

Article

Improved Adaptive Cascade Predictive Control for Trajectory Tracking of a Crawler Hydraulic Drill-Anchoring Robot with Slippage Compensation

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Abstract

In the complex operational environment of coal mine shafts, trajectory tracking control of crawler hydraulic drill-anchor robots is susceptible to track slippage and internal–external uncertain disturbances, leading to low tracking accuracy. This issue hinders the implementation of efficient and precise coal mine roadway support operations. To address these challenges, enhance the automation level of coal mine roadway support, and improve operational safety and reliability, research on high-precision trajectory tracking control for crawler hydraulic drill-anchor robots is imperative. Therefore, this paper takes crawler hydraulic drill-anchor robots as the research object and focuses on the trajectory tracking control of such robots. First, a kinematic model incorporating track slippage was established for the crawler hydraulic drill-anchor robot. Second, a cascade predictive control strategy is proposed. The upper-layer trajectory tracking control adopts an adaptive model predictive controller, which adjusts controller weights according to tracking error variations and provides reference rotational speeds for the lower-layer predictive controller. Simulation results of linear and sinusoidal trajectory tracking show that the proposed strategy can effectively compensate for the effects of track slippage and improve trajectory tracking accuracy. Finally, a friction-compensated predictive control method was designed to regulate the rotational speeds of the left and right track drive wheels, and the proposed method achieves optimal control performance with a minimum MEAE of 0.12292 rpm, SDAE of 0.44366 rpm, ITAE of 4.9168, MEACI of 3.0607 mA, SDACI of 1.2497 mA, and ITACI of 122.4283. This performance is significantly superior to that of the conventional PID, ADRC, and MPC methods, thereby realizing high-precision track speed control.

Keywords: cascade predictive control; slippage compensation; trajectory tracking; drill-anchor robot



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1. Introduction

Crawler hydraulic drill-anchor robots, as typical engineering machinery in coal mine support equipment, play a crucial role in support operations. Due to dynamically changing working surfaces—such as uneven terrain and rocky ground—the trajectory of crawler hydraulic drill-and-anchor robots often deviates from preset paths. Trajectory deviations require manual adjustments by operators, which are time-consuming, labor-intensive, and challenging. Such adjustments increase unnecessary travel distances and reduce operational

efficiency. To enhance operational efficiency, adapt to complex and harsh environments, and minimize human intervention, and to address the inherent trajectory deviation tendency of crawler hydraulic drill-and-anchor robots during operations, a controller was designed to continuously adjust the robot position during operation. This controller achieves robot attitude control, providing theoretical guidance and an application foundation for unmanned operation and autonomous navigation of crawler hydraulic drill-and-anchor robots. As tracked vehicles, crawler hydraulic drill-and-anchor robots benefit from extensive global research on track-following control, which provides valuable references.

Common trajectory tracking control algorithms include proportional-integral-derivative (PID) control, active disturbance rejection control, fuzzy control, and linear quadratic regulator (LQR) control. Dai et al. [1] established a multibody dynamics model for hydrodynamic effects and vehicle–sediment mechanical interactions of mining vehicles. A trajectory tracking controller based on a fuzzy adaptive PID method was designed to control both position and heading angle, enabling underwater tracked mining vehicles to follow preset trajectories. However, the trajectory tracking accuracy of this method requires further improvement. Aiming at the trajectory tracking control problem of wheeled mobile robots under unknown slip rates, Li et al. [2] proposed a trajectory tracking controller based on an improved rapid variable exponential reaching law and a sliding mode observer, which can effectively eliminate the influence of slip disturbances, enhance tracking accuracy and smoothness, significantly accelerate the convergence speed, and realize reliable tracking of complex trajectories. Hang et al. [3] designed a linear parameter-varying controller for path tracking, which enhances tracking performance by allocating required torque to both sides of the wheels. Sabiha et al. [4] investigated trajectory tracking control for tracked vehicles and proposed a backstepping controller. Chen et al. [5] incorporated external disturbances and nonlinear dynamics of parameter uncertainties into the control strategy. Qin et al. [6] addressed the slip characteristics between tracked vehicles and terrain by employing a sliding mode based on the instantaneous center of rotation of the track and a two-layer adaptive Kalman filter. Estimated positions and current vehicle states were input into the model to predict future trajectories, enhancing tracking accuracy. Wang et al. [7] established a system-coupled dynamic model considering the kinematic characteristics of unmanned tracked vehicles and proposed an adaptive robust controller to achieve precise trajectory tracking under uncertain disturbances. Song et al. [8] addressed external disturbances and time-varying parameters in mowing robots by employing a sliding mode controller based on an improved fractional power convergence law- an observer was used to track uncertain disturbances, further enhancing trajectory tracking accuracy. Zhang et al. [9] focus on precise path tracking for unmanned crawler harvesters. They design an adaptive system integrating a multi-parameter optimized Crawlers Heading Feedback (CHF) algorithm and nonlinear PID control. Xu et al. [10] proposed an improved hybrid trajectory optimization method for tracked robots to generate efficient, stable, and smooth motion trajectories. Zhang et al. [11] designed a fuzzy logic algorithm for tracked mobile robots to achieve trajectory tracking control. Zhang et al. [12] tackle low trajectory tracking accuracy and poor stability of continuous miners' crawler maneuvering structures in complex roadways. They design a backstepping control law based on Lyapunov stability, using a hyperbolic tangent function as virtual feedback. Particle Swarm Optimization (PSO) optimizes control parameters to enhance performance.

Model Predictive Control (MPC) is a model-based optimal control algorithm that derives control inputs by predicting future system states. Compared to error-feedback-based control algorithms that generate inputs based on error feedback, MPC offers superior control performance and is widely applied in the field of tracked vehicle control. Liu et al. [13] proposed a nonlinear model predictive path tracking control method to enhance the robust

stability of agricultural vehicle trajectory tracking, which demonstrated superior robustness compared to traditional linear model predictive controllers. Zhou et al. [14] propose extending center-driven AGVs' kinematic model to front-wheel drive, design a control scheme based on Lyapunov stability theory and backstepping, and verify its superior trajectory tracking performance via simulations. Gu et al. [15] proposed a trajectory tracking control algorithm based on linear model predictive control for tracked mobile robots, improving tracking accuracy and demonstrating good performance on paths with significant curvature changes. Taner et al. [16] designed a model predictive control strategy with task prioritization. Liang et al. [17] introduced MPC into the motion control of the loader's working mechanism and proposed a trajectory tracking control method. Zang et al. [18] established a kinematic model of the instantaneous center of rotation and proposed a coupled multi-objective iterative distributed model predictive control method to enhance controller accuracy. Li et al. [19] proposed a model predictive control-based track-following controller to enhance disturbance rejection during track-following operations of tracked bulldozers, improving tracking performance. Gong et al. [20] investigated the track-following problem for autonomous underwater vehicles in complex marine environments and designed a Lyapunov-based model predictive control strategy, utilizing online optimization to enhance tracking performance. Li et al. [21] employed a two-layer nonlinear model predictive controller for track-following of tracked robots. The upper nonlinear model predictive control (NMPC) layer performs policy optimization for global trajectory tracking at low frequencies, while the lower NMPC layer captures the sliding dynamics during obstacle crossing, effectively improving transient response and tracking accuracy. Yan et al. [22] designed a dual-loop trajectory tracking method based on model predictive control for autonomous underwater vehicles, ensuring closed-loop trajectory tracking throughout. Wang et al. [23] address trajectory tracking control for hydraulic-driven skid-steer mobile robots. They propose a hierarchical control strategy, which decouples position and hydraulic control, simplifying controller design. Zhao et al. [24] established a sliding kinematic model for tracked vehicles and proposed a trajectory tracking method based on linear model prediction, reducing trajectory tracking error. Gocer et al. [25] proposed a slip-based nonlinear two-track prediction model with simplified shear displacement theory to enhance the trajectory tracking accuracy of unmanned tracked vehicles on firm road surfaces. By integrating it into the model predictive control framework, they confirmed via comparative simulations with kinematic models and field tests on asphalt and stabilized road surfaces that this method can realize precise trajectory tracking with low lateral deviation. Guo et al. [26] proposed a synchronous trajectory tracking controller based on MPC for cruising conditions. Zhan et al. [27] addressed the step-by-step driving control requirements of crawler pavers, adopted quartic polynomial trajectory planning, and designed a nonlinear model predictive controller with transverse-longitudinal integrated control as well as a linear model predictive controller with horizontal and longitudinal decoupling. Confronted with the problems of unstable formation and fragmented control strategies of unmanned ground vehicle (UGV) platoons in mixed traffic flow, Wang et al. [28] proposed a cooperative longitudinal-lateral platoon control framework with dynamic lane management, which adopts a dual-stage multi-objective MPC and adaptive weighting strategy to enhance platoon efficiency and ensure traffic fluidity. Nam et al. [29] designed a model predictive controller considering the steering system model. Yang et al. [30] proposed a double-layer MPC algorithm for on-road autonomous vehicles' collision-free lane tracking, which outperforms existing MPC methods in computation time and collision avoidance efficiency through separated trajectory planning-tracking, precise obstacle avoidance, and anti-rollover constraints. Yue et al. [31] proposed a vehicle coordination control method to satisfy trajectory tracking control performance requirements. Achieving precise trajectory

control for crawler hydraulic drill-anchor robots requires simultaneous high-precision control of position and velocity. Slip occurs at the contact points between the robot tracks and the ground. Furthermore, traditional model predictive control methods employ constant output and input weights, which prevent adaptive adjustment of predictive controller parameters. Track rotational speed is driven by hydraulic motors. Hydraulic motor speed systems exhibit characteristics such as time-varying parameters, nonlinear friction, and internal-external random disturbances, leading to degraded control performance. In recent years, researchers have applied model predictive control to hydraulic control systems. Chen et al. [32] combined adaptive robust sliding mode control (ARSMC) with NMPC for electro-hydraulic systems. The NMPC controller optimally adjusts the hydraulic drive system to meet the dynamic requirements of the outer loop while addressing model nonlinearity and physical constraints. Li et al. [33] proposed an active disturbance rejection-based model predictive control method. Li et al. [34] investigated a nonlinear model predictive control-deep neural network feedforward cross-coupled controller aimed at minimizing synchronization error and counteracting disturbance effects. Zhang et al. [35] designed an adaptive model predictive controller for regulating the output force of hydraulic cylinders. Chen et al. [36] proposed an unbiased explicit model predictive controller and designed a disturbance observer to estimate lumped disturbances comprising model mismatch, time-varying parameters, and uncertain disturbances. Gu et al. [37] combined an extended state observer with MPC, enabling real-time estimation of system states and uncertain disturbances to enhance the MPC's disturbance rejection capability and control performance. Although the aforementioned MPC methods achieved satisfactory control results, their disturbance rejection capability remains relatively weak. Crawler hydraulic drill-anchor robots are driven by low-speed, high-torque hydraulic motors. During operation, these hydraulic motors require a control method with both strong disturbance robustness and high stability.

