

Article

Research on Gear Fault Detection of Planetary Reducers for Construction Robots Based on an Adaptive Observer

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Abstract

At present, construction robots have received extensive attention in construction scenarios such as wall masonry and component handling. The operational reliability of their joint transmission systems directly affects the working accuracy and construction safety of robots. To address the problem that planetary reducers in construction robots are prone to faults under complex working conditions, this paper proposes a fault-detection method based on an adaptive observer. First, a state–space model containing unknown disturbances and actuator faults is established by combining the transmission structure and typical fault mechanism of the planetary reducer. On this basis, an adaptive observer with more degrees of freedom is designed, and the L_∞ disturbance-robustness index is introduced. The augmented-output function is used to apply the H_∞ performance index to enhance fault sensitivity. The simulation results show that when the system is fault-free, the residual remains below the threshold, and no false alarm occurs. After a time-varying crack fault is introduced, the residual can quickly exceed the threshold and trigger a complete fault alarm. Compared with the L_∞ observer, the introduced H_∞ performance index significantly improves the fault sensitivity of the observer. Compared with the traditional Luenberger observer, the proposed method has better disturbance robustness and fault sensitivity, providing an effective method for early fault detection of planetary reducers in construction robots.

Keywords: adaptive observer; planetary reducer; gear; fault detection

1. Introduction

At present, construction robots are important equipment for promoting automation and intelligence on construction sites. They can replace manual labor in high-intensity and high-risk tasks such as wall masonry and component handling. Therefore, the development of robots for building construction has become an urgent demand for the modernization of construction. This paper studies a construction robot capable of performing wall masonry operations. As a key transmission component of the robotic arm, the planetary reducer drives the planet gears through the sun gear, enabling them to rotate and mesh with the fixed ring gear, thereby realizing power transmission in Figure 1. Due to the complex working conditions and harsh environment of construction sites, robots are easily affected by variable loads, frequent start-stop operations, and environmental disturbances during long-term operation. As a result, faults may occur in the planetary reducer, affecting the accuracy, torque, stiffness, transmission efficiency, and other performance indicators of the construction robot, and may even lead to project delays and major safety accidents.



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Therefore, research on condition monitoring and fault diagnosis of planetary reducers in construction robots is of great significance for improving operational reliability, reducing downtime and maintenance costs, and ensuring construction safety.

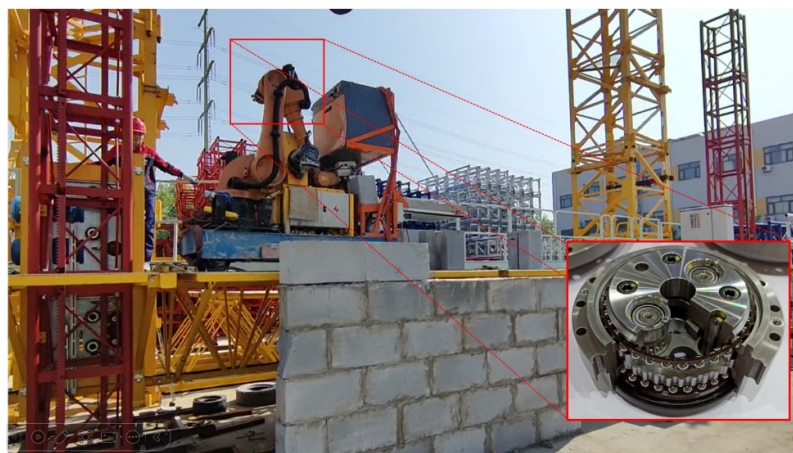


Figure 1. Application of the masonry robot and reducer in the joint transmission system (red box: reducer location).

At present, fault-diagnosis methods for reducers mainly include data-driven methods, signal-processing-based methods, and analytical model-based methods. Raouf et al. [1] developed a machine-learning-based fault-detection method using electrical current signatures, while Guo et al. [2] proposed a data-driven fault identification method for robot reducers. Qiao et al. [3] investigated nonlinear spectrum feature fusion for reducer fault diagnosis, and Wang et al. [4] analyzed reducer vibration characteristics. Han et al. [5] carried out fault diagnosis based on vibration analysis, while Jiang et al. [6] introduced a denoising time-frequency attention neural network. These methods can effectively extract fault information, but their performance may still be affected by data quality, operating-condition variations, and external disturbances.

Recent studies have further introduced physical prior knowledge, cross-condition learning, and attention mechanisms to improve the generalization and reliability of intelligent fault diagnosis. Ullah et al. [7] proposed a trust-aware domain adaptation framework based on physics-guided reliability learning for cross-condition fault diagnosis of milling machines. Lee et al. [8] developed a physics-guided self-supervised learning method for rotating-machinery fault diagnosis to enhance physically meaningful representation learning and generalization under varying conditions. Ullah et al. [9] proposed a Transformer attention-guided dual-path framework that integrates spatial and time-frequency information for bearing fault diagnosis. These studies indicate that incorporating physical knowledge and advanced feature-fusion mechanisms is becoming an important direction for improving fault-diagnosis robustness under complex operating conditions.

For analytical model-based methods, Zhao et al. [10] established a fault dynamic model for a construction robot reducer and designed an adaptive state observer for fault detection. Zhang et al. [11] studied fault feature extraction based on motor current signature analysis. Li et al. [12] investigated contact fatigue characteristics of reducer components, while Li et al. [13] analyzed the load characteristics of flexible bearing components. Wang et al. [14] established a generalized dynamic model for planetary gear transmissions, and Tang et al. [15] further considered structural flexibility and dynamic multi-tooth meshing in planetary gear modeling. These studies provide an important theoretical basis for model-based fault diagnosis of planetary reducers.

Observer-based fault detection has also received extensive attention. Venkateswaran et al. [16] studied functional observers for nonlinear systems, Hu et al. [17] investigated general-

ized proportional-integral observers for robust fault detection, and Gao et al. [18] proposed a finite-frequency H_-/L_∞ fault-diagnosis method. Azmi and Yazdizadeh [19] developed a robust adaptive fault-detection observer for nonlinear systems with uncertainty. Li et al. [20] studied finite-frequency fault detection for T-S fuzzy systems, while Wang et al. [21] considered adaptive event-triggered fault detection for LPV systems. Li et al. [22] further investigated fault detection for Lipschitz nonlinear systems with restricted frequency-domain specifications.

Finite-frequency fault-detection filters have subsequently been extended to two-dimensional nonlinear systems [23], interval type-2 T-S fuzzy systems [24], continuous-time T-S fuzzy affine systems [25], membership-function-dependent fuzzy systems [26], interconnected switched systems [27], and asynchronous switched systems [28]. Li et al. [29] also investigated unknown-input observer design for LPV systems under bounded-error conditions. These studies show that observer-based design can improve disturbance robustness and fault sensitivity under uncertain conditions.

In addition, threshold design directly affects false alarms and missed detections. Wu et al. [30] proposed an adaptive dynamic thresholding method, Zhang et al. [31] studied residual evaluation for fault detection, and Ma et al. [32] investigated interval-based adaptive thresholds combined with peak-to-peak observers. Zhao et al. [33] proposed an adaptive threshold-setting method based on KDE-EWMA, while Zhang et al. [34] studied zonotopic fault-detection observer design. However, for planetary reducers used in construction robots, the simultaneous consideration of disturbance robustness, fault sensitivity, and physically meaningful gear faults remains insufficient. Therefore, this paper develops an adaptive-observer-based fault detection method for the planetary-reducer transmission system, introduces an augmented output to enhance fault sensitivity, and employs a time-varying dynamic threshold to improve fault-detection reliability.

This paper aims to propose a fault-detection method for planetary reducers in construction robots to reduce the time and cost losses caused by reducer faults and improve the operational stability of construction robot equipment. This chapter proposes an adaptive-observer-based fault-diagnosis method with more design degrees of freedom for the transmission module of planetary reducers. The main contributions of this paper are as follows:

(1) Dynamic modeling was carried out based on the planetary reducer in the joint of the construction robot, and the fault modes of the planetary reducer during the operation of the construction robot were systematically summarized. By analyzing the failure mechanisms of the main fault modes and reasonably simplifying the planetary-reducer system, a general fault model was finally established.

(2) An adaptive observer was designed for the simplified model of the planetary reducer with faults. Compared with the traditional Luenberger observer, the proposed observer has more design degrees of freedom, which improves the upper limit of the fault-diagnosis performance of the observer.

