival}}(V_{\text{sat}}) - \frac{V_{\text{sat}}}{V_{OC}} P_{\text{arrival}}(V_{OC}) \right], \tag{10}
\]

where \( R_{\text{battery}} \) and \( R_{\text{load}} \) are the internal resistance and the load resistance of the battery, \( I_{\text{load}} \) is load current, and \( V_{OC} \) is the open-circuit voltage. The highest electrical power of approximately 200 \( \mu \)W, which was calculated by \( I_{\text{load}}(R_{\text{load}}) \times V_{\text{sat}}(R_{\text{load}}) \), was obtained if the \( R_{\text{load}} \) was set to 35 G\( \Omega \). The corresponding \( V_{\text{sat}} \) at \( R_{\text{load}} \) of 35 G\( \Omega \) was 2.6 kV. For calculating the optimum conversion efficiency, the source radioactivity \( A_{\text{source}} \) used in the experiment was estimated by using the measured short circuit current of 148 nA and a calculated \( P_{\text{escape}} \) of 0.23 for the 300 nm ScT\(_2\) source thickness:

\[
A_{\text{source}} = \frac{I_{\beta}}{e P_{\text{escape}}} = \frac{1}{1.6021 \times 10^{-19}} \frac{148 \times 10^{-9}}{0.23} \approx 108 \text{ Ci}, \tag{11}
\]
where it is noted that \( I_\beta \) in Equation (11) is equivalent to the measured short circuit current. The optimum conversion efficiency was then calculated by taking the ratio of 200 \( \mu \)W to the source thermal power:

\[
\eta_{ss,\text{ref}} = \frac{P_{\text{electrical}}}{P_{\text{thermal}}} = \frac{200 \times 10^{-6}}{108 \times 3.7 \times 10^{10} \times 5.7 \times 10^{3} \times 1.6021 \times 10^{-19}} \approx 5.5\% , \quad (12)
\]

where the average energy of a source \( \beta \) particle emitted from T is 5.7 keV.

3. Results and Discussion

Table 3 compares \( P_{\text{escape}} \) in Equation (4) calculated with and without SUS-304 emitter electrode. The use of the SUS-304 emitter electrode increased the \( P_{\text{escape}} \) by 5.4\% by reflecting \( \beta \) particles toward the collector electrode. The UIUC condition of 0.23 is a theoretical estimation, not a measured result, and does not include the reflection effect. In the UIUC experiment, ScT\(_2\) radioactivity \( A_{\text{source}} \), i.e., thermal power, was estimated directly using the theoretical value of 0.23. This means that the estimation assumed only 23\% of source \( \beta \) particles escaped from the source layer. Therefore, ScT\(_2\) thermal power was overestimated in the experiment. If the reflection effect of 5.4\% is assumed, \( P_{\text{escape}} \) in the UIUC condition is increased to 0.24, and the experimental conversion efficiency is increased to 5.80\% compared to the original result of 5.5\% at 2.6 kV. Since current and voltage were directly measured in the UIUC experiment, both of them included the reflection effect, and there was no impact on the estimated highest electrical power of 200 \( \mu \)W.

<table>
<thead>
<tr>
<th>SUS-304 Emitter Electrode</th>
<th>Reproduction Calculation</th>
<th>UIUC Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Included</td>
<td>0.247 (( \sigma ): 0.35%)</td>
<td>N/A</td>
</tr>
<tr>
<td>Not included</td>
<td>0.235 (( \sigma ): 0.34%)</td>
<td>0.23</td>
</tr>
</tbody>
</table>

