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Reliability- and Sensitivity-Guided Co-Estimation of Vehicle States, Tire Cornering Stiffness, and Tire-Road Adhesion Coefficient

Lei Liu, Jue Yang *  and Yiting Kang 

School of Mechanical Engineering, University of Science and Technology Beijing, Beijing 100083, China; ll1634723601@163.com (L.L.); kangyiting@ustb.edu.cn (Y.K.)

* Correspondence: yangjue@ustb.edu.cn

Abstract

Reliable estimation of vehicle states, tire cornering stiffness, and the tire-road adhesion coefficient are essential for vehicle lateral stability and intelligent chassis control. Under low adhesion, nonlinear tire operation, and weak excitation, lateral-force residuals are jointly affected by cornering-stiffness variation, adhesion-coefficient variation, and tire-force saturation, which may cause erroneous parameter adaptation. This paper proposes a reliability- and sensitivity-guided co-estimation method for vehicle states, tire cornering stiffness, and the tire-road adhesion coefficient. A hierarchical framework is developed based on a planar 3-DOF vehicle model and a Fiala-type nonlinear tire model. Front- and rear-axle lateral-force pseudo-measurements are reconstructed from lateral acceleration and yaw angular acceleration, without requiring additional tire-force sensors. Parameter-update reliability is evaluated by considering lateral excitation, longitudinal slip, adhesion utilization, and normalized lateral-force residual consistency. Normalized lateral-force sensitivities are then used to allocate the residual between the cornering-stiffness and adhesion-coefficient update channels. CarSim/Simulink co-simulations under high-, intermediate-, and low-adhesion double-lane-change maneuvers demonstrate that the proposed method improves sideslip-angle and lateral-velocity estimation accuracy, suppresses erroneous cornering-stiffness adaptation, and provides more stable estimates of the tire-road adhesion coefficient.

Keywords: vehicle-state estimation; sideslip angle; tire cornering stiffness; tire-road adhesion coefficient; lateral-force residual; parameter-update reliability assessment; lateral-force-sensitivity-based update allocation



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1. Introduction

Vehicle lateral stability is jointly governed by vehicle motion states, lateral tire-force generation, and tire-road adhesion conditions. The sideslip angle, longitudinal velocity, tire cornering stiffness, and peak tire-road adhesion coefficient are key quantities for describing this coupled relationship. Guo et al. indicated that vehicle-state estimation provides an essential basis for active safety control and autonomous driving systems, and that model uncertainties and tire-parameter variations may significantly degrade estimation accuracy [1]. Pusul et al. reviewed sideslip-angle estimation methods from the perspective of vehicle dynamics and pointed out that tire-model errors, vehicle-parameter uncertainties, and limitations of low-cost onboard sensors remain major factors affecting estimation

reliability [2]. Rajamani systematically presented vehicle lateral dynamics and tire-force generation mechanisms, revealing the intrinsic coupling among sideslip angle, lateral tire force, tire cornering stiffness, and tire-road adhesion capability [3]. Recent numerical and experimental research further shows that load and inflation pressure affect tire deformation, internal stress, and road-traffic safety, supporting the need for reliable online tire-parameter estimation rather than relying only on fixed nominal values [4]. Beyond chassis control, vehicle operating states are also relevant to traffic noise and traffic management, since vehicle speed, traffic flow, and acceleration can lead to rapid variations in instantaneous noise levels [5]. Therefore, reliable estimation of vehicle states, tire cornering stiffness, and the tire-road adhesion coefficient using production-vehicle sensors is essential for stability control, active safety control, and coordinated intelligent chassis control. For brevity, the peak tire-road adhesion coefficient is referred to as the tire-road adhesion coefficient hereafter.

Existing studies have proposed various vehicle-state and parameter co-estimation frameworks. Wenzel et al. employed a dual extended Kalman filter structure to separately estimate vehicle states and parameters, offering a classical framework for state-parameter co-estimation [6]. Park designed an interacting multiple-model Kalman filter using measurements from a low-cost GPS receiver and onboard sensors, in which kinematics-based and dynamics-based estimators were fused to improve the estimation accuracy of the sideslip angle and tire cornering stiffness [7]. These methods can improve the reconstruction performance of unmeasured vehicle states. Therefore, the key issue in joint state-parameter estimation is not only to reduce the residual magnitude, but also to assess whether the residual provides reliable information for parameter adaptation and to allocate it to the physically consistent tire-parameter update channel.

Tire cornering stiffness is a critical parameter affecting vehicle lateral dynamics modeling and sideslip-angle estimation accuracy. Baffet et al. estimated the sideslip angle, lateral tire forces, and tire cornering stiffness using a vehicle lateral dynamics model, providing an early example of joint estimation of vehicle-states, tire forces, and tire cornering stiffness [8]. Doumiati et al. performed online estimation of lateral tire forces and the sideslip angle based on vehicle dynamics, highlighting the value of lateral-force information in improving lateral-state estimation [9]. Berntorp and Di Cairano employed a noise-adaptive particle filtering method to estimate vehicle states and tire cornering stiffness, and indicated that tire cornering stiffness is related to the tire-road adhesion coefficient and can be affected by tire pressure, temperature, road condition, and other factors [10]. Jeon et al. combined vehicle-state estimation with tire-model learning and used data-driven information to compensate for conventional tire-model errors [11]. Wang et al. adopted a fuzzy adaptive robust cubature Kalman filter to estimate the sideslip angle and tire cornering stiffness, thereby improving estimation robustness under model uncertainties [12]. These studies indicate that variations in tire cornering stiffness should be considered in vehicle-state estimation. However, under weak lateral excitation, low-adhesion conditions, or nonlinear tire operation, the lateral-force residual does not necessarily arise mainly from variations in tire cornering stiffness alone. Therefore, continuously updating tire cornering stiffness under such conditions may lead to erroneous parameter adaptation.

Tire-road adhesion-coefficient estimation provides another important perspective for characterizing tire-road interaction. Wang et al. classified tire-road adhesion-coefficient estimation methods into external-sensor-based, vehicle-dynamics-based, and data-driven approaches, and pointed out that vehicle-dynamics-based methods can provide continuous estimates using production-vehicle signals but usually require sufficient dynamic excitation [13]. Ahn et al. integrated longitudinal and lateral vehicle dynamics for robust tire-road adhesion-coefficient estimation, indicating that combined longitudinal and lat-

eral dynamic responses can improve estimation reliability [14]. Wang et al. excited the yaw response through the left–right longitudinal tire-force difference and simultaneously estimated the tire-road adhesion coefficient and tire cornering stiffness, which is highly relevant to the topic of this study [15]. However, that method relies on active longitudinal-force excitation to enhance parameter identifiability. In contrast, this study focuses on how to determine whether the current operating condition is suitable for parameter updating and how the lateral-force residual should be allocated to different parameter-update channels, without introducing additional active excitation or tire-force sensors.

Li et al. estimated the tire-road adhesion-coefficient under complex maneuvering conditions using multi-source information fusion, showing that multi-source information can improve the reliability of tire-road adhesion-coefficient estimation in complex maneuvers [16]. Liu et al. combined an auxiliary particle filter with an interactive extended Kalman filter to address nonlinearities and uncertainties in tire-road adhesion-coefficient estimation [17]. Feng et al. employed a moving optimal estimation strategy to estimate the tire-road adhesion-coefficient of a four-wheel-drive vehicle, demonstrating the role of constrained optimization in tire-road adhesion-coefficient estimation [18]. Gözl et al. specifically investigated tire-road adhesion-coefficient estimation under low longitudinal, low lateral, and combined weak-excitation conditions, and pointed out that insufficient excitation can markedly reduce the information available from vehicle responses for tire-road adhesion coefficient estimation [19]. Although these studies have improved tire-road adhesion coefficient estimation, the coupling between adhesion-coefficient adaptation and cornering-stiffness adaptation remains insufficiently addressed. When the lateral-force residual reflects the combined effects of the adhesion limit and tire cornering stiffness, residual-magnitude-driven adaptation may lead to physically inconsistent parameter updates. Therefore, residual attribution is required before parameter adaptation to identify the dominant tire-parameter update channel.

To reduce the dependence of purely dynamics-based tire-road adhesion-coefficient estimation on sufficient excitation, vision-based and multi-source fusion methods have also been developed. Existing studies have used cameras, road images, vehicle-dynamic information, and multi-sensor fusion strategies to provide road-image priors or confidence information for peak tire-road adhesion-coefficient estimation [20–25]. These methods expand the information sources available for tire-road adhesion-coefficient estimation and improve adhesion-coefficient estimation performance when dynamic excitation is insufficient. However, this study does not introduce additional external perception information. Instead, using only standard onboard measurements, this work addresses how the lateral-force residual should be allocated between the cornering-stiffness update channel and the adhesion-coefficient update channel.

From the standpoint of tire-force generation, tire cornering stiffness and the tire-road adhesion coefficient should not be treated as independent quantities in estimation. Instead, they jointly affect vehicle lateral dynamics through nonlinear lateral tire-force generation. Zhang et al. adopted a Fiala-type tire model with a unified adhesion coefficient in vehicle lateral stability control to describe nonlinear lateral tire-force characteristics [26]. Desai and Ghaffari further investigated tire-road lateral-force estimation and tire-force saturation detection based on the Fiala brush tire model, showing that the relationship between lateral tire force and tire slip angle can reflect how closely the tire approaches the adhesion limit [27]. For the Fiala-type nonlinear tire model used in this study, when the slip angle is small, the lateral tire force is approximately proportional to the slip angle, and the variation in lateral tire force is mainly governed by tire cornering stiffness. As the tire gradually approaches saturation, the lateral tire force no longer increases approximately linearly with the slip angle; instead, it is bounded by the adhesion limit jointly determined by the vertical

load and the tire-road adhesion coefficient. In this region, the lateral tire force becomes more sensitive to the tire-road adhesion coefficient. Therefore, the physical attribution of the lateral-force residual varies with the tire operating region, and parameter adaptation should be adjusted according to both the tire operating condition and the lateral-force sensitivities to different parameters.

