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Article

Methods of Ensuring Invariance with Respect to External Disturbances: Overview and New Advances

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Abstract: In this paper, we carry out a demonstration and comparative analysis of known methods of the synthesis of various control laws ensuring the invariance of the output (controlled) variable with respect to external disturbances under various assumptions about their type and channels of acting on the control plant. Methods of the synthesis are presented on the example of a third-order nonlinear system with single input and single output (SISO-systems), dynamic feedback synthesis is presented at a descriptive level and the focus is on procedures of static feedback synthesis. For the systems in which the matching conditions are not satisfied, it is concluded that it is expedient to introduce smooth and bounded nonlinear local feedbacks. Within the framework of the block control principle, we developed an iterative procedure of synthesis of S-shaped sigmoid feedbacks for such systems. Nonlinear local feedbacks ensure stabilization of the output variable with the given accuracy and settling time as in a system with traditionally used linear local feedbacks with high gains. However, in contrast to it, sigmoid functions do not lead to a large overshoot of state variables and control actions.

Keywords: external disturbances; invariance; block control principle; decomposition; high-gain factors; sliding mode control; sigmoid function

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1. Introduction

The basic issue of automatic control theory is the tracking problem, which consists in a convergence of the output variables to the reference admissible signals with the given performance of the transient and steady-state processes. The main efforts of researchers are aimed at solving this problem for the control plants, operating under the action of external uncontrolled disturbances. The methods of the synthesis of invariant systems used at the present stage are quite diverse. However, their effectiveness and applicability depend on many factors. Firstly, our goal is to systematize the existing methods of disturbance suppressing and compensating, to formalize the requirements on the degree of certainty of the control plant, at which it is advisable to use one or another approach. We also present the methods of synthesizing the corresponding control laws. The results of the survey are presented in Sections 2 and 3. Secondly, in Section 4 we propose a new, more universal approach to the synthesis of invariant systems with nonlinear feedback, in which the advantages of classical methods are concentrated. Moreover, this approach gives an effective result in cases where classical methods are not applicable.

To strengthen the methodological component of the presented material, we will consider all the stated approaches specifically on the example of a single-channel nonlinear minimum-phase system of the third order operating under the action of external uncontrolled disturbances. The given synthesis procedures for a third-order system fully describe all the features of the presented methods. Therefore, the algorithms can be easily extended to similar systems of a higher order. In this sense, without loss of generality, the considered control plant model can be interpreted as one of the subsystems of external dynamics equations of a multichannel system [1]

For the sake of presentation simplicity, let us suppose that the mathematical model of control plant has a relative degree of three and is representable in the following canonical input-output form:

$$\begin{aligned} \dot{x}_1 &= x_2 + \eta_1(t), \\ \dot{x}_2 &= x_3 + \eta_2(t), \\ \dot{x}_3 &= f(x) + b(x)u + \eta_3(t), \end{aligned} \tag{1}$$

where $x = (x_1,\, x_2,\, x_3)^{\mathrm{T}} \in X \subset R^3$ is measured state vector, X is open bounded region; $x_1 \in R$ is controlled variable (output), $u \in R$ is control action (input); $b(x) \neq 0,\ x \in X$ is the structural requirement, needed for system controllability. In the system (1) $\eta_i(t)$ are unknown functions of time that depend on external deterministic disturbances and other uncertainties in the description of the control plant model, which are bounded in modulus by known constants:

$$|\eta_i(t)| \leq H_i = \text{const} > 0,\ t \geq 0,\ \ i = \overline{1,3}. \tag{2}$$

The assumptions about the smoothness/non-smoothness of these functions, as well as the requirements of definiteness of $f(x),\ \ b(x)$ will be refined further.

Note that the output variable $x_1(t)$ can represent a tracking error which is the residual between the controlled variable and the given signal. We can assume that the analytical form of the given signal is not known, there are only its measured current values. Then its derivative is assumed to be an unknown bounded function that is additively included in $\eta_1(t)$.

It should be understood that in the presence of persistent disturbances $\eta_{1,2}(t)$, stabilization of all state variables of the system (1) is not possible for any control law. In a closed system, the variables $x_2(t)$ and $x_3(t)$ will have to describe the external actions $\eta_1(t)$ and $\eta_2(t)$ correspondingly. Therefore, for the system (1), the problem of feedback synthesis, ensured stabilization of only the output variable $x_1(t)$, is posed, which in the general case can be achieved with some accuracy,

$$|x_1(t)| \leq \Delta_1,\ \ t \geq t_1. \tag{3}$$

Further, it is assumed that the value $\Delta_1 > 0$ is given. The settling time $t_1 > 0$ depends on the initial conditions. In addition, the requirement on the given settling time often leads to cumbersome constructions and conservative estimates on the regulator parameters selection. Therefore, in the review section, we consider sufficient conditions for solving the posed problem (3) without the given settling time. A complete solution of the problem (3) will be given in the presentation of the author's method.

Then, for the system (1) the known and new methods of solving the posed problem are considered under various assumptions. The article is structured as follows. In Section 2, we consider a particular case of system (1), when an external disturbance acts on the same channel as the control (matched disturbance). The methods of synthesis and the results of a comparative analysis of the following approaches of solving the problem (3) are presented:

- Dynamic feedback and disturbance compensation by using its estimate in combined control;
- Static feedback and complete suppression of disturbance using discontinuous controls and organizing a sliding mode;
- Static feedback and suppression of disturbance with a given accuracy using linear control with high-gain factors;
- Static feedback and suppression of disturbance with a given accuracy using piecewise linear continuous control.

In Section 3, we deal with the system (1) with unmatched disturbances. The main attention is paid to the case when external disturbances are not smooth. For the solution of the posed problem (3), a standard procedure of block synthesis of linear local feedbacks with high-gain factors is presented. The advantages and disadvantages of this method are described, and a conclusion about the advisability of introducing smooth and bounded nonlinear feedbacks in practical applications is made.

In Section 4 a new approach developed by the authors and implemented in practical applications is presented. Sufficient conditions of the posed problem (3) solution for the given settling time are formalized, and a constructive procedure of block synthesis of nonlinear sigmoid local feedbacks is developed. In the conclusion, the prospect for the further development of the results presented in Section 4 is indicated.

2. Feedback Synthesis Methods in a System with a Matched Disturbance

The most developed case in the automatic control theory means that functions with parametric uncertainties and external disturbances are affine and act in the control space. In this case, the disturbances are said to be matched with the control and the matching conditions are satisfied. For example, for a linear system

$$\dot{x} = Ax + Bu + Q\eta(t)$$

matching conditions have a form [1,2]

$$\mathrm{Im}Q \subset \mathrm{Im}B \Leftrightarrow \mathrm{rank}B = \mathrm{rank}(B\ \ Q).$$

This means that the columns of the matrix Q are a linear combination of the columns of the matrix B, therefore, the original system can be represented as

$$\dot{x} = Ax + B(u + \Lambda\eta), Q = B\Lambda.$$

For the system (1), the matching conditions take a form

$$\eta_i(t) \equiv 0,\ t \geq 0,\ i = 1, 2 \tag{4}$$

Thus, in this section, we consider a special case of the system (1) and (4)

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = x_3, \\ &\dot{x}_3 = f(x) + b(x)u + \eta_3(t), \end{aligned}$$

where the requirements on the smoothness of the functions $f(x)$, $\eta_3(t)$ are generally not imposed.

Note that if $x_1(t)$ is a tracking error, then $x_2(t)$ and $x_3(t)$ are the first and second derivatives of the tracking error, which depend on the first and second derivatives of a given signal and are supposed to be known functions of time. Uncertainty is allowed only for the third derivative of the given signal, which is bounded and additively included in $\eta_3(t)$.

In contrast to the general case, in Systems (1) and (4) with matched disturbance it is possible to ensure the stabilization of all state variables using:

(i) Dynamic feedback and disturbance compensation;
(ii) Static feedback and disturbance suppression.