Based on the international and domestic research status of trajectory tracking for crawler-type hydraulic drill-anchor robots and tracked vehicles, a critical challenge is that high-precision trajectory tracking requires simultaneously guaranteeing the body's trajectory tracking performance and high-precision rotational speed tracking of the bilateral crawlers. To address this challenge, this study aims to achieve high-precision trajectory control for the robot, and by adopting model predictive control (MPC), designs an upper-layer trajectory tracking and a lower-layer rotational speed predictive controller, thus developing an adaptive cascade predictive control method for its trajectory tracking. This proposed method makes three key contributions as follows:

- (1) For the trajectory tracking problem of crawler hydraulic drill-anchor robots, a cascaded trajectory tracking control strategy is proposed. This strategy adopts an inner-outer loop hierarchical structure: the outer-loop predictive controller is responsible for generating an optimal reference rotational speed and transmitting it to the inner-loop predictive controller, thereby ensuring the convergence of the trajectory tracking error. The inner-loop predictive controller acts as a rotational speed tracking executor, which enables the rotational speed output of the robot to accurately track the desired speed value, and further guarantees the precision of trajectory tracking.
- (2) To address the track slippage issue that easily occurs during the operation of crawler hydraulic drill-anchor robots, a kinematic model integrated with a slippage model is established. On this basis, an adaptive model predictive controller is designed, which effectively realizes high-precision trajectory tracking control of the robot.
- (3) Aiming at the control problem of the track speed system of crawler hydraulic drill-anchor robots, a model predictive control method is proposed. This method fully considers the friction characteristics of the hydraulic motor and the inherent con-

straints of the speed control system, thereby improving the control accuracy, stability, and robustness of the track speed system.

The remaining sections of this paper are organized as follows: Section 2 establishes the robot body and rotational speed models. Section 3 designs the proposed two-layer predictive control strategy. Section 4 presents simulation analysis. Section 5 provides conclusions and future research directions.

2. Kinematics Model Establishment of Crawler Hydraulic Drill-Anchor Robot

2.1. Kinematic Model of Crawler Hydraulic Drill-Anchor Robot

Assuming the geometric center of the crawler hydraulic drill-anchor robot coincides with its center of mass, the motion model of crawler hydraulic drill-anchor robot is shown in Figure 1.

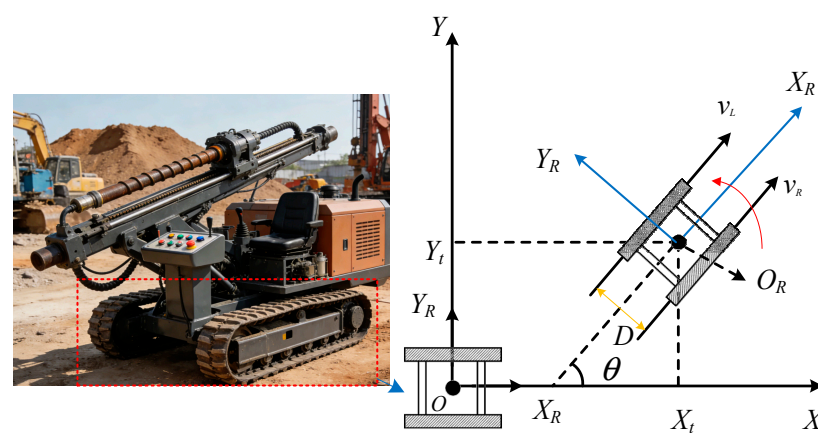


Figure 1. Motion model of crawler hydraulic drill-anchor robot.

In Figure 1, OXY represents the global coordinate system and the red arrow indicates the assumed counterclockwise rotation direction of the crawler hydraulic drill-anchor robot. At the initial moment, the geometric center of the crawler hydraulic drill-anchor robot coincides with the origin O of the global coordinate system, meaning the center-of-mass local coordinate system and the global coordinate system are coincident. After the track travel time t , the relative heading angle between the center-of-mass local coordinate system and the global coordinate system is θ . Counterclockwise rotation is defined as positive angular velocity in the global coordinate system. D denotes the distance between the centers of the two tracks. $O_R X_R Y_R$ represents the center of mass of the crawler hydraulic drill-anchor robot. The rotational speed of the left track is v_L , and the rotational speed of the right track is v_R .

Based on track kinematics, the motion state equations for the crawler hydraulic drill-anchor robot from the initial time to time t is expressed as

$$\begin{cases} x(t) = \frac{1}{2} \int_0^t (v_R + v_L) \cos \theta dt \\ y(t) = \frac{1}{2} \int_0^t (v_R + v_L) \sin \theta dt \\ \theta(t) = \frac{1}{D} \int_0^t (v_R - v_L) dt \end{cases} \quad (1)$$

where $x(t)$, $y(t)$, and $\theta(t)$ represent the position and heading angle of the robot's center of mass in the global coordinate system.

By taking the derivative of the above three equations, the velocities in the X and Y directions and the yaw angular velocity can be derived, which is given in Equation (2).

$$\begin{cases} \dot{x}(t) = \frac{1}{2}(v_R + v_L) \cos \theta \\ \dot{y}(t) = \frac{1}{2}(v_R + v_L) \sin \theta \\ \dot{\theta}(t) = \frac{v_R - v_L}{D} \end{cases} \quad (2)$$

Rewriting Equation (2) in state equation form,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \cos \theta & \frac{1}{2} \cos \theta \\ \frac{1}{2} \sin \theta & \frac{1}{2} \sin \theta \\ \frac{1}{D} & -\frac{1}{D} \end{bmatrix} \begin{bmatrix} v_R \\ v_L \end{bmatrix} \quad (3)$$

Considering track slippage during actual steering, which affects trajectory tracking performance, the left and right track speeds of the crawler hydraulic drill-anchor robot under slippage conditions are

$$\begin{cases} v_R = \omega_R(1 - \sigma_R)r \\ v_L = \omega_L(1 - \sigma_L)r \end{cases} \quad (4)$$

where ω_R and ω_L represent the angular velocities of the left and right tracks, respectively, r denotes the radius of the track drive wheel, and σ_R and σ_L denote the slip rates of the left and right tracks, respectively.

The heading angular velocity is

$$\dot{\theta} = \frac{\omega_R(1 - \sigma_R)r - \omega_L(1 - \sigma_L)r}{D} \quad (5)$$

The kinematic equations of the crawler hydraulic drill-anchor robot in the global coordinate system are

$$\begin{cases} \dot{x} = \frac{1}{2}[\omega_L r(1 - \sigma_L) + \omega_R r(1 - \sigma_R)] \cos \theta \\ \dot{y} = \frac{1}{2}[\omega_L r(1 - \sigma_L) + \omega_R r(1 - \sigma_R)] \sin \theta \\ \dot{\theta} = \frac{\omega_R(1 - \sigma_R)r - \omega_L(1 - \sigma_L)r}{D} \end{cases} \quad (6)$$

The movement speeds of the robot's left and right tracks are

$$v_L = \frac{C_w \omega_L(1 - \sigma_L)}{2\pi} \quad (7)$$

$$v_R = \frac{C_w \omega_R(1 - \sigma_R)}{2\pi} \quad (8)$$

where C_w denotes the circumference of the track drive wheel per full rotation.

2.2. Trajectory Deviation Model of Crawler Hydraulic Drill-Anchor Robot

The motion equation of the center of mass of the crawler hydraulic drill-anchor robot in the global coordinate system is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \\ r(1 - \sigma_R)/D & -r(1 - \sigma_L)/D \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} \quad (9)$$

where $s_{11} = \frac{1}{2}r(1 - \sigma_L) \cos \theta$, $s_{12} = \frac{1}{2}r(1 - \sigma_R) \cos \theta$, $s_{21} = \frac{1}{2}r(1 - \sigma_L) \sin \theta$, and $s_{22} = \frac{1}{2}r(1 - \sigma_R) \sin \theta$.

For a given reference trajectory, where the superscript p_r represents the reference quantity, we have

$$\dot{\mathbf{p}}_r = f(\mathbf{x}_r, \mathbf{u}_r) \quad (10)$$

where Equation (11) is given as

$$\begin{cases} \mathbf{x}_r = \begin{bmatrix} x_r & y_r & \theta_r \end{bmatrix}^T \\ \mathbf{u}_r = \begin{bmatrix} v_{Rr} & v_{Lr} \end{bmatrix}^T \end{cases} \quad (11)$$

Let the state variable of the crawler hydraulic drill-anchor robot be $\mathbf{x} = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$, and the left and right track speeds of the robot be $\mathbf{u} = \begin{bmatrix} v_R & v_L \end{bmatrix}^T$. Then, the corresponding expression Equation (12) is given as

$$\dot{\mathbf{p}} = f(\mathbf{x}, \mathbf{u}) \quad (12)$$

Expanding using Taylor series and neglecting higher-order terms yields Equation (13):

$$\dot{\mathbf{p}} = f(\mathbf{x}_r, \mathbf{u}_r) + \left. \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \right|_{\substack{\mathbf{x} = \mathbf{x}_r \\ \mathbf{u} = \mathbf{u}_r}} (\mathbf{x} - \mathbf{x}_r) + \left. \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}} \right|_{\substack{\mathbf{x} = \mathbf{x}_r \\ \mathbf{u} = \mathbf{u}_r}} (\mathbf{u} - \mathbf{u}_r) \quad (13)$$

Subtracting Equation (11) from, Equation (14) is expressed as

$$\dot{\bar{\mathbf{x}}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}_r = \bar{\mathbf{A}}\bar{\mathbf{x}} + \bar{\mathbf{B}}\bar{\mathbf{u}} \quad (14)$$

where $\bar{\mathbf{x}} = \mathbf{x} - \mathbf{x}_r$ is the state error, $\bar{\mathbf{u}} = \mathbf{u} - \mathbf{u}_r$ is the control error, $\bar{\mathbf{A}}$ is the state error coefficient matrix, and $\bar{\mathbf{B}}$ is the control error coefficient matrix, specifically expressed as

$$\bar{\mathbf{A}} = \begin{bmatrix} 0 & 0 & -\frac{1}{2}[\omega_{Lr}r(1 - \sigma_L) + \omega_{Rr}r(1 - \sigma_R)] \sin \theta_r \\ 0 & 0 & \frac{1}{2}[\omega_{Lr}r(1 - \sigma_L) + \omega_{Rr}r(1 - \sigma_R)] \cos \theta_r \\ 0 & 0 & 0 \end{bmatrix} \quad (15)$$

$$\bar{\mathbf{B}} = \begin{bmatrix} \frac{1}{2}(1 - \sigma_L) \cos \theta_r & \frac{1}{2}(1 - \sigma_R) \cos \theta_r \\ \frac{1}{2}(1 - \sigma_L) \sin \theta_r & \frac{1}{2}(1 - \sigma_R) \sin \theta_r \\ \frac{(1 - \sigma_R)r}{D} & -\frac{(1 - \sigma_L)r}{D} \end{bmatrix} \quad (16)$$

Discretizing Equation (14) yields

$$\dot{\bar{\mathbf{x}}}(t+1) = (T\bar{\mathbf{A}} + I)\bar{\mathbf{x}}(t) + (T\bar{\mathbf{B}} + I)\bar{\mathbf{u}}(t) \quad (17)$$

