

Article

Power-Smoothing Control Strategy for a Slope-Track Gravity Energy Storage System in Wind-Farm Applications

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Abstract

With the increasing penetration of renewable energy, smoothing wind-farm-level power fluctuations at the point of common coupling has become an important requirement for maintaining grid stability. However, the dynamic power-tracking mechanism of large-inertia slope-track solid gravity energy storage systems remains insufficiently understood. This paper presents a theoretical and control-oriented study of a slope-track solid gravity energy storage system by establishing a low-speed-branch electromechanical coupling model, deriving its steady-state power-speed characteristics, and formulating several feedforward-feedback electromagnetic torque control laws. Pure feedforward control, power feedback, speed feedback, and a gain-scheduled speed feedback law are analyzed within a unified framework. The results show that, for a given power command and under sufficient torque-control authority, increasing the moving load or slope angle enhances the gravity-driven torque and reduces the required operating speed, acceleration, and cumulative displacement. This mechanism indicates that systems with heavier loads or steeper slopes have an advantage in smoothly responding to wind-power fluctuations at a fixed power scale. Frequency-response analysis further shows that the low-speed branch is suitable for slow and medium time-scale power smoothing, whereas rapidly varying commands remain constrained by the closed-loop response time and inertial correction terms. To demonstrate a finite-track implementation of the theory, a single-track multi-unit scheme with 30 standard load units is designed and numerically tested for wind-farm-side smoothing and slow-varying command tracking. This analysis provides a theoretical basis for prototype design and parameter selection before costly full-scale experimental validation.

Keywords: solid gravity energy storage; wind power smoothing; electromechanical coupling; multi-unit coordination; electromagnetic torque control; feedforward-feedback



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1. Introduction

As the proportion of renewable energy sources, such as wind and solar photovoltaics, continues to rise, power systems are exposed to stronger power fluctuations and uncertainties across multiple time scales. Fast disturbances affect frequency quality, primary regulation, and rapid reserve requirements, whereas minute-to-hour ramping events influence reserve allocation, economic dispatch, and unit commitment. Extensive studies have indicated that, as renewable penetration enters moderate-to-high levels, the flexibility deficit becomes a critical bottleneck constraining further renewable integration, thereby increasing the demand for large-scale energy storage and other flexibility resources [1–5]. In

this context, smoothing the power injected by local renewable energy stations at the point of common coupling (PCC), so that they behave more like stable or controlled-ramping power sources, provides a promising technical route for improving grid-side power controllability and reducing the burden on system-level regulation.

Among various flexibility resources, solid gravity energy storage (SGES) stores energy by lifting heavy masses and releases energy by lowering them through an electromechanical conversion system. Compared with electrochemical storage, SGES is attractive mainly because of its long service life, low cyclic degradation, mechanical robustness, and potential compatibility with large-scale terrain-based installations [6,7]. These features make SGES a promising candidate for high-power and long-duration power balancing at local renewable energy stations. Existing solid SGES technologies can be broadly grouped into three routes. Tower/block systems, represented by standardized block-lifting schemes such as Energy Vault, provide high modularity but rely on large engineered structures and standardized loads [8–11]. Shaft/mine systems, represented by Gravitricity, Green Gravity, and abandoned-mine schemes, utilize vertical height differences in shafts or underground spaces and may reuse existing assets, but their feasibility is strongly constrained by shaft conditions, hoisting reliability, retrofit cost, and underground safety requirements [12–16]. Slope-track systems, represented by ARES-type rail systems and recent slope-track prototypes, employ heavy load units moving along inclined tracks and can make use of natural terrain, making them particularly relevant for mountainous areas, abandoned open-pit mines, and hilly renewable energy bases [7,17–20]. A route-level comparison is summarized in Table 1.

Table 1. Comparison of representative solid gravity energy storage routes.

Route	Representative Schemes	Main Advantages	Main Limitations	Relevance to This Work
Tower/block SGES	Energy Vault EVx and related standardized block-lifting schemes	High modularity, standardized construction, and relatively clear engineering replication path	Requires large engineered structures, precise block-handling systems, and a large inventory of standardized loads	Provides an important modular SGES reference, but its dominant dynamics are associated with vertical lifting and block switching rather than continuous slope-track motion
Shaft/mine SGES	Gravitricity, Green Gravity, abandoned-mine, and vertical-shaft schemes	Large vertical height difference and potential reuse of existing shafts or underground assets	Strong dependence on shaft conditions, hoisting reliability, retrofit cost, and underground safety constraints	Highlights the importance of site constraints and multi-unit coordination, but differs from the finite-track displacement constraint considered in this paper
Slope-track SGES	ARES-type rail systems, chain-rail schemes, and slope-track prototypes	Utilizes natural terrain, supports heavy-haul motion, and is suitable for continuous bidirectional operation	Limited by track length, cumulative displacement, wheel-rail impact, traction capability, and coordinated loading/unloading	Directly motivates the present study on electromechanical power tracking, feedforward-feedback torque control, and wind-farm-side power smoothing

Although slope-track SGES has attracted attention in system design, capacity assessment, efficiency evaluation, and economic analysis [19,21,22], its dynamic power-tracking mechanism remains insufficiently understood. A slope SGES system is essentially a large-inertia electromechanical system in which the output power is jointly determined by the electromagnetic torque and the mechanical speed. For a given power command, the system may operate on different steady-state branches with distinct controllability properties. Therefore, simply treating the device as an ideal power source or a quasi-static storage unit

is insufficient for evaluating its capability to smooth wind-farm-level power fluctuations over several-tens-of-seconds-to-hour time scales, which is the target range of the present study rather than sub-second turbine-scale turbulence compensation. Existing studies on gravity energy storage power control have provided valuable insights into module switching compensation, power stabilization, and coordinated operation [23,24]. Nevertheless, a systematic theoretical analysis of the low-speed operating branch, feedforward-feedback torque control laws, parameter-dependent dynamic response, and frequency-response limits of slope-track SGES is still needed.

Motivated by this gap, this paper first investigates the general electromechanical coupling dynamics of a slope-track SGES system. Because the system contains large moving masses, its output power cannot be determined by the power command alone, but is governed by the coupled effects of gravity-driven torque, electromagnetic torque, mechanical speed, and system inertia. Therefore, before evaluating its power-smoothing capability, it is necessary to clarify the steady-state power-speed relationship, the controllable operating branch, and the local dynamic response of the system. In this study, the low-speed high-torque branch is selected as the main operating branch. A corresponding electromechanical coupling model is established, and a feedforward-feedback electromagnetic torque control framework is constructed. Within this framework, pure feedforward control, power feedback, speed feedback, and a gain-scheduled speed feedback law are analyzed in a unified form. The closed-loop error dynamics and frequency-response characteristics are then derived to clarify the dynamic tracking capability and limitations of the large-inertia slope system under power commands with different time scales.

Based on the above theoretical analysis, this paper further considers an application scenario of wind-farm-side power fluctuation smoothing and scheduled command tracking at the PCC. For this purpose, a finite-track single-track SGES system with multiple load units is designed as a numerical implementation of the proposed theoretical framework. The objective of this application scenario is not to complete a long one-way charging or discharging process, but to exploit the fact that wind power over a several-hour window often fluctuates around a local mean or scheduled output. In this case, the load units can move back and forth along the slope track to compensate for positive and negative deviations from the reference power. To avoid concentrating the required total moving mass into a single extremely heavy unit, the total mass is decomposed into multiple standard load units. Soft loading/unloading and terminal inventory replacement are introduced to support approximately continuous operation within a finite track. In the numerical simulations, a single-track system with 30 standard load units is constructed, in which part of the units forms the central active motion chain and the remaining units serve as upper and lower inventories for cyclic replacement. The results show that, under the selected parameters, the system can smooth MW-scale wind-power fluctuations and track slow-varying PCC-side power commands with a finite track length and a finite load inventory.

Following the above theoretical analysis and application-oriented numerical implementation, the main contributions of this paper are summarized as follows:

1. A low-speed-branch electromechanical model of the slope-track SGES system is established. The steady-state power-speed curve is analyzed, and the low-speed high-torque branch is identified as the practical operating regime for wind-farm-level power tracking under the considered slow and medium time-scale smoothing scenario.
2. A feedforward-feedback electromagnetic torque control framework is formulated. Within this framework, pure feedforward control, power feedback, speed feedback, and a gain-scheduled speed feedback law are analyzed in a unified form. The corresponding local error dynamics and frequency-response characteristics are derived to clarify the tracking capability and its dynamic limits.

3. A scaling mechanism of heavy-load slope SGES is clarified. Under sufficient torque-control authority, increasing the load mass or slope angle increases the gravity-driven torque and reduces the speed, acceleration, and displacement excursions required for the same power regulation task. This result follows from the low-speed-branch power-speed relationship and frequency-response analysis, and explains why systems with larger moving loads or steeper slopes are more favorable for smooth power response at a fixed power scale.
4. A single-track multi-unit operation scheme is proposed as an application-oriented numerical implementation of the theoretical framework. By exploiting the fact that wind-farm power deviations over a several-hour window often fluctuate around a local reference output, the scheme allows multiple load units to move back and forth along the track, thereby providing PCC-side power balancing with a finite load inventory. The required total moving mass is decomposed into multiple standard load units to reduce track impact, traction-system stress, and loading/unloading-induced power disturbances.
5. The functional boundary of the proposed slope-track SGES system is clarified. The system is positioned primarily as a PCC-side power-shaping and short-term buffering device for local renewable energy stations, rather than as a stand-alone solution for daily large-scale energy shifting. The limitations associated with rapid command tracking, finite-track margins, and finite load inventory are analyzed to define the applicable operating range of the proposed scheme.

The present paper is intended to provide a theoretical basis for future prototype design, parameter selection, and experimental validation. Full-scale experimental validation of a heavy-haul slope-track SGES system would require substantial infrastructure construction, long-duration reliability testing, and high implementation cost. Accordingly, the present work should be understood as a theoretical prerequisite for heavy-load prototype development, rather than as a claim of field-ready engineering validation.

2. Operating Principle and Electromechanical Modeling of the Slope SGES System

Figure 1 shows the generalized electromechanical representation of the slope-track SGES considered in this study. The diagram is not intended to describe a specific site layout, but to extract the common structure shared by this type of system. The moving units are represented by an equivalent mass M traveling along an inclined track, and the motor-drum unit converts the translational motion into rotational shaft dynamics. The gravitational component, rolling resistance, electromagnetic traction/braking force, equivalent inertia, mechanical damping, and grid-side power conversion are therefore expressed in a unified electromechanical framework.

To improve the readability of the electromechanical model and the subsequent control analysis, the main symbols used in the following sections are summarized in Table 2.

For a slope-track SGES system, assume that the total moving load is M and that it moves along a track with an inclination angle of θ . A winch or traction mechanism couples the load motion to the motor-generator shaft, with an equivalent drum radius R and a shaft angular velocity ω . The corresponding load linear velocity is

$$v = R\omega. \quad (1)$$

In the following derivation, the positive direction is defined as the downhill direction of the load unit, corresponding to the generation mode. Therefore, $\omega > 0$ denotes downhill motion, and the electromagnetic torque T_e represents the controllable braking/generating

torque applied by the motor-generator against the gravity-driven motion. The generated active power is taken as positive when electrical power is delivered to the grid.

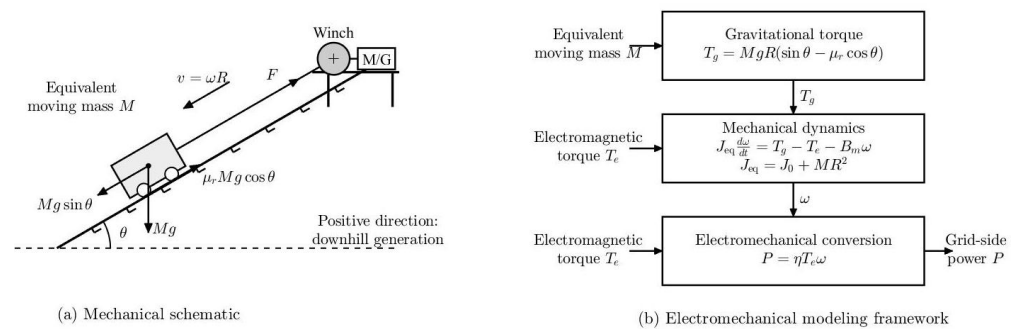


Figure 1. Schematic diagram and electromechanical modeling framework of the slope-track SGES system.