Residual-magnitude-driven simultaneous parameter adaptation may become unreliable when tire cornering stiffness, the tire-road adhesion coefficient, and nonlinear tire operation jointly affect the lateral-force residual. Under low-adhesion or nonlinear tire operating conditions, a reduction in the available lateral force may be incorrectly interpreted as a change in tire cornering stiffness. Conversely, if the tire operating region is not explicitly considered, the residual associated with tire-force saturation may also be incorrectly allocated to the adhesion-coefficient update channel. Therefore, the key challenge is not only to reduce the lateral-force residual, but also to identify its dominant physical attribution before parameter adaptation is performed. In this study, this issue is formulated as the mis-allocation of the lateral-force residual among parameter-update channels due to ambiguous physical attribution. Although existing studies have enabled the joint estimation of vehicle states, tire cornering stiffness, and the tire-road adhesion coefficient, they usually do not explicitly address two questions: whether the current vehicle response provides sufficient information for reliable parameter adaptation, and how the lateral-force residual should be allocated between the cornering-stiffness and adhesion-coefficient update channels. Accordingly, the proposed method addresses these two issues through parameter-update reliability assessment and lateral-force-sensitivity-based update allocation, rather than residual-magnitude-driven simultaneous adaptation.

To address the above issues, this paper proposes a co-estimation method for vehicle states, tire cornering stiffness, and the tire-road adhesion coefficient based on parameter-update reliability assessment and lateral-force-sensitivity-based update-allocation. First, a hierarchical estimation framework is established using a planar 3-DOF vehicle model and a Fiala-type nonlinear tire model. The front- and rear-axle lateral-force pseudo-measurements are reconstructed from lateral acceleration and yaw angular acceleration, thereby avoiding the need for additional tire-force sensors. Second, lateral excitation, longitudinal slip, adhesion utilization, and normalized lateral-force residual consistency are jointly incorporated to construct parameter-update reliability indices, which are used to suppress unreliable parameter adaptation under weak-excitation conditions. Furthermore, a selective update allocation mechanism is designed according to the normalized sensitivities of lateral tire force with respect to tire cornering stiffness and the tire-road adhesion coefficient. With this mechanism, the estimator prioritizes the cornering-stiffness update in the linear tire region while assigning a larger update weight to the adhesion-coefficient update in nonlinear or adhesion-limited regions.

The main contributions of this paper are summarized as follows.

(1) A reliability- and sensitivity-guided co-estimation framework is developed for vehicle states, tire cornering stiffness, and the tire-road adhesion coefficient. The proposed framework enables selective parameter adaptation rather than residual-magnitude-driven simultaneous adaptation.

(2) A parameter-update reliability assessment method is proposed to suppress unreliable parameter updates under weak-excitation conditions by jointly considering lateral excitation, longitudinal slip, adhesion utilization, and normalized lateral-force residual consistency.

(3) A lateral-force-sensitivity-based update-allocation mechanism is designed to identify the dominant parameter-update channel in different tire operating regions. This

mechanism prioritizes cornering-stiffness adaptation in the linear tire region and increases the adhesion-coefficient update weight in nonlinear or adhesion-limited regions.

The remainder of this paper is organized as follows. Section 2 presents the vehicle-dynamics model, the Fiala-type nonlinear tire model, the lateral-force pseudo-measurement reconstruction, and the co-estimation problem formulation. Section 3 introduces the proposed reliability- and sensitivity-guided co-estimation method, including parameter-update reliability assessment, lateral-force-sensitivity-based update allocation, and the vehicle-state and tire-road-parameter co-estimation algorithm. Section 4 describes the CarSim/Simulink co-simulation setup, including test maneuvers, comparative methods, evaluation metrics, and robustness settings. Section 5 presents the co-simulation results and discussion. Finally, Section 6 concludes this paper.

2. Vehicle-Dynamics Model and Problem Formulation

This section presents the vehicle-dynamics model, the Fiala-type nonlinear tire model, and the lateral-force pseudo-measurement reconstruction used in this study. On this basis, the vehicle-state and tire-road-parameter co-estimation problem is formulated, while the parameter-update reliability assessment and lateral-force-sensitivity-based update allocation are developed in Section 3.

2.1. Modeling Assumptions and Vehicle Dynamics

The vehicle is described by a planar 3-DOF single-track-equivalent vehicle model, as illustrated in Figure 1. The model is expressed in a body-fixed coordinate frame, where the longitudinal axis points forward, the lateral axis points to the left, and yaw rotation occurs about the vertical axis through the vehicle's center of gravity [28].

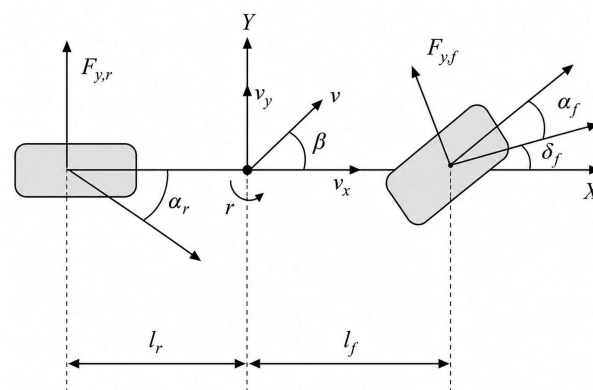


Figure 1. Single-track-equivalent vehicle model and the associated coordinate and force notation.

The vehicle is assumed to travel on an approximately flat road surface. Pitch motion, roll motion, road grade, and road bank angle are not explicitly considered in the vehicle model. In the single-track-equivalent model, the steering angles of the front-left and front-right wheels are assumed to be approximately identical and are represented by the equivalent front-wheel steering angle δ_f . The longitudinal and lateral tire forces generated by the left and right wheels on the same axle are lumped into equivalent axle-level tire forces.

The left and right tires on the same axle are assumed to have similar cornering stiffness and tire-road adhesion characteristics. Accordingly, the front and rear axles are described using equivalent axle-level cornering stiffnesses, while the tire-road adhesion condition is characterized by the tire-road adhesion coefficient. The dominant external forces acting on the planar vehicle motion are assumed to originate from the tire-road contact patches, while

aerodynamic drag, crosswind disturbances, and other higher-order effects are neglected in the vehicle model.

Under these assumptions, the longitudinal, lateral, and yaw dynamics of the vehicle are given by

$$\begin{aligned} m(\dot{v}_x - v_y r) &= F_{x,f} \cos \delta_f - F_{y,f} \sin \delta_f + F_{x,r} \\ m(\dot{v}_y + v_x r) &= F_{x,f} \sin \delta_f + F_{y,f} \cos \delta_f + F_{y,r} \\ I_z \dot{r} &= l_f (F_{x,f} \sin \delta_f + F_{y,f} \cos \delta_f) - l_r F_{y,r} \end{aligned} \quad (1)$$

where m denotes the total vehicle mass; v_x and v_y are the longitudinal and lateral velocities expressed in the vehicle body-fixed coordinate frame, respectively; r is the yaw rate; and δ_f represents the front-wheel steering angle. The variables $F_{x,f}$ and $F_{x,r}$ denote the equivalent longitudinal tire forces acting on the front and rear axles, respectively, while $F_{y,f}$ and $F_{y,r}$ are the corresponding lateral tire forces. In addition, I_z is the yaw moment of inertia about the vertical axis passing through the vehicle center of gravity, and l_f and l_r are the distances from the center of gravity to the front and rear axles, respectively.

The vehicle sideslip angle is defined as the angle between the velocity vector at the vehicle center of gravity and the longitudinal axis of the vehicle body, namely,

$$\beta = \arctan\left(\frac{v_y}{v_x}\right) \quad (2)$$

where β represents the sideslip angle. Under normal driving conditions with $|v_y| \ll |v_x|$, the approximation $\beta \approx v_y/v_x$ is commonly adopted. Owing to its direct relation to vehicle lateral motion, β is an important state variable for stability assessment, trajectory tracking, and model-based vehicle-state estimation.

2.2. Tire Model and Lateral-Force Pseudo-Measurement Reconstruction

The tire slip angles of the front and rear axles are obtained from the planar vehicle kinematic relationship as

$$\begin{aligned} \alpha_f &= \arctan\left(\frac{v_y + l_f r}{v_x}\right) - \delta_f \\ \alpha_r &= \arctan\left(\frac{v_y - l_r r}{v_x}\right) \end{aligned} \quad (3)$$

where α_f and α_r represent the slip angles of the front and rear axles, respectively.

Since the linear tire model cannot capture lateral-force saturation associated with large slip angles or limited tire-road adhesion, a Fiala-type nonlinear tire model is adopted to describe the lateral tire force over the linear, transition, and saturation regions. This model relates lateral tire force to tire slip angle, cornering stiffness, vertical load, and the tire-road adhesion coefficient, and therefore provides the basis for the sensitivity analysis developed later. For axle $j \in \{f, r\}$, the Fiala-type nonlinear lateral tire force model is expressed as

$$\begin{aligned} F_{y,j} &= \begin{cases} -C_j \tan \alpha_j + \frac{C_j^2}{3\mu F_{z,j}} |\tan \alpha_j| \tan \alpha_j - \frac{C_j^3}{27\mu^2 F_{z,j}^2} \tan^3 \alpha_j & |\alpha_j| \leq \alpha_{j,sl} \\ -\mu F_{z,j} \operatorname{sgn}(\alpha_j) & |\alpha_j| > \alpha_{j,sl} \end{cases} \\ \alpha_{j,sl} &= \arctan\left(\frac{3\mu F_{z,j}}{C_j}\right) \end{aligned} \quad (4)$$

where $j = f$ and $j = r$ correspond to the front and rear axles, respectively. C_j denotes the equivalent cornering stiffness of axle j , with C_f and C_r representing the front- and rear-axle cornering stiffnesses. μ is the peak tire-road adhesion coefficient, $F_{z,j}$ is the vertical load on axle j , $\alpha_{j,sl}$ denotes the critical slip angle, and $\operatorname{sgn}(\cdot)$ is the sign function. According to (4), the lateral tire force is mainly governed by $C_j \alpha_j$ in the linear tire region, whereas its adhesion-limited bound is primarily constrained by $\mu F_{z,j}$. This property provides the

physical basis for distinguishing cornering-stiffness-dominated and adhesion-limited tire operating regions in the subsequent lateral-force-sensitivity-based update allocation.

