

Article

Spatiotemporal Cooperative Guidance Law with Singularity-Free Obstacle Avoidance for Multiple Flight Vehicles

Shaojie Luo ¹, Le Wang ^{1,2}, Jianxiang Xi ^{1,*}, Mingxing Qin ¹ and Liyu Song ¹

¹ College of Missile Engineering, Rocket Force University of Engineering, Xi'an 710025, China; luosj06@163.com (S.L.); wangaz14@163.com (L.W.); starsnow20@mail.nwpu.edu.cn (M.Q.); songly712@163.com (L.S.)

² College of Aerospace Science and Technology, Beijing Institute of Technology, Beijing 100081, China

* Correspondence: xijx07@mails.tsinghua.edu.cn

Abstract

This paper develops a distributed cooperative guidance law for the spatiotemporal cooperative arrival of underactuated flight vehicles with uncontrollable axial acceleration under obstacle avoidance constraints. First, a singularity-free obstacle avoidance guidance law is proposed based on a linear projection function to avoid singularity-induced surges in acceleration commands. The proposed law has a simple structure and bounded magnitude, and it ensures that the flight vehicles safely avoid obstacle regions. Then, error dynamics theory is adopted to design a cooperative guidance law for arrival angle control and arrival time synchronization, which guarantees that the coordination errors of the flight vehicles converge to zero before reaching the target. Moreover, a buffer zone is constructed around each obstacle, which provides a distance-dependent transition region. Accordingly, a continuous and smooth weighting function is designed to shift the guidance priority from cooperative guidance to obstacle avoidance, thereby avoiding abrupt jumps in acceleration commands during task switching. Finally, the effectiveness of the proposed guidance law is verified through numerical simulations in typical scenarios and Monte Carlo experiments.

Keywords: cooperative guidance; spatiotemporal constraints; obstacle avoidance; singularity-free; underactuated flight vehicles

1. Introduction

With the development of aerospace science and technology, a single flight vehicle may no longer satisfy the requirements of complex missions. Benefiting from the advantages of scalability and flexible configurations, multiple flight vehicle cooperation can accomplish various missions efficiently, which has attracted widespread attention from researchers [1,2]. In practical flight missions, flight vehicles may be subject to disturbances and uncertainties. Under these conditions, robust control of flight vehicles has been extensively studied, with various intelligent and adaptive strategies developed to enhance tracking performance and disturbance rejection capabilities [3–6]. Nevertheless, in complex scenarios, flight vehicles are inevitably exposed to multiple threats, including terrain obstacles and no-fly zones. Hence, cooperative obstacle avoidance is required, whereby

Received: 29 May 2026

Revised: 1 August 2026

Accepted: 6 August 2026

Published: 7 August 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

flight vehicles evade such threat regions through detection, perception, and autonomous decision-making to ensure flight safety [7,8]. However, due to limited sensing ranges and communication capabilities, it is difficult for flight vehicles to acquire global information on obstacles and individual vehicle states during obstacle avoidance. Moreover, complex aerodynamic uncertainties and external disturbances pose challenges for the control of vehicles in such scenarios [9,10]. Accordingly, it is challenging to achieve cooperative obstacle avoidance.

Considerable research has been conducted to address the problem of cooperative obstacle avoidance for flight vehicles. On the one hand, some studies adopt optimal control methods, in which obstacle avoidance is formulated as a constraint to generate optimal acceleration commands [11–14]. On the other hand, the artificial potential field (APF) method has been applied, which can establish a repulsive potential field around obstacles to realize obstacle avoidance [15–17]. Compared with optimal control methods, the APF method has low computational complexity and enables online generation of obstacle avoidance acceleration commands, which has been widely adopted in cooperative obstacle avoidance. In ref. [18], a distance-based APF function was adopted to design the sliding mode surface, which enabled flight vehicles to generate repulsive force for obstacle avoidance. Ref. [19] further addressed the trajectory oscillation problem by designing an adaptive artificial potential field algorithm, enabling flight vehicles to achieve obstacle avoidance along curved paths. Note that the APF-based obstacle avoidance strategies proposed in refs. [15–19] constructed repulsive potential functions to generate concise acceleration commands for obstacle avoidance. Nevertheless, such repulsive functions may introduce singularity problems, which can lead to unsolvable acceleration commands or actuator input saturation. Accordingly, it is valuable to develop an obstacle avoidance approach that can efficiently generate bounded nonsingular acceleration commands online.

Consequently, while the requirement of obstacle avoidance is fulfilled, flight vehicles are also expected to perform cooperative guidance. For this purpose, spatiotemporal cooperative guidance laws have been developed, which allow flight vehicles to reach the target simultaneously at preset arrival angles [20]. Consensus theory provides an important foundation for distributed cooperative control of multiagent systems. In this context, refs. [21,22] investigated the distributed consensus tracking problem of multiagent systems under communication-network constraints, providing useful insights into the influence of communication topology on the convergence of coordination errors. To adapt to dynamic environments, refs. [15–18] adopted consensus-based coordination mechanisms to adjust flight vehicle states online via communication networks among vehicles, thereby accomplishing spatiotemporal cooperative guidance. To further address the multi-target arrival problems, ref. [23] proposed a grouped cooperative guidance law, which enabled different subgroups to achieve cooperative arrival at multiple targets with preset times. Ref. [24] extended the research to the engagement of maneuvering targets and developed a cooperative guidance law considering dynamic communication topologies. Note that the flight vehicle dynamics in the above studies are assumed to be fully actuated such that the axial acceleration of each vehicle is controllable. However, only the lateral acceleration of the flight vehicle can be controlled in many practical situations [25], so it is necessary to explore the spatiotemporal cooperation approach for the above-mentioned underactuated model.

To address cooperative guidance with obstacle avoidance, the APF method and multi-agent consensus control were adopted in ref. [26], and a distributed cooperative guidance law based on proportional navigation was proposed, which effectively addressed these problems. Along this line, ref. [27] designed a piecewise continuous repulsive potential field function, which resolved the abrupt jumps of acceleration commands

during task transition. In contrast, ref. [28] developed a threshold-based numerically continuous weighting function to achieve switching between cooperation and obstacle avoidance. Furthermore, ref. [29] proposed a distance-based weighting function with both numerical continuity and first-order derivative continuity and designed a predefined-time cooperative guidance law for a fully actuated model to ensure that coordination errors converge to zero within a specified time. It should be emphasized that besides achieving continuity of the weighting function as shown in the above-mentioned works, smooth transition of acceleration commands at task switching boundaries between cooperation and obstacle avoidance is equally essential. Overall, although scholars have explored some cooperative guidance laws that satisfy the obstacle avoidance constraint, several challenges still persist. For one thing, the design of weighting functions to achieve smoother switching at the boundaries between cooperation and obstacle avoidance requires further exploration. For another, how to realize cooperation and obstacle avoidance through bounded nonsingular normal acceleration remains an essential open problem.

Inspired by the above works, this paper develops a spatiotemporal cooperative guidance method with obstacle avoidance constraints, where the axial acceleration of flight vehicles is regarded as uncontrollable, and a directed communication network is adopted among the vehicles. Firstly, a linear projection function is developed to evaluate collision risk, which is adopted in the design of the obstacle avoidance guidance law. Subsequently, a nonsingular spatiotemporal cooperative guidance law is proposed, where the error convergence rate is guaranteed by the error dynamics theory. Finally, a smooth weighting function with a concise form is designed for the construction of guidance laws. The main contributions are as follows:

(1) A linear projection function is developed to evaluate collision risk and construct the obstacle avoidance guidance law. Compared with the APF-based methods in refs. [26–29], the proposed obstacle avoidance command contains no singular distance-related denominator and remains bounded when the flight vehicle approaches the obstacle safety boundary. Thus, excessive control inputs caused by rigid obstacle avoidance constraints can be avoided.

(2) A spatiotemporal cooperative guidance law is proposed for flight vehicles with uncontrollable axial acceleration, which can guarantee the convergence of arrival time and arrival angle errors before reaching the target. Compared with ref. [25], this paper further incorporates arrival angle control. Moreover, ref. [29] realizes spatiotemporal cooperation under the assumption that the axial acceleration of each vehicle is controllable, whereas the proposed guidance law only requires normal acceleration commands.

(3) A buffer-zone-based smooth weighting mechanism is designed to fuse obstacle avoidance and cooperative guidance commands. Compared with the switching mechanisms in refs. [28,29], the proposed cubic weighting function preserves value continuity and first-order derivative continuity, with its derivative approaching zero at the switching boundaries. Therefore, the proposed mechanism enables a smoother acceleration command transition when the flight vehicle crosses the boundary of the obstacle avoidance transition region, without introducing abrupt jumps caused by task switching.

To further clarify the distinctions between the proposed method and the most related recent works, a comparison is provided in Table 1.

Table 1. Comparisons with existing methods.

Comparison	Ref. [28]	Ref. [29]	Proposed Method
Fusion function	Value continuity	Value continuity	Value continuity and first-order derivative continuity
Obstacle avoidance strategy	APF-based avoidance with potential singularity	Distance-dependent avoidance with potential singularity	Linear projection-based avoidance without singularity

Axial acceleration assumption	Uncontrollable	Controllable	Uncontrollable
Cooperative performance	Temporal cooperative	Spatiotemporal cooperative	Spatiotemporal cooperative
Convergence property	Asymptotic convergence	Predefined-time convergence	Finite-time convergence before target arrival

The remainder of this paper is arranged as follows. The communication topology, necessary lemmas, and problem statement are presented in Section 2. Section 3 provides a spatiotemporal cooperative guidance law with obstacle avoidance and its associated proof process. In Section 4, the numerical simulation results are presented. Section 5 summarizes the conclusions.

2. Preliminaries and Problem Statement

This section first presents the relevant lemmas for graph theory and guidance law design. Then, a kinematic model for the cooperative arrival of underactuated flight vehicles to the target is established.

2.1. Preliminaries

Consider an underactuated flight vehicle swarm comprising n vehicles, and its communication network is characterized by the topological graph $\mathcal{G} = \{V, E, A\}$. Let $V = [v_1, v_2, \dots, v_n]$ denote the node set of the topological graph, with each node representing an individual flight vehicle in the swarm. $E \subseteq \{(v_i, v_j) : v_i, v_j \in V\}$ represents the edge set of the graph, which can describe the communication links among the vehicles. $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is the adjacency matrix. For any two nodes v_i and v_j , if $(v_i, v_j) \in E$, then node v_i can receive information transmitted from node v_j , in which case the element $a_{ij} = 1$; otherwise, $a_{ij} = 0$. Let the Laplacian matrix of graph $\mathcal{G}$ be $L = [l_{ij}] \in \mathbb{R}^{n \times n}$, where the element l_{ij} is given by $l_{ij} = \sum_{j=1}^n a_{ij}$ for $i = j$, and $l_{ij} = -a_{ij}$ otherwise. It is assumed that the topological graph $\mathcal{G}$ is a directed graph and contains at least one directed spanning tree.