Among these, I represents the unit matrix, T denotes the sampling time, $\bar{\mathbf{x}}(t)$ and $\bar{\mathbf{u}}(t)$ are expressed respectively as

$$\bar{\mathbf{x}} = \mathbf{x} - \mathbf{x}_r = \begin{bmatrix} x - x_r \\ y - y_r \\ \varphi - \varphi_r \end{bmatrix}, \bar{\mathbf{u}} = \mathbf{u} - \mathbf{u}_r = \begin{bmatrix} v_R - v_{Rr} \\ v_L - v_{Lr} \end{bmatrix} \quad (18)$$

2.3. Hydraulic Motor Model

2.3.1. Dynamic Characteristics of Proportional Directional Valve

When the spool displacement is $x_v \geq 0$, the flow rate into the inlet chamber Q_1 and the flow rate out of the return chamber Q_2 of the hydraulic motor are expressed as

$$\begin{cases} Q_1 = c_d \omega x_v \sqrt{2/\rho} \sqrt{P_s - P_1} \\ Q_2 = c_d \omega x_v \sqrt{2/\rho} \sqrt{P_2} \end{cases} \quad (19)$$

When the spool displacement is $x_v < 0$, the flow rate entering the hydraulic motor Q_1 and the flow rate exiting the hydraulic motor Q_2 are expressed respectively as

$$\begin{cases} Q_1 = c_d \omega x_v \sqrt{2/\rho} \sqrt{P_1} \\ Q_2 = c_d \omega x_v \sqrt{2/\rho} \sqrt{P_s - P_2} \end{cases} \quad (20)$$

where c_d is the proportional directional valve orifice flow coefficient, ω is the proportional directional valve throttle orifice area gradient, x_v is the proportional directional valve spool displacement, ρ is the hydraulic oil density, P_1 denotes the outlet pressure of the proportional directional valve, P_2 is the proportional directional valve return pressure, and P_s is the proportional valve supply pressure.

The relationship between the input signal and spool displacement of the proportional directional valve is

$$x_v = k_v i \quad (21)$$

where k_v is the gain coefficient between the control input and spool displacement, i is the control input applied to the proportional directional valve.

Defining the load flow as $Q_L = (Q_1 + Q_2)/2$ and the load pressure as $P_L = P_1 - P_2$, the linearized flow equation for the proportional directional valve can be derived as follows:

$$Q_L = k_a x_v \sqrt{P_s - P_L} \operatorname{sgn}(u) \quad (22)$$

where k_a is the flow gain coefficient, $k_a = c_d \omega \sqrt{1/\rho}$, P_L is the external load pressure, and sgn denotes the sign function.

2.3.2. Flow Characteristics of Hydraulic Motor

The total compression volume of the hydraulic motor V_t is given by

$$V_t = V_1 + V_2 \quad (23)$$

where V_1 and V_2 are the initial volumes of the two chambers of the hydraulic motor, respectively.

The flow continuity equation for the hydraulic motor is as follows:

$$Q_L = D_m \frac{d\theta_m}{dt} + c_{ip} P_L + \frac{V_t}{4\beta_e} \dot{P}_L + \Delta Q \quad (24)$$

where D_m is the displacement of the hydraulic motor, θ_m is the angular displacement of the hydraulic motor, c_{ip} is the internal leakage coefficient of the hydraulic motor, β_e is the effective volumetric elastic modulus of the hydraulic fluid, and ΔQ is the modeling error, typically arising from internal leakage and difficult-to-model factors.

2.3.3. Load Characteristics of Hydraulic Motor

The torque balance equation between the output torque and external load is expressed as

$$D_m P_L = J_t \frac{d^2 \theta_m}{dt^2} + B \frac{d\theta_m}{dt} + G\theta_m + T_L + T_f + \Delta T \quad (25)$$

where J_t is the total moment of inertia of the hydraulic motor and load, B is the viscous damping coefficient, G is the load elastic stiffness, T_L is the external load torque acting on the motor shaft, T_f is the friction torque, and ΔT is the sum of uncertainty disturbance and fitting friction deviation.

2.3.4. Hydraulic Motor Friction Torque Model

This paper employs the Stribeck friction model to establish the rotational speed system friction torque model:

$$T_s = \left[T_{cb} + (T_s - T_{cb})e^{-\alpha_f/v} \right] \text{sgn}\left(\frac{d\theta_m}{dt}\right) + k_{fv} \frac{d\theta_m}{dt} \quad (26)$$

where T_{cb} is the Coulomb friction torque, T_s is the maximum static friction torque, α_f is the Stribeck velocity coefficient, and k_{fv} is the viscous friction torque coefficient.

2.3.5. State Space Equation of Hydraulic Motor

The state vector is defined as $[x_1 \ x_2 \ x_3]^T = [\theta_m \ \omega \ \alpha]^T$. The state-space equation for the rotational speed system is expressed as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = a_1 x_2 + a_2 x_3 + bu + T_L + T_f + \Delta T \\ y = x_2 \end{cases} \quad (27)$$

where $V = (4\beta_e)^{-1} V_t$, $K_q = k_a \sqrt{P_s - P_L \text{sgn}(u)}$, $a_1 = D_m^2 / J_L V + c_{ip} B / J_L V$, $a_2 = c_{ip} / V + B / J_L$, $b = D_m k_q k_v / J_L V$, and $d = c_{ip} T_L / J_t V + \dot{T}_L / J_L + D_m \Delta Q / J_L V$.

3. Cascaded Trajectory Tracking Control Strategy for Crawler Hydraulic Drill-Anchor Robot

3.1. Design of Adaptive Trajectory Tracking Predictive Controller

The trajectory tracking process of the crawler hydraulic drill-anchor robot is shown in Figure 2.

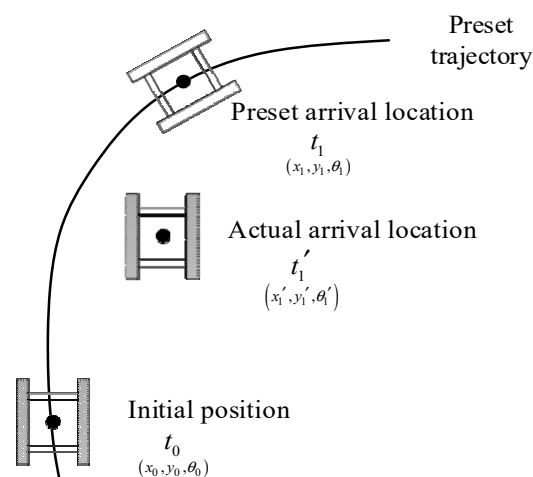


Figure 2. Trajectory tracking process of crawler hydraulic drill-anchor robot.

We conclude that

$$\hat{\mathbf{x}}(t+i|t) = \mathbf{A}^n \mathbf{x}(t) + \begin{bmatrix} \mathbf{A}^{n-1} \mathbf{A}^{n-2} & \cdots & \mathbf{I} \end{bmatrix} \mathbf{B} \begin{bmatrix} \hat{\mathbf{u}}(t|t) \\ \hat{\mathbf{u}}(t+1|t) \\ \vdots \\ \hat{\mathbf{u}}(t+n-1|t) \end{bmatrix}, i = 1, \dots, N_p \quad (30)$$

At the current time t , let

$$\Delta \hat{\mathbf{u}}(t+i|k) = \hat{\mathbf{u}}(t+i|t) - \hat{\mathbf{u}}(t+i-1|t) \quad (31)$$

A new state variable is defined:

$$\boldsymbol{\eta}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t-1) \end{bmatrix} \quad (32)$$

Substituting into Equation (29) gives

$$\boldsymbol{\eta}(t+1) = \tilde{\mathbf{A}} \boldsymbol{\eta}(t) + \tilde{\mathbf{B}} \Delta \mathbf{u}(t) \quad (33)$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{I} \end{bmatrix}, \tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \quad (34)$$

Within the prediction time domain N_p , the system prediction model is

$$\begin{cases} \hat{\boldsymbol{\eta}}(t+N_p|t) = \tilde{\mathbf{A}}^{N_p} \boldsymbol{\eta}(t) + \tilde{\mathbf{A}}^{N_p-1} \tilde{\mathbf{B}} \Delta \hat{\mathbf{u}}(t|t) + \cdots + \tilde{\mathbf{A}}^{N_p-N_c-1} \tilde{\mathbf{B}} \Delta \hat{\mathbf{u}}(t+N_c|t) \\ \hat{\mathbf{y}}(t+N_p|t) = \tilde{\mathbf{C}} \hat{\boldsymbol{\eta}}(t+N_p|t) \end{cases} \quad (35)$$

Based on the above equation, the system prediction output is

$$\mathbf{Y}(t) = \boldsymbol{\Psi} \boldsymbol{\eta}(t) + \boldsymbol{\Theta} \Delta \mathbf{U}(t) \quad (36)$$

where

$$\mathbf{Y}(t) = \begin{bmatrix} \hat{\mathbf{y}}(t+1|t) \\ \hat{\mathbf{y}}(t+2|t) \\ \vdots \\ \hat{\mathbf{y}}(t+N_c|t) \\ \vdots \\ \hat{\mathbf{y}}(t+N_p|t) \end{bmatrix}, \boldsymbol{\Psi} = \begin{bmatrix} \tilde{\mathbf{C}} \tilde{\mathbf{A}} \\ \tilde{\mathbf{C}} \tilde{\mathbf{A}}^2 \\ \vdots \\ \tilde{\mathbf{C}} \tilde{\mathbf{A}}^{N_c} \\ \vdots \\ \tilde{\mathbf{C}} \tilde{\mathbf{A}}^{N_p} \end{bmatrix}, \Delta \mathbf{U}(t) = \begin{bmatrix} \Delta u(t|t) \\ \Delta u(t+1|t) \\ \vdots \\ \Delta u(t+N_c|t) \end{bmatrix}$$

$$\boldsymbol{\Theta} = \begin{bmatrix} \tilde{\mathbf{C}} \tilde{\mathbf{B}} & \mathbf{0} & \cdots & \mathbf{0} \\ \tilde{\mathbf{C}} \tilde{\mathbf{A}} \tilde{\mathbf{B}} & \tilde{\mathbf{C}} \tilde{\mathbf{B}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{C}} \tilde{\mathbf{A}}^{N_c-1} \tilde{\mathbf{B}} & \tilde{\mathbf{C}} \tilde{\mathbf{A}}^{N_c-2} \tilde{\mathbf{B}} & \cdots & \tilde{\mathbf{C}} \tilde{\mathbf{B}} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{C}} \tilde{\mathbf{A}}^{N_p-1} \tilde{\mathbf{B}} & \tilde{\mathbf{C}} \tilde{\mathbf{A}}^{N_p-2} \tilde{\mathbf{B}} & \cdots & \tilde{\mathbf{C}} \tilde{\mathbf{A}}^{N_p-N_c-1} \tilde{\mathbf{B}} \end{bmatrix}$$

(2) Objective Function Design

In the model predictive controller, the objective function designed in this paper is as follows:

$$\min J(t) = \sum_{i=1}^{N_p} \|\mathbf{Y}(t+i) - \mathbf{Y}_r(t+i)\|_Q^2 + \sum_{i=0}^{N_c-1} \|\Delta \mathbf{U}(t+i)\|_R^2 \quad (37)$$

$$\begin{cases} \mathbf{Y}_{\min} \leq \mathbf{Y}(t+i) \leq \mathbf{Y}_{\max} & 0 \leq i < N_p \\ \Delta \mathbf{U}_{\min} \leq \Delta \mathbf{U}(t+i) \leq \Delta \mathbf{U}_{\max} & 0 < i \leq N_c \end{cases} \quad (38)$$

where $\mathbf{Y}_r$ is the desired system state, $\mathbf{Y}$ is the current system state prediction, $\Delta \mathbf{U}$ is the control input increment, Q is the system error weighting matrix, R is the system control weighting matrix, $\mathbf{Y}_{\min}$ and $\mathbf{Y}_{\max}$ are output state constraints, and $\Delta \mathbf{U}_{\min}$ and $\Delta \mathbf{U}_{\max}$ are control input constraints.

