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Nonlinearities and Chaos: A New Analysis of CEE Stock Markets

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Abstract: After a long transition period, the Central and Eastern European (CEE) capital markets have consolidated their place in the financial systems. However, little is known about the price behavior and efficiency of these markets. In this context, using a battery of tests for nonlinear and chaotic behavior, we look for the presence of nonlinearities and chaos in five CEE stock markets. We document, in general, the presence of nonlinearities and chaos which questions the efficient market hypothesis. However, if all tests highlight a chaotic behavior for the analyzed index returns, there are noteworthy differences between the analyzed stock markets underlined by nonlinearity tests, which question, thus, their level of significance. Moreover, the results of nonlinearity tests partially contrast the previous findings reported in the literature on the same group of stock markets, showing, thus, a change in their recent behavior, compared with the 1990s.

Keywords: nonlinearities; chaos; stock markets; efficient market hypothesis; CEE countries

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1. Introduction

An adequate perception of stochastic processes followed by stock index returns represents the core of the efficient market hypothesis (EMH) theory developed by [1], which shows that financial markets are informationally efficient. The weak form of this hypothesis asserts that prices already reflect all information and, therefore, in order to prove the EMH, there should be no predictability in stock prices. This means that it is impossible for the investors to achieve abnormally high returns; that is, financial markets are efficient if stock price returns follow a random walk.

Nevertheless, noteworthy studies have shown that stock returns do not follow random walks, being characterized by nonlinearities and chaos (for a review of the literature see [2]). Consequently, recent developments in investigating the nonlinear dependence and deterministic chaos of financial variables altered the traditional view of their erratic behavior [3,4]. If the stock market returns are characterized by nonlinearities and chaos, then the market is inefficient. This means that stock prices do not incorporate all existing information and might be over- or under-evaluated. Thus, the investors can record excess returns or losses.

Even if the nonlinear properties of developed stock markets were well highlighted in the literature [5], little was done for the Central and Eastern European (CEE) economies. While ref. [6] proved the presence of a long memory in eight CEE stock markets, more recently, ref. [7] provided evidence for nonlinear dependencies, nonlinear patterns and chaotic dynamics for the Czech Republic, Hungary, and Poland stock markets. However, ref. [7] included in the analysis only the 1990s' period, when CEE countries underwent major changes in their economic and political systems. These changes impacted on the stock markets functioning, and created difficulties associated with low liquidity and high volatility, but also with the lack of hedging opportunities and low price to earnings ratios. Consequently, all this evidence could affect the estimations, as the EMH can hardly be proved in transition markets.

Against this background, the aim of this study is to investigate the presence of nonlinearities and chaos in five CEE stock markets with a focus on the pre- and post-accession period to the European Union (EU). For this purpose, we use daily returns series of BET (Romania), BUX (Hungary), PX (Czech Republic), SAX (Slovak Republic) and WIG20 (Poland) indexes and we cover the period from June 2000 to September 2015, using Datastream statistics. This period is large enough to allow for nonlinear tests estimations, and it is also indicated for comparison between the selected markets, as we report to the same period for all analyzed stock markets.

Another contribution of our paper is represented by the analysis of two stock markets (Romania and Slovak Republic) which are usually not included in the advanced CEE stock markets group. However, following the EU accessions, their behavior is similar to the Czech, Hungarian or Polish stock markets, which recommends their inclusion in the same group of CEE stock markets.

Finally, in addition to tests used by [7], we also employ the McLeod–Li test for nonlinearities, a commonly used diagnostic tool for the presence of Auto Regressive Conditional Heteroskedasticity (ARCH) effects, and the recent test proposed by [8]. Further, we employ two different developments of the Lyapunov exponent to test the chaos. We also use the 0–1 test for assessing the chaotic behavior of CEE stock markets. The nonlinearities and chaos tests complement each other. Indeed, chaos is related to nonlinear dynamics of stock markets [9]. On the one hand, the chaotic dynamics are necessarily nonlinear. On the other hand, nonlinear models can generate much richer types of behavior. We therefore use this battery of tests for two reasons: first, to check the robustness of our findings; second, to compare our findings with those reported by [7]. A run test for randomness is used as a benchmark for nonlinearities and chaos tests. If this test rejects the hypothesis of random walks, it questions the EMH for the CEE stock markets.