Lemma 1 [30]. For the graph $\mathcal{G}$, there exists a positive vector $p = [p_1, p_2, \dots, p_n]^T$ and matrix $P = \text{diag}\{p_1, p_2, \dots, p_n\}$ satisfying $\sum_{i=1}^n p_i = 1$, $p^T L = 0^T$ and $\lambda_{\max}(P - pp^T) \leq 1$, where $\lambda_{\max}(\cdot)$ denotes the maximum singular value. Define $\hat{L} = PL + L^T P$, then it follows that:

$$0 = \lambda_1(\hat{L}) < \lambda_2(\hat{L}) \leq \dots \leq \lambda_n(\hat{L}), \quad (1)$$

$$\min_{\mu^T \psi = 0} \mu^T \hat{L} \mu > [\lambda_2(\hat{L})/n] \mu^T \mu, \quad (2)$$

where $\mu, \psi \in \mathbb{R}^n$ and ψ is a positive vector, and $\lambda_i(\cdot)$ stands for the i -th eigenvalue.

Lemma 2 [31]. Consider a scalar error $\varepsilon(t)$. If there exists an error dynamics equation such that

$$\dot{\varepsilon}(t) + \frac{C}{t_{go}} \varepsilon(t) = 0, \quad (3)$$

where $C > 0$ and $t_{go} = t_f - t$ with t_f the terminal arrival time, then the error $\varepsilon(t)$ will converge to zero at t_f .

2.2. Problem Statement

This paper focuses on the problem of cooperative arrival at the target for an under-actuated flight vehicle swarm subject to obstacle avoidance constraints. Its 2D planar engagement geometry, in which there exist circular obstacles, is illustrated in Figure 1. Let $M_i (i = 1, 2, \dots, n)$ and T denote the i -th flight vehicle and the target, respectively.

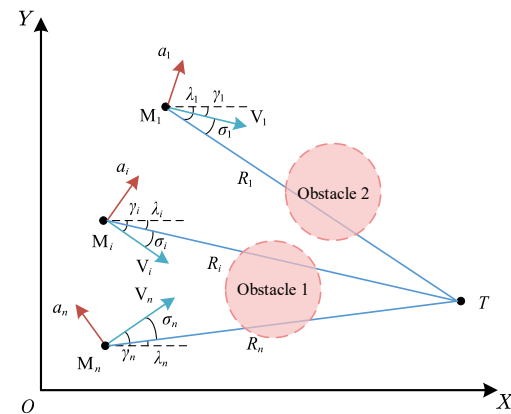


Figure 1. 2D plane engagement geometry.

The relative motion equation between M_i and target T is given by:

$$\dot{R}_i = -V_i \cos \sigma_i, \quad (4)$$

$$\dot{\lambda}_i = -\frac{V_i \sin \sigma_i}{R_i}, \quad (5)$$

$$\dot{\gamma}_i = \frac{a_i}{V_i}, \quad (6)$$

$$\sigma_i = \gamma_i - \lambda_i, \quad (7)$$

$$\dot{\sigma}_i = \dot{\gamma}_i - \dot{\lambda}_i = \frac{V_i \sin \sigma_i}{R_i} + \frac{a_i}{V_i}, \quad (8)$$

where R_i denotes the line-of-sight (LOS) range between M_i and T , V_i and a_i stand for the velocity and acceleration of M_i , respectively. In practical gliding flight, the vehicle generates aerodynamic acceleration primarily by adjusting its attitude. Since the vehicle's speed varies only slightly over the considered guidance horizon, the contribution of axial acceleration is relatively small. Therefore, it is reasonable to regard the resulting acceleration as predominantly normal to the velocity vector [32]. σ_i , γ_i and λ_i represent the leading angle, flight path angle, and line-of-sight angle of M_i , respectively. Note that the positive direction of each angle is defined in accordance with the right-hand rule.

The objective of this paper is to design a distributed cooperative guidance law, which enables the flight vehicles to comply with spatiotemporal cooperation and obstacle avoidance constraints. To this end, a guidance law is proposed in the following form.

$$a_i = \omega_{ik} a_i^{\text{ob}} + (1 - \omega_{ik}) a_i^{\text{coop}}, \quad (9)$$

where $\omega_{ik} \in [0, 1]$ denotes the weighting coefficient, a_i^{ob} and a_i^{coop} represent the obstacle avoidance guidance law and the cooperative guidance law, respectively, which are employed to achieve obstacle avoidance and cooperative arrival. For multiple obstacles, only one obstacle is active at each time instant. Let k denote the index of the active obstacle, and the $\omega_{ik} a_i^{\text{ob}}$ in the guidance law (9) is computed with respect to k -th obstacle.

Let $t_0 = 0$, t_i^f and γ_i^d represent the initial time, terminal arrival time, and desired arrival angle of M_i , respectively. Then, for the guidance system depicted in (4)–(9), the objective of the designed guidance law can be described as

$$\begin{cases} t_i^f = t_j^f, i, j \in \{1, 2, \dots, n\}, \\ \gamma_i(t_i^f) = \gamma_i^d, i \in \{1, 2, \dots, n\}, \\ r_{ik} \geq R_{sk}, \forall t \geq 0, i \in \{1, 2, \dots, n\}, k \in \{1, 2, \dots, m\}, \end{cases} \quad (10)$$

where R_{sk} is the safety boundary radius of the k -th circular obstacle, and the required safety margin is incorporated in R_{sk} . Let m denote the number of obstacles, and r_{ik} represent the distance from M_i to the center of the k -th obstacle. In this paper, obstacle avoidance refers to the avoidance of external obstacle areas, and inter-vehicle collision avoidance among flight vehicles is not considered.

3. Design of Cooperative Guidance Law with Obstacle Avoidance

In this section, a linear projection evaluation method is proposed to design the obstacle avoidance guidance law, and a smooth weighting variable is introduced for command fusion. Meanwhile, the spatiotemporal cooperative guidance law is designed to ensure that the flight vehicles can reach the target simultaneously at preset arrival angles.

3.1. Obstacle Avoidance Guidance Law

For the design of the obstacle avoidance guidance law, the geometric relationship between the i -th vehicle and the k -th obstacle is described, as illustrated in Figure 2.

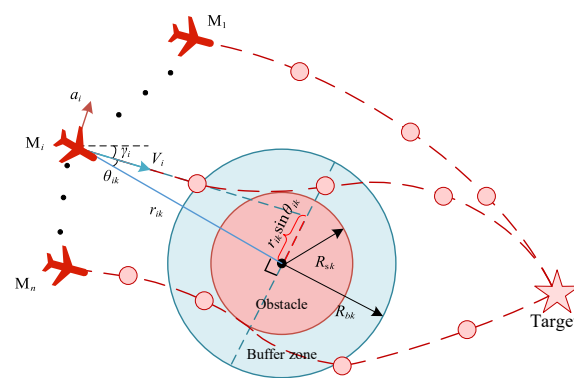


Figure 2. Obstacle avoidance geometry.

As shown in Figure 2, an obstacle avoidance mechanism is proposed according to the defined buffer zone. The radius of the buffer zone is denoted as $R_{bk} = (1 + 1/\rho)R_{sk}$ with $\rho > 0$. To avoid abrupt command changes at the switching boundaries between cooperative guidance and obstacle avoidance, the weighting coefficient of (9) is designed as the following smooth variable

$$\omega_{ik} = \begin{cases} 1, & 0 \leq r_{ik} \leq R_{bk}, \\ 1 - 3\left(\frac{r_{ik} - R_{bk}}{\hat{R}_{bk} - R_{bk}}\right)^2 + 2\left(\frac{r_{ik} - R_{bk}}{\hat{R}_{bk} - R_{bk}}\right)^3, & R_{bk} < r_{ik} \leq \hat{R}_{bk}, \\ 0, & r_{ik} > \hat{R}_{bk}, \end{cases} \quad (11)$$

where $\hat{R}_{bk} > R_{bk}$. Since R_{bk} is related to ρ , the variation of ω_{ik} is affected by the selection of ρ . For multiple obstacles, each obstacle is assigned its own buffer zone and transition region. By properly designing the buffer-zone radius of each obstacle, the avoidance regions can be arranged to avoid overlap, so that obstacle avoidance is triggered sequentially as the flight vehicle enters different transition regions. Figure 3 shows the curves of the weighting coefficient ω_{ik} versus r_{ik} for different selections of ρ , indicating that ω_{ik} can transition smoothly from 0 to 1 and that the transition region expands as ρ increases.

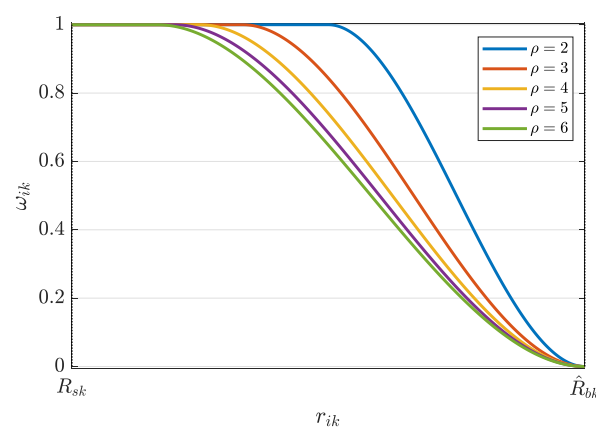


Figure 3. Curves of the weighting coefficient ω_{ik} .

The buffer zone is introduced to provide a transition region between cooperative guidance and obstacle avoidance. The size of the buffer zone has a direct influence on safety, convergence, and control smoothness. A larger buffer zone increases the available maneuvering space before the vehicle reaches the safety boundary, which allows obstacle avoidance to be activated earlier. At the same time, the variation in the weighting coefficient becomes more gradual over a wider transition region, which is beneficial to the smoothness of the acceleration command. However, if the buffer zone is excessively large, obstacle avoidance may be triggered too early so that the cooperative guidance task is interrupted for a longer period, which is unfavorable to the convergence of the cooperative arrival process. In contrast, a smaller buffer zone shortens the transition region, making the switching between cooperation and obstacle avoidance more abrupt, which may degrade command smoothness. Accordingly, ρ determines the inner buffer boundary R_{bk} , while $\hat{R}_{bk}$ determines the extent of the transition region; these two parameters should be selected to achieve a compromise between task-switching smoothness and cooperative-convergence performance.