The objective function is solved to obtain the control input sequence within the control cycle:

$$\Delta \mathbf{U}(t) = [\Delta u(t), \Delta u(t+1) \dots \Delta u(t+N_c-1)] \quad (39)$$

The first control increment of the control sequence $\Delta u(t)$ is applied to the system:

$$u(t) = u(t-1) + \Delta u(t) \quad (40)$$

After obtaining the state variables and output at the next time step, the objective function is re-evaluated to generate a new control sequence. The first control quantity is applied to the system and the process is rolled forward until the trajectory is completed.

(3) Adaptive Weight Design

When the tracking error is large, the output weighting matrix Q is increased to enhance response speed, enabling the system to track the target trajectory more rapidly. When the tracking error is small, the input weighting matrix R is increased to smooth the control signal. By dynamically adjusting Q and R based on error magnitude, the following adaptive function is designed:

$$\begin{cases} Q = Q_0 \cdot (1 + \alpha \cdot e_{xy} + \beta \cdot |e_\theta|) \\ R = R_0 \cdot (1 + \gamma / (e_{xy} + |e_\theta|)) \end{cases} \quad (41)$$

where Q_0 and R_0 are the base weights, α and β are adjustment coefficients controlling the influence of position error and heading angle error on Q , γ is an adjustment coefficient controlling the influence of error on R , $e_{xy} = \left| \sqrt{(x-x_d)^2 + (y-y_d)^2} \right|$ is the position tracking error, and $|e_\theta|$ is the absolute value of the heading angle tracking error.

3.2. Design of Predictive Controller Based on Extended State Observer

To achieve high-precision control of the hydraulic motor, a predictive control method based on state feedback is proposed. During design, a known friction model is incorporated and the system state and disturbances estimated by the observer are fed back to the predictive model. The control law is then solved to realize high-precision control of the hydraulic motor speed.

3.2.1. Design of Speed System Extended State Observer

For the state-space equation in (28), since the rotational speed is known, the parameter uncertainty and external disturbance are set as the total disturbance $\hat{x}_4$. Let ζ denote the derivative of the disturbance estimate, $\zeta = \dot{\hat{x}}_4$, and $\hat{x}_i$ denote the state estimate of

the extended state observer. A reduced-order linear extended state observer (LESO) is constructed as follows:

$$\begin{cases} e_{22} = \hat{x}_2 - x_2 \\ \dot{\hat{x}}_2 = \hat{x}_3 - \beta_1 e_{22} \\ \dot{\hat{x}}_3 = a_1 \hat{x}_2 + a_2 \hat{x}_3 + bu + T_f + \hat{x}_3 - \beta_2 e_{22} \\ \dot{\hat{x}}_4 = -\beta_3 e_{22} \end{cases} \quad (42)$$

where $\hat{x}_2$ is the observed angular velocity of x_2 , $\hat{x}_3$ is the observed load pressure of the system, $\hat{x}_4$ is the estimated total disturbance of the system, and $\beta_i (i = 1, 2, 3)$ are the LESO gains.

The stability analysis of the reduced-order linear extended state observer is conducted, with a detailed proof provided.

3.2.2. Design of Speed System Predictive Controller

(1) Prediction Model

By discretizing the continuous-time model, the predictive model is represented in discrete form:

$$\begin{cases} \dot{x}(k) = A_1 x(k-1) + B_1 u(k-1) + C_1 \Delta T(k) + C_2 T_f(k) \\ y(k) = D_1 x(k) \quad k = 1, 2, \dots \end{cases} \quad (43)$$

where, A_1, B_1, C_1, C_2 and D_1 are coefficient matrices; $\dot{x}(k)$ is the state vector in the predictive model, $y(k)$ is the state output, $T_f(k)$ is the friction torque, and $\Delta T(k)$ represents unmodeled terms, uncertain disturbances, and friction torque modeling errors.

(2) Optimization Solution

Within the prediction time domain, the state variables and predicted outputs are computed based on the system's current state variables and control inputs. The design optimization objective function is formulated as follows:

$$J(x(k), U(k)) = \sum_{k=1}^{N_p} \left[(y(k) - y_r(k))^T Q_m (y(k) - y_r(k)) + \Delta u(k)^T R_m \Delta u(k) \right] \quad (44)$$

where $y(k)$ is the rotational speed output prediction, $y_r(k+i|k), i = 1, \dots, N_p$ is the desired setpoint signal, Q_m is the output weighting matrix, R_m is the control increment weighting matrix, $U(k) = [u(k), \dots, u(k+N_u-1)]^T$ is the control input sequence within the control time domain, and $x(k)$ is the state variable of the system under the action of the input vector sequence $U(k)$.

The control variable constraints for the objective function in the time domain are

$$u_{\min}(k+i|k) \leq u(k+i|k) \leq u_{\max}(k+i|k) \quad (45)$$

By solving the objective function, the optimal control sequence $U_k^*(k) = [u_k^*(k), \dots, u_k^*(k+N_u-1)]^T$ is obtained. The first element of this control sequence is used as the actual control input.

4. Simulation Verification

To validate the tracking performance of the designed controller, simulations were conducted for both straight-line and continuous curved-trajectory tracking using the proposed trajectory-following controller. Under slip conditions, trajectory tracking performance with and without the slip-model predictive controller was analyzed to verify the controller

performance and effectiveness. All results presented in this article are derived exclusively from numerical simulations conducted using MATLAB (Version R2024a).

4.1. Trajectory Tracking Simulation Performance Analysis with and Without Track Slip Modeling

4.1.1. Straight-Line Trajectory Tracking Simulation

Primary simulation parameters are set as follows: The center-of-mass linear velocity of the target crawler hydraulic drill-anchor robot is 0.5 m/s. Velocity constraints are defined as $v_{\max} = 2.5$ m/s, $v_{\min} = 0$, and $\Delta v_{\max} = 1.2$ m/s. The center-of-mass linear velocity of the crawler hydraulic drill-anchor robot is set to 2 m/s, track gauge to 1.5 m, track width to 0.46 m, sampling time to 5 ms, prediction step size to 30, control step size to 10, state output weight to 15, and control input weight to 0.1. Considering the actual working conditions of crawler hydraulic drill-anchor robots, we set the track slip ratio to 20–40% [38].

Comparison results between actual trajectory and set trajectory from simulations are shown in Figure 4.

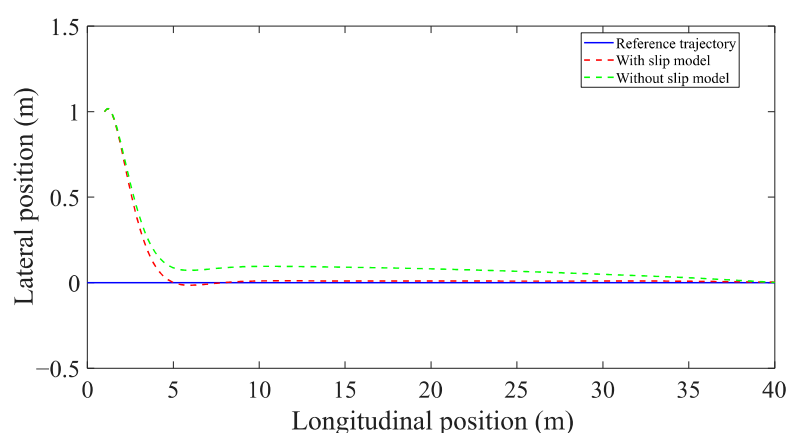


Figure 4. Straight-line trajectory tracking simulation results.

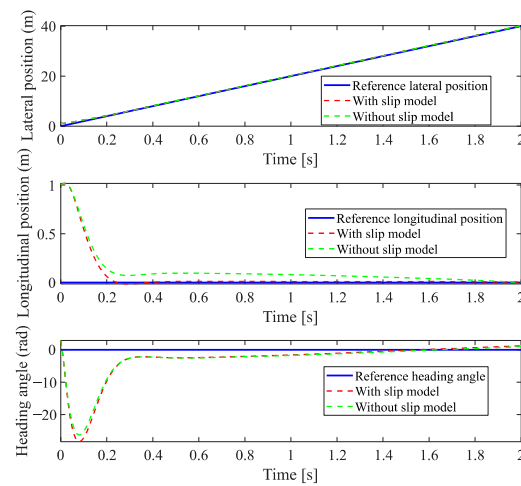
At simulation initiation, initial lateral deviation between actual position and target position is 1 m. After trajectory tracking control implementation, the actual trajectory gradually converges toward the reference trajectory.

As shown in Figure 5a,b, lateral position deviation of the crawler hydraulic drill-anchor robot varies continuously over time before 0.4 s, approaching 0 m at 0.4 s. Heading angle deviation of the robot decreases rapidly at trajectory tracking initiation, reducing to a maximum deviation of 30 deg at 0.2 s. After continuous adjustments, the heading angle deviation ultimately approaches 0 deg after 0.4 s. Table 1 lists the calculated RMSE and ME indices that quantify the straight-line trajectory tracking performance.

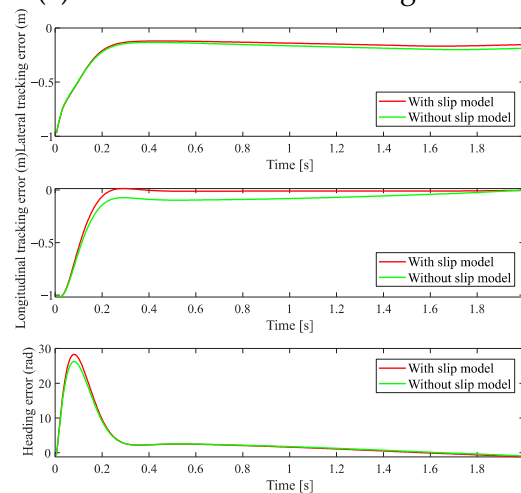
Comparison of root mean square position error results between the two trajectory tracking controllers indicates that introducing a slip rate improves tracking accuracy by 7.03%.

Table 1. RMSE and ME for straight-line trajectory tracking.

