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# Time-Varying Comovement of Foreign Exchange Markets: A GLS-Based Time-Varying Model Approach

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**Abstract:** How strongly are foreign exchange markets linked in terms of their similarities in long-run fluctuations? Are they cointegrating? To analyze such "comovements," we present a time-varying cointegration model for the foreign exchange rates of the currencies of Canada, Japan, and the UK vis-à-vis the U.S. dollar from May 1990 through July 2015. Unlike previous studies, we allow the loading matrix in the vector error-correction (VEC) model to be varying over time. Because the loading matrix in the VEC model is associated with the speed at which deviations from the long-run relationship disappear, we propose a new degree of market comovement based on the time-varying loading matrix to measure the strength or robustness of the long-run relationship over time. Since exchange rates are determined by macrovariables, cointegration among exchange rates implies these variables share common stochastic trends. Therefore, the proposed degree measures the degree of market comovement. Our main finding is that the market comovement has become stronger over the past quarter-century, but at a decreasing rate with two major turning points: one in 1995 and the other one in 2008.

**Keywords:** foreign exchange markets; market comovement; time-varying vector error correction model



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## 1. Introduction

It is well understood among researchers that exchange rate dynamics are highly complex, and there is little consensus on which econometrics model best describes the time-series process of exchange rates. Although many exchange rates are known to have unit roots, finding a cointegrating relationship among several exchange rates has attracted relatively little interest, with some exceptions such as Baillie and Bollerslev [1]. Because an exchange rate cointegrating with another exchange rate means they share a common stochastic trend, discovering such cointegration is tantamount to discovering common trends therein. As Baillie and Bollerslev [1] argue, many exchange rates are found to have a unit root or stochastic trend. If those exchange rates are not cointegrating, then any shock to an exchange rate has a permanent effect, irrespective of other exchange rates, thereby diverging from them. This situation is somewhat awkward because it is hard to imagine that interest rate differentials, one of the main determinants of exchange rate dynamics, do not exhibit international comovement. Yet, a skeptical view on cointegrating exchange rates is presented by, for example, Diebold et al. [2], who claim that the martingale model is better at out-of-sample forecast than is the cointegrating model. They also find weaker evidence of cointegration in exchange rates than do Baillie and Bollerslev [1]. As explained in Engel et al. [3], an important implication of Engel and West [4] is that a large discount factor could obscure the cointegrating relationship in exchange rates when the exchange rates are generated by the present value model, even when they are indeed cointegrated.

All in all, whether several exchange rates are cointegrated is an empirical question. However, investigating whether the cointegrating relationship is stable over time is more meaningful in economics because exchange rate dynamics might not be described by a single model, or because an unstable relationship could reflect extraordinary events that alter the global financial environment. It is also possible that the global financial markets are constantly changing, and hence, the relationship between exchange rates too are constantly changing. Therefore, our attempt in this paper is to estimate the extent to which exchange rates move together in the long run, considering the possibility that the cointegrating relationship may not be stable over time presumably owing to a time-varying market environment. In this way, we can shed new light on the debate on whether cointegrating relations exist in foreign exchange rates.

This idea of a time-varying market environment or market integration is in line with Ito et al. [5], who show that the global stock markets were efficient (in the sense of Fama [6]) for sometimes, but inefficient for the rest of the post-World War II period. To reach this conclusion, Ito et al. [5] define a time-varying degree of market efficiency and then they use this degree to determine whether the markets were efficient. Employing the same idea of time-varying nature of the global financial market, in this study, we propose a degree of market comovement and then, we show that it has monotonically increased over time in the past quarter-century. However, this increase is found to have occurred at a diminishing rate, suggesting that the cointegrating relationship may have a ceiling that prevents it from strengthening any further. There are some studies on foreign exchange market assuming parameter values to be time-varying. Kapetanios et al. [7] and Clarida et al. [8] adopt the STAR (smooth transition autoregressive) and the Markov-switching model, respectively. Those models are believed to be more suited to the situation where the parameter value moves within or toward several states. Our approach, on the other hand, allows parameter values to move in any direction at each time, thereby giving the model great flexibility.

This paper is organized as follows. In Section 2, we present our error-correction model that allows some key parameters to be time-varying. In the same section, we propose a new measure for foreign exchange markets, namely the degree of market comovement. The exchange rate data and preliminary unit root test results are given in Section 3. Section 4 provides the main results and a discussion of the time-varying nature of the global exchange market. Our conclusion is in Section 5. Detailed computations are provided in Appendices A–D.

## 2. Model

This section presents our method of capturing the time-varying nature of foreign exchange markets. The main building block of our model is a vector error-correction (VEC) model, which supposes that there are cointegrating relationships or long-run relationships among the variables in our model. In particular, our idea stems from the fact that the VEC model elucidates the adjustment process to the long-run relationships. As the following subsections explain, the crux of our model is that the adjustment process or the speed of adjustment to the long-run relationships can vary over time, reflecting the changing environment in the global foreign exchange markets.