(3) In the observer design, the L_∞ performance index is introduced to reduce the influence of disturbances on the residual. Since H_- can provide a constraint on the minimum detectable fault, it can improve the fault sensitivity of the observer to a greater extent. However, it cannot realize fault detection for systems with only actuator faults. To address this problem, this paper designs an augmented-output function to apply the H_- performance index, thereby enhancing the fault sensitivity of the adaptive observer.

2. Dynamic Modeling

2.1. Structure of the Planetary Reducer

As a key component in the joint transmission system of the construction robot, the planetary reducer is mainly composed of the sun gear, planet gears, ring gear, planet carrier, input shaft, output shaft, supporting bearings, and other components. It has the

characteristics of compact structure, strong load-bearing capacity, and large transmission ratio. The structural schematic diagram of the planetary reducer is shown in Figure 2.

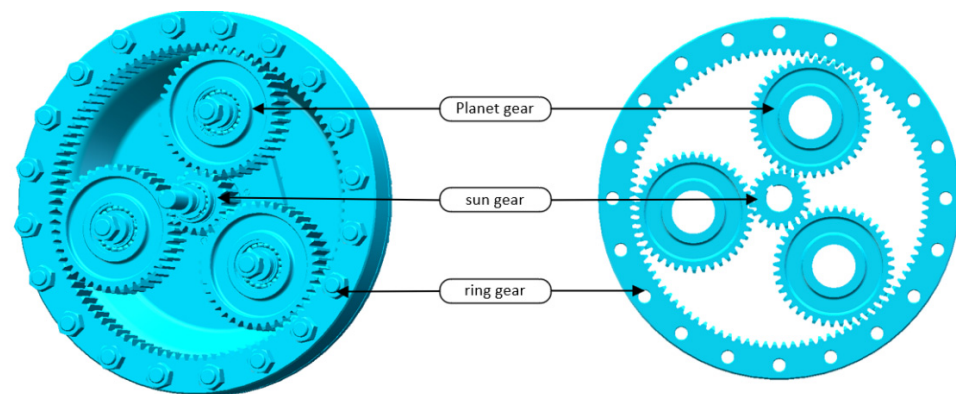


Figure 2. Structural schematic diagram of the planetary reducer.

In a typical planetary reducer, the sun gear usually acts as the input component, the planet gears mesh with both the sun gear and the ring gear, and the planet carrier acts as the output component. When the sun gear rotates, the planet gears rotate around their own axes while revolving around the center of the sun gear, and the planet carrier outputs low-speed and high-torque motion. Since the construction robot is subject to frequent start-stop operations, impact loads, and variable loads during wall masonry, component handling, and positioning, there are complex motion coupling and load transmission relationships among the internal components of the planetary reducer. The input torque is transmitted from the sun gear to multiple planet gears, and then converges at the output end of the planet carrier under the constraint of the ring gear, thereby realizing speed reduction and torque increase. The transmission model of the planetary reducer considering internal structural characteristics is shown in Figure 3.

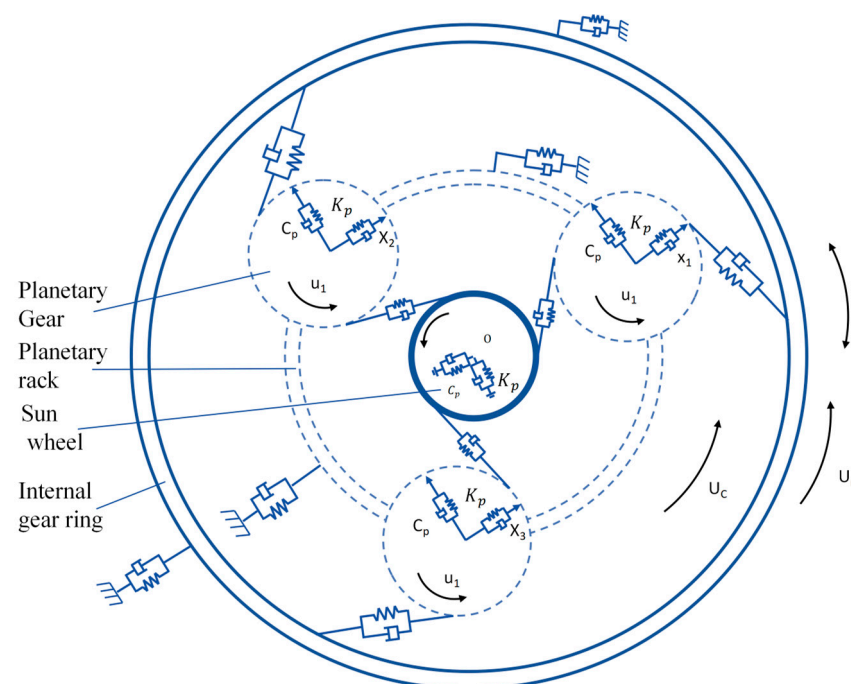


Figure 3. Transmission model of the planetary reducer.

2.2. Fault Types and Mechanisms

Gear-related faults are among the representative failure modes of planetary reducers and may directly affect meshing stiffness, transmission accuracy, vibration response, and operational reliability. Considering the structural characteristics of the planetary reducer and the fault forms involved in this study, the subsequent analysis focuses on typical gear faults. A schematic classification of representative planetary-reducer fault categories is illustrated in Figure 4.

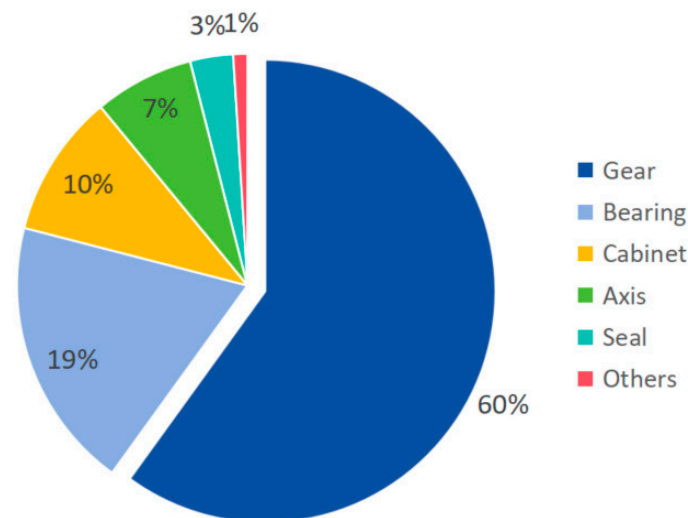


Figure 4. Pie chart of gearbox failure.

Among the typical gear fault modes, crack faults are selected as the representative fault type in this study. Gear cracks commonly originate from local fatigue damage at the tooth root under repeated meshing loads and may gradually propagate during long-term operation. Compared with sudden catastrophic failures, crack propagation can better characterize the progressive degradation process of gear transmission performance. In addition, a crack may further develop into a broken-tooth fault under continuous cyclic loading. Therefore, the crack fault is considered a representative incipient gear fault and is adopted to verify the effectiveness of the proposed observer-based fault-detection method.

1. Gear Crack and Tooth Breakage

During the transmission process of the planetary reducer, the gear meshing parts are subjected to alternating loads and contact impacts for a long time. When the tooth root area is subjected to large bending stress, local stress concentration is likely to occur. With the increase in operation time, fatigue cracks may gradually form at the tooth root. If the cracks continue to propagate, they will further develop into tooth breakage faults. Even under normal load conditions, the gear teeth are continuously affected by periodic cyclic stress, and crack initiation and propagation may still occur after long-term operation. Therefore, gear cracks can usually be regarded as the early stage before the formation of tooth breakage faults. For the planetary reducer, tooth breakage is a typical fault with a high degree of hazard, which will significantly affect transmission smoothness and system safety. According to the different fault locations, gear faults can be further divided into sun gear faults, planet gear faults, and ring gear faults. According to the different damage forms, they mainly include cracks, tooth breakage, pitting, and wear.

Gear cracks mainly affect the dynamic characteristics of the planetary gear transmission by reducing the local meshing stiffness and increasing the transmission error of the cracked gear pair. When the cracked tooth enters the meshing region, the reduction in tooth stiffness causes additional fluctuations in meshing force and transmission torque. Therefore, in the

following dynamic model, the influence of crack-induced meshing-stiffness degradation on the transmission system is represented by an equivalent fault input $f(t)$.

