

Learning Impulsive Pinning Control of Complex Networks

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Abstract: In this paper, we present an impulsive pinning control algorithm for discrete-time complex networks with different node dynamics, using a linear algebra approach and a neural network as an identifier, to synthesize a learning control law. The model of the complex network used in the analysis has unknown node self-dynamics, linear connections between nodes, where the impulsive dynamics add feedback control input only to the pinned nodes. The proposed controller consists of the linearization for the node dynamics and a reorder of the resulting quadratic Lyapunov function using the Rayleigh quotient. The learning part of the control is done with a discrete-time recurrent high order neural network used for identification of the pinned nodes, which is trained using an extended Kalman filter algorithm. A numerical simulation is included in order to illustrate the behavior of the system under the developed controller. For this simulation, a 20-node complex network with 5 different node dynamics is used. The node dynamics consists of discretized versions of well-known continuous chaotic attractors.



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1. Introduction

The study of complex networks is of great importance for the scientific community due to the interconnected nature of the modern world and the wide range of meaningful applications [1–4]. In the dynamical systems area, a complex network can be seen as a group of interconnected systems, which have their own characteristics and dynamics. When working with these systems, it is often desired that the system follows a reference signal by an adequate controller. Methods and techniques in control theory used to drive the system to stability around an equilibrium point are numerous and varied [5]. Selecting a particular method to accomplish this goal depends solely on the designer or the considered system. For complex networks, a technique named pinning control is often used, in which only a fraction of the network nodes is controlled [6]. This control method for complex networks has been intensively studied. In [7], for example, a complex network with different chaotic node dynamics was controlled using inverse optimal control and V-stability; the latter changes the problem to a linear algebra one. This change performs the gain selection process more easily.

A control technique not exclusive to complex networks is the impulsive one, in which the control input is applied at a defined time instant [8]. Published works on synchronization and control of complex networks via impulsive control have previously been carried out. In [9], the authors propose a control algorithm where the complex network has the same node dynamics in all of its nodes, which is similar to the proposal in [10]. In [11], the algorithm considers different node dynamics in its nodes, without pinning control. A previous study was undertaken where we examined the impulsive control of discrete-time complex dynamical networks with an experimental approach rather than an analytical one [12].

Alternatively, learning control algorithms are capable of estimating unknown information of a given system. This is particularly useful since we do not always have all system information at our disposal. With an optimal learning technique, we can emulate the performance of the deterministic approach. Neural networks have a wide range of applications depending on their configuration [13], and one of them is the learning control approach that recreates the dynamics of nonlinear systems using the corresponding measurements as the neural identifiers [14,15]. The output of these neural identifiers can then be used to compute the desired control input. Previous studies have used neural networks for controlling complex networks, as in [16–18], focusing on continuous-time applications.

The aim of this research is pin control of a discrete-time complex network with different node dynamics and an impulsive control input, introducing a linear algebra approach for easy gain selection, which is the main contribution of this paper. Additionally, to synthesize the control law a learning control methodology is used based on a neural identifier. Studies of impulsive pin control of discrete-time complex networks with different nodes are scarce, and the objective of this research is to fill that gap. Moreover, a priori knowledge of node dynamics is not required because of the use of neural identifiers. The proposed scheme was tested under stressful conditions, such as chaotic dynamics. Our previous work on this topic did not include stability analysis of the whole network, so this study represents a considerable advance.

This paper is organized as follows: Firstly, we summarize required mathematical preliminaries where we establish the model used for the complex network, the concept of the Rayleigh quotient and the model of the used neural network. Then, we present the analysis for the proposed controller. Later, we introduce the recurrent high order neural network and its parameters. After that, we include a numerical simulation in order to illustrate the applicability of the proposed controller. Finally, we state the respective conclusions.

2. Preliminaries

In this section, we describe the mathematical tools used for analysis and simulations.