To preview our findings, we discover that CEE stock markets are characterized by a chaotic behavior and nonlinear dynamics of price returns. Further, we discover that the results obtained in the case of the updated sample partially contradicts those reported by [7] for the 1990s, a result explained by the development of CEE stock markets. However, we also notice a strong heterogeneity of CEE stock markets behavior.

The rest of the paper is structured as follows. Section 2 presents some stylized facts and a short review of the literature assessing the CEE stock markets' behavior. Section 3 describes the methodology while Section 4 presents the results. The last section concludes.

2. Stylized Facts and Review of the Literature Related to the CEE Stock Markets' Behavior

2.1. Stylized Facts

The CEE stock exchanges were closed during the communist period and re-emerged afterwards, establishing legal structures for contracts and transparency in accounting and transactions [10]. Starting with the 1990s, the CEE stock markets unregistered remarkable changes and developments toward the status of mature markets, reached in the 2000s. Even if the CEE countries' financial systems largely remain bank-dominated, their stock markets appear to be well integrated with world financial markets [11], although loosely correlated with European developed markets [12].

Due to the fact that CEE countries witnessed major structural changes in the 1990s, including privatization, institutional reforms, and creation of financial systems [13], their stock markets developed accordingly. As ref. [14] show, the CEE stock markets performance was affected by the extent of the restructuring of these economies in the post-communist period. More recently, the economic integration process and the crisis appearance also affected the market efficiency. However, the behavior of these markets and the characteristics of transactions and financial products are different during the 2000s, which confer them the status of mature markets. Some key indicators of CEE stock markets are reflected in Table 1 and confirm CEE stock markets dynamics during the 2000s.

Table 1. CEE stock markets general indicators.

		Czech Republic	Slovak Republic	Hungary	Poland	Romania
Stock exchange		CEESEG-Prague	Bratislava Stock Exchange	CEESEG-Budapest	Warsaw Stock Exchange	Bucharest Stock Exchange
Establishment year		1993	1993	1990	1991	1995
Index		PX	SAX	BUX	WIG 20	BET
Market capitalization **	2000 *	–	–	13,946.34	29,812.44	–
	2005 *	30,295.62	3798.98	30,831.00	69,219.45	–
	2010 *	32,478.07	3524.40	22,056.60	134,075.87	9751.68
	2015 *	24,078.55	4207.37 ***	14,214.09	135,351.55	17,632.70
Number of listed companies	2000 *	–	–	61	209	–
	2005 *	42	243	43	236	–
	2010 *	26	161	48	547	67
	2015 *	24	128 ***	45	907	82

Note: (i) * September; (ii) ** Value at Month End (EUROm); (iii) *** Data for September 2014; (iv) "–" means data not available. Source: Federation of European Securities Exchanges, monthly statistics.

Several aspects of CEE stock markets are noticed in Table 1. On the one hand, Romanian, Polish and Slovak Republic stock markets recorded an increase in capitalization after the crisis outburst, contrasting with the Hungarian and Czech markets, where the capitalization decreased. Further, the stock market capitalization in Poland, and to a smaller extent in Romania, considerably increased during the last decade.

On the other hand, we notice that the number of listed companies considerably varies between the CEE stock markets. First, a relatively reduced number of listed companies is recorded in Czech Republic and Hungary. Second, in Poland, the number of listed companies is considerably higher and follows the trend of market capitalization. Third, compared with Romania for example, the number of listed companies on Bratislava Stock Exchange is doubled, while the market capitalization is considerably smaller. All these elements question the CEE stock market integration.

2.2. Literature Review

In general, international investors consider the CEE stock markets as a homogenous group, given their location and characteristics [15]. However, ref. [16] reported a poor integration of these stock markets. While most of existing literature assesses the CEE stock market integration in relation with old European Union (EU) members [10,11,17–24], or assesses the CEE stock markets' co-movements and contagion [13,25], less is done in terms of the investigation of CEE stock markets' efficiency. An exception is the paper by [26] which shows that intraday price movements present important deviations from a random walk in the case of CEE markets.