Remark 1. The proposed buffer zone plays two roles in the guidance framework. First, it provides an advanced response region outside the obstacle safety boundary so that the vehicle can start obstacle avoidance before reaching the forbidden region. Second, it serves as a distance-dependent task-priority transition region between cooperative guidance and obstacle avoidance. Unlike hard switching based on a single distance threshold, the buffer-zone mechanism allows the obstacle avoidance weight to increase continuously as the vehicle approaches the obstacle and decrease

continuously as the vehicle leaves the obstacle. The design of ω_{ik} essentially constructs a continuous cubic Hermite interpolation function satisfying the prescribed boundary conditions with the lowest polynomial degree, which makes $\dot{\omega}_{ik}(R_{bk}) = \dot{\omega}_{ik}(\hat{R}_{bk}) = 0$. Consequently, the obstacle avoidance command and the cooperative guidance command can be blended smoothly without abrupt acceleration jumps caused by task switching.

Let θ_{ik} denote the leading angle between M_i and the k -th obstacle. Then, the following linear projection function is proposed to evaluate the collision risk.

$$h_{ik} = \begin{cases} R_{sk} - |r_{ik} \sin \theta_{ik}|, & |r_{ik} \sin \theta_{ik}| \leq R_{sk}, \\ 0, & |r_{ik} \sin \theta_{ik}| > R_{sk}. \end{cases} \quad (12)$$

The quantity $|r_{ik} \sin \theta_{ik}|$ represents the perpendicular distance from the obstacle center to the current straight-line path of the flight vehicle. Thus, h_{ik} measures the remaining projected clearance with respect to the safety radius R_{sk} , and a larger h_{ik} indicates a higher local collision risk. The physical meaning of h_{ik} is valid for the approaching configuration. For a stationary obstacle, the vehicle-obstacle range rate is

$$\dot{r}_{ik} = -V_i \cos \theta_{ik} \quad (13)$$

Therefore, $\cos \theta_{ik} > 0$ means that the vehicle is moving toward the obstacle, while $\cos \theta_{ik} \leq 0$ means that the vehicle is moving away from the obstacle or that the obstacle is behind the vehicle. Accordingly, the obstacle avoidance risk can be identified by $\cos \theta_{ik}$ and h_{ik} .

When $\theta_{ik} \neq 0$, differentiating (12) with respect to t yields

$$\dot{h}_{ik} = \begin{cases} -\frac{a_i r_{ik} \cos \theta_{ik} \operatorname{sgn}(\theta_{ik})}{V_i}, & |r_{ik} \sin \theta_{ik}| \leq R_{sk}, \\ 0, & |r_{ik} \sin \theta_{ik}| > R_{sk}, \end{cases} \quad (14)$$

where the function $\operatorname{sgn}(x)$ is defined as follows

$$\operatorname{sgn}(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases} \quad (15)$$

When $r_{ik} < \hat{R}_{bk}$, the obstacle avoidance mechanism is enabled, and a_i^{ob} is constructed as

$$a_i^{\text{ob}} = \frac{\rho V_i^2 h_{ik}^\alpha \operatorname{sgn}(\theta_{ik})}{(1 - \alpha) r_{ik} R_{sk}^\alpha}, \quad (16)$$

where $0 < \alpha < 1$.

Remark 2. The linear projection function in (12) provides a bounded geometric measure of collision risk by projecting the vehicle-obstacle relative geometry onto the possible collision direction. Different from APF repulsive functions, this projection-based measure does not rely on inverse-distance terms.

Remark 3. The design of $\operatorname{sgn}(\theta_{ik})$ in (16) enables the flight vehicle to avoid obstacles along more efficient paths with a continuous reduction in h_{ik} , that is, the vehicle will fly along the increasing direction of θ_{ik} for $\theta_{ik} \geq 0$ and along the decreasing direction of θ_{ik} otherwise. Furthermore, it can be concluded that as the buffer zone decreases, the available space for obstacle avoidance

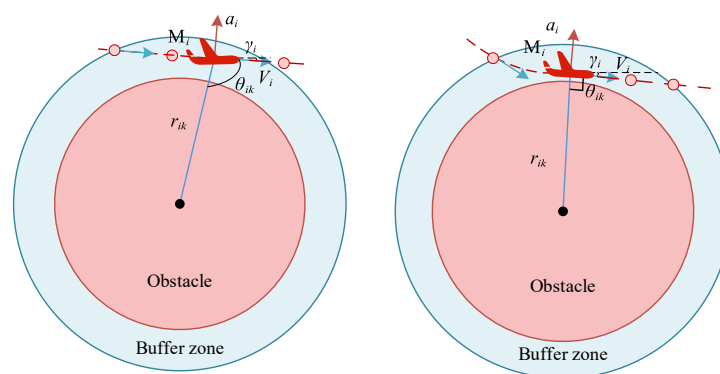
diminishes, and the radius of the buffer zone is related to parameter ρ . Note that the introduction of ρ in (16) enables the obstacle avoidance guidance gain to be adaptively adjusted according to the buffer-zone scale.

According to the weighting function in (11), the cooperative guidance law and the obstacle avoidance guidance law are simultaneously involved only in the transition region $R_{bk} < r_{ik} \leq \hat{R}_{bk}$. When the flight vehicle enters the buffer-zone $r_{ik} \leq R_{bk}$, the weighting coefficient satisfies $\omega_{ik} = 1$, and the actual guidance command is completely determined by the obstacle avoidance guidance law. Therefore, the safety property of the proposed guidance law can be established by analyzing the vehicle behavior under the obstacle avoidance guidance law within the buffer zone.

Theorem 1. For the guidance system depicted in (4)–(9), the flight vehicles can achieve obstacle avoidance, namely $r_{ik} \geq R_{sk}$, $\forall t \geq 0$.

Proof. From the definition of ω_{ik} , when $r_{ik} \leq R_{bk}$, one has $\omega_{ik} = 1$. Hence, the actual guidance command is reduced to $a_i = a_i^{ob}$. Since $R_{sk} < R_{bk}$, the safety boundary has been strictly contained in the buffer zone. Thus, before the constraint $r_{ik} \geq R_{sk}$ could be violated, the cooperative guidance term had already been eliminated from the applied command. The proof can then be reduced to showing that a_i^{ob} prevents the vehicle from crossing the safety boundary. When the flight vehicle approaches the buffer-zone area, there exist two possible scenarios of vehicle-obstacle avoidance illustrated in Figure 4, where the blue arrows denote the flight velocity vectors.

Case 1: If the flight vehicle enters the buffer zone with $h_{ik} = 0$ and $\omega_{ik} = 1$, then $a_i \equiv 0$ holds, which drives the vehicle to fly out of the buffer zone along a straight line to achieve obstacle avoidance.



(a) Obstacle avoidance in Case 1 (b) Obstacle avoidance in Case 2

Figure 4. Obstacle avoidance scenarios.

Case 2: If the flight vehicle enters the buffer zone with $h_{ik} \neq 0$ and $\omega_{ik} = 1$, it yields $a_i = a_i^{ob}$. When $\theta_{ik}(R_{bk}) \neq 0$, since $\dot{r}_{ik} = -V_i \cos \theta_{ik}$, one can obtain

$$\frac{dh_{ik}}{dr_{ik}} = \frac{\rho h_{ik}^\alpha}{(1-\alpha)R_{sk}^\alpha}. \quad (17)$$

Solving the above differential equation yields

$$h_{ik}^{1-\alpha}(r_{ik}) = \frac{\rho}{R_{sk}^\alpha} r_{ik} - \frac{\rho+1}{R_{sk}^{\alpha-1}} + h_{ik}^{1-\alpha}(R_{bk}). \quad (18)$$

Then, it follows that

$$h_{ik}^{1-\alpha}(R_{sk}) = h_{ik}^{1-\alpha}(R_{bk}) - R_{sk}^{1-\alpha} \leq 0, \quad (19)$$

since $h_{ik} \geq 0$, it holds that $h_{ik}(R_{sk}) = 0$ and $\theta_{ik}(R_{sk}) = \pm \pi/2$, which indicates that the flight vehicle can avoid obstacles effectively.

When $\theta_{ik}(R_{bk}) = 0$, it can be obtained that

$$\dot{\theta}_{ik} = \frac{\rho V_i h_{ik}^\alpha}{(1-\alpha)r_{ik}R_{sk}^\alpha} \geq 0. \quad (20)$$

In this case, the linear projection function and its derivative can be written in the following form:

$$h_{ik} = \begin{cases} R_{sk} - r_{ik} \sin \theta_{ik}, & r_{ik} \sin \theta_{ik} \leq R_{sk}, \\ 0, & r_{ik} \sin \theta_{ik} > R_{sk}, \end{cases} \quad (21)$$

$$\dot{h}_{ik} = \begin{cases} -\frac{a_i r_{ik} \cos \theta_{ik}}{V_i}, & r_{ik} \sin \theta_{ik} \leq R_{sk}, \\ 0, & r_{ik} \sin \theta_{ik} > R_{sk}. \end{cases} \quad (22)$$

One can further obtain that the form of dh_{ik}/dr_{ik} is consistent with (17), which implies that obstacle avoidance is guaranteed for $\theta_{ik}(R_{bk}) = 0$.

At the time of closest approach, the above result gives $h_{ik} = 0$. Based on the definition of h_{ik} and the boundary condition established above, the closest point between the flight vehicle and the obstacle satisfies $\theta_{ik}(R_{sk}) = \pm \pi/2$. Thus, there is no collision threat at the time of closest approach.

Overall, the flight vehicle can achieve obstacle avoidance under the effect of the obstacle avoidance guidance law (16) both in Case 1 and Case 2. $\square$

Remark 4. Theorem 1 proves that the flight vehicle can achieve obstacle avoidance, namely $r_{ik} \geq R_{sk}$. According to $0 \leq h_{ik} \leq R_{sk}$, the proposed obstacle avoidance command satisfies $|a_i^{\text{ob}}| \leq \rho V_i^2 / ((1-\alpha)R_{sk})$. This conservative upper bound can be adjusted through the parameters ρ and α . It should be noted that the obstacle avoidance guidance laws in refs. [26,29] contain the term $r_{ik} - R_{sk}$ in the denominator, which may approach infinity when the flight vehicle approaches obstacles. In contrast, the design of a_i^{ob} in (16) contains no such singular terms. Hence, the proposed design avoids singularity-induced unbounded obstacle avoidance commands and provides a tunable conservative upper bound to reduce the risk of actuator saturation.

