

## Article

# Dimensional Analysis-Based Modeling of Cutting and Thrust Forces in Orthogonal Cutting of AISI 1045 Steel

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## Abstract

Accurate prediction of cutting forces remains a key challenge in metal machining due to the strong coupling between geometry, material behavior, friction, and thermal effects, which limits the generality of purely empirical models. In this study, physically interpretable models with predictive capability for cutting and thrust forces are developed for orthogonal cutting of AISI 1045 steel using a dimensional analysis framework based on the Buckingham  $\Pi$  theorem. Experimental data collected from the literature are used to construct dimensionless formulations incorporating geometrical, kinematic, mechanical, and thermal parameters. The proposed models are calibrated and evaluated using statistical performance metrics and residual analysis, and subsequently validated against independent experimental datasets not used during model development. A correction factor associated with the tool–chip contact length is optimized during validation to improve predictive accuracy. Results show that both force components can be consistently represented through a reduced set of governing dimensionless groups, providing physically meaningful scaling across a wide range of cutting conditions. The validation results confirm the robustness of the proposed formulation, while also revealing different sensitivities of cutting and thrust forces to contact, thermal, and geometrical effects. A physical interpretation of the dimensionless groups is presented, framing the machining process as a case of severe plastic deformation under high strain rates and strong thermomechanical coupling. The study demonstrates that dimensional analysis offers a physically consistent, scalable, and transferable approach for modeling cutting forces, with potential applicability to other materials and machining configurations.

**Keywords:** dimensional analysis; orthogonal cutting; cutting force; thrust force; buckingham  $\Pi$  theorem; scaling laws; severe plastic deformation



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## 1. Introduction

Metal machining processes play a central role in modern manufacturing industries, where productivity, dimensional accuracy, and surface integrity are critical requirements [1]. Among the variables governing machining performance, cutting forces are widely recognized as fundamental indicators because they directly influence tool wear, surface quality, dimensional accuracy, process stability, and energy consumption. Consequently, the analysis and prediction of cutting forces have become essential for the design, optimization, and control of machining operations [2,3].

Cutting forces are strongly associated with several key aspects of machining performance. Excessive force levels accelerate tool wear, promote tool deflection, and may lead to

premature tool failure [4]. In addition, force fluctuations are closely related to vibration phenomena and process instability, which degrade surface finish and limit achievable productivity [5]. Beyond tool performance, variations in cutting forces influence surface integrity by affecting surface roughness, residual stresses, and subsurface damage, which are particularly critical in structural and load-bearing components [6]. Reliable estimation of cutting forces is therefore a prerequisite for both process optimization and ensuring the functional performance of machined parts [7].

From a modelling perspective, the prediction of cutting forces remains challenging because machining is a highly coupled multiphysics and multivariable process [8]. Force generation results from the interaction of cutting parameters, workpiece material properties, tool geometry, and process phenomena such as friction, severe plastic deformation, strain hardening, and thermal effects. These mechanisms interact in a nonlinear manner and may evolve during machining due to tool wear, temperature gradients, and local material heterogeneity. As a consequence, developing models capable of capturing both the physical mechanisms and the quantitative behaviour of cutting forces remains an open challenge [9].

Several modelling approaches have been proposed to address this problem, including analytical models derived from shear-plane mechanics, mechanistic formulations calibrated from cutting experiments, and more recently data-driven methods based on machine learning [10–14]. Analytical and mechanistic models provide valuable physical insight but often require detailed knowledge of material behaviour, friction conditions, and tool geometry parameters that may not always be available. Empirical formulations, such as the Victor–Kienzle model, rely on experimentally determined coefficients that are frequently material- and condition-specific, which limits their transferability across different cutting regimes. Data-driven approaches can achieve high predictive accuracy, but they typically require large training datasets and may lack clear physical interpretability [13,14].

In this context, dimensional analysis offers a modelling framework that is particularly well suited for complex engineering systems involving multiple interacting variables. By applying the Buckingham  $\pi$  theorem, the dimensional variables describing a physical phenomenon can be systematically transformed into a reduced set of dimensionless groups that capture the dominant scaling behaviour of the system. These groups represent physically meaningful ratios between governing parameters and enable the identification of fundamental relationships between process variables. Dimensional analysis has been widely used in engineering to reveal scaling laws and to construct predictive models for phenomena characterized by strong coupling between mechanical, thermal, and geometrical effects [15–17].

It should be noted that classical references such as Buckingham [15], Boothroyd [18], and Zhang et al. [19] are intentionally retained due to their foundational role in the formulation of the  $\pi$ -theorem, its early application to machining processes, and the modeling of tool–chip contact length. These works provide the theoretical and physical basis upon which the present dimensional analysis framework is constructed.

When applied to metal machining, dimensional analysis provides a rational framework to represent the mechanisms governing force generation associated with chip formation, material strength, kinematic conditions, and tool geometry [20,21]. One of the key advantages of this approach is that it reduces the number of independent variables while preserving dimensional consistency and the underlying physical relationships. This reduction not only simplifies the structure of predictive models but also significantly decreases the number of experimental trials required for model identification [22]. As a result, dimensional-analysis-based models can provide both explanatory insight into the governing mechanisms of the cutting process and reliable predictive capability across a range of machining conditions [23,24].

Recent studies have further demonstrated the applicability of physically based and dimensionless modeling approaches in manufacturing processes, reinforcing their relevance for developing scalable and interpretable models [22,25].

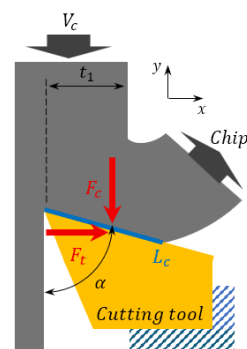
AISI 1045 steel was selected due to its widespread industrial use and its relevance as a representative medium-carbon steel. Its extensive application in mechanical components and its well-documented behavior make it a suitable material for studying machining processes and validating predictive models [3].

In contrast to conventional modelling approaches, the present work adopts a dimensional-analysis-based formulation that emphasizes physical scaling and model transferability. Unlike empirical models, which are often restricted to specific conditions, the proposed approach constructs dimensionless groups that capture the dominant thermomechanical mechanisms and enable the integration of heterogeneous datasets. Compared to analytical and mechanistic models, it avoids the need for detailed constitutive descriptions while preserving physical interpretability. In contrast to data-driven methods, it does not require large training datasets and provides direct insight into the role of each governing parameter. In particular, the explicit inclusion of the tool–chip contact length and the Peclet number allows the model to capture the coupled effects of contact mechanics and thermal transport, resulting in a compact and physically consistent framework for predicting cutting and thrust forces.

