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Magnetically Induced Two-Phonon Blockade in a Hybrid Spin–Mechanical System

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Abstract: Phonon blockade is an important quantum effect for revealing the quantum behaviors of mechanical systems. For a nitrogen-vacancy center spin strongly coupled to a mechanical resonator via the second-order magnetic gradient, we show that the qubit driving can lead to the implementation of the two-phonon blockade, while the usual mechanical driving only allows for the appearance of a single-phonon blockade. As a signature, we investigate three-phonon antibunching with a simultaneous two-phonon bunching process by numerically calculating the second-order and third-order correlation functions. We also analyze in detail the influence of the system parameters (including the qubit driving strength, the dephasing rate of the qubit, as well as the thermal phonon number) on the quality of the two-phonon blockade effect. Our work provides an alternative method for extending the concept of a phonon blockade from a single phonon to multiphonon. It is of direct relevance for the engineering of multiphonon quantum coherent devices and thus has potential applications in quantum information processing.

Keywords: spin–phonon coupling; second-order magnetic gradient; phonon statistics; two-phonon blockade



Citation: Liu, H.-Y.; Yin, T.-S.; Chen, A. Magnetically Induced Two-Phonon Blockade in a Hybrid Spin–Mechanical System. *Magnetochemistry* **2024**, *10*, 41. <https://doi.org/10.3390/magnetochemistry10060041>

Academic Editor: Roberto Zivieri

Received: 11 April 2024

Revised: 28 May 2024

Accepted: 30 May 2024

Published: 31 May 2024



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1. Introduction

The study of the quantum nature of mechanical systems has attracted extensive attention in the last decade, which plays a crucial role in exploring the validity of quantum mechanics at macroscopic scales [1,2]. In particular, the manipulation and control of the mechanical quantum (i.e., phonons) has found many promising applications in quantum information processing and quantum measurement [3–8]. With the significant advances of micro- and nano-mechanical fabricating technologies, many achievements have been made, including cooling the mechanical resonators to their quantum ground state [9–12], the observations of mechanical nonclassical states [3,13–16], and a generation of mechanical squeezing and quantum entanglement [17–20]. Phonon blockade, as a typical quantum effect, can serve as a signature of quantum behavior in nanomechanical resonators. A photon blockade [21–33] is a phenomenon in which the appearance of one phonon prevents the excitation of the second phonon in a nonlinear mechanical oscillator. The realization of a phonon blockade generally requires a strong mechanical nonlinearity, which can be realized via quadratical optomechanical coupling [34–37], the coupling between the nanomechanical resonator and a qubit [38–41], or via the magnetic-gradient-induced two-phonon nonlinear coupling [42,43]. In these cases, the physical mechanism for the generation of the phonon blockade essentially originates from the energy-level anharmonicity of a system. Differently, in the presence of weak mechanical nonlinearity, one can also obtain the phonon blockade, which is based on the destructive quantum interference between different excitation pathways [40,44–48]. However, most of the research on the phonon blockade mainly focuses on the case of a single-phonon blockade [34–48], while a multiphonon blockade is rarely involved [49]. It is known that the single-phonon blockade phenomenon is closely associated with the experimental realization of a single-phonon

source. It should be noted that the multiphonon blockade process could be used to produce a phonon stream with more than one phonon and multiphonon bundle emission, which have important applications for the engineering of multiphonon quantum coherent devices. Therefore, it is desirable to extend the concept of a phonon blockade from a single phonon to multiphonon.

On the other hand, with the recent experimental advances, mechanical systems with a high quality factor can be interfaced with a variety of physical quantum systems to form hybrid systems [50–59]. These systems enable extended capabilities by exploiting the complementary functionalities of the individual components [60]. For example, mechanical resonators could coherently interact with electronic or nuclear spins, single charges, and photons [1,50]. Due to the outstanding coherence properties and the ability to coherently couple to various external fields, nitrogen-vacancy (NV) centers in diamond are appealing candidates for integration into a hybrid quantum system [61,62]. The electronic ground state of the negatively charged NV center is a spin $S = 1$ triplet $|m_s\rangle$ ($m_s = 0, \pm 1$) with the zero-field splitting $D \simeq 2.88$ GHz. In the presence of external static magnetic fields, the degeneracy between the states $|\pm 1\rangle$ is shifted. The coupling between NV spins and mechanical resonators can be induced by strain or magnetic field gradients [63–73]. The strain-induced spin–mechanical coupling has an intrinsic nature, which has been utilized to engineer spin squeezed states of an NV ensemble [63], to enhance cooling and lasing effects for phonons [64] and to characterize the spin–strain sensitivities [65,66]. Differently, the magnetically induced spin–phonon coupling is associated with the time-varying magnetic field yielded by the oscillation of mechanical resonators, which leads to the position-dependent Zeeman shift of the NV spin. Then, the spin qubit can be parametrically coupled to the position of a mechanical oscillator. This strong first-order magnetic gradient-induced coupling can be utilized to sense the motion of mechanical resonators [69,70] and observe a phononic Mollow triplet [71]. In addition, recent works have illustrated specific geometries of NV–mechanical systems, which could result in a null first-order magnetic gradient about the position of a mechanical oscillator [74–76]. Thus, one needs to consider the quadratic dependence of the coupling on the position, namely, the second-order magnetic gradient coupling. This type of coupling has been proposed to cool a mechanical resonator to the quantum regime by heating it [74] and generate two-mode squeezing [75]. In particular, the second-order magnetic gradient-induced spin–phonon coupling can be used to implement two-phonon nonlinearity of the Jaynes–Cummings type [76]. Related works show that this two-phonon nonlinearity enables the generation of nonclassical mechanical states [76] and phonon blockade [42].