2.1. Complex Networks

The model for an impulsively controlled discrete-time complex network can be written as [12]:

$$X(k + 1) = f(X(k)) + (G \otimes B)X(k) + U(k, X(k)), \tag{1}$$

where $X(k) = [x_1^\top(k) \ x_2^\top(k) \ \dots \ x_N^\top(k)]^\top$ is the state of the complex network with N nodes, $x_i(k) \in \mathbb{R}^n$, for $i = 1, \dots, N$, is the state of each i -th node. The function $f(X(k)) = [f_1(x_1(k))^\top \ f_2(x_2(k))^\top \ \dots \ f_N(x_N(k))^\top]^\top$ contains all of the nodes self-dynamics $f_i(x_i(k))$. Matrix $G = [g_{ij}] = [c_{ij}\hat{a}_{ij}]$ represents the connections of the network nodes, with c_{ij} as the connection strength between nodes i and j . Constant $\hat{a}_{ij} = 1$ if there is a connection between node i and node j , and $\hat{a} = 0$ if there is no such connection. Matrix $B \in \mathbb{R}^{n \times n}$ represents the way the state components are connected between the different nodes. Control input is $U(k, X(k)) = K(k)X(k)$, where $K(k)$ is a block diagonal matrix of the form:

$$K(k) = \begin{bmatrix} \kappa_1(k)I_n & & & \\ & \kappa_2(k)I_n & & \\ & & \ddots & \\ & & & \kappa_N(k)I_n \end{bmatrix}, \tag{2}$$

where I_n is an identity matrix of the n -th order, $\kappa_i(k)$ is a real constant at $k = t_k$, which is the time instant defined for the control impulse, and $\kappa_i(k) = 0$ when $k \neq t_k$. Furthermore, in accord with pinning control, some $\kappa_i(k)$ are always zero.

Equation (1) is obtained through a finite difference discretization of a first order time derivative as:

$$\frac{dX}{dt} \approx \frac{X(k + 1) - X(k)}{T}, \tag{3}$$

where T is a sampling time. For notation simplicity, the corresponding summed term $X(k)$ and sampling time T are absorbed by the rest of the terms in (1).

2.2. The Rayleigh Quotient

The Rayleigh Quotient R_A is defined as:

$$R_A = \frac{x^* Ax}{x^* x}, \tag{4}$$

where x is some vector and A is a square matrix of appropriate dimensions. The quotient (4) has a real value satisfying $\lambda_{min} \leq R_A \leq \lambda_{max}$, with λ_{min} and λ_{max} as the minimum and maximum eigenvalues of the Hermitian matrix A [19]. This means that for a real symmetric matrix A , the quadratic form $x^T Ax$ is bounded as follows:

$$\lambda_{min} x^T x \leq x^T Ax \leq \lambda_{max} x^T x. \tag{5}$$

2.3. Discrete-Time Recurrent High Order Neural Network (RHONN) Model

The learning part of the control algorithm is done with the use of neural networks, with a neural identifier using a discrete-time RHONN, which is described by [15]:

$$\hat{x}_i(k+1) = w_i^T(k) z_i(x(k), u(k)), \quad i = 1, \dots, n, \tag{6}$$

where $\hat{x}_i(k)$ is the state of the i -th neuron at time-step k , n is the dimension of the state, $w_i(k)$ is the adaptable weight vector, and $z(x(k), u(k))$ is a high order non-linear function defined by the designer. This type of network can be trained by using the extended Kalman filter (EKF) algorithm, implemented with the following equations [15,20]:

$$K(k) = P(k)H^T(k) [R(k) + H(k)P(k)H^T(k)]^{-1}, \tag{7}$$

$$w(k+1) = w(k) + K(k)e(k), \tag{8}$$

$$P(k+1) = P(k) - K(k)H(k)P(k) + Q(k) \tag{9}$$

where matrix $K(k)$ is the Kalman gain, $P(k)$ is the error covariance matrix, $R(k)$ is the measurement error covariance, $Q(k)$ is the process error covariance, $e(k) = [y(k) - \hat{y}(k)]$ is the error signal between the desired output $y(k)$ and the neural network output $\hat{y}(k)$, and we will define $H^T(k) = z(x(k), u(k))$ [12,15].

3. Proposed Control Structure

Consider a complex network as the one described by (1). We can linearize the nodes self-dynamics using the Jacobian, so Equation (1) is rewritten as:

$$X(k+1) = FX(k) + (G \otimes B)X(k) + K(k)X(k), \tag{10}$$

where F is a block diagonal matrix of the form:

$$F = \begin{bmatrix} F_1 & & & \\ & F_2 & & \\ & & \ddots & \\ & & & F_N \end{bmatrix}, \tag{11}$$

where matrix $F_i \in \mathbb{R}^{n \times n}$ represents the linearized dynamics of node i .

