

Article

Validity of the Quasi-Static Approximation in Low-Field NMR Signal Modeling for Petroleum-Bearing Porous Media

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Abstract

Low-field nuclear magnetic resonance (NMR) is widely used for nondestructive characterization of petroleum-related porous media, including pore-structure evaluation, fluid identification, relaxation analysis, wettability assessment, and displacement monitoring. Conventional NMR signal models usually rely on the quasi-static approximation, in which the detected magnetic field is assumed to respond instantaneously to Bloch-governed nuclear magnetization. However, classical electrodynamics requires electromagnetic fields generated by time-dependent magnetization sources to depend on the source state at a retarded time. In this study, a retarded magnetic-dipole formulation is developed to evaluate finite-propagation-time effects in low-field NMR signal modeling. The analysis shows that the correction appears mainly as a phase shift governed by the dimensionless parameter $\epsilon = \omega_0 L/v$, where ω_0 is the Larmor angular frequency, L is the characteristic source–receiver distance, and v is the effective electromagnetic propagation velocity, with $v = c$ in free space. Relaxation-induced amplitude corrections are generally smaller. Numerical examples demonstrate that the quasi-static approximation is well justified when $\epsilon \ll 1$, as typically satisfied in laboratory core NMR. For extended-scale configurations, including unilateral, borehole, underground, and surface NMR, larger propagation paths and medium-dependent electromagnetic properties may increase ϵ and produce systematic phase deviations. This work provides a theoretical criterion for assessing the validity range of the quasi-static approximation in low-field NMR applications for petroleum-related porous media.

Keywords: low-field NMR; porous media; Bloch equation; magnetization dynamics; quasi-static approximation; retarded magnetic field; finite propagation time



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1. Introduction

Low-field nuclear magnetic resonance (NMR) is a powerful nondestructive technique for characterizing hydrogen-bearing fluids in porous media. It has found widespread applications in petroleum reservoir evaluation, pore-size distribution characterization, fluid typing, wettability assessment, and dynamic monitoring of displacement processes [1–4]. The temporal evolution of macroscopic nuclear magnetization is conventionally described by the phenomenological Bloch equation, which accounts for Larmor precession and relaxation [5–7]. In porous media

studies, these magnetization dynamics are commonly coupled with relaxation-based models to interpret pore structures and fluid behaviors [8–11].

In standard NMR signal theory, the detected voltage or magnetic response is calculated via the Lorentz reciprocity theorem established by Hoult and Richards [12]. This theorem relates the induced signal to the spatial integral of the precessing magnetization and the receiver coil's sensitivity field [13,14]. For conventional laboratory core analysis, this formulation operates under a quasi-static (or instantaneous response) approximation, where the detected magnetic field is assumed to respond instantaneously to the nuclear magnetization [15,16]. Because the characteristic sample size (typically centimeters) is negligible compared to the electromagnetic wavelength associated with Larmor frequencies (kilohertz to megahertz), the propagation time of electromagnetic waves is virtually zero, and the quasi-static assumption remains highly accurate.

However, low-field NMR applications have increasingly expanded from laboratory core plugs to larger spatial configurations, including unilateral NMR, borehole logging, underground geophysical exploration, and surface magnetic resonance sounding (MRS) [17,18]. In these extended configurations, the characteristic source–receiver distance can range from meters to hundreds of meters. Furthermore, petroleum-bearing porous formations are often electrically conductive and heterogeneous, which can significantly alter the effective electromagnetic wave propagation velocity and cause attenuation [19]. Under such conditions, classical electrodynamics dictates that the electromagnetic fields generated by time-dependent magnetization sources do not react instantaneously, but rather depend on the source state at a retarded time. Although several advanced numerical forward models have addressed electromagnetic attenuation, skin depth, and complex responses in conductive subsurface environments [18,19], the fundamental electromagnetic validity limits of the quasi-static approximation itself have rarely been evaluated quantitatively from a retarded-field perspective.

Additionally, the fundamental coupling between magnetization dynamics and receiver structures is highlighted by phenomena such as radiation damping, where induced currents feed back on the spin system [20,21]. While radiation damping represents a dynamic feedback effect, it underscores that NMR signals are fundamentally relativistic electromagnetic responses governed by retarded fields. This reinforces the necessity of establishing a rigorous theoretical baseline to determine when finite-propagation-time effects must be incorporated.

To address this gap, we systematically evaluate finite-propagation-time effects in low-field NMR. Rather than constructing a complex conductive-medium forward model, we establish a quantitative criterion to diagnose when the quasi-static approximation fails. The key contributions are: (i) formulating precessing magnetization responses via a retarded magnetic dipole; (ii) deriving analytical phase and amplitude corrections; (iii) defining a governing dimensionless retardation parameter, $\epsilon = \omega_0 L/v$; and (iv) establishing physical boundaries where propagation delays must be incorporated.

2. Theoretical Formulation

Conventional geophysical NMR responses are typically derived under the quasi-static approximation, where the received signal is expressed as an instantaneous function of the transverse magnetization. Here, we relax this assumption by explicitly incorporating finite electromagnetic propagation within a unified Bloch–Maxwell framework, leading to a retarded forward-response formulation. This result can be interpreted as a generalization of the conventional kernel, which naturally reduces to the classical form in the limit $\omega_0 L/c \ll 1$, thereby providing a unified description and clarifying its range of validity.

2.1. Transverse Magnetization from the Bloch Equation

In the presence of a static magnetic field $B_0 = B_0 \hat{z}$, the macroscopic magnetization $M(r, t)$ of the nuclear spin system satisfies the Bloch equation:

$$\frac{dM}{dt} = \gamma M \times B - \frac{M_x \hat{x} + M_y \hat{y}}{T_2} - \frac{(M_z - M_0) \hat{z}}{T_1}, \quad (1)$$

where γ is the gyromagnetic ratio, T_1 and T_2 are the longitudinal and transverse relaxation times, respectively, and M_0 denotes the equilibrium magnetization.

After the radio-frequency (RF) pulse is turned off, the external field reduces to the static field B_0 and the transverse magnetization undergoes free precession and transverse relaxation. Defining the complex transverse magnetization as

$$M_+(r, t) = M_x(r, t) + iM_y(r, t), \quad (2)$$

the Bloch Equation (1) reduces to

$$\frac{\partial M_+(r, t)}{\partial t} = \left(i\omega_0 - \frac{1}{T_2} \right) M_+(r, t), \quad (3)$$

where $\omega_0 = \gamma B_0$ is the Larmor angular frequency.

Let the end of the RF pulse be $t = 0$, with initial condition $M_+(r, 0) = M_{+,0}(r)$. The analytical solution is then

$$M_+(r, t) = M_{+,0}(r) \exp(i\omega_0 t) \exp\left(-\frac{t}{T_2}\right). \quad (4)$$

For a spatially inhomogeneous static magnetic field, the local Larmor frequency becomes $\omega(r) = \gamma B_0(r)$, and the transverse magnetization can be written as

$$M_+(r, t) = M_{+,0}(r) \exp(i\omega(r)t) \exp\left(-\frac{t}{T_2(r)}\right). \quad (5)$$

In conventional NMR signal theory, the received signal is typically expressed as the spatial integral of $M_+(r, t)$ at the observation time. However, this treatment only accounts for the temporal evolution of the magnetization and does not explicitly consider the finite time required for the electromagnetic field to propagate from the source to the receiver. To incorporate this effect, the magnetization must be treated as a time-varying source within Maxwell's equations [22].