Combining the above two aspects, here, we propose a scheme to achieve the two-phonon blockade effect in a hybrid spin–mechanical system via the second-order magnetic gradient. Specifically, the second-order magnetic gradient could induce the two-phonon nonlinear coupling between spin qubits and phonons. This strong two-phonon nonlinearity allows for the generation of a two-phonon blockade when the spin qubit is weakly driven in the two-phonon resonant case. This quantum effect essentially originates from the energy-level anharmonicity of the spin–mechanical system. By numerically calculating the second-order and third-order correlation functions, we characterize the statistical properties of phonons. Our work provides a method to yield the typical quantum effects of phonons, which is of direct relevance for realizations of multiphonon sources. Note that for the experimental feasibility of our proposal, the qubit can be given by the electronic spin of NV centers [77]. The coupling between a single spin and a mechanical oscillator is induced by the position-dependent Zeeman shift [68–73]. In addition, the quadratic dependence of the coupling on the mechanical position can be realized by particular geometries, which has been analyzed by some works [74–76]. Based on this theoretical and experimental progress, we propose the magnetically induced hybrid spin–mechanical system to study the effect of two-phonon blockade.

This paper is organized as follows: In Section 2, we describe the hybrid spin–mechanical system with the second-order magnetic gradient and derive the Hamiltonian with the two-

phonon nonlinear coupling between the mechanical resonator and an effective two-level system. In Section 3, we study the two-phonon blockade effect induced by the two-phonon nonlinearity when the effective two-level system is weakly driven. Specifically, we investigate the phonon statistics by numerically calculating the second-order and third-order correlation functions and compare the results in the two cases of weakly driving the mechanical resonator and the spin qubit. In addition, we also discuss the influence of the system parameters on the two-phonon blockade effect in detail. The conclusions are given in Section 4.

2. Theoretical Model

We consider a hybrid system consisting of a mechanical resonator coupled to an NV center spin through the second-order magnetic gradient. This second-order magnetic-gradient-induced spin–phonon coupling can be realized by symmetrically positioning two cylindrical nanomagnets on the two sides of the resonator [76]. We assume the mechanical oscillator oscillates only along the z axis. A single NV center in a diamond film [77] is positioned on top of the mechanical oscillator. Due to the mechanical oscillation, the NV spin can produce the position-dependent Zeeman shift and then couple to the mechanical motion. The magnetic field $\mathbf{B}$ experienced by the NV center can be expanded up to the second order in terms of the mechanical displacement z , i.e., $\mathbf{B} \approx \partial\mathbf{B}/\partial z(0)z + \frac{1}{2}\partial^2\mathbf{B}/\partial z^2(0)z^2$. When the NV center is positioned at an extremum of the magnetic field, i.e., the first-order magnetic gradient is zero, one needs to consider the second-order magnetic-gradient-induced coupling. Here, we consider applying two oscillating magnetic fields, one along the x axis with frequency ω_x and another in the z axis with frequency ω_d . Then, the system Hamiltonian reads ($\hbar = 1$)

$$H = DS_z^2 + \varepsilon_x \cos(\omega_x t)S_x + \varepsilon_d \cos(\omega_d t)S_z + H_m + g(b^\dagger + b)^2 S_z, \quad (1)$$

where S_x and S_z are the x and z components of the spin operator, respectively. The first three terms in Equation (1) represent the Hamiltonian of the NV center including the two oscillating magnetic fields. The fourth term is the free Hamiltonian of the mechanical oscillator with $H_m = \omega_m b^\dagger b$, where ω_m is the mechanical resonant frequency, and b ($b^\dagger$) is the annihilation (creation) operator of the mechanical mode. The displacement z can be expressed as $z = z_{\text{zpf}}(b^\dagger + b)$, where z_{zpf} is the zero-point fluctuation of the mechanical resonator. The last term in Equation (1) describes the second-order magnetically induced spin–phonon coupling with the coupling strength $g = \frac{1}{2}\mu_B g_s z_{\text{zpf}}^2 G$. Here, μ_B is the Bohr magneton, $g_s \simeq 2$ is the Landé factor of the NV center, and $G = \partial^2\mathbf{B}/\partial z^2(0)$ is the second-order magnetic gradient.

