mathematics

Article

Multi-Product Production System with the Reduced Failure Rate and the Optimum Energy Consumption under Variable Demand

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Received: 31 December 2018; Accepted: 26 February 2019; Published: 24 May 2019

Abstract: The advertising of any smart product is crucial in generating customer demand, along with reducing sale prices. Naturally, a decrease in price always increases the demand for any smart product. This study introduces a multi-product production process, taking into consideration the advertising- and price-dependent demands of products, where the failure rate of the production system is reduced under the optimum energy consumption. For long-run production systems, unusual energy consumption and machine failures occur frequently, which are reduced in this study. All costs related with the production system are included in the optimum energy costs. The unit production cost is dependent on the production rate of the machine and its failure rate. The aim of this study is to obtain the optimum profit with a reduced failure rate, under the optimum advertising costs and the optimum sale price. The total profit of the model becomes a complex, non-linear function, with respect to the decision variables. For this reason, the model is solved numerically by an iterative method. However, the global optimality is proved numerically, by using the Hessian matrix. The numerical results obtained show that for smart production, the maximum profit always occurs at the optimum values of the decision variables.

Keywords: inventory; smart production; variable demand; advertisement; energy; rework; system reliability

1. Introduction

Any production system may produce both perfect and imperfect products. During the long-run production process of a smart production system, there may be a chance of machine failure, due to machine breakdown, unskilled laborers, or interrupted energy suppliers, and so on. Due to these reasons, a defective product may be produced in two ways: at a constant rate or at a random rate. For a constant defect rate, the total number of defective items is constant and, for a random defect rate, the total number of defective products is a random variable. Many studies based on constant defect rates in single-item production systems have been carried out (see, for instance, [1]); however, very little research based on random defect rates (see, for instance, [2]) is available. Sarkar [3] proposed a model for a multi-item production system with a random defect rate and budget and space constraints. There has been no research, to our knowledge, on multi-item smart production systems with the optimum consumption of energy and a random defect rate for smart products, with advertisement, price-dependent demand patterns, and reduced failure rates. Therefore, this proposed model gives a new direction for production systems with budget and space constraints under the effect of energy.

For this type of (perfect and imperfect) production system, rework has a vital role in smart production, where the production system becomes out-of-control (from an in-control state) within a random time interval. It provides a reliable system by reworking the defective items. Some studies of imperfect production systems with deterioration are already available in the literature, such as Rossenblat and Lee [4], who discussed imperfect production systems; which was extended by Kim and Hog [5] by considering the deterioration of products in a production system to find the optimal production length. They indicated three types of deterioration methods—constant deterioration, linearly increasing deterioration, and exponentially increasing deterioration—for system moving from an in-control to an out-of-control state. Giri and Dohi [6] proposed a model which highlights random machine failure rates for single-item production systems, where the machine breakdown is stochastic and the preventive maintenance time for machine failure is also a random variable.

Sana et al. [7] gave an idea for a research model of an imperfect production system, by considering defective products with reduced prices, although this type of idea is now common in the market. Chiu et al. [8] proposed a model which assumes a rework policy of the defective products, using extra costs for the reworking of imperfect products. In the literature, there are few models based on energy in an imperfect smart production system. No study, to our knowledge, has considered the optimum energy consumption and its profit for a smart production system, under advertising- and price-dependent demands. Egea et al. [9] expressed, in a model, how to measure energy during the loading of a smart machine. González et al. [10] proposed a model for turbo-machinery components, using a total energy consideration.

Sarkar [11] considered an inventory model involving stock-dependent demand with delayed payments. This model indicated the replenishment policy for an imperfect production system with a finite replenish rate. A production-inventory model with deterioration and a finite replenishment rate was developed by Sarkar [12]. In this model, to maximize the profit, several discount offers for customers, to attract a large order size, were considered. Generally, imperfect product production depends on the production system reliability. Sarkar [13] developed an economic manufacturing quantity (EMQ) model with an investment in the production system for the development of a high system reliability with lower imperfect production. This model first considered an advertisement policy, where the demand depends on the price of products. An imperfect production model was considered by Chakraborty and Giri [14] with some imperfect products produced in an out-of-control system during preventive maintenance. An inspection was considered in this model to detect the defective items for the reworking process, although some defective products cannot be repaired.

Sarkar [15] investigated an economic production quantity (EPQ) model with imperfect products, where a back-ordering policy was included in the model, along with a reworking policy. To calculate the rate of defective items, three different distribution functions were used and the results are compared in this model. Sarkar and Saren [16] introduced a production model in an imperfect production system with an inspection policy. In their model, the production system becomes out-of-control in a random time interval. This model considered a quality inspector for choosing falsely an imperfect product and making a decision about quality, and vice versa. Over a fixed time period, the warranty policy also makes this model more realistic.

Pasandideh et al. [17] extended a production model for multiple products in a single-machine imperfect production system, where the imperfect products were classified by their nature, to consider whether to rework or scrap them. To make this model more realistic, they considered fully backlogging all shortages. An inventory model for a system with a non-stationary stochastic demand with a detailed analysis of the lot-size problem was developed by Purohit et al. [18]; a carbon-emissions mechanism was included with this model to make it a more generalized study. This model discussed labor issues with the training required involving work related to the machine. Sana [19] considered an EPQ lot-size model for imperfect production with defective items when the system becomes out-of-control. An optimal inventory for a repair model was initiated by Cárdenas-Barrón et al. [20].

Another two research models, based on deterioration and partial backlogging, were developed by Tiwari et al. [21,22]

Storage capacity for an inventory system plays an important role in any production house. Limited storage makes increasing production problematic. Due to this reason, shortages may occur. Huang et al. [23] developed an inventory model in which a rental warehouse was considered, with an associated cost, for fulfilling the required capacity in addition to the available warehouse. This inventory model investigated optimal retailer lot-size policies with delayed payments and space constraints. An inventory model for multiple products with limited space was proposed by Pasandideh and Niaki [24]. This model contained a non-linear integer programming problem and found an optimal solution for the available warehouse by adding space constraints. Through a genetic algorithm with a non-linear cost function and space constraint, a multi-stage inventory model was discussed by Hafshejani et al. [25]. An inventory model considering demand and limited space availability, where reliability depends on the unit production costs, was introduced by Mahapatra et al. [26].

In a production model, the budget for a production system is initially required for the manufacturer. Instead of a periodic budget, a limited budget is preferable. There is some on-going research into budget constraints, such as Taleizadeh et al. [27], who proposed a model, in a multi-item production system, for a reworking of defective items policy. They found the global minimum of the total cost by considering a service level and a budget constraint. Minimizing the total annual cost with a limited capital budget and calculating the optimal lot size and capital investment in a setup-costs model was explained by Hou and Lin [28]. Mohan et al. [29] introduced an inventory model, considering delayed payments, budget constraints, and permissible partial payment (with penalty) for a multi-item production system with a replenishment policy. Todde et al. [30] and Du et al. [31] published the basic energy models, based on energy consumption and energy analysis. Cárdenas-Barrón et al. [32] proposed an inventory model with an improved heuristic algorithm solving method for a just-in-time (JIT) system with the maximum available budget. Xu et al. [33] presented a bio-fuel model for the pyrolysis products of plants. Tomić and Schneider [34] discussed a method for recovering energy from waste using a closed-loop supply chain. Haraldsson and Johansson [35] developed an energy model, based on different energy efficiencies during production. Similarly, Dey et al. [36] and Sarkar et al. [37–39] developed their models based on energies, but did not consider advertising for smart products, reducing the sale price of products, or reducing the failure rate of a production system. Yao et al. [40] and Gola [41] put forth the valuable idea of formulating a model considering the reliability of a manufacturing system.

