

Article

The Behavior of Long Thin Rectangular Plates under Normal Pressure—A Thorough Investigation

Gilad Hakim and Haim Abramovich * 

Aerospace Structures Lab, Faculty of Aerospace Engineering, Technion—Israel Institute of Technology, I.I.T., Haifa 32000, Israel; ghakim@outlook.com

* Correspondence: haim@technion.ac.il

Abstract: Thin rectangular plates are considered basic structures in various sectors like aerospace, civil, and mechanical engineering. Moreover, isotropic and laminated composite plates subjected to transverse normal loading and undergoing small and large deflections have been extensively studied and published in the literature. Yet, it seems that the particular case of long thin plates having a high aspect ratio appears to be almost ignored by various scholars despite its engineering importance. The present study tries to fill this gap, yielding novel findings regarding the structural behavior of long thin plates in the small- and large-deflection regimes. In contrast to what is normally assumed in the literature, namely that a long plate with a high aspect ratio can be considered an infinitely long plate, the present results clearly show that the structural effects of the ends continue to exist near the remote ends of the long plate. An innovative finding is that long plates would (only on movable boundary conditions for the large-deflection regime) exhibit a larger mid-width displacement in comparison with deflections of infinitely long plates. This innovative higher deflection appears for both small and large-deflection regimes for both all-around simply supported and all-around clamped boundary conditions. This new finding was shown to be valid for both isotropic and orthotropic materials and presents a novel engineering approach for the old assumption well quoted in the literature that a relatively long plate on any boundary condition can be considered an infinite plate. Based on the present research, it is recommended that this assumption should be used carefully as the largest plate mid-deflection might occur at finite aspect ratios.



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Keywords: square thin plate; aspect ratio; small linear deflection; large deflection; membrane stress; finite element analysis; movable and immovable edges

1. Introduction

Thin and thick plates are known to be the basic blocks in various engineering sectors, like aerospace, civil, and mechanical engineering. Moreover, these plates may undergo large deflections, and for some applications, their length-to-width ratio or aspect ratio (AR) tends to be large.

As a result, many research articles and books have been published on thin rectangular plates with transverse loads, for both small and large deflections. Many of them include (in various forms) a general statement that “long plates with a high AR can be treated as an infinitely long plate”; i.e., the remote ends do not affect the plate’s mechanical performance and the plate would experience its highest out-of-plane deflection. For instance, Timoshenko [1] (p. 422) states that “the deflections of finite plates with $b/a < 2/3$ (where b and a are the width and length of the plate, respectively) are very close to those obtained for an infinitely long plate”. Note that this statement would relate to $AR > 1.5$. Also, the stresses for $AR = 2$ are said to be 10% lower than those for an infinitely long plate. Reddy [2] (p. 246) provides information for the midpoint deflection and bending moments for plates with $AR = 2$ and less. On p. 249 of Reddy’s work [2], information is given for $AR = 3$ and less for the bending moment load, while on p. 314, a buckling case is considered for $AR = 6$

and less. Bakker et al. [3], in their article, refer to an AR of up to 2. Longer plates are not considered. Further information regarding the behavior of plates under loading can be found in [4–9]. Wang and El-sheikh [10] present deflection information up to $AR = 10$, restricting themselves to immovable boundary conditions (namely, the plate edges cannot move relative to their support frame). Shao [11], in his thesis, investigated high- AR plates up to $AR = 5$, with movable edges, but the edges were forced to remain straight. The reason for this $AR = 5$ limit is that (p. 2) “*beyond this value, the behavior of the plate is nearly as a strip*”. Razdolsky [12] also investigated thin rectangular plates up to $AR = 4$, with simply supported immovable edges. He presents deflections and stresses in a graphical form for specific points on the plate. For the general case, a rather complex algorithm is suggested using multiple summations of trigonometric functions. Ostiguy and Evan-Iwanowski [13] also checked the influence of the aspect ratio on the dynamic response and stability of rectangular plates under the large-deflection regime. They found that “ AR plays a crucial role in determining the stability of rectangular plates”.

The present study presents a thorough investigation on the behavior of long plates under normal pressure, challenging the above-quoted statement by checking the performance of high- AR plates, for both deflections and stresses. Innovative, interesting results suggest another perspective for these long plates, showing higher deflections at specific mid-width points.

2. Materials, Methods, and Results

To investigate the behavior of long plates with small deflections, the classical Navier solution for the midpoint small deflection of a thin rectangular plate was analyzed for large AR s. The solution presented in Reddy’s work [2] on p. 230 is brought here again for clarification and is presented in Equation (1).

$$w_{\max} = \frac{16qb^4}{D\pi^6} \sum_{n=1,3,5,\dots}^{\infty} \sum_{m=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{m+n}{2}-1}}{mn \left(m^2 \left(\frac{b}{a} \right)^2 + n^2 \right)^2} \quad (1)$$

Then, the midpoint deflection expression was calculated for several AR s (a/b) using the polycarbonate data presented in Table 1. The E and ν values of this isotropic linear elastic material were taken from manufacturer’s commercial data sheets [14,15].

Table 1. Structural and material data used to evaluate Equation (1) for various AR values.

Young’s modulus E	2.4 GPa
Poisson’s ratio ν	0.38
Thickness h	0.005 m
Length a	varies
Width b	1 m
Distributed load q	100 Pa
The calculated bending rigidity $D \quad D = Eh^3/12(1 - \nu^2)$	29.219 Nm
Summation indexes m and n	1, 3, 5, ..., 31

The calculated midpoint deflection for a square plate was found to be $w_{\max} = 13.9$ mm.

The relative mid-deflection, defined as the ratio between the actual lateral displacement and the maximal deflection of a square plate, as a function of AR is depicted in Figure 1.

From the graphs presented in Figure 1, it is obvious that plates with a high finite AR (in the region of $AR = 10$) would deflect more than an infinitely long plate. The difference is just 1%, and can be considered negligible, but it still contradicts the well-known general

statement that “long plates with a high aspect ratio AR , can be treated as an infinitely long plate”.

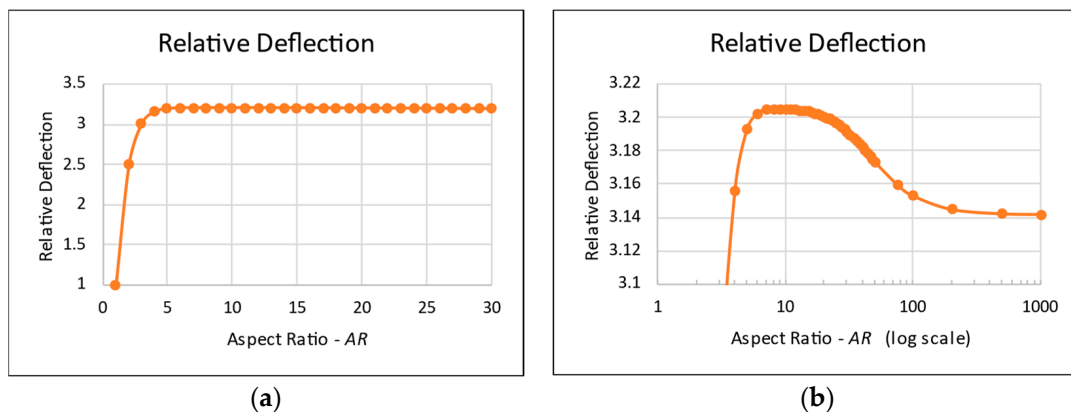


Figure 1. Navier small deflection theory—relative deflection vs. AR : (a) full scale; (b) peak vicinity.