Considering the longitudinal load transfer induced by vehicle acceleration or deceleration, the vertical loads on the front and rear axles are approximated as

$$\begin{aligned} F_{z,f} &= \frac{mgl_r - ma_x h}{l_f + l_r} \\ F_{z,r} &= \frac{mgl_f + ma_x h}{l_f + l_r} \end{aligned} \quad (5)$$

where $F_{z,f}$ and $F_{z,r}$ denote the vertical loads acting on the front and rear axles, respectively. h is the height of the vehicle center of gravity, g is the gravitational acceleration, and a_x is the longitudinal acceleration of the vehicle. This axle-level load-transfer approximation accounts for the variation in vertical loads in the lateral tire-force model while retaining a compact axle-level representation.

Since lateral tire forces are generally not directly measurable using standard production-vehicle sensors, the front- and rear-axle lateral forces are reconstructed from the measured lateral acceleration and yaw angular acceleration. According to the vehicle dynamics in (1), the equivalent axle-level lateral-force pseudo-measurements are obtained as

$$\begin{aligned} \tilde{F}_{y,f} &= \frac{1}{\cos\delta_f} \left(\frac{ml_f a_y + I_z \dot{r}}{l_f + l_r} - F_{x,f} \sin\delta_f \right) \\ \tilde{F}_{y,r} &= \frac{ml_r a_y - I_z \dot{r}}{l_f + l_r} \end{aligned} \quad (6)$$

where $\tilde{F}_{y,f}$ and $\tilde{F}_{y,r}$ denote the reconstructed lateral-force pseudo-measurements of the front and rear axles, respectively. a_y is the lateral acceleration of the vehicle, and $\dot{r}$ represents the yaw angular acceleration. These reconstructed quantities serve as pseudo-measurements of the unmeasured lateral tire forces, which are generally difficult to obtain directly using standard production-vehicle sensors. Under small front-wheel steering angles and negligible longitudinal-force projection at the front axle, (6) can be simplified to the commonly used front- and rear-axle lateral-force reconstruction form. It should be noted that $\dot{r}$ is obtained from the yaw rate r through low-pass filtering and numerical differentiation, so as to avoid direct amplification of high-frequency measurement noise.

2.3. Co-Estimation Problem Formulation

Based on the vehicle dynamics and lateral-force reconstruction, the vehicle-state vector and the parameter vector to be estimated are defined as

$$x_s = \begin{bmatrix} v_x & v_y & r \end{bmatrix}^T, \quad \theta = \begin{bmatrix} C_f & C_r & \mu \end{bmatrix}^T \quad (7)$$

where x_s denotes the vehicle-state vector, and θ represents the parameter vector to be estimated. Specifically, C_f and C_r are the equivalent cornering stiffnesses of the front and rear axles, respectively, and μ is the peak tire-road adhesion coefficient.

The measurement vector available from standard onboard sensors is defined as

$$z = \begin{bmatrix} a_x & a_y & r & \delta_f & \omega_i \end{bmatrix}^T \quad (8)$$

where z denotes the measurement vector, and ω_i is the angular speed of the i ($i = 1, 2, 3, 4$)-th wheel. This sensor configuration is consistent with commonly used onboard sensing setups for vehicle-state and tire-parameter estimation, where accelerations, yaw rate, steering angle, and wheel-speed information are used for real-time estimation.

The objective of this study is to estimate the vehicle states and tire-road parameters online under the availability of z , namely,

$$\hat{x}_s = [\hat{v}_x \quad \hat{v}_y \quad \hat{r}]^T, \quad \hat{\theta} = [\hat{C}_f \quad \hat{C}_r \quad \hat{\mu}]^T \quad (9)$$

where $\hat{x}_s$ and $\hat{\theta}$ denote the estimated state and parameter vectors, respectively. The co-estimation problem considered in this study is to obtain reliable estimates of vehicle states and tire-road parameters from standard onboard measurements while avoiding physically inconsistent parameter adaptation caused by the ambiguous attribution of the lateral-force residual. Specifically, the estimator should determine whether the current vehicle response contains sufficient information for parameter adaptation and whether the lateral-force residual should be mainly allocated to the cornering-stiffness update channel or the adhesion-coefficient update channel.

3. Reliability- and Sensitivity-Guided Co-Estimation Method

3.1. Overall Framework

Figure 2 illustrates the overall framework of the proposed reliability- and sensitivity-guided co-estimation method. The framework consists of five main functional modules: vehicle speed and sideslip-state estimation, lateral-force pseudo-measurement reconstruction, parameter-update reliability assessment, lateral-force-sensitivity-based update allocation, and selective parameter adaptation. The standard onboard measurements and known vehicle parameters are first used to reconstruct the vehicle states and lateral-force pseudo-measurements. Then, the reconstructed lateral-force residual is evaluated together with the Fiala-type nonlinear tire model to determine whether the current operating condition provides sufficient information for parameter adaptation.

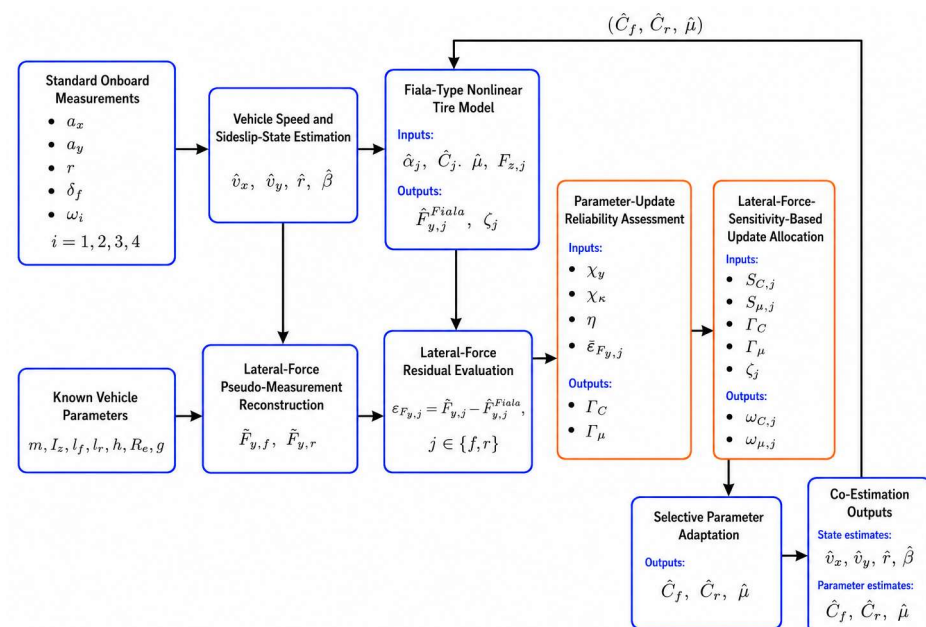


Figure 2. Overall framework of the proposed reliability- and sensitivity-guided co-estimation method.

Different from residual-magnitude-driven simultaneous parameter adaptation, the proposed framework first evaluates the parameter-update reliability and then allocates the lateral-force residual to the physically consistent update channel according to the tire operating region and normalized lateral-force sensitivities. In this way, the front- and rear-axle cornering stiffnesses C_f and C_r , together with the tire-road adhesion coefficient μ ,

can be updated selectively rather than being adjusted simultaneously without distinction. The parameter-update reliability assessment and lateral-force-sensitivity-based update allocation are detailed in Section 3.2, while the vehicle-state and tire-road-parameter co-estimation algorithm is developed in Section 3.3.

3.2. Reliability Assessment and Sensitivity-Based Update Allocation

Dynamic-model-based parameter estimation can provide reliable information for parameter adaptation only when sufficient excitation is present in the measured vehicle response. Under weak-excitation conditions, the lateral-force residual may not contain sufficient information to distinguish variations in tire cornering stiffness from variations in the tire-road adhesion coefficient. Therefore, parameter adaptation should be performed in a condition-dependent manner. In this study, four indicators are introduced to evaluate the parameter-update reliability of the current operating condition and are defined as

$$\begin{aligned}\chi_y &= \text{sat}_{[0,1]} \left(w_a \frac{|a_y|}{a_{y0}} + w_r \frac{|r|}{r_0} + w_{\dot{r}} \frac{|\dot{r}|}{\dot{r}_0} \right) \\ \chi_\kappa &= \text{sat}_{[0,1]} \left(\frac{\max_i |\kappa_i|}{\kappa_0} \right) \\ \eta_j &= \frac{\sqrt{F_{x,j}^2 + \tilde{F}_{y,j}^2}}{\hat{\mu} F_{z,j} + \varepsilon_F} \eta_j = \max_{j \in \{f,r\}} \eta_j \\ \bar{v}_y(k) &= \frac{1}{N_w} \sum_{l=k-N_w+1}^k \sum_{j \in \{f,r\}} \frac{(\tilde{F}_{y,j}(l) - \hat{F}_{y,j}(l))^2}{\sigma_{\tilde{F}_j}^2 + \varepsilon_F}\end{aligned}\quad (10)$$

In (10), χ_y and χ_κ quantify the lateral excitation and longitudinal slip levels, respectively. η_j denotes the adhesion utilization ratio of axle j , and η is the maximum utilization over the front and rear axles. $\bar{v}_y$ represents the moving-window normalized residual consistency index, where N_w is the window length and $\sigma_{\tilde{F}_j}^2$ is the variance of the lateral-force pseudo-measurement. The saturation operator $\text{sat}_{[0,1]}(\cdot)$ restricts the corresponding variable to the interval $[0, 1]$. A larger χ_y indicates stronger lateral excitation, a larger η reflects a higher tire-force utilization level, and an increased $\bar{v}_y$ implies a more significant mismatch between the reconstructed and model-predicted lateral forces. These indicators are used to evaluate the parameter-update reliability of the current operating condition and provide the basis for the subsequent sensitivity-based update allocation.