Table 2. Main symbols used in the electromechanical model and control analysis.

Symbol	Description	Unit
M	Equivalent moving mass	kg
R	Equivalent drum radius	m
θ	Track inclination angle	rad or deg
μ_r	Rolling-resistance coefficient	-
v	Translational velocity of the moving mass	m/s
ω	Angular velocity of the drum/motor shaft	rad/s
J_0	Intrinsic rotational inertia of the motor-drum unit	kg m ²
J_{eq}	Total equivalent rotational inertia	kg m ²
B_m	Equivalent viscous damping coefficient	N m s/rad
F_e	Electromagnetic traction/braking force	N
T_g	Effective gravity-driven torque	N m
T_e	Electromagnetic torque	N m
P	Grid-side output power	W
η	Lumped one-way electromechanical conversion efficiency	-
P_{ref}	Reference power command	W
P_{max}	Maximum steady-state output power	W
ω_m	Critical angular velocity at maximum power	rad/s
K_P	Power-feedback gain	-
K_v	Speed-feedback gain	N m s/rad

The purpose of this section is to establish a minimal lumped-parameter electromechanical model for the generation-side operating point and to identify the steady-state branches that are relevant to power tracking. The storage mode in the later wind-power smoothing scheme is represented at the system level by reversing the power-flow requirement and the motion tendency of the load units. In the present theoretical model, rolling resistance is treated as a small equivalent correction to the slope component of gravity, and other speed-dependent, direction-dependent, or contact-state-dependent friction effects are not explicitly resolved. This treatment keeps the analytical model focused on the dominant electromechanical coupling mechanism and is sufficient for deriving the low-speed branch properties emphasized in this work.

2.1. Derivation of the Electromechanical Coupling Dynamic Equation

According to Newton's second law, the translational dynamic equation along the positive direction of the track can be written as

$$M\dot{v} = Mg(\sin \theta - \mu_r \cos \theta) - F_e - c_v v, \quad (2)$$

where μ_r is the rolling-resistance coefficient, $F_e = T_e/R$ is the equivalent traction/braking force associated with the electromagnetic torque, and c_v is the equivalent linear viscous damping coefficient. The term $Mg(\sin \theta - \mu_r \cos \theta)$ represents the effective gravity-driven force along the downhill direction after including the small rolling-resistance correction. This simplified resistance treatment is used to derive the basic electromechanical response and does not attempt to describe detailed wheel-rail contact, static-dynamic friction transition, or bidirectional friction asymmetry.

Considering the inherent moment of inertia J_0 of the mechanical transmission and motor-side components, the above translational equation can be referred to the motor shaft side. Defining the total equivalent moment of inertia J_{eq} , the effective gravity-driven torque T_g , and the equivalent viscous damping coefficient B_m as

$$J_{eq} = J_0 + MR^2, \quad T_g = MgR(\sin \theta - \mu_r \cos \theta), \quad B_m = c_v R^2, \quad (3)$$

the electromechanical coupling dynamic equation of the system is obtained as

$$J_{eq}\dot{\omega} = T_g - T_e - B_m\omega. \quad (4)$$

The active power output to the grid is defined as

$$P = \eta T_e \omega, \quad (5)$$

where η is the lumped one-way electromechanical conversion efficiency. In Sections 2 and 3, the analytical model is used to characterize the local generation-side low-speed branch with $P \geq 0$. In the wind-power smoothing scheme of Sections 4 and 5, storage operation is represented at the system level by a negative SGES power command and the corresponding reversal of the load-motion tendency. Equations (4) and (5) jointly constitute the state-output model used in the subsequent control analysis.

2.2. Steady-State Power-Speed Curve and Maximum Power Condition

In the steady state where $\dot{\omega} = 0$, the electromagnetic torque satisfies

$$T_e^* = T_g - B_m\omega^*. \quad (6)$$

Substituting this relation into Equation (5), the output power and angular velocity exhibit a downward-opening parabolic relationship:

$$P^* = \eta(T_g - B_m\omega^*)\omega^* = \eta T_g \omega^* - \eta B_m(\omega^*)^2. \quad (7)$$

This curve reaches its maximum steady-state power

$$P_{\max} = \frac{\eta T_g^2}{4B_m} \quad (8)$$

at the critical angular velocity

$$\omega_m = \frac{T_g}{2B_m}. \quad (9)$$

For a given target power $P_{\text{ref}} \leq P_{\max}$, there exist two real roots for the steady-state angular velocity:

$$\omega_{\pm}^*(P_{\text{ref}}) = \frac{T_g \pm \sqrt{T_g^2 - 4B_m P_{\text{ref}}/\eta}}{2B_m}. \quad (10)$$

The local slope of the steady-state power-speed curve is

$$\frac{dP^*}{d\omega^*} = \eta (T_g - 2B_m\omega^*). \quad (11)$$

Therefore, the sign of the power-speed coupling changes at ω_m . This derivative plays a central role in distinguishing the controllability of the low-speed and high-speed branches.

2.3. Physical Meaning of the Low-Speed and High-Speed Branches

As illustrated in Figure 2, the system presents two steady-state branches under the same target power. The lower-speed solution corresponds to a low-speed and high-torque operating branch, whereas the higher-speed solution corresponds to a high-speed and low-torque branch. For the slope-track SGES considered in this study, the low-speed branch is preferred because it reduces the required travel distance of the moving mass within a finite track length and keeps the mechanical motion within a lower velocity range. Therefore, the following dynamic analysis and control design are mainly developed around the low-speed operating point. Low-speed operation is also common in other solid gravity storage concepts, including vertical-lifting systems, because finite height and space constraints usually limit the mechanical velocity to a low range. Therefore, the associated high-torque motor-generator operation is a general design issue for solid gravity storage rather than a feature unique to the slope-track configuration.

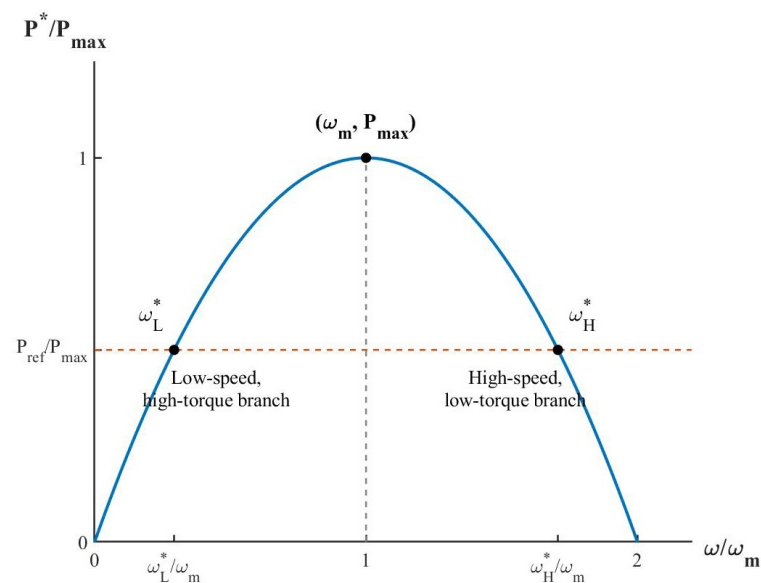


Figure 2. Schematic diagram of steady-state power vs. angular velocity.

1. Low-Speed, High-Torque Branch

The negative root of Equation (10) corresponds to the low-speed branch:

$$\omega_L^* = \frac{T_g - \sqrt{T_g^2 - 4B_m P_{\text{ref}}/\eta}}{2B_m}. \quad (12)$$

When the operating point is far from the maximum power point, namely $P_{\text{ref}} \ll P_{\text{max}}$, an asymptotic expansion gives

$$\omega_L^* \approx \frac{P_{\text{ref}}}{\eta T_g}, \quad T_{e,L}^* \approx T_g - \frac{B_m P_{\text{ref}}}{\eta T_g}. \quad (13)$$

Since $\omega_L^* < \omega_m$, Equation (11) gives $dP^*/d\omega^* > 0$ on this branch. Thus, an increase in speed corresponds to an increase in output power, which is compatible with a speed-feedback interpretation. This branch is also characterized by a low operating speed and a small mechanical displacement requirement, making it suitable as the nominal operating regime for wind-farm-level power tracking.

2. High-Speed, Low-Torque Branch

The positive root of Equation (10) corresponds to the high-speed branch:

$$\omega_H^* = \frac{T_g + \sqrt{T_g^2 - 4B_m P_{\text{ref}}/\eta}}{2B_m}. \quad (14)$$

On this branch, $\omega_H^* > \omega_m$, and therefore Equation (11) gives $dP^*/d\omega^* < 0$. This means that an increase in angular velocity is accompanied by a decrease in steady-state output power. Such reverse power-speed coupling complicates the design of a straightforward feedback loop and makes the high-speed branch unsuitable as the preferred operating regime for the control strategy considered in this paper.

Therefore, the low-speed branch is not merely a low-displacement operating regime. More importantly, it is the branch on which the sign of the power-speed coupling is consistent with speed-based power regulation. This observation provides the physical basis for converting the power-tracking problem into a speed-tracking problem in the subsequent feedforward-feedback control design.

2.4. Parameter Sensitivity at the Low-Speed Operating Point

This study focuses on the low-speed branch. For a given target power, increasing the total load M or the slope angle θ enhances the gravity-driven torque T_g . Based on Equation (12), the sensitivity of the low-speed operating point to the target power is

$$\frac{d\omega_L^*}{dP_{\text{ref}}} = \frac{1}{\eta \sqrt{T_g^2 - 4B_m P_{\text{ref}}/\eta}}. \quad (15)$$

In the low-power region where $P_{\text{ref}} \ll P_{\text{max}}$, this reduces to

$$\frac{d\omega_L^*}{dP_{\text{ref}}} \approx \frac{1}{\eta T_g}. \quad (16)$$

Consequently, a larger M or a larger θ not only reduces the steady-state operating speed, but also decreases the speed variation required per unit of power disturbance. Since the track displacement is the time integral of the load velocity, this reduced speed excursion directly suppresses the cumulative displacement demand during wind-power smoothing. This scaling property is one of the key reasons why a heavy-load slope SGES system can become more favorable for power tracking when sufficient electromagnetic torque-control authority is available. The frequency-dependent tracking boundary associated with this low-speed branch is further analyzed in Section 3.

2.5. Generation-Side Loss Decomposition Under the Simplified Model

To clarify the generation-side energy flow of the slope-track SGES system, the gross gravitational torque and the rolling-resistance torque are written as

$$T_{g,0} = MgR \sin \theta, \quad T_r = \mu_r MgR \cos \theta. \quad (17)$$

The effective gravity-driven torque used in the preceding analysis is therefore

$$T_g = T_{g,0} - T_r = MgR(\sin \theta - \mu_r \cos \theta). \quad (18)$$

Multiplying the mechanical dynamic equation by the angular velocity gives the instantaneous power balance in the generation direction:

$$T_{g,0}\omega = \frac{d}{dt} \left(\frac{1}{2} J_{eq} \omega^2 \right) + T_e \omega + T_r \omega + B_m \omega^2. \quad (19)$$

Here, $T_r \omega$ is the rolling-resistance loss, $B_m \omega^2$ is the equivalent viscous mechanical loss, and $T_e \omega$ is the mechanical power delivered to the motor-generator. With the lumped one-way electromechanical conversion efficiency η , the grid-side output power and the corresponding conversion loss are

$$P_{grid} = \eta T_e \omega, \quad P_{conv} = (1 - \eta) T_e \omega. \quad (20)$$

The kinetic-energy term represents temporary energy storage or release during acceleration and deceleration and is not a dissipative loss.

For steady-state generation, the efficiency from the released gravitational power to the grid-side output power can be written as

$$\eta_{sys} = \frac{P_{grid}}{T_{g,0}\omega} = \eta \left[1 - \mu_r \cot \theta - \frac{B_m \omega}{MgR \sin \theta} \right]. \quad (21)$$

This expression separates the lumped electromechanical conversion efficiency from the rolling-resistance and viscous-loss contributions. On the low-speed branch, under the approximation $T_e \approx T_g$, the operating speed can be estimated as

$$\omega_L \approx \frac{P_{grid}}{\eta T_g}, \quad P_{vis} = B_m \omega_L^2 \approx \frac{B_m P_{grid}^2}{\eta^2 T_g^2}. \quad (22)$$

Thus, for the same grid-side output power, a larger moving mass or a larger slope angle increases T_g , reduces the required operating speed, and consequently decreases the viscous mechanical loss. A larger slope angle also reduces the relative rolling-resistance contribution through the factor $\mu_r \cot \theta$. The constant η used here should be interpreted as a lumped one-way conversion efficiency rather than a complete round-trip efficiency or a detailed motor-converter efficiency map. Therefore, the loss decomposition in this subsection is intended to clarify the theoretical energy balance of the simplified model, while torque-dependent converter losses and detailed friction characteristics should be incorporated in future prototype-level studies.