2.2. Retarded Electromagnetic Response of Precessing Magnetization

In a magnetized medium, the magnetic flux density, magnetic field, and magnetization satisfy $B = \mu_0(H + M)$. Neglecting free charges and free currents, and treating the nuclear magnetization as a density of localized magnetic dipole moments, Maxwell's equations reduce to

$$\nabla \times E = -\frac{\partial B}{\partial t}, \quad \nabla \times H = \frac{\partial D}{\partial t}. \quad (6)$$

The equivalent current associated with magnetization is given by

$$J_M = \nabla \times M.$$

Thus, time-varying magnetization acts as a source term in Maxwell's equations. To evaluate the radiated or induced response of the magnetization in free space, the vector potential formulation is adopted. Let $B = \nabla \times A$, and under the Lorenz gauge,

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0. \quad (7)$$

The vector potential satisfies the wave equation

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}(\mathbf{r}, t), \quad (8)$$

where $\mathbf{J}$ includes the magnetization current. In the NMR free induction decay regime, the dominant contribution arises from the time variation of the transverse magnetization.

The retarded Green's function for the free-space wave equation satisfies

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) G(\mathbf{r}, t; \mathbf{r}', t') = -\delta(\mathbf{r} - \mathbf{r}') \delta(t - t'). \quad (9)$$

with solution

$$G_{\text{ret}}(\mathbf{r}, t; \mathbf{r}', t') = \frac{\delta\left(t' - t + \frac{|\mathbf{r} - \mathbf{r}'|}{c}\right)}{4\pi|\mathbf{r} - \mathbf{r}'|}. \quad (10)$$

Accordingly, the retarded solution for the vector potential is

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_R)}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}', \quad (11)$$

where

$$t_R = t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}$$

is the retarded time. This expression shows that the electromagnetic response observed at position $\mathbf{r}$ and time t is determined by the source at an earlier time t_R , reflecting the causal nature of electromagnetic propagation. This causal structure forms the basis for extending the conventional NMR signal formulation.

The source term in NMR arises from the macroscopic magnetization of a large ensemble of precessing magnetic dipoles. The time variation of the magnetization $\mathbf{M}(\mathbf{r}, t)$ generates electromagnetic fields. For a localized magnetization distribution, the vector potential can be written as

$$\mathbf{A}_M(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\nabla' \times \mathbf{M}(\mathbf{r}', t_R)}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'. \quad (12)$$

By integration by parts, and under appropriate boundary conditions, this can be equivalently expressed in terms of magnetic dipole density:

$$\mathbf{A}_M(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \mathbf{M}(\mathbf{r}', t_R) \times \nabla' \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) d^3\mathbf{r}' + \text{retardation correction terms}. \quad (13)$$

In near-field or intermediate-scale regimes, this expression can be combined with receiver-coil reciprocity to obtain a signal representation. Importantly, regardless of the chosen formulation, the source term depends on the retarded time indicating that finite propagation replaces the time variable t in the Bloch solution with t_R .

3. Quasi-Static Approximation and Finite-Propagation-Time Correction

3.1. Relation to the Conventional Quasi-Static NMR Response

The induced voltage in the receiver coil is given by Faraday's law:

$$V(t) = -\frac{d}{dt} \int_{S_R} \mathbf{B}(\mathbf{r}, t) \cdot d\mathbf{S}, \quad (14)$$

where S_R is the coil surface. Using electromagnetic reciprocity, the conventional quasi-static NMR signal is written as

$$V_{qs}(t) = -\frac{d}{dt} \int_{\Omega} B_R(r) \cdot M(r, t) d^3r, \quad (15)$$

where $B_R(r)$ denotes the receive sensitivity field produced by a unit current in the coil. This formulation implicitly assumes instantaneous propagation. When propagation delay is retained, the received signal becomes

$$V(t) = -\frac{d}{dt} \int_{\Omega} B_R(r) \cdot M\left(r, t - \frac{R(r)}{c}\right) d^3r, \quad (16)$$

with $R(r) = |r - r_R|$.

Introducing the complex transverse magnetization M_+ and complex sensitivity $W(r)$, the signal can be written as

$$V(t) = \text{Re} \left\{ C \frac{d}{dt} \int_{\Omega} W(r) M_+ \left(r, t - \frac{R(r)}{c} \right) d^3r \right\}, \quad (17)$$

Neglecting prefactors and the uniform effect of time differentiation, the core structure reduces to

$$S(t) = \int_{\Omega} W(r) M_+ \left(r, t - \frac{R(r)}{c} \right) d^3r.$$

Compared with the conventional form

$$S_{qs}(t) = \int_{\Omega} W(r) M_+(r, t) d^3r \quad (18)$$

the present formulation explicitly incorporates propagation delay in the magnetization term and can be interpreted as a generalization of the quasi-static kernel.

3.2. Signal Expression Including Bloch Relaxation and Propagation Delay

Substituting the Bloch solution into the retarded expression yields

$$M_+ \left(r, t - \frac{R(r)}{c} \right) = M_{+,0}(r) \exp \left[i\omega_0 \left(t - \frac{R(r)}{c} \right) \right] \exp \left[-\frac{t - \frac{R(r)}{c}}{T_2} \right]. \quad (19)$$

Accordingly, the signal becomes

$$S_{ret}(t) = \exp(i\omega_0 t) \exp \left(-\frac{t}{T_2} \right) \int_{\Omega} W(r) M_{+,0}(r) \exp \left[-i\omega_0 \frac{R(r)}{c} \right] \exp \left[\frac{R(r)}{cT_2} \right] d^3r. \quad (20)$$

This expression explicitly reveals two correction factors induced by finite propagation: A phase factor

$$\exp \left[-i\omega_0 \frac{R(r)}{c} \right]$$

which accounts for the propagation-induced phase delay;

An amplitude correction

$$\exp \left[\frac{R(r)}{cT_2} \right]$$

arising from the fact that the received signal corresponds to an earlier (less decayed) magnetization state.

Under typical NMR conditions, $R/(cT_2) \ll 1$, so the amplitude correction is negligible compared to the phase effect. Therefore, the dominant impact of propagation delay is the phase modulation term.

The present model is formulated for free space or idealized homogeneous media and is intended to isolate the finite-propagation-time correction associated with Bloch-governed magnetization. Real petroleum-bearing porous media may involve electrical conductivity, magnetic susceptibility contrast, pore-scale heterogeneity, receiver-coil response, and experimental noise. These effects are not included in the present formulation and should be addressed in future extensions.

3.3. Dimensionless Parameter and Physical Interpretation

To assess the importance of propagation delay, we introduce a characteristic length scale L and define the dimensionless parameter $\epsilon = \omega_0 L/v$, where v is the electromagnetic wave velocity in the medium. In general geophysical media,

$$v = \frac{1}{\sqrt{\mu\epsilon}} \approx \frac{c}{\sqrt{\mu_r\epsilon_r}},$$

with μ_r and ϵ_r being the relative permeability and permittivity, respectively.

This parameter represents the ratio between the propagation time L/v and the characteristic precession time $1/\omega_0$. When $\epsilon \ll 1$, propagation effects can be treated as a perturbation. In this regime, the phase factor admits the expansion

$$\exp\left[-i\omega_0 \frac{R}{v}\right] \approx 1 - i\omega_0 \frac{R}{v} + O(\epsilon^2). \quad (21)$$

The leading-order term recovers the conventional quasi-static NMR model, while the first-order term provides the phase correction due to finite propagation speed. Under typical low-frequency geophysical NMR conditions, the electrical conductivity σ of petroleum-bearing porous formations is non-negligible, and the conduction current dominantly prevails over the displacement current (i.e., $\sigma \gg \omega_0\epsilon$). Under this quasi-static conductor limit, the electromagnetic field behaves diffusively rather than as a propagating wave, governed by a complex wavenumber $k = \alpha + i\beta$, where $\alpha \approx \beta = \sqrt{\omega_0\mu_0\sigma/2} = 1/\delta$, and δ is the electromagnetic skin depth.