At the international level, there are noteworthy papers exploring the integration level of stock markets (for a literature review, see [27]). Other papers investigate nonlinearities and chaos on stock markets, in order to confirm or to infirm the EMH, the Adaptive Market Hypothesis (AMH), and the Heterogeneous Market Hypothesis (HMH) [28–38]. Recently, ref. [3] shows that the returns series of six Indian stock market indexes do not follow a random walk process, while ref. [39] tests the level noisy chaos in the Standard & Poor's 500 index returns over four different frequencies and reports that the dynamics in all frequencies are non-chaotic. In the same spirit, ref. [40] use different linear and nonlinear tests for the Tehran stock exchange and provide evidence in the favor of the AMH. In addition, ref. [41] investigates the presence of nonlinearities in the Athens Composite Share Price Index high-frequency returns and find that the filtered return process does not exhibit deterministic or higher-order stochastic nonlinearity. Similar, ref. [42] apply multiple state-of-the-art efficiency tests for developed stock markets and validate the idea of dynamic and time–variant efficiency. Other recent papers investigating the EMH and using nonlinearities and chaos tests are those of [43–45].

However, in respect to the CEE countries, few works reported nonlinearities and chaos [6,7]. Therefore, our purpose is to use a large battery of tests for nonlinearities and chaos to analyze the behavior and efficiency of mature CEE stock markets. As far as we know, this is the first paper which uses different developments of the Lyapunov exponent and the 0–1 test for studying the chaotic behavior of these markets. To contribute to the existing literature, and to see if the CEE markets are really integrated, we also use an appreciable number of nonlinearity tests as those advanced by [8,46–51].

3. Methodology

An important number of tests were used in the literature for assessing nonlinearities and chaos in financial markets: the McLeod–Li test, the runs test, the variance ratio test, the White test, the Teraesvirta test, the Keenan test, the Tsay test, the Engle LM test, the BDS test, the Lyapunov exponent and the noise titration test. The technical details of these tests are provided by [3,52,53], while ref. [54–57] realized a comparison of their efficiency. In this section we provide only a brief description of the characteristics of each retained test in our analysis.

3.1. Runs Test for Randomness

The runs test is a non-parametric test used to decide if a time series follows a random process. It is usually considered a linear test which allows, however, for the identification of nonlinearities in the data series. A run is defined as a series of increasing values or a series of decreasing values. If the randomness assumption is not valid, we can interpret this as a lack of efficiency of stock markets. If a data series is random, in the runs test, the actual number of runs (sequences of positive or negative returns) in the series should be close to the expected number of runs, irrespective of the signs [2]:

$$E(u) = \frac{2PN(P+N)}{(P+N)} + 1, \tag{1}$$

where: P denotes the number of positive runs, while N means the number of negative runs.

The variance of runs is given by:

$$\sigma^2 = \frac{2PN(2PN - P - N)}{(P+N)^2(P+N-1)}. \tag{2}$$

3.2. BDS Test for Independence

The BDS test proposed by [51] is a test used for independence but also for nonlinear dependences. It tests the null hypothesis that the elements of a time series are independently and identically distributed (*iid*):

$$W_m(\varepsilon) = \frac{\sqrt{n}\{C_m(\varepsilon) - C_1(\varepsilon)\}}{\sigma_m(\varepsilon)} \tag{3}$$

where: $W_m(\varepsilon)$ is known as the BDS test, $C_m(\varepsilon)$ represents the fractions of m-dimensions in the series; and $\sigma_m(\varepsilon)$ is the standard deviation under the null of *iid*.

The BDS test rejects the null if the test statistic is large (usually larger than 1.96). If the null is rejected, the residuals contain a nonlinear structure. As in [7], we use a range of dimensions (m) from 2 to 7 and four values for the distance (ε), namely 0.5σ, 1σ, 1.5σ and 2σ.

3.3. *White and Teräsvirta Tests for Neglected Nonlinearities*

An alternative way to look for nonlinearities is with the neural network models in which the network output y_t is determined based on input x_t:

$$y_t = x_t\beta + \sum_{j=1}^{q} \delta_j \psi(x_t\gamma_j) + \varepsilon_t, \tag{4}$$

where: β is a column vector connecting the strength from the input to the output layers, y_j is a column vector connecting the strength from the input layer to the hidden unit (j = 1, ... q), δj represents a scalar connecting the strength from the hidden unit j to the output unit, and ψ is a logistic squashing function.