Here, we consider the system parameters to satisfy the conditions of $\varepsilon_x \ll \{\omega_x, D, \omega_x - D\}$. In this case, the NV spin can be treated as an effective two-level system with transition frequency $\Omega \approx \Omega_x^2/(\omega_x - D)$. The reader is referred to Refs. [69,73,76] for details on the specific derivation of how the effective two-level system is defined. In the following, we focus on the two-phonon resonant case $\Omega = 2\omega_m$ and assume the condition of $\omega_m, \omega_d \gg g, \varepsilon_d$ and $\varepsilon_d \ll \Omega$. In a frame rotating with frequency ω_d for the mechanical mode and $\omega_d/2$ for the effective two-level system, one can safely ignore the terms that oscillate with high frequencies, i.e., $\pm\omega_d$ and $\pm 2\omega_d$, under the rotating-wave approximation. Then, the system Hamiltonian in Equation (1) can be approximately written as

$$H_{\text{eff}} = \Delta\sigma_+\sigma_- + \Delta_m b^\dagger b + g(b^{\dagger 2}\sigma_- + b^2\sigma_+) + \varepsilon_d(\sigma_+ + \sigma_-), \quad (2)$$

where the detunings are $\Delta = \Omega - \omega_d$ and $\Delta_m = \omega_m - \omega_d/2$, σ_+ and σ_- are the raising and lowering operators for the effective two-level system, and ε_d is the driving amplitude for the effective two-level system. Here, we obtain an effective coherently driven two-phonon Jaynes–Cummings Hamiltonian, which is the starting point of our discussion.

Including the dissipation caused by the system–bath coupling, the dissipative dynamics of the hybrid quantum system are described by the master equation

$$\begin{aligned}\dot{\rho} = & -i[H_{\text{eff}}, \rho] + \gamma_m(n_{\text{th}} + 1)\mathcal{D}[b]\rho \\ & + \gamma_m n_{\text{th}}\mathcal{D}[b^\dagger]\rho + \gamma_z\mathcal{D}[\sigma_+\sigma_-]\rho,\end{aligned}\quad (3)$$

where $\mathcal{D}[o]\rho = o\rho o^\dagger - \frac{1}{2}(o^\dagger o\rho + \rho o^\dagger o)$ is the standard Lindblad superoperator, γ_m and γ_z are the decay rate of the mechanical oscillator and dephasing rate of the qubit, respectively, and n_{th} is the thermal phonon occupation number with $n_{\text{th}} = [\exp(\hbar\omega_m/k_B T) - 1]^{-1}$, where T is the temperature of the thermal reservoir.

To exhibit the quantum behavior of the phonons, we investigate the statistical properties of the phonons, which can be characterized by the delay-time second-order correlation function of phonons

$$g^{(2)}(t, \tau) = \frac{\langle b^\dagger(t)b^\dagger(t+\tau)b(t+\tau)b(t) \rangle}{\langle b^\dagger(t)b(t) \rangle \langle b^\dagger(t+\tau)b(t+\tau) \rangle} \quad (4)$$

and the delay-time third-order correlation function of phonons

$$g^{(3)}(t, \tau_1, \tau_2) = \frac{\langle b^\dagger(t)b^\dagger(t+\tau_1)b^\dagger(t+\tau_2)b(t+\tau_2)b(t+\tau_1)b(t) \rangle}{\langle b^\dagger(t)b(t) \rangle \langle b^\dagger(t+\tau_1)b(t+\tau_1) \rangle \langle b^\dagger(t+\tau_2)b(t+\tau_2) \rangle}. \quad (5)$$

In this work, we focus on the case of phonon statistics in the steady state. Therefore, here, we assume $\tau_1 = \tau_2$ and define the steady state delay-time correlation function as $g^{(2)}(\tau) = \lim_{t \rightarrow \infty} g^{(2)}(t, \tau)$ and $g^{(3)}(\tau) = \lim_{t \rightarrow \infty} g^{(3)}(t, \tau_1, \tau_2)$. For the case of zero delay between phonons, i.e., $\tau = \tau_1 = \tau_2 = 0$, the equal-time second-order and third-order correlation functions in the steady state can be expressed as

$$g^{(n)}(0)_{\text{ss}} = \lim_{t \rightarrow \infty} \frac{\langle b^{\dagger n} b^n \rangle(t)}{\langle b^\dagger b \rangle^n(t)} = \frac{\text{Tr}(b^{\dagger n} b^n \rho_{\text{ss}})}{[\text{Tr}(b^\dagger b \rho_{\text{ss}})]^n}, \quad (6)$$

where $n = 2, 3$, and ρ_{ss} is the steady state density operator of the hybrid coupled system (see Equation (3)). Typically, $g^{(n)}(\tau) < g^{(n)}(0)_{\text{ss}}$ characterizes the bunching effect of an n -phonon, where n phonons tend to distribute themselves preferentially in bunches rather than at random, and the phonon fields show super-Poisson statistics with $g^{(n)}(0)_{\text{ss}} > 1$. Instead, $g^{(n)}(\tau) > g^{(n)}(0)_{\text{ss}}$ correspond to the phenomenon of n -phonon antibunching with sub-Poisson statistics $g^{(n)}(0)_{\text{ss}} < 1$. In the following text, we are concerned mainly with the single-phonon blockade effect and two-phonon blockade effect. The former could be presented by the result $g^{(2)}(0)_{\text{ss}} \rightarrow 0$, while the generation of the latter requires both conditions of two-phonon bunching with $g^{(2)}(0)_{\text{ss}} > 1$ and three-phonon antibunching $g^{(3)}(0)_{\text{ss}} < 1$.