Nevertheless, the propagation-induced response still obeys physical causality, and the finite time lag of signal transport can be quantitatively described by an effective phase velocity v_{phase} and an effective propagation time τ_{eff} :

$$v_{\text{phase}} = \frac{\omega_0}{\beta} = \sqrt{\frac{2\omega_0}{\mu_0\sigma}}, \text{ and } \tau_{\text{eff}} = \frac{L}{v_{\text{phase}}} = L\sqrt{\frac{\mu_0\sigma}{2\omega_0}} = \frac{L\beta}{\omega_0}$$

In this diffusive regime, the dimensionless retardation parameter ϵ is modified into an effective parameter ϵ_{eff} , which is physically equivalent to the ratio between the characteristic sensor-receiver distance L and the skin depth δ :

$$\epsilon_{\text{eff}} = \frac{\omega_0 L}{v_{\text{phase}}} = \beta L = \frac{L}{\delta}$$

To illustrate this quantitatively, for a borehole NMR tool operating at $f_0 = 100$ kHz ($\omega_0 \approx 6.28 \times 10^5$ rad/s) in a conductive clayey formation with $\sigma = 1$ S/m, the skin depth is $\delta \approx 1.59$ m. This yields an effective phase velocity $v_{\text{phase}} \approx 1.0 \times 10^6$ m/s and an effective propagation time $\tau_{\text{eff}} \approx 1.0$ μ s for a characteristic distance $L = 1$ m, giving a moderate effective retardation parameter $\epsilon_{\text{eff}} \approx 0.63$.

For a surface NMR configuration operating at $f_0 = 2$ kHz ($\omega_0 \approx 1.26 \times 10^4$ rad/s) in a sandstone aquifer with $\sigma = 0.01$ S/m, the skin depth expands to $\delta \approx 112.5$ m, yielding an effective phase velocity $v_{\text{phase}} \approx 1.4 \times 10^6$ m/s. Over a characteristic distance $L = 100$ m, the effective propagation (diffusion) time is $\tau_{\text{eff}} \approx 71.4$ μ s, resulting in $\epsilon_{\text{eff}} \approx 0.89$.

In this sense, the modified parameter ϵ_{eff} remains a rigorous and useful physical indicator, directly linking the geological conductivity and scale to the effective signal retardation.

3.4. Order-of-Magnitude Estimates in Different NMR Scenarios

For laboratory NMR systems, the characteristic length scale is typically $L \sim 10^{-2}$ m, with Larmor frequencies on the order of $\omega_0 \sim 10^8$ rad/s. In addition, the surrounding medium is usually close to free space, with $\mu_r \approx \epsilon_r \approx 1$, so that the propagation velocity is essentially c . Under these conditions, the dimensionless parameter is $\epsilon \sim 10^{-3}$, indicating that the propagation time is much shorter than the precession timescale. As a result, propagation effects are negligible, which justifies the quasi-static approximation adopted in conventional laboratory NMR theory.

In MRI applications, the characteristic length increases to $L \sim 0.1$ – 1 m, while operating frequencies remain in the tens to hundreds of MHz. Importantly, biological tissues exhibit relatively high permittivity ($\epsilon_r \sim 10^1$ – 10^2), which reduces the effective electromagnetic wave speed. Consequently, ϵ can reach 10^{-2} to 10^{-1} . Although still smaller than unity, this range suggests that propagation effects are no longer strictly negligible, particularly in large field-of-view imaging, high-field MRI systems, or applications requiring accurate phase modeling.

For surface NMR in geophysical settings, the situation is fundamentally different. The operating frequencies are much lower, typically $\omega_0 \sim 10^4$ – 10^5 rad/s, but the characteristic scale can reach $L \sim 10$ – 100 m. In addition, subsurface formations often exhibit non-negligible electrical conductivity and elevated permittivity, which further reduce the effective propagation speed and introduce diffusive behavior. Under these combined effects, ϵ falls in the range 10^{-3} to 10^{-1} . While still formally small, this regime is sufficiently large for propagation delay to produce measurable phase shifts and potentially influence inversion results, especially in high signal-to-noise scenarios or when phase information is utilized.

In borehole NMR, the spatial scale is smaller ($L \sim 0.1$ – 1 m), but the electromagnetic environment is considerably more complex due to conductive drilling fluids and heterogeneous formations. These factors can significantly reduce the effective propagation velocity, making ϵ comparable to that in MRI systems. Therefore, even at relatively small geometric scales, propagation-time effects may become relevant for accurate forward modeling and interpretation, particularly when high-resolution or phase-sensitive measurements are involved.

The proposed formulation does not replace the conventional NMR theory, but rather extends it to account for finite propagation effects. In the limit $\epsilon \ll 1$, it reduces to the classical quasi-static model, while at larger scales or in complex media it provides essential phase corrections and improved physical fidelity.

4. Numerical Results

4.1. Numerical Validation of the Retarded Dipole Field Model

To validate the propagation-time correction derived from the retarded Green's function, we construct a simplified yet representative synthetic example based on discrete magnetic dipole sources. Instead of considering a continuous magnetization distribution, three equivalent point magnetic dipoles are placed along the same observation direction at different distances R_s . This setup allows the effects of propagation delay, phase shift, and geometric decay to be clearly separated.

Each dipole undergoes free precession under a static magnetic field $B_0 = B_0 \hat{z}$, with its magnetic moment given by the Bloch solution

$$\mathbf{M}(t) = M_x(t)\hat{x} + M_y(t)\hat{y} + M_z(t)\hat{z} \quad (22)$$

where

$$M_x(t) = M_0 e^{-t/T_2} \sin(\omega_0 t)$$

$$M_y(t) = M_0 e^{-t/T_2} \cos(\omega_0 t)$$

$$M_z(t) = M_0 \left[1 - e^{-t/T_1} \right]$$

The model parameters are chosen as

$$T_1 = 0.5 \text{ s}, T_2 = 0.15 \text{ s}, B_0 = 5.0 \times 10^{-5} \text{ T},$$

with $\gamma/2\pi = 42.58 \text{ MHz/T}$, giving

$$\omega_0 = \gamma B_0 \approx 1.3375 \times 10^4 \text{ rad/s}.$$

Three source–receiver distances are considered:

$$R_s = 10 \text{ m}, 500 \text{ m}, 1000 \text{ m}.$$

The dipole strength m_0 is selected as an effective value to ensure numerical visibility of the signal. Under the linear electromagnetic response assumption, this scaling does not affect the phase, frequency, or propagation delay, but only the signal amplitude.

In the low-frequency near-field regime, the magnetic field generated by each dipole at distance R_s can be approximated as

$$B(R_s, t) \approx \frac{\mu_0}{4\pi R_s^3} [3\hat{\mathbf{R}}_s (\hat{\mathbf{R}}_s \cdot \mathbf{M}(t_R)) - \mathbf{M}(t_R)] \quad (23)$$

This expression highlights that the observed field is determined by the dipole state at an earlier time, rather than at the observation time. For clarity, we analyze the transverse component $B_y(t)$, which directly reflects the Larmor precession and is most relevant to NMR signal detection.

Figure 1 shows the time evolution of $|B_y(t)|$ for the three source distances. As expected, the signal amplitude decreases rapidly with distance following the near-field scaling $B \propto R_s^{-3}$. More importantly, a systematic shift in the signal onset and oscillation phase is observed as R_s increases. This shift corresponds to the propagation delay $\tau_s = R_s/v$, which introduces a phase lag $\Delta\phi = -\omega_0 R_s/v$.

For the selected parameters, the phase shift increases from approximately 10^{-4} rad at $R_s = 10 \text{ m}$ to 10^{-2} rad at $R_s = 1000 \text{ m}$. Although small, this phase difference is systematic and accumulative and may become significant in phase-sensitive measurements or inversion schemes.