3.2. Spatiotemporal Cooperative Guidance Law

In this paper, arrival angle control and simultaneous target arrival are realized through the spatiotemporal cooperative guidance law in (9), which is designed as follows.

$$a_i^{\text{coop}} = a_i^{\text{IACG}} + a_i^{\text{BT}}, \quad (23)$$

where a_i^{IACG} and a_i^{BT} denote the optimal arrival angle control guidance law and the temporal cooperative guidance law, respectively. These two terms are used to achieve arrival angle control and arrival time cooperation. The basic form of a_i^{IACG} is given by

$$a_i^{\text{IACG}} = NV_i \dot{\lambda}_i - \frac{K(N-1)V_i}{t_{\text{go},i}^{\text{IACG}}} e_{\gamma,i}, \quad (24)$$

where N and K are two positive constants satisfying $N \geq 2$, $K \geq 1$, $N + K > 3$ and $N \neq K + 1$. The parameter N denotes the proportional-navigation gain, which dictates the convergence rate of the line-of-sight (LOS) angle. The parameter K represents the angle-control gain governing the convergence rate of the angle error. Larger guidance gains improve the convergence rate of the associated errors while simultaneously increasing the required acceleration magnitude. $t_{go,i}^{IACG}$ and $e_{\gamma,i}$ respectively denote the time-to-go before terminal arrival and the error of arrival angle, as follows:

$$t_{go,i}^{IACG} = -\frac{R_i}{\dot{R}_i}, \quad (25)$$

$$e_{\gamma,i} = \gamma_i^d - \frac{N\lambda_i}{N-1} + \frac{\gamma_i}{N-1}. \quad (26)$$

With the optimal arrival angle control guidance law given by (24), the time-to-go of M_i can be estimated by [33]

$$t_{go,i} = \frac{R_i}{V_i} \left[1 + \frac{\sigma_i^2}{2(2N-1)} - \frac{K(N-1)\sigma_i e_{\gamma,i}}{(2N-1)(N+K)} + \frac{K^2(N-1)^2 e_{\gamma,i}^2}{(2N-1)(N+K)(2K+1)} \right], \quad (27)$$

in this case, the arrival time of M_i is estimated as

$$t_i^f = t_{go,i} + t. \quad (28)$$

In the sequel, the temporal cooperative guidance law a_i^{BT} is designed. Differentiating (28) with respect to t and substituting (23) into the derived expression yields that

$$\dot{t}_i^f = \frac{NR_i \xi_i}{(2N-1)(N+K)V_i^2} a_i^{BT}, \quad (29)$$

where

$$\xi_i = \sigma_i - \frac{K(N-1)}{N(2K+1)} e_{\gamma,i}. \quad (30)$$

Define the arrival time error $e_i = \sum_{j=1}^n a_{ij} (t_i^f - t_j^f)$. Then, the temporal cooperative guidance law a_i^{BT} is proposed as

$$a_i^{BT} = -\frac{\kappa_1(2N-1)(N+K)V_i^2}{NR_i \xi_i t_{go,i}} e_i - \beta e_i \operatorname{sgn}(\xi_i), \quad (31)$$

where $\kappa_1 \geq 1$ and $\beta > 0$. The parameter κ_1 acts as the temporal coordination gain and determines the convergence rate of the arrival time synchronization error. The parameter β contributes to avoiding complete singularity and accelerates the convergence of the arrival time errors. In general, larger values of these gains accelerate the error convergence but may increase the control effort and degrade command smoothness. It can be observed from (31) that a_i^{BT} becomes singular when $\xi_i = 0$. To avoid the singularity in (31), an auxiliary function is designed in the following form

$$\chi(\bar{\xi}_i) = \begin{cases} |\bar{\xi}_i|^{\kappa_2}, & |\bar{\xi}_i| \leq 1, \\ 1, & |\bar{\xi}_i| > 1, \end{cases} \quad (32)$$

where $\bar{\xi}_i = \xi_i/\xi_{\max}$, $\kappa_2 \geq 2$. Then, the nonsingular temporal cooperative guidance law a_i^{BT} can be reconstructed as

$$a_i^{\text{BT}} = - \frac{\kappa_1 (2N-1) (N+K) V_i^2 \chi(\bar{\xi}_i)}{NR_i \xi_i t_{\text{go},i}} e_i - \beta e_i \text{sgn}(\xi_i) \quad (33)$$

Consequently, the complete form of the spatiotemporal cooperative guidance law can be described as

$$a_i^{\text{coop}} = NV_i \dot{\lambda}_i - \frac{K(N-1) V_i}{t_{\text{go},i}^{\text{IACG}}} e_{\gamma,i} - \frac{\kappa_1 (2N-1) (N+K) V_i^2 \chi(\bar{\xi}_i)}{NR_i \xi_i t_{\text{go},i}} e_i - \beta e_i \text{sgn}(\xi_i). \quad (34)$$

It should be noted that the introduction of $t_{\text{go},i}$ in (33) plays a vital role in guaranteeing the convergence rate of arrival time errors. Let T and t_1 denote the actual convergence time of arrival time errors and the time instant at which the flight vehicles depart from the buffer zone, respectively. If the complete singularity holds, namely, there exists $t_0 \in [t_1, T)$ satisfying $\xi_i(t) \equiv 0$ for $t \in [t_0, T)$, then $\chi(\bar{\xi}_i)/t_{\text{go},i} \equiv 0$ is deduced.

To ensure the convergence rate of the nonsingular temporal cooperative guidance law (33), the following theorem is presented.

Theorem 2. For the guidance system described in (4)–(9), adopting the auxiliary function structured as (32) ensures that the temporal cooperative guidance law (33) avoids singularity and eliminates the complete singularity phenomenon, i.e., there does not exist any $t_0 \in [t_1, T)$ such that $\xi_i(t) = 0, \dot{\xi}_i(t) \equiv 0$ hold on $t \in [t_0, T)$.

Proof. Setting $\xi_i(t) = 0$, one can obtain that

$$(2NK + N - K) \gamma_i(t) = K(N-1) \gamma_i^d + (NK + N) \lambda_i(t), \quad (35)$$

thus, $\xi_i(t) = 0$ if and only if $\gamma_i(t) = \lambda_i(t) = \gamma_i^d$.

If there exists $t_0 \in [t_1, T)$ satisfying $\gamma_i(t_0) = \lambda_i(t_0) = \gamma_i^d$, then $a_i(t_0) = -(1 - \omega_{ik}) \beta e_i$, and it can be further obtained that

$$\begin{aligned} \dot{\xi}_i(t_0) &= \dot{\gamma}_i(t_0) - \dot{\lambda}_i(t_0) + \frac{K(N-1)}{N(2K+1)} \left(\frac{N\dot{\lambda}_i(t_0)}{N-1} - \frac{\dot{\gamma}_i(t_0)}{N-1} \right) \\ &= - \left(1 - \frac{K}{2NK+N} \right) (1 - \omega_{ik}) \frac{\beta e_i}{V_i}. \end{aligned} \quad (36)$$

Therefore, there exists no time instant $t_0 \in [t_1, T)$ satisfying $\xi_i(t) = 0, \dot{\xi}_i(t) \equiv 0$ for $t \in [t_0, T)$, namely the temporal cooperative guidance law is free from complete singularity before arrival time errors converge. $\square$

Remark 5. If there exists $t_0 \in [t_1, T)$ such that $\xi_i(t) = 0, \dot{\xi}_i(t) \equiv 0$ for $t \in [t_0, T)$, then $\chi(\bar{\xi}_i)/t_{\text{go},i} \equiv 0, \forall t \in (t_0, +\infty)$ holds, and the first term of the temporal cooperative guidance law may be ineffective. Note that the $\text{sgn}(\xi_i)$ function defined in (15) ensures that $\dot{\xi}_i \neq 0$ when $\xi_i = 0$, thereby avoiding complete singularity and maintaining the effectiveness of the temporal cooperative guidance law.

Based on the error dynamics theory, the convergence of coordination errors in the guidance system is analyzed in the following theorem.

Theorem 3. For the guidance system depicted in (4)–(9), the arrival time errors of the flight vehicles will converge to zero before reaching the target under the effect of the guidance law (34).

Proof. A Lyapunov function is constructed as follows.

$$V = \frac{1}{2} (Le)^T P (Le), \quad (37)$$

with P defined in Lemma 1 and $e = [e_1, e_2, \dots, e_n]^T$.

During the smooth switching process, the flight vehicle is located in the transition region between cooperative guidance and obstacle avoidance; the weighting coefficient satisfies $0 < \omega_{ik} < 1$. The switched guidance command is

$$a_i = a_i^{\text{IACG}} + (1 - \omega_{ik}) a_i^{\text{BT}} + \omega_{ik} (a_i^{\text{ob}} - a_i^{\text{IACG}}). \quad (38)$$

According to the time-to-go derivative relation, one has

$$\begin{aligned} \dot{t}_i^f &= \frac{NR_i \xi_i}{(2N-1)(N+K)V_i^2} [(1 - \omega_{ik}) a_i^{\text{BT}} + \omega_{ik} (a_i^{\text{ob}} - a_i^{\text{IACG}})] \\ &= (1 - \omega_{ik}) \left[-\frac{\kappa_1 \chi(\bar{\xi}_i)}{t_{go,i}} e_i - \frac{NR_i \xi_i \beta e_i \text{sgn}(\xi_i)}{(2N-1)(N+K)V_i^2} \right] + d_{ik}, \end{aligned} \quad (39)$$

where

$$d_{ik} = \frac{NR_i \xi_i \omega_{ik}}{(2N-1)(N+K)V_i^2} (a_i^{\text{ob}} - a_i^{\text{IACG}}), \quad (40)$$

Since the obstacle avoidance guidance command a_i^{ob} is bounded and the impact-angle-control guidance command a_i^{IACG} is nonsingular, there exists a positive constant $\bar{d}_i$ such that $|d_{ik}| \leq \bar{d}_i$.

Set $D = [d_{1k}, d_{2k}, \dots, d_{nk}]^T$, $\bar{\omega} = \max\{\omega_{1k}, \omega_{2k}, \dots, \omega_{nk}\}$, $\bar{t}_{go} = \max\{t_{go,1}, t_{go,2}, \dots, t_{go,n}\}$ and $\bar{\chi} = \min\{\chi(\bar{\xi}_1(t)), \chi(\bar{\xi}_2(t)), \dots, \chi(\bar{\xi}_n(t))\}$. Differentiating (37) with respect to t yields that

$$\begin{aligned} \dot{V} &= (Le)^T P (\dot{L}e) \\ &\leq -\frac{\kappa_1(1-\bar{\omega})\bar{\chi}}{\bar{t}_{go}} (Le)^T P L L e + e^T P L D \\ &\leq -\frac{\kappa_1(1-\bar{\omega})\bar{\chi}}{2\bar{t}_{go}} (Le)^T \hat{L} L e + e^T P L D \\ &\leq -\frac{\kappa_1(1-\bar{\omega})\bar{\chi}\lambda_2(\hat{L})}{2n\lambda_{\max}(P)\bar{t}_{go}} V + e^T P L D. \end{aligned} \quad (41)$$

Since D is bounded in the switching region, there exists a positive constant $\bar{D}$ such that $\|D\| \leq \bar{D}$. Let $\delta_D = \|PL\|\bar{D}$, it follows from Young's inequality that

$$\dot{V} \leq -\frac{\kappa_1(1-\bar{\omega})\bar{\chi}\lambda_2(\hat{L})}{4n\lambda_{\max}(P)\bar{t}_{go}} V + \frac{2n\lambda_{\max}(P)\bar{t}_{go}}{\kappa_1(1-\bar{\omega})\bar{\chi}\lambda_2(\hat{L})\lambda_{\min}(P)} \delta_D^2, \quad (42)$$

it shows that $V(t)$ is bounded during the smooth switching process.