According to the first approach, firstly, the complete definiteness of the factor $b(x)$ before control is required. Secondly, we need to estimate the unknown disturbance $\eta_3(t)$ using any method to ensure asymptotically decreasing of the estimation error $\Delta\eta(t) = \eta_3(t) - \hat{\eta}_3(t)$ or its convergence to some small vicinity of zero rather quickly,

$$\lim_{t\to+\infty} \Delta\eta(t) = 0 \text{ or } |\Delta\eta(t)| \leq \delta,\ t \geq t_0,\ 0 < t_0 < t_1.$$

The obtained estimate $\hat{\eta}_3(t)$ is used for the synthesis of combined control

$$u = -(\phi(x) + \hat{\eta}_3(t))/b(x),$$

where $\phi(x)$ is a stabilizing component. If the function $f(x)$ is complete defined, then we can linearize the closed system by feedback

$$u = -\frac{1}{b(x)}\left(f(x) + \hat{\eta}_3(t) + \sum_{i=1}^{3} c_i x_i\right), \quad (5)$$

where $c_i > 0$ are the coefficients of stable polynomial $\lambda^3 + c_3\lambda^2 + c_2\lambda + c_1$.

The closed system (1), (4) and (5) has a form

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -c_1x_1 - c_2x_2 - c_3x_3 + \Delta\eta(t), \end{aligned}$$

where, in the general case, the given control accuracy (3) is ensured. Particularly, when estimate error decreases, an asymptotic stabilization of all state vector and, hence, the output variable

$$\lim_{t\to+\infty} x_1(t) = 0, \quad (6)$$

is occurred. In both cases by the selection of $c_i > 0$ we can ensure the required characteristics of the transient process of the output variable.

The standard approach of obtaining an estimate of an external disturbance $\hat{\eta}_3(t)$ is to expand the state space using a dynamic model, simulating the action of external disturbance, and construction of extended observer [3–5]. In the case of parametric uncertainty of the control plant model, the identification and adaptation algorithms are additionally used to estimate the unknown parameters [6–8].

However, the implementation of these approaches will lead to large estimation errors if the parameters and disturbances vary significantly during the operation of the control plant, and the used model does not describe these changes adequately. On the other side, taking into account all possible variations of external disturbances will lead to an unacceptable expansion of the dynamic model, a significant complication of the controller, and an increase in computing time of the control signal. An alternative to introducing a model of external influences is the construction of an observer based on the model of the control plant, which allows to obtain the estimates of unknown inputs without their dynamical model under certain conditions [9–12].

The second approach of invariance ensuring does not require the external disturbance estimation and consists in it suppressing by discontinuous controls with the organization of sliding modes or continuous feedbacks with high-gain factors. As a rule, these are linear controls.

To organize the sliding mode in the system (1) and (4), it is necessary to specify the switching surface (plane)

$$s = c_1x_1 + c_2x_2 + x_3,$$

where $c_{1,2} > 0$ are the coefficients of the stable polynomial $\lambda^2 + c_2\lambda + c_1$, and introduce the discontinuous control law

$$u = -M\mathrm{sign}(b(x))\mathrm{sign}(s),\ \mathrm{sign}(b(x)) = \mathrm{const},$$

where $M = \mathrm{const} > 0$ is the amplitude of discontinuous control, $\mathrm{sign}(s)$ is the sign function

$$\mathrm{sign}(s) = \left[\begin{array}{l} -1,\ s < 0, \\ +1,\ s > 0, \end{array}\right.$$

which value is undefined when $s = 0$, but it bounded on interval $[-1; 1]$.

Within the framework of this method, complete certainty of $f(x)$, $b(x)$ is not required, but the boundaries of their varying are assumed to be known

$$|f(x(t))| \le F, \\ 0 < b_{\min} \le |b(x)| \le b_{\max}, \ x \in X, \ t \ge 0 \quad (7)$$

A sufficient condition of sliding mode occurrence on the plane $s = 0$ has the form of inequality $s\dot{s} < 0$ [12–14], where

$$\dot{s} = c_2 s - c_2 c_1 x_1 + (c_1 - c_2^2) x_2 + f(x) - M|b(x)|\text{sign}(s) + \eta_3(t), \\ \left| c_2 s(t) - c_2 c_1 x_1(t) + (c_1 - c_2^2) x_2(t) \right| \le C, \ t \ge 0.$$

They are satisfied when we select amplitude from the inequality

$$M > (C + F + H_3)/b_{\min}. \quad (8)$$

When determining the upper estimate C of the admissible region of initial conditions $|x(0)| \le X_0$, it is necessary to estimate the region of variation of closed system variables

$$\dot{x}_1 = x_2, \\ \dot{x}_2 = -c_1 x_1 - c_2 x_2 + s$$

with respect $|s(t)| \le |s(0)|, \ t \ge 0$.

When (8) is valid, the requirement $s\dot{s} \le |s|(C + F + H_3 - M b_{\min}) < 0$ is satisfied, and the sliding mode arises on the plane $s = 0$ in a finite time $t > t_0$, $0 < t_0 < t_1$. In the sliding mode, the dynamic order of the closed system decreases

$$\dot{x}_1 = x_2, \\ \dot{x}_2 = -c_1 x_1 - c_2 x_2, \ s(t) = 0, \ t \ge t_0,$$

and the stability of the accepted polynomial implies asymptotic stabilization of the output variable (6).

Thus, according to this method, the synthesis problem is divided into two successively solved subproblems of lower dimension:

(i) The selection of switching plane parameters $c_{1,2} > 0$ at which the reduced second-order system is stable;
(ii) Selection of the amplitude of discontinuous control (8), at which stabilization of the virtual elementary system of the first order is ensured. This decomposition simplifies the synthesis of a controller for the multidimensional system with vector control. The main advantage of the method is that motion in the sliding mode does not depend on the operator of the control plan, external matched disturbances and is determined by the selection of the switching surface. The disadvantages include the need to calculate the upper estimate C in systems with a constant amplitude of discontinuous control, if the factor $b(x)$ before control contains undefined parameters. Such estimates are always conservative and lead to excessive consumption of control resources in a sliding mode.

$$\dot{s} = c_2 s - c_2 c_1 x_1 + (c_1 - c_2^2) x_2 + f(x) - M|b(x)|\text{sign}(s) + \eta_3(t)$$

Note that the use of discontinuous controls is natural in the presence of electrical inertia-less actuators that function exclusively in the key mode. In this case, the implementation of constant amplitude is a standard technical solution. Now let us consider systems in which there are no electrical actuators and only continuous control is permissible. Another method, based on disturbance suppression, is to use linear controls with high-gain factors [14–16]. For system (1) and (4) we introduce linear feedback instead of discontinuous control

$$u = -k\text{sign}(b(x))s,\ \text{sign}(b(x)) = \text{const},$$

where $k = \text{onst} > 0$ is a high-gain factor inversely proportional to the desired accuracy of suppression of matched disturbances and uncertainties: $|s(t)| \le \Delta$, $t \ge t_0$, $0 < t_0 < t_1$. With respect,

$$\begin{array}{l} \dot{s} = c_2 s - c_2 c_1 x_1 + (c_1 - c_2^2)x_2 + f(x) - k|b(x)|s + \eta_3(t), \\ \left|(c_1 - c_2^2)x_2(t) - c_2 c_1 x_1(t)\right| \le C_1,\ t \ge 0 \end{array}$$

the selection of the high-gain factor from inequality

$$k > \frac{C_1 + F + H_3}{\Delta b_{\min}} + \frac{c_2}{b_{\min}} \tag{9}$$

will ensure that sufficient condition $s\dot{s} \le |s|(C_1 + F + H_3 - (kb_{\min} - c_2)|s|) < 0$ is satisfied outer the region $|s(t)| \le \Delta$, in which the variable $s(t)$ converges in a finite time. When $t \ge t_0$, the closed system can be represented in the form

$$\begin{array}{l} \dot{x}_1 = x_2, \\ \dot{x}_2 = -c_1 x_1 - c_2 x_2 + s,\ |s(t)| \le \Delta, \end{array} \tag{10}$$

which ensures the control goal (3), where Δ_1 depend on Δ and accepted $c_{1,2}$.