3. Feedforward-Feedback Torque Control for Low-Speed-Branch Power Tracking

Based on the low-speed branch identified in Section 2, the power-tracking problem can be converted into the problem of tracking a reference angular velocity mapped from the target power. This conversion is valid because the low-speed branch satisfies $dP^*/d\omega^* > 0$, so that the signs of speed correction and power correction are consistent. In this section, a feedforward-feedback electromagnetic torque control framework is formulated for the low-speed operating branch. The objective is to clarify how different torque-control laws modify the local electromechanical response, how the gain-scheduled speed feedback law

changes the effective feedback gain with the operating point, and how the finite closed-loop response time constrains rapid power tracking.

3.1. Low-Speed-Branch Feedforward Mapping and Feedback Control Laws

Selecting the low-speed branch as the nominal operating branch, the target power command $P_{\text{ref}}(t)$ is first mapped into a reference angular velocity and a quasi-steady feedforward torque:

$$\omega_{\text{ref}}(t) = \frac{T_g - \sqrt{T_g^2 - 4B_m P_{\text{ref}}(t)/\eta}}{2B_m}, \quad T_{e,\text{ff}}(t) = T_g - B_m \omega_{\text{ref}}(t). \quad (23)$$

This mapping ensures that, in the absence of dynamic lag and parameter mismatch, the operating point remains on the low-speed branch of the steady-state power-speed curve. Define the speed tracking error as

$$e(t) = \omega(t) - \omega_{\text{ref}}(t). \quad (24)$$

The following feedforward-feedback torque control laws are then considered.

3.1.1. Pure Feedforward Control

The simplest control law is

$$T_e = T_{e,\text{ff}}. \quad (25)$$

Substituting Equation (25) into the dynamic equation gives an error dynamics governed only by the inherent mechanical damping. Therefore, the characteristic response time is

$$\tau_0 = \frac{J_{\text{eq}}}{B_m}. \quad (26)$$

For large-inertia heavy-haul systems, this response is generally too slow for rapid power tracking. Pure feedforward control can provide the correct quasi-steady torque, but it cannot actively suppress the transient tracking error induced by the finite inertia of the moving mass.

3.1.2. Power-Feedback Control

A direct way to correct the output power error is to introduce power feedback:

$$T_e = T_{e,\text{ff}} + K_P(P - P_{\text{ref}}). \quad (27)$$

Since the instantaneous power satisfies $P = \eta T_e \omega$, Equation (27) should be interpreted as an algebraic feedback relation when the instantaneous power is directly used. It can be rewritten as

$$T_e = \frac{T_{e,\text{ff}} - K_P P_{\text{ref}}}{1 - K_P \eta \omega}. \quad (28)$$

Linearizing Equation (27) around a steady-state operating point (ω^*, T_e^*) yields

$$\delta T_e = \frac{K_P \eta T_e^*}{1 - K_P \eta \omega^*} \delta \omega. \quad (29)$$

Thus, the equivalent damping coefficient and time constant are

$$a_P = B_m + \frac{K_P \eta T_e^*}{1 - K_P \eta \omega^*}, \quad \tau_P = \frac{J_{\text{eq}}}{a_P}. \quad (30)$$

This control can accelerate the response, but its effective damping depends explicitly on the operating point. Moreover, the denominator $1 - K_P \eta \omega^*$ should be kept away from zero, and the local stability condition requires $a_P > 0$. These features make direct power feedback more sensitive to parameter tuning and operating-point variation.

3.1.3. Speed-Feedback Control

A more transparent approach is to directly feed back the speed error:

$$T_e = T_{e,ff} + K_v e. \quad (31)$$

Because T_e acts as a braking/generating torque in the downhill generation direction, a positive speed error should increase T_e to decelerate the load. Therefore, the positive sign in Equation (31) corresponds to negative feedback in the mechanical speed dynamics. The equivalent damping and closed-loop time constant are

$$a_v = B_m + K_v, \quad \tau_v = \frac{J_{eq}}{B_m + K_v}. \quad (32)$$

Once a target first-order response time τ_v is specified, the feedback gain can be selected as

$$K_v = \frac{J_{eq}}{\tau_v} - B_m. \quad (33)$$

Thus, speed feedback provides a direct way to prescribe the local closed-loop time scale, provided that the required electromagnetic torque remains within the available torque-control authority.

3.1.4. Gain-Scheduled Speed Feedback Control

To reduce the need for repeated gain tuning under different operating conditions, a gain-scheduled speed feedback law is further considered:

$$T_e = T_{e,ff} \frac{\omega}{\omega_{ref} + \omega_{reg}}, \quad (34)$$

where $\omega_{reg} > 0$ is a small regularization angular velocity introduced to avoid the singularity that would occur when ω_{ref} approaches zero. Without this regularization, a control law proportional to ω / ω_{ref} would become ill-conditioned near the zero-speed operating point. Equation (34) therefore uses a small positive base value in the denominator to keep the gain finite.

It should be noted, however, that this regularization also introduces a small target offset. The commanded torque equals the quasi-steady feedforward torque only when

$$\omega = \omega_{ref} + \omega_{reg}, \quad (35)$$

rather than exactly when $\omega = \omega_{ref}$. Equivalently, Equation (34) can be rewritten as

$$T_e = T_{e,ff} + K_v^{raw} (\omega - \omega_{ref} - \omega_{reg}), \quad K_v^{raw} = \frac{T_{e,ff}}{\omega_{ref} + \omega_{reg}}. \quad (36)$$

Thus, the regularized law behaves like a speed-feedback controller whose nominal equilibrium is shifted by ω_{reg} . When $\omega_{reg} \ll \omega_{ref}$, this offset is negligible and the controller approaches the ideal target-speed feedback law. When ω_{ref} is close to zero, the offset becomes more visible, but the regularization prevents the control gain from diverging.

Another consequence is that the raw equivalent gain K_v^{raw} can still become very large when $\omega_{\text{ref}} + \omega_{\text{reg}}$ is small. Therefore, the regularized gain-scheduled law should be used together with gain clipping:

$$K_v^{\text{eff}} = \text{clip}(K_v^{\text{raw}}, K_{\min}, K_{\max}), \quad (37)$$

where $K_{\min}$ provides the required minimum tracking speed and $K_{\max}$ is determined by the available torque authority, actuator bandwidth, and stability margin. In implementation, Equation (36) is therefore replaced by

$$T_e = T_{e,\text{ff}} + K_v^{\text{eff}}(\omega - \omega_{\text{ref}} - \omega_{\text{reg}}). \quad (38)$$

If K_v^{raw} lies inside the admissible interval, Equation (38) reduces to the regularized gain-scheduled law in Equation (34). If the raw gain exceeds the admissible range, the clipped gain prevents unrealistically large equivalent damping and avoids excessive torque correction near very low reference speeds.

For the clipped gain-scheduled controller, the local damping and response time are

$$a_{\text{gs}} = B_m + K_v^{\text{eff}}, \quad \tau_{\text{gs}} = \frac{J_{\text{eq}}}{a_{\text{gs}}}. \quad (39)$$

The local speed-error dynamics also contain a small regularization-induced bias term:

$$J_{\text{eq}}\dot{e} + (B_m + K_v^{\text{eff}})e = -J_{\text{eq}}\dot{\omega}_{\text{ref}} + K_v^{\text{eff}}\omega_{\text{reg}}. \quad (40)$$

For slowly varying commands, the corresponding bias is approximately

$$e_{\text{reg}} \approx \frac{K_v^{\text{eff}}}{B_m + K_v^{\text{eff}}}\omega_{\text{reg}}. \quad (41)$$

Therefore, ω_{reg} should be chosen as a small positive value: small enough to keep the speed offset negligible relative to the operating speed and expected speed fluctuation, but large enough to avoid numerical singularity and to keep the raw gain within a reasonable range before clipping. In this sense, ω_{reg} is not an independent tuning parameter for improving tracking performance, but a regularization parameter that must be used together with gain clipping near the zero-speed limit.

3.2. Unified Local Error Dynamics and Periodic Tracking Analysis

Near the low-speed operating point, the above control laws can be unified into the following local error equation:

$$J_{\text{eq}}\dot{e} + ae = -J_{\text{eq}}\dot{\omega}_{\text{ref}}, \quad (42)$$

where the equivalent damping coefficient a takes different forms for different control strategies:

$$a = \begin{cases} B_m, & \text{pure feedforward,} \\ B_m + \frac{K_P\eta T_e^*}{1 - K_P\eta\omega^*}, & \text{power feedback,} \\ B_m + K_v, & \text{speed feedback,} \\ B_m + K_v^{\text{eff}}, & \text{gain-scheduled speed feedback with clipping.} \end{cases} \quad (43)$$

The corresponding local closed-loop time constant is

$$\tau = \frac{J_{eq}}{a}. \quad (44)$$

Equation (42) shows that the speed tracking dynamics are primarily determined by τ . For the gain-scheduled speed-feedback law, the exact error equation also contains the small regularization-induced bias shown in Equation (40). In the normal low-speed operating range where ω_{reg} is much smaller than ω_{ref} and the expected speed swing, this bias is negligible, and the first-order form in Equation (42) provides the local dynamic approximation. This provides a convenient way to compare different feedback designs using a unified time-scale parameter rather than separate gain values.

To analyze the tracking limit under periodic power commands, assume that the target power consists of a constant component and a sinusoidal disturbance:

$$P_{ref}(t) = P_0 + P_1 \sin(\Omega t). \quad (45)$$

On the low-speed branch, the corresponding reference speed can be approximated as

$$\omega_{ref}(t) \approx \omega_0 + \frac{P_1}{\eta T_g} \sin(\Omega t), \quad \omega_0 \approx \frac{P_0}{\eta T_g}. \quad (46)$$

This expression shows that, for the same power fluctuation amplitude P_1 , the required speed swing is inversely proportional to the gravity-driven torque T_g .

Taking the Laplace transform of Equation (42), the transfer function from the reference speed to the actual speed is

$$H_\omega(s) = \frac{\omega(s)}{\omega_{ref}(s)} = \frac{1}{1 + \tau s}. \quad (47)$$

Thus, speed tracking is governed by the dimensionless parameter $\tau\Omega$. However, power tracking is not determined by speed tracking alone, because the output power also depends on the electromagnetic torque. Using the low-speed approximation and the dynamic relation $T_e = T_g - B_m\omega - J_{eq}\dot{\omega}$, the small-signal power response can be approximated as

$$H_P(s) = \frac{\delta P(s)}{\delta P_{ref}(s)} \approx \frac{1 - \varepsilon s}{1 + \tau s}, \quad \varepsilon \approx \frac{J_{eq}P_0}{\eta T_g^2}. \quad (48)$$

The amplitude ratio and phase lag are therefore

$$A_P(\Omega) = \frac{\sqrt{1 + (\varepsilon\Omega)^2}}{\sqrt{1 + (\tau\Omega)^2}}, \quad (49)$$

$$\phi_P(\Omega) = -\arctan(\varepsilon\Omega) - \arctan(\tau\Omega). \quad (50)$$

Equations (48)–(50) indicate that angular velocity tracking is mainly limited by τ , whereas power tracking is additionally affected by ε . Since

$$\varepsilon = \frac{(J_0 + MR^2)P_0}{\eta M^2 g^2 R^2 (\sin\theta - \mu_r \cos\theta)^2}, \quad (51)$$

increasing the slope angle reduces ε through the squared effective gravity component. Increasing the moving mass also reduces ε in the large-mass regime, although approximately in an inverse-mass manner because J_{eq} increases with M . This scaling indicates that, provided sufficient electromagnetic torque-control authority is available, heavier loads and

steeper slopes can reduce the speed excursion, displacement demand, and power-tracking distortion for the same regulation task.