The slip ratio of the i -th wheel is defined as

$$\kappa_i = \frac{R_e \omega_i - v_{x,i}}{\max(|v_{x,i}|, R_e \omega_i | \varepsilon_v)} \quad (11)$$

where κ_i denotes the slip ratio of wheel i , R_e is the effective tire rolling radius, and $v_{x,i}$ represents the longitudinal velocity of the wheel center expressed in the tire coordinate frame. The small positive constant ε_v is introduced to avoid numerical singularity at very low speeds. This definition provides a normalized measure of the relative motion between the tire circumferential speed and the longitudinal velocity of the wheel center, and is used to characterize the longitudinal excitation level in tire-road adhesion coefficient and vehicle-state estimation.

Based on (10), the adaptation confidence levels associated with the tire cornering stiffness and tire-road adhesion coefficient are defined as

$$\begin{aligned}\Gamma_C &= \chi_y \left[1 - \text{sat}_{[0,1]} \left(\frac{\eta - \eta_{lin}}{\eta_{sat} - \eta_{lin}} \right) \right] \\ \Gamma_\mu &= \text{sat}_{[0,1]} \left(w_\eta \eta + w_v \frac{\bar{v}_y}{v_0} + w_\kappa \chi_\kappa \right)\end{aligned}\quad (12)$$

In (12), Γ_C and Γ_μ represent the adaptation confidence levels for tire cornering stiffness and tire-road adhesion coefficient, respectively. η_{lin} and η_{sat} denote the boundaries of the linear and saturated tire operating regions, while w_η , w_v , and w_κ are nonnegative weighting coefficients. The value of Γ_C increases when sufficient lateral excitation is available and the tire does not operate in a deeply saturated region. By contrast, Γ_μ becomes larger as the tire-force utilization level, lateral-force residual inconsistency, or longitudinal slip level increases. These two confidence levels provide the basis for determining whether the subsequent parameter adaptation should be mainly assigned to C_j or μ .

To determine whether the lateral-force residual should be mainly attributed to variations in tire cornering stiffness or changes in the tire-road adhesion coefficient, the local sensitivities of $F_{y,j}$ with respect to C_j and μ are defined as

$$S_{Cj} = \left| \frac{\partial F_{y,j}}{\partial C_j} \right|, \quad S_\mu = \left| \frac{\partial F_{y,j}}{\partial \mu} \right| \quad (13)$$

where S_{Cj} and S_μ denote the local sensitivities of the axle lateral force to the cornering stiffness C_j and the tire-road adhesion coefficient μ , respectively. Since C_j and μ have different physical dimensions, S_{Cj} and S_μ cannot be compared directly. Therefore, the dimensionless sensitivity indices are defined as

$$\begin{aligned} \bar{S}_{Cj} &= \frac{S_{Cj}\hat{C}_j}{S_{Cj}\hat{C}_j + S_\mu\hat{\mu} + \varepsilon_s} \\ \bar{S}_\mu &= \frac{S_\mu\hat{\mu}}{S_{Cj}\hat{C}_j + S_\mu\hat{\mu} + \varepsilon_s} \end{aligned} \quad (14)$$

In (14), $\bar{S}_{Cj}$ and $\bar{S}_\mu$ are the normalized sensitivity indices associated with C_j and μ , respectively, and ε_s is introduced to avoid numerical singularity. For small slip angles, the Fiala model can be locally approximated by the linear relation $F_{y,j} \approx -C_j\alpha_j$, indicating that the lateral force is mainly influenced by the cornering stiffness. Under this condition, $S_{Cj} \approx |\alpha_j|$ and $S_{\mu j} \approx 0$. When the tire approaches saturation, the lateral force is dominated by the adhesion constraint $F_{y,j} \approx -\mu F_{z,j} \text{sgn}(\alpha_j)$, leading to $S_{Cj} \approx 0$, $S_\mu \approx F_{z,j}$. This property implies that the linear tire region provides more reliable information for updating C_j , whereas the saturated region is more informative for estimating μ .

The update-allocation weights for tire cornering stiffness and the tire-road adhesion coefficient are determined by combining the parameter-update reliability indices with the normalized lateral-force sensitivities, as follows:

$$\omega_{Cj} = \Gamma_C \bar{S}_{Cj} \quad \omega_\mu = \Gamma_\mu \bar{S}_\mu, \quad j \in \{f, r\} \quad (15)$$

where ω_{Cj} denotes the update weight assigned to the cornering stiffness of axle j , and $\omega_{\mu j}$ represents the corresponding update weight for the tire-road adhesion coefficient. These two weights jointly determine how the lateral-force residual is distributed between the cornering-stiffness adaptation channel and the adhesion-coefficient update channel. Specifically, Γ_C and Γ_μ evaluate whether the current driving condition provides sufficient information for parameter adaptation, while $\bar{S}_{Cj}$ and $\bar{S}_\mu$ quantify the relative sensitivity of the lateral tire force to C_j and μ . Therefore, (15) addresses two key issues simultaneously: whether the current operating condition is suitable for parameter adaptation, and whether the lateral-force residual should be mainly attributed to cornering-stiffness variation or variations in the tire-road adhesion coefficient. This allocation mechanism helps suppress erroneous parameter adaptation under weak-excitation, low-adhesion, and nonlinear tire conditions.

3.3. Vehicle-State and Tire-Road-Parameter Co-Estimation

The vehicle-state-estimation module is formulated in a recursive nonlinear state-space form compatible with Kalman-type filtering. The state-transition and measurement equations are given by

$$\begin{aligned}x_{s,k+1} &= f_s(x_{s,k}u_k\hat{\theta}_k) + w_{s,k} \\ z_{s,k} &= h_s(x_{s,k}u_k\hat{\theta}_k) + v_{s,k}\end{aligned}\quad (16)$$

In (16), $x_{s,k}$ denotes the vehicle-state vector at the k -th sampling instant, u_k is the input vector, and $z_{s,k}$ is the measurement vector used for state estimation. The functions $f_s(\cdot)$ and $h_s(\cdot)$ represent the nonlinear state-transition and measurement mappings, respectively. The vectors $w_{s,k}$ and $v_{s,k}$ are the process and measurement noises. The parameter estimate $\hat{\theta}_k$ is incorporated into the state-space model to account for the effects of tire cornering stiffness and the tire-road adhesion coefficient on vehicle dynamics. This recursive formulation provides the basis for estimating unmeasured lateral states, such as lateral velocity and sideslip angle, using standard production-vehicle measurements.

When the extended Kalman filter (EKF) is employed, the prediction and correction steps of the vehicle-state estimator are written as

$$\begin{aligned}\hat{x}_{s,k|k-1} &= f_s(\hat{x}_{s,k-1|k-1}u_k\hat{\theta}_{k-1}) \\ P_{s,k|k-1} &= F_{s,k}P_{s,k-1|k-1}F_{s,k}^T + Q_s \\ K_{s,k} &= P_{s,k|k-1}H_{s,k}^T(H_{s,k}P_{s,k|k-1}H_{s,k}^T + R_s)^{-1} \\ \hat{x}_{s,k|k} &= \hat{x}_{s,k|k-1} + K_{s,k}[z_{s,k} - h_s(\hat{x}_{s,k|k-1}u_k\hat{\theta}_{k-1})] \\ P_{s,k|k} &= (I - K_{s,k}H_{s,k})P_{s,k|k-1}\end{aligned}\quad (17)$$

In (17), $F_{s,k}$ and $H_{s,k}$ denote the Jacobian matrices of the nonlinear state-transition and measurement functions with respect to the state vector, respectively. $K_{s,k}$ is the Kalman gain, while $P_{s,k|k-1}$ and $P_{s,k|k}$ represent the predicted and updated state-estimation error covariance matrices. The matrices Q_s and R_s are the process-noise and measurement-noise covariance matrices, respectively. This recursive estimation structure is consistent with standard model-based state-estimation frameworks for vehicle lateral dynamics and sideslip-angle estimation.

After the longitudinal and lateral-velocity estimates are obtained, the vehicle sideslip-angle estimate is computed as

$$\hat{\beta}_k = \arctan\left(\frac{\hat{v}_{y,k}}{\hat{v}_{x,k}}\right)\quad (18)$$

where $\hat{\beta}_k$ denotes the estimated sideslip angle at the k -th sampling instant. Therefore, the proposed recursive estimator enables unmeasured lateral states, including lateral velocity and sideslip angle, to be inferred from the nonlinear vehicle model and standard onboard measurements.