2. Gear Pitting

During the gear meshing process, the contact area continuously shifts with the change in the meshing position, and the tooth surface contact stress also presents periodic variation. When the gear lubrication condition is poor, or the contact surface bears excessive load, fatigue damage is likely to occur in the local area of the tooth surface, further causing small metal particles to peel off and form pit-like depressions. This phenomenon is usually called gear pitting. In the initial stage of pitting, its influence on transmission is relatively small. However, as the damage area expands, it will intensify the change in tooth surface roughness, cause increased vibration and noise, and further affect the gear meshing quality.

3. Uniform Tooth Surface Wear

Tooth surface wear is mainly caused by long-term meshing friction of gears in Figure 5. When lubrication is insufficient, machining errors exist on the tooth surface, or external impurities enter the meshing area, tooth surface wear will be further intensified. The impurities usually include metal abrasive particles, dust particles, and other contaminants. These particles are repeatedly squeezed and slid between the tooth surfaces, which damages the original morphology of the tooth surface and causes gradual loss of tooth surface material. As the degree of wear increases, the gear meshing clearance and transmission error may change, thereby reducing the transmission accuracy and operational stability of the planetary reducer.

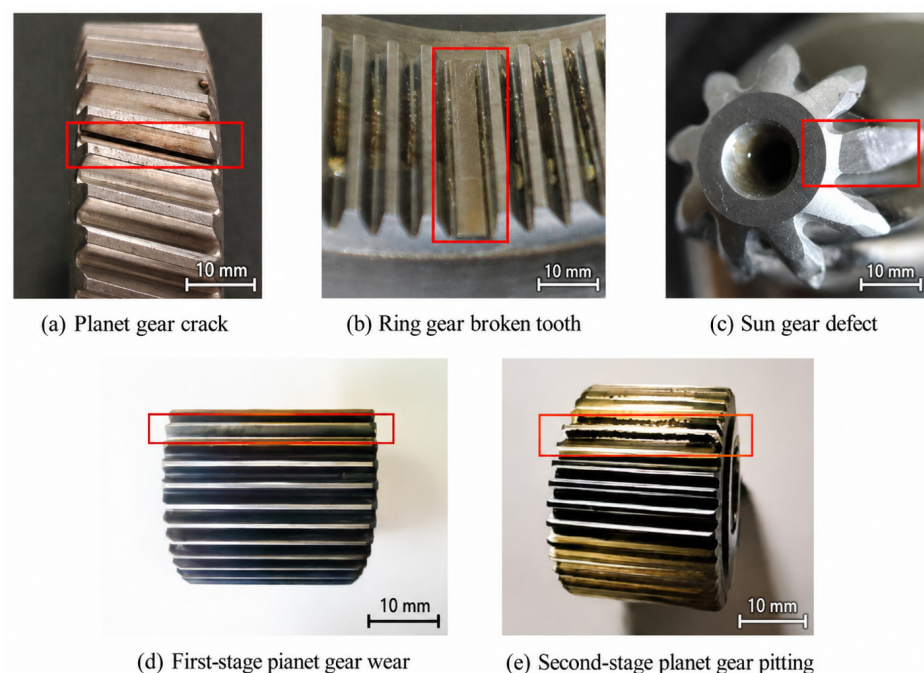


Figure 5. Typical gear fault morphologies of a planetary reducer, they should be listed as: (a) Planet gear crack; (b) Ring gear broken tooth; (c) Sun gear defect; (d) First-stage planet gear wear; (e) Second-stage planet gear pitting (red markings: fault locations).

2.3. Fault Dynamic Model

During the operation of the planetary reducer, the drive motor applies power to the sun gear of the planetary gear train in the planetary reducer through the input shaft, and the planetary reducer transmits power to the load through the output shaft. The rotational inertias of the input shaft and output shaft of the planetary reducer are denoted as J_1 and

J_0 , respectively. θ_1 is the angle of the input shaft at the drive motor end, T_1 is the torque obtained by the sun gear of the planetary gear train, and B_1 is the friction coefficient of the input shaft. The schematic diagram of the transmission system is shown in Figure 6.

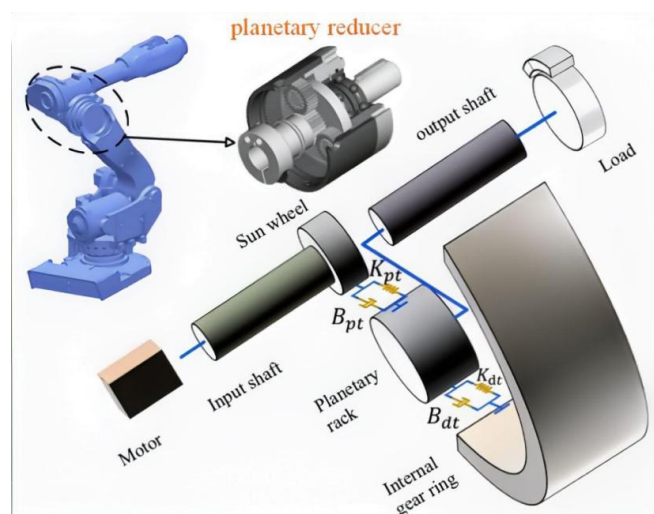


Figure 6. Transmission system schematic diagram.

K_{pt} is the stiffness between the planet gear and the sun gear, B_{pt} is the damping between the planet gear and the sun gear, K_{dt} is the stiffness between the planet gear and the ring gear, and B_{dt} is the damping between the planet gear and the ring gear. Thus, we obtain:

$$K_t = \frac{1}{(N^2 K_{pt} l_{pt}^2)^{-1} + (K_{dt} l_{dt}^2)^{-1}} \quad (1)$$

$$B_t = N^2 B_{pt} + B_{dt} \quad (2)$$

K_t is the torsional stiffness coefficient, B_t is the torsional damping coefficient, l_{pt} is the arm length of the contact force between the sun gear and the planet gear, namely the base-circle radius of the sun gear, and l_{st} is the arm length of the contact force between the cycloidal gear and the pin gear, which is equal to the distance from the axis of the cycloidal gear to the contact action line. θ_Δ is the torsional angle. Considering the damping coefficient and the torsional force of the torsional spring, we obtain:

$$\omega(t) = \dot{\theta}(t) \quad (3)$$

$$T_l(t) = K_t \theta_\Delta(t) + B_t \left(\omega_r(t) - \frac{\omega_g(t)}{N_g} \right) \quad (4)$$

The dynamic equation of the input shaft of the planetary reducer can be expressed as:

$$J_l \ddot{\theta}_l(t) = \tau_l(t) - T_l(t) - B_l(t) \dot{\theta}_l(t) \quad (5)$$

Let θ_0 be the angle of the output shaft at the load end, T_0 be the torque of the output shaft, and B_0 be the friction coefficient of the output shaft. Considering the rotor inertia factor, the dynamic equation of the output shaft of the planetary reducer can be expressed as:

$$J_o \ddot{\theta}_o(t) = \tau_o(t) - T_o(t) - B_o(t) \dot{\theta}_o(t) \quad (6)$$

where τ_1 is the drive motor torque, and τ_0 is the load torque.

Let N_g be the transmission ratio of the planetary reducer, and η_{dt} be the transmission system efficiency. The form of the transmission ratio can be expressed as:

$$T_o(t) = \eta_{dt} \frac{T_l(t)}{N_g} \quad (7)$$

Through the above parameter introduction, the model of the planetary reducer in the joint of the construction robot can be obtained as:

$$J_l \dot{\omega}_l(t) = \tau_l(t) - K_t \theta_\Delta(t) - (B_t + B_l) \omega_l(t) + \frac{B_t}{N_g} \omega_o(t) \quad (8)$$

$$J_o \dot{\omega}_o(t) = \frac{\eta_{dt} K_t}{N_g} \theta_\Delta(t) + \frac{\eta_{dt} B_t}{N_g} \omega_l(t) - \left(\frac{\eta_{dt} B_t}{N_g^2} + B_o \right) \omega_o(t) - \tau_o(t) \quad (9)$$

$$\theta_\Delta(t) = \omega_l(t) - \frac{1}{N_g} \omega_o(t) \quad (10)$$

To establish the physical correspondence between gear cracking and the equivalent fault input, the degradation of meshing stiffness caused by gear cracks is considered. Let $k_m^0(t)$ denote the meshing stiffness under the healthy condition. When a gear crack occurs, the meshing stiffness can be expressed as

$$k_m(t) = k_m^0(t) - \Delta k_m(t) \quad (11)$$

where $\Delta k_m(t)$ denotes the reduction in meshing stiffness caused by the gear crack. The corresponding variation in meshing force can be equivalently expressed as

$$\Delta F_m(t) = -\Delta k_m(t) \delta_m(t) \quad (12)$$

where $\delta_m(t)$ denotes the relative meshing deformation. Accordingly, the equivalent fault torque caused by the crack can be expressed as

$$T_f(t) = r_b \Delta F_m(t) \quad (13)$$

where r_b denotes the effective base-circle radius. To further characterize the physical severity of the crack-induced degradation, the relative meshing-stiffness degradation coefficient is defined as $\eta_c(t) = \Delta k_m(t) / \Delta k_m^0(t)$. A larger $\eta_c(t)$ indicates a greater loss of local meshing stiffness and consequently produces a larger perturbation in the meshing force and equivalent fault torque. In the following state-space model, $Ff(t)$ is used to represent this equivalent dynamic effect. Therefore, the amplitude of $f(t)$ qualitatively reflects the relative crack severity rather than the physical crack length itself.