The same frequency-response expression also clarifies the inherent tracking limitation of the low-speed branch. For low-distortion power tracking, both the speed response and the inertial correction term should remain small on the time scale of the command:

$$\Omega\tau \ll 1, \quad \Omega\varepsilon \ll 1. \quad (52)$$

Equivalently, the fluctuation period should be much longer than $2\pi \max(\tau, \varepsilon)$. When this condition is violated, the finite response speed of the SGES unit leads to amplitude distortion and phase lag, and the quasi-steady low-speed-branch prediction becomes less accurate. Therefore, the proposed low-speed operating strategy is suitable for slow and medium time-scale power smoothing, whereas very rapidly varying power commands are outside the intended operating range of the present control framework.

3.3. Effect of Feedforward Parameter Mismatch

The feedforward mapping in Equation (23) depends on the model parameters T_g and B_m . In practical implementation, these parameters may deviate from their nominal values because of load variation, slope-angle uncertainty, rolling-resistance variation, or unmodeled mechanical losses. To examine the robustness of the feedback structure, let $\hat{T}_g$ and $\hat{B}_m$ denote the parameters used in the feedforward mapping, and let $\hat{\omega}_{\text{ref}}$ and $\hat{T}_{e,\text{ff}}$ be the corresponding reference speed and feedforward torque. For a speed-feedback-type controller with a local equivalent gain K_{fb} , the torque command can be written as

$$T_e = \hat{T}_{e,\text{ff}} + K_{\text{fb}}(\omega - \hat{\omega}_{\text{ref}}), \quad (53)$$

where $K_{\text{fb}} = K_v$ for constant speed feedback and $K_{\text{fb}} = K_v^{\text{eff}}$ for gain-scheduled speed feedback. Since

$$\hat{T}_{e,\text{ff}} = \hat{T}_g - \hat{B}_m \hat{\omega}_{\text{ref}}, \quad (54)$$

the error $e = \omega - \hat{\omega}_{\text{ref}}$ satisfies

$$J_{\text{eq}}\dot{e} + (B_m + K_{\text{fb}})e = -J_{\text{eq}}\dot{\hat{\omega}}_{\text{ref}} + (T_g - \hat{T}_g) + (\hat{B}_m - B_m)\hat{\omega}_{\text{ref}}. \quad (55)$$

Equation (55) shows that feedforward parameter mismatch appears as an additive disturbance in the local error dynamics. For a slowly varying command, the quasi-steady error bias is approximately

$$e_{\text{bias}} \approx \frac{(T_g - \hat{T}_g) + (\hat{B}_m - B_m)\hat{\omega}_{\text{ref}}}{B_m + K_{\text{fb}}}. \quad (56)$$

Thus, feedback reduces the influence of feedforward parameter mismatch by increasing the effective damping $B_m + K_{\text{fb}}$. The feedforward term should therefore be interpreted as a nominal low-speed-branch compensation rather than an exact open-loop command. Unmodeled rolling-resistance variation, moderate friction uncertainty, and engagement-induced torque perturbations enter the local dynamics as disturbance terms and are partly suppressed by the feedback channel, provided that the required torque remains within the actuator authority. Pure feedforward control corresponds to $K_{\text{fb}} = 0$, so the mismatch-induced bias is suppressed only by the inherent damping B_m . In contrast, speed feedback and gain-scheduled speed feedback provide an additional correction channel. This analysis explains why the proposed control framework does not rely solely on an exact feedforward model: the feedforward term places the operating point near the low-speed branch, while the feedback term compensates for residual dynamic lag and moderate parameter

perturbations. Large parameter errors, actuator saturation, or strongly nonlinear friction, however, would require recalibration or more detailed prototype-level modeling.

3.4. Numerical Verification of Step and Periodic Power Tracking

Numerical simulations are conducted to validate the theoretical analysis above. Table 3 summarizes the baseline physical parameters used in the simulations. Since the total load and slope angle are varied in the step and periodic tracking tests, they are not listed as fixed baseline parameters in the table. For feedback control, the gains are selected through the prescribed closed-loop time constant τ , which is marked in the corresponding figures. Therefore, separate values of K_v or K_p are not listed. For the gain-scheduled speed feedback law, ω_{reg} is taken as a small positive regularization parameter, and the corresponding raw gain is clipped within the admissible range discussed above.

Table 3. Key parameter configuration for slope SGES system simulation.

Symbol	Parameter Name	Value	Unit
g	Gravitational acceleration	9.81	m/s ²
μ_r	Rolling resistance coefficient	0.002	-
R	Equivalent radius	1.0	m
η	Electromechanical efficiency	0.95	-
B_m	Mechanical damping coefficient	1.0×10^4	N m s/rad
J_0	System inherent inertia	1.0×10^4	kg m ²

Figure 3 illustrates the system response to step power commands under pure feedforward control, speed-feedback control, and gain-scheduled speed feedback control.

Under pure feedforward control, the response is governed by the inherent mechanical time constant $\tau_0 = J_{\text{eq}}/B_m$. As shown in Figure 3a,b, the pure feedforward response is very sluggish: even when the target power changes abruptly, the output power exhibits only a weak change in slope rather than a rapid transition to the new command. Figure 3a further shows that increasing the total load increases J_{eq} , thereby lengthening the response time. In Figure 3b, the power-response curves for different slope angles almost completely overlap. This is consistent with the pure feedforward dynamics: when M , J_{eq} , and B_m are fixed, changing θ mainly shifts the low-speed steady-state operating speed through T_g , whereas the feedforward-only response time remains governed by $\tau_0 = J_{\text{eq}}/B_m$. Therefore, the slope-angle dependence of the output-power transient is weak on the low-speed branch.

With speed feedback, the closed-loop response time can be prescribed through $\tau_v = J_{\text{eq}}/(B_m + K_v)$. Figure 3c,d show that, after the feedback time scale is specified, larger total load and larger slope angle reduce the speed excursion and transient power deviation required for the same power step. This is because a larger M or θ increases the gravity-driven torque T_g , so the same power increment can be achieved with a smaller change in speed. The electromagnetic torque no longer remains at the quasi-steady value; instead, it temporarily deviates from the feedforward torque to accelerate or decelerate the load, thereby realizing faster dynamic regulation. Figure 3e further shows that the gain-scheduled speed feedback law adjusts its equivalent gain according to the operating point. The small regularization velocity prevents singular behavior near zero reference speed, while the gain clipping avoids unrealistically large correction gains. In this way, the controller provides a practical gain-scheduling effect without manually assigning a constant feedback gain for each case.

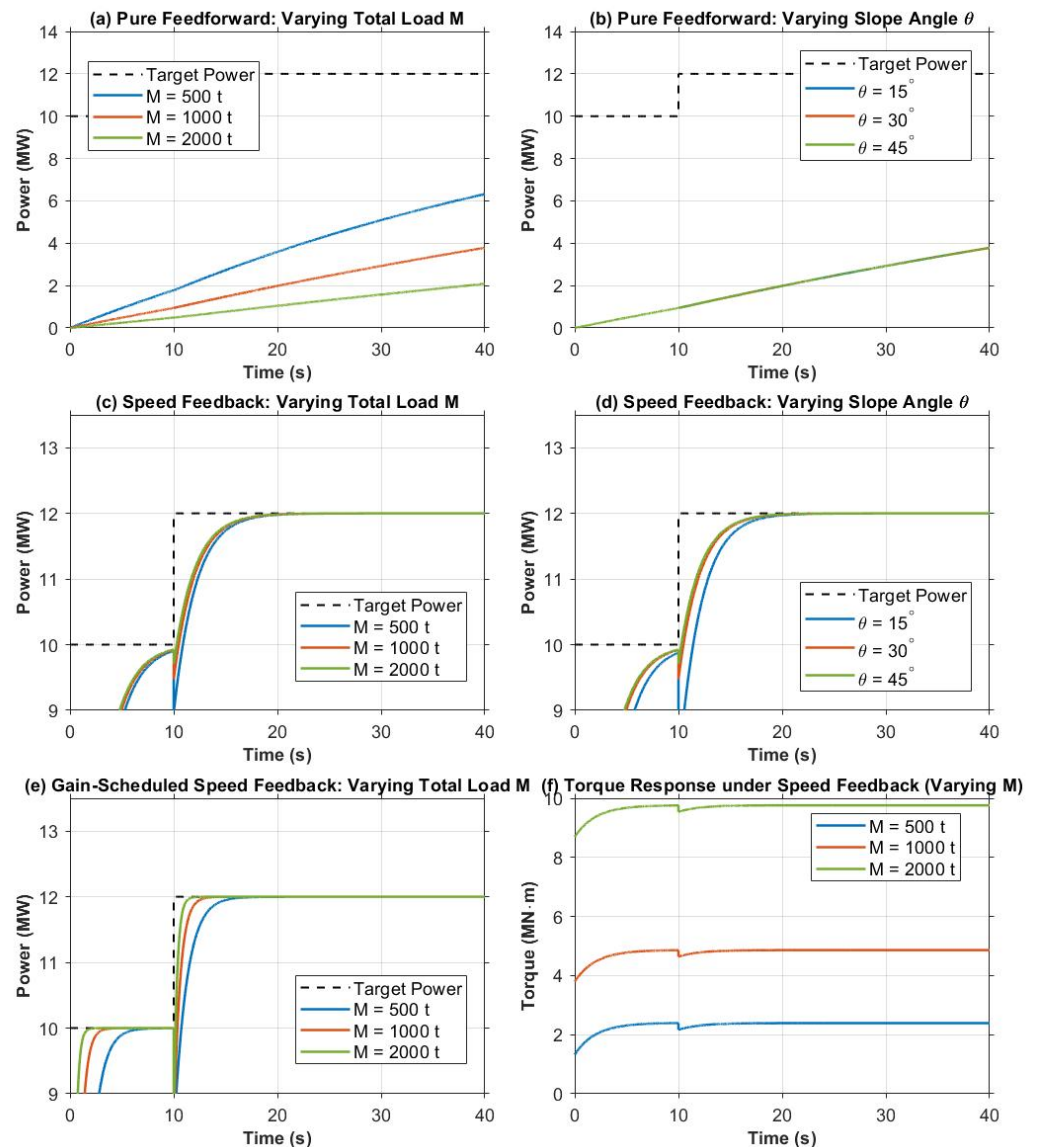


Figure 3. Effects of total load M and slope angle θ on the dynamic response under different control strategies.

To further examine tracking limits under periodic power fluctuations, Figure 4 compares the power, speed, and torque responses under different masses, slope angles, and fluctuation periods.

Comparing the cases with $T = 15$ s in Figure 4, increasing the load mass or the slope angle reduces the speed variation required to follow the same power fluctuation. This is consistent with Equation (46). At the same time, the reduction of ε in Equation (51) alleviates the amplitude distortion and phase lag of the output power. In contrast, for the ultra-short-period case with $T = 1$ s, both $\tau\Omega$ and $\varepsilon\Omega$ become large, and obvious tracking lag appears. This confirms that the power-tracking capability of the system is frequency-dependent and remains bounded by the closed-loop response time and inertial correction term.