Therefore, the present study adopts a dimensional analysis framework to model cutting and thrust forces under orthogonal cutting conditions. The proposed methodology aims to develop a physically interpretable and predictive formulation capable of integrating cutting parameters, material properties, and tool geometry through dimensionless groups. By combining dimensional analysis with experimental data compiled from the literature, the study seeks to establish compact force models that capture the dominant scaling behavior of the process while maintaining practical applicability for machining analysis and process optimization.

## 2. Materials and Methods

The entire analysis and force modeling process presented in this work is carried out under the orthogonal cutting configuration shown in Figure 1. Orthogonal cutting represents a simplified machining configuration that enables the systematic study of cutting variables, input parameters, and their causal relationships.



**Figure 1.** Orthogonal cutting configuration and main variables used in this study: cutting speed  $V_c$ , uncut chip thickness  $t_1$ , width of cut  $w$  in the  $z$ -axis direction, absolute rake angle  $\alpha$ , tool–chip contact length  $L_c$ , and forces  $F_c$  (cutting) and  $F_t$  (thrust).

AISI 1045 steel was selected in this study due to its widespread industrial use and its relevance as a representative medium-carbon steel. Carbon steels account for approximately 80–90% of global steel production, with medium-carbon grades such as AISI 1045

being extensively employed in structural and mechanical components, including shafts, gears, and machine elements. This broad industrial application makes AISI 1045 a suitable reference material for studying machining processes [3]. From a technical perspective, AISI 1045 exhibits a balanced combination of mechanical strength, ductility, and machinability, making it particularly suitable for analyzing cutting forces, tool–chip interaction, and thermomechanical effects. Its well-characterized material properties and stable plastic behavior allow the development of physically interpretable models [2]. From a scientific standpoint, AISI 1045 is widely used in machining research, enabling direct comparison with existing studies and facilitating model validation [26].

### 2.1. Data Sources and Availability

The experimental data used in this study were collected from peer-reviewed literature reporting orthogonal cutting experiments on AISI 1045 steel. The dataset includes measured values of cutting force  $F_c$  ( $n = 314$ ) and thrust force  $F_t$  ( $n = 250$ ), together with the corresponding cutting parameters. All data used for model development and validation are fully available to readers, and no restrictions apply to their use (modeling: [27–29]; validation: [26,30–38]).

A broad range of machining conditions representative of orthogonal cutting of AISI 1045 steel is included in the dataset. The ranges of the main process parameters used for model development and validation are summarized in Table 1. Variations in cutting speed, uncut chip thickness, and tool geometry are included, providing sufficient variability to support the dimensional analysis framework and the regression of the proposed  $\Pi$ -based force models. The absolute rake angle ( $\alpha$ ) denotes the rake angle of the cutting tool measured with respect to the machined surface and is equivalent to the conventional rake angle  $\gamma$  used in orthogonal cutting analysis. By defining the rake angle in absolute terms, the sign convention is removed, simplifying its use in the modeling formulation.

**Table 1.** Ranges of experimental machining parameters compiled from the literature datasets used for model development and validation.

| Parameter (Symbol)                 | Range               | Units      |
|------------------------------------|---------------------|------------|
| Cutting speed ( $V_c$ )            | 10–613; 0.167–10.22 | m/min; m/s |
| Uncut chip thickness ( $t_1$ )     | 0.01–0.50           | mm         |
| Width of cut ( $w$ )               | 0.5–4.0             | mm         |
| Absolute rake angle ( $\alpha$ )   | 1.396–1.745         | rad        |
| Tool–chip contact length ( $L_c$ ) | 0.15–1.22           | mm         |
| Cutting force ( $F_c$ )            | 36.3–2509           | N          |
| Thrust force ( $F_t$ )             | 23.3–1659           | N          |

$V_c$  is reported in both original literature units and corresponding SI units used in the dimensional analysis.

To ensure transparency and reproducibility, the experimental dataset used in this study was compiled exclusively from peer-reviewed literature reporting orthogonal cutting experiments on medium-carbon steel (AISI 1045). The selection criteria required that each study reported measured cutting and thrust forces together with the process variables necessary for the application of dimensional analysis, including cutting speed, uncut chip thickness, tool geometry parameters, and material properties. Studies lacking sufficient information to reconstruct these variables were excluded. Although the dataset originates from three independent experimental campaigns, it contains a large number of individual measurements ( $n = 234$  for cutting force and  $n = 199$  for thrust force), which ensures sufficient variability in cutting conditions to support the regression analysis of the proposed dimensionless force models. The validation phase includes an additional  $n = 80$  tests for  $F_c$  and  $n = 51$  for  $F_t$ .

## 2.2. Material Properties

All models were developed assuming constant thermomechanical properties corresponding to AISI 1045 steel. Material density, yield strength, thermal conductivity, and specific heat capacity were taken from validated literature sources [39].

Representative constant values corresponding to typical properties of AISI 1045 steel at room temperature were selected to ensure consistency in the dimensional analysis and comparability across datasets (Table 2). This assumption is consistent with classical and modern machining models and enables the formulation of dimensionless parameters that capture the dominant physical mechanisms while avoiding unnecessary model complexity.

**Table 2.** Material properties used for AISI 1045 steel in the present study.

| Property               | Symbol     | Value | Unit              |
|------------------------|------------|-------|-------------------|
| Density                | $\rho$     | 7850  | kg/m <sup>3</sup> |
| Thermal conductivity   | $k$        | 49    | W/(m K)           |
| Specific heat capacity | $c_p$      | 470   | J/(kg K)          |
| Yield strength         | $\sigma_y$ | 370   | MPa               |

## 2.3. Dimensional Analysis

Dimensional analysis was performed using the Buckingham  $\Pi$  theorem to identify the governing dimensionless parameters of the cutting process. This approach provides a systematic framework to reduce complex multiphysics problems into a smaller set of dimensionless groups that capture the dominant scaling relationships between variables. While dimensional analysis does not determine the explicit functional form of these relationships, it enables the construction of physically interpretable models that can be calibrated using experimental data.

## 2.4. Dimensionless Groups and Model Formulation

The starting point of the dimensional analysis is the assumption that the cutting force  $F_c$  and the thrust force  $F_t$  can be expressed as functions of the main variables governing the orthogonal cutting process, including geometrical, kinematic, mechanical, and thermal parameters. To avoid redundancy in the thermal description, thermal conductivity and specific heat are combined into the thermal diffusivity  $\alpha_{th} = k/(\rho c_p)$ , which represents the relevant physical scale for heat transport in the cutting process. This functional dependence is expressed in Equation (1):

$$f(F, V_c, t_1, w, L_c, \alpha, \rho, \sigma_y, \alpha_{th}) = 0, \quad (1)$$

where  $F$  represents either  $F_c$  or  $F_t$ .