To check the importance of the boundary conditions, an all-around clamped plate was investigated with increasing aspect ratios, in a similar way to that shown above for all-around simply supported boundary conditions. Since a simple solution for the case of all-around clamped boundary conditions is not available, a finite element analysis (FEA) method was used. The analysis code was Simcenter Femap with Nastran ver. 2021.1 from Siemens [16]. More details about the FEA are given in Appendix A.

The dimensions and the mechanical properties, presented in Table 1, were used to yield the two graphs presented in Figure 2. A pressure load of 25 Pa was used to have a small midpoint deflection of about half-plate thickness, while the midpoint deflection for a square plate was found to be $w_{\max} = 1.1035$ mm.

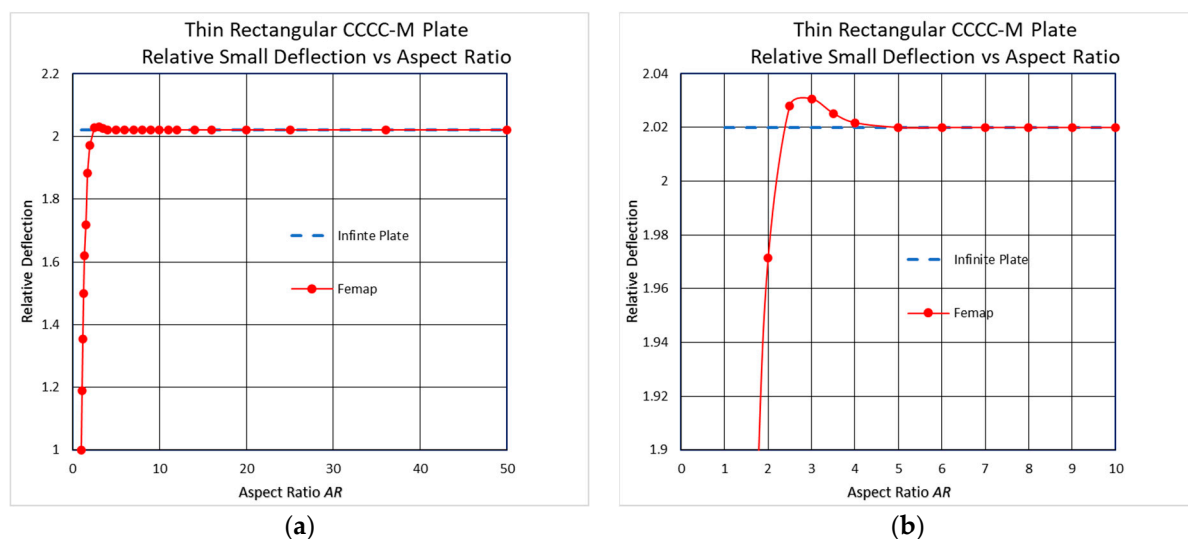


Figure 2. Thin rectangular all-around clamped conditions in the small-deflection regime (FE results)—relative deflection vs. AR : (a) full scale; (b) peak vicinity.

As for the all-around simply supported case, as well as for the all-around clamped case, there is a small peak at $AR = 3$, which is about 0.5% larger than the infinite-length deflection case. So again, although the difference is just 0.5% and can be considered negligible, it still contradicts the well-known general statement that “long plates with a high aspect ratio, AR , can be treated as an infinitely long plate”.

The present study will now proceed to investigate the large-deflection regime. While, for a small-deflection regime, the in-plane effects are generally negligible, for a large-

deflection regime that exceeds the plate thickness, the in-plane effects play a major role and must be considered. The additional elastic energy required to strain the plate in its plane causes the plate to deflect less than expected when using the linear small-deflection theory.

To investigate the behavior of a thin plate with a high AR , in the large-deflection regime, rectangular plates were addressed. The geometry and the material data are presented in Table 1. Note that the length is defined in the x direction, while the width is in the y direction, as shown in Figure 3. To enable large deflections, the plate was loaded with a uniform pressure, $q = 1000$ Pa.

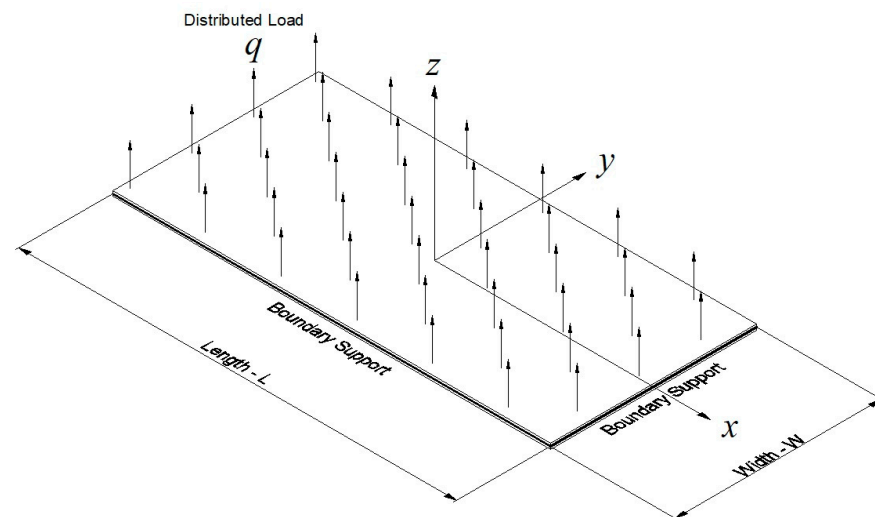


Figure 3. The plate schematic model for the large-deflection regime.

In the plate's large-deflection regime, the in-plane boundary conditions (BCs) have a significant influence on both the deflection and the generated membrane stresses. Since there are many possible BC combinations, in the present study, we dealt with only two representative BCs: a four-side simply supported designated SSSS where there are no bending moments at the various edges, and a four-side clamped designated CCCC where no rotations are allowed at the various edges.

For the in-plane BC, we used two types, which are described hereafter. The movable four sides, where the plate edges can freely move relative to the support frame in both the parallel and perpendicular directions, are designated as M. The immovable four sides, where the plate edges cannot move relative to the support frame, are written as I.

Finite element analyses (FEAs) for several types of rectangular plates were performed. As for the case of small deflections presented above, the analysis code was Femap 2021.1 from Siemens, with Simcenter Nastran [16] as the code processor (see Appendix A for details). A nonlinear static analysis was performed for quad-type plate elements with a 50 mm element size. The nonlinear code increased the load in 20 steps, while in each step, the deflections and stresses were recalculated and used as a starting point for the next step. Each of these steps had internal iterations to verify its convergence. Upon completion of the run, all final deflections and membrane forces of the entire plate were transferred to an Excel (v2405) sheet. Also, graphic pictures of the FEA results were saved for further processing. Then, the data and figures were further evaluated for significant findings. For further details, see Hakim and Abramovich [17].