3. The Two-Phonon Coupling Induced the Two-Phonon Blockade via Driving the Effective Two-Level System

To observe phonon quantum behaviors, it is highly desirable to realize a strong coupling regime. That is, the two-phonon-coupling strength $g = \frac{1}{2}\mu_B g_s z_{\text{zpf}}^2 G$ needs to be much larger than the mechanical dissipation rate. This can be realized by enhancing the zero-point fluctuation z_{zpf} of the mechanical oscillator and the magnetic gradient G . The strong spin–phonon nonlinear coupling will induce the energy-level anharmonicity of the system. As shown in Figure 1, the ground state of the coupled system is $|0, g\rangle$, and the first excited state is $|1, g\rangle$. However, the second excited state is the superposition state of $|2, g\rangle$ and $|0, e\rangle$ with the energy splitting $2\sqrt{2}g$. The corresponding dressed states of the hybrid system can be denoted as $|m, \pm\rangle = (|m, g\rangle \pm |m-2, e\rangle)/\sqrt{2}$ ($m = 2, 3, \dots$). Here, $|m\rangle$ and $|g\rangle$ ($|e\rangle$) represent the eigenstates of the mechanical mode and the ground state (excited state) of the effective two-level system, respectively.

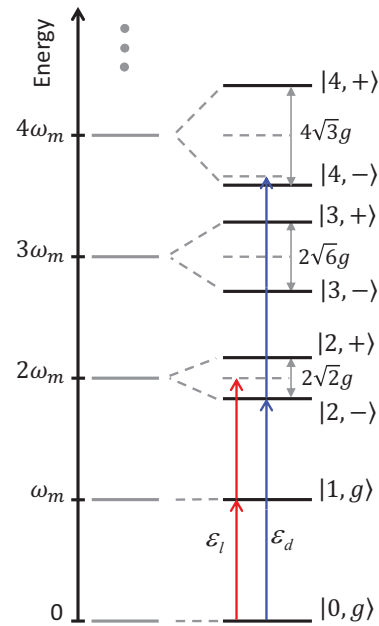


Figure 1. Energy-level diagram of the hybrid spin-resonator system with a strong two-phonon nonlinearity. Here, $|m, \pm\rangle$ denote the corresponding dressed states with $|m, \pm\rangle = (|m, g\rangle \pm |m - 2, e\rangle) / \sqrt{2}$ ($m \geq 2$); $|m\rangle$ and $|g\rangle$ ($|e\rangle$) represent the eigenstates of the mechanical mode and the ground state (excited state) of the effective two-level system, respectively. The red arrows describe the mechanical resonant driving $\Delta'_m = 0$ when the mechanical resonator is driven with the driving strength ε_l , which shows the appearance of the single-phonon blockade. The blue arrows correspond to the case of two-phonon resonant driving $\Delta_m/g = \pm\sqrt{2}$ when the spin qubit is driven with the driving strength ε_d . This process indicates the generation of the two-phonon blockade effect.

Note that when the mechanical resonator is weakly driven, the two-phonon Jaynes–Cummings model can be utilized to achieve the single-phonon blockade effect, which has been studied in [42]. In order to study whether the two-phonon blockade can be generated in this case and compare it with the case of weakly driving the qubit, we first consider the case of weakly driving the mechanical mode with the Hamiltonian $H_b = \varepsilon_l(b^\dagger e^{-i\omega_l t} + b e^{i\omega_l t})$. In a frame rotating with frequency ω_l for the mechanical mode and $2\omega_l$ for the effective two-level system, the effective Hamiltonian can be written as $H'_{\text{eff}} = \Delta' \sigma_+ \sigma_- + \Delta'_m b^\dagger b + g(b^{\dagger 2} \sigma_- + b^2 \sigma_+) + \varepsilon_l(b + b^\dagger)$ with the detunings $\Delta' = \Omega - 2\omega_l$ and $\Delta'_m = \omega_m - \omega_l$.

In Figure 2a,c, we calculate the average phonon number n_b and the steady-state equal-time second-order correlation function $g^{(2)}(0)_{\text{ss}}$ and three-order correlation function $g^{(3)}(0)_{\text{ss}}$ versus the driving detuning Δ'_m/g by numerically solving the master Equation (3) with the effective Hamiltonian H'_{eff} . Namely, Figure 2a,c correspond to the case of driving the mechanical resonator. It is shown that when the mechanical resonator is resonantly driven, i.e., $\Delta'_m = 0$, the curve of the average phonon number presents a single peak, as shown in Figure 2a. However, in Figure 2c, when $\Delta'_m = 0$, the curves of $g^{(2)}(0)_{\text{ss}}$ and $g^{(3)}(0)_{\text{ss}}$ have a dip, where $g^{(2)}(0)_{\text{ss}} \rightarrow 0$ and $g^{(3)}(0)_{\text{ss}} \rightarrow 0$. This indicates the occurrence of a single-phonon blockade, which can be explained from the energy-level diagram in Figure 1. When the driving laser is on resonance with the $|0, g\rangle \rightarrow |1, g\rangle$ transition, i.e., $\Delta'_m = 0$, the same $|1, g\rangle \rightarrow |2, \pm\rangle$ transition is detuned and will be suppressed for $g \gg \gamma_m$, as indicated by the red arrow in Figure 1. In addition, in Figure 2c there are two peaks in the curves of $g^{(2)}(0)_{\text{ss}}$ and $g^{(3)}(0)_{\text{ss}}$ at the two-phonon resonant driving, i.e., $\Delta'_m = \pm\sqrt{2}g$, where $g^{(2)}(0)_{\text{ss}} \gg 1$ and $g^{(3)}(0)_{\text{ss}} \gg 1$, which predicts the phonon-induced tunneling. The above results indicate that when the mechanical resonator is weakly driven, the single-phonon blockade can be observed, while the two-phonon blockade effect can not be generated. In contrast, Figure 2b,d describe the situation where the effective two-