After the obstacle avoidance task is completed, the flight vehicles leave the transition region. Then,

$$\dot{V} \leq -\frac{\kappa_1\bar{\chi}\lambda_2(\hat{L})}{2n\lambda_{\max}(P)\bar{t}_{go}} V. \quad (43)$$

Then, the above inequality can be rewritten as

$$\dot{V} \leq -\frac{C_v}{t_{go}} V, \quad (44)$$

where

$$C_v = \frac{\kappa_1 \bar{\lambda} \lambda_2(\hat{\mathbf{L}})}{2n\lambda_{\max}(\mathbf{P})}. \quad (45)$$

According to Lemma 1, $\lambda_2(\hat{\mathbf{L}}) > 0$ and $\lambda_{\max}(\mathbf{P}) > 0$ hold under the directed spanning tree condition. In addition, the cooperative gain κ_1 is selected as a positive constant in the guidance law design. Therefore, the above inequality satisfies the convergence condition required by Lemma 2. It follows from Lemma 2 and the comparison principle that $V(t)$ converges to zero before the target is reached and the arrival time errors of the flight vehicles converge to zero before reaching the target.

After the temporal cooperation error converges, the guidance command is governed by the optimal arrival angle control term. Substituting $a_i = a_i^{\text{IACG}}$ into the above equation gives

$$\dot{e}_{\gamma,i} + \frac{K}{t_{go,i}^{\text{IACG}}} e_{\gamma,i} = 0. \quad (46)$$

Since $K \geq 1$, the above equation satisfies the convergence condition in Lemma 2. Therefore, $e_{\gamma,i}$ converges to zero before terminal arrival, which implies $\gamma_i(t_i^f) = \gamma_i^d$. Thus, the desired arrival angle constraint is satisfied. $\square$

Remark 6. Inspired by the principle of error dynamics, $t_{go,i}$ is adopted in the design of temporal cooperative guidance law to establish a time-varying gain, which ensures that the coordination errors converge to zero within the remaining flight duration [31]. Meanwhile, the $\text{sgn}(\xi_i)$ function in (33) allows the construction of a negative definite term related to V , which also accelerates the convergence of arrival time errors. Accordingly, even though the transition between obstacle avoidance and cooperation tasks may enlarge the coordination errors, a_i^{BT} can guarantee the convergence of coordination errors before reaching the target.

From an implementation perspective, the proposed guidance law is suitable for real-time onboard computation. It has a closed-form solution and only needs basic algebraic operations and simple logic switching. In addition, each vehicle generates its guidance command only from the obstacles it faces and the information of its local neighbors, which keeps the computational burden modest as the mission scale increases. Therefore, the computational burden increases mainly with the number of vehicles and the number of locally relevant obstacles, indicating good scalability for large-scale cooperative missions.

4. Numerical Simulations

In this section, four flight vehicles are considered in different cases to demonstrate the effectiveness of the proposed guidance law. The main focus of this paper is vehicle-obstacle collision avoidance. Nevertheless, since the flight vehicles may adjust their trajectories and arrival times during the cooperative process, the inter-vehicle distances are also examined in the simulations. Specifically, for each pair of flight vehicles, the minimum distance before terminal arrival is calculated to check whether inter-vehicle conflicts occur during obstacle avoidance and arrival time adjustment. The simulation time step is set to 0.001 s. When the line-of-sight range between any flight vehicle and the target is less than 1 m, the primary guidance phase terminates, and the vehicle enters the inertial coasting phase. A smaller time step of 0.0001 s is adopted to further calculate the miss distance,

arrival time, and flight path angle error. For each flight vehicle, the simulation terminates once the line-of-sight distance becomes negative for the first time. The miss distance is defined as the minimum line-of-sight range between the flight vehicle and the target during the inertial coasting phase. The arrival time error is defined as the absolute value of the difference between each vehicle's arrival time and the average arrival time of the four vehicles. The flight path angle error is defined as the absolute value of the deviation between the desired terminal flight path angle and the actual terminal flight path angle.

Consider a flight vehicle swarm consisting of four vehicles that reach a stationary target in the presence of two obstacles. The initial conditions of the flight vehicles are given in Table 2. The topology of the swarm is illustrated in Figure 5, where each number stands for the flight vehicle index, and the corresponding weight coefficient matrix is given as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (47)$$

Table 2. The initial states of the four flight vehicles.

M_i	M_1	M_2	M_3	M_4
$R_i/(m)$	9000	8500	8300	8700
$V_i/(m \cdot s^{-1})$	350	340	360	340
$\lambda_i/(^{\circ})$	-10	-15	-20	-25
$\gamma_i/(^{\circ})$	-20	-30	-40	-50
$\gamma_i^d/(^{\circ})$	-50	-40	-30	-20

The centers O_1 and O_2 of the two obstacles are respectively located at $(-4000, 1300)$ m and $(-4000, 3000)$ m, and both have a radius of 500 m. The target is located at $(0, 0)$ m.

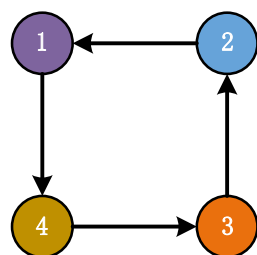


Figure 5. Swarm topology.

4.1. Cooperation Without Obstacle Avoidance

In this case, obstacle avoidance is not considered, and the main objective is to verify whether spatiotemporal coordination can be achieved under the proposed cooperative guidance law. The parameters of the cooperative guidance law are selected as follows: $N = K = 3$, $\xi_{\max} = 10^{\circ}$, $\kappa_1 = 8$, $\beta = 2$.

Figure 6a illustrates the cooperative trajectories of the flight vehicles. The pink circles denote obstacle regions. It can be observed that several flight-vehicle trajectories pass through these regions, indicating that collisions with obstacles occur during flight. Figure 6b shows the variation in the line-of-sight range between the flight vehicles and the target.

Combining Figure 6a,b, it can be seen that the flight vehicles are capable of reaching the target. Figure 6c illustrates the variation in the swarm–obstacle distance. The red lines denote the safety boundaries for obstacles. It shows that distances between flight vehicles and obstacle centers fall below the safety boundaries. Combined with Figure 6a, it is obvious that the vehicles fly straight through the obstacle without the obstacle avoidance guidance law. The swarm arrival times are shown in Figure 6d, which implies that the arrival time errors of the flight vehicles converge to zero before reaching the target. The flight path angles of the flight vehicles are presented in Figure 6e, which indicates that the flight path angles converge to the desired values. Figure 6f shows the acceleration commands of the flight vehicles. It can be seen from Figure 6f that the acceleration commands are free of singularities.

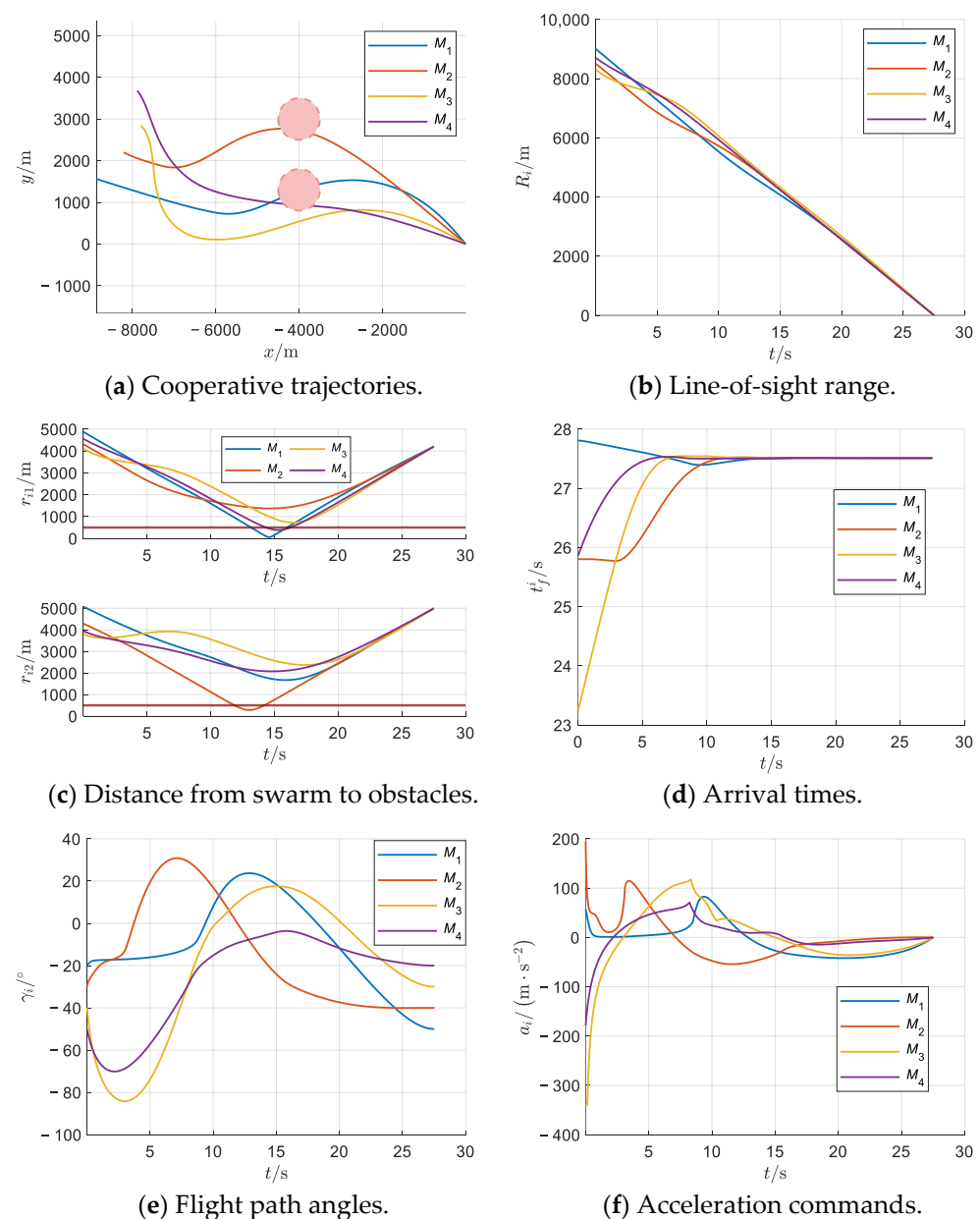


Figure 6. Simulation results without obstacle avoidance.