In the linear tire region, the lateral-force pseudo-measurement can be locally approximated by the following linear-in-parameter regression relation $\tilde{F}_{y,jk} \approx \varphi_{jk}C_{jk}$, where $\varphi_{jk} = -\hat{\alpha}_{jk}$. Based on this relation, a recursive least-squares estimator with a forgetting factor is adopted to identify the time-varying axle cornering stiffness C_j . To suppress erroneous cornering-stiffness adaptation under nonlinear or adhesion-limited tire operating conditions, the lateral-force-sensitivity-based update weight ω_{Cjk} is introduced into the RLS recursion. The resulting update law is written as

$$\begin{aligned}
K_{Cjk} &= \frac{P_{Cjk-1}\varphi_{jk}}{\lambda_C + \varphi_{jk}^2 P_{Cjk-1}} \\
e_{Cjk} &= \tilde{F}_{y,jk} - \varphi_{jk} \hat{C}_{jk-1} \\
\hat{C}_{jk} &= \Pi_{[C_{j,min}, C_{j,max}]} \left(\hat{C}_{jk-1} + \omega_{Cjk} K_{Cjk} e_{Cjk} \right) \\
P_{Cjk} &= \lambda_C^{-1} \left(P_{Cjk-1} - \omega_{Cjk} K_{Cjk} \varphi_{jk} P_{Cjk-1} \right)
\end{aligned} \tag{19}$$

In (19), K_{Cjk} denotes the RLS gain for the cornering stiffness of axle j , P_{Cjk} is the covariance associated with the stiffness estimate, and e_{Cjk} represents the residual between the reconstructed lateral force and the prediction from the local linear tire model. The parameter λ_C is the forgetting factor, and $\Pi_{[C_{j,min}, C_{j,max}]}(\cdot)$ denotes a projection operator used to keep the estimated cornering stiffness within a physically admissible range. By embedding ω_{Cjk} into the recursion, the adaptation of C_j is enhanced under informative linear-region conditions and weakened when the lateral-force residual is more likely to originate from tire saturation or variations in the tire-road adhesion coefficient.

The tire-road adhesion coefficient is updated according to the residual between the reconstructed lateral-force pseudo-measurement and the nonlinear Fiala-model-based lateral-force prediction. For axle j , the adhesion-related residual and the corresponding sensitivity term are defined as

$$\begin{aligned}
e_{\mu jk} &= \tilde{F}_{y,jk} - F_y(\hat{C}_{jk} \hat{\mu}_{k-1} F_{z,jk} \hat{\alpha}_{jk}) \\
H_{\mu jk} &= \left. \frac{\partial F_y}{\partial \mu} \right|_{\hat{C}_{jk} \hat{\mu}_{k-1} F_{z,jk} \hat{\alpha}_{jk}}
\end{aligned} \tag{20}$$

In (20), $e_{\mu jk}$ denotes the residual used for adhesion-coefficient update, which reflects the mismatch between the reconstructed lateral force and the nonlinear tire-model prediction. $H_{\mu jk}$ represents the local sensitivity, or equivalently the local Jacobian, of the lateral tire force with respect to the tire-road adhesion coefficient μ , evaluated at the current operating point determined by C_j , μ , $F_{z,j}$, and α_j . This formulation enables the adhesion-coefficient update to explicitly account for the sensitivity of the lateral tire force to variation in the tire-road adhesion coefficient, which is consistent with model-based tire-force and vehicle-state-estimation frameworks.

The Kalman-type update of the tire-road adhesion coefficient is written as

$$\begin{aligned}
K_{\mu jk} &= \frac{P_{\mu k-1} H_{\mu jk}}{H_{\mu jk}^2 P_{\mu k-1} + R_{\mu jk}} \\
\hat{\mu}_k &= \Pi_{[\mu_{min}, \mu_{max}]} \left(\hat{\mu}_{k-1} + \sum_{j \in \{f, r\}} \omega_{\mu jk} K_{\mu jk} e_{\mu jk} \right) \\
P_{\mu k} &= \left(1 - \sum_{j \in \{f, r\}} \omega_{\mu jk} K_{\mu jk} H_{\mu jk} \right) P_{\mu k-1}
\end{aligned} \tag{21}$$

In (21), $K_{\mu jk}$ denotes the Kalman-type gain associated with the tire-road adhesion coefficient, $P_{\mu k}$ is the covariance of the adhesion-coefficient estimate, and $R_{\mu jk}$ represents the measurement-noise variance used in the adhesion adaptation. The projection operator $\Pi_{[\mu_{min}, \mu_{max}]}(\cdot)$ constrains the estimated adhesion coefficient within the physically admissible interval, such that $0 < \mu_{min} \leq \hat{\mu}_k \leq \mu_{max}$. By introducing the lateral-force-sensitivity-based update weight $\omega_{\mu jk}$, the adhesion adaptation is enhanced when the lateral-force residual is more informative about variation in the tire-road adhesion coefficient, while unnecessary updates are suppressed when the residual is mainly related to cornering-stiffness mismatch or insufficient excitation. This formulation follows a model-based vehicle-state and tire-road-parameter estimation structure while incorporating sensitivity-weighted update allocation.

To characterize the uncertainty of the tire-road adhesion-coefficient estimate, a physically bounded confidence interval is constructed based on the covariance $P_{\mu k}$ of the adhesion-coefficient estimate. Considering the feasible range of the tire-road adhesion coefficient, the confidence interval is computed as

$$CI_{\mu k}^p = \left[\max\left(\mu_{\min}\hat{\mu}_k - 1.96\sqrt{P_{\mu k}}\right), \min\left(\mu_{\max}\hat{\mu}_k + 1.96\sqrt{P_{\mu k}}\right) \right] \quad (22)$$

In (22), $CI_{\mu k}^p$ denotes the confidence interval after enforcing the physical bounds of the tire-road adhesion coefficient. The constants $\mu_{\min}$ and $\mu_{\max}$ define the admissible lower and upper limits of μ , respectively. The coefficient 1.96 corresponds to an approximate 95% confidence level under the Gaussian error assumption. Therefore, (22) complements the point estimate $\hat{\mu}_k$ with an uncertainty-aware and physically bounded interval of the estimated tire-road adhesion coefficient while ensuring physical feasibility.

3.4. Implementation Procedure

The complete implementation procedure of the proposed reliability- and sensitivity-guided co-estimation method is summarized in Algorithm 1. At each sampling instant, the estimator first updates the vehicle states using standard onboard measurements and then reconstructs the lateral-force pseudo-measurements. The parameter-update reliability indices and normalized lateral-force sensitivities are subsequently evaluated to determine whether the current operating condition supports reliable parameter adaptation and how the lateral-force residual should be allocated between the cornering-stiffness and adhesion-coefficient update channels. Finally, the front- and rear-axle cornering stiffnesses and the tire-road adhesion coefficient are updated selectively, and a physically bounded confidence interval for the tire-road adhesion-coefficient estimate is computed.

Algorithm 1. Reliability- and Sensitivity-Guided Co-Estimation Procedure

Input: $a_x, a_y, r, \delta_f, \omega_i$; vehicle parameters m, I_z, l_f, l_r, h, R_e , and g .

Output: $\hat{v}_x, \hat{v}_y, \hat{r}, \hat{\beta}, \hat{C}_f, \hat{C}_r, \hat{\mu}$, and CI_{μ}^p .

Initialize: Set $\hat{x}_{s,0}, \hat{C}_{f,0}, \hat{C}_{r,0}, \hat{\mu}_0, P_{s,0}, P_{C,f,0}, P_{C,r,0}$, and $P_{\mu,0}$.

1: for $k = 1$ to T **do**

2: Estimate $\hat{v}_{x,k}$ using wheel-speed measurements and longitudinal acceleration.

3: Compute $\kappa_i, F_{z,f}, F_{z,r}, \hat{a}_f$, and $\hat{a}_r$.

4: Update $\hat{v}_{y,k}, \hat{r}_k$, and $\hat{\beta}_k$ using the vehicle-state estimator in (16)–(18).

5: Construct $\tilde{F}_{y,f}$ and $\tilde{F}_{y,r}$ according to (6).

6: Compute $\chi_y, \chi_x, \eta, \bar{v}_y, \Gamma_C$, and Γ_{μ} using (10)–(12).

7: Calculate $\bar{S}_{Cj}, \bar{S}_{\mu j}, \omega_{Cj}$, and $\omega_{\mu j}$ using (13)–(15), where $j \in \{f, r\}$.

8: Update $\hat{C}_{f,k}$ and $\hat{C}_{r,k}$ using the sensitivity-weighted RLS adaptation law in (19).

9: Update $\hat{\mu}_k$ using the sensitivity-weighted Kalman-type update law in (20) and (21).

10: Compute $CI_{\mu k}^p$ according to (22).

11: end for

The algorithm is organized in a modular and sequential manner. The vehicle-state-estimation module provides the velocity and sideslip-angle estimates, while the lateral-force pseudo-measurement reconstruction and parameter-update reliability assessment modules evaluate whether the current operating condition is informative for parameter adaptation. The update-allocation weights ω_{Cj} and $\omega_{\mu j}$ then allocate the lateral-force residual between the cornering-stiffness and adhesion-coefficient update channels. This structure suppresses erroneous cornering-stiffness updates under nonlinear or adhesion-

limited tire operating conditions and mitigates unreliable parameter adaptation under weak-excitation conditions.

4. CarSim/Simulink Co-Simulation Setup

The proposed estimator is evaluated in a CarSim/Simulink co-simulation environment, where the CarSim full-vehicle model serves as the plant and the estimator is implemented in MATLAB/Simulink R2018b. This setup separates the plant model from the estimation algorithm and enables the estimator to be evaluated against high-fidelity full-vehicle dynamics and nonlinear tire characteristics.

4.1. Co-Simulation Platform and Signal Configuration

Figure 3 shows the CarSim/Simulink co-simulation architecture for vehicle-state and tire-road-parameter co-estimation. The CarSim full-vehicle model generates vehicle responses under prescribed double-lane-change maneuvers and tire-road adhesion conditions. The estimator receives only standard onboard measurements, including longitudinal acceleration, lateral acceleration, yaw rate, front-wheel steering angle, and wheel angular speed measurements. The reference sideslip angle, lateral velocity, tire forces, and prescribed tire-road adhesion coefficient are used only for offline performance evaluation and are not provided to the estimator.