### *2.1. Exchange Rate Dynamics and Cointegration*

Let us suppose the natural log of the spot Japanese yen per U.S. dollar exchange rate $s_t^J$ and the natural log of the spot Canadian dollar per U.S. dollar $s_t^C$ are cointegrated. Then, we have the following error-correction model representation:

$$\begin{aligned} \Delta s_t^J &= \mu_1 + \alpha_1 \left( s_{t-1}^J - \beta_1 s_{t-1}^C \right) + \gamma_{1,1} \Delta s_{t-1}^J + \gamma_{1,2} \Delta s_{t-1}^C + e_{1t} \\ \Delta s_t^C &= \mu_2 + \alpha_2 \left( s_{t-1}^J - \beta_1 s_{t-1}^C \right) + \gamma_{2,1} \Delta s_{t-1}^J + \gamma_{2,2} \Delta s_{t-1}^C + e_{2t}, \end{aligned}$$

where $\mu_1$ and $\mu_2$ are drifts; $\beta = (1, \beta_1)$ is the cointegrating vector; and $s^J_{t-1} - \beta_1 s^C_{t-1} = 0$ is the long-run relationship, which is stationary. The coefficients on the long-run relationship, $\alpha_1$ and $\alpha_2$ are interpreted as the speed at which any deviations from the long-run relationship disappear (More precisely, coefficient $\alpha_1$ or $\alpha_2$ is associated with the half-life of the deviation from the cointegrating relationship). As we shall see in the following subsections, it is possible that the speed of adjustment changes over time, perhaps because the environment of the international financial markets is changing.

### 2.2. The Vector Error-Correction Model

Assuming there are some cointegrating relationships, let us consider a vector error-correction (VEC) model for $m$-vector time-series $X_t$, as follows:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_k \Delta X_{t-k} + \Pi_k X_{t-k} + \mu + \varepsilon_t, \tag{1}$$

where $\Delta X_t = X_t - X_{t-1}$, $\mu$ is a vector of intercepts, and $\varepsilon_t$ is a vector of error terms.

The VEC model (1) suggests that $\Delta X_t$ consists of a stationary part, $\Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_k \Delta X_{t-k}$, and an error-correction term, $\Pi_k X_{t-k}$, which is also a stationary process when each variable in the vector $X_t$ is an integrated process of order one (often denoted as $I(1)$). Because we apply the VEC model to a set of variables whose first differences are stationary, $\Pi_k X_{t-k}$ is a vector of stationary processes with a zero mean vector; and $\Pi_k X_{t-k}$ includes some long-run relationships among the variables in $X_t$. Please note that the $m \times m$ matrix $\Pi_k$ is a singular matrix, i.e., the rank of $\Pi_k$ is $r$, which is less than $m$. Hence, if we decompose matrix $\Pi_k$ such that $\Pi_k = \alpha\beta'$, then $\alpha$ is an $m \times r$ matrix; $\beta'$ is an $r \times m$; and both are rank-$r$ matrices.

The long-run relationships among the variables in the system, or the cointegration is described as $\beta' X_{t-k} = 0$, and hence, $\beta$ is called the cointegrating matrix. We pay special attention to $\alpha$, which indicates how quickly the exchange rates in the system restore the long-run relationships when deviations from such relationships occur. This is because a rapid adjustment implies a strong or robust cointegrating relation. Thus, the determinants of exchange rate dynamics such as interest rate differentials, risk premia, and price level differentials should follow common stochastic trends. In this sense, the quicker the adjustment, the stronger is the comovement. Furthermore, by allowing $\alpha$ to be time-varying, we capture the time-varying nature of market comovement by estimating the change in the matrix over time using the time-varying VEC model (Although cointegration in foreign exchange rates does not mean that foreign exchange markets are efficient per se (See Engel [9]), it is also possible to interpret $\alpha$ as the speed at which arbitrage occurs in some cases. Suppose that there are arbitrage opportunities in the foreign exchange market due to relatively large shocks. Consider a case where the cross (indirect) exchange rate does not equal the bilateral (direct) exchange rate. We do not believe such opportunities to appear often, nor do we believe them to be long lasting. However, under the assumption that the long-run relationship or comovement are on the dynamic equilibrium path (or efficient market), the speed at which market participants exploit the arbitrage opportunities or the speed at which deviations from the long-run relationship are corrected is captured by $\alpha$. On the other hand, the fluctuations of an exchange rate due to such corrections also appear in the corresponding element in $\varepsilon$) .

### 2.3. The Time-Varying VEC Model

It is not obvious whether the loading matrix $\alpha$ is, in fact, time-varying. As an assessment, we apply the Hansen [10,11] parameter constancy test to the VEC model. We impose the following parameter dynamics to the VEC model so that we can estimate the loading matrix changes over time. For given constant $\beta$, we estimate $\Gamma$ and $\alpha$, assuming

$$\begin{aligned} \Gamma_t &= \Gamma_{t-1} + u_t \\ \alpha_t &= \alpha_{t-1} + v_{1t} \end{aligned}$$

Please note that simultaneous estimation of $\alpha_t$ and $\beta_t$ for each $t$ is infeasible owing to an identification problem of a linear regression (See Appendices A–D for a more detailed discussion). As explained in the appendix, one can estimate the time-varying VEC model without using the Bayesian method (MCMC). Instead, we employ an GLS-based method proposed by Ito et al. [12].

### 2.4. The Degree of Market Comovement

When the loading matrix $\alpha$ is not stable for the whole sample period, then it is reasonable to assume the speed of adjustment changes over time. Since the long-run relationship in exchange rates is presumably due to comovement in interest rate differentials, risk premia, or price level differentials, a rapid adjustment to the long-run relationship is associated with a strong comovement among those variables. For this, we propose a new measure of international comovement, called the degree of market comovement, which can be computed from the loading matrix $\alpha$:

$$\zeta_t = \sqrt{\max \lambda(\alpha_t \alpha_t')}$$

where $\max \lambda(A)$ is the largest eigenvalue of a matrix $A$. Please note that the degree of market comovement is similar to the one proposed by Ito et al. [5], who examine international stock market efficiency and quantify its time-varying nature. The greater the degree of market comovement, the faster foreign exchange markets return to their long-run relationship when deviations from the cointegrating relationship arise.