level system is weakly driven. In Figure 2b,d, we plot the average phonon number n_b and the steady-state equal-time correlation functions $g^{(2)}(0)_{ss}$ and $g^{(3)}(0)_{ss}$ versus the driving detuning Δ_m/g by numerically solving the master equation but with the effective Hamiltonian H_{eff} in Equation (2). Different from that in Figure 2a,b, the curve of n_b is characterized by a double-peak structure, which corresponds to the vacuum Rabi splitting. Moreover, it is found in Figure 2d that when the detuning is $\Delta_m = 0$, the values of $g^{(2)}(0)_{ss}$ are much larger than 1. Note that the transition $|0, g\rangle \rightarrow |1, g\rangle$ will be forbidden when the qubit is weakly driven. This is due to the absorption of two phonons by a qubit, which is induced by the nonlinear interaction. However, in the case of two-phonon resonant driving, i.e., $\Delta_m = \pm \frac{\sqrt{2}}{2}g$, the curve of $g^{(2)}(0)_{ss}$ has two dips with $g^{(2)}(0)_{ss} > 1$, which indicates the two-phonon bunching. Meanwhile, one can have $g_{ss}^{(3)}(0) < 1$ with three-phonon antibunching when $\Delta_m/g = \pm \frac{\sqrt{2}}{2}$. This means that the two-phonon blockade effect can be achieved via weakly driving the qubit instead of the mechanical resonator. This quantum phenomenon can also be explained from the energy-level diagram in Figure 1. When the qubit driving satisfies the two-phonon resonant condition for the mechanical resonator, i.e., $\Delta_m/g = \pm \frac{\sqrt{2}}{2}$, the ground state $|0, g\rangle$ can be transitioned to the two-phonon state $|2, \pm\rangle$, while the $|2, \pm\rangle \rightarrow |3, \pm\rangle$ transition will be suppressed due to the frequency detuning, as shown by the blue arrow in Figure 1.

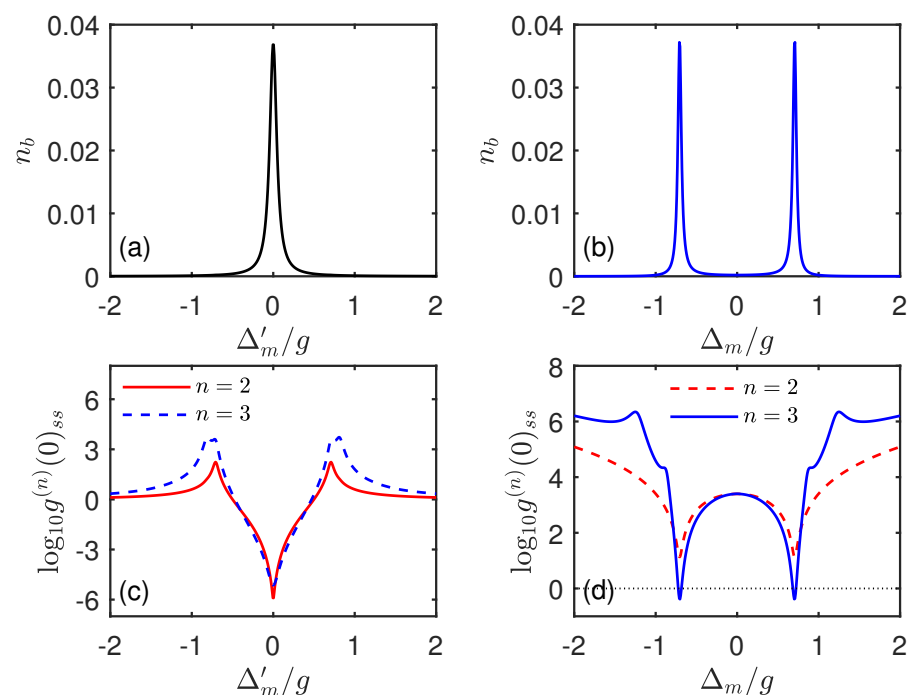


Figure 2. (a,b) The average phonon number n_b and (c,d) the steady-state equal-time second-order correlation function $g^{(2)}(0)_{ss}$ and three-order correlation function $g^{(3)}(0)_{ss}$ versus the driving detuning Δ'_m/g and Δ_m/g . Here, (a,c) and (b,d) correspond to the cases of driving the mechanical resonator and the qubit, respectively. The system parameters we take are $g = 10\gamma_m$, $\gamma_z = 0$, $n_{th} = 0$, (a,c) $\varepsilon_l = 0.1\gamma_m$, (b,d) $\varepsilon_d = 0.1\gamma_m$.

To clearly present the generation of the two-phonon blockade effect, the steady-state equal-time second-order correlation function $g^{(2)}(0)_{ss}$ and three-order correlation function $g^{(3)}(0)_{ss}$ are plotted as a function of the mechanical driving detuning Δ_m/γ_m and the coupling strength g/γ_m , as shown in Figure 3.