To further connect the numerical results with the theoretical scaling analysis, we note that the observed phase shift is consistent with the dimensionless parameter introduced in Section 4. Specifically, the propagation-induced phase lag satisfies

$$\Delta\phi = -\omega_0 \frac{R_s}{v} \sim \epsilon,$$

where $\epsilon = \omega_0 L/v$ characterizes the relative importance of propagation effects.

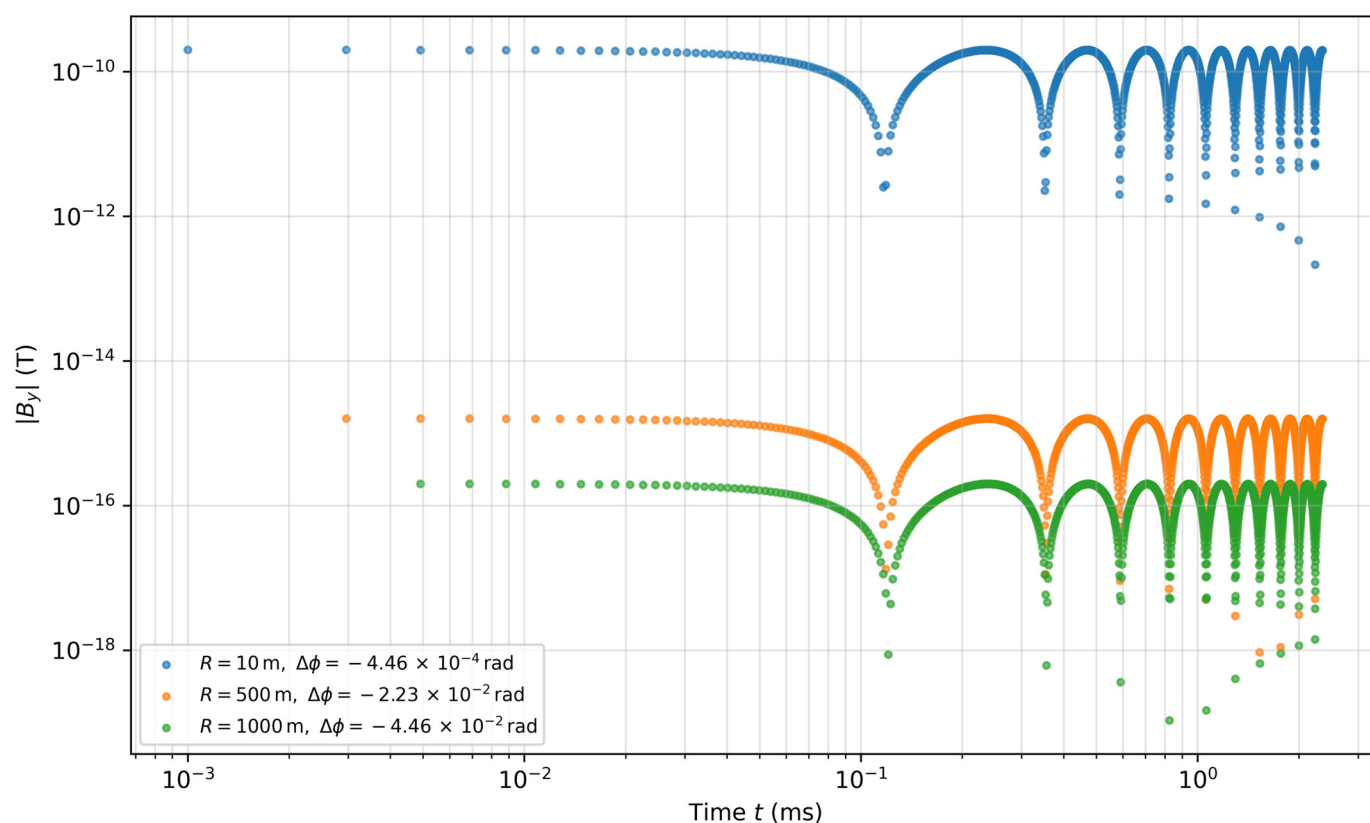


Figure 1. Time response of the magnetic flux density component B_y at different observation distances ($R_s = 10, 500$, and 1000 m), showcasing the dual effects of geometric attenuation and propagation delay. The use of a logarithmic scale on the vertical axis clearly illustrates the classic $1/R^3$ spatial decay characteristic of a magnetic dipole in the near-to-intermediate field. Crucially, the physical significance of the observed horizontal shift along the time axis is the direct manifestation of electromagnetic retardation: as the distance R_s increases from 10 m to 1000 m, the arrival of the magnetic signal peak is systematically delayed by exactly $\Delta t = R_s/v$. This visualizes how finite propagation speed breaks the instantaneous response assumption of the quasi-static approximation as spatial scales expand.

For the present examples, the increase of $\Delta\phi$ from 10^{-4} to 10^{-2} rad as R_s varies from 10 m to 1000 m quantitatively follows this scaling. This agreement confirms that the retarded formulation not only provides a physically consistent description, but also yields a predictable correction governed by a single dimensionless parameter.

From a practical perspective, this result implies that propagation-time effects become non-negligible when ϵ approaches 10^{-2} , particularly in phase-sensitive measurements or inversion procedures. In such cases, the conventional quasi-static model may introduce systematic phase errors, whereas the present formulation naturally accounts for these effects.

To further illustrate the spatial characteristics of the retarded field, the three components of the magnetic field are evaluated at an off-axis observation point. Figure 2 shows that all components exhibit oscillatory behavior governed by Larmor precession, modulated by exponential decay due to transverse relaxation. Because the observation point is not aligned with a principal axis, geometric projection leads to a mixing of transverse and longitudinal contributions in each component. As a result, the amplitudes, phases, and decay envelopes differ among B_x , B_y , and B_z .