### 2.5. Confirming Our Assumption of Constant Cointegrating Vectors

Among the several assumptions we make about our model, we need to justify one substantial assumption, namely that the cointegrating vectors are constant over time while the loading matrix varies over time. It is conceivable that the fact that matrix $\Pi$ is time-varying, implies that the number of cointegrating vectors (as well as the cointegrating vectors themselves) is time-varying. To confirm the stability of the cointegrating vectors, we apply the state-of-the-art econometrics test proposed by Qu [13] to the exchange rate data (As detailed in Juselius [14], there are several econometrics tests that investigate the constancy of cointegrating vectors. In our view, however, the Qu [13] test is best suited for our purpose because it considers a variety of test statistics and their asymptotic properties, without assuming the timing of possible structural breaks). In essence, the Qu [13] test allows a researcher to assess whether the number of cointegrating vectors has changed in a subsample, say, the time between $T_a$ and $T_b$, where neither $T_a$ nor $T_b$ is known to the researcher. As the null hypothesis, the Qu [13] test states the number of cointegrating vectors is constant for the whole sample. Hence, not being able to reject the null hypothesis can be interpreted as justification of our assumption that the cointegrating vectors are stable over time.

## 3. Data

We use average monthly nominal data on spot and forward exchange rates for three developed countries (Canada, Japan, and UK) from May 1990 to July 2015, taken from the Thomson Reuters Datastream.

We choose these three exchange rates because they trade frequently worldwide (or they have high liquidity). In addition, they are the national currencies of three of the Group of Seven (G7) countries, presumably related to large macroeconomic fundamentals. The euro, however, has only existed since 1999. Hence, we exclude the euro, despite its high liquidity and the large economic fundamentals behind it.

For the forward exchange rate data, we use a nearby (one month) contract month following previous studies. We take the natural log of the spot and forward exchange rates to obtain the level data, and we also take the first difference of the natural log of the level data to compute the returns on the spot and forward exchange rates.

Table 1 provides some descriptive statistics and the results of the ADF-GLS tests. For the unit root tests, the ADF-GLS test of Elliott et al. [15] is applied. We employ the modified Bayesian information criterion (MBIC) instead of the modified Akaike information criterion (MAIC) to select the optimal lag length. This is because we are unable to find evidence of size-distortions (see Elliott et al. [15]; Ng and Perron [16]) in the estimated coefficient of the detrended series, $\widehat{\psi}$. It is widely known that the logarithmic spot and forward exchange rates are both integrated of order one (or $I(1)$ process), so that the differences are stationary (or $I(0)$ process) variables.

**Table 1.** Descriptive Statistics and Unit Root Tests.

| | | Mean | SD | Min | Max | ADF-GLS | Lags | $\hat{\varphi}$ | $\mathcal{N}$ |
|---|---|---|---|---|---|---|---|---|---|
| Level | | | | | | | | | |
| | $S_{CA}$ | 0.2175 | 0.1452 | −0.0454 | 0.4697 | −1.2364 | 1 | 0.9948 | 308 |
| | $F_{CA}$ | 0.2180 | 0.1449 | −0.0447 | 0.4699 | −1.2470 | 1 | 0.9947 | |
| | $S_{JP}$ | 4.6918 | 0.1486 | 4.3396 | 5.0352 | −1.6915 | 1 | 0.9902 | |
| | $F_{JP}$ | 4.6899 | 0.1483 | 4.3393 | 5.0344 | −1.6844 | 1 | 0.9903 | |
| | $S_{UK}$ | −0.4965 | 0.0905 | −0.7279 | −0.3387 | −2.9910 ** | 1 | 0.9700 | |
| | $F_{UK}$ | −0.4953 | 0.0900 | −0.7269 | −0.3373 | −2.9969 ** | 1 | 0.9698 | |
| First Difference | | | | | | | | | |
| | $\Delta S_{CA}$ | 0.0005 | 0.0167 | −0.0626 | 0.1083 | −11.8715 *** | 0 | 0.3648 | 307 |
| | $\Delta F_{CA}$ | 0.0005 | 0.0167 | −0.0626 | 0.1077 | −11.8337 *** | 0 | 0.3676 | |
| | $\Delta S_{JP}$ | −0.0007 | 0.0261 | −0.1095 | 0.0823 | −11.5044 *** | 0 | 0.3948 | |
| | $\Delta F_{JP}$ | −0.0007 | 0.0261 | −0.1094 | 0.0831 | −11.5039 *** | 0 | 0.3948 | |
| | $\Delta S_{UK}$ | 0.0004 | 0.0228 | −0.0583 | 0.1066 | −12.2763 *** | 0 | 0.3385 | |
| | $\Delta F_{UK}$ | 0.0004 | 0.0227 | −0.0585 | 0.1080 | −12.2221 *** | 0 | 0.3424 | |

Notes: (1) "ADF-GLS" denotes the ADF-GLS test statistics, "Lags" denotes the lag order selected by the MBIC, and "$\hat{\varphi}$" denotes the coefficients vector in the GLS detrended series (see Equation (6) in Ng and Perron [16]). (2) In computing the ADF-GLS test, a model with a time trend and a constant is assumed. (3) "***" and "**" indicate statistically significant at 1% and 5% level, respectively. (4) "$\mathcal{N}$" denotes the number of observations. (5) R version 4.0.5 was used to compute the statistics.

## 4. Empirical Results

### *4.1. Preliminaries*

First we determine whether our data on exchange rates exhibit non-stationarity, and more specifically, have a unit root. As shown in Table 1, the level data (upper panel) exhibit non-stationarity because the ADF-GLS test uniformly fails to reject the null hypothesis of the series possessing a unit root. However, the same test can reject the null hypothesis once the first differences of each series are computed (lower panel).