Error Type	Position Error	
	RMSE	ME
With slip model	0.30108 m	0.20684 m
Without slip model	0.32384 m	0.24453 m



(a) Three-directional tracking results

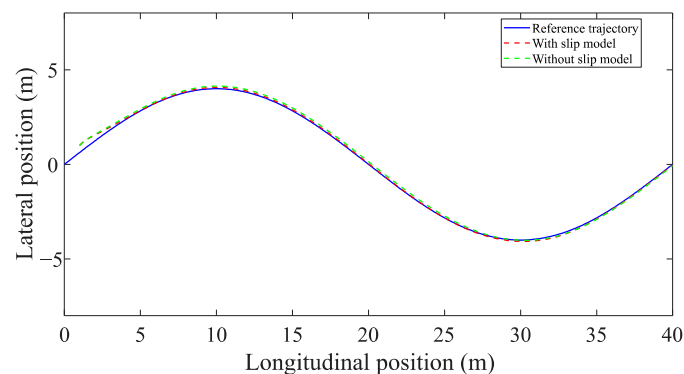


(b) Three-directional tracking errors

Figure 5. Straight-line trajectory tracking simulation result comparison.

4.1.2. Curved Trajectory Tracking Simulation

Simulations were conducted for the target crawler hydraulic drill-anchor robot navigating continuous curved paths. Trajectory tracking simulation comparison results are shown in Figure 6.

**Figure 6.** Trajectory tracking simulation results.

At simulation initiation, an initial 1 m deviation exists between actual and target positions in the lateral coordinate. After trajectory tracking control implementation, the trajectories gradually converge, achieving accurate tracking after 8 s of travel in the longi-

tudinal coordinate. Curved trajectory tracking simulation results comparison, as shown in Figure 7.

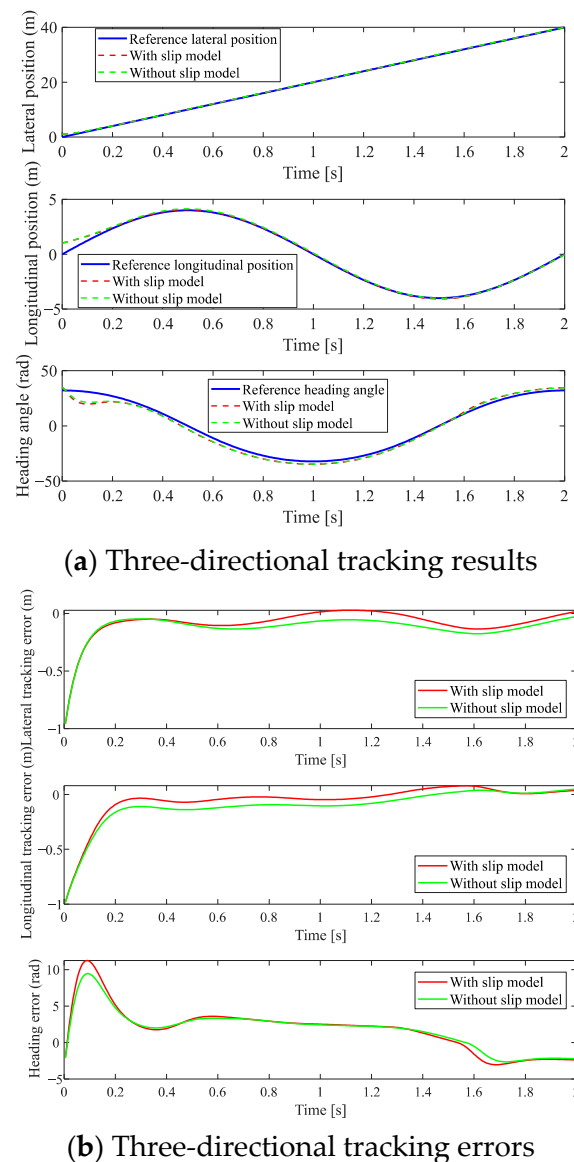


Figure 7. Curved trajectory tracking simulation result comparison.

Lateral position deviation of the crawler hydraulic drill-anchor robot decreases continuously over time, ultimately converging near 0 m. Vertical position error decreases initially before stabilizing, with slight fluctuations at the second curve. Heading angle deviation of the crawler hydraulic drill-anchor robot decreases rapidly at trajectory tracking initiation, reaching a maximum deviation of 12 deg around 0.1 s before continuous reduction. After continuous adjustments, the heading angle deviation ultimately approaches near 0 deg, with two minor subsequent fluctuations. Table 2 lists the quantitative values of RMSE and ME that characterize the sinusoidal trajectory tracking.

Table 2. RMSE and ME for sinusoidal trajectory tracking.

Error Type	Position Error Index	
	RMSE	ME
With slip model	0.221910 m	0.12329 m
Without slip model	0.24827 m	0.1773 m

Based on the above comparison results, track slippage significantly impacts tracking accuracy for curved trajectories requiring continuous heading angle adjustments. The trajectory tracking controller without slip model exhibits a maximum position error of 0.1773 m and a root mean square error of 0.24827 m. After introducing the slip model, compensated speeds of both tracks approach desired trajectory speeds more closely, effectively compensating for slip-induced velocity deviations. The root mean square position error is reduced to 0.221910 m, representing an accuracy improvement of approximately 10.62%. Thus, the predictive controller incorporating the slip model effectively resolves the issue of track slip affecting trajectory tracking accuracy.

4.2. Adaptive Model Predictive Controller Performance

The adaptive weight model predictive controller designed in this paper was validated through simulation experiments for straight-line and continuous curved trajectory tracking of the crawler hydraulic drill-anchor robot. Key simulation parameters are set as follows: target center-of-mass linear velocity of the crawler hydraulic drill-anchor robot is 2 m/s, velocity constraints are $v_{\max} = 2.5$ m/s, $v_{\min} = 0$, and $\Delta v_{\max} = 1.2$ m/s, and a track gauge of 1.5 m, track width of 0.46 m, sampling time of 5 ms, prediction step size of 30, control step size of 10, output weight of 15, control input weight of 0.1, position error output weight influence coefficient of 1.5, heading angle error output weight influence coefficient of 2, and error influence coefficient on control input of 0.3.

4.2.1. Straight-Line Trajectory Tracking Simulation

The preset trajectory is a horizontal straight line with initial pose $x(t_0) = [1 \ 0 \ 0]^T$. At simulation initiation, actual track position deviates 1 m laterally from the target track. Comparison between target and actual trajectories is shown in Figure 8.

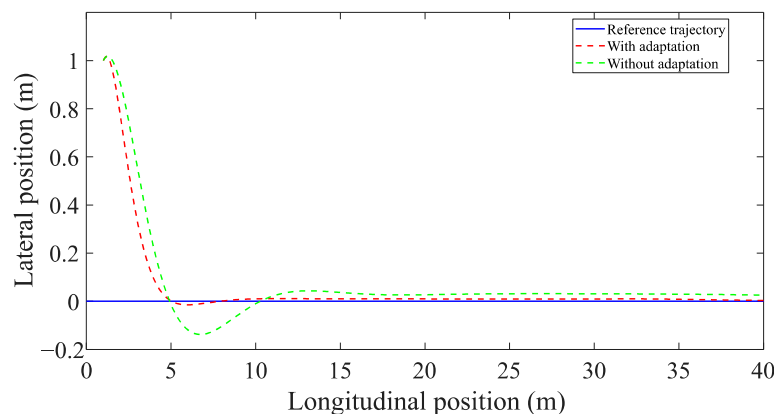


Figure 8. Straight-line trajectory tracking simulation results.

In Figure 8, the solid blue line represents the desired trajectory, the dashed green line shows the tracking path of the controller without adaptive weights, and the dashed red line indicates the tracking path of the adaptive weight controller. Prior to 10 s, lateral position deviation of the crawler hydraulic drill-anchor robot fluctuates over time. At 10 s, the deviation stabilizes and ultimately maintains at 0 m. This demonstrates that the adaptive weight controller designed in this paper can rapidly approach the desired trajectory.

Error curves during straight-line trajectory tracking are shown in Figure 9a,b.

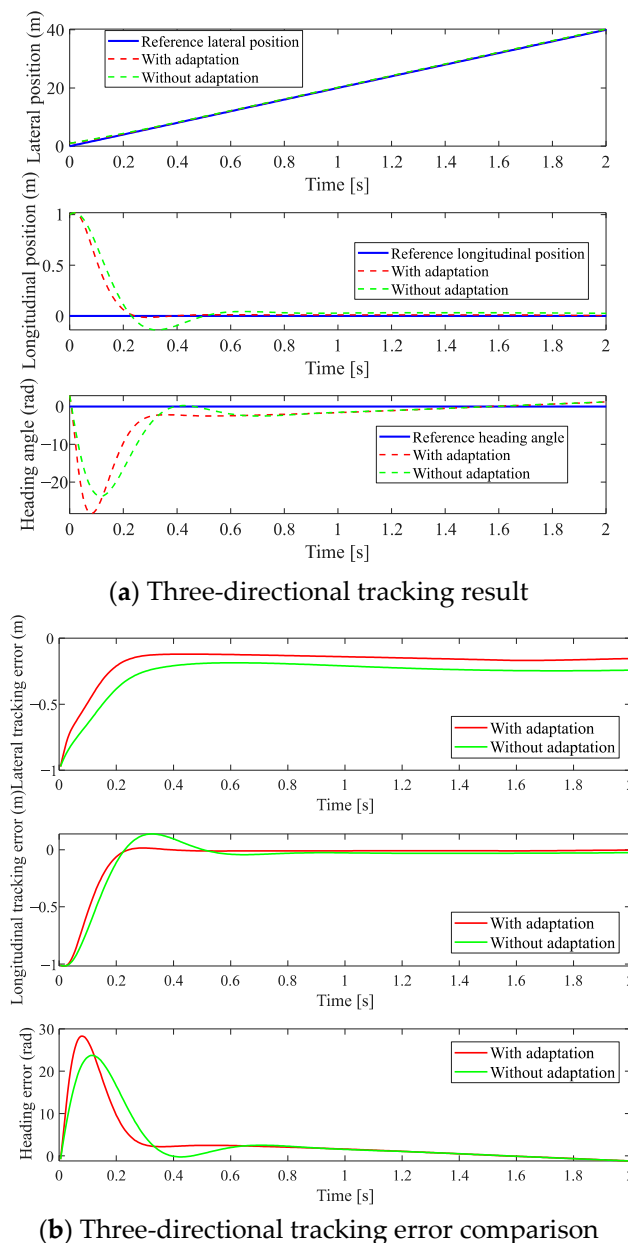


Figure 9. Straight-line trajectory tracking simulation result comparison.

Figure 9 reveals that after the crawler hydraulic drill-anchor robot first contacts the desired trajectory, a slight overshoot occurs, followed by position error convergence through adjustments. The actual heading angle error of the crawler hydraulic drill-anchor robot decreases rapidly at trajectory tracking initiation, reaching a maximum deviation of 28 deg at 0.2 s. After continuous adjustments, the heading angle error ultimately approaches 0 deg after 0.4 s.

Table 3 lists the calculated RMSE and ME for straight-line trajectory tracking.