Ref. [49] proposes a test for neglected nonlinearity, designed to be more powerful compared with other neural network models. The neural network test by [49] is based on a test function $h(x_t)$ which activates the hidden units $\psi(x_t\Gamma_j)$. Under the null we have:

$$E\left[\psi(x_t\Gamma_j)\varepsilon_t^*|\Gamma_j\right] = E\left[\psi(x_t\Gamma_j)\varepsilon_t^*\right] = 0, \tag{5}$$

so that

$$E(\psi_t\varepsilon_t^*) = 0, \tag{6}$$

where: Γ_j are random column vectors independent of x_t, and $\psi_t = \left(\psi(x_t\Gamma_1), \ldots, (x_t\Gamma_q)\right)$ represents a hidden unit activation vector.

Ref. [49] shows that in the presence of correlation, the network performance improves by including in the model an additional hidden unit with the activation $\psi(x_t\Gamma_j)$. The test is thus based on sample correlations of affine network errors:

$$n^{-1}\sum_{t=1}^{n} \psi_t\hat{\varepsilon}_t = n^{-1}\sum_{t=1}^{n} \psi_t\left(y_t - x_t\hat{\beta}\right). \tag{7}$$

Different from [49], ref. [50] replace $\delta_j\psi$ with a Taylor expansion approximation, in order to solve the linearity testing problem, and to use a score testing framework.

3.4. *Keenan and Tsay Tests for Nonlinearities*

The null hypothesis of the Keenan test [47] is that of a linear model against a nonlinear specification. Ref. [48], building upon [47], explicitly tests for quadratic serial dependence in the data. It represents, thus, a more general form of the Keenan test.

The [48] test can be specified as follows. If we have $K = k(k-1)/2$ column vectors $V_1, \ldots, V_k$ which contain all possible cross-products $e_{t-i}e_{t-j}$ (where $i \in [1,k]$ and $j \in [i,k]$), then:

$$v_{t,1} = e_{t-1}^2, v_{t,2} = e_{t-1}e_{t-2}, v_{t,3} = e_{t-1}e_{t-3}, v_{t,k+1} = e_{t-2}e_{t-3}, v_{t,k+2} = e_{t-2}e_{t-4}, \ldots, v_{t,k} = e_{t-k}^2. \tag{8}$$

If $\hat{v}_{t,j}$ represents the projection of $v_{t,i}$ on the orthogonal subspace $e_{t-1}, \ldots, e_{t-k}$ (meaning the residuals of the $v_{t,j}$ regression on $e_{t-1}, \ldots, e_{t-k}$), then the parameters $\gamma_1, \ldots, \gamma_k$ are the Ordinary Least Squares (OLS) estimates of the regression equation:

$$e_t = \gamma_0 + \sum_{i=1}^{K} \gamma_t\hat{v}_{t,j} + \eta_t. \tag{9}$$

Note that, if p exceeds K then the projection is unnecessary and the Tsay test is equivalent to a classic F statistic for testing the null that $\gamma_1, \ldots, \gamma_k$ are zero.

3.5. McLeod–Li Test for Nonlinearity

The McLeod–Li test [46] uses the following statistic to test for nonlinear effects in time series data:

$$Q(m) = \frac{n(n+2)}{n-k} \sum_{k=1}^{m} r_a^2(k). \quad (10)$$

where $r_a^2(k) = \sum_{t=k+1}^{n} \varepsilon_t^2 \varepsilon_{t-k}^2 / \sum_{t=1}^{n} \varepsilon_t^2$ (with $k = 0, 1, \ldots, n-1$) are the autocorrelations of the squared residuals ε_t^2 generated by fitting the model to the data. If the residuals are *iid*, χ^2 with m degrees of freedom represents the asymptotic distribution of *Q(m)*.

3.6. Harvey Test for Nonlinearities

A more powerful test for assessing the nonlinearities in data series is proposed by [8]. This test does not require an assumption of I(0) behavior of data series as the previous tests do, and represents a simple data-dependent weighted average (W_λ) of two Wald test statistics (computed for an I(0), and an I(1) process respectively). For a time series $y_t = \mu + v_t$, the nonlinearity is assumed to enter through the level of y_t for an I(0) series, or through the first differences of y_t if the series is I(1):

$$\begin{cases} v_t = \delta_1 v_{t-1} + \delta_2 v_{t-1}^2 + \delta_3 v_{t-1}^3 + \varepsilon_t, \ if \ I(0) \\ \Delta v_t = \lambda_1 \Delta v_{t-1} + \lambda_2 (\Delta v_{t-1})^2 + \lambda_3 (\Delta v_{t-1})^3 + \varepsilon_t, \ if \ I(1) \end{cases}. \quad (11)$$

If the series y_t is (I0), the null hypothesis of linearity is $H_{0,0} : \delta_2 = \delta_3 = 0$, while the alternative of nonlinearity is expressed as $H_{1,0} : \delta_2 \neq 0, \ and/or \delta_3 \neq 0$. The standard Wald statistic for testing these restrictions is given by W_0. The similar applies if y_t is (I1), situation in which the Wald statistic is represented by W_1.