The world becomes smarter every day. Several researchers have discussed imperfect production processes, such as Tayyab et al. [42], Sarkar [43], Kim et al. [44], and Sarkar et al. [45], but the effect of a smart manufacturing system in any production model has not been discussed. The effects of energy and failure rate in a multi-item smart production system was first discussed by Sarkar et al. [3]. In reality, the demand for a particular product depends on various key factors, with advertisement of a particular product being one of them. Ideally, advertisement of a particular product increases the demand for that product. Thus, the total profit can be optimized when the demand depends on the advertisement of products. The relationship between product quality and advertising, in an analytical model, was developed by Chenavaz and Jasimuddin [46]; they also explained the positive and negative advertising–quality relationships. A two-level supply chain model was developed by Giri and Sharma [47], where the demand depends on the advertising cost. In this model, they considered a single-manufacturer, two-retailer system, where the retailers compete. In the same direction, Xiao et al. [48] formulated a two-echelon supply chain model for a single manufacturer and multiple retailers. In this model, they studied co-operative issues in advertising. Recently, Noh et al. [49] developed a two-echelon supply chain model, where the demand depends on the advertisement. They used the Stackelberge game policy to solve this model. Sale price, also, has a great impact on the demand for a product. The demand for a product gradually increases if the sale price is less, and vice versa. Constant demand is a business service that helps customers to find new consumers and penetrate new markets, which can optimize the inside-sales and marketing activities to achieve high-quality sales. Variable demand can predict and quantify changes that are caused by transportation conditions on the demand. As a high price negatively affects how likely clients are to buy products or services, assuming the demand to be price-dependent is more realistic. Karaoz et al. [50] considered an inventory model with price- and time-dependent demand, under the influence of complementary and substitute product sale prices. The finite replenishment inventory model was developed by considering the demand to be sensitive to changes in time and sale price. Sana [51] introduced the price-sensitive demand for perishable items in an inventory model. The demand for any inventory system is not always constant and may depend on time, sale price, and inventory. Pal et al. [52] developed a multi-item inventory model where the demand was sensitive to the sale price and price-break. Sarkar et al. [53] developed an EMQ model with price- and time-dependent demand, under the effects of reliability and inflation. Sarkar and Sarkar [54] established an inventory model, where the demand was inventory-dependent, and an algorithm was developed to maximize the profit. To maximize the vendor profit, the optimal ordering quantity and sale price were optimized by an analytical procedure. An integrated model, with development lead time and production rate, was discussed by Azadeh and Paknafs [55]. Several researchers have developed many models where the demand depends on the sale price of the products or advertising-dependent demand; however, price- and advertising-dependent demand in a smart production system has still not been considered, to our knowledge. Thus, this new direction is considered in this research. Table 1 shows the contribution of previous author(s).

Table 1. Author(s) contribution table.

Author(s)	Development Cost	Inspection	Demand	Advertisement	Energy	Rework	Reliability
Cárdenas-Barrón [1]			√			√	
Sana et al. [2]			√				
Sarkar et al. [3]	√	√	√		√	√	√
Rosenblatt and Lee [4]			√				
Sarkar [12]	√		√				√
Sarkar and Saren [16]			√				
Sana [19]	√		√				√
Taleizadeh et al. [27]			√				√
Mohan et al. [29]	√		√				
Cárdenas-Barrón [32]			√			√	√
This Paper	√	√	√	√	√	√	√

2. Problem Definition, Notation, Assumptions

2.1. Problem Definition

Smart production systems are modern production systems, in which the main aim is to produce smart products (such as mobile phones, computers, and so on) through some smart machines by some smart, skilled labors. The whole smart production process is controlled under the optimum energy consumptions. For example, a smart machine can produce more woolen clothes than an expert labor; in this case, quality and quantity will also be more developed than in simple production systems. An imperfect production system is a production system in which a defective product is produced in a long-run process, due to machine breakdown or any other problem. Energy consumption has an important role in imperfect production systems. In smart production systems, production of imperfect products can be controlled with sufficient consumption of energy. Our model is based on a smart production system for multiple products, where machine failure occurs in random time intervals, γ_i, in which there is an unusual amount of energy consumption and less reliable conditions and production completed in random time intervals, T_i . For this situation, the development cost used in this model is reliability-dependent, to reduce the machine failure rate and to minimize the consumption of energy. An inspection cost is used to detect defective items. For the popularity of the products in this model, advertisement costs and price-dependent demand patterns are used. To obtain more reliability, there are a several possibilities for the reworking of defective items before being sold to the customer. The aim of this model is to obtain the maximum profit for a multi-item production system with the conditions of energy consumption and a random defect rate, under the effects of advertisement and sale price. Process flow for multi-product production system are shown in the following Figure 1.

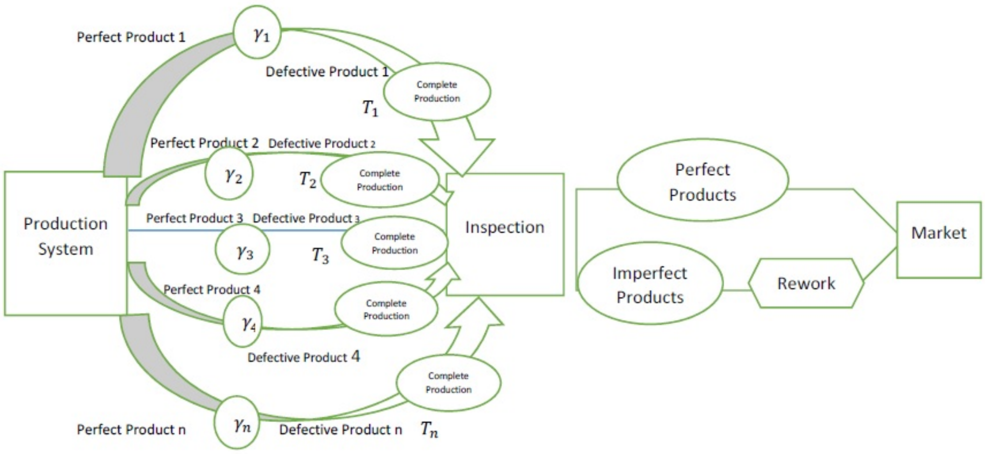

Figure 1. Complex production management with system reliability.