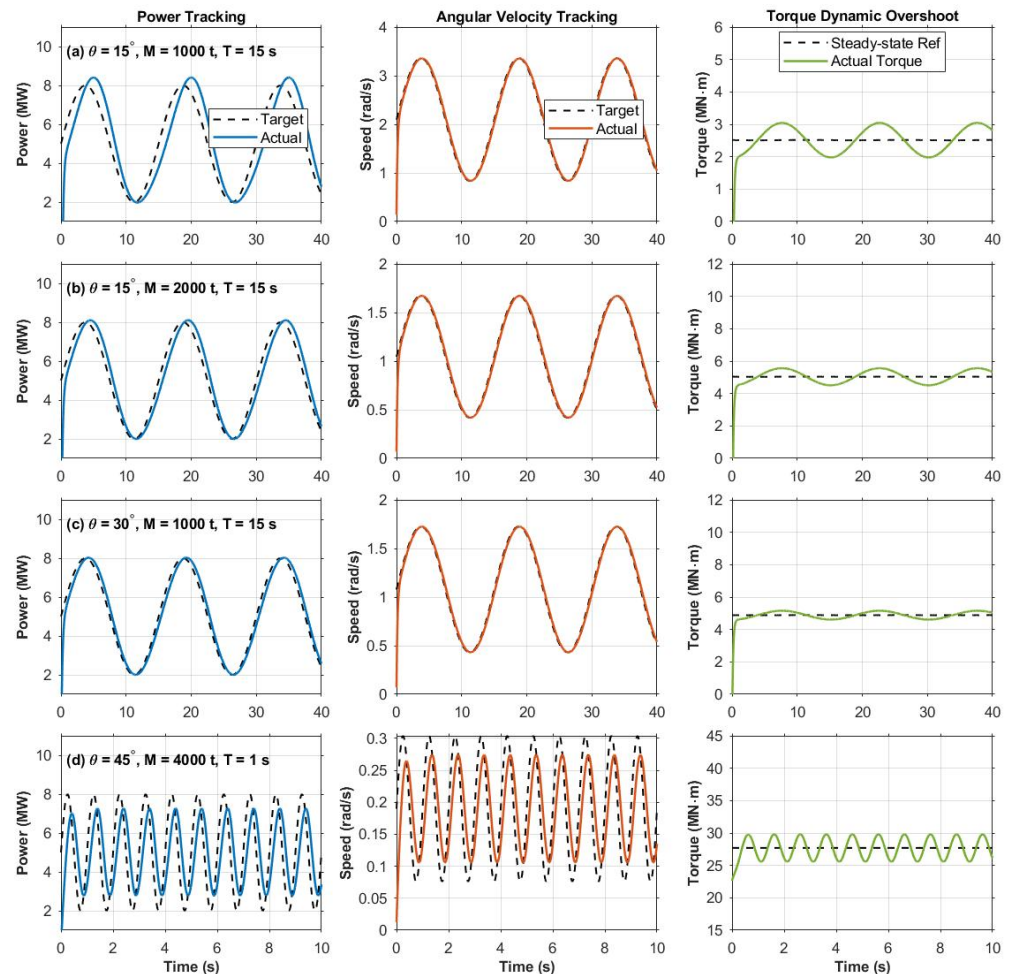


Figure 4. Comparison of periodic power tracking, velocity tracking, and torque overshoot characteristics under different operating conditions.

The torque curves in Figure 4 illustrate the dynamic compensation mechanism behind the tracking behavior. The actual electromagnetic torque deviates from the quasi-steady torque reference during acceleration and deceleration intervals. When the system needs to increase output power, the controller temporarily reduces the braking torque or creates a torque deviation that allows gravity to accelerate the load. When the system needs to suppress speed or power, the electromagnetic torque is increased to decelerate the load. Therefore, accurate power smoothing is not maintained by static torque balance alone, but by dynamic torque correction around the low-speed operating branch. This also indicates that sufficient torque-control authority is a prerequisite for realizing the predicted tracking performance.

Figure 4 also provides an estimate of the applicable time scale of the theoretical tracking model. The theoretical prediction agrees well with the numerical response when the power fluctuation period is sufficiently longer than the electromechanical response time. Noticeable deviations appear only when the imposed fluctuation approaches the second-level time scale, where the finite response speed and nonlinear power-speed relation become more important. This boundary is not a fixed value, but depends on the system parameters and operating point. In general, increasing the moving mass or the slope angle increases the available gravity-driven torque and reduces the required operating speed for the same output power, which can extend the valid theoretical prediction toward shorter fluctuation periods. Therefore, after this applicable range has been identified, the wind-power smoothing cases considered later, whose dominant fluctuation periods are on

the order of tens of seconds or longer, remain within the reliable operating range of the proposed model and control framework.

4. Single-Track Multi-Unit Operation for Wind-Farm Power Smoothing

The theoretical analysis in the preceding sections shows that low-speed-branch feedforward-feedback torque control can reduce the speed and displacement excursions required for power tracking when sufficient gravity-driven torque and torque-control authority are available. Building on this result, this section proposes a single-track multi-unit operation scheme for slope SGES in wind-farm-side power smoothing, as illustrated in Figure 5. The scheme arranges multiple standard load units on a finite-length slope track and divides the track into an upper parking zone, a central motion zone, and a lower parking zone. Through coordinated scheduling and soft loading/unloading of terminal units, the system can provide bidirectional regulation around the PCC reference power within a finite-track constraint.

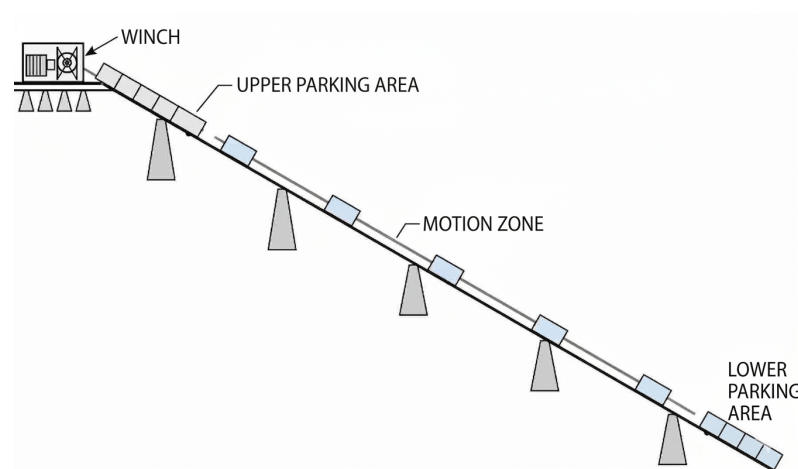


Figure 5. Schematic diagram of the single-track multi-unit SGES operation scheme.

The operation mode considered here is designed for wind-farm fluctuation smoothing rather than full-day energy shifting. In a daily peak-valley balancing task, the load units would move predominantly in one direction over a long interval, and the required height difference and load inventory would become very large. By contrast, wind-farm power over a several-hour window often contains substantial fluctuations around a local mean or scheduled output. In this case, the SGES system can compensate for positive and negative power deviations through reciprocating motion of load units along the track. Therefore, the key constraint is not only the energy capacity of a single full-stroke cycle, but also the cumulative displacement and inventory margin within the finite track length.

The core objective of the scheme is to compensate for wind-power deficits or absorb surplus wind power. Defining the required SGES compensation command as

$$P_{g,ref}(t) = P_{PCC,ref}(t) - P_w(t), \quad (57)$$

the SGES output is controlled to satisfy

$$P_g(t) \approx P_{g,ref}(t), \quad (58)$$

so that the final PCC-side power approaches the reference target:

$$P_{PCC}(t) = P_w(t) + P_g(t) \approx P_{PCC,ref}(t). \quad (59)$$

Here, $P_{g,\text{ref}}(t) > 0$ means that the wind-farm output is lower than the reference value and the SGES system should generate power; $P_{g,\text{ref}}(t) < 0$ means that the wind-farm output exceeds the reference value and the SGES system should absorb power.

In the numerical implementation of this section, the power variables are expressed in deviation form around the scheduled PCC power, so that the nominal target is set to zero. In this deviation-coordinate implementation, $P_w(t)$ denotes the wind-power deviation relative to the scheduled PCC output rather than the absolute wind-farm output. This does not imply a zero-output wind farm; rather, it represents the fluctuating component to be compensated by the SGES system. The SGES system therefore acts as a local balancing source that offsets the wind-power deviation and keeps the PCC-side output close to the scheduled value.

4.1. Power-Balance Formulation and Motion-Range Scaling

To estimate the required track margin, define the wind-power deviation around the reference value as

$$\delta P(t) = P_w(t) - P_{\text{PCC},\text{ref}}(t) = -P_{g,\text{ref}}(t). \quad (60)$$

Thus, $\delta P(t) > 0$ corresponds to surplus wind power and storage operation, while $\delta P(t) < 0$ corresponds to wind-power deficit and generation operation. On the low-speed operating branch, the equivalent position $s(t)$ of the active load cluster is predominantly driven by the power deviation. Taking the positive direction of s as the downhill generation direction, one obtains

$$\dot{s}(t) = -\frac{\delta P(t)}{\eta M_{\text{eff}} g (\sin \theta - \mu_r \cos \theta)}, \quad (61)$$

where M_{eff} is the effective moving mass of the active load chain. Equation (61) is consistent with the sign convention above: when wind power is in surplus, $\delta P(t) > 0$, the system absorbs power and the active load cluster tends to move uphill, giving $\dot{s}(t) < 0$.

The cumulative displacement over a time interval is therefore

$$s(t) - s(0) = -\frac{1}{\eta M_{\text{eff}} g (\sin \theta - \mu_r \cos \theta)} \int_0^t \delta P(\tau) d\tau. \quad (62)$$

For a sinusoidal power-deviation component $\delta P_k(t) = A_k \sin(\Omega_k t + \phi_k)$, the corresponding displacement amplitude is approximately

$$\Delta s_k \approx \frac{A_k}{\eta M_{\text{eff}} g (\sin \theta - \mu_r \cos \theta) \Omega_k}. \quad (63)$$

This expression shows that the motion range is mainly controlled by the effective gravity factor

$$M_{\text{eff}} (\sin \theta - \mu_r \cos \theta). \quad (64)$$

Therefore, increasing the active moving mass or the slope angle reduces the displacement required for the same fluctuation-smoothing task. This scaling relation provides the physical basis for using a heavy but modular active load chain within a finite track.

In this work, the baseline track layout is specified as

$$L = 1000 \text{ m}, \quad \theta = 30^\circ, \quad H = L \sin \theta = 500 \text{ m}. \quad (65)$$

This geometry is used as a representative numerical configuration for demonstrating the finite-track operation principle. The track is divided into an upper parking zone, a central motion zone, and a lower parking zone:

$$L_{\text{upper}} = 150 \text{ m}, \quad L_{\text{mid}} = 700 \text{ m}, \quad L_{\text{lower}} = 150 \text{ m}. \quad (66)$$

This finite-track configuration imposes requirements on motion-range control, module scheduling, and soft loading/unloading. The purpose of the multi-unit design is to maintain bidirectional regulation within this finite geometry rather than to rely on a very long single-stroke motion.

To generate a representative wind-power fluctuation for simulation, the following multi-time-scale synthetic signal is used:

$$S(t) = c_0 \sin\left(\frac{2\pi t}{T_0} + \phi_0\right) + c_1 \sin\left(\frac{2\pi t}{T_1} + \phi_1\right) + c_2 \sin\left(\frac{2\pi t}{T_2} + \phi_2\right) + c_3 \sin\left(\frac{2\pi t}{T_3} + \phi_3\right). \quad (67)$$

The present study mainly focuses on smoothing wind-power fluctuations with time scales of several tens of seconds and longer. Therefore, high-frequency stochastic noise components that may be included in more detailed wind-power models are not explicitly considered in the synthetic command used here. In the simulation, the characteristic periods are

$$T_0 = 4400 \text{ s}, \quad T_1 = 1803 \text{ s}, \quad T_2 = 603 \text{ s}, \quad T_3 = 60 \text{ s}, \quad (68)$$

and the corresponding coefficients are

$$c_0 = 0.70, \quad c_1 = 0.70, \quad c_2 = 0.50, \quad c_3 = 0.15. \quad (69)$$

The normalized fluctuation signal is defined as

$$\tilde{S}(t) = \frac{S(t)}{\max_{t \in [0, T_{\text{sim}}]} |S(t)|}, \quad (70)$$

and the wind-power deviation is constructed as

$$P_w(t) = P_{w,0} + A_w \tanh\left(1.2\tilde{S}(t)\right). \quad (71)$$

In the simulation, $P_{w,0} = 0$ in deviation coordinates, $A_w = 4.2$ MW, and the simulation horizon is $T_{\text{sim}} = 7200$ s. This synthetic signal is used to test the mechanism of multi-scale fluctuation smoothing under the finite-track constraint, rather than to represent a site-specific measured wind profile. The fastest deterministic component used here has a period of 60 s; sub-second turbulence and converter-level compensation are outside the target bandwidth of the proposed SGES scheme.

Figure 6 compares the cumulative orbital motion range under different total moving masses and slope angles.

The comparison indicates that increasing either M_{eff} or θ reduces the speed variation required for the same power compensation and thereby suppresses cumulative displacement. When different combinations of M_{eff} and θ produce similar displacement ranges, this should be interpreted as a consequence of the effective gravity factor $M_{\text{eff}}(\sin \theta - \mu_r \cos \theta)$, rather than as an equivalence between mass and angle themselves. This scaling provides a design guideline for matching load scale and track inclination under finite-track conditions.