Note, if we exactly know $f(x),\ b(x)$, the combined control laws can be formed, which resources will be used only on the suppression of external disturbances. Selected based on the virtual system $\dot{s} = c_1 x_2 + c_2 x_3 + f(x) + b(x)u + \eta_3(t)$ combine control law

$$u = -(c_1 x_2 + c_2 x_3 + f(x) + M\text{sign}(s)\ [\text{or } ks])/b(x)$$

leads to the closed system

$$\begin{array}{l} \dot{x}_1 = x_2, \\ \dot{x}_2 = -c_1 x_1 - c_2 x_2 + s, \\ \dot{s} = -M\text{sign}(s)\ \ [\text{or } \ -ks] + \eta_3(t) \end{array}$$

and when $M > H_3$ [or $k > H_3/\Delta$] ensure the fulfillment (6) [or (3)].

The main restriction of the synthesis method of systems with high-gain factors is that it is unrealizable in practical applications. To satisfy the constraints on control actions, the continuous piecewise linear controls in the form of saturation functions are used [17,18], which are the hybrid of linear and discontinuous controls. These functions are bounded, and they tend to a sign function with the increasing of high-gain factors. Consequently, in the closed system saturation functions ensure similar properties as in the systems, operating in sliding mode, and with some accuracy.

For system (1) and (4) let us consider feedback in the form of saturation function

$$u = -M\text{sign}(b(x))\text{sat}(\bar{k}s),\ \text{sign}(b(x)) = \text{const},$$

where $M = \text{const} > 0$ is the amplitude, $\bar{k} = \text{const} > 0$ is the high-gain factor

$$M\text{sat}(\bar{k}s) = \left[\begin{array}{l} M\text{sign}(\bar{k}s),\ \ |s| > 1/\bar{k}, \\ M\bar{k}s,\ \ |s| \le 1/\bar{k}. \end{array}\right.$$

Amplitude is selected so as in a system with discontinuous control (8), that ensures $|s(t)| \le 1/\bar{k} \le \Delta,\ \ t > t_0$, when $|s(0)| > 1/\bar{k}$. Selection of $\bar{k} \ge 1/\Delta$ ensures the desired stabilization accuracy, and as a result, the fulfillment of (10) and (3).

Significantly, that in contrast to a discontinuous control law with constant amplitude, which value does not vary in modulus during all control process, the values of saturation control automatically decrease in modulus in the steady-state mode (this fact is also valid for linear continues control). It occurs due to the stabilization of state variables, and when $t > t_1$, the control signal describes only external disturbance $\eta_3(t)$ with small not-decreasing components.

Thus, the combined control makes it possible to compensate for external matched disturbances, but for this, it is necessary to obtain their estimates and identify the unknown parameters of the system. In the case when the combined control cannot be realized, it remains to use the control aimed at suppressing external disturbances and model uncertainties. The selection of the type of control depends on the properties of the system and the existing design requirements on the smoothness and boundness.

3. Block Synthesis of Linear Local Feedbacks in System with Unmatched Disturbances

The most difficult are the control plants with unmatched disturbances (when conditions (4) are not satisfied), which cannot be compensated or suppressed by true control. In the tracking problem, these disturbances also include the derivatives of the reference signals. In addition, the problem of ensuring invariance with respect to disturbances is posed only for controlled outputs (tracking errors), since the remaining variables have to describe the corresponding external influences. According to the classical approach of the synthesis of a tracking system under the assumption of the smoothness of external influences, the state space is expanded due to the generators of reference and external influences, as well as the corresponding dynamic observers and identifiers of parameters [1]. In this case, the dynamic order of the closed system can increase by a factor of five or more times (in comparison with the dimension of the control plant model) if the external disturbances (and the corresponding autonomous models) vary significantly during the control process. If it is possible to formulate a model that accurately describes the dynamics of external disturbances, then asymptotic stabilization of tracking errors is theoretically achieved by expanding the state space.

Another approach is to represent the model of the control plant in the canonical or block input-output form, with the differentiation of external signals. In the process of obtaining this form, mixed variables are generated, which are the functions of state variables with additive external influences and their derivatives [19,20]. For system (1) under the assumption of differentiability of external disturbances $\eta_{1,2}(t)$, the canonical system in mixed variables has the form

$$\begin{aligned} \dot{\overline{x}}_1 &= \overline{x}_2, \\ \dot{\overline{x}}_2 &= \overline{x}_3, \\ \dot{\overline{x}}_3 &= f(x) + b(x)u + \overline{\eta}_3(t), \end{aligned} \tag{11}$$

where

$$\overline{x}_1 = x_1,\ \overline{x}_2 = x_2 + \eta_1(t),\ \overline{x}_3 = x_3 + \eta_2(t) + \dot{\eta}_1(t),\ \overline{\eta}_3(t) = \eta_3(t) + \dot{\eta}_2(t) + \ddot{\eta}_1(t).$$

In the last equation of the system (11), the initial variables x are left in the arguments of the functions $f(x)$, $b(x)$ for the convenience of synthesis. Structurally, system (11) repeats system (1), (4) with matched disturbances, since all uncertainties are concentrated in the control space and are subject to compensation or suppression using the control laws presented in Section 2.

The feature of this approach of ensuring invariance is that the problem of evaluating external influences separately is not considered, the autonomous models that generate them are not introduced into the constructions. Assuming that the output variable $\overline{x}_1 = x_1$, is measured, an observer is constructed based on the transformed system (11) with an indefinite input. Due to the suppression function of corrective action of the observer, it gives an estimate of mixed variables and uncertainties to form feedback and leads to an increase in the dynamic order of closed system by no more than twice. As a rule, in this case, ε-invariance of the output variable with respect to external unmatched disturbances is achieved.

However, the mentioned approaches are not applicable in the case when external unmatched disturbances and other model uncertainties are not smooth enough and cannot be differentiated. An example is shock loads and dry friction forces when controlling mechanical objects, taking into account the dynamics of actuators [21–25]. In the particular case, when a non-smooth disturbance is separated from the true control by one integrator, it can be suppressed using "vortex" control with continuous and discontinuous components. The result is achieved due to the organization of an oscillatory transient process in the system, in which part of the state variables automatically compensates for the influence of unknown terms [26].

In the general case, when external unmatched disturbances act on the control plant, the estimation, and compensation or suppression of which are not possible by true control, it remains to use the possibilities of disturbance suppression using local feedbacks. The methodological basis for the implementation of this approach is the decomposition methods and the block control principle [16,27]. According to this approach, using a non-degenerate change of variables, the equations of external dynamics are reduced to a block input-output form with an affine occurrence of fictitious and true controls. It consists of elementary blocks, in each of which the dimension of the controlled variables is equal to the rank of the matrix before the fictitious controls, which are the variables of the next block. For the general case of a controllable minimum-phase nonlinear system of the n-th order with affine external influences η, the block form is the following [20]:

$$\begin{aligned}
&\dot{x}_1 = f_1(x_1) + B_1(x_1)x_2 + Q_1(x_1)\eta; \\
&\dot{x}_i = f_i(x_1, \ldots, x_i) + B_i(x_1, \ldots, x_i)x_{i+1} + Q_i(x_1, \ldots, x_i)\eta, \; i = \overline{2, r-1}; \\
&\dot{x}_r = f_r(x_1, \ldots, x_r) + B_r(x_1, \ldots, x_r)u + Q_r(x_1, \ldots, x_r)\eta,
\end{aligned}$$

where $B_i \in R^{p_i \times p_{i+1}}$, $i = \overline{1, r-1}$, $B_r \in R^{p_r \times p_r}$, $Q_i \in R^{p_i \times 1}$, $\dim x_i = \operatorname{rank} B_i = p_i$, $i = \overline{1, r}$, $p_1 + p_2 + \ldots + p_r = n$.

Sequentially (from top to bottom) formed stabilizing local feedbacks in each block are provided by the selection of true control. When a block form is obtained, external influences are not differentiated and do not participate in transformations, but with a block organization, they become matched with fictitious controls. Then, with an appropriate selection of fictitious controls, it is possible to stabilize the output variables with some accuracy.

Let us explain the essence of the block control principle using the example of system (1), which, as we see, is a special case of the block form and consists of three elementary blocks of the first order. In the first and second equations, the variables x_2 and x_3, respectively, are treated as fictitious controls, with which the bounded disturbances η_1 and η_2 are matched, respectively. The smoothness requirement is not imposed on external disturbances. The question arises about the selection of the form of stabilizing functions in fictitious and true controls, that would ensure the invariance of the output variable with respect to external disturbances by suppressing them.