Theorem 1. *The discrete-time complex network with pinning control described in (1) is stable at equilibrium point zero if*

$$\lim_{k \rightarrow \infty} \sum_{j=0}^k \ln \alpha_j = -\infty, \tag{12}$$

where α_k is the largest eigenvalue of matrix $A^\top(k)A(k)$, with

$$A(k) = F + G \otimes B + K(k). \tag{13}$$

Proof. By substituting with (13), linearized network dynamics in (10) can be written as:

$$X(k + 1) = A(k)X(k). \tag{14}$$

Now, we propose the quadratic Lyapunov function $V(X(k)) = X^\top(k)X(k)$, so we have:

$$V(X(k + 1)) = X^\top(k)A^\top(k)A(k)X(k). \tag{15}$$

Substituting (5) in (15) we obtain:

$$V(X(k + 1)) \leq \alpha_k X^\top(k)X(k), \tag{16}$$

where α_k is the largest eigenvalue of matrix $A(k)^\top A(k)$. Inequality (16) can be written as:

$$V(X(k + 1)) \leq \alpha_k V(X(k)). \tag{17}$$

By solving for $V(X(k))$ we get:

$$V(X(k)) \leq \prod_{j=0}^k \alpha_j V(X(0)) = e^{\ln \prod_{j=0}^k \alpha_j} V(X(0)) = e^{\sum_{j=0}^k \ln \alpha_j} V(X(0)). \tag{18}$$

Hence, if the sum $\sum_{j=0}^k \ln \alpha_j$ approaches $-\infty$, $V(X(k))$ decreases, and the system (1) is stable at the equilibrium point zero. This finishes the proof of Theorem. \square

We can modify the eigenvalues of matrix $A(k)^\top A(k)$ by varying the elements of the design matrix $K(k)$ until the condition stated in (12) is fulfilled for a desired impulsive time scheme.

The proposed scheme for the neural control is as shown in Figure 1, where the identified state $\hat{X}(k)$ is substituted in the control input $U(k, X(k)) = K(k)X(k)$ in (1), so the control input can be selected as:

$$U(k, \hat{X}(k)) = K(k)\hat{X}(k), \tag{19}$$

where $\hat{X}(k) = [\hat{x}_1^\top(k) \quad \hat{x}_2^\top(k) \quad \dots \quad \hat{x}_N^\top(k)]^\top$, and $\hat{x}_i(k)$ is obtained with the equations of Section 2.3 if node i is a pinned node and $\hat{x}_i(k) = 0$ if node i is not a pinned node.

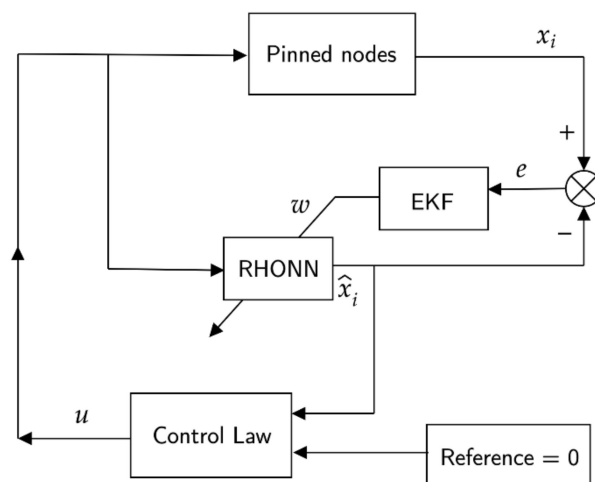


Figure 1. Proposed control scheme using a neural identifier for the pinned nodes.

4. Numerical Simulation and Discussion

To illustrate the proposed controller performance, we do a simulation following the control scheme of Figure 1 using the 20-node complex network, illustrated in Figure 2, with 5 different discretized versions of known continuous chaotic attractors. These systems are selected because of their apparent randomness which can be useful to test our learning control scheme.