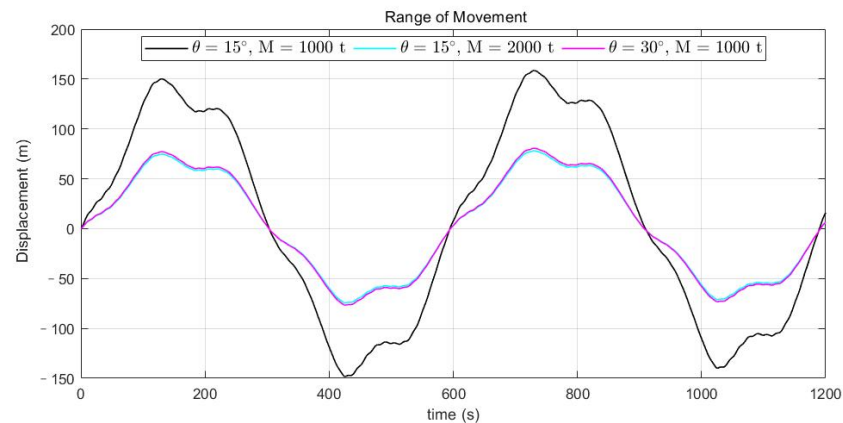


Figure 6. Comparison of cumulative orbital motion range under different slope angles and effective moving masses.

4.2. Soft Loading/Unloading and Effective-Mass Transition

For wind-farm-level fluctuation smoothing, a large total moving mass is required to provide sufficient regulation power and to reduce the speed excursion associated with a given power deviation. However, concentrating this mass into a single extremely heavy unit would impose severe impact loads on the track and traction system. In addition, the engagement or disengagement of a single large unit would introduce a large transient jump in the equivalent moving mass and gravity-driven torque. Therefore, the required total mass is decomposed into multiple standard load units, so that both mechanical impact and power disturbance can be distributed.

Although modularization reduces the impact associated with each individual unit, it also increases the frequency of engagement and disengagement events. Therefore, the transition of each unit into or out of the active moving chain must be smoothed. For standard load units with masses on the order of 100 t, rail engagement, rail-clamping or rail-gripping action, disengagement, and traction-force build-up cannot occur as ideal step changes. In implementation, this transition can be supported by variable-speed drives, braking devices, traction-force control, rail-clamping mechanisms, and mechanical buffers. These mechanisms provide a finite transition interval during which the load participation and force transmission are gradually established or released.

In the system-level dynamic model, this process is represented using continuous participation coefficients. Let $\alpha(t)$ denote the loading participation coefficient and $\beta(t)$ denote the unloading participation coefficient. A simple exponential transition model is adopted:

$$\alpha(t) = \begin{cases} 0, & t < t_0, \\ 1 - \exp\left[-\frac{t-t_0}{\tau_s}\right], & t \geq t_0, \end{cases} \quad \beta(t) = \begin{cases} 1, & t < t_0, \\ \exp\left[-\frac{t-t_0}{\tau_s}\right], & t \geq t_0, \end{cases} \quad (72)$$

where t_0 is the trigger time of the loading or unloading action, and τ_s is the transition time constant determined by the mechanical actuation and torque-control loop. In the simulation, the generation-side loading and unloading time constants are both taken as 0.7 s. During storage operation, the upper unloading time constant is 0.7 s, the lower loading time constant is 1.0 s, and the speed-matching time constant of a newly engaged lower unit is 1.0 s.

For a multi-unit system, define $\gamma_i(t)$ as the participation coefficient of the i -th load unit. It can take values between 0 and 1, including the soft-loading and soft-unloading forms in Equation (72). The effective moving mass is then

$$M_{\text{eff}}(t) = \sum_{i=1}^N m_i \gamma_i(t), \quad (73)$$

where m_i is the mass of the i -th load unit. The corresponding gravity-driven torque is

$$T_g(t) = M_{\text{eff}}(t)gR(\sin \theta - \mu_r \cos \theta). \quad (74)$$

The equivalent inertia can also be updated according to the participating mass,

$$J_{\text{eq}}(t) \approx J_0 + M_{\text{eff}}(t)R^2, \quad (75)$$

or evaluated at the nominal active mass when the number of active units is maintained approximately constant. In the scheduling strategy below, the active chain is designed to keep about ten units in motion, so the nominal active mass remains close to 1000 t.

Compared with an ideal step model, the continuous transition model helps smooth the variation of $M_{\text{eff}}(t)$ and $T_g(t)$, thereby reducing abrupt disturbances in the torque-control loop.

The transition time constant used in the soft loading and unloading functions is selected according to both mechanical-response and control-time-scale considerations. The selected range, $\tau_s = 0.7\text{--}1.0$ s, is consistent with the expected response time scale of mechanical engagement and rail-clamping devices in a heavy-load track system. For the exponential transition in Equation (72), the participation coefficient reaches approximately 98% of its final value after about $4\tau_s$. Thus, the corresponding saturation time is approximately 2.8–4.0 s, which is much shorter than the several-tens-of-seconds fluctuation scale targeted by the present smoothing strategy. Therefore, this soft transition is slow enough to avoid an ideal stepwise mass change, while still fast enough not to weaken the smoothing of the dominant wind-power fluctuations considered here. In an actual system, the detailed realization of this soft transition would need to be determined together with the specific design of the traction system, rail-clamping system, braking devices, and mechanical buffers. In the present control-level model, deviations in these transition dynamics are treated as effective-mass and torque perturbations, whose influence is reduced by the feedback part of the torque controller within the admissible bandwidth and torque limits.

4.3. 30-Unit Layout and Scheduling Logic

The proposed system consists of a 1000 m single-track slope with a baseline inclination angle of 30° , corresponding to a vertical height difference of approximately 500 m. The track is divided into a 150 m upper parking zone, a 700 m central motion zone, and a 150 m lower parking zone, as specified in Equation (66).

A baseline modular configuration consists of 30 standard load units, each with a mass of 100 t. Initially, 10 units are parked in the upper zone, 10 units participate in the central active motion chain, and 10 units are parked in the lower zone. The corresponding nominal moving mass is

$$M_{\text{eff},0} = 10 \times 100 \text{ t} = 1000 \text{ t}. \quad (76)$$

The initial positions of the upper parking units are distributed from 7.5 m to 142.5 m with a spacing of 15 m; the active units in the central motion zone are distributed from 185 m to 815 m with a spacing of 70 m; and the lower parking units are distributed from 857.5 m to

992.5 m with a spacing of 15 m. These values define the baseline module arrangement for the 1000 m track.

The operational rule is as follows. During generation, the active load chain moves downhill and supplies the power deficit. When the leading active unit approaches the lower boundary, it is softly unloaded into the lower inventory, while a unit from the upper inventory is softly loaded into the active chain. During storage, the process is reversed: the active chain moves uphill, the leading unit near the upper boundary is softly unloaded into the upper inventory, and a unit from the lower inventory is softly loaded into the active chain. In this way, the number of active units and the effective moving mass are kept approximately constant.

For clarity, the scheduling logic can be summarized as the following event-driven procedure:

1. Compute the SGES compensation command $P_{g,ref}(t)$ from the PCC power-balance requirement.
2. Determine the operation mode. If $P_{g,ref}(t) > 0$, the active chain operates in generation mode and moves downhill. If $P_{g,ref}(t) < 0$, the active chain operates in storage mode and moves uphill.
3. Track the power command using the low-speed-branch feedforward-feedback torque controller.
4. Monitor the leading active unit near the downstream terminal of the current motion direction.
5. When the trigger distance is reached, softly unload the terminal unit into the corresponding parking inventory and softly load one unit from the opposite inventory into the active chain.
6. Update $M_{eff}(t)$, $T_g(t)$, and, if necessary, $J_{eq}(t)$ according to the participation coefficients $\gamma_i(t)$.

The soft-switching trigger distance is set to 15 m at both terminals. This trigger distance is consistent with the 150 m parking zones, while leaving sufficient space for the gradual change of participation coefficients. The simulation adopts a deadband of 0.02 MW around the target power, a torque filter cutoff frequency of 0.5 Hz, a storage-mode tracking time constant of 3.0 s, and a stopping time constant of 1.2 s. These parameters are used to illustrate cyclic terminal-unit replacement and power smoothing under the finite-track constraint.

The 1000 m configuration does not eliminate the finite-stroke constraint. Rather, it extends the effective operating duration under finite-track conditions by combining bidirectional reciprocating motion, terminal inventory replacement, and soft effective-mass transitions. As long as the cumulative displacement remains within the available track and inventory margins, the system can maintain sustained PCC-side power smoothing within the considered multi-hour horizon.

5. Numerical Verification of the Single-Track Multi-Unit Operation Scheme

This section numerically verifies the single-track multi-unit operation scheme proposed in Section 4. The purpose is to examine whether the finite-track scheduling logic, soft loading/unloading process, and low-speed-branch feedforward-feedback torque control can work together to smooth wind-farm-side power fluctuations within a finite load inventory. The simulations focus on the dynamic consistency of the proposed operation scheme, its finite-track feasibility under the selected parameters, and the influence of slope angle and scheduled PCC-side power commands.

The simulations use the 1000 m track layout described in Section 4. The track consists of a 150 m upper parking zone, a 700 m central motion zone, and a 150 m lower parking

zone. A total of 30 standard 100 t load units are considered, with 10 units initially parked in the upper zone, 10 units forming the active motion chain, and 10 units parked in the lower zone. Therefore, the nominal effective moving mass is

$$M_{\text{eff},0} = 1000 \text{ t.} \quad (77)$$

The simulations include the soft-loading and soft-unloading logic during both generation and storage phases, so that cyclic replacement of terminal units can be represented within the finite-track constraint. As in Section 4, the power variables are expressed in deviation coordinates around the scheduled PCC output. Thus, $P_{\text{PCC,ref}} = 0$ represents a constant scheduled export in deviation coordinates rather than zero wind-farm output.

5.1. Sustained Smoothing Under Different Slope Angles

The first set of simulations examines how the slope angle affects the sustained smoothing capability of the single-track multi-unit system. According to the scaling analysis in Sections 2–4, the cumulative displacement of the active load chain is mainly controlled by the effective gravity factor

$$M_{\text{eff}}(\sin \theta - \mu_r \cos \theta). \quad (78)$$

For a given wind-power fluctuation and a fixed active moving mass, a smaller slope angle reduces the available gravity-driven torque and therefore requires a larger speed excursion to provide the same power compensation. Since the cumulative motion range is determined by the time integral of the velocity, this larger speed excursion can more rapidly consume the available finite-track margin and increase the frequency of terminal-unit replacement.

To demonstrate this effect, two slope angles are compared: $\theta = 15^\circ$ and $\theta = 30^\circ$. The case of $\theta = 15^\circ$ is used as a stress case to reveal the finite-track limitation under weaker gravity drive, whereas $\theta = 30^\circ$ is adopted as the baseline slope angle of the proposed numerical configuration. Other parameters, including the 1000 m track layout, the 30 standard load units, the soft-loading/soft-unloading settings, and the wind-power fluctuation signal, are kept unchanged.

It can be seen that, during the approximately two-hour simulation window, switching between storage and generation, corresponding to reversals of the motion direction, occurs about 14 times. This switching frequency is well within the realizable range of an electromagnetic torque-control system. These reversals are system-level changes of motion direction over several-minute intervals, rather than repeated second-scale braking or engagement events.

Figures 7 and 8 show that both cases can switch between generation and storage according to the sign of the wind-power deviation and can suppress PCC-side power fluctuations. However, their sustained operation over the multi-hour simulation horizon differs significantly. In the $\theta = 15^\circ$ case, the smaller gravity component requires a larger motion response of the active load chain. As a result, the terminal inventories are more frequently involved in cyclic replacement, and the system approaches the available finite-track and inventory margins more rapidly. This behavior indicates that an insufficient slope angle may lead to excessive cumulative displacement even when the short-time power response appears acceptable.