This functional dependence reflects the coupled thermomechanical nature of the cutting process, where force generation arises from plastic deformation, friction, and heat transfer mechanisms. Considering the set of variables and the fundamental dimensions  $M$ ,  $L$ ,  $T$ , and  $\Theta$ , the dimensional information is summarized in Table 3. The problem therefore involves  $n = 9$  variables and  $r = 4$  fundamental dimensions, leading to  $n - r = 5$  independent dimensionless groups.

**Table 3.** Variables used in the dimensional analysis and their corresponding units and base dimensions.

| Variable                   | Symbol        | Unit         | Base Dimensions   |
|----------------------------|---------------|--------------|-------------------|
| Force                      | $F$           | N            | $M L T^{-2}$      |
| Width of cut               | $w$           | m            | $L$               |
| Uncut chip thickness       | $t_1$         | m            | $L$               |
| Cutting speed              | $V_c$         | $m s^{-1}$   | $L T^{-1}$        |
| Tool–chip contact length   | $L_c$         | m            | $L$               |
| Material density           | $\rho$        | $kg m^{-3}$  | $M L^{-3}$        |
| Yield strength (reference) | $\sigma_y$    | Pa           | $M L^{-1} T^{-2}$ |
| Absolute rake angle        | $\alpha$      | rad          | 1                 |
| Thermal diffusivity        | $\alpha_{th}$ | $m^2 s^{-1}$ | $L^2 T^{-1}$      |

Thermal conductivity ( $k$ ), expressed in  $W m^{-1} K^{-1}$  with base dimensions  $M L T^{-3} \Theta^{-1}$ , and specific heat capacity ( $c_p$ ), expressed in  $J kg^{-1} K^{-1}$  with base dimensions  $L^2 T^{-2} \Theta^{-1}$ , are combined to define the thermal diffusivity  $\alpha_{th} = k/(\rho c_p)$ . Using  $\sigma_y$ ,  $t_1$ ,  $\rho$ , and  $\alpha_{th}$  as repeating variables, a set of independent dimensionless groups is obtained following the Buckingham  $\Pi$  theorem. These groups, defined in Equations (2)–(6), represent the dominant geometrical, mechanical, dynamic, and thermal mechanisms governing force generation in orthogonal cutting.

The normalized force group, given by Equation (2), is defined as

$$\Pi_F = \frac{F}{\sigma_y w t_1}, \quad (2)$$

and represents the mechanical loading relative to the material resistance and the characteristic uncut chip area.

Geometrical effects associated with the tool–chip interaction are described by the contact ratio in Equation (3):

$$\Pi_{L_c} = \frac{L_c}{t_1}, \quad (3)$$

which characterizes the relative extent of the contact zone and is directly related to frictional interaction and stress distribution along the rake face.

The influence of cutting tool geometry is incorporated through the dimensionless rake angle defined in Equation (4):

$$\Pi_\alpha = \alpha, \quad (4)$$

which governs chip flow and force partitioning at the tool–chip interface.

Dynamic effects associated with cutting speed are represented by the inertial group in Equation (5):

$$\Pi_V = \frac{\rho V_c^2}{\sigma_y}, \quad (5)$$

which quantifies the relative magnitude of inertial stresses compared to the material flow strength.

Thermal effects are captured through the Peclet number defined in Equation (6):

$$\Pi_{Pe} = \frac{V_c t_1}{\alpha_{th}}, \quad (6)$$

where  $\alpha_{th}$  is the thermal diffusivity. The Peclet number represents the balance between advective heat transport and thermal diffusion [18].

Based on these dimensionless groups, the force model is formulated as a multiplicative power-law relationship in Equation (7):

$$\Pi_F = C \Pi_{L_c}^a \Pi_{\alpha}^b \Pi_V^c \Pi_{P_e}^d, \quad (7)$$

where  $C$  and the exponents  $a$ ,  $b$ ,  $c$ , and  $d$  are determined through regression.

For regression purposes, Equation (7) is transformed into logarithmic form, yielding Equation (8):

$$\ln(\Pi_F) = \ln C + a \ln(\Pi_{L_c}) + b \ln(\Pi_{\alpha}) + c \ln(\Pi_V) + d \ln(\Pi_{P_e}), \quad (8)$$

which enables linear fitting in log-space.

### 2.5. Tool–Chip Contact Length Estimation

The tool–chip contact length  $L_c$  is a key variable in orthogonal cutting, as it governs frictional interaction and heat transfer at the tool–chip interface, which are fundamental mechanisms in force generation [40]. However,  $L_c$  is not systematically reported in experimental studies, and its direct measurement remains challenging due to the small spatial scales and transient nature of the contact zone.

Consequently, in the present work  $L_c$  is treated as an estimated variable and is reconstructed using the empirical model proposed by Zhang et al. [19], which relates the contact length to cutting conditions and tool geometry. This model is complemented with a correction factor  $\eta$  to account for systematic deviations reported in the literature [41,42].

According to this formulation, the contact length is expressed in Equation (9) as

$$L_c = 4.534 \times 10^{-2} \eta t_1^{0.515} V_c^{-0.065} \alpha^{0.733}, \quad (9)$$

where  $t_1$  is the uncut chip thickness,  $V_c$  is the cutting speed, and  $\alpha$  is the absolute rake angle (see Figure 1). All parameters are expressed in SI units.