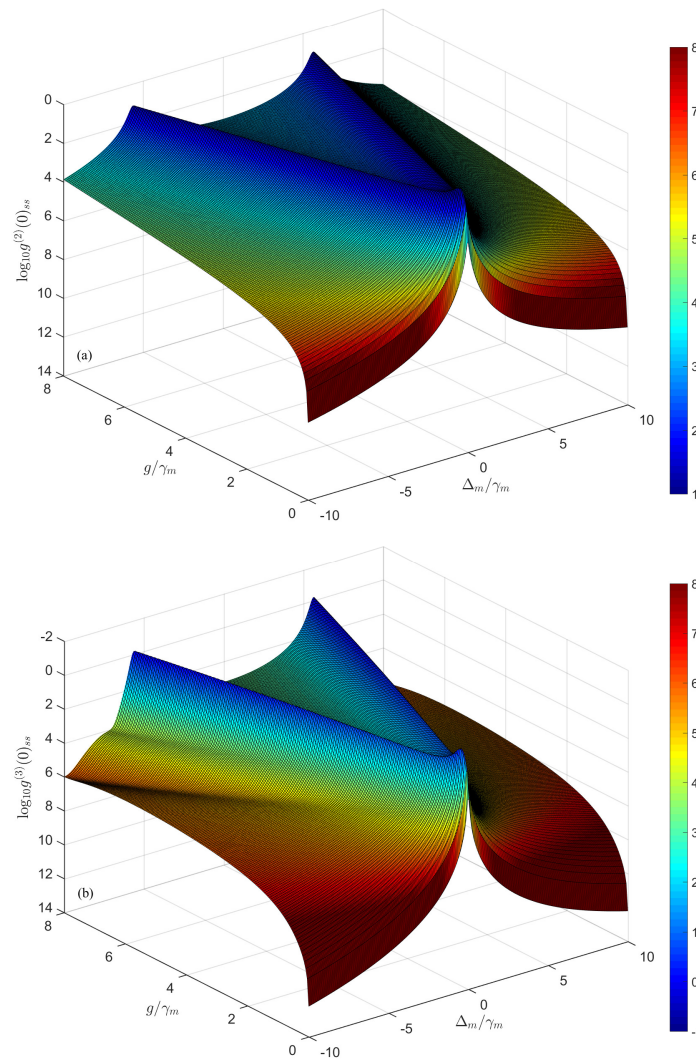


Figure 3. (a) Steady-state equal-time second-order correlation function $g^{(2)}(0)_{ss}$ and (b) steady-state equal-time three-order correlation function $g^{(3)}(0)_{ss}$ versus the mechanical driving detuning Δ_m/γ_m and the coupling strength g/γ_m . The system parameters are the same as that in Figure 2.

It can be seen from Figure 3a that in a wide range of parameters g and Δ_m , the values of $g^{(2)}(0)_{ss}$ are always larger than 1. However, the values of $g^{(3)}(0)_{ss}$ can be smaller than 1 around the two-phonon resonant driving, as depicted by the deep blue region in Figure 3b. Actually, the deep blue regions with the two “ridge” shapes in Figure 3a,b correspond to the case of two-phonon resonant driving $\Delta_m/g = \pm \frac{\sqrt{2}}{2}$, where $g^{(3)}(0)_{ss} < 1 < g^{(2)}(0)_{ss}$. These results show the appearance of the two-phonon blockade effect, which is consistent with the discussion in Figure 2d. In addition, to study how the driving strength affects the phonon statistical characteristic, in Figure 4, we plot $g^{(2)}(0)_{ss}$ and $g^{(3)}(0)_{ss}$ versus the qubit driving strength ε_d/γ_m in the case of two-phonon resonant driving. One can find that when the driving strength ε_d/γ_m increases, the values of $g^{(2)}(0)_{ss}$ gradually decrease and finally tend to 1, while the values of $g^{(3)}(0)_{ss}$ first decrease, then increase, and finally approach 1. To obtain the desired two-phonon blockade effect with $g^{(2)}(0)_{ss} > 1$ and $g^{(3)}(0)_{ss} < 1$, the qubit driving field is required to be weak, i.e., $\varepsilon_d \ll \gamma_m$, as shown by the gray region in Figure 4. For the chosen system parameters, when the values of ε_d/γ_m range from 0.06 to 0.73, the two-phonon blockade effect can be generated [see the inset in Figure 4].

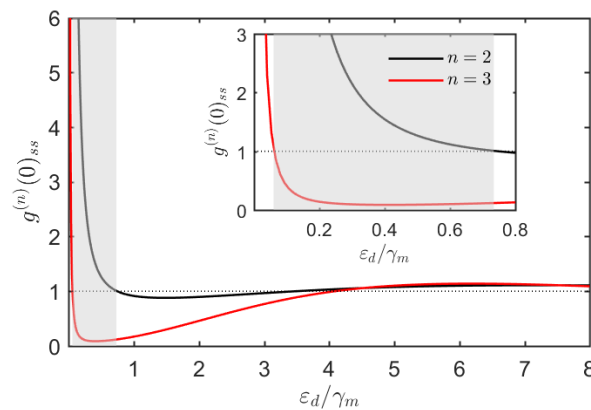


Figure 4. Steady-state equal-time second-order correlation function $g^{(2)}(0)_{ss}$ and steady-state equal-time three-order correlation function $g^{(3)}(0)_{ss}$ versus the qubit driving strength ε_d/γ_m . The system parameters are the same as that in Figure 2 except for $\Delta_m = g/\sqrt{2}$.