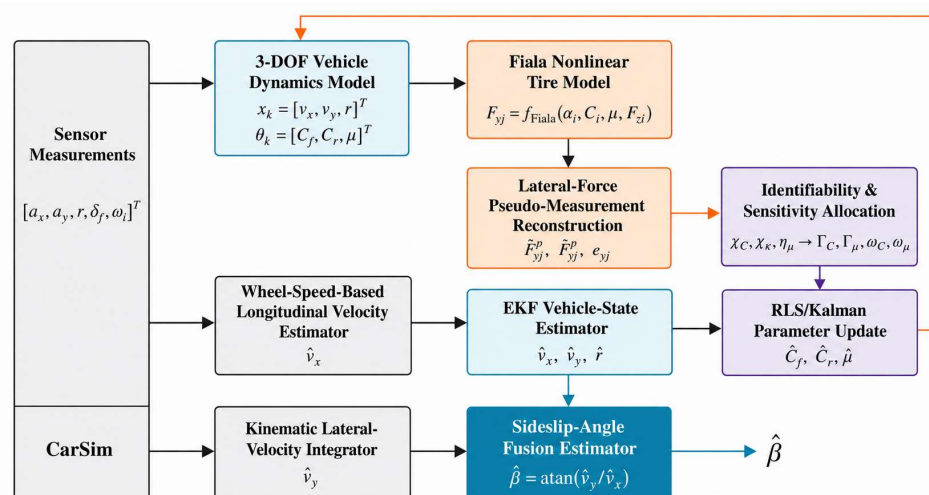


Figure 3. CarSim/Simulink co-simulation architecture for co-estimation of vehicle states, tire cornering stiffness, and the tire-road adhesion coefficient.

The estimator is developed based on the planar 3-DOF single-track-equivalent vehicle model and the Fiala-type nonlinear tire model presented in Section 2. In contrast, the plant is represented by the high-fidelity CarSim full-vehicle model with built-in nonlinear tire characteristics. Therefore, plant–estimator model mismatch is retained in the co-simulation, so that the estimator operates with simplified vehicle dynamics while being evaluated against high-fidelity vehicle responses.

Front- and rear-axle lateral-force pseudo-measurements are reconstructed from measurable vehicle responses and then used to estimate vehicle states, tire cornering stiffnesses, and the tire-road adhesion coefficient. The parameter-update allocation follows the reliability- and sensitivity-guided scheme developed in Section 3.

4.2. Test Maneuvers and Tire-Road Adhesion Conditions

The main vehicle parameters and simulation configuration are listed in Table 1. These parameters are adopted as nominal values for the estimator and co-simulation. The simulation step size is set to 0.001 s.

Table 1. Main vehicle parameters and simulation configuration.

Symbol	Description	Value and Unit
m	Vehicle mass	1413 kg
l_f	Distance from the center of gravity to the front axle	1.015 m
l_r	Distance from the center of gravity to the rear axle	1.895 m
L	Wheelbase	2.910 m
d	Track width	1.675 m
R	Wheel rolling radius	0.325 m
I_z	Yaw moment of inertia	1536.7 kg·m ²
h	Height of the center of gravity	0.540 m
T_s	Simulation step size	0.001 s

Four double-lane-change scenarios are considered, as summarized in Table 2, to evaluate the estimator under different tire-road adhesion and speed conditions. Case 1 corresponds to a high-adhesion condition with $\mu = 0.85$ and a vehicle speed of 120 km/h. Case 2 represents a low-adhesion condition with $\mu = 0.35$ at the same vehicle speed. Case 3 uses an intermediate tire-road adhesion coefficient of $\mu = 0.50$ and a vehicle speed of 100 km/h. Case 4 adopts the same tire-road adhesion coefficient as Case 2, $\mu = 0.35$, but with a lower vehicle speed of 80 km/h. The four scenarios cover high-, intermediate-, and low-adhesion conditions with different vehicle speeds.

Table 2. Test maneuvers and tire-road adhesion-coefficient settings.

Case	Maneuver	Tire-Road Adhesion Coefficient	Speed (km/h)
Case 1	Double-lane-change	0.85	120
Case 2	Double-lane-change	0.35	120
Case 3	Double-lane-change	0.50	100
Case 4	Double-lane-change	0.35	80

4.3. Comparative Methods, Evaluation Metrics, and Robustness Settings

Five estimation methods are compared under identical co-simulation conditions. As summarized in Table 3, Method A is a fixed-parameter EKF, Method B updates the tire cornering stiffnesses while keeping the tire-road adhesion coefficient fixed, Method C updates all tire-road parameters simultaneously, Method D uses sensitivity-based update allocation, and Method E is the proposed reliability- and sensitivity-guided co-estimation method. All methods use the same sampling period, input signals, initial estimates, and evaluation intervals to ensure a fair comparison.

The estimation accuracy is evaluated using P -RMSE $_{\beta}$, M -RMSE $_{\beta}$, P -RMSE $_{v_y}$, and the final error of μ . P -RMSE $_{\beta}$ and P -RMSE $_{v_y}$ quantify the peak transient errors of the sideslip angle and lateral velocity, respectively, whereas M -RMSE $_{\beta}$ evaluates the overall sideslip-angle estimation accuracy over the selected evaluation interval. Since the tire-road adhesion coefficient is prescribed as a constant in each scenario, the final error of μ is defined as the absolute deviation between the reference tire-road adhesion coefficient and the average estimated value in the final steady-state window. For Methods A and B, μ is not estimated online; therefore, this metric is not reported. Because the true axle-level cornering stiffnesses

are not directly measurable from the CarSim full-vehicle plant and vary implicitly with tire operating conditions, a direct numerical error metric is not reported for C_f and C_r . Instead, the cornering-stiffness estimates are evaluated in terms of convergence behavior, boundedness, and physical consistency.

Table 3. Baseline estimators and the proposed method used for comparison.

Method	Description	Purpose
Method A	Fixed-parameter EKF	Evaluates model mismatch when C_f , C_r , and μ are fixed
Method B	EKF with conventional RLS stiffness adaptation	Examines conventional stiffness update under nonlinear tire operation
Method C	Unconstrained joint update of C_f , C_r , and μ	Investigates parameter-coupling errors without update allocation
Method D	Sensitivity-only update	Evaluates sensitivity-based allocation without reliability assessment
Method E	Proposed method	Complete reliability- and sensitivity-guided co-estimation method

Robustness tests are performed under measurement noise and estimator-parameter-uncertainty. Case 2 is selected as the baseline because low-adhesion operation is more demanding for both vehicle-state estimation and tire-road-parameter adaptation. In the measurement-noise test, zero-mean Gaussian noise is injected into the measured signals listed in Table 4. In the estimator parameter uncertainty test, representative perturbations are applied only to the vehicle parameters used inside the estimator, while the CarSim plant remains unchanged. Thus, this test introduces plant–estimator parameter mismatch rather than modifying the reference vehicle dynamics. The robustness-test settings are summarized in Table 4. The robustness evaluation includes the nominal condition, the measurement-noise condition, the estimator-parameter-uncertainty condition, and the combined-disturbance condition.

Table 4. Robustness-test settings for measurement noise and estimator parameter uncertainty.

Category	Item	Setting
Measurement noise	Longitudinal acceleration	0.0500 m/s ²
	Lateral acceleration	0.0500 m/s ²
	Yaw rate	0.0035 rad/s
	Front-wheel steering angle	0.0017 rad
	Wheel angular speed	0.1000 rad/s
Estimator parameter uncertainty	Vehicle mass	−10%
	Yaw moment of inertia	+10%
	Distance from CG to front axle	−10%
	Distance from CG to rear axle	+10%

5. Results and Discussion

This section presents the CarSim/Simulink co-simulation results. Time-domain estimation performance, quantitative accuracy, update-allocation behavior, robustness, and real-time feasibility are analyzed under the driving scenarios and evaluation settings defined in Section 4.

5.1. Estimation Performance Under Representative Maneuvers

Figures 4 and 5 present the co-simulation results for representative high- and low-adhesion double-lane-change maneuvers, respectively. The estimated lateral velocity,

longitudinal velocity, sideslip angle, tire-road adhesion coefficient, front-tire cornering stiffness, and rear-tire cornering stiffness are compared for the five methods defined in Section 4.3.