The weights (W_0 and W_1) are determined by considering the switch between the two efficient Wald statistics, based on an auxiliary test. The new weighted statistic has a standard chi-squared limiting null distribution in both the I(0) and I(1) cases:

$$W_\lambda = \{1 - \lambda\} W_0 + \lambda W_1. \quad (12)$$

where λ represents a function which converges in probability to zero when the series y_t is I(0), and to one when y_t is I(1).

3.7. The Lyapunov Exponent

The Lyapunov exponent is explicitly used in the literature to test whether a time series is chaotic. In a chaotic system, if an infinitesimal change $\delta x(0)$ appears in the initial conditions, the corresponding change iterated through the system will grow exponentially with the time t. Technically, the largest Lyapunov exponent is considered the only test explicitly devised for testing chaos and measures the rate at which information is lost from a system [3]. A process shows chaotic behavior if the maximum Lyapunov exponent is positive [58].

Considering an infinitesimally small hypersphere of radius ϵ, the maximum Lyapunov exponent is measured by the extent of the deformation as follows [55]:

$$\lambda_i = \lim_{t \to \infty} \lim_{\epsilon(0) \to \infty} \left\{ \frac{1}{T} log_2 \left[\frac{\epsilon_i(t)}{\epsilon_i(0)} \right] \right\}. \quad (13)$$

where $\epsilon_i(t)$ represents the length of the ith principal axis of the ellipsoid at time t.

The literature recorded several developments of the Lyapunov exponent, based on a neural network, or on a non-neural network. In the first category, we can find the maximum Lyapunov exponent proposed by [59], which allows for the identification of chaotic dynamics in short noisy systems. In the second category of models, we mention [60], who propose an alternative way of calculating the largest Lyapunov exponent, which takes advantage of all the available data. We use both tests in our empirical analysis.

3.8. The 0–1 Test for Chaos

The 0–1 test for chaos proposed by proposed by [61] is based on a Euclidean extension, instead on a phase space reconstruction as the Lyapunov exponent. It tests the chaos in a deterministic dynamical system $\{x_t\}$, by studding the asymptotic behavior of the translation variables $p_c(n) = \sum_{j=0}^{n-1} \cos(jc) g(x_j)$ and $q_c(n) = \sum_{j=0}^{n-1} \sin(jc) g(x_j)$, with $n \in \mathbb{N}$ and $c \in (0, \pi)$ representing an arbitrary but fixed frequency. Both $p_c(n)$ and $q_c(n)$ remain bounded as $n \to \infty$ if the system does not exhibit chaos.

Ref. [61]'s test static is represented by the asymptotic Bravais–Pearson correlation coefficient between $n = (1, 2, \ldots, n)^{\mathrm{T}}$ and $\Delta_n = (D_c(1), \ldots, D_c(n))^{\mathrm{T}}$:

$$K_c = \lim_{n \to \infty} \frac{\mathbf{n}^{\mathrm{T}} \Delta_n - \frac{1}{n} 1_n^{\mathrm{T}} n 1_n^{\mathrm{T}} \Delta_n}{\sqrt{\left[n^{\mathrm{T}} n - \frac{1}{n}(1_n^{\mathrm{T}} n)^2\right]\left[\Delta_n^{\mathrm{T}} \Delta_n - \frac{1}{n}(1_n^{\mathrm{T}} \Delta_n)^2\right]}}. \tag{14}$$

where $1_n = (1, 1, \ldots, 1)^{\mathrm{T}} \in \mathbb{R}^n$.

4. Results and Discussion

4.1. Results of the Runs Test for Randomness

The results presented in Table 2 indicate the lack of randomness for the BET, SAX and WIG20 indexes, while different results are reported for the BUX and PX indexes. In the case of the PX index, the results contrast to those reported by [7]. It seems that after the year 2000, the Czech Republic stock market is closer to the efficient market hypothesis.

Table 2. Results of the runs test.