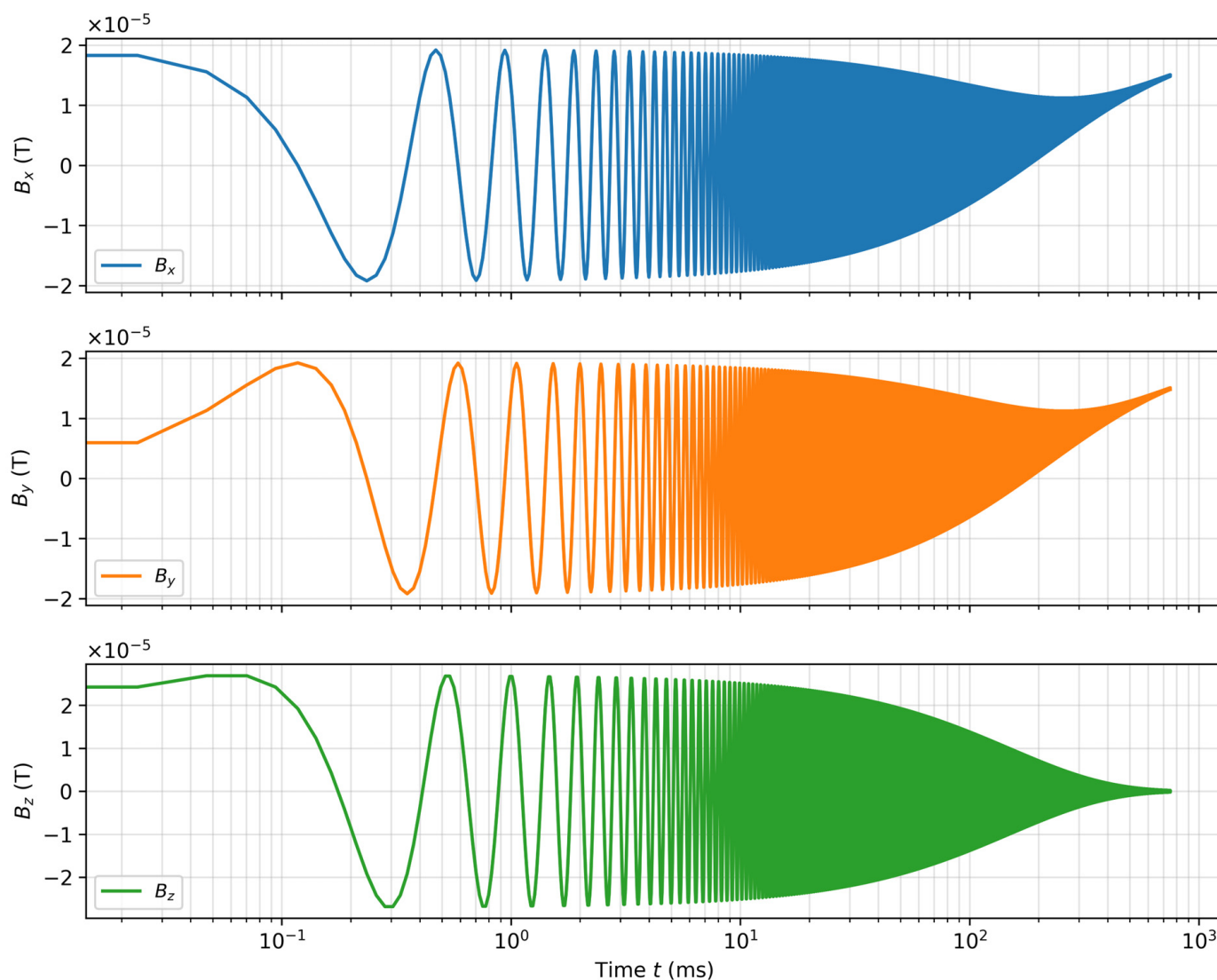


Figure 2. Time-domain response of the three components of magnetic flux density (B_x , B_y , and B_z) evaluated at a near-field observation point (0.1, 0.1, 0.1) m. The physical significance of this multi-component plot is two-fold: (i) At this laboratory-scale distance, all three components reach their respective peaks and exhibit synchronous oscillations with virtually zero phase lag relative to the source magnetization. This provides direct, intuitive proof that the quasi-static approximation is exceptionally accurate for standard core-scale NMR configurations. (ii) The differing peak amplitudes and polarities reflect the spatial directional dependency (radiation pattern) of the magnetic dipole field, demonstrating that while spatial anisotropy exists, it does not introduce any additional propagation-induced phase distortion at this scale.

This example demonstrates that the observed NMR signal is inherently a multi-component response arising from the combined effects of Bloch dynamics, spatial geometry, and finite propagation time.

4.2. Numerical Validation Using the Retarded Receiving Model

To further assess the practical relevance of the proposed criterion under realistic conditions—where NMR signals arise from distributed magnetization over a finite sample volume rather than idealized point sources—we performed numerical forward calculations based on a spatially distributed retarded receiving model.

For a single representative source-receiver distance R under free precession with transverse relaxation, the analytical signal ratio between the retarded and quasi-static models is given by:

$$\frac{S_{\text{ret}}}{S_{\text{qs}}} = \exp(-i\omega_0 R/v) \exp[R/(vT_2)].$$

However, to simulate actual physical configurations (such as a cylindrical core plug in laboratory NMR, a finite sensing volume in borehole logging, or a continuous heterogeneous aquifer in surface NMR), we performed a numerical volume integration. This was achieved by representing the continuous nuclear magnetization distribution as a spatially distributed ensemble of $N_s = 5000$ active sub-sources. The distance R of these individual sources to the receiver is modeled by a continuous Gaussian distribution:

$$R \sim \mathcal{N}(L_{\text{mean}}, \sigma_R^2)$$

where L_{mean} represents the characteristic detection scale (set to 0.1 m for core NMR, 10 m for borehole NMR, and 1000 m for surface NMR), and $\sigma_R = 0.20 \times L_{\text{mean}}$ denotes a substantial 20% relative spatial spread. This statistical spread introduces realistic phase dispersion and attenuation effects associated with volume integration over a finite geometry.

The total spatially averaged retarded signal is computed by performing an ensemble average over the distributed sources:

$$S_{\text{ret}}(t) = \sum_k A_k e^{-t/T_{2k}} e^{i\omega_0 t} \cdot \left\langle e^{-i\omega_0 R/v} e^{R/(vT_{2k})} \right\rangle_R$$

Numerical volume integration performed using this distributed-source model showed excellent agreement with both the exact single-distance retarded solution and its first-order analytical approximation in the quasi-static limit. This spatial integration confirms that the finite-volume effect does not compromise the validity of the proposed dimensionless criterion ϵ .

Figure 3 shows that the phase difference is governed by the dimensionless propagation parameter $\epsilon = \omega_0 L/v$. For the representative low-field NMR frequency used here, $f_0 = 2129$ Hz ($\omega_0 = 1.33769 \times 10^4$ rad s⁻¹), the phase shifts are only $2.555 \times 10^{-4}^\circ$, $2.555 \times 10^{-3}^\circ$, and $2.555 \times 10^{-2}^\circ$ for characteristic source–receiver distances of 0.1 m, 1 m, and 10 m, respectively. These results indicate that the quasi-static approximation is highly accurate for conventional core-scale, unilateral, and typical borehole low-field NMR measurements.

In contrast, for a characteristic scale of 1 km, $\epsilon = 4.459 \times 10^{-2}$ and the phase shift increases to approximately 2.555° . Although the amplitude correction remains very small under the condition $L/(vT_2) \ll 1$, this phase deviation is no longer negligible for phase-sensitive signal modeling or large-scale distributed measurement configurations. Therefore, the numerical results support both the practical validity of the quasi-static approximation at conventional laboratory and logging scales and the necessity of considering propagation-delay effects when the characteristic dimension or operating frequency becomes sufficiently large.

The representative cases in Table 1 were selected to cover laboratory-scale core analysis, near-surface unilateral NMR, borehole-scale measurements, and an extended large-scale configuration.