As shown above, the classical methods of suppressing external and parametric bounded disturbances acting in the control space are: (1) continuous linear feedbacks with high-gain factors; (2) discontinuous controls bounded in modulus with the organizations of sliding modes. In addition, only controls of the first type (due to their smoothness) can be used to form local feedbacks. We emphasize once again that with the help of linear local feedbacks in a system with unmatched disturbances, it is possible to ensure stabilization of the controlled variable only with certain accuracy (3).

For system (1), let us consider the standard step-by-step procedure of block synthesis of linear local feedbacks with high-gain factors under the action of unmatched bounded disturbances [16]. It consists of the following stages: (1) introduction of local feedbacks (stabilizing fictitious controls) by non-degenerate change of variables of the original system (1) to residuals between real and adopted fictitious controls; (2) the selection of the control law; (3) setting the parameters of the feedback that meets the control goal. We represent the first stage in the form of the following procedure, which for system (1) consists of three steps and is similarly extended to systems of any order presented in the block form of controllability.

Procedure 1 : Non-degenerate transformation with the introduction of linear local feedbacks.

Step 1. In the first equation of system (1), we introduce linear local feedback $x_2^* = -k_1 x_1$, $k_1 = \text{const} > 0$ and the residual between the actual and the selected fictitious control

$$e_2 = x_2 - x_2^* = x_2 + k_1 x_1. \tag{12}$$

Taking into account the notation $e_1 = x_1$ and (12), the first equation of System (1) takes the form

$$\dot{e}_1 = -k_1 e_1 + e_2 + \eta_1. \tag{13}$$

Step 2. Let us write the differential equation for the residual (12) by (1) and (13)

$$\dot{e}_2 = x_3 + \eta_2 + k_1 \dot{e}_1 = -k_1^2 e_1 + k_1 e_2 + x_3 + \eta_2 + k_1 \eta_1,$$

where we form a combined fictitious control with a linear stabilizing component $x_3^* = k_1^2 e_1 - k_2 e_2$, $k_2 = \text{const}$, $k_2 > k_1$ and a residual between the actual and the selected fictitious control

$$e_3 = x_3 - x_3^* = x_3 - k_1^2 e_1 + k_2 e_2. \tag{14}$$

With respect (14), the second equation of the system (1) takes the form

$$\dot{e}_2 = -(k_2 - k_1) e_2 + e_3 + \eta_2 + k_1 \eta_1. \tag{15}$$

Step 3. Let us write the differential equation for the residual (14) by (1) and (15)

$$\begin{aligned} \dot{e}_3 &= f(x) + b(x)u + \eta_3(t) - k_1^2 \dot{e}_1 + k_2 \dot{e}_2 = k_1^3 e_1 - (k_1^2 + k_2^2 - k_2 k_1) e_2 + k_2 e_3 + \\ &+ f(x) + b(x)u + \eta_3 + k_2 \eta_2 + (k_2 k_1 - k_1^2) \eta_1. \end{aligned} \tag{16}$$

The procedure is over.

Thus, we have obtained system (13), (15) and (16) using a nondegenerate linear transformation of the system (1). The final transformation matrix is obtained as a result of the product of the transformation matrices performed at the first (12) and second (14) steps of the procedure (in the indicated order)

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -k_1^2 & k_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ k_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ k_1 & 1 & 0 \\ k_1 k_2 - k_1^2 & k_2 & 1 \end{pmatrix}, \ \det P \neq 0$$

For simplicity of presentation, we will consider the case of complete definiteness of functions $f(x)$, $b(x)$, which allows us to accept a combined true control in the form

$$u = -(k_1^3 e_1 - (k_1^2 + k_2^2 - k_2 k_1) e_2 + k_2 e_3 + f(x) + \varphi(e_3))/b(x). \tag{17}$$

The closed system (13) and (15)–(17) takes the form

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 + e_2 + \eta_1, \\ \dot{e}_2 &= -(k_2 - k_1) e_2 + e_3 + \eta_2 + k_1 \eta_1, \\ \dot{e}_3 &= -\phi(e_3) + \eta_3 + k_2 \eta_2 + (k_2 k_1 - k_1^2) \eta_1. \end{aligned} \tag{18}$$

The stabilizing component $\phi(e_3)$ of the control law (17) must ensure the suppression of the linear combination of disturbances $\eta_3 + k_2 \eta_2 + (k_2 k_1 - k_1^2) \eta_1$ and the stabilization of the variable e_3. For this, either discontinuous control or linear with high-gain factors or their piecewise-linear continuous hybrid in the form of a saturation function is applied, namely:

$$\begin{aligned}&(1)\ \phi(e_3) = M\mathrm{sign}(e_3);\\&(2)\ \phi(e_3) = k_3 e_3;\\&(3)\ \phi(e_3) = M\mathrm{sat}(\bar{k}_3 e_3),\end{aligned} \tag{19}$$

where $M, k_3, \bar{k}_3 = \mathrm{const} > 0$. Note that the control laws (19) are formed by the variable e_3 (14), which is a linear combination of the measured state variables of the original system (1).

As shown above, in the first case (19) when the amplitude is selected based on the inequality

$$M > H_3 + k_2 H_2 + (k_2 k_1 - k_1^2) H_1 \tag{20}$$

the sufficient condition $e_3 \dot{e}_3 < 0$ is satisfied. The sliding mode arises on the plane $e_3 = 0$ in a finite time $t > t_0 > 0$, and the dynamical order of the system is reduced. In the second case (19), a high-gain factor is selected taking into account the specified stabilization accuracy similarly to (9), namely:

$$k_3 > \frac{H_3 + k_2 H_2 + (k_2 k_1 - k_1^2) H_1}{\Delta_3}. \tag{21}$$

In the third case (19), the lower bounds of the parameter's selection of the piecewise linear control have the form (20) and $\bar{k}_3 > 1/\Delta_3$, $\Delta_3 > 0$. In both the second and third cases, the convergence of the variable to some neighborhood of zero is ensured

$$|e_3(t)| \le \Delta_3,\ t > t_0. \tag{22}$$

Using (22), let us consider the procedure of selection the high-gain factors $k_1 > 0$, $k_2 > 0$ based on the second Lyapunov method. We introduce a candidate on the Lyapunov function as the sum of two terms $V = V_1 + V_2$, $V_i = \frac{1}{2} e_i^2$, $i = 1, 2$, and estimate their derivatives by (2), (13) and (15):

$$\begin{aligned}&e_1 \dot{e}_1 \le |e_1|(|e_2| + H_1 - k_1 |e_1|),\\&e_2 \dot{e}_2 \le |e_2|(|e_3| + H_2 + k_1 H_1 - (k_2 - k_1)|e_2|).\end{aligned} \tag{23}$$

It follows from inequalities (23) that sufficient stability conditions $\dot{V} < 0$ are met if the high-gain factors satisfy the inequalities

$$\begin{aligned}&k_1 > \tfrac{H_1 + \Delta_2}{\Delta_1}, |e_3| \le \Delta_3,\ |e_2| \le \Delta_2,\ |e_1| > \Delta_1,\\&k_2 > \tfrac{H_2 + k_1 H_1 + \Delta_3}{\Delta_2} + k_1,\ |e_3| \le \Delta_3,\ |e_2| > \Delta_2.\end{aligned} \tag{24}$$

Thus, first, we set the desired accuracy of the stabilization Δ_i, $i = \overline{1, 3}$ of the virtual variables $e = (e_1, e_2, e_3)^{\mathrm{T}}$. Then, with a sequential (from top to bottom) selection of high-gain factors based on inequalities (24) and (21), the variables of the closed system (18) and (19) sequentially (from bottom to top) converge into the given neighborhoods of zero

$$|e_3(t)| \le \Delta_3 \Rightarrow |e_2(t)| \le \Delta_2 \Rightarrow |e_1(t)| \le \Delta_1, \tag{25}$$

and the control goal (3) is achieved. When selecting the high-gain factors, one should take into account that as the k_1 increases, the accuracy improves (3) (in the limited case $\Delta_1 \to 0$ when $k_1 \to +\infty$) and the settling time decreases. However, due to the unboundedness of linear controls, this leads to the well-known problem of large overshoot [28]. On the other hand, in practical applications control resources are always bounded, so there is an upper bound of the selection of $k_1 \le k_{1\max}$ and the corresponding minimum achievable tracking error $\Delta_{1,\min} \le \Delta_1$.