Index	Standard Normal	*p*-Value
BET	−4.954	0.000
BUX	−0.146	0.883
PX	−1.345	0.178
SAX	3.236	0.001
WIG20	2.324	0.020

4.2. Results of the BDS Test for Independence

For all selected markets, the BDS test results clearly indicate that the independence is rejected (Table 3). These results are recorded for all values for the distance (ε), namely 0.5σ, 1σ, 1.5σ and 2σ.

4.3. The Results of the White and Teräsvirta Tests for Neglected Nonlinearities

The null hypothesis of the White neural network test is the linearity in the mean. As shown in Table 4, except from BUX daily returns for which the null hypothesis of linearity cannot be rejected, all other indexes present nonlinear features. In the case of WIG20 daily returns, our results are different once again from those reported by [7], facts which prove that the nonlinear characteristics are influenced by the selected period.

Table 3. Results of the BDS test.

	σ/m	0.5σ	1σ	1.5σ	2σ
BET	2	20.612	21.342	21.820	21.707
	3	25.429	26.070	25.898	25.058
	4	31.419	30.854	29.548	27.743
	5	38.558	36.703	33.473	30.225
	6	51.442	45.031	38.432	33.137
	7	74.107	56.772	44.931	36.704
BUX	2	8.467	9.295	10.096	11.140
	3	11.387	12.138	12.909	13.982
	4	13.571	14.315	15.036	16.206
	5	15.202	16.022	16.893	18.055
	6	16.740	17.854	18.771	19.785
	7	19.016	19.996	20.912	21.728
PX	2	11.496	12.570	14.223	15.725
	3	15.768	16.748	18.484	20.032
	4	18.846	19.404	21.007	22.393
	5	22.922	22.281	23.330	24.266
	6	28.717	25.587	25.732	26.038
	7	36.473	30.328	28.828	28.033
SAX	2	9.527	8.916	8.342	7.477
	3	11.987	11.367	10.186	9.192
	4	14.092	12.830	11.471	10.593
	5	16.045	14.000	12.170	11.238
	6	18.518	16.052	13.365	11.801
	7	20.896	17.631	14.106	12.015
PX	2	4.253	4.896	5.787	6.465
	3	6.548	7.340	8.206	8.720
	4	8.700	10.043	10.917	11.338
	5	10.956	12.619	13.409	13.804
	6	13.424	15.795	16.205	16.300
	7	15.951	18.911	18.900	18.605

Table 4. Results of the White test.

Index	Test Statistics (Chi-Squared)	*p*-Value
BET	29.53	0.000
BUX	0.178	0.914
PX	5.392	0.067
SAX	5.276	0.071
WIG20	9.970	0.006

Table 5 presents the results for the Teraesvirta test. This test uses a Taylor series expansion of the activation function to arrive at a suitable test statistic. The null hypothesis is similar to the White test. However, the Teraesvirta test demonstrates that beside the BUX index, the null hypothesis of linearity also cannot be rejected for the SAX index. Thus, our results are mixed, as in [7].

Table 5. Results of the Teräsvirta test.

Index	Test Statistics (Chi-Squared)	*p*-Value
BET	34.87	0.000
BUX	4.171	0.124
PX	26.08	0.000
SAX	2.503	0.286
WIG20	7.794	0.020

4.4. The Results of the Keenan and Tsay Tests for Nonlinearities

The results of the two tests are presented in Table 6. The two tests provide opposite results. While the Keenan test shows that all index returns are linear, the Tsay test proves the opposite. Only in the case of the WIG20 index, the two tests show consistence and indicate linearity. As the Tsay test is considered a more powerful test, we conclude that the series are nonlinear.

Table 6. Results of Keenan and Tsay tests.

Index	**Keenan Test**		**Tsay Test**	
	Test Statistics	***p*-Value**	**Test Statistics**	***p*-Value**
BET	0.664	0.415	3.483	0.000
BUX	1.079	0.298	3.833	0.000
PX	1.991	0.158	3.311	0.000
SAX	1.019	0.312	2.528	0.000
WIG20	0.938	0.332	0.426	0.734

4.5. The Results of the McLeod–Li Test for Nonlinearity

The test checks for the presence of conditional heteroscedascity by computing the Ljung–Box (portmanteau) test with the squared data or with the squared residuals from an Autoregressive Integrated Moving Average (ARIMA) model. As shown in Figure 1, all series prove to be nonlinearities.