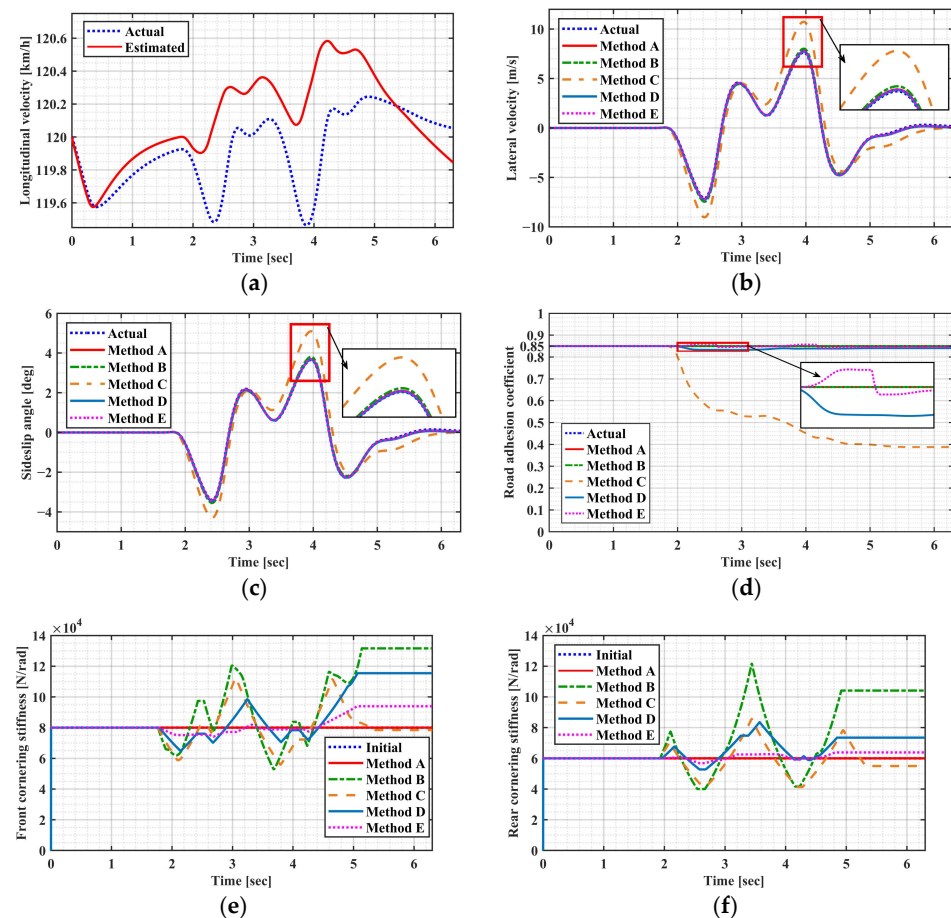


Figure 4. Co-simulation results under the high-adhesion double-lane-change maneuver with $\mu = 0.85$ and $v_x = 120$ km/h: (a) lateral velocity, (b) longitudinal velocity, (c) sideslip angle, (d) tire-road adhesion coefficient, (e) front-tire cornering stiffness, and (f) rear-tire cornering stiffness.

Under the high-adhesion condition with $\mu = 0.85$ and $v_x = 120$ km/h, most methods provide acceptable vehicle-state tracking because the nominal tire parameters and tire-road adhesion coefficient are close to the actual high-adhesion condition. However, Method C exhibits noticeable parameter drift in μ , C_f , and C_r , indicating that unconstrained simultaneous parameter updates may introduce erroneous adaptation in the linear tire operating region. In contrast, Method E keeps the tire-road adhesion coefficient close to the reference value and maintains the front- and rear-axle cornering stiffnesses within physically reasonable ranges.

Under the low-adhesion condition with $\mu = 0.35$ and $v_x = 120$ km/h, the differences among the compared methods become more pronounced. Method A suffers from increased estimation errors because the preset tire parameters and tire-road adhesion coefficient no longer match the actual low-adhesion tire response. Method B partially improves vehicle-state estimation by updating tire cornering stiffness, but it cannot distinguish stiffness variation from adhesion reduction. Method C produces large fluctuations in the estimated C_f , C_r , and μ , whereas Method D alleviates this problem only to a limited extent. By combining parameter-update reliability assessment with sensitivity-based residual allocation, Method E provides more stable estimates of C_f , C_r , and μ , leading to improved sideslip-angle and lateral-velocity tracking.

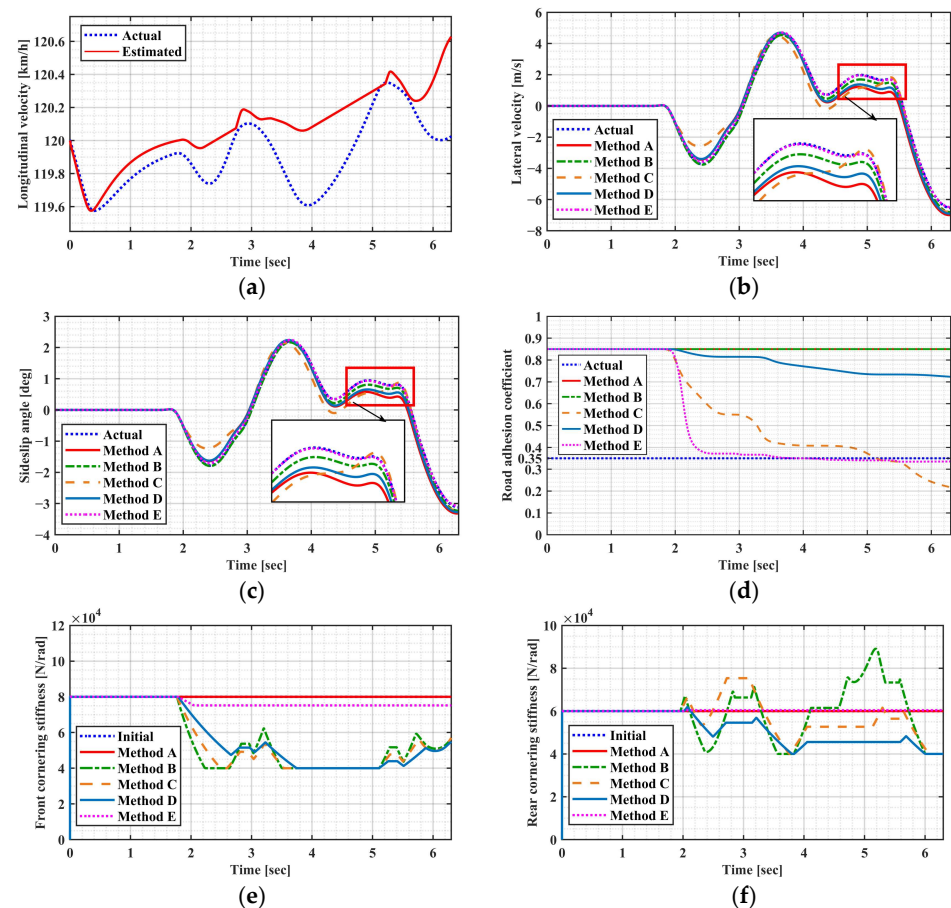


Figure 5. Co-simulation results under the low-adhesion double-lane-change maneuver with $\mu = 0.35$ and $v_x = 120$ km/h: (a) lateral velocity, (b) longitudinal velocity, (c) sideslip angle, (d) tire-road adhesion coefficient, (e) front-tire cornering stiffness, and (f) rear-tire cornering stiffness.

The locally enlarged views in Figures 4 and 5 further show that Method E reduces transient estimation deviations during the main steering phases, where lateral excitation and nonlinear tire effects are more pronounced. These results indicate that the proposed method improves vehicle-state-estimation accuracy while suppressing physically unreasonable tire-parameter and adhesion-coefficient drift.

5.2. Quantitative Comparison Under Extended Driving Conditions

Table 5 summarizes the quantitative comparison under the four driving scenarios defined in Section 4.2. The error metrics $P\text{-RMSE}_{\beta}$, $M\text{-RMSE}_{\beta}$, $P\text{-RMSE}_{v_y}$, and the final error of μ follow the definitions given in Section 4.3. The four scenarios cover different tire-road adhesion and speed conditions, allowing the estimation performance to be evaluated under different lateral excitation levels.

In Case 1, the improvement of Method E over Method A is moderate because the nominal tire parameters and tire-road adhesion coefficient are close to the high-adhesion condition. Nevertheless, Method E achieves the smallest state-estimation errors and reduces the final error of μ from 0.4617 for Method C to 0.0054. In Case 2, where the mismatch between the nominal and actual tire-road conditions becomes more significant, Method E provides clear reductions in $P\text{-RMSE}_{\beta}$, $M\text{-RMSE}_{\beta}$, and $P\text{-RMSE}_{v_y}$ compared with Methods A–D, while maintaining a small final μ error of 0.0132.

Table 5. Quantitative comparison of estimation accuracy under extended driving conditions.

Case	Method	$P\text{-RMSE}_\beta$ (Deg)	$M\text{-RMSE}_\beta$ (Deg)	$P\text{-RMSE}_{v_y}$ (km/h)	Final Error of μ
1	A	0.0324	0.0554	0.0742	/
	B	0.0788	0.1659	0.1739	/
	C	0.5785	1.443	1.228	0.4617
	D	0.0379	0.0722	0.0847	0.0096
	E	0.0304	0.0527	0.0705	0.0054
2	A	0.2081	0.4054	0.4378	/
	B	0.0790	0.1383	0.1668	/
	C	0.2634	0.5827	0.5509	0.0547
	D	0.1659	0.2964	0.3491	0.3816
	E	0.0407	0.0990	0.0852	0.0132
3	A	0.0498	0.1149	0.0867	/
	B	0.0507	0.1290	0.0899	/
	C	0.3568	1.2650	0.6280	0.2606
	D	0.0400	0.0824	0.0711	0.2844
	E	0.0267	0.0709	0.0472	0.0453
4	A	0.1290	0.3060	0.1796	/
	B	0.0383	0.1012	0.0548	/
	C	0.3791	1.2090	0.5344	0.1189
	D	0.0964	0.2242	0.1357	0.4202
	E	0.0443	0.1101	0.0630	0.0139

The results in Cases 3 and 4 further confirm the applicability of Method E under different adhesion and speed conditions. In Case 3, Method E achieves the lowest state-estimation errors and a smaller final error of μ than Methods C and D. In Case 4, Method B yields slightly smaller state-estimation errors because of the weaker low-speed excitation, but it does not provide online adhesion-coefficient estimation. Method E still maintains competitive vehicle-state-estimation accuracy and reduces the final error of μ to 0.0139.

Overall, Table 5 shows that Method E provides the most balanced estimation performance among the compared methods. It improves vehicle-state-estimation accuracy under most conditions and, more importantly, provides physically more reliable tire-parameter and tire-road adhesion estimates. These results indicate that increasing the number of adaptive parameters does not necessarily improve estimation performance unless the residual attribution between cornering stiffness and the tire-road adhesion coefficient is properly handled.

5.3. Mechanism Analysis of Reliability and Sensitivity Allocation

Figure 6 reports the relative reductions in $P\text{-RMSE}_\beta$, $M\text{-RMSE}_\beta$, and $P\text{-RMSE}_{v_y}$ achieved by Method E compared with Methods A–D under representative high- and low-adhesion conditions. The adhesion-coefficient estimation accuracy is evaluated separately using the final error of μ in Table 5.