3. Adaptive-Observer Design

In this section, dynamic modeling is carried out for the planetary-reducer transmission system.

$$\text{Let: } x = [\omega_l \quad \omega_o \quad \theta_\Delta]^T, u = [\tau_l \quad \tau_o]^T,$$

$$A = \begin{bmatrix} -\frac{B_t+B_l}{J_l} & \frac{B_t}{J_l N_g} & -\frac{K_t}{J_l} \\ \frac{\eta_{dt} B_t}{J_o N_g} & -\frac{\eta_{dt} B_t - N_g^2 B_o}{N_g^2 J_o} & \frac{\eta_{dt} K_t}{J_o N_g} \\ 1 & -\frac{1}{N_g} & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{J_l} & 0 \\ 0 & -\frac{1}{J_o} \\ 0 & 0 \end{bmatrix}.$$

The dynamic model of the planetary reducer with unknown disturbances and faults can be expressed as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t) + Dd(t) \\ y(t) = Cx(t) \end{cases} \quad (14)$$

$$F = \begin{bmatrix} J_1^{-1} \\ J_0^{-1} \\ 1 \end{bmatrix}, D = \begin{bmatrix} J_1^{-1} \\ J_0^{-1} \\ 1 \end{bmatrix}, d(t) \in R^q \text{ denotes the unknown disturbance, and } f(t) \in R^m$$

denotes the equivalent fault input representing the dynamic effect caused by crack-induced degradation of gear meshing stiffness and transmission error.

Assumption 1. The disturbance $d(t)$ and the fault signal $f(t)$ are continuously differentiable, and their amplitudes and first-order derivatives are bounded, i.e.,

$$\begin{aligned}\|d(t)\| &\leq \bar{d}, \|\dot{d}(t)\| \leq \bar{d}_1, \\ \|f(t)\| &\leq \bar{f}, \|\dot{f}(t)\| \leq \bar{f}_1,\end{aligned}\quad (15)$$

where $\bar{d}$, $\bar{d}_1$, $\bar{f}$ and $\bar{f}_1$ are finite positive constants. This assumption guarantees the existence and boundedness of the derivative terms involved in the augmented-output formulation.

When only actuator faults occur in the system.

When only actuator faults $\bar{d}$, $\bar{d}_1$, $\bar{f}$ and $\bar{f}_1$ occur in the system, the H_-/H_∞ fault-detection observer for Equation (11) cannot be designed in the full-frequency domain. To address this problem, this paper first differentiates the original measured output of the system, yielding:

$$\dot{y}(t) = C\dot{x}(t) = CAx(t) + CBu(t) + CFf(t) + CDd(t) \quad (16)$$

The original measured output vector $y(t)$ of the system is augmented with its first-order derivative $\dot{y}(t)$, and the auxiliary output vector is obtained as:

$$\tilde{y}(t) = \tilde{C}x(t) + Rf(t) + D_y d(t) \quad (17)$$

The augmented system can be obtained as:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t) + Dd(t) \\ \tilde{y}(t) = \tilde{C}x(t) + Rf(t) + D_y d(t) \end{cases} \quad (18)$$

where:

$$\tilde{y}(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) - CBu(t) \end{bmatrix} \tilde{C} = \begin{bmatrix} C \\ CA \end{bmatrix}, R = \begin{bmatrix} 0 \\ CF \end{bmatrix}, D_y = \begin{bmatrix} 0 \\ CD \end{bmatrix}$$

In practical applications, the measured output generally contains measurement noise and can be expressed as

$$y_m(t) = y(t) + n(t) \quad (19)$$

where $n(t)$ denotes the measurement noise. Direct numerical differentiation of $y_m(t)$ may amplify high-frequency measurement noise. Therefore, in numerical implementation, the derivative of the measured output is estimated using a first-order filtered differentiator rather than direct differentiation:

$$\hat{\dot{y}}(s) = \frac{\omega_d s}{s + \omega_d} Y_m(s) \quad (20)$$

where ω_d denotes the bandwidth parameter of the filtered differentiator. This implementation approximates the output derivative within the frequency range of interest while limiting the amplification of high-frequency measurement noise.

Assume that the system satisfies the rank condition:

$$\text{rank}(CF) = \text{rank}(F) = n_f \quad (21)$$

Remark 1. From Equation (14), it can be seen that matrix $R \neq 0$ and satisfies the full-column-rank condition. Therefore, the newly generated auxiliary output vector explicitly contains actuator fault information, and the H_- design can be carried out for the augmented system.

For system (14), the designed adaptive observer is given as follows:

$$\begin{cases} \dot{z}(t) = TA\hat{x}(t) + TBu(t) + L(y(t) - \tilde{C}\hat{x}(t)) \\ \hat{x}(t) = z(t) + N\tilde{y}(t) \end{cases} \quad (22)$$

Moreover, the T matrix and N matrix satisfy:

$$T + N\tilde{C} = I \quad (23)$$

Remark 2. The proposed adaptive observer (16) has more design degrees of freedom than the traditional Luenberger observer.

According to the lemma: $\begin{bmatrix} T & \Theta \end{bmatrix} = \begin{bmatrix} I \\ C \end{bmatrix}^\dagger + S \left(I - \begin{bmatrix} I \\ C \end{bmatrix} \begin{bmatrix} I \\ C \end{bmatrix}^\dagger \right)$, the gain parameter T and the parameter N are represented by the matrix S as follows.

$$\begin{cases} T = \begin{bmatrix} I \\ C \end{bmatrix}^\dagger \begin{bmatrix} I \\ 0 \end{bmatrix} + S \left(I - \begin{bmatrix} I \\ C \end{bmatrix} \begin{bmatrix} I \\ C \end{bmatrix}^\dagger \right) \begin{bmatrix} I \\ 0 \end{bmatrix} \\ N = \begin{bmatrix} I \\ C \end{bmatrix}^\dagger \begin{bmatrix} 0 \\ I \end{bmatrix} + S \left(I - \begin{bmatrix} I \\ C \end{bmatrix} \begin{bmatrix} I \\ C \end{bmatrix}^\dagger \right) \begin{bmatrix} 0 \\ I \end{bmatrix} \end{cases} \quad (24)$$

The state estimation error equation is:

$$\begin{aligned} \dot{e}_x(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= \dot{x}(t) - \dot{z}(t) - N\dot{y}(t) \\ &= Ax(t) + Bu(t) + Ff(t) + Dd(t) - \dot{z}(t) - N\dot{y}(t) \\ &= (TA - L\tilde{C})e_x(t) + (TF - LR)f(t) - NR\dot{f}(t) \\ &\quad + (TD - LD_y)d(x) - ND_y\dot{d}(x) \end{aligned} \quad (25)$$

The residual equation is:

$$\begin{aligned} r(t) &= \hat{y}(t) - \tilde{C}\hat{x} \\ &= \tilde{C}x(t) + Rf(t) + D_yd(t) - \tilde{C}\hat{x} \\ &= \tilde{C}e_x(t) + Rf(t) + D_yd(t) \end{aligned} \quad (26)$$

To facilitate the analysis of the disturbance robustness and fault sensitivity of the residual, the system is decomposed into the following two subsystems:

When there is no fault in the planetary-reducer system $\hat{f}(t) = 0$:

$$\begin{aligned} \dot{e}_d(t) &= (TA - L\tilde{C})e_d(t) + (TD - LD_y)d(x) - ND_y\dot{d}(x) \\ r_d(t) &= \tilde{C}e_d(t) + D_yd(t) \end{aligned} \quad (27)$$

When there is no disturbance in the planetary-reducer system $\hat{d}(t) = 0$:

$$\begin{aligned} \dot{e}_f(t) &= (TA - L\tilde{C})e_f(t) + (TF - LR)f(t) - NR\dot{f}(t) \\ r_f(t) &= \tilde{C}e_f(t) + Rf(t) \end{aligned} \quad (28)$$

The main design idea of the adaptive observer (16) for reducing the influence of disturbances on the planetary reducer and improving sensitivity is as follows:

L_∞ performance: the influence of disturbance d on residual r .

$$\|r_d(t)\| < \sqrt{\gamma_1^2 V(0) \exp(-\alpha t) + (\gamma_1^2 + \gamma_2^2) \|\varphi(t)\|_\infty^2} \quad (29)$$

where $\|r_d(t)\|$ denotes the 2-norm of the residual vector r_d at time t , and $V(0)$ is the initial value of the Lyapunov function.