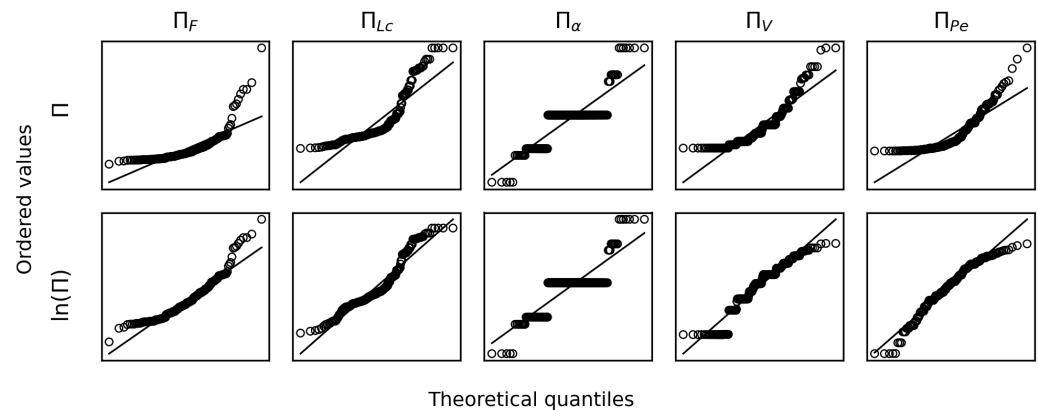
This formulation captures the coupled effects of chip thickness, cutting speed, and tool geometry on the extent of the tool–chip contact region. Although originally developed for medium-carbon steels, the model has been shown to overestimate  $L_c$ , which justifies the introduction of the correction factor  $\eta$  to improve consistency with experimental observations [42]. The resulting values of  $L_c$  are subsequently used to construct the dimensionless contact-length ratio  $\Pi_{L_c} = L_c/t_1$ , which represents the relative extent of the contact zone and its role in frictional dissipation and energy transfer during machining.

### 2.6. Machining Parameters and Variables

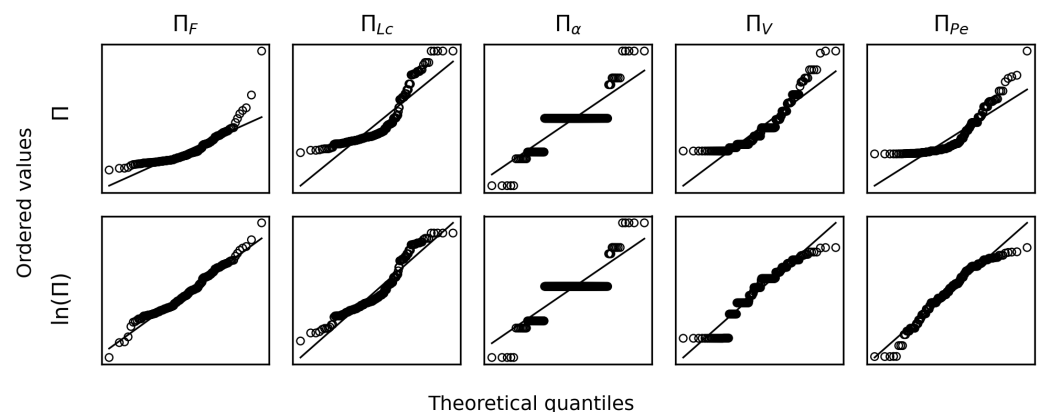
The cutting and thrust forces were modeled as functions of the main variables governing orthogonal cutting. The dependent variables are the cutting force  $F_c$  and the thrust force  $F_t$ , both expressed in newtons. The independent variables considered are the cutting speed  $V_c$  (m/s), the uncut chip thickness  $t_1$  (m), the width of cut  $w$  (m), the absolute tool rake angle  $\alpha$  (rad), and the estimated tool–chip contact length  $L_c$  (m). These variables are commonly reported in the literature and represent the dominant geometrical and kinematic parameters influencing force generation in orthogonal cutting [11].

Normality diagnostics were performed directly on the dimensionless groups  $\Pi_i$  and on their logarithmic transforms  $\ln(\Pi_i)$ , since the regression model is formulated in log-space. Figures 2 and 3 show compact QQ-plots for  $\Pi_F$ ,  $\Pi_{L_c}$ ,  $\Pi_{\alpha}$ ,  $\Pi_V$ , and  $\Pi_{P_e}$ . The raw dimensionless groups exhibit pronounced deviations from normality, with strong skewness and heavy tails, particularly for  $\Pi_F$ . Applying the logarithmic transformation reduces skewness and improves symmetry in the central region of the QQ-plots for most dimen-

sionless groups. Although deviations from normality persist in the tails, the transformation enhances numerical stability and supports the use of the linear regression model given by Equation (8), while model adequacy is subsequently assessed through residual diagnostics rather than strict normality assumptions.



**Figure 2.** Normal probability plots (QQ-plots) for the dimensionless groups (**top row**) and their logarithmic transforms (**bottom row**) used in the  $\Pi$ -based **cutting-force** ( $F_c$ ) model. Open circles represent the empirical sample quantiles, while the solid line denotes the theoretical normal-distribution reference.



**Figure 3.** Normal probability plots (QQ-plots) for the dimensionless groups (**top row**) and their logarithmic transforms (**bottom row**) used in the  $\Pi$ -based **thrust-force** ( $F_t$ ) model. Open circles represent the empirical sample quantiles, while the solid line denotes the theoretical normal-distribution reference.

### 2.7. Sensitivity Analysis of the Dimensionless Model

The model derived from dimensional analysis is expressed by Equation (7), where  $C$ ,  $a$ ,  $b$ ,  $c$ , and  $d$  are model coefficients. From Equation (8), the sensitivity indices are defined in Equation (10) as

$$S_{\Pi_i} = \frac{\partial \ln \Pi_F}{\partial \ln \Pi_i}, \quad (10)$$

which yields Equation (11):

$$S_{\Pi_{Lc}} = a, \quad S_{\Pi_{\alpha}} = b, \quad S_{\Pi_V} = c, \quad S_{\Pi_{Pe}} = d. \quad (11)$$

These indices quantify the sensitivity of the normalized force to changes in the governing dimensionless groups and provide a direct measure of the relative importance of contact, geometrical, dynamic, and thermal mechanisms in force generation. The sensitivity indices summarized in Table 4 provide a direct ranking of the governing mechanisms.

**Table 4.** Sensitivity indices for the dimensionless force model.

| Dimensionless Group | Sensitivity Index | Dominant Mechanism                |
|---------------------|-------------------|-----------------------------------|
| $\Pi_{L_c}$         | $a$               | Contact length and friction       |
| $\Pi_{P_e}$         | $d$               | Thermal regime and heat transport |
| $\Pi_V$             | $c$               | Inertial and strain-rate effects  |
| $\Pi_\alpha$        | $b$               | Tool geometry effects             |

### Physical Interpretation

The sensitivity indices in Equation (11) and Table 4 provide a direct ranking of the mechanisms controlling force generation in orthogonal cutting. Larger absolute values of the exponents indicate dimensionless groups that dominate uncertainty propagation and model response. In the present framework,  $\Pi_{L_c}$  captures the effect of tool–chip contact extent and frictional interaction along the rake face, whereas  $\Pi_{P_e}$  quantifies the thermal transport regime through the balance between advection and diffusion. The group  $\Pi_V$  represents the contribution of inertial and strain-rate related effects relative to the material flow strength, and  $\Pi_\alpha$  accounts for the influence of tool geometry on chip flow and force partitioning. Overall, this interpretation supports machining as a coupled thermomechanical process governed by severe plastic deformation, frictional dissipation, and localized heat generation.