Table 3 lists the miss distances, arrival time errors, flight path angle errors, and minimum distances to the obstacles of the four flight vehicles without obstacle avoidance. The miss distances range from 0.002 m to 0.018 m, the arrival time errors range from

2.2×10^{-3} s to 2.3×10^{-3} s, and the flight path angle errors range from $0.1 \times 10^{-5}^\circ$ to $5.3 \times 10^{-5}^\circ$. These results show that the cooperative guidance law achieves spatiotemporal coordination before the vehicles reach the target. However, without obstacle avoidance, the vehicle trajectories pass through the obstacle regions.

Table 3. Terminal states and errors without obstacle avoidance.

M_i	M_1	M_2	M_3	M_4
Miss distance/(m)	0.018	0.002	0.006	0.014
Arrival time errors/(10^{-3} s)	2.2	2.3	2.3	2.2
Flight path angle errors/(10^{-5}°)	0.9	0.1	5.3	1.4
Minimum distance to O_1 /(m)	55	1372	726	383
Minimum distance to O_2 /(m)	1671	278	2371	2076

4.2. Cooperation with Obstacle Avoidance

Considering obstacle avoidance during the cooperation phase, the effectiveness of the obstacle avoidance guidance law is mainly verified. When collision risk between the flight vehicles and obstacles is detected, obstacle avoidance is implemented first, and then the cooperation task continues. The parameters of the cooperative guidance law with obstacle avoidance are selected as follows: $\rho = 3$, $\alpha = 0.5$, $R_{s1} = R_{s2} = 500$ m, $\hat{R}_{b1} = \hat{R}_{b2} = 1000$ m. The remaining parameters are consistent with those in Section 4.1.

Figure 7a presents the trajectories of the flight vehicles. As the collision risk between the flight vehicles and obstacles increases, the weighting of the obstacle avoidance guidance law gradually increases to drive vehicles away from obstacles, and the cooperation task is resumed after avoidance is accomplished. Figure 7b shows the line-of-sight range between the flight vehicles and the target, which implies that vehicles can successfully reach the target. Figure 7c illustrates the relative distance between the flight vehicles and obstacles. Compared with Figure 6c, the results demonstrate that the proposed obstacle avoidance guidance law enables the flight vehicles to avoid obstacles. The arrival times and flight path angles of the flight vehicles are shown in Figure 7d and Figure 7e, respectively. It can be observed that the vehicles execute obstacle avoidance between 10 s and 20 s. As the obstacle avoidance weight increases gradually, the cooperative term weakens temporarily, and this variation raises the coordination errors. After obstacle avoidance is completed, the cooperative weight gradually recovers. The errors then decrease and converge to zero before the vehicles reach the target. Figure 7f presents the acceleration commands of the flight vehicles. The proposed weighting function enables smooth switching of acceleration commands at the boundaries between obstacle avoidance and cooperation tasks.

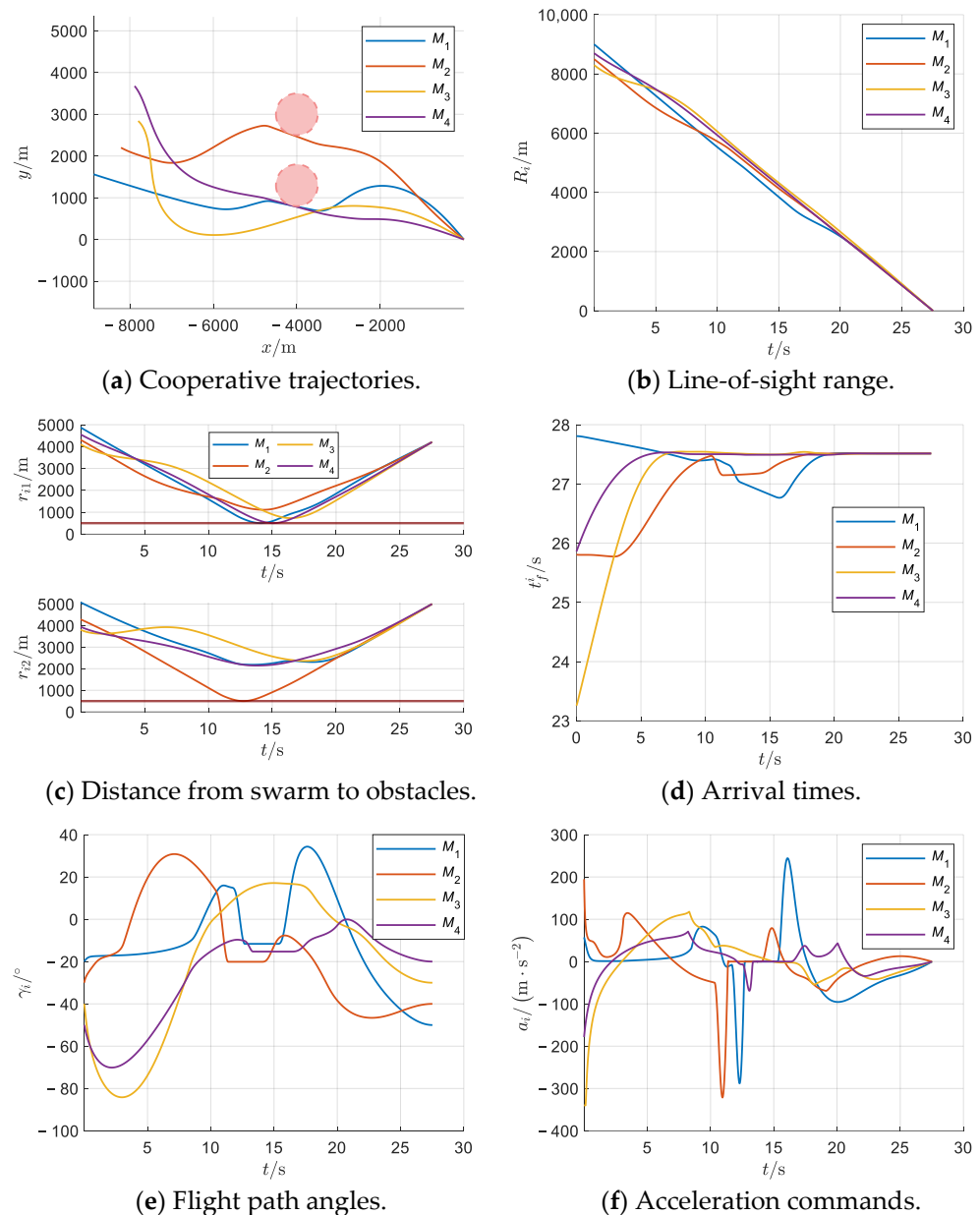


Figure 7. Simulation results with obstacle avoidance.

Table 4 lists the miss distances, arrival time errors, flight path angle errors, and minimum distances to the obstacles of the four flight vehicles with obstacle avoidance. The miss distances range from 0.010 m to 0.027 m, the arrival time errors range from 0.72×10^{-3} s to 0.82×10^{-3} s, and the flight path angle errors range from $0.6 \times 10^{-5}^\circ$ to $1.6 \times 10^{-5}^\circ$. These results show that the guidance law enables the flight vehicles to avoid obstacles and achieve spatiotemporal cooperative arrival. Furthermore, the minimum inter-vehicle separation distance observed in simulations is 0.234 m, which occurs only at the instant of target arrival. This phenomenon stems from the requirement of time coordination, and no collisions between flight vehicles occur prior to arrival at the target.

Table 4. Terminal states and errors with obstacle avoidance.

M_i	M_1	M_2	M_3	M_4
Miss distance/(m)	0.010	0.012	0.027	0.026

Arrival time errors/(10^{-3} s)	0.77	0.77	0.82	0.72
Flight path angle errors/(10^{-5} °)	1.1	0.6	1.6	1.3
Minimum distance to O_1 /(m)	500.029	1111.251	729.602	500.013
Minimum distance to O_2 /(m)	2185.679	500.034	2361.154	2144.367
Minimum safety distance deviation/(m)	0.029	0.034	229.602	0.013

4.3. Comparative Simulations

To verify the advantages of the proposed obstacle avoidance strategy, we adopt the obstacle avoidance mechanism from ref. [29] for comparison on the basis of the proposed spatiotemporal cooperative guidance law. The obstacle avoidance cooperative guidance law is designed as follows.

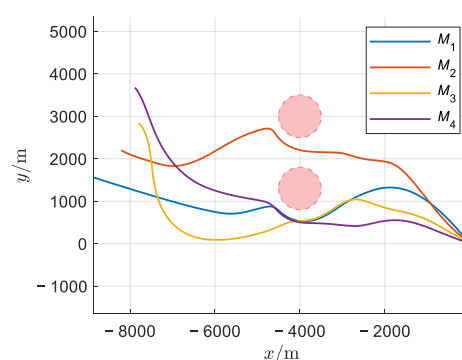
$$a_i = \omega_{ik} a_i^{\text{coop}} + (1 - \omega_{ik}) a_i^{\text{avo}} \quad (48)$$

$$a_i^{\text{avo}} = k_a \left(\frac{(R_k - r_{ik})^2}{(r_{ik} - R_{sk})^2} - \frac{(R_k - r_{ik})}{(R_k - R_{sk})^2} \right) \cos(\theta_{ik}) \text{sgn}(\theta_{ik}) \quad (49)$$

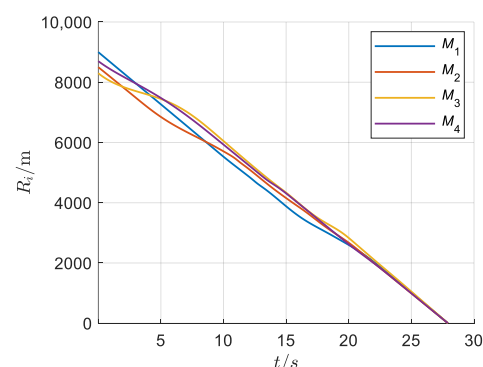
$$\omega_{ik} = \frac{\ln\left(\frac{(e^a - 1)(R_k - r_{ik})}{R_k - R_{sk}} + 1\right)}{\ln((e^a - 1) + 1)} \quad (50)$$

where $k_a > 0$, $R_k > R_{sk}$, $a > 0$. In the simulation, the parameters contained in a_i^{coop} are consistent with those in Section 4.1. The parameters contained in a_i^{avo} are selected as follows: $k_a = 300$, $R_1 = R_2 = 1300$, $a = 2$.