2.2. *Assumptions*

In developing the proposed model, the following assumptions are considered:

1. This is a multi-item production model, where defective products are produced after a random time. The system moves to an out-of-control state at a random time, γ_i, at a failure rate of α, and produces defective items. These items are then reworked to make them as new products.
2. This system is controlled, under energy consumption, for completely finished products only.
3. No shortages are allowed for this multi-item smart production system, as variable production rate is greater than the demand and the lead time is considered to be negligible.
4. The demand is assumed to be advertising- and price-dependent, to increase the demand pattern. It is taken as $D(S,y) = \frac{S_{max}-S}{S-S_{min}} + xy^{\mu}$, where x is the scaling parameter and μ is the shape parameter.
5. If the failure rate α decreases, then the system will be more reliable, and vice versa.
6. The system contains multiple items and, thus, there will be a possibility for space problems for the products, which affects the total budget. Thus, to make this model more realistic under the sufficient energy consumption, space, and budget constraints are considered in this model.

3. Model Formulation

The model studies about a smart production system for multi-item. During the time $t_i = 0$ to $t_i = t_{1i}$, the production continues with upward direction graph and for the time $[t_{1i}, T]$, there is no production, thus, the holding inventories positions are downstream direction with demand D. The production system becomes out-of-control from in-control in random time interval γ_i. See Figure 2 for the description of the production system. The governing differential equation of the inventory is

$$\frac{dI_{1i}(t_i)}{dt_i} = p_i - D, 0 \leq t_i \leq t_{1i} \tag{1}$$

with initial condition $I_{1i}(0) = 0$ and

$$\frac{dI_{2i}(t_i)}{dt_i} = -D, t_{1i} \leq t_i \leq T \tag{2}$$

with initial condition $I_{2i}(T) = 0$.

Solving the above differential equations, one can obtain

$$I_{1i} = (p_i - D)t_i, 0 \leq t_i \leq t_{1i} \tag{3}$$
$$I_{2i} = D(T - t_i), t_{1i} \leq t_i \leq T \tag{4}$$

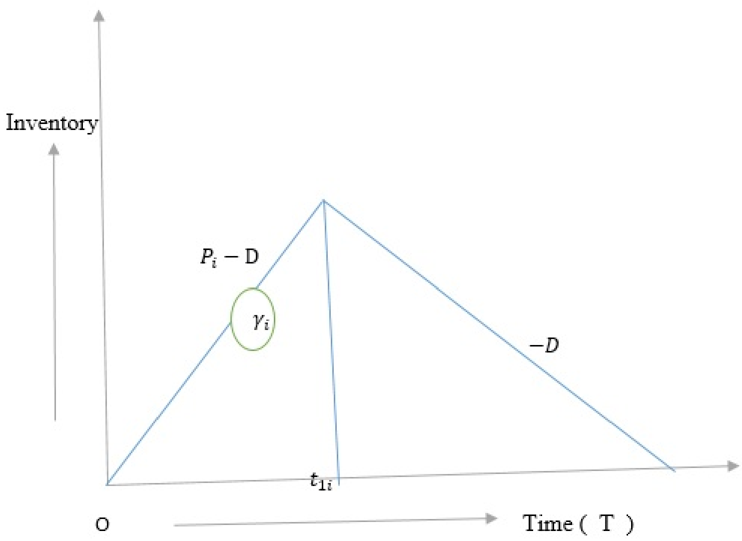

Figure 2. Economic Production quantity.

Considering the following costs in a multi-item smart production system, we must optimize for those which have a low failure rate and maximum profit.

Setup Cost (SC)

In this model, setup cost for product i is S_{ci} per setup and per setup energy consumption cost be S'_{ci}. Then, the average setup cost per cycle is

$$\text{SC} = \sum_{i=1}^{n}(S_{ci} + S'_{ci})\frac{D}{q_i}. \tag{5}$$

Holding Cost (HC)

For this production system, to calculate the holding cost it is necessary to find the total inventory by summation from $i = 1$ to $i = n$; further, to get the average inventory, the total inventory can be divided by cycle length. Thus, the total inventory can be calculated as

$$\begin{aligned}\text{Total Inventory} &= \sum_{i=1}^{n}\left[\int_0^{t_{1i}} I_{1i}(t_i)dt_i + \int_{t_{1i}}^{T} I_{2i}(t_i)dt_i\right] \\ &= \sum_{i=1}^{n}\left[\int_0^{t_{1i}} (p_i - D)t_i dt_i + \int_{t_{1i}}^{T} D(T - t_i)dt_i\right]. \end{aligned} \tag{6}$$

Hence, the total holding cost, with sufficient consumption of energy, can be calculated, as follows:

$$\begin{aligned}\mathrm{HC} &= \sum_{i=1}^{n}\frac{(H_{ci}+H_{ci}^{'})D}{2q_i}\left[\int_0^{t_{1i}}(p_i-D)t_i dt_i+\int_{t_{1i}}^{T}((p_i-D)\frac{q_i}{p_i}-Dt_i)dt_i\right] \\ &= \sum_{i=1}^{n}\frac{(H_{ci}+H_{ci}^{'})q_i}{2}\left(1-\frac{D}{p_i}\right). \end{aligned} \tag{7}$$

Development Cost (DC)

The system becomes more reliable based on development costs. A high investment in the production system, in terms of development costs, makes for a low failure rate, and vice versa. The labor cost and energy resources cost are included in the development cost. Thus, the total development cost per unit time is

$$\mathrm{DC} = G + Ze^{r^{\frac{\alpha_{max}-\alpha}{\alpha-\alpha_{min}}}}. \tag{8}$$

Inspection Cost (IC)

A multi-item smart production system is considered to be a long-run process, where the process is imperfect. Thus, an inspection cost plays an important role in detecting defective items, such that the defective products can be reworked easily. Thus, the inspection cost per unit cycle, under energy consumption, is

$$\begin{aligned}\mathrm{IC} &= \sum_{i=1}^{n}(I_c+I_c^{'})q_i\times\frac{D}{q_i} \\ &= \sum_{i=1}^{n}(I_c+I_c^{'})D. \end{aligned} \tag{9}$$

Rework Cost (RC)

For reworking the defective products, at first, inspection is needed for all defective products. To find the RC, it is important to know how many defective products there are, and how much the RC is for defective items. The rate of defective items is considered (see, for instance, [3]) to be $\beta P_i^{\delta}(t_i-\gamma_i)^{\gamma}$, where $\delta \geq 0, \tau \geq 0$, and $t_i \geq \gamma_i$. Hence, the number of imperfect items produced by the machine is

$$\begin{aligned} E(K) &= \sum_{i=1}^{n}\left(\frac{\beta}{\tau+1}\right)p_i^{\delta+1}\int_0^{t_{1i}}(t_{1i}-\gamma_i)^{\gamma+1}dH(\gamma_i) \\ &= \sum_{i=1}^{n}p_i^{\delta+1}\left(\frac{\beta}{\tau+1}\right)e^{\frac{-\alpha q_i}{p_i}}\chi\left(\alpha,\frac{q_i}{p_i}\right),\ \ as\ \ t_{1i}=\frac{q_i}{p_i}. \end{aligned} \tag{10}$$