One should note that during nonlinear analysis, due to the large-deflection status, the plate elements would change their special direction. Therefore, one must consider the load direction acting on the plate elements. Two types of distributed loads are commonly used within the FE code. The first one is a vector-oriented load direction, in which the load direction is fixed in the space and does not follow the element direction's special changes. The second one is a pressure load type that operates perpendicularly to the element surface and follows the special change in the element direction, namely, a follower force. These

two load types represent clear physical situations. The vector-oriented distributed load represents snow load, where the load direction is the weight of the snow acting downward. The distributed pressure load represents a wind load that operates perpendicularly to the plate element and follows its special directions while loaded. In the present study, we used the vector-oriented distributed load, applying it to obtain the plate's response to distributed equal load.

Typical results are presented in Figures 4–7, presenting the calculated out-of-plane deflections, the membrane x forces, the y forces, and the shear xy forces for the SSSS-M case. The numbers near the plates are their respective ARs, while SFSF means that the two remote ends (left and right) are free and the two sides (up and down) are simply supported, representing an infinitely long side-supported plate.

The relatively high Y membrane force near the ends of the SFSF is probably related to the sudden discontinuity of the plate combined with the Poisson effect. Nevertheless, as the SFSF represents an infinitely long plate, this end effect is ignored here.

Based on the results presented in Figure 4, the influence of the AR on the out-of-plane deflections was investigated.

One may expect that the out-of-plane deflection for plates with a high aspect ratio would asymptotically approach the deflection of an infinitely long plate, with the long plate having the highest deflection.

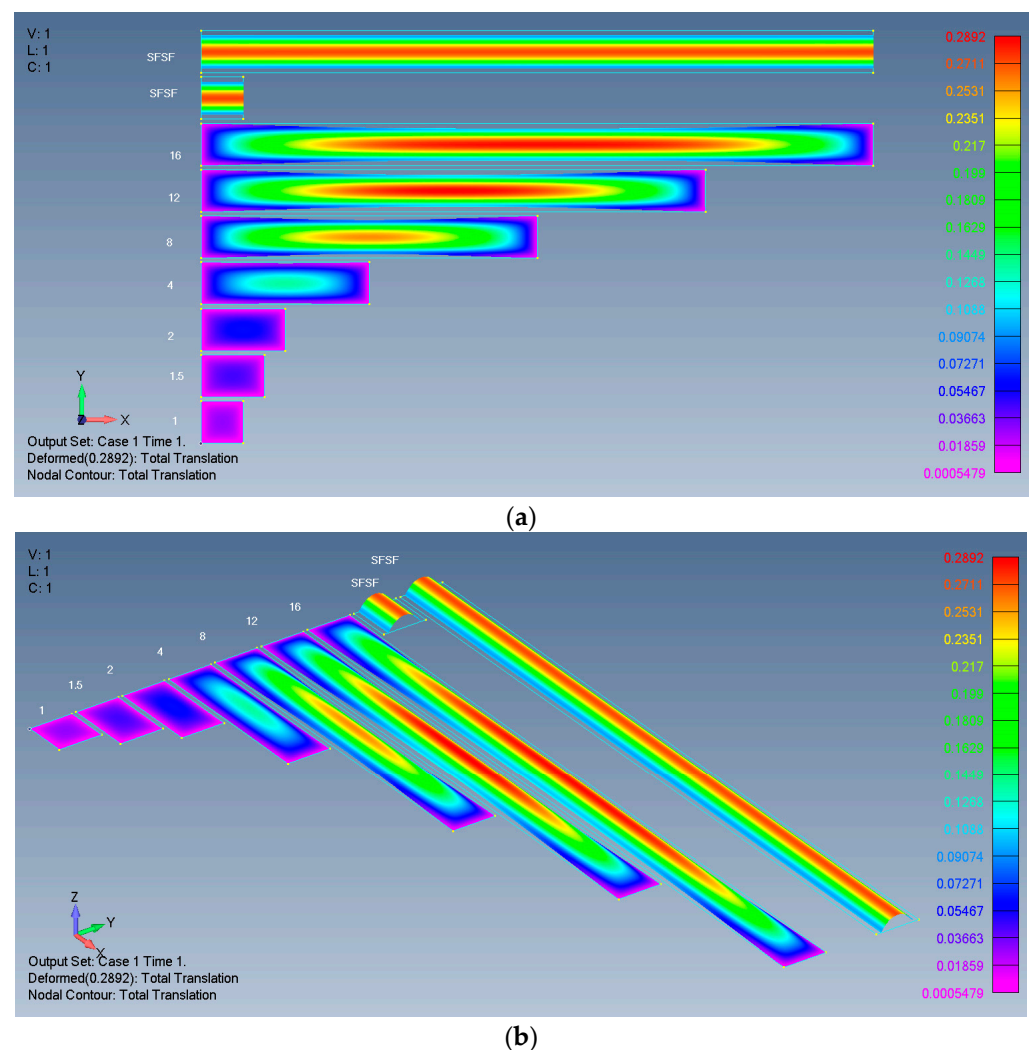


Figure 4. Deflections of rectangular isotropic plates: (a) 2D view; (b) 3D view.