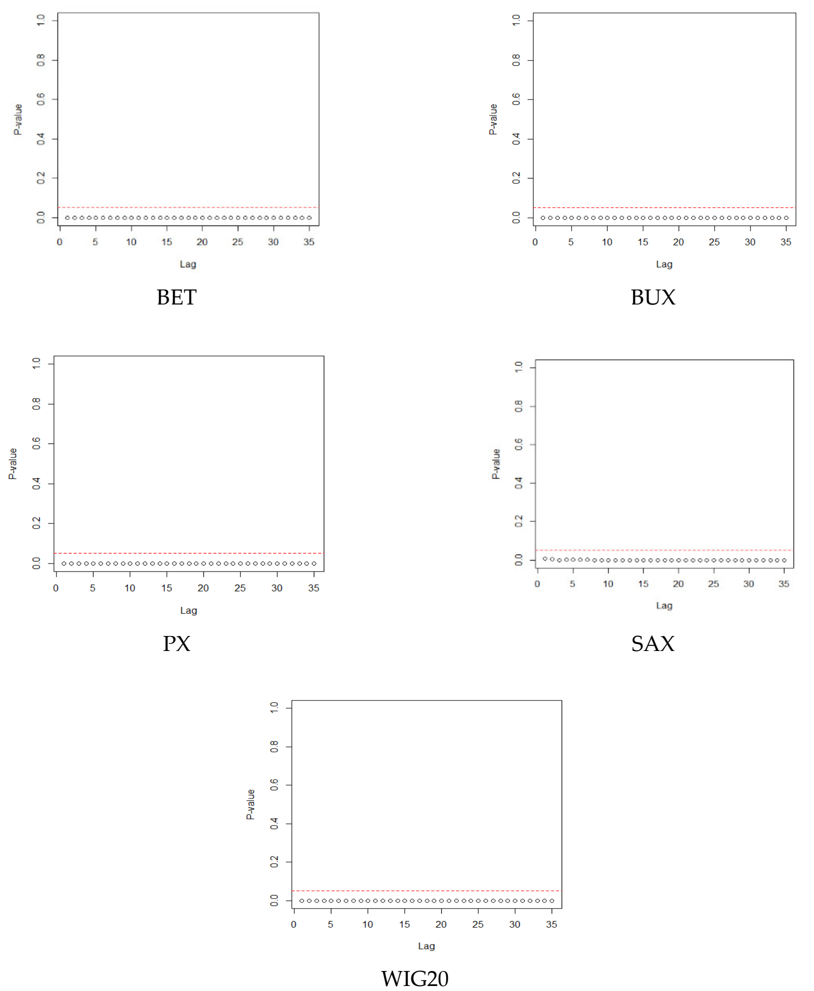

Figure 1. Results of the McLeod–Li test.

4.6. The Results of the Harvey Test for Nonlinearities

The null hypothesis of the Harvey test is the existence of linearity. We notice from Table 7 that all returns series are linear, results opposed to Tsay and McLeod–Li tests but in agreement with the Keenan test.

Table 7. Results of the Harvey test.

Index	Test Statistics (W)	Critical Values 1%	5%	10%
BET	36.57	76.79	76.62	76.53
BUX	6.500	39.77	39.71	39.67
PX	27.32	84.92	84.76	84.67
SAX	2.960	7.400	7.380	7.380
WIG20	7.750	24.26	24.22	24.20

Notes: (i) In order to reject the null of linearity, the W statistic has to be above its critical value; (ii) the critical values are simulated with 10,000 replications.

4.7. The Results of the Lyapunov Exponent

The positive, largest Lyapunov exponent is considered as one operational definition of chaotic behavior. For all the series, we observe chaotic dynamics, a fact which proves the impossibility to forecast in the medium to long run (Table 8).

Table 8. The results of Lyapunov exponent tests for chaotic behavior.