For reworking the defective products to perfect state, a RC is needed, along with a consumption of energy cost. The rework cost (RC) per cycle can, thus, be calculated as

$$\mathrm{RC} = \sum_{i=1}^{n}(R_{ci}+R_{ci}^{'})\frac{D}{q_i}E(K). \tag{11}$$

Unit Production Cost (UPC)

The production cost depends on the raw material costs, the development cost, and the tool/die cost. It is proportional to the cost of the previous indicated costs. The quality of the raw materials affects the reliability of the products. In this model, the unit production cost is considered to be as follows:

$$\mathrm{UPC} = \sum_{i=1}^{n}[M_c+\frac{\mathrm{DC}}{p_i}+\sigma p_i^{\xi}]. \tag{12}$$

Advertisement Cost (AC)

The multi-item production system based on advertisement, and demand gradually increased due to huge amount investment on advertisement.This model consider advertisement cost to make popular of the smart products.

$$\text{AC} = \frac{hy^2}{2}. \tag{13}$$

Total Expected Profit (TEP)

The TEP per cycle is

$$\begin{aligned}
\text{TEP}(q_i, p_i, S, \alpha, Y) &= \text{Revenue} - \text{HC} - \text{SC} - \text{IC} - \text{RC} - \text{AC} \\
&= \textstyle\sum_{i=1}^{n} \Big[D(S - \text{UPC}) - \frac{(H_{ci}+H'_{ci})q_i}{2}\left(1 - \frac{D}{p_i}\right) - (S_{ci} + S'_{ci})\frac{D}{q_i} - (I_c + I'_c)D - (R_{ci} + R'_{ci})\frac{D}{q_i}E(K) \Big] - \frac{hy^2}{2} \\
&= \textstyle\sum_{i=1}^{n} \Big[\left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right)(S - P_{ci}) - \frac{(H_{ci}+H'_{ci})q_i}{2}\left(1 - \frac{(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu})}{p_i}\right) \\
&- (S_{ci} + S'_{ci})\frac{\left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right)}{q_i} - (I_c + I'_c)\left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right) \\
&- (R_{ci} + R'_{ci})\frac{\left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right)}{q_i} p_i^{\delta+1}\left(\frac{\beta}{\tau+1}\right) e^{\frac{-\alpha q_i}{p_i}} \chi\left(\alpha, \frac{q_i}{p_i}\right) \Big] - \frac{hy^2}{2},
\end{aligned} \tag{14}$$

where

$$t_{2i} = \frac{\left(p_i - \left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right)\right) q_i}{p_i \left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right)}, \tag{15}$$

and

$$\begin{aligned}
\chi(\alpha, \frac{q_i}{p_i}) &= \frac{t_1^{\tau+2}}{\tau+2} + \frac{\alpha t_1^{\tau+3}}{\tau+3} + \frac{\alpha^2 t_1^{\tau+4}}{\tau+4} + \frac{\alpha^3 t_1^{\tau+5}}{\tau+5} + \ldots\ldots \\
&= \sum_{i=1}^{\infty} \frac{t_1^{\tau+j+1}\alpha^{j+1}}{(j-1)!(\tau+j+1)} \\
&= \sum_{i=1}^{\infty} \frac{(\frac{q_i}{p_i})^{\tau+j+1}\alpha^{j+1}}{(j-1)!(\tau+j+1)}.
\end{aligned} \tag{16}$$

Constraints

Capital investment in a smart production system plays an important role, although the amount is limited. A sufficient amount of investment gives an opportunity to choose good-quality raw materials within a required time. Although in this model, defective items are produced and a rework facility is available, this may differ in other situations, with different investment budgets. This model considers a budget constraint and, to separate the imperfect products, managers define a specific quality level, which may or may not be chosen for reworking. Considering A for maximum space available for storing in square feet and B for maximum budget available. Sufficient spaces are allotted for storing good-quality products and for reworking imperfect products. Therefore, the profit function, including budget and space constraints, becomes

$$\begin{aligned}
\text{TEP}(q_i, p_i, S, \alpha, Y) = \textstyle\sum_{i=1}^{n} \Big[&\left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right)(S - P_{ci}) - \frac{(H_{ci}+H'_{ci})q_i}{2}\left(1 - \frac{(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu})}{p_i}\right) \\
&- (S_{ci} + S'_{ci})\frac{\left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right)}{q_i} - (I_c + I'_c)\left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right) \\
&- (R_{ci} + R'_{ci})\frac{\left(\frac{S_{max}-S}{S-S_{min}} + xy^{\mu}\right)}{q_i} p_i^{\delta+1}\left(\frac{\beta}{\tau+1}\right) e^{\frac{-\alpha q_i}{p_i}} \chi\left(\alpha, \frac{q_i}{p_i}\right) \Big] - \frac{hy^2}{2},
\end{aligned} \tag{17}$$

$\sum_{i=1}^{n} \phi_i q_i \leq A$, and $\sum_{i=1}^{n} \psi_i q_i \leq B$.

The model cannot be solved analytically. Thus, this model is solved through a numerical tool. The global maximum values of the decision variables are proved numerically.

4. Numerical Examples

We use three numerical examples to validate the model.

4.1. Example 1

In this section, numerical examples are provided to validate this model. In Table 2, for Example 1, the parametric values of the material, holding, rework, and inspection costs; maximum and minimum sale prices; maximum and minimum failure rate; energy costs due to different materials; and other shifting and scaling parameters are shown, which are used in the model and solved using the Mathematica 9.0 software (Wolfram Research, Champaign, IL, USA). The output values of the decision variables (production rate, production lot size, average sale price, advertising variable, and failure rate) are shown in Table 3.

Table 2. Input parameters of Example 1.

I_c ($/unit)	$I_c^{'}$ ($/unit)	H_{c1} ($/unit)	H_{c2} ($/unit)	M (Square Feet)
6	4	2	3.05	130
S_{ci} ($/unit)	$S_{ci}^{'}$ ($/unit)	R_{ci} ($/ defective items)	$R_{ci}^{'}$ ($/defective items)	β (unit)
1000	1100	110	120	5.9
S_{max} ($/unit)	S_{min} ($/unit)	α_{max} (unit)	α_{min} (unit)	δ (unit)
2000	100	0.90	0.10	0.8
ξ	h ($/year)	r	σ	G ($/item)
0.7	20,000	0.85	0.02	200
τ	x	μ	M_c ($/unit)	Z ($)
3	10	1.85	100	30

Table 3. Optimum results of Example 1.

p_1	p_2	q_1	q_2	**S**	y	α	**Total Profit**
598.50 (units/year)	530.29 (units/year)	60.17 (units)	51.70 (units)	565.31 ($/unit)	0.25 ($/year)	0.50 (unit)	2619.20 ($/unit)

The TEP is maximum, as the values of the Hessian at the optimal values of the decision variables are $H_{11} = -0.0388796 < 0$; $H_{22} = 0.00254255 > 0$; $H_{33} = -1.01444 \times 10^{-7} < 0$; $H_{44} = 4.75811 \times 10^{-12} > 0$; $H_{55} = -1.5627 \times 10^{-10} < 0$; $H_{66} = 2.66536 \times 10^{-13} > 0$; $H_{77} = -1.87583 \times 10^{-8} < 0$.