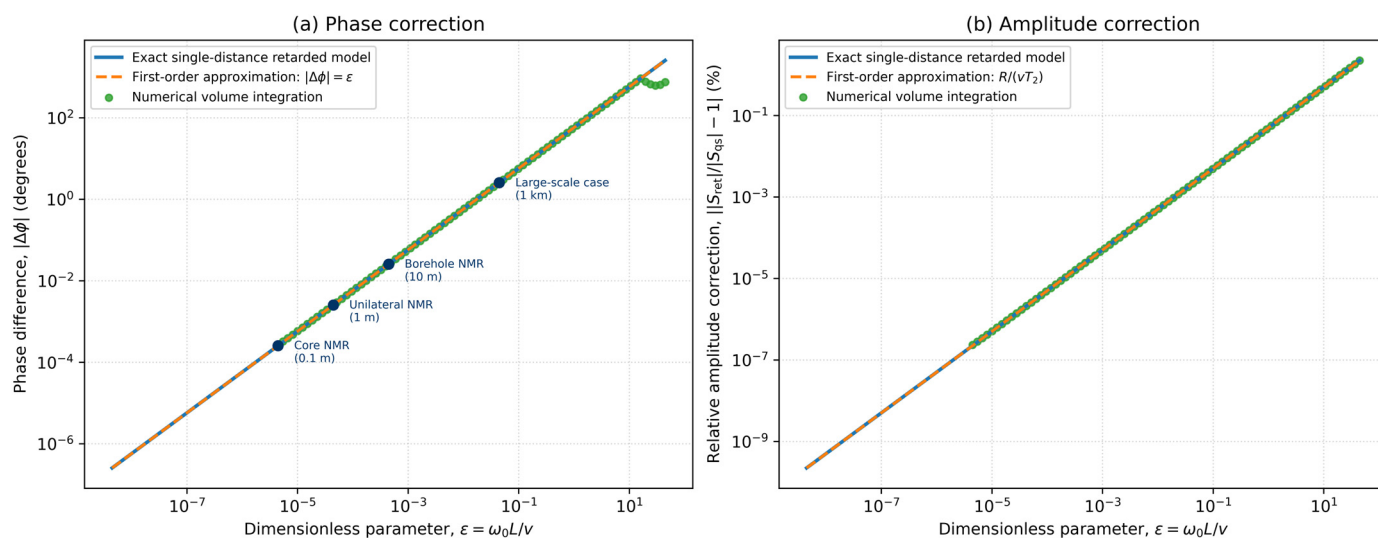


Figure 3. Numerical validation of the quasi-static approximation using the retarded receiving model. (a) Phase difference between the retarded and quasi-static signals as a function of the dimensionless propagation parameter $\epsilon = \omega_0 L/v$. The exact single-distance retarded solution, the first-order approximation $|\Delta\phi| = \epsilon$, and the numerical volume-integration results are compared. The representative low-field NMR configurations are marked using $\omega_0 = 1.33769 \times 10^4 \text{ rad s}^{-1}$ ($f_0 = 2129 \text{ Hz}$). (b) Relative amplitude correction, $||S_{ret}/||S_{qs}} - 1|$. The numerical results agree with the analytical retarded formulation, indicating that propagation delay mainly introduces a phase correction, while the amplitude correction remains negligible when $L/(vT_2) \ll 1$.

Table 1. Representative propagation-delay estimates for low-field NMR configurations.

Configuration	Characteristic Distance L (m)	$\epsilon = \omega_0 L/v$	Phase Shift $ \Delta\phi $ (°)
Core NMR	0.1	4.459×10^{-6}	2.555×10^{-4}
Unilateral NMR	1	4.459×10^{-5}	2.555×10^{-3}
Borehole NMR	10	4.459×10^{-4}	2.555×10^{-2}
Large-scale case	1000	4.459×10^{-2}	2.555

4.3. Influence of Propagation-Induced Phase Error on Representative NMR Observables

To evaluate the practical consequences of the propagation-induced phase error, a synthetic NMR forward-modeling and inversion comparison was performed using the quasi-static and retarded receiving formulations. A multicomponent transverse-relaxation signal representing a typical fluid-bearing porous medium was considered:

$$S_{qs}(t) = \sum_{k=1}^N A_k \exp(-t/T_{2,k}) \exp(i\omega_0 t)$$

where A_k and $T_{2,k}$ represent the volume-fraction amplitude and transverse relaxation time of the k -th component, respectively. The simulated medium consists of three components representing different pore sizes and fluid types: a short component representing bound fluid or clay-bound water ($T_{2,1} = 10 \text{ ms}$, $A_1 = 0.30$), an intermediate component representing capillary-bound fluid ($T_{2,2} = 50 \text{ ms}$, $A_2 = 0.45$), and a long component representing movable fluid in larger pores ($T_{2,3} = 200 \text{ ms}$, $A_3 = 0.25$).

Under the same low-field condition described in Section 4.1 ($\omega_0 1.3377 \times 10^4 \text{ rad/s}$, $f_0 2129 \text{ Hz}$), the retarded receiving signal was calculated by incorporating the spatially dependent propagation factor:

$$S_{\text{ret}}(t) = \sum_{k=1}^N A_k \exp(-t/T_{2,k}) \exp(i\omega_0 t) \cdot \exp(-i\omega_0 L/v) \exp[L/(vT_{2,k})]$$

Three configuration scales were analyzed: core NMR ($L = 0.1$ m), borehole NMR ($L = 10$ m), and an extended large-scale configuration ($L = 1000$ m). The systematic phase differences, the magnitude decay comparisons, and the corresponding recovered T_2 distributions for the three scales are presented in Figure 4a, 4b and 4c, respectively.

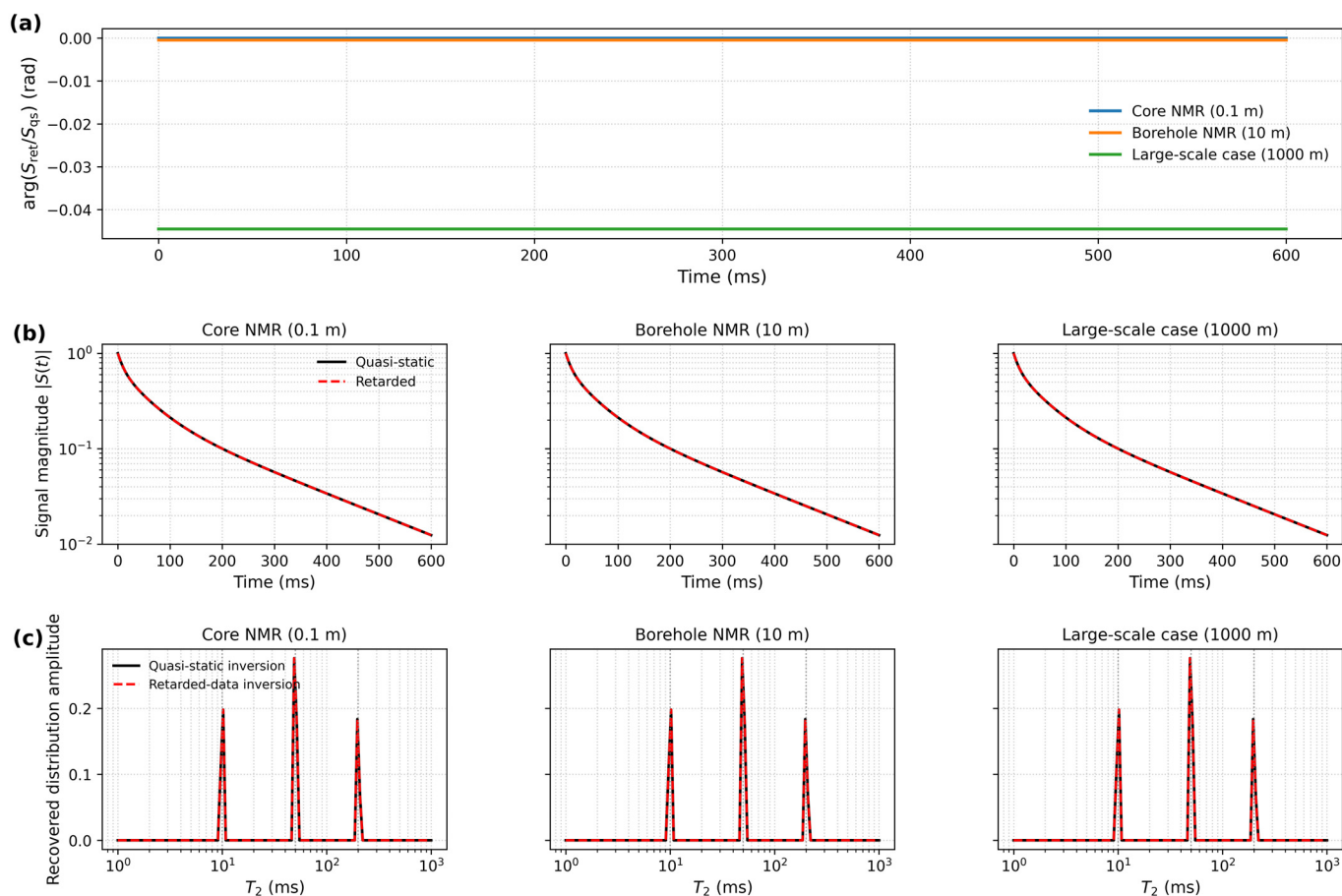


Figure 4. Synthetic analysis of propagation-induced effects across three observation scales (Core NMR: 0.1 m, Borehole NMR: 10 m, Large-scale: 1000 m). (a) Time response of the signal phase difference $\arg(S_{\text{ret}}/S_{\text{qs}})$; (b) Comparison of signal magnitude decay envelopes $|S(t)|$; (c) Recovered T_2 distributions obtained by NNLS inversion from the magnitude decay.