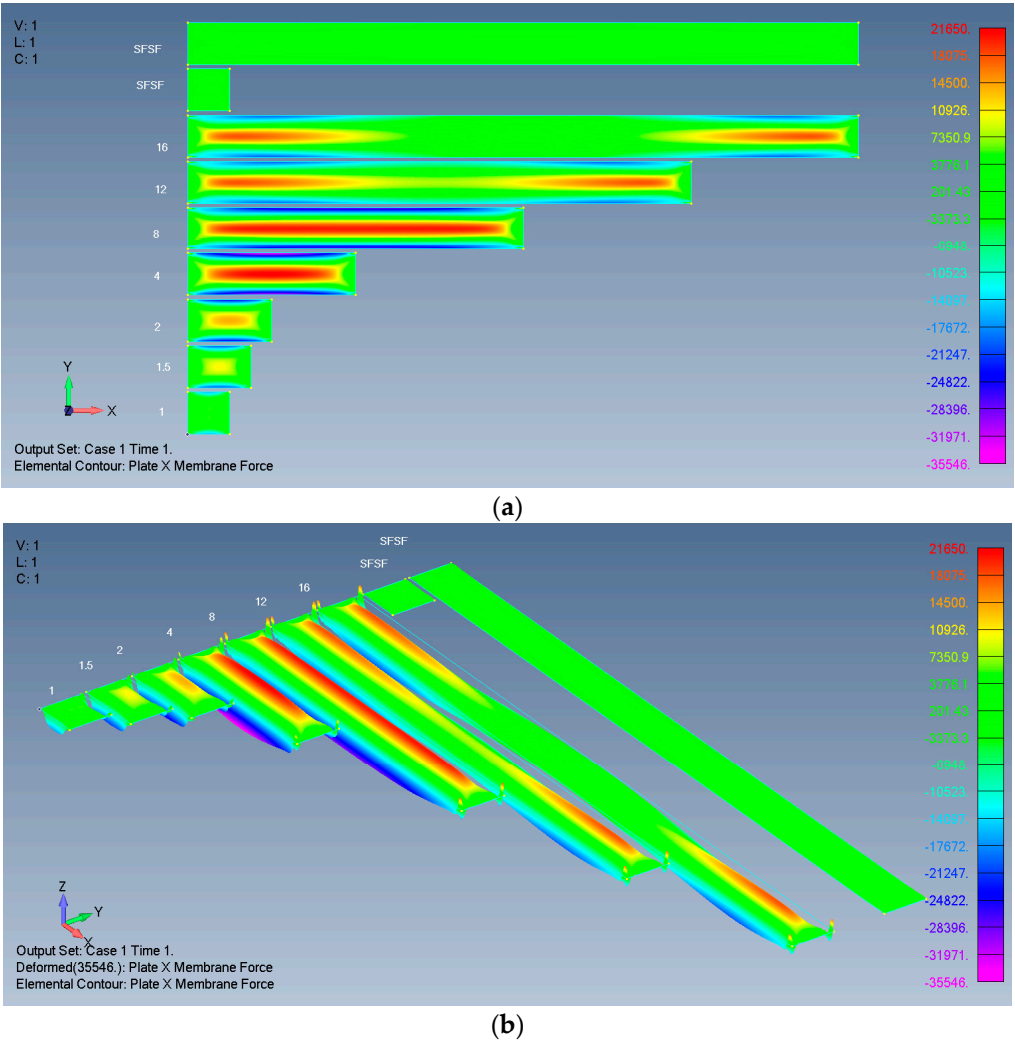


Figure 5. X membrane forces of rectangular isotropic plates: (a) 2D view; (b) 3D view.

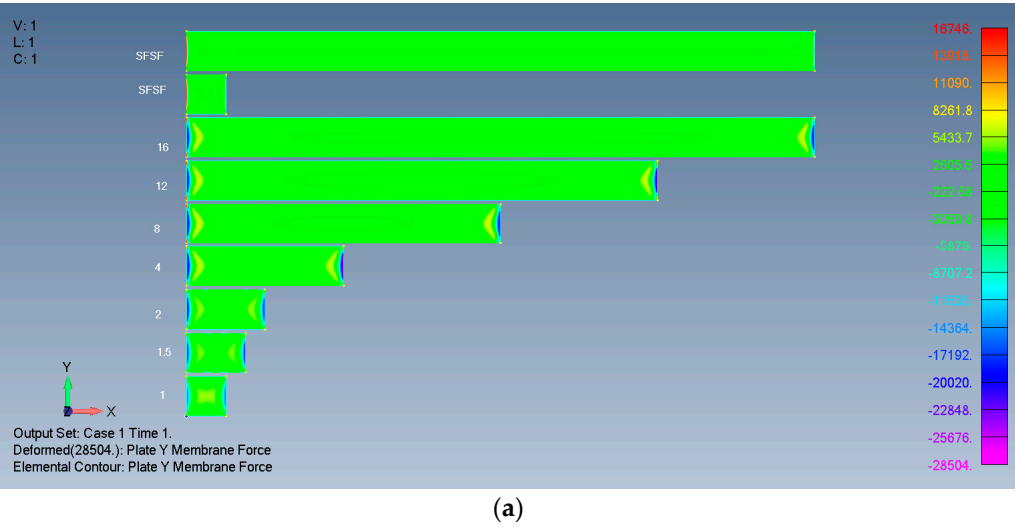
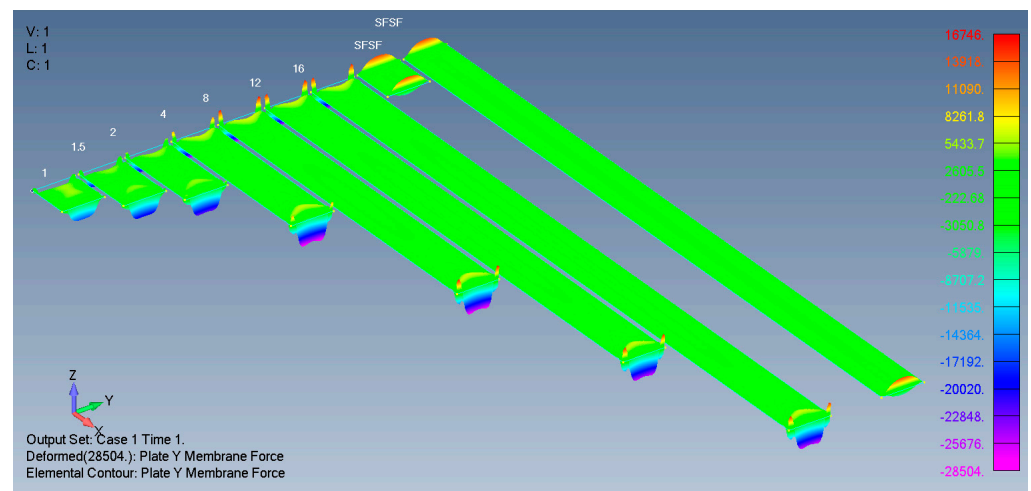
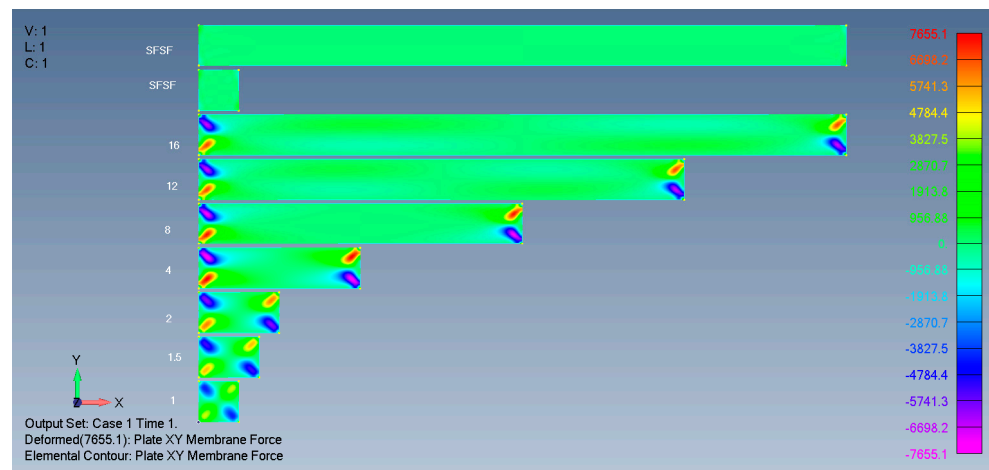


Figure 6. Cont.

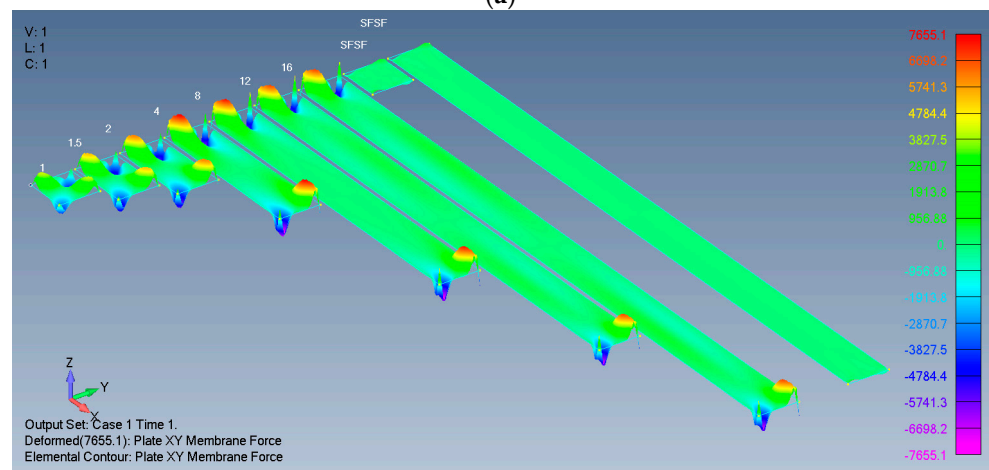


(b)

Figure 6. Y membrane forces of rectangular isotropic plates: (a) 2D view; (b) 3D view.