Having confirmed that our data have unit roots, we proceed to investigate whether the data have cointegrating relationships. To this end, we use the Johansen [17] maximum eigenvalue test and the Johansen [18] trace test.

Here, we assess the number of cointegrating vectors in Table 2. The null hypotheses for the two tests are presented in the first column of Table 2. From the first row, we can conclude that the null hypothesis of no cointegrating vector is rejected at the 5% and 10% level of significance by both the maximal eigenvalue test and the trace test. Yet, it is conclusive that there is no more than one cointegrating vector. This is because the maximal eigenvalue test whose alternative in the second row is two cointegrating vectors, cannot reject the null hypothesis, and neither can the trace test, whose alternative states that there is more than one cointegrating vectors (We also test the number of cointegrating vectors using the real exchange rates to check the robustness of our test results and obtain the almost same results as in the case of the nominal exchange rates. Thus, we conclude that there exists more than one cointegration vector in the VEC system).

We report the estimates of our VEC models in Table 3, where each column corresponds to each equation in the VEC model, showing coefficients on the error-correction term in the lower half of the table.

**Table 2.** Johansen's Cointegration Tests.

| | Eigenvalues | Max Eigen | Trace |
|---|---|---|---|
| None | 0.1352 | 44.45 ** | 100.19 * |
| At most 1 | 0.0826 | 26.39 | 55.74 |

Notes: (1) "Max Eigen" and "Trace" denote the Johansen [17] maximal eigenvalue test statistic and the Johansen [18] trace test statistic, respectively. (2) "**" and "*" indicate statistically significant at 5% and 10% level for each test, respectively. (3) R version 4.0.5 was used to compute the statistics.

**Table 3.** Time-Invariant VEC Model Estimations.

| | | $S_{CA}$ | $F_{CA}$ | $S_{JP}$ | $F_{JP}$ | $S_{UK}$ | $F_{UK}$ |
|---|---|---|---|---|---|---|---|
| Difference | | | | | | | |
| | $S_{CA}$ | −0.4209 | −0.2370 | 6.0713 | 6.1933 | 2.0254 | 2.1560 |
| | | [1.8453] | [1.8414] | [4.5597] | [4.5475] | [4.4885] | [4.4183] |
| | $F_{CA}$ | 0.7703 | 0.5885 | −6.0537 | −6.1739 | −1.6558 | −1.7854 |
| | | [1.8608] | [1.8560] | [4.5810] | [4.5684] | [4.4883] | [4.4181] |
| | $S_{JP}$ | 1.6242 | 1.6664 | −0.7974 | −0.5514 | 1.0198 | 0.9973 |
| | | [1.8997] | [1.8902] | [3.3395] | [3.3475] | [3.0337] | [3.0269] |
| | $F_{JP}$ | −1.6031 | −1.6460 | 1.1054 | 0.8580 | −0.9913 | −0.9692 |
| | | [1.9188] | [1.9093] | [3.3492] | [3.3573] | [3.0485] | [3.0412] |
| | $S_{UK}$ | 0.1174 | 0.0667 | −3.9585 | −4.1291 | −6.0241 | −5.8113 |
| | | [2.1271] | [2.1253] | [3.1513] | [3.1505] | [3.5704] | [3.5750] |
| | $F_{UK}$ | −0.1774 | −0.1263 | 3.9517 | 4.1228 | 6.2729 | 6.0556 |
| | | [2.1210] | [2.1200] | [3.1599] | [3.1593] | [3.6079] | [3.6115] |
| Level | | | | | | | |
| | Constant | −0.0334 | −0.0334 | 0.2006 | 0.1995 | 0.1467 | 0.1452 |
| | | [0.0470] | [0.0468] | [0.0680] | [0.0680] | [0.0470] | [0.0467] |
| | $S_{CA}$ | 0.8047 | 0.9465 | 3.3757 | 3.3233 | 2.2904 | 2.2845 |
| | | [1.1232] | [1.1317] | [2.6344] | [2.6460] | [2.2176] | [2.2168] |
| | $F_{CA}$ | −0.8390 | −0.9808 | −3.3467 | −3.2952 | −2.2552 | −2.2494 |
| | | [1.1254] | [1.1337] | [2.6427] | [2.6543] | [2.2191] | [2.2181] |
| | $S_{JP}$ | 1.3304 | 1.2953 | −1.0766 | −0.9938 | −1.5576 | −1.5626 |
| | | [1.0113] | [1.0109] | [1.3641] | [1.3615] | [1.1244] | [1.1277] |
| | $F_{JP}$ | −1.3189 | −1.2837 | 1.0302 | 0.9477 | 1.5173 | 1.5226 |
| | | [1.0135] | [1.0131] | [1.3695] | [1.3668] | [1.1194] | [1.1226] |
| | $S_{UK}$ | −2.0125 | −2.0557 | −1.4713 | −1.4990 | −1.8191 | −1.7604 |
| | | [1.5610] | [1.5579] | [2.2110] | [2.2115] | [2.3796] | [2.3631] |
| | $F_{UK}$ | 2.0478 | 2.0910 | 1.4478 | 1.4764 | 1.7452 | 1.6862 |
| | | [1.5691] | [1.5658] | [2.2266] | [2.2271] | [2.3855] | [2.3688] |
| $\bar{R}^2$ | | 0.0984 | 0.1005 | 0.1032 | 0.1026 | 0.1918 | 0.1900 |
| $L_C$ | | | | 65.5233 *** | | | |

Notes: (1) "$\bar{R}^2$" denotes the adjusted $R^2$, and "$L_C$" denotes the Hansen [10,11] joint $L_C$ statistic with variance. (2) Newey and West [19] robust standard errors are in brackets. (3) "***" indicates statistically significant at 1% level. (4) R version 4.0.5 was used to compute the estimates and the statistics.