In the discussion above, we mainly focused on how the equal-time correlation function was affected by the system parameters. Next, we present the delayed-time second-order correlation function $g^{(2)}(\tau)$ and three-order correlation function $g^{(3)}(\tau)$ versus the scaled time delay $\gamma_m\tau$, as shown in Figure 5a,b. First, it can be seen from Figure 5a that the curves with different dephasing rates γ_z show $g^{(2)}(\tau) < g^{(2)}(0)$, which presents the phonon bunching phenomenon.

However, the curves of the three-order correlation function show $g^{(3)}(0) < g^{(3)}(\tau)$. In addition, from the prospective of the dephasing rate, the values of $g^{(2)}(0)$ and $g^{(3)}(0)$ both increase with γ_z increasing. However, when the values of γ_z further increase, e.g., $\gamma_z = 2\gamma_m$ and $\gamma_z = 4\gamma_m$, the values of $g^{(3)}(0)$ become larger than 1, as depicted in Figure 5b. This reveals that the qubit dephasing rate will destroy the quality of the two-phonon blockade effect to some content. In order to clearly illustrate this influence, in Figure 5c, we plot $g^{(2)}(0)_{ss}$ and $g^{(3)}(0)_{ss}$ versus the qubit dephasing rate γ_z/γ_m . It is shown that when the dephasing rate γ_z/γ_m increases, one can always have $g^{(2)}(0)_{ss} > 1$, while $g^{(3)}(0)_{ss} < 1$ is obtained only for a weak dephasing rate. Therefore, it is necessary to suppress the qubit dephasing rate for the observation of the quantum effects of the two-phonon blockade.

Note that we have assumed cooling the mechanical resonator to its quantum ground state, i.e., $n_{th} = 0$, in the above discussion. In order to further clarify the dependence of the two-phonon blockade effect on the mechanical thermal noise, in Figure 6, we illustrate the correlation functions $g^{(2)}(0)_{ss}$ and $g^{(3)}(0)_{ss}$ as a function of the driving detuning Δ_m/g for different thermal phonon numbers n_{th} . When the thermal phonon number is $n_{th} = 10^{-4}$, as shown in Figure 6a, the curves of the correlation function $g^{(n)}(0)_{ss}$ ($n = 2, 3$) have two distinct dips around the two-phonon resonant driving (i.e., $\Delta_m/g = \pm\sqrt{2}/2$). In this case, one can still obtain $g^{(2)}(0)_{ss} > 1$ and $g^{(3)}(0)_{ss} < 1$. However, in Figure 6b, when $n_{th} = 10^{-3}$, although there exists two distinct dips, both values of $g^{(n)}(0)_{ss}$ ($n = 2, 3$) are larger than 1 around the detuning $\Delta_m/g = \pm\sqrt{2}/2$. When the thermal phonon number further increases, e.g., $n_{th} = 10^{-2}$, the minimum values of $g^{(2)}(0)_{ss}$ and $g^{(3)}(0)_{ss}$ are much larger than 1, as seen from Figure 6c. Meanwhile, the dips in the curve of the steady-state equal-time three-order correlation function almost disappear. The results indicate that the nonclassical property of phonons is sensitive to the thermal noise. To fully present how the quantum effect of phonons is influenced by the thermal noise, in Figure 7, the correlation functions $g^{(2)}(0)_{ss}$ and $g^{(3)}(0)_{ss}$ are both plotted as a function of the thermal phonon number n_{th} in the case of two-phonon resonant driving. It is obvious that both curves have a large change with n_{th} increasing. This reveals that the presence of the thermal phonon number has a significant effect of the phonon statistical properties. Therefore, to better obtain the desired the two-phonon blockade, it requires cooling the mechanical resonator to low temperature, which could be realized by means of laser cooling [10,11,78] or using another spin qubit [68,76].