(a)



(b)

Figure 7. XY membrane forces of rectangular isotropic plates: (a) 2D view; (b) 3D view.

However, checking the deflection against increasing values of AR reveals an interesting phenomenon for the movable BC (M) cases. To highlight it, the midpoint deflections for several plates were normalized by the deflection of a square plate ($AR = 1$). The relative

deflection is defined as the ratio of the midpoint deflection to the midpoint deflection of a square plate. The resulting graphs are presented in Figures 8 and 9.

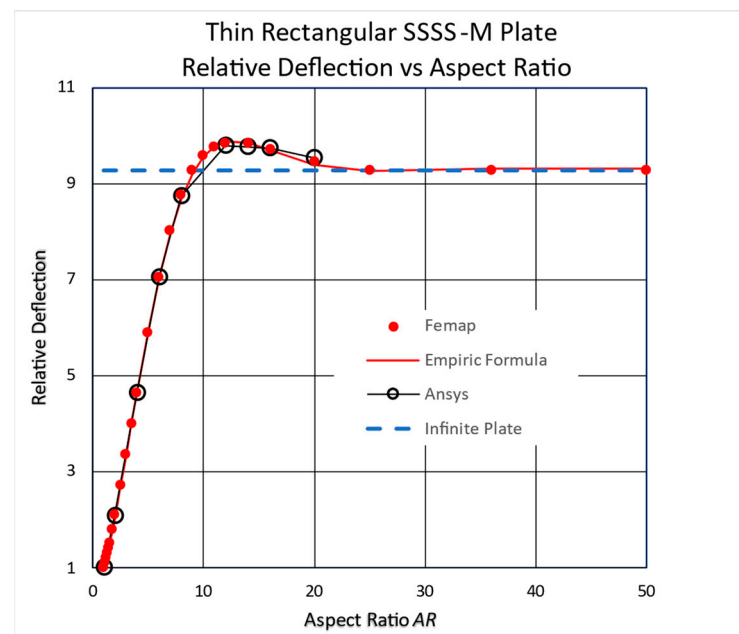


Figure 8. Relative midpoint deflection for high ARs—FEAs and empirical formula.

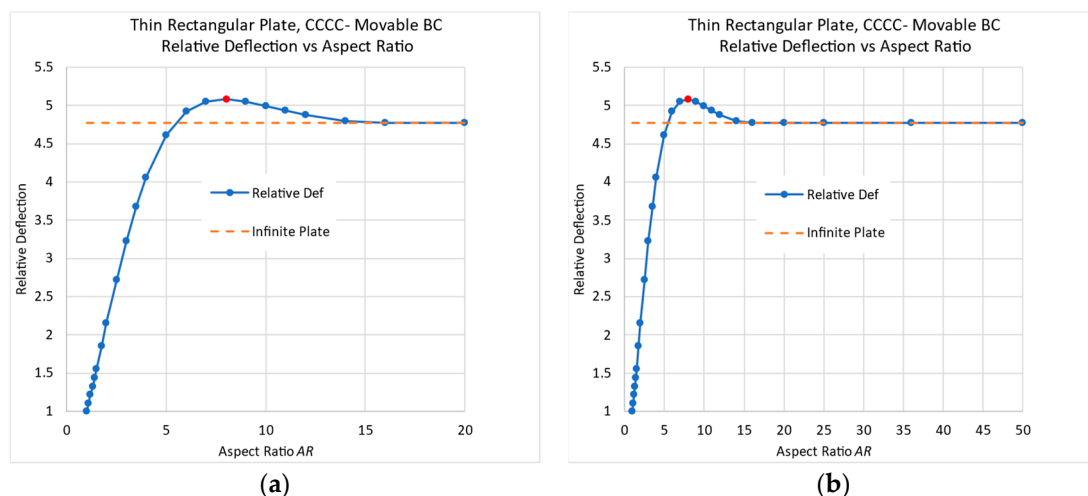


Figure 9. CCCC-M's relative midpoint deflection for high AR: (a) up to $AR = 20$; (b) up to $AR = 50$.

As shown in Figure 8, the plate's deflections for $8 < AR < 20$ are higher than that of an infinitely long plate with a maximum $AR = 12$. This unexpected behavior persists with other Young's moduli and other Poisson's ratios ν , including $\nu = 0$.

Identical results were obtained using another piece of FEA software, Ansys 2023/R2, as can be seen in Figure 8.

Ansys is a large scientific analysis code described partially in [18,19]. The section used here was Ansys Workbench with its Static Structural finite element section. In the analysis, the large-deflection option was activated to correctly account for the in-plane strains.

To present the behavior shown in Figure 8 more conveniently, an empirical formula describing the relative midpoint deflection, RD , as a function of AR using a minimum squares regression was performed. Excel's Solver add-in was used to minimize the squares sum by optimizing the formula coefficients. This empirical formula for the SSSS-M BC case is presented in Equation (2):

$$RD = B + C \cdot \exp(D \cdot AR) \cdot \sin(E \cdot AR + F) \quad (2)$$

The optimal coefficients were found to be $B = 9.3078$, $C = -11.167$, $D = -0.20872$, $E = 0.24266$, and $F = 0.90972$, with an excellent correlation coefficient of $R = 0.99996$. The valid range of AR in (2) is $AR \geq 1$, while the RD value approaches B for high AR s. The influence of other material properties on this behavior is still to be checked.

The clamped movable CCCC-M case in Figure 9 shows a similar behavior, but with another high range, $6 < AR < 15$, and a lower maximum point at $AR = 8$.

To demonstrate that the above phenomenon also occurs in other types of plates made of non-isotropic materials, 16 mm multiwall plates were analyzed. Multiwall plates consist of parallel thin-walled sheets connected by vertical ribs, extruded from various plastics to yield a light and yet rigid structure, as seen in Figure 10.

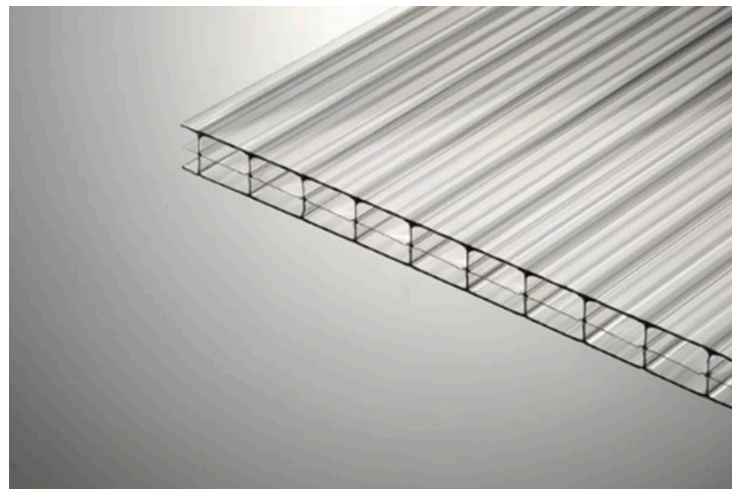


Figure 10. A 16 mm triple-wall plate.