### *4.2. The Time-Varying Model*

So far, we have assumed that the VEC model has time-invariant parameters. Let us relax this assumption and use the time-varying parameter model, which presumably better captures the dynamics of exchange rates, by taking into account a constantly changing market environment. Therefore, how different a picture can we get once we apply the time-varying VEC model? (Although we follow the results of Hansen's test for modeling the time-varying model, we do not provide the R-squared for this model because our approach is based on GLS, and hence, the R-squared is not an appropriate measure of fit.) First, we need to establish whether the cointegrating relationship, or the cointegrating vector, is preserved for the entire sample period. Table 4 shows the results of the cointegration order change test of Qu [13]. We can conclude that the number of cointegrating vectors among the Japanese yen-U.S. dollar, Canadian dollar-U.S. dollar, and pound sterling-U.S. dollar exchange rates is constant over time at the 1% level of significance (In addition to the Qu [13] test, we also confirm the stability or robustness of the cointegrating vector using the Hansen and Johansen [20] test, which rejects the null hypothesis of time-varying parameters in the cointegrating vector at the 1% significance level).

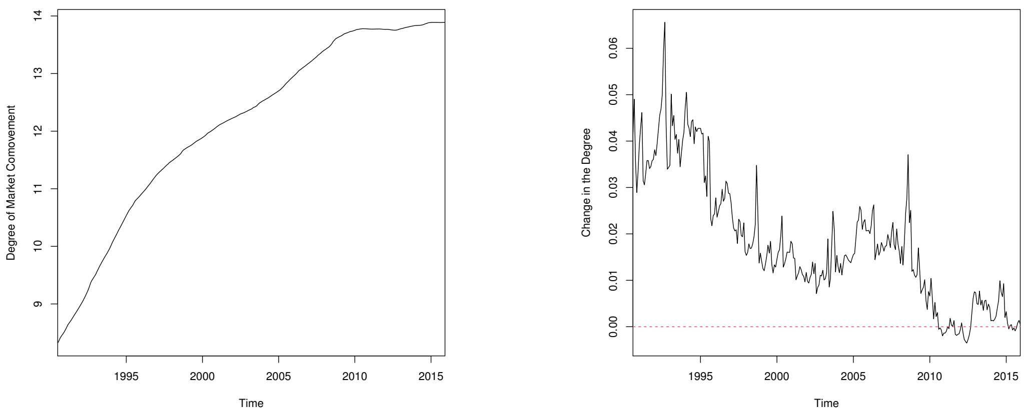
**Table 4.** Qu's Cointegration Order Change Tests.

| | $SupQ^1$ | $SupQ^2$ | $WQ$ | $SQ$ |
|---|---|---|---|---|
| Test Stats | 8.55 *** | 10.81 *** | 8.55 *** | 15.94 *** |

Notes: (1) "$SupQ^1$" and "$SupQ^2$" denote the values allowing for one break and two breaks, respectively. (2) "$WQ$" and "$SQ$" also denote consistent test stats when we suppose less than three breaks based on the maximum and sum of $SupQ^1$ and $SupQ^2$, respectively. (3) "***" indicate statistically significant at 1 % level. (4) R version 4.0.5 was used to compute the statistics.

Instead of presenting detailed estimates about our time-varying VEC model (which are available in Appendices A–D), we report the degree of adjustment speed $\zeta_t$. As we discuss in Section 2, the larger $\zeta_t$ is, the quicker the foreign exchange markets adjust to their long-run relationship.

The left panel of Figure 1 shows the foreign exchange markets that are increasingly rapidly restoring the long-run relationship when unanticipated shocks hit the markets (We also estimate the time-varying degree of market comovement using the real exchange rates to explore whether our results in the main document are robust or not. As a result, we obtain almost the same results as the time-varying degree of market comovement in the main document even if we use the real exchange rates. Therefore, we can consider that our empirical results do not depend on the differences of data). The right panel provides a close look at the time-varying nature of the speed of adjustment, specifically, the changes in the speed of adjustment. It is interesting to observe that the speed at which the markets restore the long-run relationship has been increasing over time, but its rate of increase has diminished over the sample period.




Notes: R version 4.0.5 was used to compute the estimates and the calculates.

**Figure 1.** Time-Varying Degree of Market Comovement.

Notably, the changes in the rate of increase occurred at least twice, once in the vicinity of 1995, and then in 2008. The second change seemingly coincides with the global financial crisis, while the first time is likely associated with the Mexican peso crisis (An interesting extension of this approach is to assess the forecasting power of the time-varying model, relative to the time-invariant model. To formally evaluate the two models with different numbers of parameters, one would need to employ a bootstrap approach (See Clark and McCracken [21], for example)).

Our results are consistent with the findings of Baillie and Bollerslev [1], since we find cointegrating relationships in foreign exchange rates, yet we allow such relationships to be time-varying in this paper. Although it is not so simple to link the degree of market comovement with the stochastic discount factor, a claim of Engel et al. [3]—cointegrating relations obscured by a large stochastic discount factor—may be viewed as cointegration with the time-varying loading matrix that we found. Yet, the degree of market comovement is a new concept presented by this study. With it, we provide insight into a constantly changing foreign exchange market environment that affects the fluctuations of foreign exchange rates in the long run.