### 2.8. Model Evaluation

Model performance was evaluated using standard statistical metrics, including the coefficient of determination  $R^2$ , the root mean square error (RMSE), and the mean absolute error (MAE). Residual analysis was conducted to assess model adequacy and to identify parameter ranges associated with improved or degraded predictive accuracy. All analyses were performed consistently for both cutting and thrust force models.

### 2.9. Ethical Considerations

This study does not involve experiments on humans or animals and therefore does not require ethical approval.

## 3. Results

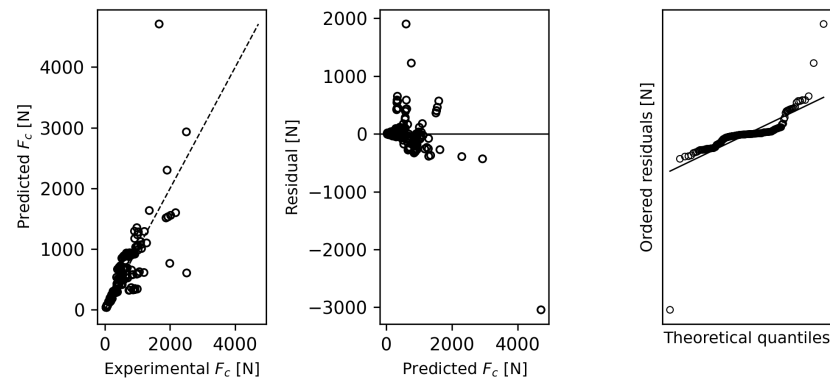
Using the dimensionless groups defined in Section 2.4, the cutting and thrust force models were formulated as power-law relationships in  $\Pi$ -space using Equation (7) and fitted by linear regression in log-space using Equation (8). For model development, the contact length was estimated using Equation (9) with  $\eta = 1$ , as in the original Zhang formulation. The fitted coefficients and corresponding performance metrics are summarized in Table 5.

**Table 5.** Fitted coefficients and performance metrics of the  $\Pi$ -based cutting ( $F_c$ ) and thrust ( $F_t$ ) force models.

| Parameter                          | $F_c$    | $F_t$   |
|------------------------------------|----------|---------|
| ln C                               | 80.8369  | −0.2250 |
| $a$ (contact length exponent)      | −16.7647 | 0.3318  |
| $b$ (rake angle exponent)          | 12.0771  | 1.8799  |
| $c$ (dynamic exponent)             | 3.5799   | 0.0669  |
| $d$ (thermal exponent)             | −8.2894  | 0.0383  |
| Number of data points $n$          | 234      | 199     |
| Coefficient of determination $R^2$ | 0.5489   | 0.408   |
| RMSE (N)                           | 302.5    | 196.3   |
| MAE (N)                            | 129.0    | 110.5   |

### 3.1. Cutting Force Results

Figure 4 presents the diagnostic plots for the  $\Pi$ -based cutting-force model. The experimental–predicted comparison shows that the model captures the global scaling of  $F_c$  over a wide range of cutting conditions, although larger deviations are observed at high force levels.

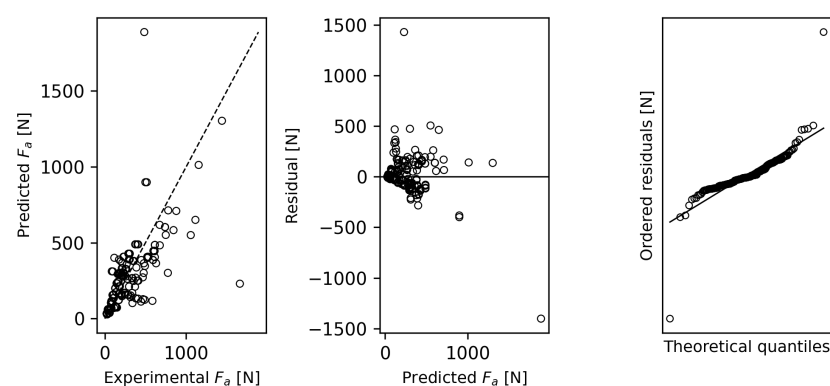


**Figure 4.** Full diagnostic plots for the  $\Pi$ -based cutting-force model. **(Left):** experimental vs. predicted  $F_c$ . **(Middle):** residuals  $F_c - \hat{F}_c$  vs. predicted  $F_c$ . **(Right):** normal probability plot (QQ-plot) of residuals.

Residuals are approximately centered around zero (mean residual  $\approx -5.9$  N), indicating negligible global bias. However, the residual spread increases with predicted force, revealing heteroscedastic behavior. The QQ-plot exhibits a linear central region with strong departures in the tails (Shapiro–Wilk  $p \ll 0.05$ ), indicating heavy-tailed residuals. This behavior is consistent with literature-compiled machining datasets, where multiple cutting regimes and unmodeled variability—such as friction conditions and temperature-dependent material response—may coexist. Overall, the statistical indicators in Table 5 show that the proposed dimensionless formulation provides a consistent representation of the experimental trends.

### 3.2. Thrust Force Results

Figure 5 shows the diagnostic plots for the  $\Pi$ -based thrust-force model. The model captures the global scaling of  $F_t$  across the dataset, although a larger dispersion is observed compared to the cutting force.



**Figure 5.** Diagnostic plots for the  $\Pi$ -based thrust-force model. **(Left):** experimental vs. predicted  $F_t$  with the 1:1 reference line. **(Middle):** residuals  $e = F_t - \hat{F}_t$  versus predicted force. **(Right):** normal probability plot (QQ-plot) of residuals.

Residuals remain centered around zero, indicating no significant bias, while the QQ-plot shows pronounced deviations in the tails (Shapiro–Wilk  $p \ll 0.05$ ), again indicating heavy-tailed error distributions. These features are consistent with the variability inherent to literature-based datasets. The lower predictive accuracy observed for  $F_t$  (Table 5) is consistent with the stronger dependence of thrust force on local contact conditions, normal stress distribution, and frictional effects at the tool–chip interface. Despite this increased variability, the dimensionless formulation reproduces the main experimental trends and improves consistency across different cutting regimes, supporting the applicability of the proposed framework for modeling thrust force in orthogonal cutting.