Under the high-adhesion condition, the improvement of Method E over Method A is relatively moderate because the nominal tire parameters and tire-road adhesion coefficient are close to the actual condition. However, Method E significantly outperforms adaptive methods that lack proper residual-attribution constraints, especially Method C. Under the low-adhesion condition, the advantages of Method E become more pronounced, indicating that the reliability assessment and sensitivity-based allocation mechanism can improve vehicle-state estimation when the tire-road adhesion coefficient differs significantly from its nominal value.

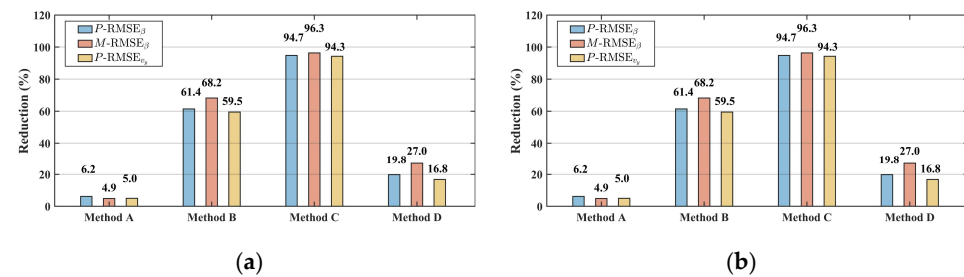


Figure 6. Relative error reductions achieved by Method E compared with Methods A–D under different tire-road adhesion conditions at $v_x = 120$ km/h: (a) high-adhesion condition with $\mu = 0.85$; (b) low-adhesion condition with $\mu = 0.35$.

Figure 7 shows the evolution of the parameter-update reliability indices, normalized lateral-force sensitivities, and update-allocation weights. For the high-adhesion case with $\mu = 0.85$, the adhesion-coefficient-related reliability index and the corresponding update weight remain close to zero during most of the maneuver. This is consistent with the Fiala-type nonlinear tire model, where the lateral tire force in the linear region is more sensitive to tire cornering stiffness than to the tire-road adhesion coefficient. Therefore, the estimator mainly allocates the lateral-force residual to the cornering-stiffness update channels, while unnecessary adhesion-coefficient updates are suppressed.

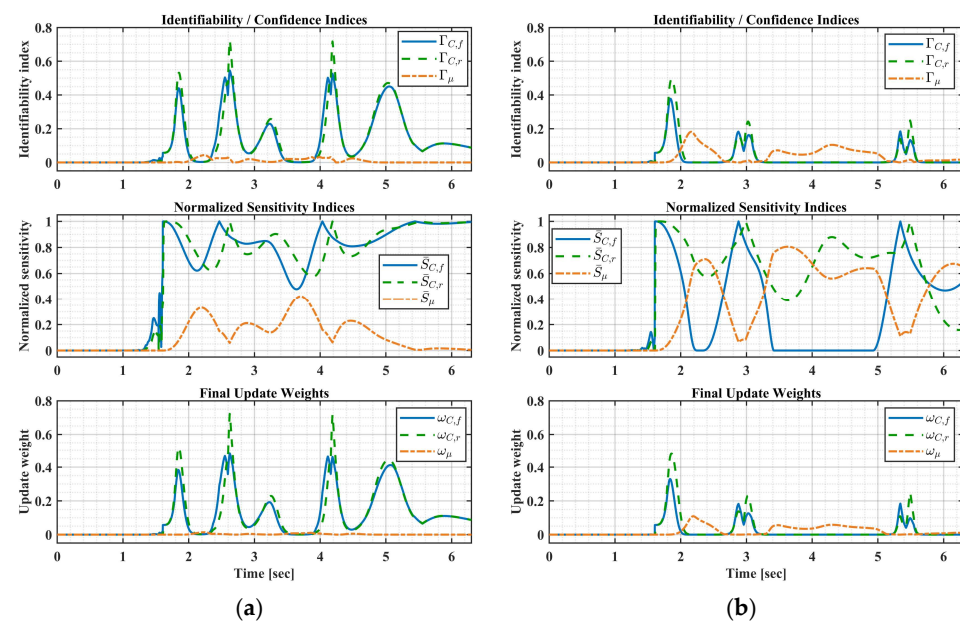


Figure 7. Evolution of parameter-update reliability indices, normalized lateral-force sensitivities, and update-allocation weights under double-lane-change maneuvers with $v_x = 120$ km/h: (a) high-adhesion condition with $\mu = 0.85$; (b) low-adhesion condition with $\mu = 0.35$.

For the low-adhesion case with $\mu = 0.35$, the adhesion-coefficient-related reliability index and normalized lateral-force sensitivity become more pronounced during intervals with strong lateral excitation. This indicates that the lateral-force residual contains useful information associated with the reduction in available tire-road adhesion. Meanwhile, the cornering-stiffness update channels are activated only when the lateral excitation is sufficiently informative and are suppressed during weakly informative intervals or nonlinear tire operation.

These results show that the proposed method improves the physical attribution of lateral-force residuals. By assigning the residual to the parameter-update channel that is more consistent with the tire-force generation mechanism, Method E mit-

igates erroneous parameter coupling and provides more reliable vehicle-state and tire-road-parameter estimates.

5.4. Robustness and Real-Time Feasibility Analysis

Robustness results obtained under the measurement-noise, estimator-parameter-uncertainty, and combined-disturbance conditions are summarized in Table 6. The nominal low-adhesion scenario, namely Case 2, is used as the baseline. The corresponding noise levels and estimator parameter perturbations are defined in Section 4.3.

Table 6. Robustness evaluation results under measurement noise and estimator parameter uncertainty.

Test Condition	$P\text{-RMSE}_\beta$ (Deg)	$M\text{-RMSE}_\beta$ (Deg)	$P\text{-RMSE}_{v_y}$ (km/h)	Final Error of μ
Nominal	0.0407	0.0990	0.0852	0.0132
Measurement noise	0.0755	0.1836	0.1615	0.0253
Parameter uncertainty	0.0448	0.1086	0.0939	0.0338
Noise + parameter uncertainty	0.0788	0.1967	0.1694	0.0426

Measurement noise and estimator parameter uncertainty increase the estimation errors, but no divergence or unrealistic parameter drift is observed. Under the combined-disturbance condition, $P\text{-RMSE}_\beta$, $M\text{-RMSE}_\beta$, and $P\text{-RMSE}_{v_y}$ remain 0.0788 deg, 0.1967 deg, and 0.1694 km/h, respectively, and the final error of μ is 0.0426. These results indicate that the proposed update allocation remains stable under noisy measurements and plant–estimator parameter mismatch.

The computational burden and real-time feasibility of the proposed method are evaluated using MATLAB tic/toc. For the 6.3 s double-lane-change maneuver, the complete CarSim/Simulink co-simulation requires 2.5730 s, corresponding to a co-simulation time ratio of 0.4084. This value includes the CarSim plant, Simulink estimator, interface communication, and data logging. The CarSim computational time ratio is 0.100952, indicating that the high-fidelity vehicle plant itself also runs faster than real time.

With a sampling period of 1 ms, the average wall-clock time per simulation step is 0.4084 ms, which is smaller than the sampling period. Therefore, the proposed estimator meets the sampling-time constraint in the MATLAB/Simulink co-simulation environment. Although this result does not replace hardware-in-the-loop or real-vehicle validation, it indicates that the proposed algorithm has a relatively low computational burden and is suitable for subsequent real-time implementation tests.

The results in Tables 6 and 7 indicate that the proposed reliability- and sensitivity-guided co-estimation framework remains stable under representative measurement noise and plant–estimator parameter mismatch, while satisfying the sampling-time constraint in the considered co-simulation environment.

Table 7. Computational cost and real-time feasibility of the proposed estimator.

Item	Value
Simulation time	6.3 s
Wall-clock time	2.573 s
Co-simulation time ratio	0.4084
CarSim computational time ratio	0.1009
Sampling period	1 ms
Average wall-clock time per step	0.4084 ms

6. Conclusions

This paper presents a reliability- and sensitivity-guided co-estimation method for vehicle states, tire cornering stiffness, and the tire-road adhesion coefficient. The key novelty is that lateral-force residuals are assessed in terms of both update reliability and physical attribution before parameter adaptation, rather than being directly used for residual-magnitude-driven simultaneous parameter adaptation. Based on a planar 3-DOF vehicle model and a Fiala-type nonlinear tire model, front- and rear-axle lateral-force pseudo-measurements are reconstructed from measured vehicle-dynamic responses. Parameter-update reliability indices are introduced to suppress unreliable parameter updates under weak-excitation conditions, while normalized lateral-force sensitivities are used to allocate the residual between the cornering-stiffness and adhesion-coefficient update channels.

CarSim/Simulink co-simulations are conducted under high-, intermediate-, and low-adhesion double-lane-change scenarios with different vehicle speeds. The results show that the proposed method improves sideslip-angle and lateral-velocity estimation accuracy, suppresses physically inconsistent cornering-stiffness adaptation, and provides more stable estimates of the tire-road adhesion coefficient. The update-allocation analysis further confirms that the lateral-force residual is mainly assigned to the cornering-stiffness update channel in the linear tire region, whereas the adhesion-coefficient update weight increases under low-adhesion or nonlinear tire operating conditions. Robustness analysis and computational-cost evaluation further indicate that the proposed method remains effective under representative measurement noise and plant-estimator parameter mismatch, while satisfying the sampling-time constraint in the considered co-simulation environment. These results suggest that the proposed method can provide reliable state and tire-road-parameter information for vehicle stability assessment, active safety control, and intelligent chassis control. Future work will focus on hardware-in-the-loop verification and real-vehicle experiments, with further consideration of roll motion, roll-induced lateral load transfer, road grade, tire degradation, and environmental disturbances.

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