H_- performance: the influence of fault f on residual r .

$$\|G_{rf}(s)\|_- > \beta \quad (30)$$

Remark 3. The main idea of this part is to describe the system stability condition, improve the disturbance robustness of the residual using the L_∞ performance, and improve the fault sensitivity of the residual using the H_- performance.

4. Fault-Detection Scheme for Planetary Reducers Based on an Adaptive Observer

In the design and operation of engineering systems, disturbance robustness and fault sensitivity are two key and mutually constrained performance requirements. On the one hand, the system must have robustness against external disturbances to avoid false alarms and ensure stable operation. On the other hand, the system also needs to maintain sufficient sensitivity to faults, so that faults can be detected timely and accurately, preventing fault evolution from causing serious consequences. In this section, the residual disturbance-robustness constraint is first established to suppress the influence of unknown disturbances on the residual signal. Meanwhile, the fault-sensitivity constraint is introduced to ensure that the residual has a sufficient response amplitude to incipient faults. To further improve detection reliability, a time-varying dynamic threshold is derived based on the bounded initial estimation error and peak-bounded disturbance to reduce false alarms during fault detection. Finally, the resulting multi-objective design contains bilinear matrix inequality (BMI) constraints. Therefore, an alternating iterative optimization strategy is adopted. At each iteration, one group of decision variables is fixed so that the remaining subproblem can be formulated and solved as a convex LMI problem. By alternately updating the two groups of variables until convergence, feasible observer parameters satisfying the prescribed disturbance-robustness and fault-sensitivity constraints are obtained.

4.1. Disturbance-Robustness Condition

In this section, the disturbance-robustness condition is derived based on the Lyapunov function to suppress the influence of unknown disturbances on the residual signal.

Theorem 1. For the state estimation error system (21), if there exist scalars $\alpha > 0$, $\gamma_1 > 0$, $\gamma_2 > 0$, and a positive definite symmetric matrix $P_d \in R^{n_x \times n_x}$, such that the following linear matrix inequalities hold simultaneously,

$$\begin{bmatrix} P_d(TA - L\tilde{C}) + (TA - LC)^T P_d + \alpha P_d & P_d D_{ed} \\ * & -\alpha I \end{bmatrix} < 0 \quad (31)$$

$$\begin{bmatrix} \tilde{C}^T \tilde{C} - \gamma_1^2 P_d & \tilde{C}^T D_y \\ * & -\gamma_2^2 I + D_y^T D_y \end{bmatrix} < 0 \quad (32)$$

then the state estimation error system satisfies the L_∞ performance index (23).

Proof. Based on model (21), consider the following Lyapunov function.

$$V_d(t) = e^T(t) P_d e(t) \quad (33)$$

where matrix P_d is a positive definite symmetric matrix. $\square$

The derivative of the Lyapunov function is:

$$\dot{V}_d(t) = e^T(t) (P_d(TA - LC) + (TA - LC)^T P_d) e(t) + 2e^T(t) P_d D_{ed} \varphi(t) \quad (34)$$

where $D_{ed} = [TD - LD_y \quad -ND_y]$, $\varphi(t) = \begin{bmatrix} d(x) \\ \dot{d}(x) \end{bmatrix}$, $D_{rd} = [D_y \quad 0]$.

If the following condition is satisfied:

$$\dot{V}_d(t) \leq -\alpha V_d(t) + \alpha \varphi^T(t) \varphi(t) \quad (35)$$

where $\alpha > 0$, and since $\varphi^T(t) \varphi(t) \leq \|\varphi\|_\infty^2$, we obtain:

$$V_d(t) \leq V(0) \exp(-\alpha t) + \|\varphi(t)\|_\infty^2 \quad (36)$$

Substituting (27) and (28) into (29) and rearranging, we obtain:

$$\begin{bmatrix} e(t) \\ \varphi(t) \end{bmatrix}^T \begin{bmatrix} P_d(TA - L\tilde{C}) + (TA - LC)^T P_d + \alpha P_d & P_d D_{ed} \\ \star & -\alpha I \end{bmatrix} \begin{bmatrix} e(t) \\ \varphi(t) \end{bmatrix} < 0 \quad (37)$$

The linear inequality (25) is obtained.

Solving (29) gives:

$$\begin{aligned} V_d(t) &\leq V(0) \exp(-\alpha t) + \alpha \|\varphi\|_\infty^2 \int_0^t \exp(-\alpha(t-\tau)) d\tau \\ &= V(0) \exp(-\alpha t) + (1 - \exp(-\alpha t)) \|\varphi\|_\infty^2 \\ &\leq V(0) \exp(-\alpha t) + \|\varphi\|_\infty^2 \end{aligned} \quad (38)$$

In addition, from inequality (26), we can obtain:

$$\begin{bmatrix} e(t) \\ \varphi(t) \end{bmatrix}^T \begin{bmatrix} \tilde{C}^T \tilde{C} - \gamma_1^2 P_d & \tilde{C}^T D_y \\ \star & -\gamma_2^2 I + D_y^T D_y \end{bmatrix} \begin{bmatrix} e(t) \\ \varphi(t) \end{bmatrix} < 0 \quad (39)$$

Inequality (33) is equivalent to:

$$r^T(t)r(t) \leq \gamma_1^2 V_d(t) + \gamma_2^2 \varphi^T(t)\varphi(t) \quad (40)$$

Substituting (32) into (34), we obtain:

$$\begin{aligned} r^T(t)r(t) &\leq \gamma_1^2 [V(0) \exp(-\alpha t) + \|\varphi(t)\|_\infty^2] + \gamma_2^2 \|\varphi(t)\|_\infty^2 \\ &= \gamma_1^2 V(0) \exp(-\alpha t) + (\gamma_1^2 + \gamma_2^2) \|\varphi(t)\|_\infty^2 \end{aligned} \quad (41)$$

Thus, system (21) satisfies the L_∞ performance index (23).

4.2. Dynamic Threshold Design

In this section, based on model (21) and matrix inequality (26), a time-varying dynamic threshold is derived by considering the bounded initial estimation error and peak-bounded disturbance.

According to the L_∞ performance index (23), the residual evaluation function is derived as follows:

$$J(t) = \|r(t)\| \quad (42)$$

Assume that the initial value of the estimation error is bounded and satisfies the following condition:

$$e(0) \leq \epsilon_e \quad (43)$$

where ϵ_e is a known positive number, indicating that the initial value of the estimation error has an upper bound.

$$V(0) = e^T(0)P_d e(0) \quad (44)$$

$$V(0) \leq \lambda_{\max}(P_d) \epsilon_e^2 \quad (45)$$

where it denotes the maximum eigenvalue of the matrix.

Since the disturbance signal is peak-bounded, we obtain:

$$\|\varphi(t)\|_\infty \leq \epsilon_d \quad (46)$$

The resulting dynamic threshold is given by:

$$J_{th}(t) = \sqrt{\gamma_1^2 \exp(-\alpha t) \lambda_{\max}(P_d) \epsilon_e^2 + (\gamma_1^2 + \gamma_2^2) \epsilon_d^2} \quad (47)$$

4.3. Fault-Sensitivity Condition

In this section, the fault-sensitivity condition of the planetary-reducer transmission system is derived based on the Lyapunov function to ensure that the residual has a sufficient response amplitude to faults and avoid missed alarms.

Theorem 2. Given a positive scalar ρ , if there exists a positive definite symmetric matrix $Q \in \mathbb{R}^{n \times n}$, such that the following inequality holds:

$$\begin{bmatrix} (TA - L\tilde{C})^T Q + Q(TA - L\tilde{C}) - \tilde{C}^T \tilde{C} & \tilde{C}^T F_{rf} - QF_{ef} \\ * & \beta^2 I - F_{rf}^T F_{rf} \end{bmatrix} < 0 \quad (48)$$

the error system model (22) is stable and satisfies the H_{∞} performance index (24).