The steady-state conversion efficiency \( \eta_{ss} \) and \( P_{\text{arrival}}(V) \) calculated by the PHITS code are shown in Figure 4a. Following the voltage increase, \( P_{\text{arrival}}(V) \) decreases due to Coulomb repulsion. On the other hand, electrostatic energy converted from the kinetic energy of \( \beta \) particles increases. According to these two competing effects, the conversion efficiency \( \eta_{ss} \) is upper-curved and becomes the maximum of 6.48\% (\( \sigma \): 0.001\%) at 3.0 kV, which has a relative error of 17.9\% to the UIUC original experimental result of 5.5\% at 2.6 kV and 11.9\% to the corrected efficiency of 5.80\%. The residual error will be attributed to no consideration of secondary, backscattering, or leakage electrons at the collector electrode in the reproduction calculation. Considering the relative error of 20\% or less in DCNB conversion efficiency shown by preceding studies, the 11.9\% relative error is comparable, and the PHITS code is sufficiently applicable to DCNB conversion efficiency analysis. The difference in the optimum voltage of 0.4 kV, which is equivalent to an energy increase of 0.4 keV, would be attributed to the complete vacuum condition between electrodes in the calculation. The optimum voltage depends on the energy and angular distribution of \( \beta \) particles escaping from the source layer. The energy spectrum of the escaped \( \beta \) particles, which is normalized to a single escaped \( \beta \) particle, is shown in Figure 4b. The spectrum was calculated on the upper surface of the \( \beta \) source layer. According to the spectrum, one can expect that the optimum voltage will be between 3.0 and 5.0 kV. It should be noted that the spectrum is shifted to the right of the initial spectrum of source \( \beta \) particles. The average energy of the escaped \( \beta \) particles is 6.48 keV, compared to 5.70 keV for the source \( \beta \) particles. This is because \( \beta \) particles with lower energy are self-absorbed in the source layer and are not able to escape from the source.

The conversion efficiency \( \eta \) of DCNB is known to be relatively higher than that of other types of nuclear batteries such as RTG, \( \beta \)- and \( \gamma \)-voltaics. The combinational use of cylindrical or spherical electrodes and 4-\( \pi \) sources will further increase the efficiency. In fact, \( \eta \) of 14\% has already been achieved by experiment [9], where a thin planer 4-\( \pi \) source
of Pm-147 was placed at the center of a surrounding cylindrical collector electrode coated with polyimide insulation for increasing $P_{\text{escape}}$ and $P_{\text{cross}}$. Applications of nuclear batteries are not limited to space missions and have been proposed in the emerging fields of isolated sensors and wireless network systems [4,23]. Even though the kV-class optimum voltage of DCNB will require a step-down transformer for conventional electrical devices, DCNB will be suitable and applicable as a low-power long-life battery to isolated harsh environments such as the deep seabed or the geographic poles for long-term monitoring operations.

![Figure 4](image)

**Figure 4.** (a) Conversion efficiency of the reference T-loaded DCNB; (b) Normalized spectrum of β particles escaped from the source layer. There is no escaped β particles with energy lower than 1.0 keV due to the PHITS code cut-off energy setting.

4. Conclusions

This work presented preliminary verification of a methodology for calculating DCNB steady-state conversion efficiency $\eta_{ss}$ by using the PHITS code. It was confirmed that the efficiency reproduced by the PHITS code for the reference UIUC T-loaded parallel plates DCNB had a relative error of 11.9%, which was comparable to the relative error of 20% or less in DCNB conversion efficiency shown by preceding studies using other Monte Carlo codes or theoretical estimations. This means that the PHITS code is sufficiently applicable to the DCNB conversion efficiency analysis. Consideration of secondary, backscattering, and leakage electrons at the collector electrode will further improve DCNB efficiency calculations.

**Author Contributions:** Conceptualization, H.T.; methodology, H.T.; software, H.T., R.K. and H.U.; validation, H.T., R.K. and H.U.; formal analysis, H.T.; investigation, H.T.; resources, H.T.; data curation, H.T.; writing—original draft preparation, H.T.; writing—review and editing, H.T., F.T., Y.U. and K.T.; visualization, H.T.; supervision, H.T., F.T., Y.U. and T.K.; project administration, H.T. All authors have read and agreed to the published version of the manuscript.

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**References**


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