The bounded control in the form of a saturation function is not smooth. On the one hand, it is not an obstacle when these functions are used in corrective actions of observers of the state of systems with disturbances [20,29]. However, on the other hand, it narrows the possibilities of its application as fictitious controls in practical problems.

Summing up, we can conclude that for the universal formation of invariant local feedbacks and the practical realizability of control Algorithms, it is advisable to use smooth analogs of the saturation function. These include transcendental S-shaped functions: arctangent, hyperbolic tangent, logistic function, etc. The odd hyperbolic tangent $\text{th}(x) = 1 - 2/(\exp(2x) + 1)$ appears to be a constructive tool for the analysis and synthesis of nonlinear control. This bounded function depends on the exponent, its derivatives are also bounded everywhere and are recursively expressed through the antiderivative.

In this paper, a modification of the hyperbolic tangent, which is more convenient for constructions, is used in the form of a sigmoid function $\sigma(x) = -\text{th}(-x/2)$. Its properties and the corresponding synthesis procedure developed by the authors are presented in the next section and constitute the main result of this work.

4. Block Synthesis of Nonlinear Local Feedbacks in Systems with Unmatched Disturbances

Let us consider a smooth and bounded sigmoid function

$$\sigma(kx) = \frac{2}{1 + \exp(-kx)} - 1,\ k = \text{const} > 0,$$

which is defined on the whole number axis and has the following properties: $\sigma(-kx) = -\sigma(kx)$, $\sigma(kx) \underset{x \to 0}{\sim} kx/2$, $\sigma(kx) \underset{k \to +\infty}{\sim} \text{sign}(x)$. In its argument, a factor k is specially introduced, which plays a role of a high-gain factor in a small neighborhood of zero in further constructions. The derivative of the sigmoid function has a recursive form:

$$\sigma'(kx) = k(1 - \sigma^2(kx))/2 > 0,\ x \in R, \sigma'(-kx) = \sigma'(kx).$$

To simplify the analysis of a nonlinear sigmoid function, let us establish its analogy with a piecewise linear saturation function. Consider some neighborhood of zero with radius $\Delta > 0$. The following estimates

$$\begin{array}{l} \sigma(k\Delta) < |\sigma(kx)| < 1,\ 0 < \sigma'(kx) < \sigma'(k\Delta),\ |x| > \Delta; \\ \frac{\sigma(k\Delta)|x|}{\Delta} \le |\sigma(kx)| \le \sigma(k\Delta),\ \sigma'(k\Delta) \le \sigma'(kx) \le \sigma'(0) = \frac{k}{2},\ |x| \le \Delta \end{array} \quad (26)$$

are valid for the sigmoid function and its derivative in the indicated intervals. Inequalities (26) demonstrate that when $|x| > \Delta$ the sigmoid function is close to a constant, and when $|x| \le \Delta$ it is close to a linear function. To formalize the abscissa of the specified division, we introduce the parameter $c = \text{const} > 0$: $|x| = \Delta = c/k$, which is advisable to select from the interval

$$k\Delta = c \in [1.3; 3], \quad (27)$$

where ± 1.3 are the abscissas of the inflection points of the first derivative $\sigma'''(\pm 1.3) = 0$, and $\sigma(\pm 1.3) \approx \pm 0.57$, $\sigma'(\pm 1.3) \approx 0.34k$; ± 3 are the abscissas of the vertices of the sigmoid function, in which its curvature reaches its maximum, while $\sigma(\pm 3) \approx \pm 0.9$, $\sigma'(\pm 0.9) \approx 0.095k$ [19].

For the convenience of calculations, we take

$$c = 2.2;\ \sigma(c) \approx 0.8;\ \frac{1}{\sigma(c)} \approx 1.25;\ \sigma'(c) \approx 0.18k. \quad (28)$$

Using (28), estimates (26) take the following form:

$$\begin{array}{l} 0.8 < |\sigma(kx)| < 1,\ 0 < \sigma'(kx) < 0.18k,\ |x| > c/k,\ c = 2.2; \\ \frac{0.8k|x|}{c} = 0.36k|x| \le |\sigma(kx)| \le 0.8,\ 0.18k \le \sigma'(kx) \le \sigma'(0) = \frac{k}{2},\ |x| \le c/k. \end{array} \quad (29)$$

Let us explain the idea of using sigmoid feedback and the selection of its parameters in the problem of ensuring invariance using the example of an elementary system with external disturbance

$$\dot{x} = \eta(t) + u, \tag{30}$$

where $x \in R$ is the state variable, $\eta(t)$ is the external disturbance, which is described by a deterministic, unknown, but bounded function of a time. The requirement of smoothness is not imposed on it, it is sufficient that it be piecewise continuous. The problem of stabilizing system (30) with a given accuracy using the sigmoid control

$$u = -m\sigma(kx) \tag{31}$$

with a constant amplitude $m = \text{onst} > 0$ and high-gain factor $k = \text{const} > 0$ is posed.

Lemma 1. *If in system (30), (31) the external disturbance is bounded by a known constant* $|\eta(t)| \le H = \text{const} > 0$, $t \ge 0$, *then for any arbitrary small* $\Delta > 0$, $T > 0$ *and any initial values* $x(0)$ *from some bounded domain* $X_0 \ge |x(0)|$ *there are positive real numbers* $\bar{k}$ *u* $\overline{m}$, *such that for any* $k \ge \bar{k}$, $m \ge \overline{m}$, *the following inequality is valid*

$$|x(t)| \le \Delta, \ \ t \ge T. \tag{32}$$

Proof. Let us introduce the parametric dependence (27), then, with respect to (28) and (29), the following lower estimates are valid for control (31) on the indicated intervals. To analyze the stability of closed system (30) and (31), we use the second Lyapunov method. Let us introduce a candidate on the Lyapunov function $V = x^2/2$ and estimate its derivative on the indicated intervals taking into account (33).

$$|u(x)| = |m\sigma(kx)| \ge \left[\begin{array}{l} 0.8m, \ |x| > \Delta, \\ 0.8mk|x|/2.2, \ |x| \le \Delta \end{array} \right. \tag{33}$$

$$\dot{V} = x(\eta(t) - m\sigma(kx)) \le \left[\begin{array}{l} |x|(H - 0.8m), \ |x| > \Delta, \\ |x|(H - 0.8mk|x|/2.2), \ |x| \le \Delta. \end{array} \right. \tag{34}$$

It follows from (34) that the derivative of the Lyapunov function is negative if the feedback parameters satisfy the following conditions:

$$\begin{array}{l} 0.8m > H \Leftrightarrow m > 1.25H, \\ k > \frac{H}{0.8m} \cdot \frac{2.2}{\Delta}. \end{array} \tag{35}$$

The fulfillment of the first inequality (35) means that the state variable will converge into the region $|x| \le \Delta$ or will not leave it if it was there initially. In addition, the fulfillment of the second inequality guarantees stabilization with a given accuracy (32), namely:

$$|x| \le \frac{H}{0.8m}\Delta < \Delta.$$

Using $0 < H/(0.8m) < 1$, it is possible to simplify the lower bound for selection a high-gain factor in comparison with the second inequality (35) and take

$$k \ge \bar{k} = 2.2/\Delta. \tag{36}$$

In the general case $|x(0)| > \Delta$ to guarantee the achievement of the state variable of a given region in a given time $T > 0$, let us increase the lower bound of selection of amplitude. With respect to the estimate of the solution of system (30) and (31) on the interval $t \in [0; T]$

$$|x(t)| \le |x(0)| + (H - 0.8m)T \le \Delta. \tag{37}$$

we obtain

$$m \ge \overline{m} = 1.25\left(\frac{X_0 - \Delta}{T} + H\right), \ X_0 > \Delta. \tag{38}$$

Thus, we defined such $\bar{k}$ (36) and $\overline{m}$ (38) that for any $k \geq \bar{k}$, $m \geq \overline{m}$, the stabilization of the state variable with the given accuracy and for the given time (32) is ensured in the closed system (30) and (31). Lemma 1 is proved. □

As you can see, the sigmoid control, as well as the piecewise-linear saturation function, is bounded everywhere and contains two adjustable parameters. The selection of the amplitude provides the given time of convergence of the controlled variable to a certain neighborhood of zero, and the selection of a high-gain factor provides the radius of this area, i.e., the given stabilization accuracy. In a first-order system, the transient process is monotonic.