4.2. Example 2

In Table 4, for Example 2, the parametric values of the material, holding, rework, and inspection costs; maximum and minimum sale prices; maximum and minimum failure rate; energy costs due to different materials; and other shifting and scaling parameters are shown, which are used in the model and solved using the Mathematica 9 software. The output values of the decision variables (production rate, production lot size, average sale price, advertising variable, and failure rate) are shown in Table 5.

Table 4. Input parameters of Example 2.

I_c (\$/unit)	$I_c^{'}$ (\$/unit)	H_{c1} (\$/unit)	H_{c2} (\$/unit)	M (Square feet)
8	5	3	5.05	120
S_{ci} (\$/unit)	$S_{ci}^{'}$ (\$/unit)	R_{ci} (\$/defective items)	$R_{ci}^{'}$ (\$/defective items)	β (unit)
1100	1300	110	120	3.8
S_{max} (\$/unit)	S_{min} (\$/unit)	α_{max} (unit)	α_{min} (unit)	δ (unit)
2000	250	0.50	0.10	0.65
ξ	h (\$/year)	r	σ	G (\$/item)
0.7	5000	0.75	0.01	350
τ	x	μ	M_c (\$/unit)	Z (\$)
4	9	1.35	150	60

Table 5. Optimum results of Example 2.

p_1	p_2	q_1	q_2	S	y	α	Total Profit
707.48 (units/year)	596.59 (units/year)	125.82 (units)	106.539 (units)	379.61 (\$/unit)	1.04 (\$/year)	0.50 (unit)	6133.76 (\$/unit)

The TEP is maximum, as the values of the Hessian at the optimal values of the decision variables are $H_{11} = -0.0267288 < 0$; $H_{22} = 0.00137255 > 0$; $H_{33} = -1.37276 \times 10^{-7} < 0$; $H_{44} = 1.88938 \times 10^{-11} > 0$; $H_{55} = -8.46171 \times 10^{-10} < 0$; $H_{66} = 2.25542 \times 10^{-10} > 0$; $H_{77} = -1.21061 \times 10^{-6} < 0$.

4.3. Example 3

In Table 6, for Example 3, the parametric values of the material, holding, rework, and inspection costs; maximum and minimum sale prices; maximum and minimum failure rate; energy costs due to different materials; and other shifting and scaling parameters are shown, which are used in the model and solved using the Mathematica 9 software. The output values of the decision variables (production rate, production lot size, average sale price, advertising variable, and failure rate) are shown in Table 7.

Table 6. Input parameters of Example 3.

I_c (\$/unit)	$I_c^{'}$ (\$/unit)	H_{c1} (\$/unit)	H_{c2} (\$/unit)	M (Square Feet)
8	5	2	4.05	120
S_{ci} (\$/unit)	$S_{ci}^{'}$ (\$/unit)	R_{ci} (\$/defective items)	$R_{ci}^{'}$ (\$/defective items)	β (unit)
1100	1400	110	120	4.8
S_{max} (\$/unit)	S_{min} (\$/unit)	α_{max} (unit)	α_{min} (unit)	δ (unit)
1600	400	0.50	0.10	0.25
ξ	h (\$/year)	r	σ	G (\$/item)
0.7	6000	0.65	0.01	350
τ	x	μ	M_c (\$/unit)	Z (\$)
4	10	1.35	150	60

Table 7. Optimum results of Example 3.

p_1	p_2	q_1	q_2	S	y	α	Total Profit
689.67 (units/year)	547.70 (units/year)	193.80 (units)	156.36 (units)	508.92 ($/unit)	1.96 ($/year)	0.50 (unit)	11914.6 ($/unit)

The TEP is maximum, as the values of the Hessian at the optimal values of the decision variables are $H_{11} = -0.0119313 < 0$; $H_{22} = 0.000332921 > 0$; $H_{33} = -5.68341 \times 10^{-8} < 0$; $H_{44} = 1.57781 \times 10^{-11} > 0$; $H_{55} = -9.86172 \times 10^{-10} < 0$; $H_{66} = 8.62069 \times 10^{-10} > 0$; $H_{77} = -4.44002 \times 10^{-6} < 0$.

Table 8 compares the total expected profits (TEP) of the three given examples. There is are another scope to maximize the TEP—by increasing sale price, which depends on the total cost of making the products; although production rate and production lot size also give opportunity to maximize the TEP. Setup cost and energy consumption costs due to setup also have impacts on the TEP. Furthermore, advertising plays an important role; due to increased investment into advertisement, the examples show how the total profit can be extended.

Table 8. Comparative Study.

	Example 1	Example 2	Example 2
TEP	2619.20 $/unit	6133.76 $/unit	11914.6 $/unit

The following three dimension Figures 3–9 are total expected profit (TEP) versus different pair decision variables. The concavity of the figures indicated the profit of the model in different cases.

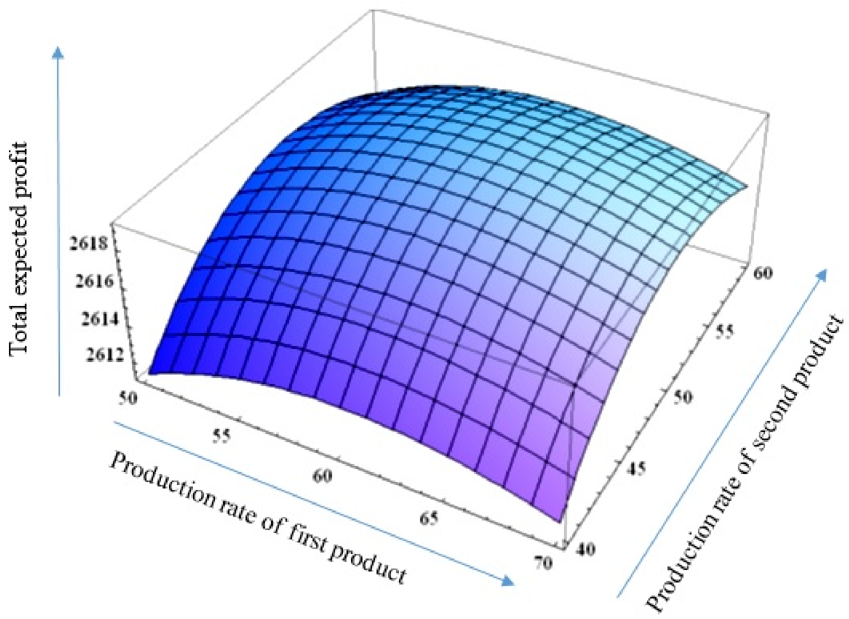

Figure 3. The total expected profit versus the production rate of two products (P_1, P_2).