## 5. Concluding Remarks

A novel model for exchange rates, taking into account the time-varying nature of the market environment, is presented. Our approach enables us to use the cointegrating relationship among exchange rates, with the loading matrix in the VEC model changing over time. Since the loading matrix can be interpreted as the speed of adjustment or the strength of the cointegrating relationship in exchange rates determined by various macroeconomic variables sharing common stochastic trends, we call the new degree derived from the loading matrix the "degree of market comovement." With the new degree, we find that market comovement has become stronger, but at a decreasing rate with large turning points in 1995 and in 2008.

In summary, the contribution of this paper is as follows. First, we develop a multivariate time-varying parameter model that allows the loading matrix of the cointegrating relationship to change over time. Secondly, we define the concept of the degree of market comovement, using the estimated loading matrix. Third, we find that the degree of market comovement has increased, albeit not monotonically, over time. Because of our novel approach, we can shed new light on the debate about the existence of cointegrating relations in foreign exchange rates.

The time-varying degrees of comovement uncovered in this paper have, at least, the following two implications. First, a multivariate approach is more appropriate when the dynamics of foreign exchange rates are considered. Second, care must be taken if one wants to quantify the persistence of a shock to the exchange rate using techniques such as the standard impulse-response functions computed from a vector autoregressive model.

However, it is worth mentioning that the limitation of our study stems from the assumption that cointegrating vectors remain unchanged over time. Therefore, as a new direction of research, one may further the idea of time-varying cointegration by considering an exchange rate system that has a cointegration relation at a certain time, but not for the whole sample period. Such models can assess whether the claim made by Engel and West [4] is empirically valid.

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## Abbreviations

The following abbreviations are used in this manuscript:

| | |
|---|---|
| ADF | Augmented Dickey-Fuller |
| GLS | Generalized least squares |
| MAIC | modified Akaike information criterion |
| MBIC | Modified Bayesian information criterion |
| MCMC | Markov chain Monte Carlo |
| OLS | Ordinary least squares |
| STAR | Smooth transition autoregressive |
| TV-VEC | time-varying vector error correction |
| VEC | Vector error correction |

## Appendix A. Vector Error-Correction Model

First, we present a framework of a vector error-correction (VEC) model to use simple least square techniques. Please note that a standard VEC model is derived from the following vector autoregression (VAR) equation for $m$-vector time-series $X_t$ ($t = 1, \cdots, T$).

$$X_t = \Pi_1 X_{t-1} + \cdots + \Pi_k X_{t-k} + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t, \tag{A1}$$

where $\boldsymbol{\mu}$, $D_t$ and $\boldsymbol{\varepsilon}_t$ denote a drift term, $n$ exogenous vector (i.e., trend term) and a $m$ random vector, respectively. Thus, $\Phi$ is $m \times n$ matrix; each $\Pi_j$'s are $m \times m$ square matrices. Through tedious algebra, we obtain the following VEC equation from provides

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_k \Delta X_{t-k+1} + \Pi_k X_{t-k} + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t. \tag{A2}$$

This equation suggests that the change of the time-series, $\Delta X_t$, is a sum of a stationary part, $\Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_k \Delta X_{t-k+1} + \boldsymbol{\varepsilon}_t$, and an error correction $\Pi_k X_{t-k}$, which is stationary process when each component of $X_t$ is an integrated process.

Supposing $T$ sample periods, we can rewrite Equation (A2) in an extended linear regression form. For example,

$$\begin{bmatrix}\Delta X_1 & \cdots & \Delta X_T\end{bmatrix} = \begin{bmatrix}\boldsymbol{\mu} & \Gamma_1 & \cdots & \Gamma_k & \Pi\end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1} \\ X_{1-k} & \cdots & X_{T-k} \end{bmatrix} + \begin{bmatrix}\boldsymbol{\varepsilon}_1 & \cdots & \boldsymbol{\varepsilon}_T\end{bmatrix}.$$

Technically speaking, we can break down the above linear system to the following three cases according to what type of long-run relations are supposed.

**Case 1:** *Error Correction Terms without Drift*

The first case corresponds to the long-run equilibrium equations without constant terms.

$$\begin{aligned}\begin{bmatrix}\Delta X_1 & \cdots & \Delta X_T\end{bmatrix} = \begin{bmatrix}\boldsymbol{\mu} & \Gamma_1 & \cdots & \Gamma_k\end{bmatrix} &\begin{bmatrix} 1 & \cdots & 1 \\ \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1} \end{bmatrix} \\ &+ \Pi_{\mathrm{n}} \begin{bmatrix}X_{1-k} & \cdots & X_{T-k}\end{bmatrix} + \begin{bmatrix}\boldsymbol{\varepsilon}_1 & \cdots & \boldsymbol{\varepsilon}_T\end{bmatrix}.\end{aligned} \tag{A3}$$

**Case 2:** *Error Correction Terms with Drift*

The second case corresponds to the long-run equilibrium equations with constant terms.

$$\begin{bmatrix}\Delta X_1 & \cdots & \Delta X_T\end{bmatrix} = \begin{bmatrix}\Gamma_1 & \cdots & \Gamma_k\end{bmatrix} \begin{bmatrix}\Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1}\end{bmatrix} + \Pi_{\mathrm{c}} \begin{bmatrix} X_{1-k} & \cdots & X_{T-k} \\ 1 & \cdots & 1 \end{bmatrix} + \begin{bmatrix}\varepsilon_1 & \cdots & \varepsilon_T\end{bmatrix}. \quad \text{(A4)}$$

**Case 3:** *Error Correction Terms with Linear Time Trend*

The third case corresponds to the long-run equilibrium equations with linear time trend.

$$\begin{bmatrix}\Delta X_1 & \cdots & \Delta X_T\end{bmatrix} = \begin{bmatrix}\boldsymbol{\mu} & \Gamma_1 & \cdots & \Gamma_k\end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1}\end{bmatrix} + \Pi_{\mathrm{t}} \begin{bmatrix} X_{1-k} & \cdots & X_{T-k} \\ 1 & \cdots & T \end{bmatrix} + \begin{bmatrix}\varepsilon_1 & \cdots & \varepsilon_T\end{bmatrix}, \quad \text{(A5)}$$

where the last row of the second term in (A5) presents a time trend $(1,\ 2,\ \cdots,\ T)$.