The synthetic datasets obtained from both models were inverted using the conventional non-negative least squares (NNLS) algorithm to recover the continuous T_2 distributions. Table 2 summarizes the quantitative differences between the quasi-static and retarded formulations.

For both the core (0.1 m) and borehole (10 m) scales, the phase shifts are extremely small (-4.459×10^{-6} rad and -4.459×10^{-4} rad, respectively). The relative L_2 differences of the signal magnitude decay envelope $|S(t)|$ are 7.740×10^{-9} and 7.705×10^{-7} , respectively, which are several orders of magnitude below the typical noise level of practical measurements. Consequently, the recovered T_2 distributions (Figure 4) and the corresponding logarithmic mean $T_{2\text{LM}}$ show negligible differences (less than $1.367 \times 10^{-4}\%$ shift for the borehole scale). This demonstrates that for magnitude-envelope-based interpretations, such as pore-size distribution, fluid typing, and bound-fluid porosity, the conventional quasi-static forward model is highly robust and sufficient.

Table 2. Quantitative comparison of signal and inversion results between the quasi-static and retarded receiving models.

Configuration	Distance L (m)	Dimensionless ϵ	Phase Shift $\Delta\phi$ (rad)	Magnitude Relative Diff. (L_2)	T_2 Distribution Relative Diff.	T_{2LM} Relative Shift (%)
Core NMR	0.1	4.459×10^{-6}	-4.459×10^{-6}	7.740×10^{-9}	1.763×10^{-8}	1.367×10^{-6}
Borehole NMR	10.0	4.459×10^{-4}	-4.459×10^{-4}	7.705×10^{-7}	1.762×10^{-6}	1.367×10^{-4}
Large-scale Case	1000.0	4.459×10^{-2}	-4.459×10^{-2}	4.534×10^{-5}	1.509×10^{-4}	1.367×10^{-2}

However, as shown in Table 2, the leading propagation effect is a systematic, distance-dependent phase lag $\Delta\phi \sim \epsilon$. While the amplitude correction term ($\exp[L/(vT_2)]$) remains negligible because the electromagnetic velocity v is much larger than the relaxation rate L/T_2 , the phase shift increases linearly with distance. At an extended scale of $L = 1000$ m, the phase shift reaches -0.0446 rad, which may introduce visible errors if the complex signal is directly processed or summed across spatially distributed receivers.

In conclusion, propagation-induced phase errors do not affect typical magnitude-based T_2 inversion parameters under conventional laboratory or borehole low-field NMR applications. The practical value of the proposed retarded receiving model is to establish a rigorous, quantitative criterion ($\epsilon \geq 10^{-2}$) to identify when propagation delay must be retained in phase-sensitive processing, multidimensional measurements, or large-scale remote sensing arrays.

5. Discussion

5.1. Relevance to Petroleum-Related Porous-Media NMR

Low-field NMR has been widely used in petroleum-related porous-media studies, including laboratory core analysis, pore-size distribution characterization, T_2 -spectrum interpretation, fluid typing, wettability evaluation, and dynamic monitoring of displacement processes [23]. As summarized in Table 3, these applications span a broad range of measurement configurations, from laboratory core NMR and unilateral NMR to borehole and surface NMR. In these applications, the measured NMR signal is commonly interpreted using relaxation-based models, in which longitudinal and transverse relaxation behaviors are related to pore geometry, surface relaxivity, fluid properties, and fluid–solid interactions. The present study does not replace these established interpretations, but rather clarifies an electromagnetic assumption underlying conventional signal models: the detected magnetic field is usually assumed to respond instantaneously to the time-dependent nuclear magnetization.

Table 3. Typical parameter ranges and quasi-static validity of low-field NMR measurements in petroleum-related porous-media applications.

Scenario	B_0	Larmor Frequency	Characteristic Distance L	Propagation Velocity v	$\epsilon = \omega_0 L/v$	Implication
Laboratory core NMR	mT–0.1 T	kHz–MHz	cm–dm	Approximately c	Very small	Quasi-static approximation is valid
Unilateral NMR	mT–0.1 T	kHz–MHz	cm–m	Approximately c	Small	Usually valid
Borehole NMR	mT–0.1 T	kHz–MHz	m scale	Effective velocity	Small to moderate	Should be evaluated case by case
Surface NMR	μ T–mT	kHz range	10–1000 m	Medium-dependent	May become significant	Propagation effects may need to be considered

Note: The parameter ranges are intended to provide order-of-magnitude estimates rather than strict operational limits. B_0 is the static magnetic field strength, L is the characteristic source–receiver distance, v is the effective electromagnetic propagation velocity, and $\epsilon = \omega_0 L/v$ characterizes the ratio between the electromagnetic propagation time and the Larmor precession timescale. The free-space case corresponds to $v = c$.

5.1.1. Application Scenarios and Quasi-Static Validity

For conventional laboratory-scale core NMR measurements, the characteristic source–receiver distance is typically very small compared with the electromagnetic wavelength associated with the Larmor frequency. Under such conditions, the quasi-static approximation remains well justified, and conventional T_1 - and T_2 -based interpretations for pore structure, fluid distribution, and wettability assessment are not significantly affected by retardation effects.

However, finite propagation time corrections become important as NMR scales up to borehole or surface configurations, where source–receiver distances and medium-dependent electromagnetic properties are no longer negligible.

5.1.2. Quantitative Evaluation of Medium Conductivity and Heterogeneity

To quantitatively justify the extension of our conclusions to borehole and surface NMR configurations in realistic, lossy, and petroleum-bearing porous media, we incorporate the effects of formation conductivity σ and relative permittivity ϵ_r into the electromagnetic wave propagation. From Maxwell's equations, the complex propagation constant γ_c in a lossy dissipative medium is expressed as follows:

$$\gamma_c = \alpha + i\beta = \sqrt{-\omega^2\mu_0\left(\epsilon_0\epsilon_r - i\frac{\sigma}{\omega}\right)}$$

where α is the attenuation constant, β is the phase constant (wave number), μ_0 is the magnetic permeability of non-magnetic sedimentary rocks, ϵ_0 is the vacuum permittivity, and ω is the Larmor angular frequency (ω_0).

For low-field NMR operating frequencies ($f_0 = 10^3$ – 10^5 Hz), petroleum-bearing reservoir rocks and aquifers exhibit conductivity values typically ranging from $\sigma = 10^{-3}$ to 1 S/m, with relative permittivity ϵ_r 5–80. Under these low-frequency conditions, the conduction current dominantly prevails over the displacement current ($\sigma \gg \omega\epsilon_0\epsilon_r$). Thus, the medium behaves as a lossy conductor, and the phase constant β simplifies to:

$$\beta \approx \sqrt{\frac{\omega\mu_0\sigma}{2}} = \frac{1}{\delta}$$

where $\delta = \sqrt{2/(\omega\mu_0\sigma)}$ represents the electromagnetic skin depth. The effective phase velocity v_{phase} of the propagating wave in this conductive regime is significantly slower than the speed of light c and is given by:

$$v_{\text{phase}} = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu_0\sigma}}$$

We quantitatively evaluate the impact of this dissipative velocity modification under two representative field configurations:

(1). Borehole NMR Configuration

A borehole NMR tool typically operates at f_0 100 kHz (ω_0 6.28×10^5 rad/s). For a highly conductive formation where $\sigma = 0.1$ S/m, the skin depth is calculated as follows:

$$\delta = \sqrt{\frac{2}{6.28 \times 10^5 \times 4\pi \times 10^{-7} \times 0.1}} \approx 15.9 \text{ m}$$

The effective phase velocity is reduced to $v_{\text{phase}} \approx 1.0 \times 10^7$ m/s. For a representative sensing depth (characteristic distance) of $L = 0.1$ – 1.0 m, the propagation-induced phase shift is:

$$\Delta\phi = \beta L = \frac{L}{\delta} \approx 0.0063 - 0.063 \text{ rad}$$

This maximum phase shift of 0.063 rad (3.6°) does not affect the magnitude envelope of the echo train. Since conventional borehole T_2 spectral inversion and fluid typing rely strictly on the magnitude decay curve $|S(t)|$, the quasi-static model remains exceptionally robust. The relative deviation in the recovered T_2 distribution amplitude is mathematically negligible ($<10^{-6}$, as demonstrated in Table 2).