### 3.3. Model Validation

The validation of the proposed models was carried out by comparing predicted values with independent experimental data collected from the literature, which were not used during the model development stage. A total of 80 experimental data points for the cutting force ( $F_c$ ) and 51 for the thrust force ( $F_t$ ) were considered. The range of experimental cutting parameters used for model validation is presented in Table 6.

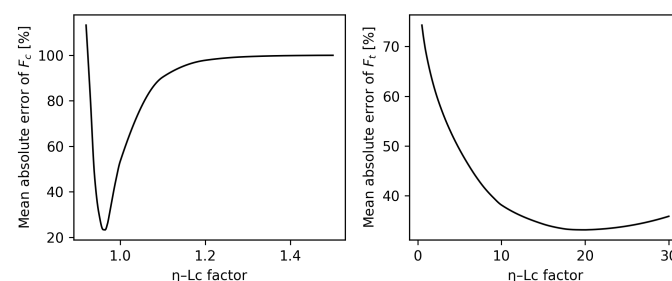
**Table 6.** Ranges of experimental machining parameters used for model validation [26,30–38].

| Parameter (Symbol)                 | Range              | Units      |
|------------------------------------|--------------------|------------|
| Cutting speed ( $V_c$ )            | 10–500; 0.167–8.33 | m/min; m/s |
| Uncut chip thickness ( $t_1$ )     | 0.05–0.5           | mm         |
| Width of cut ( $w$ )               | 0.5–4              | mm         |
| Absolute rake angle ( $\alpha$ )   | 1.396–1.745        | rad        |
| Tool–chip contact length ( $L_c$ ) | 0.33–1.27          | mm         |
| Cutting force ( $F_c$ )            | 175–2509           | N          |
| Thrust force ( $F_t$ )             | 115–1659           | N          |

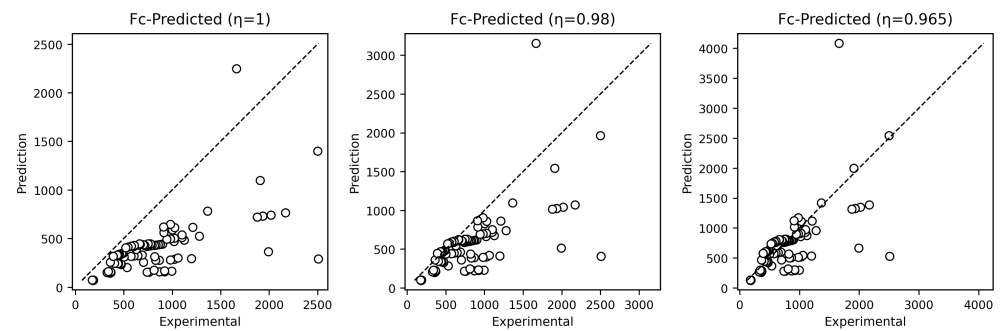
$V_c$  is reported in both original literature units and corresponding SI units used in the dimensional analysis.

The model predictions were obtained using the dimensionless formulations proposed in this work, incorporating the tool–chip contact length correction factor  $\eta$  as a calibration parameter. The optimal value of  $\eta$  was determined by minimizing the mean absolute error over the entire validation dataset.

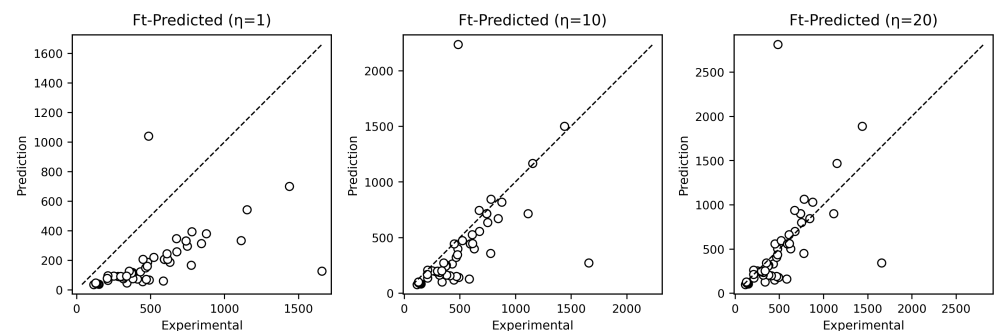
The mean absolute error was evaluated separately for  $F_c$  and  $F_t$ , and the corresponding trends as a function of  $\eta$  are presented in Figure 6. The minimum error values were obtained for  $\eta = 0.965$  in the case of the cutting force and  $\eta = 20$  for the thrust force. The comparison between experimental and predicted values for different  $\eta$  levels is shown in Figures 7 and 8.



**Figure 6.** Mean prediction error for the cutting force  $F_c$  and thrust force  $F_t$  models.



**Figure 7.** Cutting force  $F_c$  validation for different  $\eta$  values. Open circles represent experimental–predicted data pairs, and the dashed line represents the ideal agreement ( $y = x$ ). The results show a good overall agreement between experimental and predicted values.



**Figure 8.** Thrust force  $F_t$  validation for different  $\eta$  values. Open circles represent experimental–predicted data pairs, and the dashed line represents the ideal agreement ( $y = x$ ). A higher dispersion is observed compared to the cutting force, indicating greater sensitivity of the model to process variables.

The error analysis performed for both cutting and thrust forces reveals distinct sensitivities of the proposed model to the governing dimensionless parameters. For the cutting force, the prediction error increases with the magnitude of the predicted force, indicating reduced accuracy under high-force regimes. This behavior suggests that nonlinear effects become dominant under severe cutting conditions and are not fully captured by the current formulation. A moderate dependence of the error is observed with respect to the geometric parameter  $\Pi_\alpha$ , highlighting the influence of tool geometry. In contrast, the contact-length parameter  $\Pi_{L_c}$  exhibits only a weak correlation with the error, indicating that its effect is adequately represented in the model. The dynamic and thermal parameters,  $\Pi_V$  and  $\Pi_{Pe}$ , show negligible influence on the prediction accuracy for the cutting force.

For the thrust force, a different trend is observed. The prediction error shows a weaker dependence on force magnitude, suggesting that large force values are not the primary source of inaccuracy. Instead, a more noticeable influence of the thermal parameter  $\Pi_{Pe}$  is identified, indicating that heat-related effects play a more significant role in the prediction of thrust forces. The geometric parameter  $\Pi_\alpha$  also presents a moderate correlation with the error. As in the case of the cutting force, the influence of the contact length  $\Pi_{L_c}$  remains limited.