Table 3. RMSE and ME for straight-line trajectory tracking.

Error Type	Position Error	
	RMSE	ME
With adaption	0.3011 m	0.20684 m
Without adaption	0.3787 m	0.29908 m

The comparison of RMSE and ME for straight-line trajectory tracking in Table 3 demonstrates that the adaptive model predictive control (AMPC) controller designed in this paper achieves superior tracking performance for straight-line trajectory following.

4.2.2. Curved Trajectory Tracking Simulation

Simulation analysis was conducted for the crawler hydraulic drill-anchor robot navigating continuous curves. Initial pose of the robot was set to $x(t_0) = [1 \ 0 \ 0]^T$. Comparison between simulated trajectory and actual trajectory is shown in Figure 10.

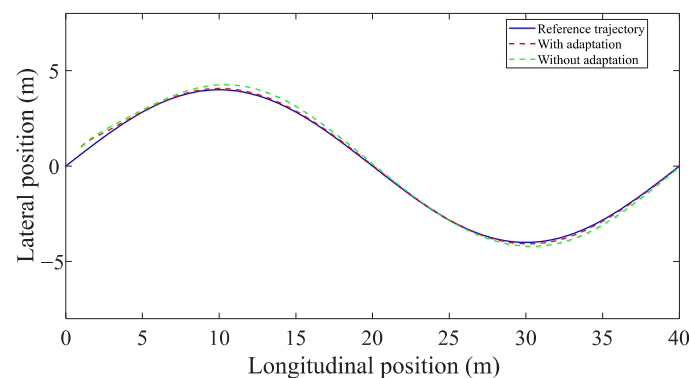


Figure 10. Curved trajectory tracking simulation results.

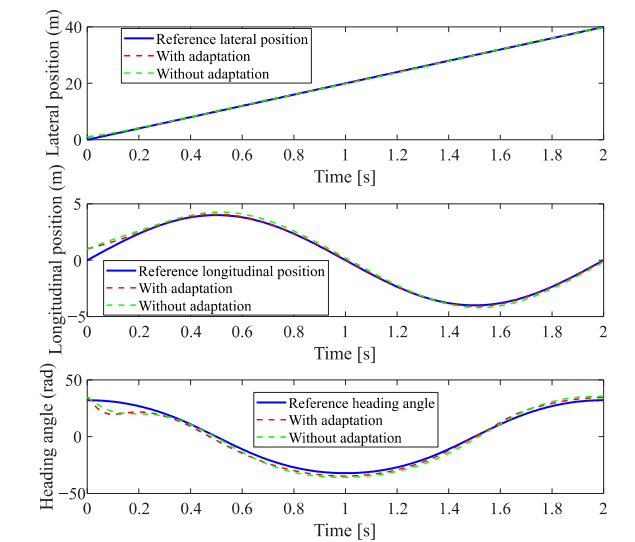
As shown in the tracking comparison results of Figure 10, during curved trajectory following, the AMPC controller enables the crawler hydraulic drill-anchor robot to reach the desired path relatively quickly and track it with minimal error.

As shown in Figure 11a, due to the initial heading angle deviation of the crawler hydraulic drill-anchor robot, the controller rapidly adjusts left and right track speeds to track the desired trajectory, demonstrating the AMPC controller's ability to stably control tracking to the desired path. As shown in Figure 11b, an initial 1 m deviation exists between the actual and target positions in the lateral coordinate at simulation initiation. After trajectory tracking control implementation, the trajectory gradually converges. Through continuous adjustments, accurate tracking is achieved after approximately 0.4 s of travel in the longitudinal coordinate. Lateral position deviation of the crawler hydraulic drill-anchor robot decreases continuously over time, ultimately stabilizing at 0 m. Longitudinal position error decreases somewhat initially, stabilizing after 0.4 s. Minor fluctuations occur at the second bend but have negligible impact. Actual heading angle deviation of the crawler hydraulic drill-anchor robot decreases rapidly at trajectory tracking initiation. After reaching a maximum deviation of 15 deg around 0.2 s, the deviation continues to decrease. Through continuous adjustments, the heading angle deviation eventually approaches 0 deg, with two minor subsequent fluctuations. Although position error and heading angle error do not converge to zero, they fluctuate regularly within a small range, indicating good tracking performance.

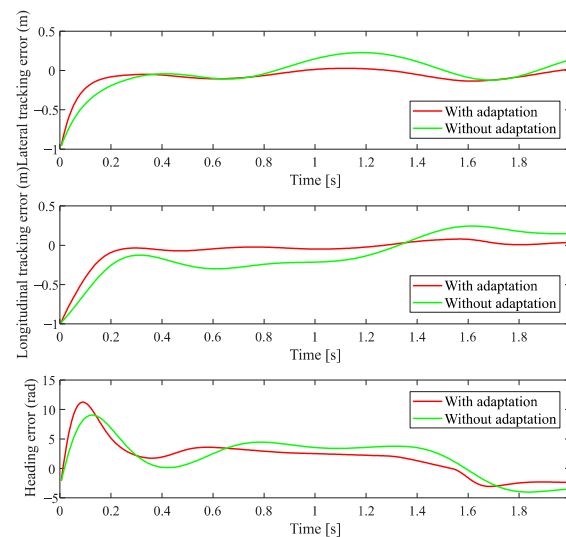
Table 4 lists the calculated RMSE and ME indices characterizing the curved path tracking performance.

Table 4. RMSE and ME for curved path tracking.

Error Type	Position Error	
	RMSE	ME
With adaption	0.22191 m	0.12329 m
Without adaption	0.34245 m	0.28194 m



(a) Three-directional tracking result comparison



(b) Three-directional tracking error comparison

Figure 11. Curved trajectory tracking simulation result comparison.

As shown in Table 4, position RMSE with the adaptive mechanism is 0.22191 m, while position RMSE without the adaptive mechanism is 0.34245 m. This demonstrates that the AMPC controller designed in this paper exhibits excellent performance for tracking dynamically adjusting direction curved trajectories.

The proposed adaptive predictive controller effectively guides the crawler hydraulic drill-anchor robot along the preset continuous curved trajectory, achieving remarkable trajectory tracking performance and maintaining precise tracking.

4.3. Simulation Verification of Speed Prediction Control Based on Friction Compensation

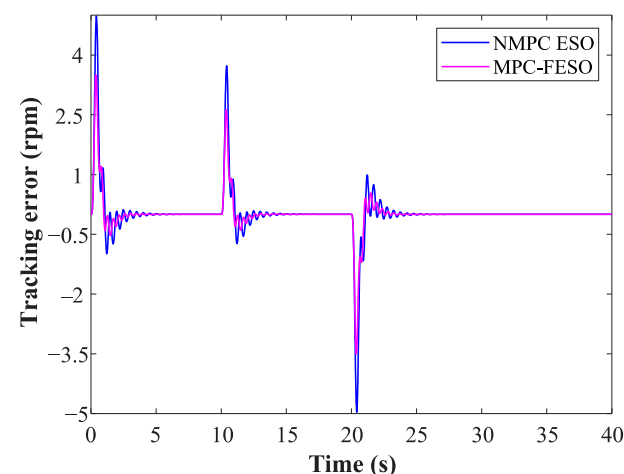
To validate the effectiveness of the proposed method, two sets of simulation experiments were conducted: (1) comparison between the extended state observer based on the friction model and a traditional extended state observer, and (2) analysis of control performance and effectiveness of the proposed control algorithm versus other control methods. Main parameters of the hydraulic motor model are selected as shown in Table 5.

Table 5. Hydraulic motor model parameters.

Parameter	Value	Unit
Hydraulic fluid bulk modulus β_e	700	(MPa)
Viscous damping coefficient B_m	0.353	(m·N·s/rad)
Hydraulic motor displacement D_m	1.2×10^{-4}	(m ³ /rad)
Load elastic stiffness G	0	(N/m)
External load torque T_L	10	(N·m)
Total cavity volume V_t	1.15×10^{-4}	(m ³)
Total moment of inertia J_t	50	(Kg·m ²)
Total leakage coefficient C_t	2.336×10^{-12}	(m ³ /(s·Pa))
Return oil pressure P_L	0.3	(MPa)
Supply pressure P_s	10	(MPa)
Hydraulic oil density ρ	880	(Kg/m ³)
Area gradient ω	1.7×10^{-3}	-
Orifice flow coefficient c_d	0.67	-
Gain coefficient k_v	0.25	(m/A)
Flow-pressure coefficient K_{ce}	7×10^{-12}	(m ³ /(s·Pa))

4.3.1. Observer Comparison Results

Figure 12 shows the disturbance estimation performance of the frictionless predictive observer and the proposed friction-based predictive observer.

**Figure 12.** Performance comparison of the proposed control method with and without friction compensation.

As shown in Figure 12, the proposed observer in this paper can estimate disturbances more rapidly and accurately. The performance evaluation indices employed for controller assessment include the mean absolute error (MEAE), standard deviation of absolute error (SDAE), and integral of time-weighted absolute error (ITAE). Table 6 presents comparison results based on six control performance indices.

Table 6. Control performance metric comparison for different extended state observers.

Observer	MEAE (rpm)	SDAE (rpm)	ITAE
Frictionless compensation	0.17525	0.62377	7.0103
Friction-compensated	0.12292	0.44366	4.9168

In summary, comparison of control performance indices between frictionless prediction and friction prediction observers indicates that the friction prediction-based observer, which accounts for friction torque, achieves higher estimation accuracy.

4.3.2. Speed Prediction Controller Effectiveness

To further validate the performance of the proposed controller, comparisons were conducted with three control methods—PID, active disturbance rejection control (ADRC), and MPC—in conjunction with the proposed NMPC with friction-based extended state observer (FESO) algorithm. A speed setpoint signal was applied to the controlled object, and all compared control methods were subjected to varying friction torque and uncertain disturbances. Parameter settings for the four control methods are as follows:

- PID controller [39]: $K_P = 50$, $K_I = 30$, $K_D = 0.01$.
- ADRC controller [40]: Observer gain $\beta = [4\omega, 6\omega^2, 4\omega^3, \omega^4]^T$, $\omega = 30$, control law gain $k_1 = 52$, $k_2 = 40$, $k_3 = 0.01$.
- MPC controller [41]: $N_p = 10$, $N_c = 5$, output weight matrix main diagonal set to 2×10^3 , control increment matrix main diagonal set to 1.
- MPC-FESO controller: $N_p = 10$, $N_c = 5$, output weight matrix main diagonal set to 2×10^3 , control matrix main diagonal set to 1, observer parameters $\beta = [3\omega, 3\omega^2, \omega^3]^T$, $\omega = 30$.

Constraints are set as follows:

$$\begin{cases} -0.1(\text{mA}) \leq \Delta u(k+i|k) \leq 0.1(\text{mA}), i = 0, \dots, N_c - 1 \\ 0(\text{r/min}) \leq y(k+i|k) \leq 360(\text{r/min}), i = 1, \dots, N_p \end{cases} \quad (46)$$

The crawler hydraulic drill-anchor robot primarily drives track wheels via hydraulic motors. Single-track travel speed is expressed as

$$v_{ld} = \frac{2\pi(r + h_{ld})n}{60i_{ld}} \quad (47)$$

In the formula, v_{ld} represents single-track travel speed, r denotes track drive wheel radius, h_{ld} indicates track thickness, n signifies hydraulic motor rotational speed, and i_{ld} represents gear transmission ratio, typically set at 1:4.