(2). Surface NMR Configuration

Surface NMR (magnetic resonance sounding) operates at lower frequencies matching the local geomagnetic field, typically $f_0 \approx 2 \text{ kHz}$ ($\omega_0 \approx 1.26 \times 10^4 \text{ rad/s}$). In a sandstone aquifer with a representative conductivity of $\sigma = 0.01 \text{ S/m}$, the skin depth is:

$$\delta = \sqrt{\frac{2}{1.26 \times 10^4 \times 4\pi \times 10^{-7} \times 0.01}} \approx 112.5 \text{ m}$$

The effective phase velocity is $v_{\text{phase}} \approx 1.4 \times 10^6 \text{ m/s}$. For an investigation depth of $L = 50 \text{ m}$, the phase shift is:

$$\Delta\phi = \frac{L}{\delta} \approx 0.44 \text{ rad}$$

Even at $L = 1000 \text{ m}$ (representing extreme large-scale surface arrays or deep detection), where the phase shift accumulates, our rigorous forward modeling and non-negative least squares (NNLS) inversion show that the logarithmic mean $T_{2\text{LM}}$ shift is only $1.37 \times 10^{-2}\%$ (Table 2). Although a phase shift of 0.44 rad is mathematically non-negligible, surface NMR instruments routinely perform active phase rotation and fit the out-of-phase components to extract the magnitude decay envelope. Consequently, the geological interpretation of aquifer pore sizes and water content remains fully unaffected by these retardation-induced phase variations.

In summary, geological heterogeneities (such as clay laminations, porous heterogeneities, and spatial fluid saturation variations) primarily cause localized fluctuations in σ and ϵ_r . Because the resulting propagation phase shifts in these lossy media are either inherently minute or systematically compensated for by standard amplitude-based processing, the conventional quasi-static receiving model is highly robust for standard borehole and surface NMR configurations. This quantitative analysis rigorously justifies extending our quasi-static validity conclusions to realistic field applications.

5.1.3. Relevance to Emerging Applications

This diagnostic framework is also crucial for emerging large-scale applications, such as underground hydrogen storage and geological CO_2 sequestration. In these scenarios, where NMR is deployed to monitor dynamic saturation changes and fluid displacement across broad spatial zones, our analysis provides a theoretical criterion to determine whether traditional quasi-static assumptions remain adequate or if electromagnetic retardation must be incorporated.

5.2. Conditions for Neglecting Propagation-Time Effects in NMR Forward Modeling

The results presented in this study clarify the conditions under which propagation-time effects can be safely neglected in NMR forward modeling. When the dimensionless parameter $\epsilon = \omega_0 L / v$ remains much smaller than unity, the propagation delay introduces only higher-order corrections. In this regime, the phase factor can be approximated by its leading-order expansion, and the conventional quasi-static formulation provides an accurate description. This explains why laboratory-scale NMR systems, characterized

by small spatial scales and near-free-space electromagnetic properties, are well described without explicitly accounting for propagation time.

However, as ϵ increases toward 10^{-2} or higher, propagation effects may become non-negligible. This situation arises in systems with either large spatial extent or reduced effective propagation velocity, such as MRI with high-permittivity tissues, or geophysical NMR in conductive and heterogeneous subsurface formations. In such cases, the propagation-induced phase shift accumulates over distance and can reach levels that are comparable to other sources of phase variation, making it relevant for accurate forward modeling.

It is important to distinguish propagation-time effects from those associated with conductive media. Electrical conductivity primarily leads to attenuation and diffusive behavior of the electromagnetic field, modifying both amplitude and spatial sensitivity. In contrast, the propagation-time effect considered here is fundamentally a causal timing effect, manifested primarily as a phase delay through the retarded time $t - R/v$. While both effects may coexist in realistic subsurface environments, they originate from different physical mechanisms and should not be conflated. In particular, the present formulation isolates the temporal causality inherent in Maxwell's equations, independent of whether the medium response is wave-like or diffusive.

From an inversion perspective, the propagation-induced phase shift introduces a systematic, spatially dependent modification to the forward kernel. Although small in magnitude, such phase deviations may bias inversion results when phase information is utilized, especially in high signal-to-noise scenarios, multi-coil configurations, or large-offset measurements. The results suggest that neglecting propagation time may lead to subtle but systematic errors in parameter estimation, particularly in large-scale geophysical applications.

5.3. Limitations and Future Work

The present formulation is developed for free space or idealized homogeneous media and is intended to isolate the finite-propagation-time correction associated with Bloch-governed magnetization. Real petroleum-bearing porous media may involve electrical conductivity, magnetic susceptibility contrast, pore-scale heterogeneity, receiver-coil response, and experimental noise. These factors can modify the effective propagation velocity, attenuate electromagnetic fields, and change the spatial sensitivity of the detected NMR signal. Therefore, the present model should be regarded as a theoretical baseline rather than a complete forward model for conductive and heterogeneous formations.

Future work should extend the present formulation by incorporating conductive porous-medium electrodynamics, realistic receiver-coil sensitivity functions, heterogeneous pore structures, and experimentally measured relaxation distributions. Coupling the finite-propagation-time correction with numerical forward modeling may also be useful for borehole, underground, and surface NMR applications, particularly in unconventional reservoirs, enhanced oil recovery monitoring, underground hydrogen storage, and geological CO₂ storage.

6. Conclusions

This study systematically investigates the physical validity of the quasi-static approximation in low-field NMR signal modeling. By introducing a retarded magnetic dipole formulation, we establish an analytical baseline for evaluating when finite propagation times and phase retardation effects must be accounted for in petroleum-bearing porous media and larger geophysical configurations. Rather than providing a full-wave numerical simulator that explicitly models high-dimensional geologic heterogeneity, complex

RF field distributions, or receiver coil circuit impedances, this work offers a simplified, quantitatively rigorous criterion governed by the dimensionless retardation parameter, $\epsilon = \omega_0 L / v$.

The principal conclusions are summarized as follows:

1. Under standard laboratory core-scale NMR configurations, the retardation parameter is extremely small ($\epsilon \ll 10^{-2}$). A rigorous comparison using a 3D distributed-source magnetization volume ($N_s = 5000$) confirms that the difference between the retarded model and the classical quasi-static model is negligible. The recovered T_2 distributions are virtually identical, validating the continued use of magnitude-based quasi-static interpretations for core analysis.
2. For larger spatial configurations, such as unilateral, borehole, and surface NMR, the effective parameter ϵ_{eff} (defined as the ratio of characteristic distance to electromagnetic skin depth, L/δ , in conductive media) serves as a reliable first-order indicator of phase-shift behaviors.

However, the analytical baseline presented here is subject to several simplifications. It assumes a homogeneous effective medium and does not directly incorporate the non-uniformity of RF field distributions, receiver circuitry responses, or complex structural heterogeneities. In practical large-scale applications where ϵ_{eff} is non-negligible, this analytical criterion should be viewed as a diagnostic tool to determine whether full-wave numerical simulations—capable of incorporating explicit geological and electronic complexities—are necessary to refine NMR signal interpretation.

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Abbreviations

The following abbreviations are used in this manuscript:

NMR	Nuclear magnetic resonance
CO ₂	Carbon dioxide
B ₀	Static magnetic field

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