In terms of cutting conditions, the model demonstrates higher predictive accuracy for both force components under moderate to high uncut chip thickness ( $t_1$ ) and cutting width ( $w$ ), as well as for higher absolute rake angles. Larger deviations are observed for small  $t_1$  values and low absolute rake angles, where frictional effects and nonlinear tool–chip interactions become more pronounced. While the cutting speed ( $V_c$ ) does not significantly affect the prediction error for the cutting force, its indirect influence through thermal effects becomes more relevant in the case of the thrust force. Overall, the results indicate that

the proposed model is robust under conventional machining conditions, but its accuracy decreases in regimes dominated by frictional and thermal nonlinearities.

### 3.4. Sensitivity Analysis Results

The sensitivity analysis clarifies the relative importance of the governing mechanisms in the proposed  $\Pi$ -based models. Since the models are formulated in dimensionless form, the fitted exponents directly represent the sensitivity of the normalized force to each governing group. For the cutting-force model, the largest sensitivities correspond to the contact-length ratio  $\Pi_{L_c}$  and the thermal Peclet number  $\Pi_{Pe}$ , showing that contact mechanics and thermomechanical coupling dominate the response. The dynamic group  $\Pi_V$  has a secondary influence, whereas the rake-angle term  $\Pi_\alpha$  mainly affects chip flow and force partitioning. For the thrust-force model, the sensitivities are smaller overall, with  $\Pi_\alpha$  providing the largest contribution. This indicates that thrust force is more strongly governed by tool geometry, while contact length, dynamic effects, and thermal transport have more limited influence within the analyzed range. Overall, the sensitivity analysis supports the interpretation of orthogonal cutting as a coupled process controlled primarily by contact, thermal, and geometrical effects.

## 4. Discussion

The proposed  $\Pi$ -based formulation provides a physically consistent framework for interpreting force generation in orthogonal cutting. The sensitivity analysis indicates that contact mechanics and thermomechanical coupling dominate the scaling of cutting forces, while tool geometry primarily governs force partitioning, particularly for the thrust component.

It should be emphasized that the proposed model is primarily explanatory in nature, aiming to reveal physically meaningful relationships between dimensionless groups and cutting forces. Therefore, the objective is not to maximize predictive accuracy, but to provide a transparent and physically grounded framework that supports interpretation and transferability across different machining conditions.

The strong influence of the contact-length ratio  $\Pi_{L_c}$  highlights the central role of tool–chip interaction in controlling frictional dissipation and stress distribution along the rake face. However, its limited correlation with prediction error suggests that, although global scaling is well captured, local variations associated with contact conditions remain only partially resolved.

Thermal effects, represented by the Peclet number  $\Pi_{Pe}$ , play a critical role in force scaling, supporting the interpretation of machining as a thermoviscoplastic process governed by localized heating and strain-rate effects. In contrast, dynamic effects represented by  $\Pi_V$  contribute as a secondary mechanism within the analyzed range.

The observed heteroscedasticity and heavy-tailed residuals reflect the intrinsic variability of literature-compiled machining datasets and the coexistence of multiple cutting regimes. These features indicate that the reduced  $\Pi$  set captures the dominant scaling behavior while leaving secondary mechanisms, such as friction variability and temperature-dependent material response, unresolved. Overall, the formulation achieves a balance between physical interpretability and predictive capability, providing robust scaling laws across heterogeneous machining conditions.

### 4.1. Interpretation of the Governing $\Pi$ Groups

Within the proposed formulation, the contact-length ratio  $\Pi_{L_c} = L_c/t_1$  emerges as a key descriptor of the tool–chip interaction scale. An increase in  $L_c$  at fixed  $t_1$  enlarges the contact region and modifies the stress distribution along the rake face, directly affecting

frictional dissipation and the partition between cutting and thrust forces, in agreement with classical orthogonal cutting models.

In the present work,  $L_c$  was estimated using the model of Zhang et al. [19], modified by a correction factor  $\eta$  to account for systematic deviations. Previous studies have reported that Zhang's model tends to overestimate  $L_c$  [41,42], and accordingly the best agreement for  $F_c$  is obtained for  $\eta < 1$ .

In contrast, for  $F_t$ , the best agreement is obtained for  $\eta \gg 1$ , indicating that the thrust-force model underestimates the actual force levels. Larger values of  $\eta$  compensate for additional contact effects not captured by the original formulation, particularly those associated with the tertiary contact zone and increased tool–workpiece interaction due to flank wear [43,44].

This formulation introduces an explicit dependence of  $L_c$  on cutting speed, uncut chip thickness, and rake angle. While this enables a consistent treatment of literature data, any bias or limitation in the estimation of  $L_c$  may propagate into  $\Pi_{L_c}$  and influence the fitted exponents.

Thermal effects are captured by the Peclet number  $\Pi_{Pe} = V_c t_1 / \alpha_{th}$ , which compares advective heat transport with thermal diffusion. High values of  $\Pi_{Pe}$  are associated with thermal localization and limited heat dissipation, leading to temperature rises that promote thermal softening and modify the effective flow stress. The strong sensitivity of  $\Pi_F$  to  $\Pi_{Pe}$  therefore supports the interpretation of orthogonal cutting as a coupled thermomechanical process dominated by localized heating and severe plastic deformation.

The inertial group  $\Pi_V = \rho V_c^2 / \sigma_y$  represents the ratio between dynamic stress and material flow strength. Although inertia is not dominant in conventional machining, this group captures strain-rate effects and dynamic trends associated with cutting speed. Its moderate influence compared to  $\Pi_{L_c}$  and  $\Pi_{Pe}$  indicates that dynamic effects act as a secondary scaling mechanism.

Finally, the rake angle  $\Pi_\alpha$  accounts for tool geometry effects on chip flow, shear plane orientation, and stress distribution at the interface. Its comparatively lower sensitivity is consistent with its role in modulating force partitioning rather than defining the overall force scale. Nevertheless, its inclusion is essential to preserve physical interpretability and to prevent geometry-induced effects from being absorbed into other fitted parameters.

#### 4.2. Residual Structure, Heteroscedasticity, and Tail Behavior

The diagnostic plots show that residuals are approximately centered, indicating limited global bias, while the error variance increases with predicted force, revealing heteroscedasticity. This behavior is expected in literature-compiled machining datasets, where high-force conditions are associated with large chip thickness, extended contact length, and thermal regimes that amplify nonlinearities in material response and friction.

The normal probability plots exhibit heavy-tailed residuals, with pronounced deviations in the extremes. These tails likely arise from the coexistence of multiple cutting regimes and from unobserved variables not included in the present  $\Pi$  set.