With a track drive wheel radius of 233 mm, track thickness of 40 mm, and maximum travel speed of 9 km/h for the crawler hydraulic drill-anchor robot, calculations yield a maximum hydraulic motor speed of 350 r/min. To mitigate abrupt changes in control input signals, a transition process is introduced to convert the tracking signal into a continuous desired input, as shown in Figure 13.

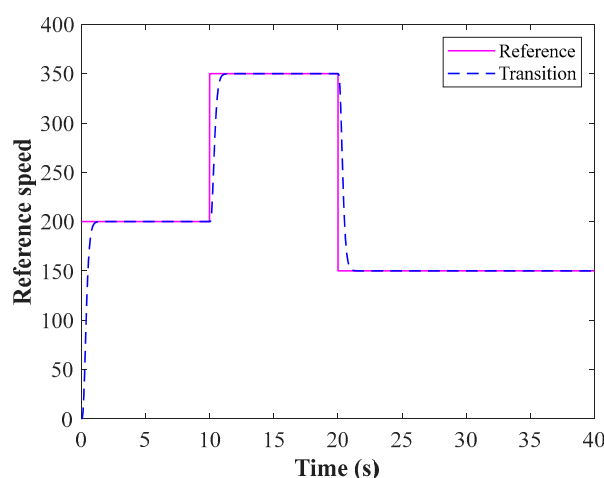


Figure 13. Speed tracking setpoint curve.

A sinusoidal disturbance as shown in Figure 14 was applied to the rotary system of the drill rig for drill-anchor robots. The tracking errors for the proposed method and comparison methods are presented in Figure 15, respectively. The corresponding performance metrics for the five control methods are summarized in Table 7.

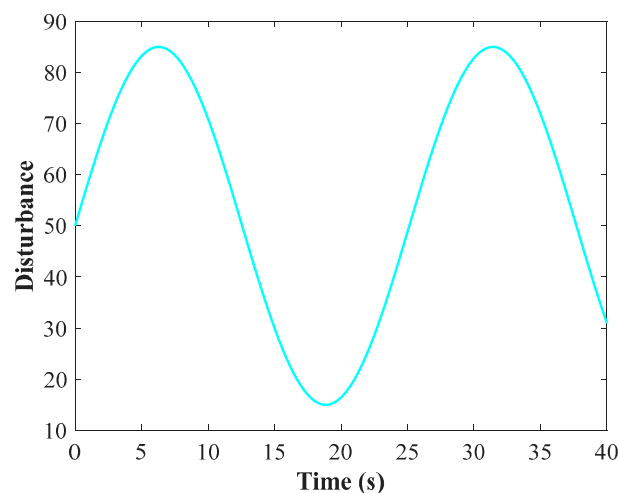


Figure 14. Sinusoidal disturbance configuration.

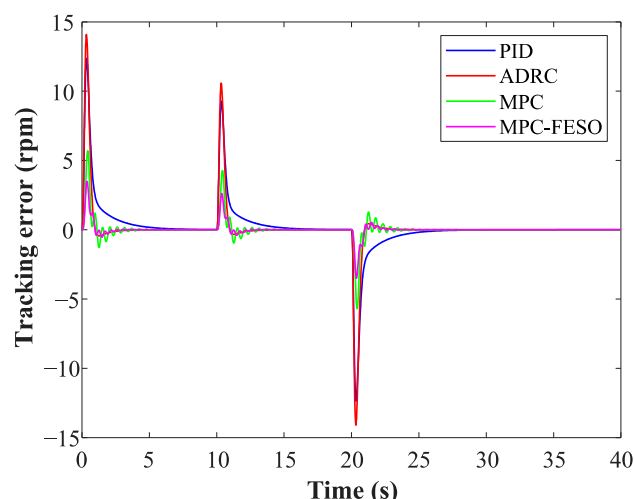


Figure 15. Speed tracking error of four comparison methods.

Table 7. Control performance metric comparison.

Control Method	MEAE (rpm)	SDAE (rpm)	ITAE
PID	0.66103	1.8154	26.4419
ADRC	0.44199	1.9137	17.68
MPC	0.20628	0.7167	8.2514
MPC-FESO	0.12292	0.44366	4.9168

Based on simulation results in Figure 15, during given signal tracking, the proposed control method achieves faster target setpoint signal tracking compared to other control methods. Additionally, tracking error is significantly reduced relative to other control methods, demonstrating superior tracking performance.

Based on tracking error and corresponding control performance indices for different control methods in Table 7, experimental results demonstrate that the proposed control strategy effectively tracks the setpoint signal and exhibits superior control performance.

5. Conclusions

This paper primarily investigates trajectory tracking for crawler hydraulic drill-anchor robots. A kinematic model of the robot was established, and a novel cascade predictive control framework was proposed to ensure high-precision trajectory tracking performance. Addressing the impact of track slippage on tracking accuracy, an adaptive model predictive controller accounting for slippage was designed. Simulation results demonstrate that the proposed control strategy achieves excellent tracking performance. To address track speed control, friction torque in the speed system is estimated and a speed prediction controller is designed. Simulation experiments demonstrate that friction estimation significantly improves track speed control accuracy. However, the proposed trajectory tracking predictive controller does not yet account for other factors, such as track ground resistance and center-of-mass displacement. Furthermore, the lower-layer predictive controller employs nonlinear model predictive control, which suffers from high computational complexity. Therefore, future research will focus on establishing a mathematical model that better reflects actual track operating conditions, designing a lower-layer speed model predictive controller with simpler computation and lower computational complexity, and conducting experimental and hardware-in-the-loop (HIL) verification to validate the proposed model and controller in actual engineering scenarios.

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Nomenclature

The following abbreviations and symbols are used in this manuscript:

Abbreviations

RMSE	Root mean square error
ME	Maximum error
PID	Proportional-integral-derivative
ADRC	Active disturbance rejection control
MPC	Model predictive control
AMPC	Adaptive model predictive control
FESO	Friction based extended state observer
ME	Maximum error

Symbols and physical properties

v_L	The left track
v_R	The right track
$x(t)$	The position
$y(t)$	The position
$\theta(t)$	The heading angle
ω_R	The angular velocities of the right
ω_L	The angular velocities of the left
r	The radius of the track drive wheel
σ_R	The slip rates of the right tracks

σ_L	The slip rates of the left tracks
$\dot{\theta}$	The heading angular velocity
C_w	The circumference of the track drive wheel Per full rotation
p_r	The reference quantity
x	The state variable of the crawler hydraulic Drill-anchor robot
$\bar{x}$	The state error
$\bar{u}$	The control error
$\bar{A}$	The state error coefficient matrix
$\bar{B}$	The control error coefficient matrix
I	The unit matrix
T	The sampling time
Q_1	The flow rate into the inlet chamber
Q_2	The flow rate out of the return chamber
c_d	The proportional directional valve orifice flow coefficient
ω	The proportional directional valve throttle orifice area gradient
x_v	The proportional directional valve spool Displacement
ρ	The hydraulic oil density
P_1	The outlet pressure of the proportional directional valve
P_2	The proportional directional valve return pressure
P_s	The proportional valve supply pressure
k_v	The gain coefficient between the control inPut and spool displacement
i	The control input applied to the proporTional directional valve
Q_L	The load flow
P_L	The load pressure
k_a	The flow gain coefficient
P_L	The external load pressure
sgn	The sign function
V_t	The total compression volume of the hydraulic motor
Q_L	The flow continuity equation for the hydraulic motor
D_m	The displacement of the hydraulic motor
θ_m	The angular displacement of the hydraulic motor
c_{ip}	The internal leakage coefficient of the hydraulic motor
β_e	The effective volumetric elastic modulus of The hydraulic fluid
ΔQ	The modeling error
$D_m P_L$	The torque balance equation between the Output torque and external load
J_t	The total moment of inertia of the hydraulic motor and load
B	The viscous damping coefficient
G	The load elastic stiffness
T_L	The external load torque acting on the motor shaft
T_f	The friction torque
ΔT	The sum of uncertainty disturbance and fitTing friction deviation
T_{cb}	The Coulomb friction torque
T_s	The maximum static friction torque
α_f	The Stribeck velocity coefficient
k_{fv}	The viscous friction torque coefficient
r	The preset pose
p	The actual pose
N_P	The system prediction time domain length
N_c	The system control time domain length
Y_r	The desired system state
Y	The current system state prediction
ΔU	The control input increment

Q	The system error weighting matrix
R	The system control weighting matrix
$Y_{\min}$	Output state constraints
$Y_{\max}$	Output state constraints
$\Delta U_{\min}$	Control input constraints
$\Delta U_{\max}$	Control input constraints
Q_0	The base weights
R_0	The base weights
e_{xy}	The position tracking error
$ e_\theta $	The absolute value of the heading angle Tracking error
$\hat{x}_4$	The total disturbance
$\dot{\xi}$	The derivative of the disturbance estimate
$\hat{x}_i$	The state estimate of the extended state observer
$\hat{x}_2$	The observed angular velocity of x_2
$\hat{x}_3$	The observed load pressure of the system
$\hat{x}_4$	The estimated total disturbance of the system
$\beta_i (i = 1, 2, 3)$	The LESO gains
$\dot{x}(k)$	The state vector in the predictive model
$y(k)$	The state output
$T_f(k)$	The friction torque
$\Delta T(k)$	Unmodeled terms, uncertain disturbances, and friction torque modeling errors
$J(x(k), U(k))$	The design optimization objective function
$y(k)$	The rotational speed output prediction
$y_r(k + i k), i = 1, \dots, N_p$	The desired setpoint signal
Q_m	The output weighting matrix
R_m	The control increment weighting matrix
$U(k)$	The control input sequence within the control time domain
$x(k)$	The state variable of the system under the action of the input vector sequence
$u_{\min}(k + i k) \leq u(k + i k) \leq u_{\max}(k + i k)$	The control variable constraints
$x(t_0)$	initial pose
β_e	Hydraulic fluid bulk modulus
B_m	Viscous damping coefficient
D_m	Hydraulic motor displacement
G	Load elastic stiffness
T_L	External load torque
V_t	Total cavity volume
J_t	Total moment of inertia
C_t	Total leakage coefficient
P_L	Return oil pressure
P_s	Supply pressure
ρ	Hydraulic oil density
ω	Area gradient
c_d	Orifice flow coefficient
k_v	Gain coefficient
K_{ce}	Flow-pressure coefficient
v_{ld}	Single-track travel speed
r	Track drive wheel radius
h_{ld}	Track thickness
n	Hydraulic motor rotational speed
i_{ld}	Gear transmission ratio