Figure 8 illustrates the comparative simulation results adopting the obstacle avoidance mechanism proposed in ref. [29]. Figure 8a,b show that the flight vehicles are capable of evading obstacles and reaching the target. By comparing Figures 7c and 8c, it can be seen that the obstacle avoidance strategy in ref. [29] keeps the flight vehicles farther away from the obstacle safety boundaries. This is because its distance-dependent obstacle avoidance command increases rapidly as the flight vehicle approaches the safety boundary. In contrast, the proposed linear-projection-based obstacle avoidance command gradually decreases as the flight vehicle approaches the safety boundary. Since the required safety margin is incorporated into the prescribed safety-boundary radius R_{sk} , the proposed method avoids overly conservative obstacle avoidance while satisfying the safety requirement. Figure 8d,e show that the flight vehicles achieve arrival time synchronization and arrival angle control. Figure 8f displays the acceleration commands of flight vehicles. By comparing Figures 7f and 8f, it can be concluded that the proposed method requires lower acceleration commands during obstacle avoidance.



(a) Cooperative trajectories.



(b) Line-of-sight range.

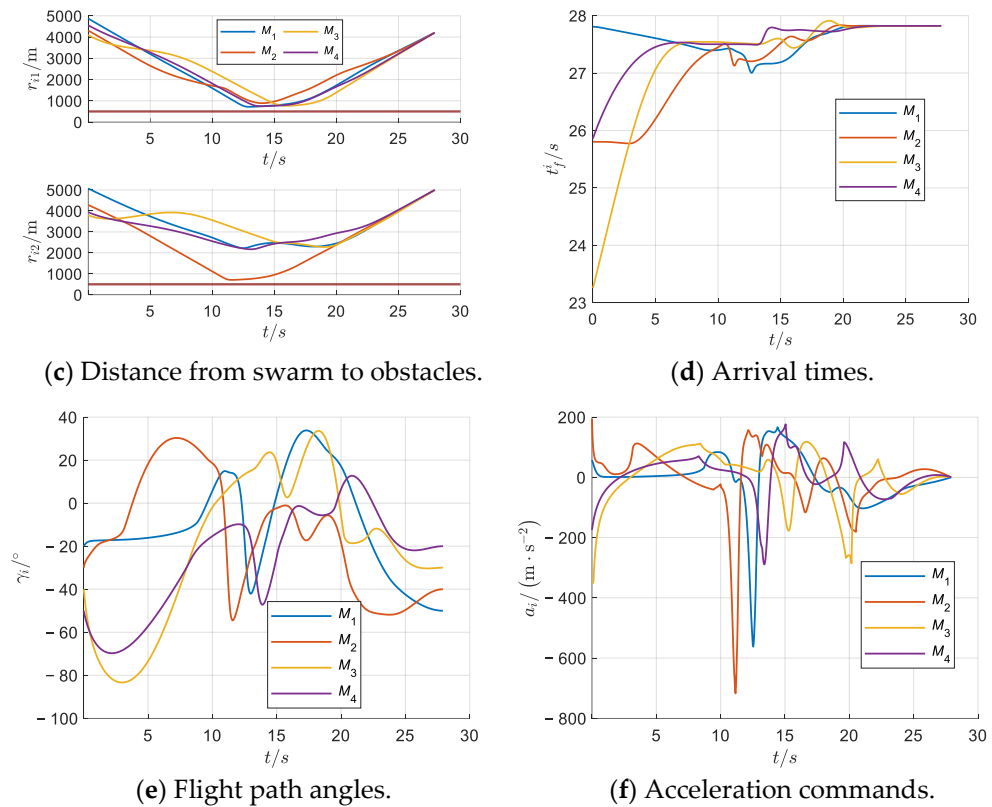


Figure 8. Comparative simulation results.

Table 5 lists the miss distance, coordination errors, and minimum distance to obstacles of the four flight vehicles with obstacle avoidance. In summary, under the cooperative guidance law with the obstacle avoidance strategy in ref. [29], all flight vehicles can avoid obstacles and achieve spatiotemporal cooperative arrival, whereas the strategy remains highly conservative for the given safety threshold in obstacle avoidance.

Table 5. Terminal states and errors with obstacle avoidance strategy in ref. [29].

M_i	M_1	M_2	M_3	M_4
Miss distance/(m)	0.008	0.022	0.035	0.017
Arrival time errors/($10^{-3}s$)	0.36	0.48	0.17	0.95
Flight path angle errors/(10^{-5}°)	0.8	1.4	0.6	2.1
Minimum distance to O_1 /(m)	718.023	890.025	766.457	744.558
Minimum distance to O_2 /(m)	2224.735	709.897	2278.521	2171.083
Minimum safety distance deviation/(m)	218.023	209.897	266.457	244.558

4.4. Monte Carlo Simulations

In this section, Monte Carlo simulations are conducted to further verify the robustness of the proposed method. In this work, `rng('shuffle')` is utilized to initialize the random number generator, which automatically sets the random seed according to the current system time. The initial position errors of the flight vehicles are set to randomly vary within $[-100, 100]$ m, and the initial path angle errors are set to randomly vary within $[-3, 3]^\circ$. The LOS angle measurement noise was configured to follow a zero-mean Gaussian distribution with a standard deviation of 0.1° , and a communication delay of

0.05 s was incorporated in simulations. With other parameters identical to those in Section 4.2, a total of 100 Monte Carlo simulations were conducted. Simulation results are depicted in Figure 9.

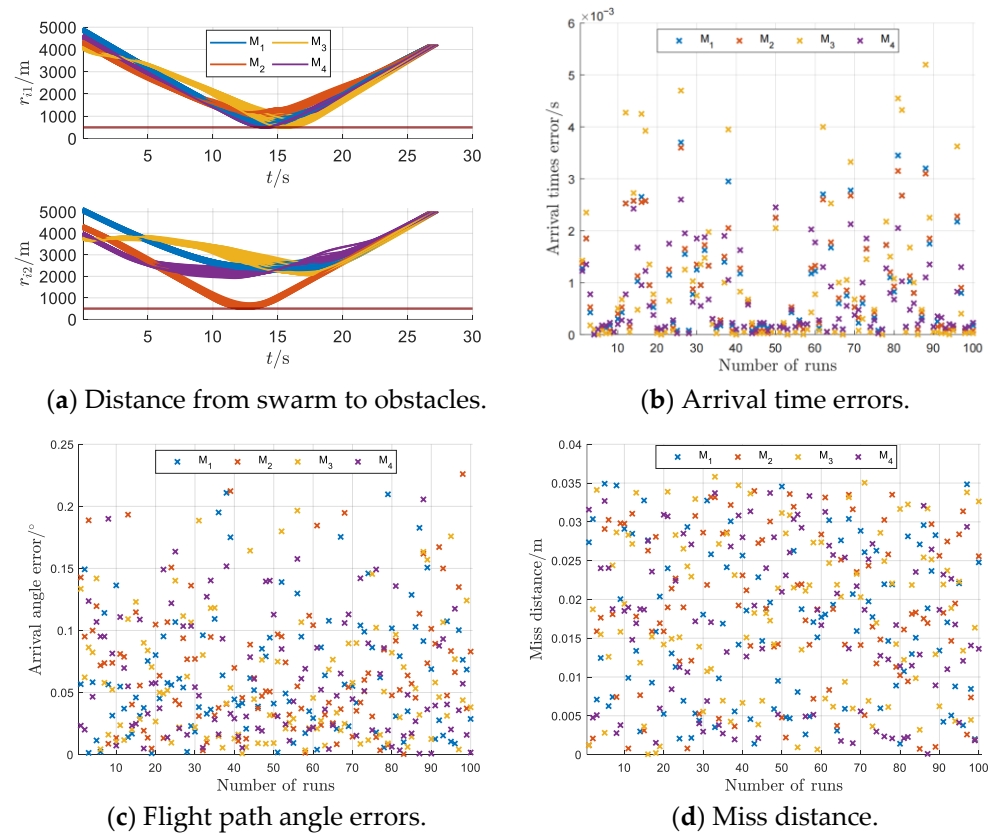


Figure 9. Monte Carlo simulation results.—

Table 6 presents the statistical results obtained from Monte Carlo simulations. It can be observed that the maximum miss distance of the flight vehicles is 0.036 m, the maximum arrival time error is 5.2×10^{-3} s, and the maximum impact angle error is 0.237° . It implies that the flight vehicle swarm is capable of realizing spatiotemporal coordinated arrival under the action of the proposed guidance law. Moreover, the minimum safety distances from the flight vehicles to O_1 and O_2 are 0.015 m and 0.031 m, respectively, which demonstrates that the vehicles avoid obstacles successfully. The results in Figure 9 and Table 6 illustrate that the proposed guidance law is robust to initial errors and measurement noise. It should be noted that in the 100 Monte Carlo simulations, the minimum separation distance between flight vehicles reaches 0.105 m. In each trial, the minimum inter-vehicle distance only appears at the instant of target arrival, and no collisions occur before flight vehicles reach the target.

Table 6. Terminal states and errors in Monte Carlo simulations.

Metric	Maximum	Mean	Standard Deviation	95th Percentile
Miss distance/(m)	0.036	0.018	0.010	0.033
Arrival time errors/(10^{-3} s)	5.20	0.78	0.99	2.70
Flight path angle errors/($^\circ$)	0.237	0.064	0.051	0.166

5. Conclusions

This paper proposed a distributed guidance law for underactuated flight vehicles under the constant-speed assumption, achieving spatiotemporal cooperative arrival while avoiding static circular obstacles. First, a linear projection function was introduced to evaluate collision risk, and a singularity-free obstacle avoidance guidance law was constructed. Then, the error dynamics theory was applied to design a spatiotemporal cooperative guidance law for accurate arrival time and arrival angle control. Finally, a continuous and smooth weighting function was adopted to fuse obstacle avoidance and cooperative guidance laws, ensuring the continuity of acceleration commands during task switching. The proposed guidance law successfully achieved singularity-free obstacle avoidance, arrival time synchronization, and arrival angle control under the considered assumptions. Numerical simulations and Monte Carlo experiments showed that all flight vehicles successfully avoided obstacles, with the arrival time errors less than 0.01 s, arrival angle errors limited to 0.25° , and miss distance lower than 0.04 m. The results indicate that the flight vehicles can avoid obstacle regions and reach the target simultaneously at the preset arrival angles. The proposed cooperative guidance law with obstacle avoidance focuses on vehicle-obstacle safety and does not explicitly address inter-vehicle collision avoidance. Although the weighting function ensures a smooth transition between cooperative guidance and obstacle avoidance at the task-switching boundaries, the sgn function in the guidance law may still cause discontinuities in the guidance command. The proposed method is also limited to static circular obstacles and does not consider irregularly shaped or dynamic obstacles. Future research will focus on designing smoother cooperative guidance laws with obstacle avoidance while incorporating inter-vehicle collision avoidance and accommodating more complex obstacle environments.

Author Contributions: Conceptualization, S.L. and L.W.; formal analysis, S.L. and M.Q.; methodology, S.L. and L.W.; resources, L.W. and J.X.; validation, S.L., L.S. and M.Q.; writing—original draft, S.L.; writing—review and editing, L.W., J.X., M.Q. and L.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the National Natural Science Foundation of China (Grant Number U23B2064, 62176263, 62103434, 62573423, 62506378, 62503439), in part by the Shaanxi Natural Science Basic Research Program-Youth Project (Grant Number 2025JC-YBQN-824), in part by the China Postdoctoral Science Foundation (Grant Number 2024M754095).