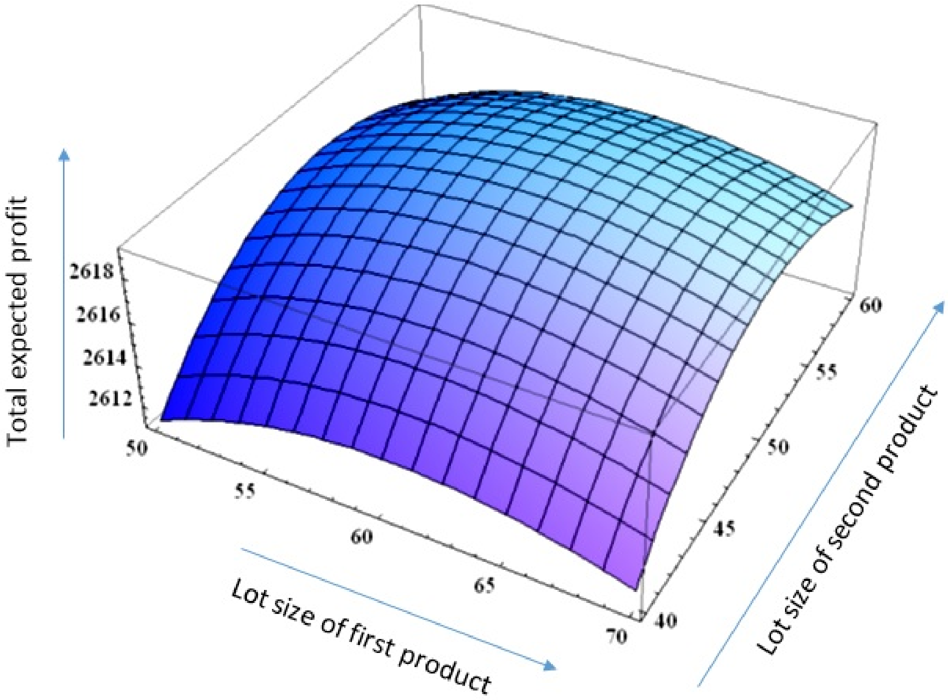

Figure 4. The total expected profit versus lot size of two products (q_1, q_2).

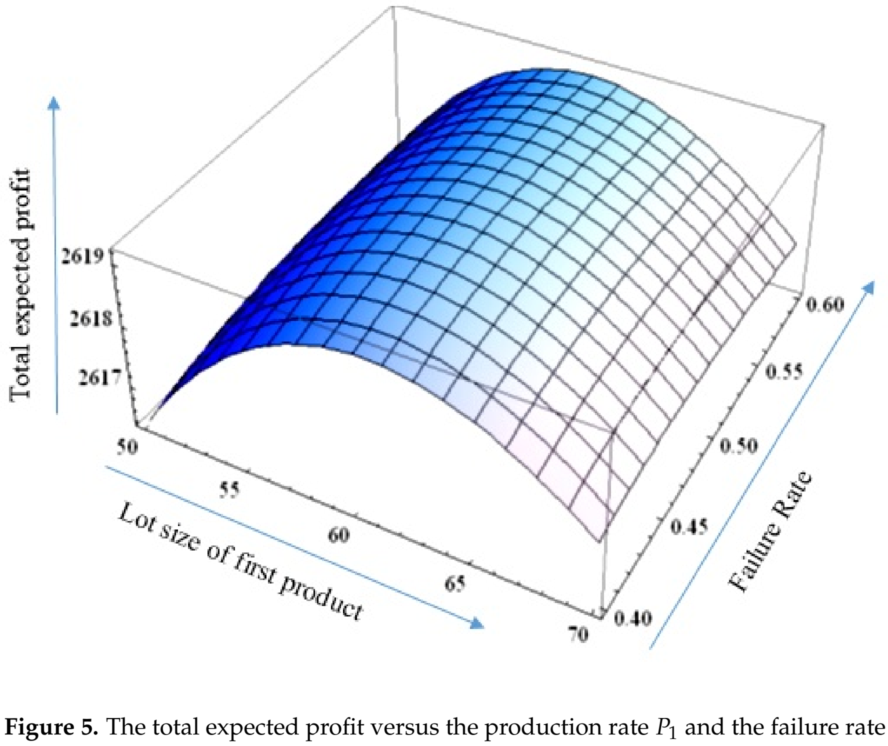

Figure 5. The total expected profit versus the production rate P_1 and the failure rate α.

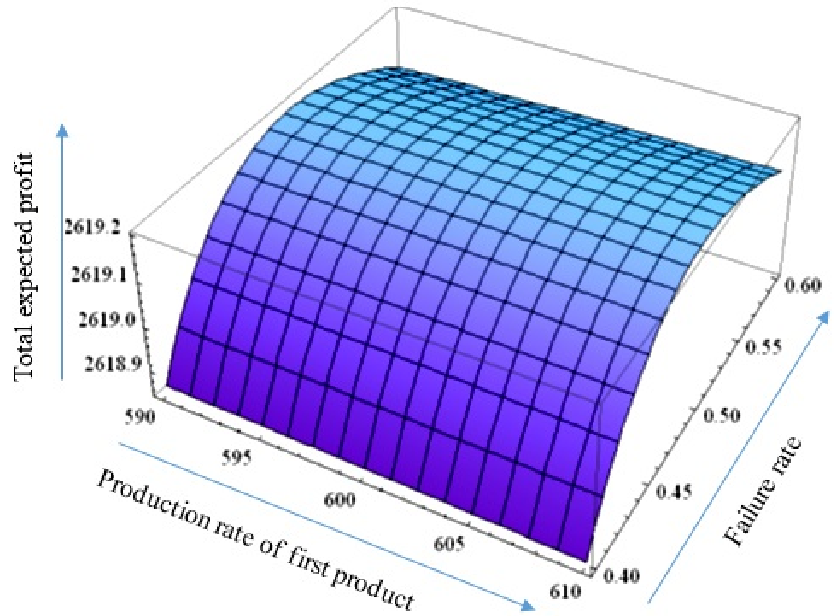

Figure 6. The total expected profit versus the production lot size q_1 and failure rateα.