Let $F_{ef} = [TF - LR \quad -NR]$, $F_{rf} = [R \quad 0]$, and $\psi(t) = \begin{bmatrix} f(t) \\ \dot{f}(t) \end{bmatrix}$. System (22) can be transformed into:

$$\begin{aligned} \dot{e}_f(t) &= (TA - L\tilde{C})e_f(t) + F_{ef}\psi(t) \\ r_f(t) &= \tilde{C}e_f(t) + F_{rf}\psi(t) \end{aligned} \quad (49)$$

Since:

$$\|G_{rf}(s)\|_{\infty} > \beta \Leftrightarrow \int_0^t r_f^T r_f dt > \beta^2 \int_0^t \psi^T \psi dt \quad (50)$$

Based on model (43), consider the following Lyapunov function:

$$V(e_f) = e_f^T Q e_f \quad (51)$$

where matrix Q is a positive definite symmetric matrix.

$$\begin{aligned} J' &= \int_0^t r_f^T r_f dt - \beta^2 \int_0^t \psi^T \psi dt \\ &= \int_0^t (r_f^T r_f - \beta^2 \psi^T \psi) dt \\ &= \int_0^t (r_f^T r_f - \beta^2 \psi^T \psi - V(e_f)) dt + V(e_f) \\ &= \int_0^t [e_f^T \psi^T] \Pi \begin{bmatrix} e_f \\ \psi \end{bmatrix} dt + V(e_f) \end{aligned} \quad (52)$$

if

$$\Pi = \begin{bmatrix} \tilde{C}^T \tilde{C} - Q(TA - L\tilde{C})^T - (TA - L\tilde{C})Q & \tilde{C}^T F_{rf} - QF_{ef} \\ * & -\beta^2 I + F_{rf}^T F_{rf} \end{bmatrix} > 0 \quad (53)$$

It can be obtained that $J' > 0$.

The linear matrix inequality can be written in the following form:

$$\begin{bmatrix} (TA - L\tilde{C})^T Q + Q(TA - L\tilde{C}) - \tilde{C}^T \tilde{C} & \tilde{C}^T F_{rf} - QF_{ef} \\ * & \beta^2 I - F_{rf}^T F_{rf} \end{bmatrix} < 0 \quad (54)$$

4.4. Fault-Detection Decision for the Planetary-Reducer Transmission Module

In this part, the fault-detection scheme based on the adaptive observer is described in detail. The observer gain parameters are obtained through the alternating iterative optimization procedure described below and are subsequently used for residual generation and fault detection of the planetary-reducer transmission module.

It should be noted that the optimization constraints derived above contain products between different decision variables, such as $P_d T$, $P_d L$, QT , and QL . Therefore, when all decision variables are optimized simultaneously, the resulting problem is not a standard

convex optimization problem. To handle these bilinear couplings, an alternating iterative optimization strategy is adopted. The decision variables are divided into two groups, and one group is fixed while the other group is optimized in each iteration. When P_d and Q are fixed, the bilinear terms involving $P_d T$, $P_d L$, $Q T$, and $Q L$ become linear with respect to the observer-related variables. The corresponding constraints can then be formulated as linear matrix inequalities, and a convex subproblem is solved to update the observer parameters. Subsequently, the obtained observer-related variables are fixed, while P_d and Q are updated by solving another convex LMI subproblem. These two optimization steps are performed alternately until the variation of the objective function between two consecutive iterations is smaller than a prescribed tolerance or the maximum number of iterations is reached. In this manner, the original optimization problem containing bilinear matrix inequalities is handled through a sequence of convex LMI subproblems, and observer parameters satisfying the prescribed performance constraints are obtained.

The observer-gain design is formulated as the following optimization problem:

$$\min \gamma_1 + \gamma_2 - \beta$$

$$s.t. (25) (26) (42)$$

Fault-detection decision scheme: When the residual and the dynamic threshold satisfy the inequality, the occurrence of a fault is detected, and then a fault alarm is issued in Figure 7.

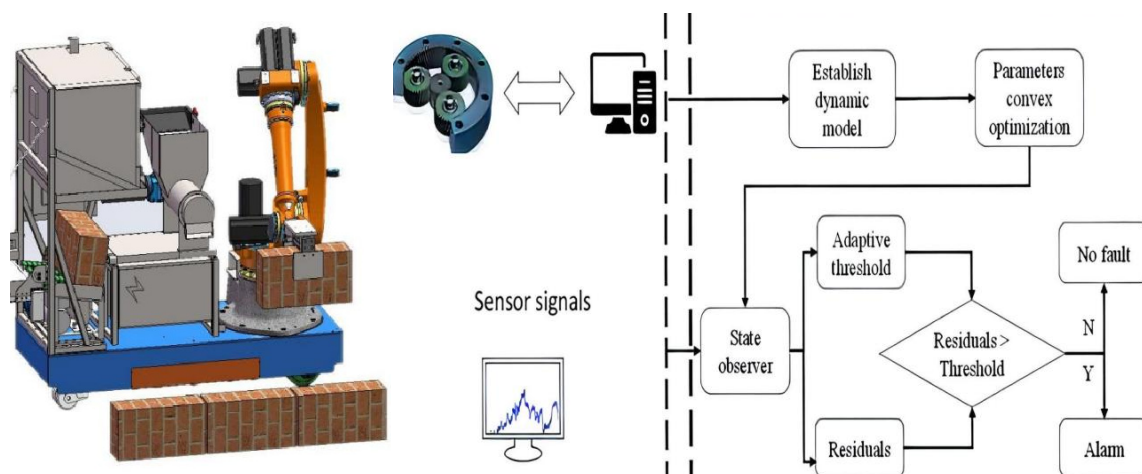


Figure 7. Fault-detection scheme.

5. Numerical Simulation Verification

In this section, the fault-detection scheme for the planetary-reducer transmission system is verified through numerical simulation. The previous analysis shows that when only actuator faults occur in the system over the full-frequency domain, the performance index cannot be applied to improve the fault sensitivity of the observer. To address this problem, this paper uses the auxiliary output function to successfully apply the H_{∞} performance index to improve the fault sensitivity of the adaptive observer. Meanwhile, the L_{∞} performance index is introduced to improve the disturbance robustness of the observer. The parameters of the planetary-reducer transmission system in Table 1.

Table 1. The parameters of the planetary-reducer transmission system.

Parameters	Value	Unit
J1	25.2	kg·mm
Jo	270.2	kg·mm
K	7.54×10^6	N/mm
K	7.54×10^6	N/mm
B	5.379	Ns/mm
B	5.379	Ns/mm
N	1/7	
η_t	0.99	

By introducing the system parameters into the planetary-reducer system model, the following state equation is obtained:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t) + Dd(t) \\ \tilde{y}(t) = \tilde{C}x(t) + Rf(t) + D_y d(t) \end{cases} \quad (55)$$

where:

$$A = \begin{bmatrix} -0.4313 & 1.5247 & -9.428 \times 10^4 \\ 0.1408 & -1.0053 & 6.0935 \times 10^4 \\ 1 & -7 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0397 & 0 \\ 0 & -0.0037 \\ 0 & 0 \end{bmatrix},$$

$$F = D = \begin{bmatrix} 0.0397 & 0 & 0 \\ 0 & 0.0037 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\tilde{y}(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) - CBu(t) \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C \\ CA \end{bmatrix}, \quad R = \begin{bmatrix} 0 \\ CF \end{bmatrix}, \quad D_y = \begin{bmatrix} 0 \\ CD \end{bmatrix}, \quad |D_{ed} = [TD - LD_y \quad -ND_y]$$

$$\varphi(t) = \begin{bmatrix} d(x) \\ \dot{d}(x) \end{bmatrix}, \quad D_{rd} = [D_y \quad 0], \quad F_{ef} = [TF - LR \quad -NR], \quad F_{rf} = [-R \quad 0], \quad \psi(t) = \begin{bmatrix} f(t) \\ \dot{f}(t) \end{bmatrix}$$

Solving this equation gives:

$$P_d = \begin{bmatrix} 6201 & -156 & -0.687 \\ -156 & 6202 & -0.305 \\ -0.687 & -0.305 & 7190 \end{bmatrix}, \quad Q = \begin{bmatrix} 7371 & -1520 & -6.95 \\ -1520 & 9832 & -9.48 \\ -6.95 & -9.48 & 8545 \end{bmatrix},$$

$$T = \begin{bmatrix} -0.073 & -34.998 & 0.0079 \\ 0.0041 & 637.486 & 0.0264 \\ -0.0027 & -0.1644 & -0.004 \end{bmatrix}$$

$$L = \begin{bmatrix} 11689 & 111.08 & 0.2316 & -1.144 & 12.658 & 0.15 \\ 7.916 & 965.28 & 0.8062 & 5.033 & -98 & 0.37 \\ -0.049 & -0.407 & 224.02 & 0.394 & 0.643 & -0.17 \end{bmatrix}$$

Based on the fault-mechanism analysis in Section 2.2, a time-varying equivalent crack fault is selected as the representative validation object. The following numerical simulation is performed under a representative nominal operating condition. The purpose of this simulation is to provide proof-of-concept verification of the proposed observer-based fault-detection method and to evaluate whether the designed residual can effectively respond to a progressive gear crack fault while maintaining robustness against the considered unknown disturbance.