Notice that the dimension of the $\Pi_{\mathrm{n}}$ matrix for the third case differs from the other two cases. It is $m \times m$ whereas the others are $m \times (m+1)$. Because a VEC model is algebraically derived from a certain VAR model, a linear stochastic system, it is also such a system. Thus, we can estimate parameters $\mu$, $\Gamma_i$'s and $\Pi$ using some regression techniques, say, OLS or GLS. Let $Z_{0a}$, $Z_{1a}$ and $Z_{ka}$, $(a = \mathrm{n}, \mathrm{c}, \mathrm{t})$ denote appropriate data matrices representing for Equations (A3)–(A5). Furthermore, $\boldsymbol{\varepsilon}$ denotes a matrix of exogenous shock vectors, i.e.,

$$Z_0' = \Gamma_{\mathrm{n}} Z_{1\mathrm{n}}' + \Pi_{\mathrm{n}} Z_{k\mathrm{n}}' + \boldsymbol{\varepsilon}, \quad \text{(A6)}$$

where

$$Z_0' = \begin{bmatrix}\Delta X_1 & \cdots & \Delta X_T\end{bmatrix},$$

$$Z_{1\mathrm{n}}' = \begin{bmatrix} 1 & \cdots & 1 \\ \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1}\end{bmatrix},$$

$$Z_{k\mathrm{n}}' = \begin{bmatrix} X_{1-k} & \cdots & X_{T-k}\end{bmatrix},$$

$$\Gamma_{\mathrm{n}} = \begin{bmatrix}\boldsymbol{\mu} & \Gamma_1 & \cdots & \Gamma_k\end{bmatrix},$$

and

$$\boldsymbol{\varepsilon} = \begin{bmatrix}\varepsilon_1 & \cdots & \varepsilon_T\end{bmatrix}.$$

Similarly, as for (A4), its expression is as follows:

$$Z_0' = \Gamma_{\mathrm{c}} Z_{1\mathrm{c}}' + \Pi_{\mathrm{c}} Z_{k\mathrm{c}}' + \boldsymbol{\varepsilon}, \quad \text{(A7)}$$

where

$$Z_{1\mathrm{c}}' = \begin{bmatrix} \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1}\end{bmatrix},$$

$$Z'_{k\mathrm{c}} = \begin{bmatrix} 1 & \cdots & 1 \\ X_{1-k} & \cdots & X_{T-k} \end{bmatrix},$$

and

$$\Gamma_{\mathrm{c}} = \begin{bmatrix} \Gamma_1 & \cdots & \Gamma_k \end{bmatrix}.$$

Finally, as for (A4), its expression is as follows:

$$Z'_0 = \Gamma_{\mathrm{t}} Z'_{1\mathrm{t}} + \Pi_{\mathrm{t}} Z'_{k\mathrm{t}} + \boldsymbol{\varepsilon}, \tag{A8}$$

where

$$Z'_{1\mathrm{t}} = Z'_{1\mathrm{n}},$$

$$Z'_{k\mathrm{t}} = \begin{bmatrix} X_{1-k} & \cdots & X_{T-k} \\ 1 & \cdots & T \end{bmatrix},$$

and

$$\Gamma_{\mathrm{n}} = \Gamma_{\mathrm{t}}.$$

The three matrices, $\Gamma_{\mathrm{n}}$, $\Gamma_{\mathrm{c}}$ and $\Gamma_{\mathrm{t}}$ might include $\boldsymbol{\mu}$ as well as $\Gamma_1, \cdots, \Gamma_k$. They provide us with information about stationary aspect of the time-series $X_t$. On the other hand, the three matrices, $\Pi_{\mathrm{n}}$, $\Pi_{\mathrm{c}}$ and $\Pi_{\mathrm{t}}$, take a crucial role in a VEC model. They are decomposed into the loading matrix and the cointegration matrix such that $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$ (Three identifier, "n", "c" and "t", for the above three options are omitted here). In particular, $\boldsymbol{\beta}' Z'_k$ signifies some long-run relationship among the observation. One can select the lag order $k$ using usual information criteria such as SBIC for each linear model above; it is easy to compute them.

Given some cointegration order $r$, we can decompose the estimated $\Pi$ in the above linear models with respect to $Z_0$, $Z_1$ and $Z_K$ such that $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are called the loading and cointegration matrices, respectively. The cointegration order $r$ is usually selected through well-known the Johansen [17,18] test. Johansen's procedure on the rank helps ones to obtain all the estimates and statistics for applied econometrician.

## Appendix B. Parameter Constancy Test

Since we can regard a VEC model as a simultaneous linear regression system, (A6), (A7) or (A8), we can examine the possibility of parameter constancy on $\Pi$ and $\Gamma$ using the Hansen [10] parameter constancy test. Its null hypothesis is that parameters are time-invariant; the alternative hypothesis is that they are martingale.