A detailed description of these plates and their mechanical behavior under uniform pressure is presented in Hakim and Abramovich's work [17].

To correctly account for the complex internal structure, a homogenization procedure was applied, in which 10 independent elastic constants represented the global plate's elastic response as a solid plate, ignoring the small internal structure features. In Table 2, these constants are listed (see [20] for the homogenization process of the plate).

Table 2. The 16 mm multiwall elastic equivalent constants.

Equivalent E_y^t modulus	228.56	MPa
Equivalent E_x^t modulus	318.77	MPa
Equivalent G_{xy}^t modulus	83.417	MPa
Poisson's ratio ν_{xy}^t	0.37	
Equivalent E_x^b modulus	647.69	MPa
Equivalent E_y^b modulus	370.51	MPa
Equivalent G_{xy}^b modulus	13.55	MPa
Poisson's ratio ν_{xy}^b	0.37	
Shear rigidity S_x	136.27	N/mm
Shear rigidity S_y	3.8697	N/mm

The superscript t stands for "tension" and b stands for "bending".

The above extruded plate, supported with the SSSS-M BC, was analyzed using the Femap code. The chart in Figure 11 shows the same effect as that discussed above.

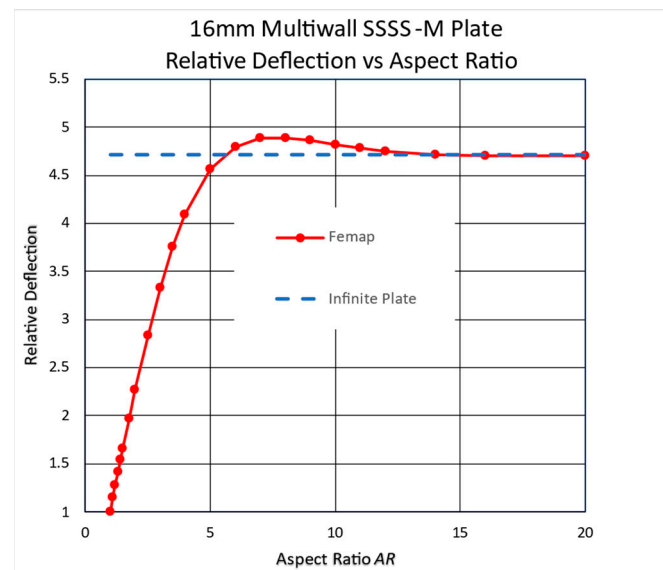


Figure 11. A multiwall plate's higher deflection.

One should note that this behavior occurs only in plates with a movable BC (M), while plates with an immovable BC (I) do not show this phenomenon, as is presented in Figure 12 for isotropic plates with properties and loading identical to the ones used for movable SSSS boundary conditions.

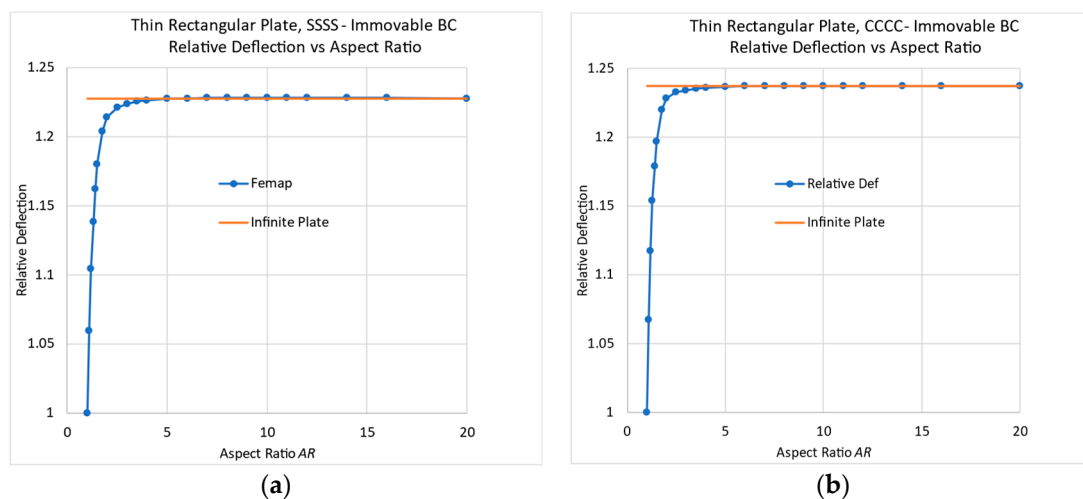


Figure 12. Deflections of rectangular plates on immovable edges: (a) simply supported: SSSS-I; (b) clamped: CCCC-I.

While trying to identify the origin of this behavior, the width midpoint deflections were checked along several long plates. The plates were identical but with different lengths. Figure 13 shows the mid-width deflection along the plates.

From these graphs, it is obvious that the high-deflection red points are located about six widths away from the plate ends. This might explain the maximum seen in Figure 8 at $AR = 12$, where the two maxima meet each other.

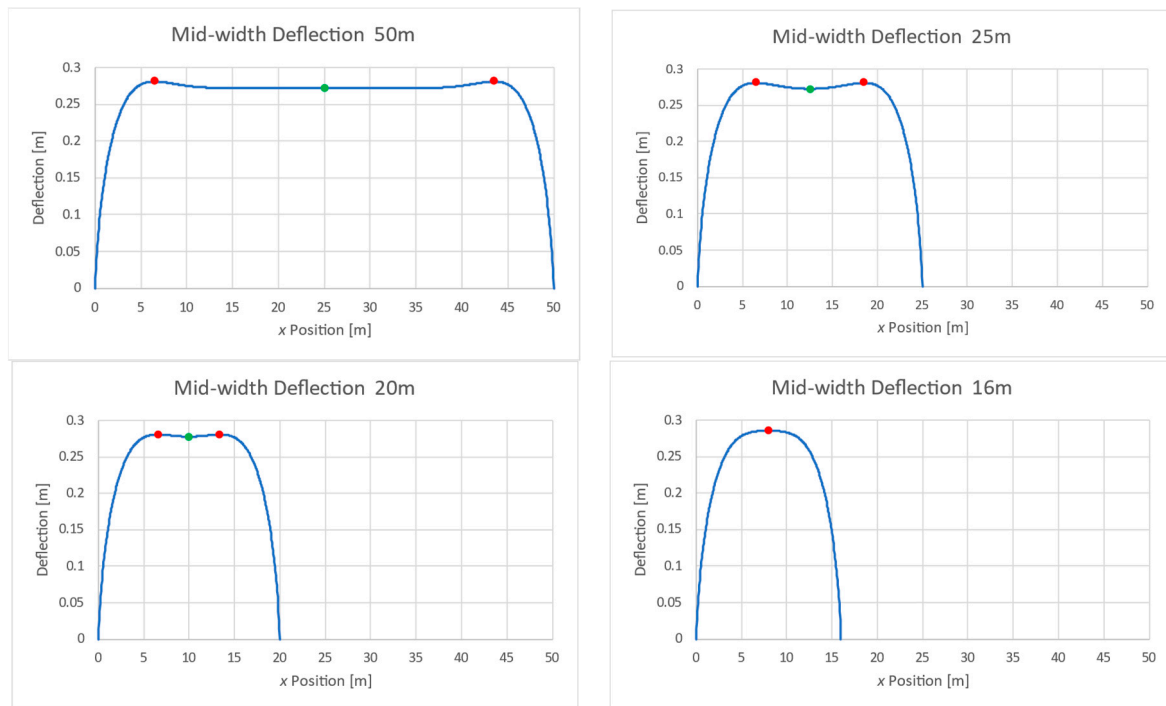


Figure 13. Mid-width deflections of SSSS-M plate with various lengths.