We use the results obtained in Lemma 1 to stabilize the output variable of system (1) taking into account (2) and (7) under the following assumptions: the requirements of smoothness of external disturbances are not imposed, the functions $f(x)$, $b(x)$ are not required to be completely defined, the sign of $b(x)$ is constant and known. In further constructions, we will take into account the given settling time (3), which is guaranteed for all initial values of the variables from the bounded admissible region

$$|x_1(0)| \leq X_1,\ |x_2(0)| \leq X_2,\ |x_3(0)| \leq X_3. \tag{39}$$

As a methodological basis of the synthesis procedure, we use the block control principle, demonstrated in Section 3 for the synthesis of linear feedbacks. Let us emphasize that the idea of the approach proposed below is similar to the backstepping [30]. The main differences of our approach are that it does not require smoothness of functions $f(x)$, $b(x)$, and $\eta_i(t)$, $i = \overline{1,3}$; we use static feedback, do not expand the state space, and do not aim to obtain estimates of the existing uncertainties. To avoid large overshoot, which is typical for linear feedbacks with high-gain factors, we will select stabilizing fictitious controls in the form of smooth and bounded sigmoid functions

$$x_i^* = -m_{i-1}\sigma(k_{i-1}e_{i-1}),\ k_{i-1} = \text{const} > 0,\ m_{i-1} = \text{const} > 0,\ i = 2,\ 3,$$

where e_2 and e_3 are the residuals between the variables x_2 and x_3, respectively, and the selected fictitious controls

$$e_i = x_i - x_i^* = x_i + m_{i-1}\sigma(k_{i-1}e_{i-1}),\ i = 2,\ 3, e_1 = x_1. \tag{40}$$

For uniformity, true control is also accepted as a sigmoid function

$$u = -\text{sign}(b)m_3\sigma(k_3e_3),\ k_3 = \text{const} > 0,\ m_3 = \text{const} > 0 \tag{41}$$

Note that to simplify the computational implementation, instead of (41), one can also use a continuous, bounded, but non-smooth saturation function or discontinuous control in systems with electric actuators as a true control.

Also note that, unlike Procedure 1 with linear transformations in changes of variables (40) and control law (41), we did not compensate the nonlinear components that do not depend on external disturbances in order not to complicate the control function.

Let us rewrite closed system (1), (41) with respect to residuals (40)

$$\begin{aligned} \dot{e}_1 &= -m_1\sigma(k_1e_1) + e_2 + \eta_1, \\ \dot{e}_2 &= -m_2\sigma(k_2e_2) + e_3 + \eta_2 + \Lambda_1, \\ \dot{e}_3 &= -|b(x)|m_3\sigma(k_3e_3) + f(x) + \eta_3 + \Lambda_2, \end{aligned} \tag{42}$$

where terms

$$\Lambda_i = m_i\frac{k_i(1-\sigma^2(k_ie_i))}{2}\dot{e}_i,\ i = 1,\ 2, \tag{43}$$

are the derivatives of the corresponding fictitious controls, which arise in the transition to the new coordinate basis (40).

There is no need to change the arguments of functions $b(x)$ and $f(x)$ in the last equation of transformed system (42) since constraints (7) are specified in terms of the variables of the original system (1), and the specific of these functions do not matter for the formation of control law (41).

We will perform feedback synthesis according to the block approach in terms of virtual system (42). The idea is that sigmoid fictitious and true controls introduced into each subsystem using non-degenerate change of variables (40) and feedback (41) serve to suppress external uncontrolled disturbances. This will ensure the stabilization of the residuals e_i, $i = \overline{1,3}$ with any given accuracy. By virtue of the inverse change of variables (40), namely, this means that in the closed system (1) with nonlinear control (41), which is realized in the form

$$u = -\text{sign}(b) m_3 \sigma(k_3(x_3 + m_2\sigma(k_2(x_2 + m_1\sigma(k_1 x_1))))) \,, \tag{44}$$

in the steady-state, the variables $x_2(t)$ and $x_3(t)$ describe external disturbances $\eta_1(t)$ and $\eta_2(t)$, accordingly. In addition, the stabilization accuracy of the output variables of both systems will be the same. Thus, the fulfillment of the objective condition in closed system (42) and (41)

$$|e_1(t)| \le \Delta_1, \, t \ge t_1, \tag{45}$$

is equivalent to solving the problem (3).

As shown in Section 3, the block approach in multidimensional systems consists in sequentially solving elementary synthesis problems in subsystems (blocks) similar to (30). However, only the last subsystem is directly regulated by the true control, and in the rest, the variables of the next block act as fictitious controls. As a consequence, in the general case of nonzero initial conditions only in the last block, a monotonic transient process is guaranteed.

Sufficient conditions of the existence of feedback parameters $m_i, k_i, \; i = 1, 2, 3$ that ensure the fulfillment of objective condition (45) in the system (42) are formulated in Lemma 2. In the process of constructive proof, a step-by-step procedure of adjusting the amplitudes of sigmoid controls was formalized, in which the decomposition principle is implemented [31,32].

Lemma 2. *Let us consider closed system (1), (44), presented in the form (42) using non-degenerate changes of variables. If conditions (2) and (7) are satisfied for this system, then for any initial values of variables from the bounded domain (39) and for any, arbitrarily small* $\Delta_1 > 0$, $t_1 > 0$, *there are real numbers* $\bar{k}_i > 0$, $i = \overline{1,3}$, $0 < \overline{m}_i < \overline{\overline{m}}_i$, $i = 1, 2$, $\overline{m}_3 > 0$, *such that for any* $k_i \ge \bar{k}_i$, $m_i : \; \overline{m}_i < m_i \le \overline{\overline{m}}_i$, *inequality (45) is satisfied.*

Constructive Proof. According to the ideology of the block approach, in the closed system (42) it is necessary to provide the following sequence of convergence of residuals:

$$|e_3| \le \Delta_3(\, t \ge t_3 > 0) \Rightarrow |e_2| \le \Delta_2(\, t \ge t_2 > t_3) \Rightarrow |e_1| \le \Delta_1(\, t \ge t_1 > t_2), \tag{46}$$

where $\Delta_1 > 0$, $t_1 > 0$ are the given (45), $\Delta_{2,3} > 0$ are assigned arbitrarily. The dependences $t_{2,3}$ on the initial conditions and accepted $\Delta_{2,3} > 0$ are established in the course of the proof.

Lemma 1 demonstrates the existence of $\bar{k}_i > 0$, $i = \overline{1,3}$ such as for any $k_i \ge \bar{k}_i$, $i = \overline{1,3}$ the desired radii $\Delta_{1,2,3} > 0$ (46) of neighborhoods of zero are guaranteed, at which the residuals converge in the indicated times (46). With respect (28) and similarly to (36), we fix the values of high-gain factors based on the inequalities

$$k_i^* \ge \bar{k}_i = 2.2/\Delta_i, \, i = \overline{1,3}. \tag{47}$$

In (47) and below, using the symbol $*$ in the superscript, we will denote specific accepted numerical values of the parameters.

Increasing the accepted values k_i^* leads to a decrease in the stabilization errors of residuals. The convergence of the residuals into the established areas in the specified time (46) is ensured by selection m_i, $i = \overline{1,3}$.

The stabilization of system (42) is carried out "from the bottom up" (46). Sufficient conditions of the selection of amplitudes, similar to the first inequality in (35), are valid when the indicated conditions are met:

$$\begin{array}{l} 0.8m_1 > H_1 + \Delta_2,\ |e_2| \le \Delta_2 \\ 0.8m_2 > H_2 + |\Lambda_1| + \Delta_3,\ |e_3| \le \Delta_3, \\ 0.8b_{\min}m_3 > F + H_3 + |\Lambda_2|. \end{array} \tag{48}$$

The fulfillment of (47) and (48) ensures the sequential stabilization of the residuals with a given accuracy without taking into account the convergence time, which depends on the initial conditions. In a particular case, the fulfillment of (47) and (48) will ensure $|e_i(t)| \le \Delta_i$, $i = \overline{1,3}$ at $t \ge 0$, i.e., the goal of control (45) is achieved.