Two main sources of variability are identified. First, friction conditions at the tool–chip interface are rarely reported consistently and may vary with tool material, surface condition, or lubrication history. Second, the effective flow stress depends on strain, strain rate, and temperature, whereas the present normalization uses a constant yield strength  $\sigma_y$ . While appropriate for a compact similarity framework, this simplification cannot capture all thermoviscoplastic variations.

Therefore, heavy-tailed residuals should be interpreted as evidence that the reduced  $\Pi$  set captures dominant scaling while leaving secondary mechanisms unresolved. The observed heteroscedasticity explains the moderate RMSE and MAE values, particularly for the

cutting force, where large errors concentrate at high force levels. Nevertheless, the absence of systematic bias indicates that the models remain statistically consistent, with errors driven primarily by regime-dependent variability rather than structural deficiencies.

#### 4.3. Implications for Model Transferability and Practical Use

From a practical standpoint, the  $\Pi$ -based formulation provides two main advantages. First, it establishes a physically interpretable hierarchy of mechanisms: tool–chip contact and thermal transport dominate uncertainty propagation, while dynamic and geometric effects contribute with lower relative weight. This ranking can guide experimental design, emphasizing the importance of accurately characterizing contact length and thermal conditions.

Second, the dimensionless representation enables comparison of experiments performed under different cutting conditions, facilitating the integration of multi-source datasets and the transfer of models across machines and laboratories.

However, the results indicate that the present  $\Pi$  set is more effective at capturing global trends than at minimizing prediction error in extreme regimes. This trade-off is inherent to compact similarity models and suggests that applications requiring high accuracy at the extremes may require either regime-dependent calibration or the inclusion of additional dimensionless descriptors.

The reported statistical metrics should therefore be interpreted in light of the modeling objective. Unlike highly calibrated models developed for specific datasets, the present formulation prioritizes physical scaling and transferability. In this context,  $R^2$  values in the range 0.4–0.55 reflect a robust and physically meaningful framework rather than limited predictive capability. This behavior is consistent with models that prioritize physical interpretability over purely empirical fitting, where higher predictive accuracy is often achieved at the expense of generality and physical insight.

#### 4.4. Limitations and Future Work

This study presents several limitations that motivate future work. The dataset is compiled from the literature, introducing variability in experimental conditions and reporting practices. The tool–chip contact length was estimated using a published correlation [19], which may introduce systematic uncertainty outside its original validity range. Additionally, constant thermophysical properties were assumed, whereas machining involves strong temperature and strain-rate dependence.

Future work may focus on: (i) incorporating a friction-related dimensionless group based on measured or estimated interface conditions; (ii) replacing the constant strength scale  $\sigma_y$  with an effective flow stress derived from constitutive models such as Johnson–Cook; and (iii) introducing regime segmentation to reduce heteroscedasticity and improve accuracy in extreme conditions. Validation using controlled experiments with direct measurement of contact length and friction would further strengthen the physical interpretation of  $\Pi_{L_c}$ .

### 5. Conclusions

This study developed predictive models for cutting and thrust forces in orthogonal cutting of AISI 1045 steel using a dimensional analysis framework based on the Buckingham  $\Pi$  theorem. The main findings and contributions are summarized as follows:

- A compact and physically interpretable formulation was obtained by expressing the forces in terms of a reduced set of dimensionless groups, namely  $\Pi_{L_c}$ ,  $\Pi_\alpha$ ,  $\Pi_V$ , and  $\Pi_{Pe}$ , demonstrating the effectiveness of dimensional analysis for simplifying complex machining phenomena.
- The proposed  $\Pi$ -based models successfully capture the global scaling behavior of both cutting and thrust forces across a wide range of machining conditions, despite

moderate statistical dispersion ( $R^2 = 0.4\text{--}0.55$ ), which is consistent with the variability of literature-compiled datasets.

- Sensitivity analysis identified the contact-length ratio  $\Pi_{L_c}$  and the thermal Peclet number  $\Pi_{Pe}$  as the most influential dimensionless groups, highlighting the dominant role of tool–chip contact mechanics and thermomechanical coupling in force generation. The rake angle  $\Pi_\alpha$  showed a significant influence on force partitioning, particularly for the thrust force, while the dynamic group  $\Pi_V$  contributed as a secondary scaling parameter.
- In terms of physical variables, the most influential parameters were the tool–chip contact length  $L_c$ , the cutting speed  $V_c$ , and the uncut chip thickness  $t_1$ , which together govern frictional interaction, thermal effects, and deformation intensity. Tool geometry, represented by the rake angle  $\alpha$ , was found to play a key role in modulating force components.
- Residual analysis revealed heteroscedasticity and heavy-tailed distributions, indicating increased prediction error under high-force regimes and the presence of multiple cutting conditions. These features confirm that the proposed formulation captures dominant scaling while leaving secondary mechanisms, such as friction variability and temperature-dependent material behavior, partially unresolved.
- The use of logarithmic transformation improved numerical stability and linearity in the regression process, enabling robust parameter estimation and consistent model fitting in  $\Pi$ -space.
- Model validation using independent experimental data, distinct from those used for model calibration, demonstrated the predictive capability and robustness of the proposed formulation. The optimization of the correction factor  $\eta$  further confirmed the importance of accurately representing tool–chip contact length in force prediction.
- Overall, the dimensional analysis approach proved to be an effective and scalable methodology for modeling machining forces, providing a balance between physical interpretability and predictive capability. The framework is suitable for integration of heterogeneous datasets and for extension to other materials and cutting configurations.
- Future work should focus on incorporating additional dimensionless descriptors related to friction and temperature-dependent material behavior, as well as on developing regime-dependent formulations to improve accuracy under extreme cutting conditions.

## 6. Use of Generative Artificial Intelligence

Generative artificial intelligence tools were used to assist in the organization of the manuscript text and in the linguistic refinement of the presentation. The scientific content, data analysis, modeling methodology, interpretation of results, and conclusions were developed and validated by the authors. No generative artificial intelligence tools were used to generate experimental data or to replace scientific reasoning.

**Author Contributions:** Conceptualization, F.R.-P.; methodology, F.R.-P. and J.C.P.; validation, E.L.; formal analysis, J.C.P.; investigation, F.R.-P.; data curation, J.C.P.; writing—original draft preparation, F.R.-P.; writing—review and editing, J.C.P. and F.R.-P. All authors have read and agreed to the published version of the manuscript.

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**Data Availability Statement:** Data are contained within the article. All data used for model development and validation are available in the references cited throughout the manuscript. The data

supporting the findings of this study can be obtained directly from the original sources referenced in the article.

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