To further investigate the topic in the large-deflection regime and the influence of the type of loading, namely the vector and pressure types described above, a comparison was made between the two cases. Figure 14 demonstrates that both load types present the same behavior, with the pressure load type presenting a smaller deflection.

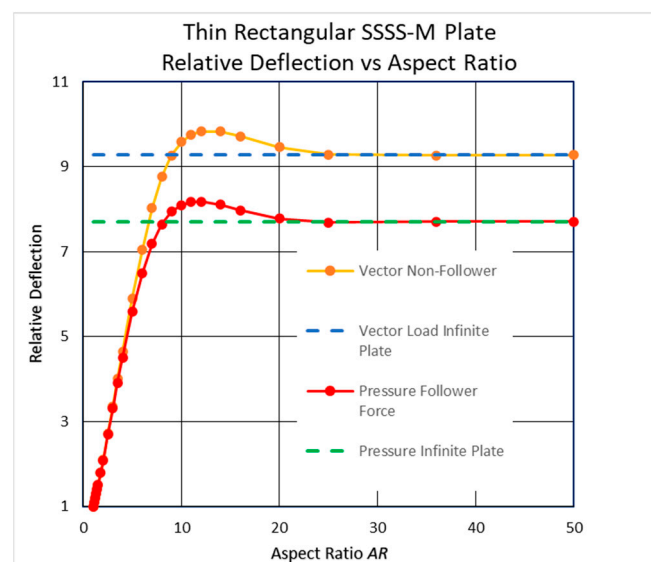


Figure 14. Relative deflection vs. AR for a fixed vector load and a follower pressure load.

To understand the difference in the lateral deflections when applying the two types of loading, as presented in Figure 14, the sum of the vertical (in the direction of the z coordinate—see Figure 3) nodal reaction forces at the plate edges was calculated. The plate dimensions were $1 \text{ m} \times 12 \text{ m}$ and the load value was 1000 Pa (one time with a fixed-direction vector load and one time with a pressure follower load). The reaction with the fixed vector load was exactly 12 kN (load multiplied by the area), while the reaction with the pressure follower load was only 10.7 kN . The reduced reaction in the second case

was caused by the change in the element direction with the pressure force that followed the change. This reduction induced the smaller deflections seen in Figure 14, as the vertical force was reduced.

3. Discussion

The classical question regarding thin rectangular plates, namely, “*Above which aspect ratio (AR) would the plate behave like an infinitely long plate?*”, cannot be answered intuitively. The present study clearly demonstrates that the influence of the plate’s ends on the deflection and on the in-plane stresses penetrates from the ends into the plate’s midpoint differently. Moreover, when the plate becomes longer, the end effects do not disappear but, rather, remain stuck to the moving-away ends.

In addition, for out-of-plane deflections, the end influence penetrates towards the plate’s midpoint from each end by 5–6 times the width.

For tensile membrane X stresses (or forces), the influence also penetrates towards the midpoint by five times the width.

For tensile membrane Y stresses (or forces), the influence penetrates towards the midpoint by one width.

For shear membrane XY stresses (or force), the influence also penetrates towards the midpoint by one width.

Another significant finding stemming from the present study is that the midpoint deflection of a specific AR range is higher than that of an infinitely long plate. Higher deflections occur in SSSS-M’s case six widths away from the ends towards the plate’s midpoint. This is evident for the small-deflection linear Navier solution, all-around clamped plate with small-deflection, and large-deflection regimes with movable edges. The authors believe that this is a new finding that has not been reached before in the literature and will encourage future research on it.

This innovative finding has some implications when designing plates for given specified deflections. The common assumption is that infinitely long plates have the highest deflection, more than any rectangular plate with the same width, and therefore they are considered the most conservative design. However, the common assumption described above might be wrong since the present findings prove that a long plate ($8 < AR < 20$ for a simply supported BC and $6 < AR < 15$ for a clamped BC) in the large-deflection regime would deflect more than an infinite plate, for both simply supported and clamped movable edges. Therefore, caution is suggested when making this common assumption while dealing with long plates under normal lateral pressure.

4. Conclusions

Based on the results presented in the present study, the following conclusions can be drawn:

1. High- AR rectangular plates (above a certain value) with movable edges cannot be considered infinitely long plates, but rather, the end influences are preserved near the ends.
2. The midpoint deflections of the SSSS-M case with $10 < AR < 20$ and of the CCCC-M case with $6 < AR < 15$ are higher than that of an infinitely long plate. A plausible explanation for this phenomenon might be the unusual shape of the deflections for these unique boundary conditions.
3. This behavior has also been detected for small-deflection theory based on the classical Navier solution and all-around clamped plates in the small-deflection regime.
4. The influence of the plate’s ends on the deflection and on the in-plane stresses penetrates from the far ends into the plate’s midpoint in different ways.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. (Based on [21])

Finite element analysis (FEA) process description

FEA software is used to calculate and display the plate response to load. There are several software packages available on the marketplace that are suitable for this task. Among these, one piece of software is unique—Simcenter Femap, ver. 2021.1. This software, Femap, functions as an interface for preparing and displaying FEA models for other software packages. It can run ABAQUS, ANSYS, I-DEAS, LS-DYNA, MSC.Mark, Nastran, MSC.Patran, and more. Nevertheless, Femap itself includes a well-integrated internal solver running Simcenter Nastran. The software is called “Simcenter Femap with Nastran”, and it is the one used for this paper.

The FEA process has three main steps: model preparation, running the solver, and post-processing. A description of the details of each step is given here as user instructions.

Model preparation:

1. Geometry: create surfaces with the shape and dimensions of the requested plates—squares and rectangles.
2. Define the material for the plate: isotropic polycarbonate with an E modulus of 2.4 GPa and Poisson’s ratio of 0.38. These two constants are enough here.
3. Define the property for the plate element using the material from step 2 with a thickness of 0.005 m (5 mm); give all other inputs the default values.
4. Mesh the surface with 50×50 mm quad plate elements using the property from step 3.
5. Set the distributed load on the surface; use force per area on surface for a directional vector load and pressure on surface for a follower load. Set the load value to 1000 Pa in the Z direction.
6. Define the boundary conditions, which are called constraints in the software. TX-1, TY-2, and TZ-3 relate to translation in the x,y,z directions. RX-4, RY-5, and RZ-6 relate to rotations around the x,y,z axes. The four types of BCs (simply supported, clamped, movable, immovable) are alternatively set on the surface perimeter:
 - SSSS-M: TZ, designated 3 (in-plane movable BC);
 - CCCC-M: TZ, RX, RY, designated 345 (in-plane movable BC);
 - SSSS-I: TX, TY, TZ, designated T (in-plane immovable BC);
 - CCCC-I: TX, TY, TZ, RX, RY, RZ, designated F (in-plane immovable BC).