In the tracking system, the following variants of the initial conditions are also subject of interest: if $|e_3(0)| \le \Delta_3$,$|e_2(0)| > \Delta_2$, then the transient process of $e_2(t)$ will be monotonical; if $|e_i(0)| \le \Delta_i$, $i = 2, 3$, $|e_1(0)| > \Delta_1$, then the transient process of $e_1(t)$ will be without overshoot.

In the rest of the particulars, as well as in the general case $|e_i(t)| > \Delta_i$, $i = \overline{1,3}$, within the framework of these constructions, a monotonic transient process is guaranteed only for the variable $e_3(t)$.

Until the variables of the lower blocks of system (42) reach the specified neighborhoods (46), the variables of the upper blocks grow in absolute value and reach their maximum value no later than at the following times:

$$|e_3(t)| \le |e_3(0)| = e_{3,\max}, |e_2(t)| \le |e_2(t_3)| = e_{2,\max}, |e_1(t)| \le |e_1(t_2)| = e_{1,\max},\ t \ge 0 \tag{49}$$

By (39) and (40), we estimate the ranges of initial values of the variables of system (42)

$$|e_1(0)| \le X_1,\ |e_i(0)| \le X_i + m_{i-1},\ i = 2, 3 \tag{50}$$

Using (42), (48) and (50) and taking into account that the proper motions in closed system (42) are stable, we estimate the maximum values (49)

$$\begin{array}{l} e_{1,\max} \le X_1 + (e_{2,\max} - \Delta_2)t_2, \\ e_{2,\max} \le X_2 + m_1 + (e_{3,\max} - \Delta_3)t_3, \\ e_{3,\max} \le X_3 + m_2. \end{array} \tag{51}$$

To ensure the given convergence time, it is necessary to increase the lower bounds of the selection of amplitudes (48). First, we give estimates of the derivatives of fictitious controls (43). They differ at different intervals and depend on the corresponding estimates of the derivatives of the sigmoid functions and the derivatives of the corresponding residuals (42). Using (48), for the derivatives of the residuals, the following estimates are valid:

$$\begin{array}{l} t \in [0;\, t_2) : \left|\dot{e}_1(t)\right| \le \underbrace{H_1 + \Delta_2}_{<0.8m_1} + e_{2,\max} - \Delta_2 + m_1 < 2m_1 + e_{2,\max} - \Delta_2, \\ t \ge t_2 : \left|\dot{e}_1(t)\right| \le H_1 + \Delta_2 + m_1 < 2m_1; \\ t \in [0;\, t_3) : \left|\dot{e}_2(t)\right| = \underbrace{H_2 + |\Lambda_1| + \Delta_3}_{<0.8m_i} + e_{3,\max} - \Delta_3 + m_2 < 2m_2 + e_{3,\max} - \Delta_3, \\ t \ge\ t_3 : \left|\dot{e}_2(t)\right| = H_2 + |\Lambda_1| + \Delta_3 + m_2 < 2m_2. \end{array} \tag{52}$$

For the derivative of the sigmoid function, by (29) on the indicated intervals, we have

$$\begin{array}{l} |e_i(t)| > c/k_i,\ t \in [0;\, t_i) :\ 0 < 0.5k_i(1 - \sigma^2(k_ie_i)) < 0.18k_i, \\ |e_i(t)| \le c/k_i,\ t \ge\ t_i\ 0.18k_i \le 0.5k_i(1 - \sigma^2(k_ie_i)) \le 0.5k_i,\ i = 1, 2 \end{array} \tag{53}$$

Combining (52) and (53), we obtain estimates of the derivatives of fictitious controls (43) on the indicated intervals

$$|\Lambda_i| = m_i \frac{k_i(1-\sigma^2(k_i e_i))}{2} |\dot{e}_i| \leq \left[\begin{array}{l} 0.36 k_i m_i^2 + 0.18 k_i m_i (e_{i+1,\max} - \Delta_{i+1}),\ t \in [0;\ t_{i+1}); \\ 0.36 k_i m_i^2,\ t \in [\,t_{i+1};\ t_i); \\ k_i m_i^2,\ t \geq t_i;\ i = 1, 2 \end{array} \right.$$

To uniformly accept as an estimate

$$|\Lambda_i| \leq k_i m_i^2,\ t \geq 0;\ i = 1, 2, \tag{54}$$

we need to provide $0.18 k_i m_i (e_{i+1,\max} - \Delta_{i+1}) \leq 0.64 k_i m_i^2 \Rightarrow e_{i+1,\max} - \Delta_{i+1} \leq 3.5 m_i$, $i = 1, 2$. For this we introduce constraints on the peak values of the residuals, slightly lowering the limiting estimates for the convenience of calculations:

$$e_{i,\max} \leq 3 m_{i-1} + \Delta_i,\ i = 1, 2 \tag{55}$$

For consistency, limitation on the overshoot of the output variable also can be introduced:

$$|e_1(0)| \leq X_1 < e_{1,\max} \leq E_1. \tag{56}$$

In a particular case $|e_1(0)| < \Delta_1$, the implementation of $e_{1,\max} \leq E_1 = \Delta_1$ provides $|e_1(t)| \leq \Delta_1,\ t \geq 0$.

With respect (55) and (56), inequalities (51) take the form

$$\begin{array}{c} e_{1,\max} \leq X_1 + 3 m_1 t_2 \leq E_1, \\ e_{2,\max} \leq X_2 + m_1 + 3 m_2 t_3 \leq 3 m_1 + \Delta_2, \\ e_{3,\max} \leq X_3 + m_2 \leq 3 m_2 + \Delta_3, \end{array} \tag{57}$$

whence additional conditions follow, which must be taken into account when selecting $t_{2,3}$ $(0 < t_3 < t_2 < t_1)$ and amplitudes of fictitious controls:

$$0 < m_1 \leq \frac{E_1 - X_1}{3 t_2},\ 0 < m_2 \leq \frac{2 m_1 + \Delta_2 - X_2}{3 t_3}; \tag{58}$$

$$m_1 > \frac{X_2 - \Delta_2}{2},\ m_2 > \frac{X_3 - \Delta_3}{2}. \tag{59}$$

Note that, according to constructions (48) $m_{i-1} > \Delta_i$, $i = 2,3$, while $\Delta_i > 0$, $i = 2,3$ can be accepted less or more than values X_i. The requirement of smallness is not imposed on them. To simplify the calculations, one can initially fix $\Delta_i = X_i$, $i = 2, 3$, which removes the need to check the fulfillment of conditions (59).

In the general case $\Delta_i < X_i$, $i = 2, 3$, the inequalities of the lower bound of the selection of amplitudes m_i will contain two basic components. Due to the first component m_{i1}, as well as m_3, similarly to (38), the convergence of residuals $e_1(t)$, $e_2(t)$, $e_3(t)$ on intervals $[t_2;\ t_1]$, $[t_3;\ t_2]$, $[0;\ t_3]$, respectively, from the peak values (51), (57) into the given areas in a given time (46) is ensured. The second component m_{i2} provides the implementation of constraints (59). In addition, in contrast to the amplitude of the true control m_3, which is selected only based on the lower estimate, there are upper constraints on the selection of the amplitudes of the fictitious controls (58).

Let us formalize a step-by-step procedure of sequential, "top-down" selection of the amplitudes of sigmoid controls and admissible times $t_{2,3}$ for the given Δ_1, t_1, assigned E_1 (56), $\Delta_{2,3} > 0$, and adopted on their basis k_i^*, $i = \overline{1, 3}$ (47). During the procedure, variation of free parameters is allowed.