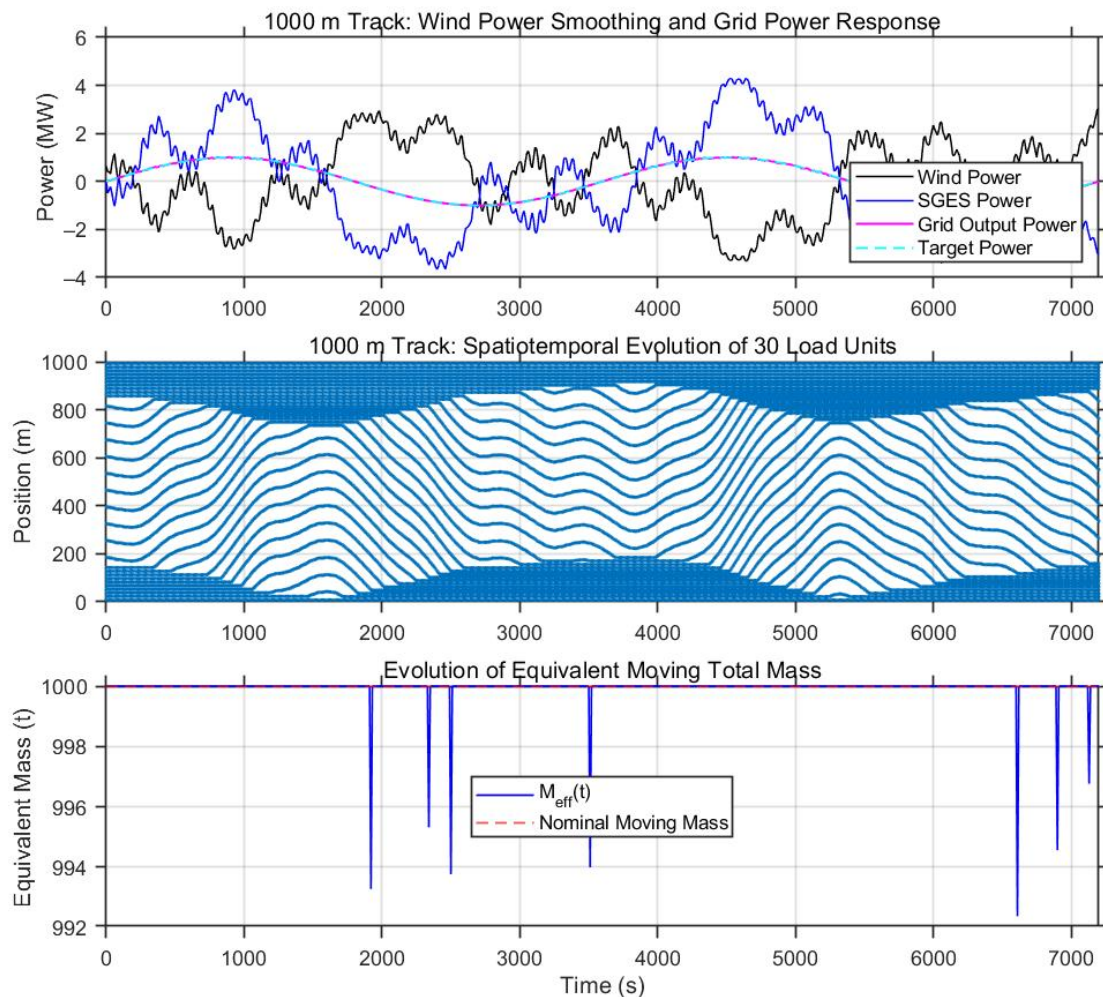


Figure 7. Long-term operation results of the single-track multi-unit operation scheme under $\theta = 15^\circ$. The panels show the wind-power deviation and SGES compensation, the resulting PCC-side power, the active-chain motion, and the terminal inventory evolution under the finite-track constraint.

By contrast, under $\theta = 30^\circ$, the larger gravity-driven torque reduces the velocity and displacement required for the same power compensation. The active load chain remains within a more favorable operating envelope during the simulated time window, and the upper/lower inventories are not exhausted. This comparison is consistent with the theoretical scaling that increasing the slope angle reduces the cumulative motion range required for a fixed power regulation task, provided that the torque-control authority is sufficient.

It should be emphasized that this result does not imply unlimited operation. The system remains constrained by the track length, the available upper and lower inventories, and the allowable displacement of the active motion chain. The numerical results indicate that, under the simulated wind-power fluctuation and the proposed 1000 m layout, the $\theta = 30^\circ$ configuration maintains near-constant PCC-side output throughout the simulated multi-hour window without exhausting the available inventory margin. This verifies the feasibility of the proposed scheduling logic under the selected theoretical parameters.

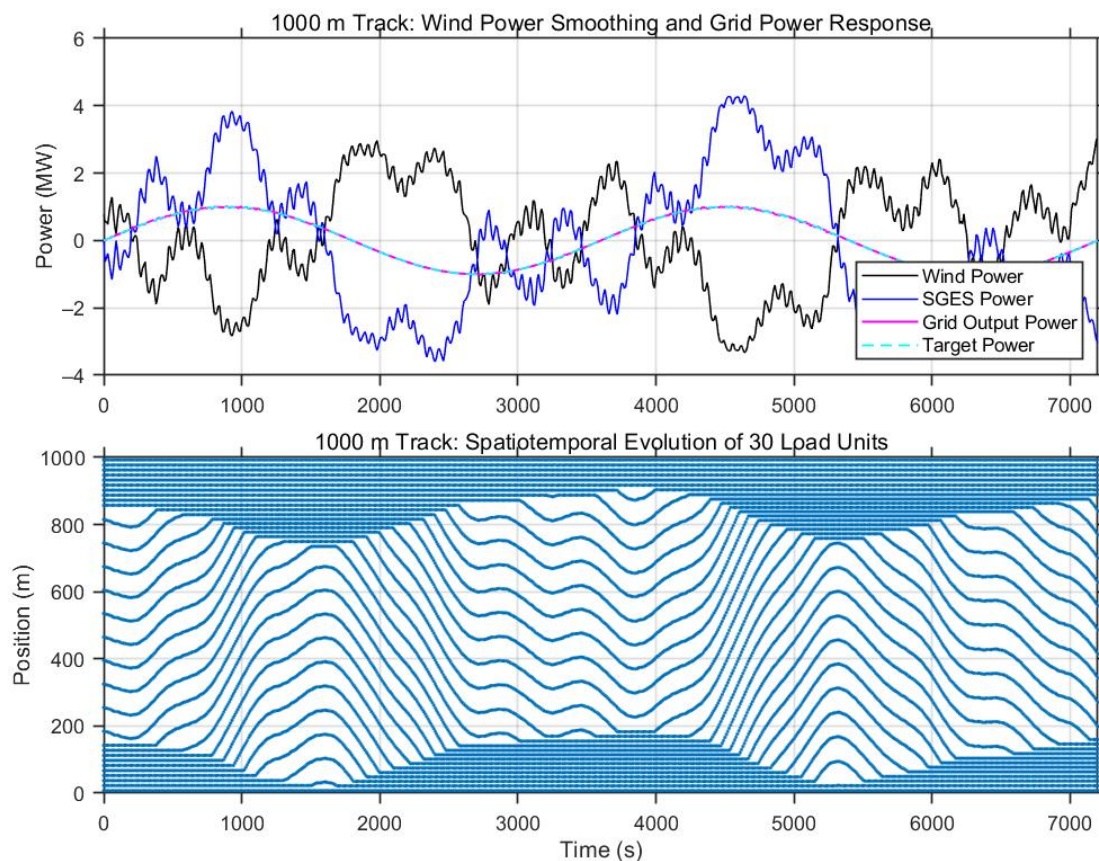


Figure 8. Long-term operation results of the single-track multi-unit operation scheme under $\theta = 30^\circ$. Compared with the 15° case, the larger gravity-driven torque reduces the required speed and displacement excursions for the same wind-power smoothing task.

5.2. Slow-Varying Target Power Tracking and Effective-Mass Evolution

Beyond constant scheduled output in deviation coordinates, the system's ability to track a slow-varying PCC-side power command is also investigated. The PCC-side reference trajectory $P_{\text{PCC,ref}}(t)$ is set as a slowly varying command with a period of approximately 1 h and an amplitude of approximately 2 MW. This test examines whether the proposed single-track multi-unit operation scheme can move from constant-value smoothing to scheduled curve tracking while still operating under the same 1000 m finite-track constraint.

Figure 9 shows that, under the baseline $\theta = 30^\circ$ configuration, the system can follow the slowly varying MW-scale PCC-side reference trajectory while smoothing the imposed wind-power fluctuation. This indicates that the proposed scheme is not limited to constant-output smoothing, but can also support slowly varying scheduling commands within the considered operating range. This result is consistent with the frequency-response analysis in Section 3: the imposed scheduled command changes on an hour-level time scale, and the dominant fluctuation components considered here are on the order of several tens of seconds or longer, which remain much slower than the local electromechanical response time selected for the controller.

This tracking capability is nevertheless bounded by the finite-track margin, the available upper/lower inventories, and the allowable active-chain displacement. If the target power deviates from its mean for a prolonged period, the SGES system will experience a net energy imbalance, causing the load units to drift preferentially in one direction and eventually approach the stroke limits. Such a limitation may be alleviated by increasing the active moving mass, increasing the slope angle where terrain permits, enlarging the

upper/lower inventory margins, or optimizing the scheduling strategy. Therefore, the present simulation verifies feasible multi-hour operation for the selected command and layout, but it should not be interpreted as a guarantee of indefinite operation under arbitrary power commands.

The bottom panel of Figure 9 further illustrates the evolution of the effective moving mass during long-term operation. The variation of $M_{\text{eff}}(t)$ reflects the soft-loading and soft-unloading events associated with terminal-unit replacement. During the storage phase, newly engaged lower-side units are smoothly accelerated from rest and incorporated into the active chain. Because this loading process does not have to be exactly symmetric with the unloading process at the opposite end, the effective moving mass may exhibit finite and gradual short-time fluctuations. These fluctuations are consistent with the participation-coefficient model introduced in Section 4 and indicate that the loading process is represented as a continuous transition rather than an ideal step change.

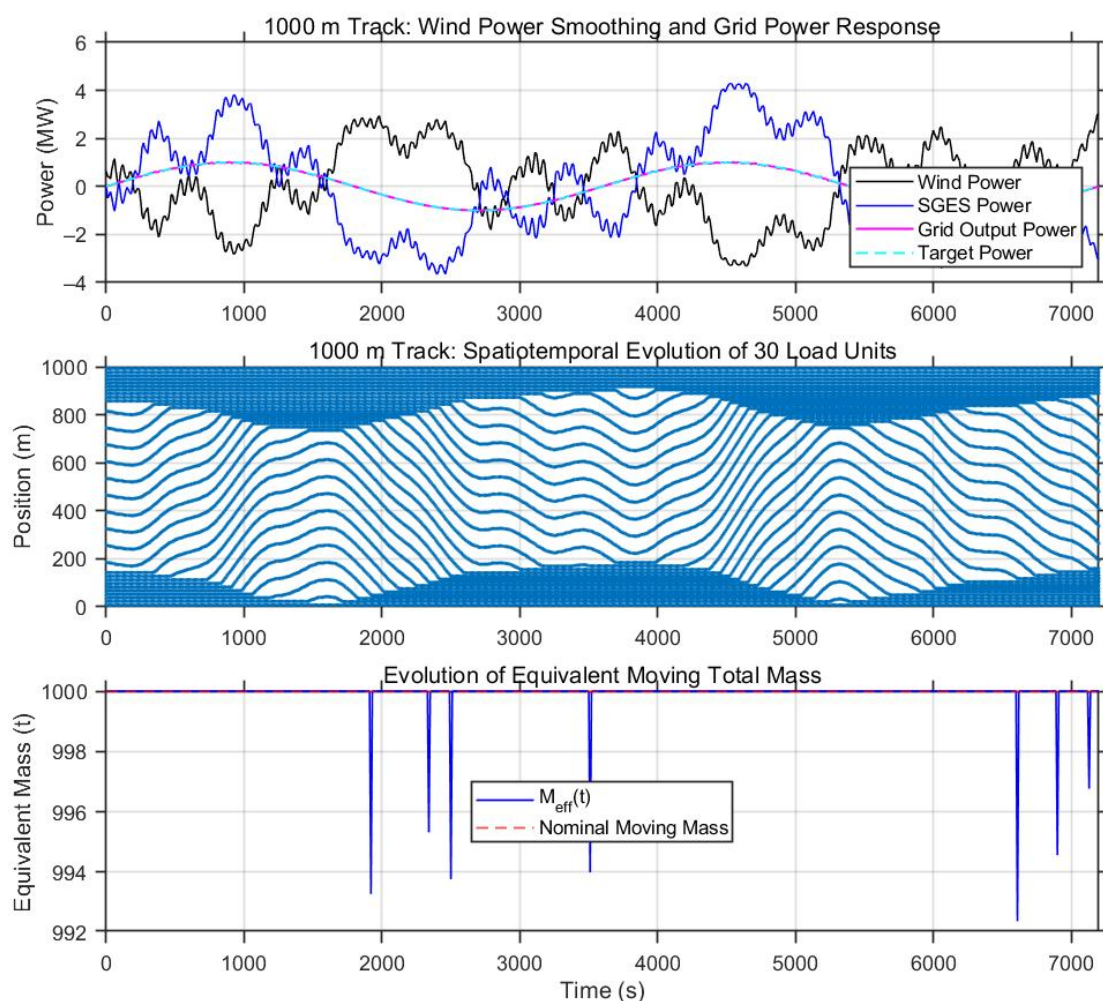


Figure 9. Numerical results of the single-track multi-unit operation scheme under slowly varying target power conditions. The simulation demonstrates simultaneous wind-power smoothing, scheduled PCC-side command tracking, and effective-mass transition during terminal-unit replacement.