Additionally, the FEA process requires that all free-body degrees of freedom (DOF) will be eliminated. For that, virtual (not participating) BCs are added where necessary: the plate midpoint may have a TX and TY designated 12, and one edge may have an additional TX or TY.

7. The last step is to define the type of analysis to run. Since large deflections with nonlinear effects are expected, we chose the analysis type “10..Nonlinear Static”. This analysis type has many input parameters that control its operation. From these, we set “Increments or Time Steps” to 20, and in “Output Control”, we collected “All” intermediate results. These settings enabled us to later see how various plate features developed as the load increased.

Analyzing the model:

Pressing the Analyze button begins the process. During the calculation, the convergence of each step is checked. In cases where the convergence is slow, Nastran automatically reduces the step size, which increases the calculation time. If the step does not converge, the run stops, but the results up to the last step are saved. Upon analysis completion, it is possible to see the results.

For this analysis, Nastran uses the PSHELL entry that uses the CQUAD4 element. The CQUAD4 element can model in-plane, bending, and transverse shear behavior. The element's behavior is controlled by the presence or absence of a material ID number in the appropriate field(s) on the PSHELL entry. This is a plane stress 2D element with four nodes.

Post-processing:

A very rich set of tools allows us to see and report almost any requested feature of the plate performance. We used four output vectors to show the Deform and Contour (color) styles of the results:

- Total translation (deflection);
- Plate X membrane force;
- Plate Y membrane force;
- Plate XY membrane force.

The resulting pictures (2D and 3D) were then saved to be used in the report.

The accuracy (convergence) of the FEA model and the comparison to previously published data are discussed in depth in [21], showing that the precision of the model built for the large deflections of plates under normal uniform pressure is high and fits the few available results presented in the literature.

References

1. Timoshenko, S.; Woinowsky-Krieger, S. *Theory of Plates and Shells*, 2nd ed.; 1959 Reissued 1987; McGraw-Hill Book Company: New York, NY, USA, 1987; ISBN 0-07-064779-8.
2. Reddy, J.N. *Theory and Analysis of Elastic Plates and Shells*, 2nd ed.; CRC Press: Boca Raton, FL, USA; Taylor & Francis Group: Abingdon, UK, 2007. [CrossRef]
3. Bakker, M.C.M.; Rosmanit, M.; Hofmeyer, H. Approximate large-deflection analysis of simply supported rectangular plates under transverse loading using plate post-buckling solutions. *Thin-Walled Struct.* **2008**, *46*, 1224–1235. [CrossRef]
4. Bhaskar, K.; Varadan, T.K. *Plates—Theories and Applications*; Springer: Berlin/Heidelberg, Germany; ANE Books Pvt, Ltd.: New Delhi, India, 2021; ISBN 978-3-030-69423-4/978-3-030-69424-1. [CrossRef]
5. McFarland, D.E.; Smith, B.I.; Bernhart, W.D. *Analysis of Plates*, 1st ed.; Spartan Books: New York, NY, USA, 1972; ISBN-10: 0876715609; ISBN-13: 978-0876715604.
6. Onyeka, F.C.; Okeke, E.T.; Nwa-David, C.D.; Mama, B.O. Exact analytical solution for static bending of 3-D plate under transverse loading. *J. Comput. Appl. Mech.* **2022**, *53*, 309–331. [CrossRef]
7. Paik, J.K. *Ultimate Limit State Analysis and Design of Plated Structures*, 2nd ed.; John Wiley & Sons Ltd.: Hoboken, NJ, USA, 2018; Chapter 4; pp. 179–270, ISBN 978-1-119-36775-8.
8. Bauer, F.; Bauer, L.; Becker, W.; Reiss, E.L. Bending of rectangular plates of finite deflections. *J. Appl. Mech.* **1965**, 821–825. [CrossRef]
9. Brown, C.J.; Goodey, R.J.; Rotter, J.M. Bending of rectangular plates subject to non-uniform pressure distributions relevant to containment structures. In Proceedings of the Eurosteel 2017, Copenhagen, Denmark, 13–15 September 2017. [CrossRef]
10. Wang, D.; El-Sheikh, A.I. Large-Deflection Mathematical Analysis of Rectangular Plates. *J. Eng. Mech.* **2005**, *131*, 809–821; ISSN 0733-9399. [CrossRef]
11. Shao, C.J. Large Deflection of Rectangular Plate with High Aspect Ratio. Master's Thesis, University of California, Berkeley, CA, USA, 1964. Available online: <https://calhoun.nps.edu/handle/10945/12540> (accessed on 6 June 2024).
12. Razdolsky, A.G. Large Deflections of Elastic Rectangular Plates. *Int. J. Comput. Methods Eng. Sci. Mech.* **2015**, *16*, 354–361. [CrossRef]
13. Ostiguy, G.L.; Evan-Iwanowski, R.M. Influence of the Aspect Ratio on the Dynamic Stability and Nonlinear Response of Rectangular Plates. *J. Mech. Des.* **1982**, *104*, 417–425. [CrossRef]
14. Covestro AG. Leverkusen, Makrolon 2407 Datasheet. Available online: https://solutions.covestro.com/-/media/covestro/solution-center/products/datasheets/imported/makrolon/makrolon-2407_en_56977361-00009645-19213223.pdf (accessed on 6 June 2024).
15. Sabic Lexan 940A Datasheet. Available online: <https://www.matweb.com/search/datasheet.aspx?matguid=cda0ab793fed4ed6a7e5df4902937a4b&ckck=1> (accessed on 6 June 2024).

16. Siemens. *Simcenter Nastran Basic Nonlinear Analysis User's Guide*; Siemens Industry Software: Plano, TX, USA, 2020. Available online: https://docs.plm.automation.siemens.com/data_services/resources/scnastran/2020_1/help/tdoc/en_US/pdf/basic_nonlinear.pdf (accessed on 6 June 2024).
17. Hakim, G.; Abramovich, H. Multiwall rectangular plates under transverse pressure—A non-linear experimental and numerical study. *Materials* **2023**, *16*, 2041. [CrossRef]
18. Lee, H.-H. *Finite Element Simulations with ANSYS Workbench 2023: Theory, Applications, Case Studies*; SDC Publications: Mission, KS, USA, 2023; ISBN 1630576158/9781630576158. Available online: <https://www.amazon.com/Finite-Element-Simulations-ANSYS-Workbench/dp/1630576158> (accessed on 6 June 2024).
19. Ansys Internet Site. Available online: <https://www.ansys.com/products/ansys-workbench> (accessed on 6 June 2024).
20. Hakim, G.; Abramovich, H. Homogenization of multiwall plates—An analytical, numerical and experimental study. *Thin-Walled Struct.* **2022**, *185*, 110583. [CrossRef]
21. Hakim, G.; Abramovich, H. Large Deflections of Thin-Walled Plates under Transverse Loading—Investigation of the Generated In-Plane Stresses. *Materials* **2022**, *15*, 1577. [CrossRef]

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