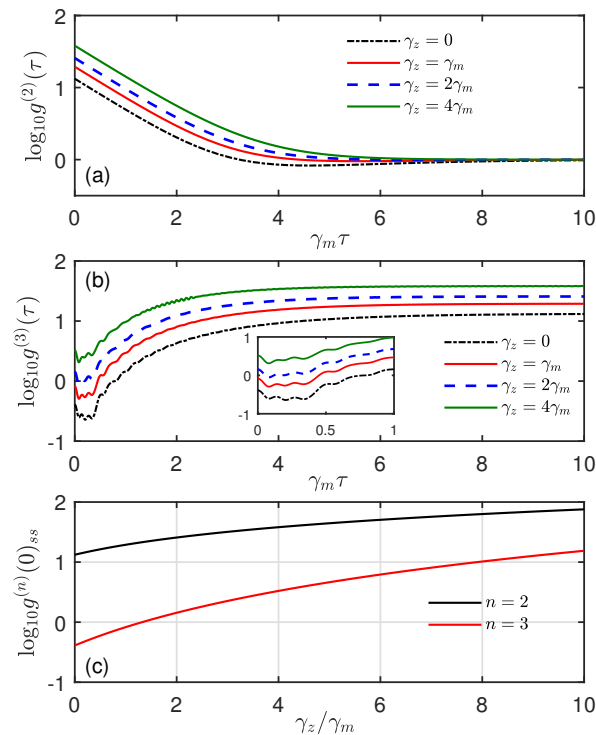


Figure 5. (a) Steady-state delayed-time second-order correlation function $g^{(2)}(\tau)$ and (b) delayed-time three-order correlation function $g^{(3)}(\tau)$ versus the scaled time delay $\gamma_m \tau$ for different dephasing rates γ_z . (c) Steady-state equal-time second-order correlation function $g^{(2)}(0)_{ss}$ and equal-time three-order correlation function $g^{(3)}(0)_{ss}$ versus the qubit dephasing rate γ_z/γ_m . The system parameters are the same as those in Figure 2, except for $\Delta_m = g/\sqrt{2}$.

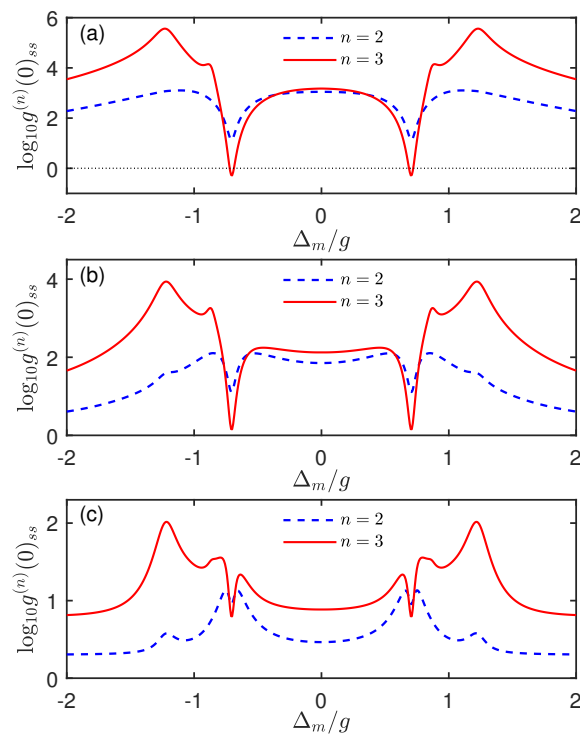


Figure 6. Steady-state equal-time second-order correlation function $g^{(2)}(0)_{ss}$ and three-order correlation function $g^{(3)}(0)_{ss}$ versus the driving detuning Δ_m/g when the thermal phonon number is (a) $n_{th} = 10^{-4}$, (b) $n_{th} = 10^{-3}$, (c) $n_{th} = 10^{-2}$. The system parameters are the same as those in Figure 2.

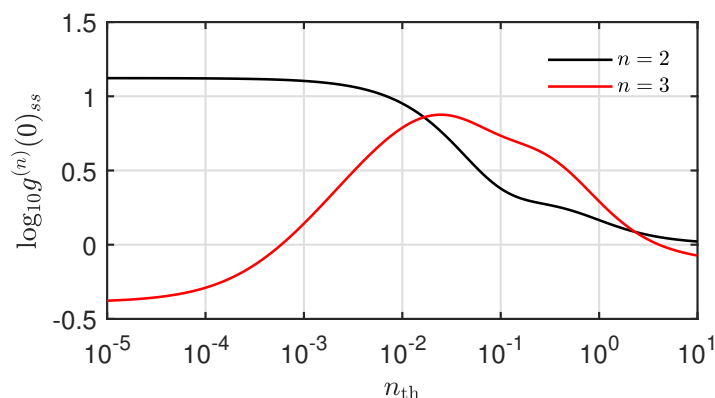


Figure 7. Steady-state equal-time second-order correlation function $g^{(2)}(0)_{ss}$ and three-order correlation function $g^{(3)}(0)_{ss}$ versus the thermal phonon number n_{th} . The system parameters are the same as those in Figure 2, except for $\Delta_m = g/\sqrt{2}$.

4. Conclusions

We have proposed an efficient method for implementing the two-phonon blockade effect in a hybrid spin–mechanical system, in which the spin qubit and the mechanical resonator are coupled by a strong two-phonon nonlinearity induced by the second-order magnetic gradient. The two-phonon blockade effect is obtained when the effective two-level system is driven. However, in the case of weakly driving the mechanical resonator instead of the spin qubit, one obtains the single-phonon blockade, while the two-phonon blockade can not be achieved. As a signature, we illustrate three-phonon antibunching with a simultaneous two-phonon bunching process by numerically calculating the second-order and third-order correlation functions. In addition, we also analyze the influence of the system parameters on the achieved two-phonon blockade. Our work extends the concept of the phonon blockade from a single phonon to multiphonon. It may inspire the development of multiphonon quantum coherent devices, which have potential applications in quantum information processing and quantum phononics.

Author Contributions: Conceptualization, T.-S.Y. and H.-Y.L.; methodology, H.-Y.L.; software, T.-S.Y. and A.C.; validation, H.-Y.L.; writing—original draft preparation, H.-Y.L.; writing—review and editing, T.-S.Y. and A.C. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Natural Science Foundation of China through Grants No. 12005189 and No. 12175199, and the Zhejiang Provincial Natural Science Foundation of China through Grant No. LZ20A040002.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

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