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding authors.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

APF	Artificial potential field
LOS	Line-of-sight

References

1. Lee, J.-I.; Jeon, I.-S.; Tahk, M.-J. Guidance Law to Control Impact Time and Angle. *IEEE Trans. Aerosp. Electron. Syst.* **2007**, *43*, 301–310. <https://doi.org/10.1109/TAES.2007.357135>.
2. Luo, S.X.; Zhang, Z.S.; Wang, S.; Zhang, S.; Dai, J.B.; Bu, X.Y.; An, J.P. Network for Hypersonic UCAV Swarms. *Sci. China Inf. Sci.* **2020**, *63*, 140311. <https://doi.org/10.1007/s11432-019-2765-7>.

3. Adde, Y.A.; Razoumny, Y.N.; Betelie, A.A.; Mohammed, T.K.; Wendemagegn, Y.A.; Abdissa, C.M. Intelligent Hybrid Control for Free-Floating Space Robots: PSO–RL–SMC with Inverse-Dynamics Tracking. *IEEE Access* **2026**, *14*, 27137–27157. <https://doi.org/10.1109/ACCESS.2026.3661861>.
4. Fikadu Assefa, D.; Andarge Gedefaw, E.; Merga Abdissa, C.; Negash Lemma, L. Adaptive Neuro-Fuzzy Inference System-Based Sliding Mode Control in the Presence of External Disturbances and Parameter Variation for Quadcopter UAV. *Eng. Rep.* **2025**, *7*, e70417. <https://doi.org/10.1002/eng2.70417>.
5. Metekia, E.W.; Asfaw, W.A.; Abdissa, C.M.; Lemma, L.N. Control of a Fixed Wing Unmanned Aerial Vehicle Using a Robust Fractional Order Controller. *Sci. Rep.* **2025**, *15*, 19954. <https://doi.org/10.1038/s41598-025-03552-0>.
6. Yareshe, F.T.; Madebo, N.W.; Abdissa, C.M.; Lemma, L.N. Trajectory Tracking of Fixed-Wing UAV Using ANFIS-Based Sliding Mode Controller. *IEEE Access* **2025**, *13*, 61986–62003. <https://doi.org/10.1109/ACCESS.2025.3557472>.
7. Merei, A.; Mcheick, H.; Ghaddar, A.; Rebaine, D. A Survey on Obstacle Detection and Avoidance Methods for UAVs. *Drones* **2025**, *9*, 203–233. <https://doi.org/10.3390/drones9030203>.
8. Huang, S.N.; Teo, R.S.H.; Tan, K.K. Collision Avoidance of Multi Unmanned Aerial Vehicles: A Review. *Annu. Rev. Control* **2019**, *48*, 147–164. <https://doi.org/10.1016/j.arcontrol.2019.10.001>.
9. Gedefaw, E.A.; Abera, N.B.; Abdissa, C.M. A Review of Modeling and Control Techniques for Unmanned Aerial Vehicles. *Eng. Rep.* **2025**, *7*, e70215. <https://doi.org/10.1002/eng2.70215>.
10. Mohammed, T.K.; Asfaw, W.A.; Abdissa, C.M.; Lemma, L.N. Constrained Robust Adaptive Control Design for Fixed Wing Uav under Parameter Uncertainties and External Disturbances. *Eng. Res. Express* **2025**, *7*, 25254. <https://doi.org/10.1088/2631-8695/add643>.
11. Weiss, M.; Shima, T. Linear Quadratic Optimal Control-Based Missile Guidance Law with Obstacle Avoidance. *IEEE Trans. Aerosp. Electron. Syst.* **2019**, *55*, 205–214. <https://doi.org/10.1109/TAES.2018.2849901>.
12. Tao, Y.H.; Du, J.L.; Lewis, F.L. Integrated Intelligent Guidance and Motion Control of USVs with Anticipatory Collision Avoidance Decision-Making. *IEEE Trans. Intell. Transp. Syst.* **2024**, *25*, 17811–17820. <https://doi.org/10.1109/TITS.2024.3419585>.
13. Wang, X.T.; Yang, M.; Wang, S.Y.; Hou, M.Z.; Chao, T. Linear-Quadratic and Norm-Bounded Combined Differential Game Guidance Scheme with Obstacle Avoidance for Attacking Defended Aircraft in Three-Player Engagement. *Def. Technol.* **2024**, *42*, 136–155. <https://doi.org/10.1016/j.dt.2024.06.018>.
14. Luis, C.E.; Vukosavljev, M.; Schoellig, A.P. Online Trajectory Generation with Distributed Model Predictive Control for Multi-Robot Motion Planning. *IEEE Robot. Autom. Lett.* **2020**, *5*, 604–611. <https://doi.org/10.1109/LRA.2020.2964159>.
15. Li, Y.W.; Zhang, P.F.; Wang, Z.L.; Rong, D.; Niu, M.Y.; Liu, C. Multi-UAV Obstacle Avoidance and Formation Control in Unknown Environments. *Drones* **2024**, *8*, 714. <https://doi.org/10.3390/drones8120714>.
16. Bai, T.T.; Tang, W.; Liu, Y.; Yu, M.G.; Jiang, Y. UAV Formation Cooperative Obstacle Avoidance Based on Improved APF Method under Variable Topology. *Sci. China Technol. Sci.* **2025**, *68*, 1920403. <https://doi.org/10.1007/s11431-024-2961-6>.
17. Sun, Z.Y.; Sun, H.B.; Li, P.; Zou, J. Cooperative Strategy for Pursuit-Evasion Problem in the Presence of Static and Dynamic Obstacles. *Ocean Eng.* **2023**, *279*, 114476. <https://doi.org/10.1016/j.oceaneng.2023.114476>.
18. Guo, Y.; Song, D.; Wang, C.Q.; Zhang, J.P.; Zhang, G.P.; Li, A.J. Robust Formation Control for Missiles with Obstacle Avoidance. *Chin. J. Aeronaut.* **2022**, *35*, 71–80. <https://doi.org/10.1016/j.cja.2021.06.003>.
19. Ma, B.; Ji, Y.; Fang, L.Y. A Multi-UAV Formation Obstacle Avoidance Method Combined with Improved Simulated Annealing and an Adaptive Artificial Potential Field. *Drones* **2025**, *9*, 391–421. <https://doi.org/10.3390/drones9060390>.
20. Liu, S.X.; Lin, Z.H.; Huang, W.; Yan, B.B. Technical Development and Future Prospects of Cooperative Terminal Guidance Based on Knowledge Graph Analysis: A Review. *J. Zhejiang Univ.-SCI. A* **2025**, *26*, 605–634. <https://doi.org/10.1631/jzus.A2400317>.
21. Li, W.; Yan, S.; Shi, L.; Yue, J.; Shi, M.; Lin, B.; Qin, K. Multiagent Consensus Tracking Control over Asynchronous Cooperation–Competition Networks. *IEEE Trans. Cybern.* **2025**, *55*, 4347–4360. <https://doi.org/10.1109/TCYB.2025.3583387>.
22. Shi, L.; Yan, S.; Li, W. Consensus and Products of Substochastic Matrices: Convergence Rate with Communication Delays. *IEEE Trans. Syst. Man Cybern. Syst.* **2025**, *55*, 4752–4761. <https://doi.org/10.1109/TSMC.2025.3559667>.
23. Wu, Y.J.; An, X.M.; Hua, Y.Y.; Li, G.F.; Li, B.H. Prescribed-Time Cooperative Guidance Law with Seeker-Less Followers against Multiple Targets. *ISA Trans.* **2025**, *164*, 75–90. <https://doi.org/10.1016/j.isatra.2025.05.024>.
24. Yu, J.W.; Song, J.; Zhao, K.; Yu, J.L.; Wei, M.J. Distributed Arbitrary-Time Cooperative Guidance Law for Multi-Aircraft with Dynamic Communication Topology. *Aerosp. Sci. Technol.* **2026**, *168*, 111033. <https://doi.org/10.1016/j.ast.2025.111033>.
25. Zhao, K.; Song, J.; Yu, J.W.; Liu, Y. Cooperative Guidance under Multiple Constraints without Active Speed Control. *Aerosp. Sci. Technol.* **2025**, *164*, 110446. <https://doi.org/10.1016/j.ast.2025.110446>.

26. Zhao, J.; Zhou, R. Obstacle Avoidance for Multi-Missile Network via Distributed Coordination Algorithm. *Chin. J. Aeronaut.* **2016**, *29*, 441–447. <https://doi.org/10.1016/j.cja.2016.01.011>.
27. Yu, J.L.; Shi, Z.X.; Dong, X.W.; Li, Q.D.; Ren, Z. Secure Cooperative Guidance Strategy for Multimissile System with Collision Avoidance. *IEEE Trans. Aerosp. Electron. Syst.* **2025**, *61*, 1034–1047. <https://doi.org/10.1109/TAES.2024.3450451>.
28. Liu, X.Y.; Li, D.G.; Wang, Y.; Zhang, Y.M.; Zhuang, X.; Li, H.Y. Research on a Distributed Cooperative Guidance Law for Obstacle Avoidance and Synchronized Arrival in UAV Swarms. *Drones* **2024**, *8*, 352–373. <https://doi.org/10.3390/drones8080352>.
29. Li, G.F.; Chang, Y.N.; Fan, Y.H.; Xiong, H. Exponential Predefined-Time Spatial-Temporal Cooperative Guidance Method with Obstacle Avoidance. *Chin. J. Aeronaut.* **2026**, *39*, 103893. <https://doi.org/10.1016/j.cja.2025.103893>.
30. Ren, W.; Beard, R.W. Consensus Seeking in Multiagent Systems under Dynamically Changing Interaction Topologies. *IEEE Trans. Autom. Control* **2005**, *50*, 655–661. <https://doi.org/10.1109/TAC.2005.846556>.
31. He, S.M.; Lee, C.-H. Optimality of Error Dynamics in Missile Guidance Problems. *J. Guid. Control Dyn.* **2018**, *41*, 1624–1633. <https://doi.org/10.2514/1.G003343>.
32. Wang, K.; Ding, C.; Wei, Z.; Wang, P.; Chen, Z. Guaranteeing Convergence in Trajectory Shaping Guidance for Impact Time Control. *Autom. J. IFAC* **2026**, *191*, 113107. <https://doi.org/10.1016/j.automatica.2026.113107>.
33. Wang, C.Y.; Dong, W.; Wang, J.N.; Xin, M. Impact-Angle-Constrained Cooperative Guidance for Salvo Attack. *J. Guid. Control Dyn.* **2022**, *45*, 684–703. <https://doi.org/10.2514/1.G006342>.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.