There are several stochastic processes that are martingale. Thus, when the null hypothesis is rejected, we must choose one of such processes that would be followed by the time-varying parameters in our model. Because we are interested in gradual changes in speed of adjustment to a certain long-run equilibrium for a VEC model, we choose a parameter dynamics in which the parameters follow random walk. The next section presents our estimation method for a VEC model with random walk parameters.

## Appendix C. Time-Varying VEC Model

As Lütkepohl [22] exactly points, the essential aim of VEC models is to decompose a multivariate time-series into a pair of stationary and non-stationary time-series. It is very similar to that of the Beveridge and Nelson [23] decomposition for a univariate time-series. Roughly speaking, we can consider that the matrices $\Gamma_i$'s for $i = 1, \cdots, k$ represents the stationary structure of the time-series to be analyzed; the matrix $\Pi$ or matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ represent its non-stationary structure, the so-called cointegrated relationship among variables of the time-series.

Although we start from a VAR model (A1), its corresponding VEC model provides more information. When we consider a time-varying nature of some multivariate time-series using VEC model, we should specify to what structure we focus: that represented by $\Gamma$, that of $\Pi$ or $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ or both. We show some options for time-varying VEC (TV-VEC) model. Regarding a VEC model as a simple linear regression model, we just consider one of (A6) through (A8). Thus, we write down it for convenience as follows.

$$Z_0' = \Gamma Z_1' + \Pi Z_k' + \boldsymbol{\varepsilon}. \tag{A9}$$

We can suppose following combinations of the parameter dynamics with the above linear model of VEC (A9). First, we estimate $\Gamma$ and $\Pi$ as time-varying parameters without considering the decomposition of $\Pi$ into $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

$$\Gamma_t = \Gamma_{t-1} + \boldsymbol{u}_t, \tag{A10}$$

$$\Pi_t = \Pi_{t-1} + \boldsymbol{v}_t, \tag{A11}$$

To this case, we can simply apply the Ito et al. [5] method to estimate a linear regression model (See their online appendix which is available at http://at-noda.com/appendix/inter_market_appendix.pdf (accessed on 10 April 2021)).

Second, we estimate $\Gamma$ and $\boldsymbol{\alpha}$ with regarding $\boldsymbol{\beta}$ as time-invariant and given.

$$\Gamma_t = \Gamma_{t-1} + \boldsymbol{u}_t, \tag{A12}$$

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \boldsymbol{v}_{1t}. \tag{A13}$$

To this case, we first build a $r$-dimensional time-series $Y = Z_k\boldsymbol{\beta}$. Then, we apply Ito et al.'s method again to a new time linear regression

$$Z_0' = \Gamma Z_1' + \boldsymbol{\alpha} Y' + \boldsymbol{\varepsilon}, \tag{A14}$$

considering (A12) and (A13).

Third, we estimate $\Gamma$ and $\boldsymbol{\beta}$ with regarding $\boldsymbol{\alpha}$ as time-invariant and given.

$$\Gamma_t = \Gamma_{t-1} + \boldsymbol{u}_t \tag{A15}$$

$$\beta_t = \beta_{t-1} + \boldsymbol{v}_{2t} \tag{A16}$$

Considering $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$, we first rewrite (A9) as follows.

$$vec(Z_0) = (\boldsymbol{\alpha} \otimes Z_k)vec(\boldsymbol{\beta}) + (I \otimes Z_1)vec(\Gamma') + vec(\boldsymbol{\varepsilon}'), \tag{A17}$$

where $\otimes$ is the Kronecker product and $vec$ operator transforms a matrix into a vector by stacking the columns. Please note that $\boldsymbol{\alpha} \otimes Z_k$ can be regarded as a data matrix because $\boldsymbol{\alpha}$ is given. Considering this equation as a linear regression model whose parameters are supposed time-varying, we apply again the Ito et al. [5] method estimate the parameters to be varying over time.

Please note that both $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}_t$ for each $t$ cannot be estimated. In particular, since $\Pi_t = \boldsymbol{\alpha}_t\boldsymbol{\beta}_t'$ for each $t$ and $\Pi_t$ is not of full rank, a decomposition of $\Pi_t$ into $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ is not unique. Thus, either $\boldsymbol{\alpha}_t$ or $\boldsymbol{\beta}_t$ is supposed time-invariant for the most general case of both $\Pi_t$ and $\Gamma_t$ supposed time-varying.

## Appendix D. Degree of Market Comovement

We regard $\boldsymbol{\beta}' Z_k' = \mathbf{0}$ as long-run equilibrium relations with respect to the multiple time-series. The loading matrix $\boldsymbol{\alpha}$, representing a speed of adjustment, is time-varying when we employ a TV-VEC model to analyze market comovement. Thus, we pay our attention to the time-varying loading matrix $\boldsymbol{\alpha}_t$, which provides information about dynamics of market comovement. The larger the absolute value of its components, the more significant their contribution to ameliorate deviation from the long-run equilibrium. Thus, we propose an index of market comovement based on the loading matrix. Then, we applied the index for the time-varying loading matrix $\boldsymbol{\alpha}_t$ to investigate how degree of market comovement varies.

We derive the index $\zeta_t$ from $\boldsymbol{\alpha}_t$ following Ito et al. [5]. In particular,

$$\zeta_t = \sqrt{\max \lambda(\boldsymbol{\alpha}_t \boldsymbol{\alpha}_t')}. \tag{A18}$$

That is, $\zeta_t$ is the square root of the largest eigen value of $\boldsymbol{\alpha}_t \boldsymbol{\alpha}_t'$, which is a non-negative semi definite matrix, for each $t$. Notice that the more the index the faster the adjustment of markets to the long-run equilibrium.

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