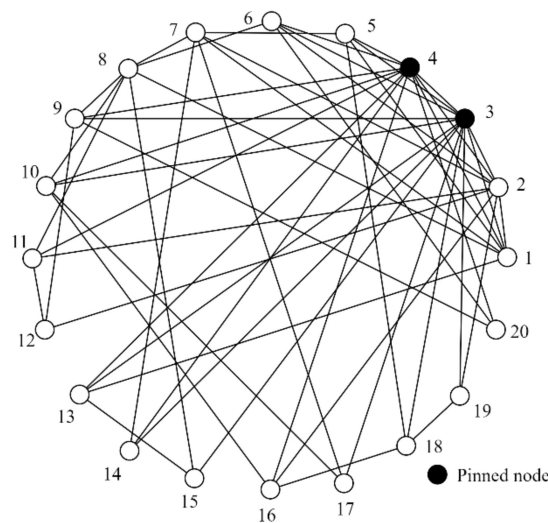


Figure 2. 20-node network used in the numerical simulation.

Using a sample time $T = 0.001$, the node self-dynamics are:

$$f_i(x_i(k)) = \begin{bmatrix} x_{i1}(k) + T[a_1x_{i2}(k) - a_1x_{i1}(k)] \\ x_{i2}(k) + T[c_1x_{i1}(k) - x_{i1}(k)x_{i3}(k) - x_{i2}(k)] \\ x_{i3}(k) + T[x_{i1}(k)x_{i2}(k) - b_1x_{i3}(k)] \end{bmatrix}, \quad (20)$$

which is a discretized version of the Lorenz model with constants $a_1 = 10$, $b_1 = \frac{8}{3}$, and $c_1 = 28$, as in [21], for $i = 1, 6, 11, 16$. Additionally, we consider:

$$f_i(x_i(k)) = \begin{bmatrix} x_{i1}(k) + T[a_2x_{i2}(k) - a_2x_{i1}(k)] \\ x_{i2}(k) + T[(c_2 - a_2 - x_{i3}(k))x_{i1}(k) - c_2x_{i2}(k)] \\ x_{i3}(k) + T[x_{i1}(k)x_{i2}(k) - b_2x_{i3}(k)] \end{bmatrix}, \quad (21)$$

$$\hat{A}_{12} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}. \tag{29}$$

Matrix B is an identity matrix, whose dimensions are equal to the state order.

Based on (6), we implement the next RHONN for identification of the pinned nodes state elements:

$$\hat{x}_{i1}(k + 1) = w_{i11}(k)\varphi(x_{i2}(k)) + w_{i21}(k)\varphi(x_{i1}(k)) + w_{i31}(k)\varphi(x_{i2}(k))\varphi(x_{i3}(k)), \tag{30}$$

$$\hat{x}_{i2}(k + 1) = w_{i12}(k)\varphi(x_{i1}(k)) + w_{i22}(k)\varphi(x_{i1}(k))\varphi(x_{i3}(k)) + w_{i32}(k)\varphi(x_{i2}(k)), \tag{31}$$

$$\hat{x}_{i3}(k + 1) = w_{i13}(k)\varphi(x_{i1}(k))\varphi(x_{i2}(k)) + w_{i23}(k)\varphi(x_{i3}(k)) + w_{i33}(k)\varphi(x_{i2}(k)), \tag{32}$$

with

$$\varphi(x_{ij}(k)) = 0.1 \tanh(0.05x_{ij}(k)). \tag{33}$$

The weights are obtained using Equations (7)–(9), with:

$$P_i(0) = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \tag{34}$$

$$H_{i1}^\top(k) = \begin{bmatrix} \varphi(x_{i2}(k)) \\ \varphi(x_{i1}(k)) \\ \varphi(x_{i2}(k))\varphi(x_{i3}(k)) \end{bmatrix}, \tag{35}$$

$$H_{i2}^\top(k) = \begin{bmatrix} \varphi(x_{i1}(k)) \\ \varphi(x_{i1}(k))\varphi(x_{i3}(k)) \\ \varphi(x_{i2}(k)) \end{bmatrix}, \tag{36}$$

$$H_{i3}^\top(k) = \begin{bmatrix} \varphi(x_{i1}(k))\varphi(x_{i2}(k)) \\ \varphi(x_{i3}(k)) \\ \varphi(x_{i2}(k)) \end{bmatrix}, \tag{37}$$

$$R_i(k) = R_i = 10, \tag{38}$$

$$Q_i(k) = Q_i = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}. \tag{39}$$