The following simulation verification is carried out using MATLAB/Simulink. The numerical simulations were performed using MATLAB/Simulink R2024a. By comparing the residual response characteristics and detection results of observers under different performance index constraints, the ability of the designed adaptive observer to improve fault sensitivity using the H_- index while suppressing the influence of disturbances is evaluated. Meanwhile, the feasibility and effectiveness of the fault-detection scheme for the planetary-reducer transmission system are verified.

During the simulation, a bounded band-limited disturbance with an amplitude limited to $[-0.01, 0.01]$ is introduced to represent the unknown disturbance. The band limitation ensures that the disturbance and its first-order derivative remain bounded, which is consistent with Assumption 1. It is assumed that a time-varying fault occurs in the planetary-reducer system at $t = 18$ s, and the fault expression is shown as follows:

$$f(t) = \begin{cases} 0 & 0 \text{ s} < t \leq 18 \text{ s} \\ 1 - e^{-0.05(t-18)} & 18 \text{ s} < t < 30 \text{ s} \end{cases}$$

where $f(t)$ denotes the equivalent fault input associated with crack-induced degradation of gear meshing stiffness. Its amplitude characterizes the increasing influence of crack propagation on the transmission dynamics. $f(t) = 0$ represents the healthy condition, whereas an increase in the amplitude of $f(t)$ indicates an increase in the equivalent fault severity. It should be noted that $f(t)$ represents the equivalent dynamic effect of crack evolution rather than the physical crack length itself.

To quantitatively evaluate the residual responses generated by different observers, the root mean square (RMS) value is adopted as the evaluation index. For the i -th residual component $r_i(t)$, its RMS value over the time interval $[t_a, t_b]$ is defined as

$$R_{\text{RMS},i} = \sqrt{\frac{1}{t_b - t_a} \int_{t_a}^{t_b} r_i^2(t) dt} \quad (56)$$

where t_a and t_b denote the beginning and end of the evaluation interval, respectively. In numerical simulation, the corresponding discrete form is

$$R_{\text{RMS},i} = \sqrt{\frac{1}{N} \sum_{k=1}^N r_i^2(k)} \quad (57)$$

where N is the number of sampled residual data points within the evaluation interval. A smaller RMS value under fault-free conditions indicates stronger disturbance robustness, whereas a larger RMS value after fault occurrence indicates higher fault sensitivity.

Figure 8 shows the dynamic threshold and the residual evaluation result generated by the observer. Figure 9 shows the residual evaluation results generated by different observers. The simulation result is shown in Figure 8, where the blue line represents the designed dynamic threshold, and the green line represents the residual evaluation result generated by the observer. When $0 \text{ s} \leq t \leq 18 \text{ s}$, the system is in the fault-free state, and the residuals generated by the observers are all lower than the dynamic threshold, indicating that no false alarm occurs. When $18 \text{ s} \leq t$, after the time-varying fault is introduced, the residual amplitude gradually increases with time and exceeds the threshold. According to the fault decision logic, the system is determined to have a fault, and the fault is successfully detected.

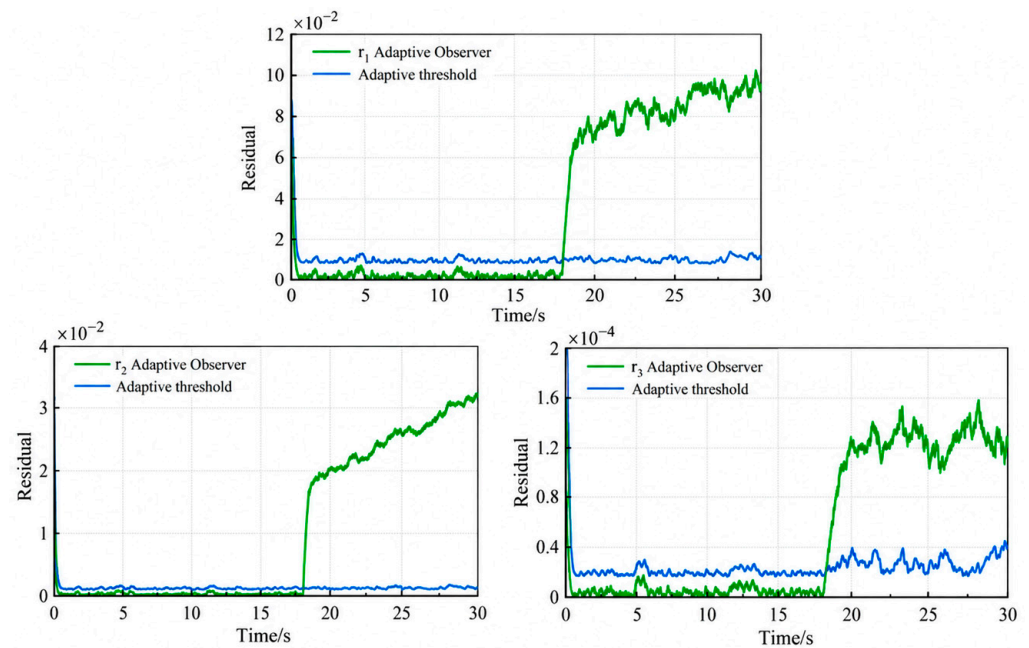


Figure 8. Dynamic threshold and residual evaluation.

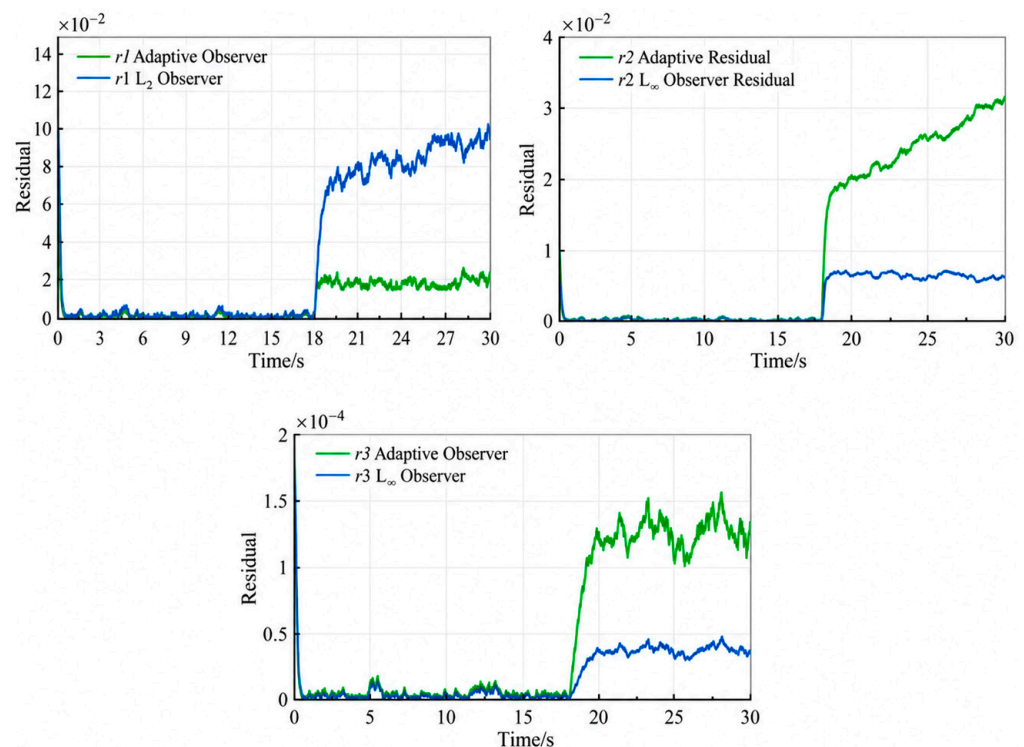


Figure 9. Residuals generated by the adaptive observer and the L_∞ observer.