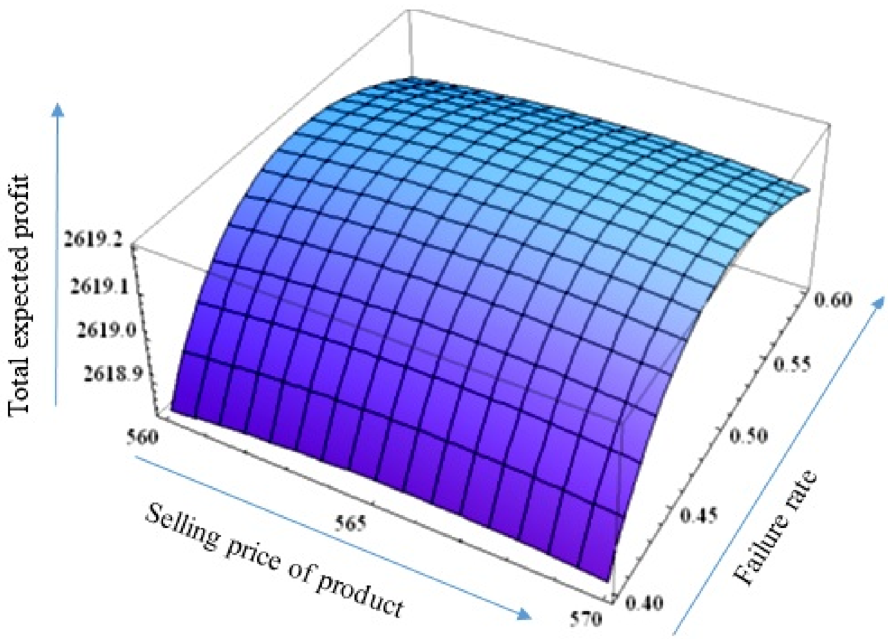

Figure 7. The total expected profit versus the advertising variable quantity and failure rate α of two products.

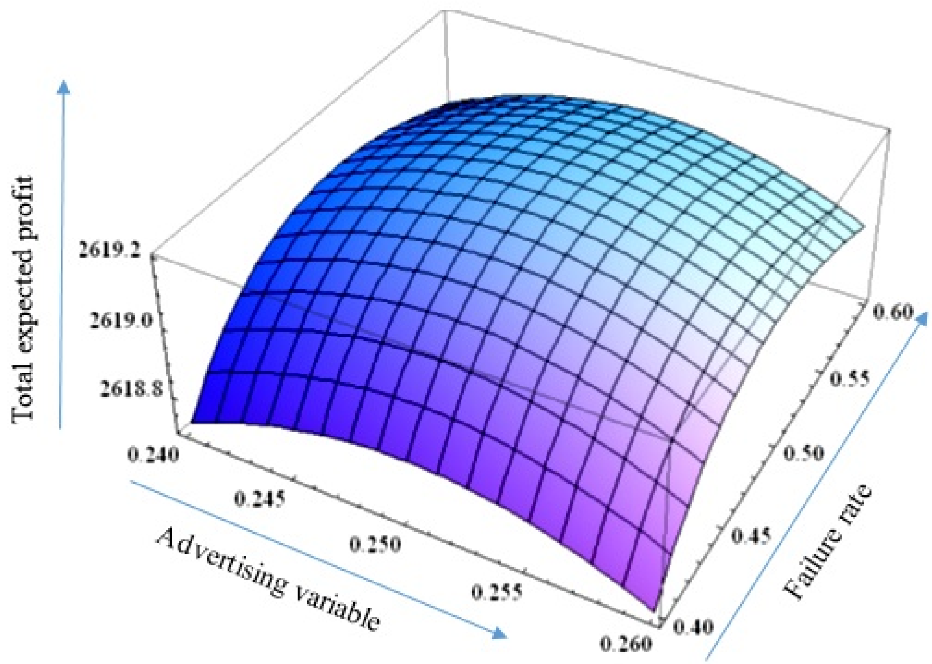

Figure 8. The total expected profit versus the selling price and the failure rate α.

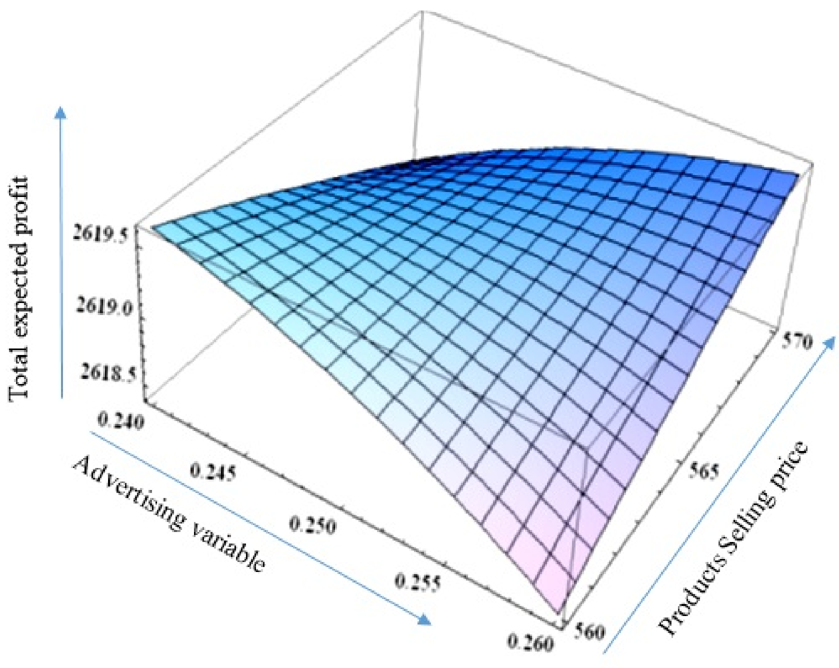

Figure 9. The total expected profit versus the advertising variable and the sale price.

5. Sensitivity Analysis

A sensitivity analysis of the cost and scaling parameters was conducted, and the major changes are summarized in Table 9 and Figure 10.

Table 9 shows how changes by certain percentages (−50%, −25%, +25%, +50%) in the cost and scaling parameters affect the total profit. Here NF indicates that the numerical result is not feasible. We can conclude the following from the sensitivity analysis.

Table 9. Sensitivity analysis of the key parameters.

Parameters	Changes (%)	TEP (%)	Parameters	Changes (%)	TEP (%)
I_c	−50	+0.44	$I_c^{'}$	−50	+0.29
	−25	+0.22		−25	+0.15
	+25	−0.22		+25	−0.15
	+50	−0.44		+50	−0.29
H_{c1}	−50	+1.32	H_{c2}	−50	+1.73
	−25	+0.61		−25	+0.18
	+25	−0.54		+25	−0.70
	+50	−1.03		+50	−1.34
S_{ci}	−50	+1.43	$S_{ci}^{'}$	−50	+1.84
	−25	+0.66		−25	+0.84
	+25	−0.58		+25	−0.74
	+50	−1.10		+50	−1.40
R_{ci}	−50	+0.03	$R_{ci}^{'}$	−50	+0.02
	−25	+0.01		−25	+0.01
	+25	−0.01		+25	−0.01
	+50	−0.02		+50	−0.02
M_c	−50	+16.18	x	−50	−30.70
	−25	+7.71		−25	−9.36
	+25	−7.03		+25	+4.74
	+50	−13.49		+50	+7.24
σ	−50	+0.13	h	−50	NF
	−25	+0.27		−25	+5.44
	+25	−0.23		+25	−6.30
	+50	−0.12		+50	−13.06

1. That changes in energy cost due to inspection and energy cost per setup has low effects on the total optimal profit as, in a smart production system, there is an optimum consumption of energy for different purposes. Some industries do not consider inspection on the final product, due to the value-added process. Due to an increase of inspection, the reworking cost and other costs also increase, which has a large impact on the total optimal profit. On the other hand, inspection maintains the quality and correctness of the products produced in a smart production system.