Index	Largest Lyapunov Exponent m	Lambda	Largest Lyapunov Index of Wolf m	Lambda	Rosenstein Largest Lyapunov Exponent m	Lambda
BET	9	0.025	9	0.017	9	0.001
BUX	8	0.053	8	0.043	8	0.000
PX	5	0.016	5	0.087	5	0.100
SAX	11	0.033	11	0.021	11	0.001
WIG20	7	0.000	7	0.051	7	0.000

4.8. The Results of the 0–1 Test for Chaos

While applying the 0–1 test for chaos, we draw 100 random frequencies from a uniform distribution on $\left[\frac{\pi}{5}, \frac{4\pi}{5}\right]$ in each filter and calculate the test statistic for each frequency. The results of [57] are presented in Table 9. Thus, Table 9 reports, for each series, the medians of these 100 single frequency test statistics. As these test statistics are close to one, they indicate that chaotic structures are inherent in the stock indexes' return series of CEE countries.

Table 9. The results of the 0–1 test for chaotic behavior.

Index	0–1 Test Statistics
BET	0.997
BUX	0.998
PX	0.996
SAX	0.997
WIG20	0.996

All in all, even though the presence of nonlinearities and chaos in the CEE stock markets is obvious, sometimes the results are mixed and contrast from those reported by [7]. Table 10 centralizes our findings and makes a comparison with those reported by [7]. The findings based on the BDS test and Lyapunov exponent are consistent for all the analyzed stock markets and show nonlinearities and chaotic behavior. The other tests provide mixed results, highlighting thus the differences in respect to the EMH between the five CEE stock markets and between the analyzed periods.

Table 10. Summary of results.

Test	Own Estimations All Indexes: June 2000–September 2015					Caraiani Results [7] BUX: June 1993–December 2010 WIG20: June 1993–December 2010 PX: April 1994–December 2010		
Index	**BUX**	**PX**	**WIG20**	**BET**	**SAX**	**BUX**	**PX**	**WIG20**
Runs test	R	R	LR	LR	LR	LR	LR	LR
BDS test	LI	LI	LI	LI	LI	LI	LI	LI
White test	L	NL	NL	NL	NL	NL	NL	L
Teräsvirta test	L	NL	NL	NL	L	NL	NL	L
Keenan test	L	L	L	L	L	L	NL	L
Tsay test	NL	NL	L	NL	NL	L	NL	NL
McLeod–Li test	NL	NL	NL	NL	NL	-	-	-
Harvey test	NL	NL	NL	NL	NL	-	-	-
Largest Lyapunov exponent	C	C	C	C	C	C	C	C
Lyapunov index of Wolf	C	C	C	C	C	-	-	-
Rosenstein Lyapunov exponent	C	C	C	C	C	-	-	-
0–1 test for chaotic behavior	C	C	C	C	C	-	-	-

Note: I = independence; LI = lack of independence; R = randomness; LR = lack of randomness; L = linearities; NL = nonlinearities; C = chaotic; NC = nonchaotic.

We also notice noteworthy discrepancies comparing our findings, obtained in the case of mature CEE markets, with those reported by [7] with a focus on the 1990s. First, in the case of BUX index, the Runs test used by [7] indicates a lack of randomness, whereas our results show the existence of randomness, which is closer to the EMH and characteristics for developed stock markets. The same applies in the case of the Tsay test, which, unlike [7], indicates the existence of a nonlinear behavior. Second, the Keenan and Tsay tests applied to the PX and WIG20 indexes indicate opposite results in our case compared to those reported by [7]. We can therefore notice a change in the behavior of CEE stock markets, which is influenced by their development level. At the same time, the outburst of the 2008–2009 global crisis might have had an impact on stock price behavior.

5. Conclusions

This paper tests for nonlinearities and chaos in five mature CEE stock markets, using a large battery of tests. Although our results generally contradict the theoretical assumption of linearity and nonchaotic behavior of stock markets, some issues appear. First, most of the selected tests for nonlinearity points to mixed evidence. Second, our results partially contrast from those reported by [7], which show that mature CEE stock market have a different behavior compared to their behavior during the 1990s and 2000s, when the efficient functioning of the selected markets was questioned. For example, the Runs test's results underline the presence of randomness for BUX and PX indexes, while the findings of [7] show the lack of randomness for the same index returns. In addition, the White and Teräsvirta tests highlight a linear behavior of the BUX index returns in our case.

To sum up, our results confirm the findings reported by the existing literature, which point, in general, in favor of nonlinear and chaotic behavior of stock markets that requires adequate forecasting techniques to predict the stock prices behavior. However, our findings partially contrast earlier reported findings for the CEE stock markets behavior in the 1990s. At the same time, our results underline the discrepancies existing between CEE stock markets, which question the idea of CEE stock markets' increasing integration.

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