Figure 9 shows the residual result of the adaptive observer and the residual result generated by the L_∞ observer that only satisfies the L_∞ disturbance-robustness index. Table 2 lists the root mean square values of the residual signals in the same time interval. It can be seen from Table 2 that within $18\text{ s} \leq t \leq 30\text{ s}$, the root mean square values of the residuals generated by the adaptive observer are 0.1003 , 0.0279 , and 1.36×10^{-4} , respectively. The corresponding root mean square values of the observer that only considers the L_∞ performance index are 0.0202 , 0.0072 , and 4.11×10^{-5} , respectively. According to the

comparison of the root mean square values, the observer designed in this paper considers the H_{∞} performance index, which significantly enhances fault sensitivity.

Table 2. Residual analysis of the adaptive observer.

Time Period		$18\text{ s} \leq t \leq 30\text{ s}$					
State Vector		x^1		x^2		x^3	
		Adaptive observer	L_{∞} observer	Adaptive observer	L_{∞} observer	Adaptive observer	L_{∞} observer
RMS		0.1003	0.0202	0.0279	0.0072	1.36×10^4	4.11×10^5

Compared with the traditional Luenberger observer, the adaptive observer designed in this section has more design degrees of freedom and lower conservatism. Under the same constraints, the solved observer gain can enable the observer to achieve better disturbance robustness and fault sensitivity. In this section, the residuals generated by the two observers are compared under the same conditions. The residuals generated by the adaptive observer and the Luenberger observer are shown in Figure 10.

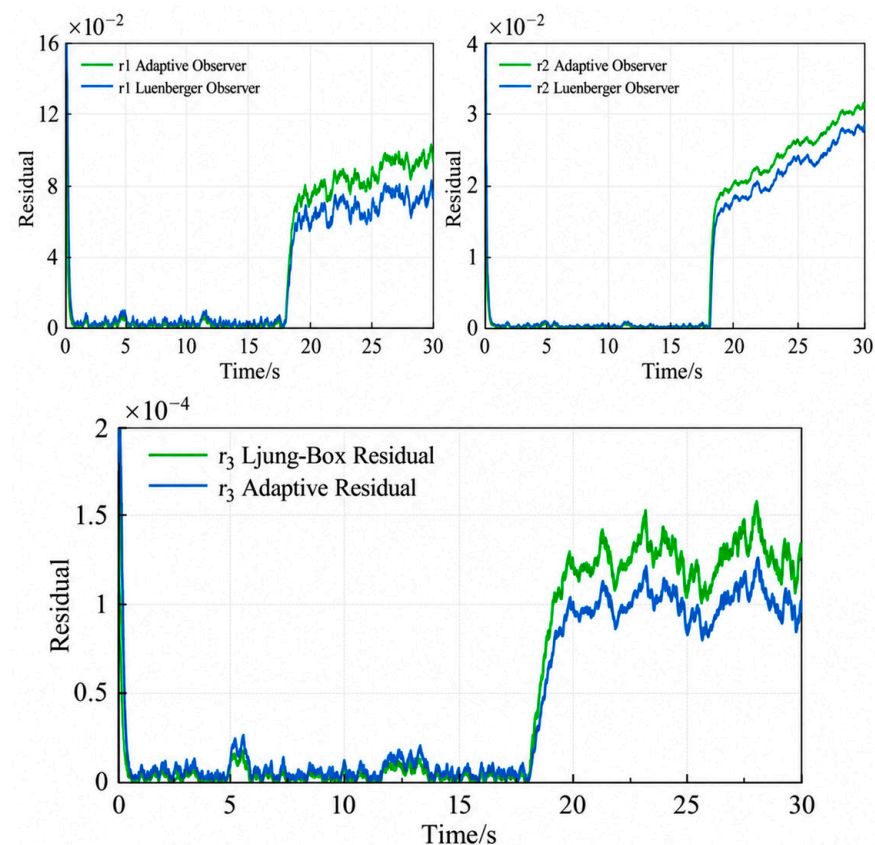


Figure 10. Residuals generated by the adaptive observer and the Luenberger observer.

In the multi-constraint and complex optimization process, the single gain parameter in the Luenberger observer makes it difficult for the generated residual to achieve better performance. Compared with the Luenberger observer, the adaptive observer designed in this paper has more design degrees of freedom, which enables the generated residual to achieve better performance. As shown in Table 3, when $0 \leq t < 18$, no fault occurs, and the residual generated by the adaptive observer is less affected by disturbances than the residual generated by the Luenberger observer. When $18 \leq t \leq 30$, after the time-varying fault signal

occurs, the residual generated by the designed adaptive observer is more sensitive to the time-varying fault signal than the residual generated by the Luenberger observer.

Table 3. Performance comparison of residuals from different observers.

Time Period	$0\text{ s} \leq t \leq 18\text{ s}$			$18\text{ s} \leq t \leq 30\text{ s}$		
State Vector	x_1	x_2	x_3	x_1	x_2	x_3
Adaptive observer	0.0429	0.0142	6.37×10^{-5}	0.0841	0.0249	1.61×10^{-4}
Luenberger observer	0.0534	0.0157	7.63×10^{-5}	0.0676	0.0225	6.74×10^{-5}

As shown in Table 3, when there is no fault, the root mean square value of the residual generated by the designed adaptive observer is smaller than that of the residual generated by the Luenberger observer. Therefore, the designed adaptive observer with more design degrees of freedom has stronger disturbance robustness. When a fault occurs, the root mean square value of the residual generated by the designed adaptive observer is larger than that of the residual generated by the Luenberger observer. Therefore, it has stronger fault sensitivity. Although the residual generated by the Luenberger observer can also detect faults, the residual generated by the adaptive observer has better disturbance robustness and fault sensitivity.

The present numerical simulation is mainly intended to provide proof-of-concept verification under a representative nominal operating condition. As discussed in Section 2.2, gear cracks are representative progressive faults that can reflect the gradual degradation of gear transmission performance. Therefore, the present study focuses on a time-varying equivalent crack fault rather than attempting to cover all possible combinations of operating conditions and fault modes. Under the considered nominal condition, the residual remains below the dynamic threshold during the fault-free stage and exceeds the threshold after the crack fault is introduced, indicating successful fault detection without false alarms in the current simulation.

It should be noted that the present verification does not constitute a comprehensive statistical assessment over the entire operating range of the planetary reducer. Variations in input speed, load torque, equivalent fault severity, and random disturbances may further affect the fault-detection performance. Systematic multi-condition verification and Monte Carlo repeated simulations, including statistical evaluation of detection delay and false alarm rate, will therefore be considered in future work.

In addition, physical experiments on a planetary-reducer test platform under variable-speed, variable-load, impact-load, and noise-disturbed operating conditions will be conducted to further evaluate the engineering applicability of the proposed method.

6. Conclusions

This chapter studies the planetary reducer of a construction robot and designs an adaptive-observer-based fault-diagnosis scheme with more design degrees of freedom for the planetary-reducer transmission system. By generating an augmented system using the auxiliary output, the actuator fault is transmitted to the augmented measured output equation, thereby improving the fault sensitivity of the observer through the H_- performance index. Meanwhile, the L_∞ performance index is introduced to improve the disturbance robustness of the observer, and Meanwhile, the disturbance-robustness performance index is introduced, and the observer gain parameters are obtained using an alternating iterative optimization procedure in which a sequence of convex LMI subproblems is solved.

In the numerical simulation, the comparison verifies that the auxiliary output method can apply the H_- performance index to the adaptive observer for fault detection of a single

actuator fault in the full-frequency domain, and effectively improves the fault sensitivity of the observer. At the same time, the effectiveness of the designed adaptive-observer-based fault-diagnosis method is verified. Furthermore, it is verified that the residual generated by the proposed method has better disturbance robustness and fault sensitivity than the residual generated by the Luenberger observer.

This method enriches the existing alternatives for fault detection of planetary reducers. Future research will further investigate unknown disturbance factors under actual operating conditions, extend the current validation to a wider range of input speeds, load torques, equivalent fault severity levels, and Monte Carlo repeated simulations, and conduct physical test-bench experiments to provide a more comprehensive assessment of the robustness, reliability, and engineering applicability of the proposed fault-detection method.

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