2. Changes in holding cost had little impact on the total profit of the smart production system. This result can be justified, as the products are not held for a long time. Similarly, variation of the setup cost also has a lesser, but significant, effect on the total optimal profit. Increasing the value of the cost parameters decreases the total optimal profit, which is clearly shown in the sensitivity analysis table and agrees with reality.

3. On the other hand, the material costs in a multi-item smart production system has a great impact on the total profit. With an increase in material costs, all other production costs will increase and, thus, the profit decreases.

4. The value of the scaling parameters for advertising and the tool/die cost function also had effects on the total profit. These parameters behave similarly to other cost parameters for controlling the total optimal profit.

5. The effect of advertisement has a sensitive impact on the total profit. The advertisement of a product is very much important for generating a high popularity and increasing the market demand for the product. Due to increases in sales, the total optimal profit gradually increases. As in Table 9, it is found that eventually, an increasing value of advertising will reduce the total profit.

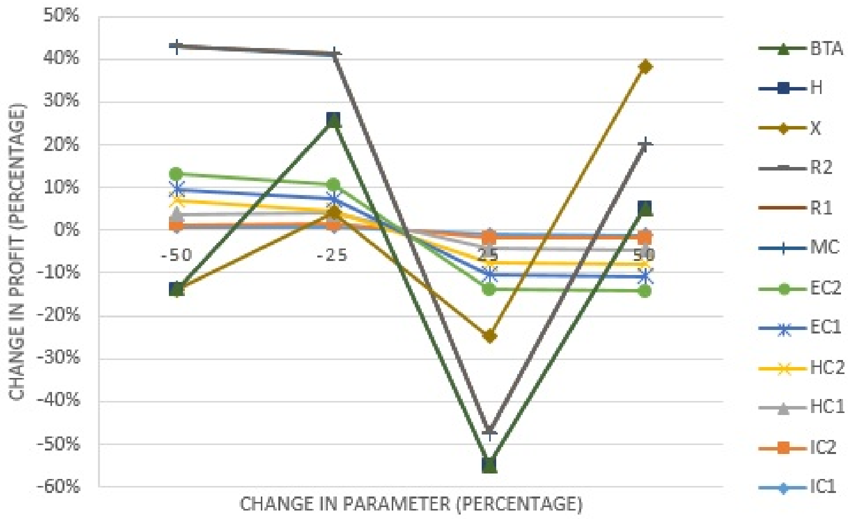

Figure 10. Change of total expected profit (TEP) in percentage versus parametric values in percentage.

6. Managerial Insights

Some recommendations for the industry are as follows:

1. The industry manager must concentrate on the matter of advertisement. This paper clearly shows the impact of advertisement on the total profit. To increase the popularity of products and, in turn, the demand for products, the industry manager should invest some costs into advertisement. In the competitive modern market, this matter plays a significant role for a smart production system. However, no research has considered the advertisement concept, until now, in the field of smart production system. Thus, the result of this study will help the industry manager to increase their profits.

2. Another support that industry managers of smart production systems can use are the strategies, obtained by this research, regarding energy consumption. If a smart production system uses a machine, instead of labor, energy consumption will be present and there will, subsequently, be energy costs to bear. Thus, this model will help the industry to maximize the profit.

3. This model considers random machine breakdowns. Using this idea, the industry manager can pay attention to the previous data of machine failures. This way, the failure rate can be reduced by using smart machines with optimum energy consumptions, and by optimizing development-cost investments.

7. Conclusions

The advertising of smart products has a large impact on the market demand, in addition to the reduction of sale prices, which has an important influence on the profit made from smart products. This study used these two ideas, along with the effects of the optimum use of energy consumption, random machine failures, and optimization of profit, in order to design a model for investigating the optimal decision variables for smart production systems, which gave strong benefits regarding the quality of the smart products. The profit equation of the model was a non-linear equation, and thus could not be solved with an analytical approach. A numerical tool was used to obtain the results, and the numerical findings gave a global optimum solution. The main limitation of the model is that the setup cost in the model is considered to be constant, which can be reduced by an initial investment. In this direction, the model can be extended (refer to the references Malik and Sarkar, [56]; Sarkar and Moon, [57]; Sarkar et al. [58]; Sarkar and Majumdar, [59]; Sarkar et al. [60]; and Majumdar et al. [61]). This model may also be extended by incorporating inspection errors, as the inspection may be done by human beings.

Author Contributions: Conceptualization, B.S. and S.B.; methodology, B.S. and S.B.; software, S.B.; validation, B.S., S.B. and S.P.; formal analysis, S.B.; investigation, B.S.; resources, B.S. and S.P.; data curation, S.B., B.S. and S.P.;

writing—original draft preparation, S.B.; writing—review and editing, B.S., S.B. and S.P.; visualization, B.S. and S.B.; supervision, B.S. and S.P.

Funding: This research received no external funding.

Acknowledgments: The authors are happy to acknowledge the support from two reviewers for revising the earlier versions of the paper.

Conflicts of Interest: The authors declare no conflict of interest.

Notations

Index	
i	number of product ($i = 1, 2, 3, \ldots\ldots, n$)
Model Decision Variables	
p_i	production rate for item i per cycle (unit/year)
q_i	production lot size for item i per cycle (units)
S	average selling price of the product ($/unit)
y	advertising variable ($/year)
α	failure rate, indicates reliability, known as system design variable
Input Parameters	
N	available maximum budget ($/planning period)
M	available maximum space for sorting in square feet
α_{max}	maximum system failure rate
α_{min}	minimum system failure rate
T	production length per cycle (year)
t_{1i}	required time for maximum inventory (year)
D_c	development cost per unit item ($/unit)
P_c	production cost per unit item ($/unit)
S_{max}	maximum selling price ($/unit)
S_{min}	minimum selling price ($/unit)
ψ_i	consumed budget of item i ($/unit)
ϕ_i	occupied space for the unit item i (square feet per item)
K	number of defective items per cycle
I_c	inspection cost per item ($/item)
D	demand for item i
$E(K)$	per unit production system expected number of defective items
H_{ci}	holding cost for item i ($/unit/unit time)
S_{ci}	setup cost for item i per setup ($/setup)
M_C	material cost for item i ($/unit)
R_{ci}	rework cost for item i ($/defective item)
R'_{ci}	energy consumption cost due to rework for item i ($/item)
x	scaling parameter for the products
μ	shape parameter for the products
Z	resources fixed cost ($)
G	fixed energy and labor cost, independent of reliability α ($/item)
TEP	total expected profit per unit time ($/year)
I'_c	energy consumption due to Inspection per unit cycle ($/cycle)
S'_{ci}	energy consumption cost per setup ($/setup)
r	shape parameter for development cost function
β	scaling parameter for defective item function
σ	scaling parameter for tool/die cost function
δ	shape parameter for defective item function
h	investment cost for advertisement
ξ	shape parameter for tool/die cost function
Random Variable	
γ_i	system random time to move to an out-of-control state from an in-control situation, for item i.

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