During the generation phase, a more symmetric “soft-unloading plus synchronized soft-loading” process can keep the effective moving mass closer to its nominal value during handover. In both operating modes, the feedforward-feedback electromagnetic torque controller compensates for the mechanical-side mass variation and helps maintain a smooth PCC-side power output. Therefore, the numerical results demonstrate not only the power

tracking behavior, but also the role of soft effective-mass transition in reducing abrupt mechanical and electrical disturbances during multi-unit operation.

Overall, the simulations verify three aspects of the proposed scheme. First, the finite-track multi-unit layout can realize cyclic terminal-unit replacement while maintaining an approximately constant active moving mass. Second, the low-speed-branch torque-control framework can compensate for wind-power deviations and slowly varying scheduled commands within the considered time-scale range. Third, increasing the slope angle reduces the required motion range, which agrees with the theoretical scaling derived from the effective gravity factor. These results support the use of the single-track multi-unit scheme as a finite-track numerical implementation of the theoretical power-smoothing framework developed in the preceding sections.

6. Discussion

The results of this study should be interpreted within the theoretical and control-oriented scope of the proposed slope-track SGES framework. The main objective is not to provide a complete engineering design of a full-scale storage plant, but to clarify the electromechanical power-tracking mechanism of a low-speed-branch slope SGES system and to demonstrate how this mechanism can be implemented in a finite-track multi-unit configuration. The numerical results in Sections 4 and 5 show that, under the selected parameters, the single-track multi-unit scheme can smooth MW-scale wind-power deviations and track slowly varying PCC-side commands while maintaining the active load chain within the available track and inventory margins. These results verify the internal consistency of the proposed modeling, control, and scheduling framework.

A key implication of the analysis is that the proposed single-track multi-unit scheme should be distinguished from a conventional long-duration energy-shifting device. For daily-scale energy balancing, the required stored energy scales with the product of the balancing power and duration, $P_{\text{bal}}T_{\text{bal}}$, whereas the gravitational energy provided by a finite slope-track system scales with the total movable mass and height difference, approximately as ηMgH . Therefore, if a slope SGES system is required to sustain a large nearly one-way power output over many hours, the required load inventory and terminal handling capacity would increase rapidly. Under finite-track constraints, this would require a much larger number of load units, a larger height difference, multiple tracks, or additional system-level coordination.

The operating mode proposed in this paper is different. The representative system contains 30 standard 100 t load units, corresponding to a total load inventory of 3000 t, and only part of these units forms the active moving chain at a given time. This load scale is intended for local wind-farm-side power smoothing rather than full-day energy shifting. In this case, the wind-power deviation fluctuates around a scheduled PCC-side output, and the SGES units move back and forth around a local operating region instead of completing a long one-way stroke. As a result, the system mainly needs to provide bidirectional compensation for positive and negative power deviations, while keeping the active chain within the finite track and inventory margins. Where suitable terrain, existing height differences, or mine-slope resources are available and cost constraints are acceptable, such a distributed finite-track SGES configuration can therefore serve as a local power-shaping resource for renewable energy stations, helping the wind farm behave more like a smoothed and controllable power source and thereby supporting grid-side stability.

The proposed scheme should also be understood as complementary to, rather than a direct replacement for, BESS or supercapacitor-based smoothing systems. Batteries and supercapacitors are more suitable for fast response and high-frequency power compensation. By contrast, the slope-track SGES scheme considered here mainly targets slow and medium

time-scale PCC-side power shaping, with potential advantages in long service life, low cyclic degradation, mechanical robustness, and the use of suitable terrain or existing height differences. A full techno-economic comparison among different storage options depends on specific site conditions, lifetime assumptions, power/energy ratings, and maintenance strategies, and is beyond the scope of the present theoretical control study.

The frequency-response analysis in Section 3 clarifies the dynamic boundary of this operating strategy. The low-speed branch is suitable for slow and medium time-scale power smoothing when both $\Omega\tau \ll 1$ and $\Omega\epsilon \ll 1$ are satisfied. Under this condition, the reference-speed tracking error and the inertial correction term remain small, and the quasi-steady low-speed-branch prediction is reliable. The numerical results show that noticeable deviations occur only when the imposed fluctuation approaches the second-level time scale, whereas the smoothing cases considered in this paper mainly involve fluctuations on the order of several tens of seconds or longer. This supports the use of the proposed controller for the intended smoothing scenario, while also indicating that very rapidly varying power commands should not be regarded as the primary operating target of the present scheme. In practical applications, the admissible tracking bandwidth should be determined together with the available electromagnetic torque authority, actuator bandwidth, and gain-clipping limits of the gain-scheduled speed feedback law.

The loss analysis in this paper separates the main generation-side energy-flow terms under a simplified lumped model. Rolling resistance, equivalent viscous mechanical loss, and lumped electromechanical conversion loss are explicitly distinguished, which helps clarify how load mass and slope angle affect the required operating speed and viscous loss on the low-speed branch. Nevertheless, the conversion efficiency η is treated as a constant, and the resistance model is limited to a simple rolling-resistance correction plus equivalent viscous damping. This treatment is appropriate for deriving the analytical low-speed-branch mechanism, but it does not represent a complete efficiency map of the motor-converter-transmission system. Future prototype-level studies should incorporate speed-dependent and torque-dependent motor/converter efficiency, traction and braking losses, transmission losses, and experimentally identified resistance parameters into the feedforward mapping and gain-scheduling design.

The present friction model is also deliberately simplified. Real slope-track systems may involve direction-dependent rolling resistance, static-dynamic friction transitions, rail-contact-state dependence, wheel-rail or clamp-rail interaction, and additional losses during loading and unloading. These effects may alter the effective gravity-driven torque, introduce asymmetric behavior between generation and storage, and affect the precise tracking performance near low-speed or near-zero-speed operating points. In the current framework, moderate parameter deviations are partly compensated by the feedback terms, as shown by the parameter-mismatch analysis. However, strongly nonlinear friction or abrupt contact-state changes would require a more detailed model and experimental parameter identification. Therefore, incorporating more realistic friction and contact models is an important direction for future work. At the control level, however, these nonidealities do not require the feedforward mapping to be exact; they mainly define the admissible uncertainty range within which feedback compensation, gain clipping, and actuator torque limits remain effective.

The soft loading/unloading process used in the multi-unit scheme is represented by continuous participation coefficients. This approximation allows the effective moving mass, gravity-driven torque, and equivalent inertia to change smoothly during terminal-unit replacement. The transition time constants used in the simulations are chosen to be compatible with the expected response time of mechanical engagement, rail-clamping, traction-force build-up, and braking devices, while their saturation time remains much

shorter than the several-tens-of-seconds fluctuation scale targeted by the smoothing strategy. In an actual system, the detailed realization of this soft transition would need to be determined together with the specific design of the traction system, rail-clamping system, braking devices, and mechanical buffers. The present model therefore provides a system-level representation of soft mass transition, rather than a detailed mechanical design of the loading/unloading hardware.

It should also be emphasized that the present verification remains numerical. No full-scale experimental measurement of a slope-track SGES prototype is included in this study. Such an experimental validation would require heavy-load track infrastructure, terminal loading/unloading devices, traction and braking systems, and long-duration reliability assessment. The model assumptions used here, including the lumped electromechanical representation, simplified rolling-resistance correction, constant conversion efficiency, and continuous participation coefficients for soft loading/unloading, are suitable for clarifying the control mechanism and finite-track scheduling logic. However, the quantitative accuracy of the model should be further examined in future prototype or subsystem experiments. Such experiments may include traction/braking response tests, rail-clamping and release tests, speed-matching tests for newly engaged units, wheel-rail or clamp-rail fatigue assessment, and parameter-identification tests for resistance, damping, and conversion efficiency. The absence of full-scale experimental validation in this work is mainly due to the large infrastructure, heavy-load equipment, and long-duration reliability tests required for such a system; this is precisely why the theoretical mechanism, parameter dependence, and control boundary need to be clarified before prototype construction.

7. Conclusions

This paper investigated the low-speed-branch power-tracking mechanism of a slope-track solid gravity energy storage system and its finite-track implementation for wind-farm-side power smoothing. The main findings are summarized as follows.

First, a lumped electromechanical model of the slope-track SGES system was established by combining the gravity-driven mechanical dynamics, motor-side electromagnetic torque, equivalent inertia, and grid-side power output. The steady-state power-speed curve was shown to contain a low-speed high-torque branch and a high-speed low-torque branch. For solid gravity energy storage systems, the low-speed branch is generally more suitable because it has a positive power-speed coupling, requires a smaller operating speed, and reduces the cumulative displacement of the moving load under finite-track constraints.

Second, a feedforward-feedback torque-control framework was formulated for low-speed-branch power tracking. Pure feedforward control, power feedback, speed feedback, and gain-scheduled speed feedback were analyzed in a unified local error-dynamics form. The feedback torque provides the required transient deviation from the quasi-steady torque, allowing the load chain to accelerate or decelerate during power-command variations. The frequency-response analysis further showed that accurate power tracking requires both $\Omega\tau \ll 1$ and $\Omega\varepsilon \ll 1$. Therefore, the proposed low-speed operating strategy is suitable for slow and medium time-scale power smoothing, whereas very rapidly varying power commands are constrained by the finite closed-loop response time and inertial correction term.

Third, the parameter-scaling mechanism of heavy-load slope SGES was clarified. For a fixed power regulation task, increasing the active moving mass or the slope angle enhances the effective gravity-driven torque. As a result, the required operating speed, acceleration demand, and cumulative displacement are reduced when sufficient electromagnetic torque-control authority is available. The generation-side loss decomposition also shows that lower operating speed can reduce the equivalent viscous mechanical loss on the low-speed

branch, while rolling resistance and lumped electromechanical conversion loss remain important factors in the overall energy balance.

Fourth, a finite-track single-track multi-unit operation scheme was proposed and numerically tested. In the representative configuration, a 1000 m track is divided into upper parking, central motion, and lower parking zones, and 30 standard 100 t load units are arranged so that approximately ten units form the active moving chain. Soft loading/unloading and terminal-unit replacement allow the system to maintain an approximately constant active moving mass and to reduce abrupt disturbances during module switching. The numerical results show that, under the selected parameters, the system can smooth MW-scale wind-power deviations and track slowly varying PCC-side power commands over a multi-hour window without exhausting the finite track and inventory margins. For the smoothing of fluctuations on the time scale of several tens of seconds or longer, the transition between storage and generation occurs roughly once every several to ten minutes in the simulated cases. This switching frequency is well within the realizable operating range of a four-quadrant converter-fed traction motor-generator system.

Finally, the functional boundary of the proposed scheme was clarified. The single-track multi-unit slope SGES system is better interpreted as a PCC-side power-shaping and short-term buffering device for local renewable energy stations, rather than as a stand-alone solution for large-scale daily energy shifting. Its practical operating range is jointly determined by the available track length, upper/lower load inventories, active moving mass, slope angle, torque-control authority, and admissible tracking bandwidth. For applications requiring larger energy-shifting capacity, the scheme would need to be extended by increasing the load inventory, enlarging the available height difference, using multiple tracks, or coordinating with other storage resources.

The present results provide a theoretical and simulation-level basis for subsequent prototype design and experimental validation. Future work should incorporate measured wind-power data, detailed efficiency maps, nonlinear and direction-dependent friction models, converter and actuator constraints, and hardware-in-the-loop or prototype-level validation to further assess the practical performance of the proposed slope-track SGES control strategy.

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