By doing the analysis described in Section 3, linearizing around equilibrium point $\bar{X} = 0$, for the uncontrolled system we get a maximum eigenvalue $\alpha_{k \neq t_k} = 1.0326$, and by pinning nodes 3 and 4 with a control gain $\kappa_i(k) = 500T$ we get a maximum eigenvalue of $\alpha_{k=t_k} = 0.9620$, setting the time of the control impulse as $t_k = 2nT$, for $n = 1, 2, \dots, \infty$, we can stabilize the system at state zero. Additionally, we replace the node state with the identified state for the pinned nodes as stated in (19). The simulation follows the next outline, at the beginning of the simulation the nodes are not connected to each other and they are also not controlled, at $Tk = 2$ the nodes are connected according to (26), and at $Tk = 4$ pinned nodes 3 and 4 are controlled as previously described.

Figure 3 shows the results of the numerical simulation done in MATLAB[®]. As can be seen in Figure 3, the proposed control scheme drives all of the network node state variables to zero. The control impulses can cause oscillation, being more noticeable for $x_{i3}(k)$ at $Tk = 4$, nonetheless, the stabilization goal is achieved.

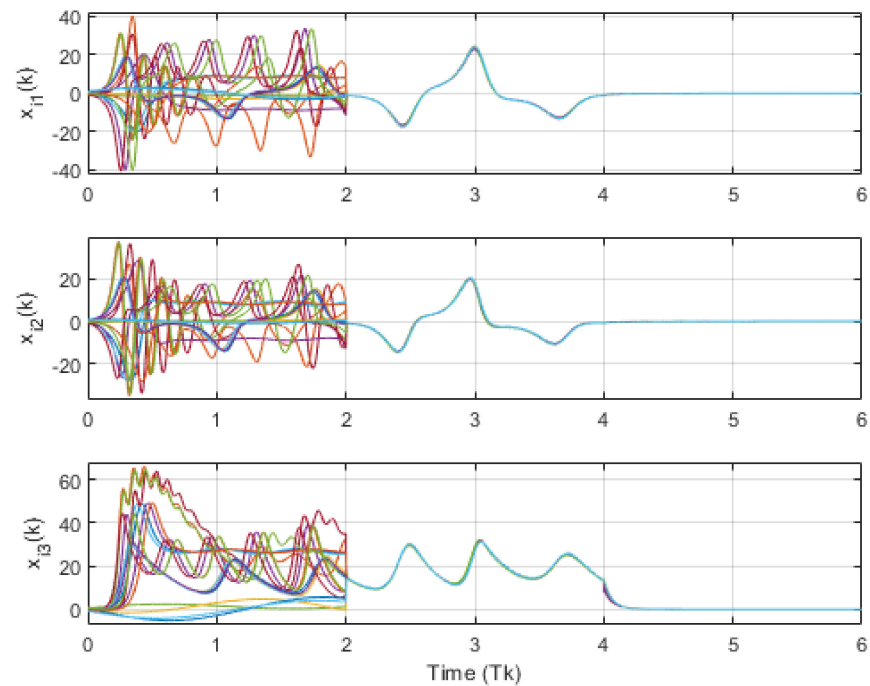


Figure 3. Complex network state variables where $x_{ij}(k)$ for $i = 1, \dots, N$ and $j = 1, 2, 3$ is the state variable of the i -th node of dimension 3.

5. Conclusions

The linearization approach proves to be a viable solution for the pinned control of complex networks as illustrated via simulations. The matrices resulting from such linearization make it easier to select the respective control gains. The neural network identifier for the pinned nodes dynamics is a powerful tool as illustrated by the included simulations; the same identifier structure is used for all the pinned nodes.

Different meaningful applications may be controlled using the proposed approach, such as robotic and biomedical systems.

Future work may center on using the passivity degree for the node dynamics as the V-stability approach [12,26] and avoid the use of linearized dynamics. Furthermore, the development of analysis for complex networks with time-varying connections is also an important topic for future research.

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