References

1. Dai, Y.; Su, Q.; Zhang, Y. A new dynamic model and trajectory tracking control strategy for deep ocean mining vehicle. *Ocean Eng.* **2020**, *216*, 108162. [\[CrossRef\]](#)
2. Li, Z.; Guo, J.; Wang, T.; Li, X.; Zhang, H. Trajectory Tracking Control for Wheeled Mobile Robots with Unknown Slip Rates Based on Improved Rapid Variable Exponential Reaching Law and Sliding Mode Observer. *Machines* **2025**, *13*, 765. [\[CrossRef\]](#)
3. Hang, P.; Chen, X.; Luo, F. LPV/ H_∞ controller design for path tracking of autonomous ground vehicles through four-wheel steering and direct yaw-moment control. *Int. J. Automot. Technol.* **2019**, *20*, 679–691. [\[CrossRef\]](#)
4. Sabiha, A.D.; Kamel, M.A.; Said, E.; El-Hawwary, A. ROS-based trajectory tracking control for autonomous tracked vehicle using optimized backstepping and sliding mode control. *Robot. Auton. Syst.* **2022**, *152*, 104058. [\[CrossRef\]](#)
5. Chen, Y.; Zhang, H.; Zou, W.; Li, J.; Wang, Y. Dynamic modeling and learning based path tracking control for ROV-based deep-sea mining vehicle. *Expert Syst. Appl.* **2025**, *262*, 125612. [\[CrossRef\]](#)
6. Qin, Z.; Chen, L.; Fan, J.; Li, Y.; Wang, H. An improved real-time slip model identification method for autonomous tracked vehicles using forward trajectory prediction compensation. *IEEE Trans. Instrum. Meas.* **2021**, *70*, 7501012. [\[CrossRef\]](#)
7. Wang, X.; Wang, Y.; Sun, Q.; Liu, Z.; Zhang, L. Adaptive robust control of unmanned tracked vehicles for trajectory tracking based on constraint modeling and analysis. *Nonlinear Dyn.* **2024**, *112*, 9117–9135. [\[CrossRef\]](#)
8. Song, L.; Huang, J.; Liang, Q.; Wang, Y.; Li, S. Trajectory tracking strategy for sliding mode control with double closed-loop for lawn mowing robot based on ESO. *IEEE Access* **2022**, *11*, 1867–1882. [\[CrossRef\]](#)
9. Zhang, S.; Wei, X.; Liu, C.; Wang, H.; Li, Y. Adaptive path tracking and control system for unmanned crawler harvesters in paddy fields. *Comput. Electron. Agric.* **2025**, *230*, 109878. [\[CrossRef\]](#)
10. Xu, Z.; Chen, Y.; Jian, Z.; Li, X.; Wang, H. Hybrid trajectory optimization for autonomous terrain traversal of articulated tracked robots. *IEEE Robot. Autom. Lett.* **2023**, *9*, 755–762. [\[CrossRef\]](#)
11. Zhang, S.; Tao, B.; Zhao, Y.; Li, J.; Wang, Y. Trajectory Tracking of Tracked Robot Using Super Twisting Algorithm with Enhanced Boundary Layer and Dynamics Incorporating Gravity Center Offset. *IEEE/ASME Trans. Mechatron.* **2024**, *30*, 3665–3675. [\[CrossRef\]](#)
12. Zhang, C.; Zhang, H.; Tian, L.; Wang, X.; Li, Y. Trajectory tracking control of a crawler maneuvering structure via backstepping–PSO parameter optimization. *J. Mech. Sci. Technol.* **2025**, *39*, 7763–7774. [\[CrossRef\]](#)
13. Liu, Z.; Zheng, W.; Wang, N.; Zhang, L.; Li, Y. Trajectory tracking control of agricultural vehicles based on disturbance test. *Int. J. Agric. Biol. Eng.* **2020**, *13*, 138–145. [\[CrossRef\]](#)
14. Zhou, J.; Zhao, C.; Chen, Y.; Wang, H.; Li, X. Trajectory tracking control of unmanned vehicles via front-wheel driving. *Drones* **2024**, *8*, 543. [\[CrossRef\]](#)
15. Gu, Q.; Bai, G.; Meng, Y.; Zhang, L.; Wang, Y. Efficient path tracking control for autonomous driving of tracked emergency rescue robot under 6G network. *Wirel. Commun. Mob. Comput.* **2021**, *2021*, 5593033. [\[CrossRef\]](#)
16. Taner, B.; Subbarao, K. Modeling of Cooperative Robotic Systems and Predictive Control Applied to Biped Robots and UAV-UGV Docking with Task Prioritization. *Sensors* **2024**, *24*, 3189. [\[CrossRef\]](#)
17. Liang, G.; Jiang, Y.; Gao, Z.; Wang, X.; Zhang, Q. Research on Operation Trajectory Tracking Control of Loader Working Mechanisms. *Machines* **2025**, *13*, 165. [\[CrossRef\]](#)
18. Zang, Z.; Gong, J.; Li, Z.; Zhang, Y.; Chen, H. Formation trajectory tracking control of UTVs: A coupling multi-objective iterative distributed model predictive control approach. *IEEE Trans. Intell. Veh.* **2022**, *8*, 2222–2232. [\[CrossRef\]](#)
19. Li, Y.; He, D.; Ma, F.; Wang, Z.; Liu, Y. MPC-based trajectory tracking control of unmanned underwater tracked bulldozer considering track slipping and motion smoothing. *Ocean Eng.* **2023**, *279*, 114449. [\[CrossRef\]](#)
20. Gong, P.; Yan, Z.; Zhang, W.; Wang, Y.; Li, X. Lyapunov-based model predictive control trajectory tracking for an autonomous underwater vehicle with external disturbances. *Ocean Eng.* **2021**, *232*, 109010. [\[CrossRef\]](#)
21. Li, Z.; Jing, X.; Sun, B.; Li, G.; Wang, Z. Autonomous navigation of a tracked mobile robot with novel passive bio-inspired suspension. *IEEE/ASME Trans. Mechatron.* **2020**, *25*, 2633–2644. [\[CrossRef\]](#)
22. Yan, Z.; Gong, P.; Zhang, W.; Li, X.; Wang, Y. Model predictive control of autonomous underwater vehicles for trajectory tracking with external disturbances. *Ocean Eng.* **2020**, *217*, 107884. [\[CrossRef\]](#)
23. Wang, J.; Liu, Z.; Chen, H.; Li, D.; Lu, K. Trajectory tracking control of a skid-steer mobile robot based on nonlinear model predictive control with a hydraulic motor velocity mapping. *Appl. Sci.* **2023**, *14*, 122. [\[CrossRef\]](#)
24. Zhao, Z.; Liu, H.; Chen, H.; Wang, J.; Li, D. Kinematics-aware model predictive control for autonomous high-speed tracked vehicles under the off-road conditions. *Mech. Syst. Signal Process.* **2019**, *123*, 333–350. [\[CrossRef\]](#)
25. Gocer, I.; Baslamisli, S.C. A Slip-Based Model Predictive Control Approach for Trajectory Following of Unmanned Tracked Vehicles. *Machines* **2025**, *13*, 817. [\[CrossRef\]](#)
26. Guo, H.; Shen, C.; Zhang, H.; Li, J.; Wang, Y. Simultaneous trajectory planning and tracking using an MPC method for cyber-physical systems: A case study of obstacle avoidance for an intelligent vehicle. *IEEE Trans. Ind. Inform.* **2018**, *14*, 4273–4283. [\[CrossRef\]](#)

27. Zhan, J.; Li, W.; Wang, J.; Zhang, L.; Liu, Z. Research on trajectory planning and tracking algorithm of crawler paver. *Machines* **2024**, *12*, 650. [[CrossRef](#)]
28. Wang, S.; Wu, Z.; Su, Y. A Cooperative Longitudinal-Lateral Platoon Control Framework with Dynamic Lane Management for Unmanned Ground Vehicles Based on a Dual-Stage Multi-Objective MPC Approach. *Drones* **2025**, *9*, 711. [[CrossRef](#)]
29. Nam, H.; Choi, W.; Ahn, C. Model predictive control for evasive steering of an autonomous vehicle. *Int. J. Automot. Technol.* **2019**, *20*, 1033–1042. [[CrossRef](#)]
30. Yang, W.; Chen, Y.; Su, Y. A double-layer model predictive control approach for collision-free lane tracking of on-road autonomous vehicles. *Actuators* **2023**, *12*, 169. [[CrossRef](#)]
31. Yue, M.; Hou, X.; Gao, R.; Wang, L.; Zhang, H. Trajectory tracking control for tractor-trailer vehicles: A coordinated control approach. *Nonlinear Dyn.* **2018**, *91*, 1061–1074. [[CrossRef](#)]
32. Chen, S.; Xie, P.; Liao, J. ARSMC-NMPC control strategy for active heave compensation in underwater operations of ship-mounted offshore crane. *Ocean Eng.* **2025**, *340*, 122295. [[CrossRef](#)]
33. Li, J.; Lei, Q.; Guo, W.; Wang, Y.; Zhang, H. Bandwidth extension of shaking table based on active disturbance rejection predictive control. *Appl. Math. Model.* **2025**, *150*, 116348. [[CrossRef](#)]
34. Li, D.; Lu, K.; Cheng, Y.; Wang, X.; Zhang, L. Nonlinear model predictive control—Cross-coupling control with deep neural network feedforward for multi-hydraulic system synchronization control. *ISA Trans.* **2024**, *150*, 30–43. [[CrossRef](#)] [[PubMed](#)]
35. Zhang, Y.; Lu, J.; Xia, G.; Wang, H.; Li, Y. Enhancing roll stability of counterbalance forklift trucks with a novel rollover index and hierarchical adaptive model predictive control. *IEEE Trans. Veh. Technol.* **2024**, *73*, 7925–7938. [[CrossRef](#)]
36. Chen, Z.; Huang, C.; Liu, Z.; Wang, Y.; Zhang, H. An offset-free explicit model predictive pressure controller for integrated braking system. *IEEE Trans. Transp. Electrification* **2023**, *10*, 2355–2366. [[CrossRef](#)]
37. Gu, W.; Yao, J.; Yao, Z.; Li, X.; Wang, H. Output feedback model predictive control of hydraulic systems with disturbances compensation. *ISA Trans.* **2019**, *88*, 216–224. [[CrossRef](#)]
38. Yan, X.; Wang, D.; Ma, A.; Zhang, L.; Liu, Z. Adaptive robust control for heavy-duty tracked transportation robots in underground mines. *Control Eng. Pract.* **2025**, *164*, 106494. [[CrossRef](#)]
39. Lopez-Sanchez, I.; Moreno-Valenzuela, J. PID control of quadrotor UAVs: A survey. *Annu. Rev. Control* **2023**, *56*, 100900. [[CrossRef](#)]
40. Zhao, J.; Zhang, Y.; Hou, H.; Wang, Y.; Li, J. Active disturbance rejection control with backstepping for decoupling control of hydraulic driven lower limb exoskeleton robot. *IEEE Trans. Ind. Electron.* **2024**, *72*, 714–723. [[CrossRef](#)]
41. Li, L.; Chen, Y.; Yan, C.; Wang, H.; Zhang, H. Motion control of negative-lift AUV based on disturbance compensation nonlinear model predictive control with extended state observation. *Ocean Eng.* **2025**, *329*, 121145. [[CrossRef](#)]

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