Procedure 2. Selection of sigmoid feedback amplitudes

Step 1. Using (57), the first inequality (48) takes the form

$$0.8m_1 \geq \frac{X_1 + 3m_1 t_2 - \Delta_1}{t_1 - t_2} + H_1 + \Delta_2 \Rightarrow m_{11} \geq \frac{X_1 - \Delta_1 + (H_1 + \Delta_2)(t_1 - t_2)}{0.8t_1 - 3.8t_2},$$

whence the constraint on the selection $0 < t_2 < t_1$ follows:

$$0.8t_1 - 3.8t_2 > 0 \Rightarrow t_2 < 0.2t_1. \tag{60}$$

Based on (60), we select $t_2^* > 0$ and substitute it into the double inequality

$$\max\{m_{11};\, m_{12}\} < \overline{m}_1 < \overline{\overline{m}}_1, \tag{61}$$

$$m_{11} = \frac{X_1 - \Delta_1 + (H_1 + \Delta_2)(t_1 - t_2^*)}{0.8t_1 - 3.8t_2^*},\ m_{12} = \frac{X_2 - \Delta_2}{2},\ \overline{\overline{m}}_1 = \frac{E_1 - X_1}{3t_2^*}. \tag{62}$$

If inequality (61) is satisfied, then we fix t_2^*, $m_1^* \in (\overline{m}_1;\, \overline{\overline{m}}_1]$ and go to the second step. If (61) is not satisfied, arbitrary parameters should be varied. This can be performed in two ways.

First way. If it is required to ensure accepted E_1 (56), then we vary Δ_2 and/or t_2. If with the initially accepted $0 < t_2^* < 0.2t_1$ inequality $m_{12} > m_{11}$ (62) is valid, then by increasing Δ_2 (up to $\Delta_2 = X_2$) it is necessary to ensure $m_{11} > m_{12}$. If with the new Δ_2^* the inequality (61) is not valid and initially $m_{11} > m_{12}$, then we decrease t_2^*. The critical value $\bar{t}_2 > 0 : m_{11}(\bar{t}_2) = \overline{\overline{m}}_1(\bar{t}_2)$ exists and equals

$$\bar{t}_2 = \frac{\sqrt{p_2^2 - 4p_1 p_3} - p_2}{2p_1},$$

$$p_1 = -3(H_1 + \Delta_2),$$
$$p_2 = 0.8(E_1 - X_1) + 3(E_1 - \Delta_1 + (H_1 + \Delta_2)t_1),$$
$$p_3 = -0.8(E_1 - X_1)t_1.$$

From the limit relation

$$\lim_{t_2 \to +0} m_{11}(t_2) = \frac{X_1 - \Delta_1 + (H_1 + \Delta_2)t_1}{0.8t_1} = \text{const} < \lim_{t_2 \to +0} \frac{E_1 - X_1}{3t_2} = +\infty, \tag{63}$$

it follows that $\overline{\overline{m}}_1$ can be made arbitrarily large and for any $t_2^* > 0 : 0 < t_2^* < \bar{t}_2$ inequality (61) will be satisfied.

Thus, by reducing t_2, it is possible to provide any sufficiently small overshoot in the output variable (56). However, this can lead to a significant increase in the lower bounds of the selection of amplitudes in the following blocks.

Second way. If we abandon the accepted E_1 (56) and increase its value

$$E_1 > \overline{E} = X_1 + 3m_1^* t_2^*, \tag{64}$$

where $\overline{E}$ is the minimum possible overshoot of the output variable with the initial accepted value $0 < t_2^* < 0.2t_1$, then one can arbitrarily increase the upper bound $\overline{\overline{m}}_1$ of the selection of the amplitude (61).

Step 2. The second inequality (46) is ensured by selection m_2. With respect (54), (57), the second inequality (48) takes the form

$$\begin{aligned} &0.8m_2 \geq \tfrac{X_2 + m_1^* + 3m_2 t_3 - \Delta_2^*}{t_2^* - t_3} + H_2 + k_1^*(m_1^*)^2 + \Delta_3 \Rightarrow \\ &m_{21} \geq \tfrac{X_2 + m_1^* - \Delta_2^* + (H_2 + k_1^*(m_1^*)^2 + \Delta_3)(t_2^* - t_3)}{0.8t_2^* - 3.8t_3}, \end{aligned} \tag{65}$$

whence follows a constraint on the selection $0 < t_3 < t_2^*$, similar to (60)

$$0.8t_2^* - 3.8t_3 > 0 \Rightarrow t_3 < 0.2t_2^*. \tag{66}$$

Based on (66), we select $t_3^* > 0$ and substitute it into the double inequality

$$\max\{m_{21}; m_{22}\} < \overline{m}_2 < \overline{\overline{m}}_2, \tag{67}$$

where $m_{21}(t_3^*)$ (65),

$$m_{22} = \frac{X_3 - \Delta_3}{2},\ \overline{\overline{m}}_2 = \frac{2m_1^* + \Delta_2^* - X_2}{3t_3^*}. \tag{68}$$

If (67) is satisfied, then we fix t_3^*, $m_2^* \in (\overline{m}_2; \overline{\overline{m}}_2]$ and go to the third step. If (67) is not fulfilled, arbitrary parameters Δ_3 and/or t_3 should be varied. If initially $m_{22} > m_{21}$, then by increasing Δ_3 (up to $\Delta_3 = X_3$) we need to ensure $m_{21} > m_{22}$. If with new Δ_3^* the inequality (67) is not satisfied or initially $m_{21} > m_{22}$, then we decrease t_3^*. The critical value $\bar{t}_3 > 0 : m_{21}(\bar{t}_3) = \overline{\overline{m}}_2(\bar{t}_3)$ exists and equals

$$\bar{t}_3 = \frac{\sqrt{q_2^2 - 4q_1q_3} - q_2}{2q_1},$$

$$\begin{aligned} q_1 &= -3(H_2 + k_1^*(m_1^*)^2 + \Delta_3), \\ q_2 &= 3(3m_1^* + (H_2 + k_1^*(m_1^*)^2 + \Delta_3)t_2^*) + 0.8(2m_1^* + \Delta_2 - X_2), \\ q_3 &= -0.8(2m_1^* + \Delta_2 - X_2)t_2^*. \end{aligned} \tag{69}$$

From a limit relation similar to (63), namely

$$\lim_{t_3 \to +0} m_{21}(t_3) = \frac{X_2 + m_1^* - \Delta_2^* + (H_2 + k_1^*(m_1^*)^2 + \Delta_3)t_2^*}{0.8t_2^*} = \text{const} < \lim_{t_3 \to +0} \frac{2m_1^* + \Delta_2^* - X_2}{3t_3} = +\infty$$

it follows that for any $t_3^* > 0 : 0 < t_3^* < \bar{t}_3$ inequality (48) is valid.

Note that at the second step (as opposed to the first), the fulfillment of (67) can be ensured only in the indicated way. Increasing the upper limit $\overline{\overline{m}}_2$ by increasing m_1^* will also lead to an increase in the lower limit $\overline{m}_2(m_{21})$, and at a faster rate.

Allowable values t_3^*, m_2^*, Δ_3^* and $k_3^*(\Delta_3^*)$ are fixed, and then we go to the third step.

Step 3. Using (54), (57), the third inequality (48) takes a form similar to (38)

$$m_3 \geq \overline{m}_3 = \frac{1.25}{b_{\min}}\left(\frac{X_3 + m_2^* - \Delta_3^*}{t_3^*} + F + H_3 + k_2^*(m_2^*)^2\right). \tag{70}$$

Based on (70), let us fix m_3^*. The amplitude adjustment procedure is complete.

Thus, there are exist such $\bar{k}_i > 0$, $i = \overline{1,3}$(47), $0 < \overline{m}_i < \overline{\overline{m}}_i$, $i = 1, 2$(61), (62) and (67) and $\overline{m}_3 > 0$ (70), that for all $k_i \geq \bar{k}_i$, $m_i :\ \overline{m}_i < m_i \leq \overline{\overline{m}}_i$, $\forall m_3 \geq \overline{m}_3$ the variables in closed system (42) sequentially converge into the indicated regions within the specified time (46), which ensures the fulfillment of the target condition. Lemma 2 is proved. □

The theoretical significance of the obtained results is as follows. It is shown that it is fundamentally possible to ensure any arbitrary small stabilization error of the output variable with any sufficiently small overshoot (56) in any arbitrary small time for any admissible initial conditions (39). However, it must be understood that a decrease in target characteristics (45) will lead to an increase in the parameters of the controller and the values of fictitious and true controls in the transient process, which is undesirable in real automatic control systems.

We can easily extend the procedure presented in the proof of Lemma 2 to n-dimensional canonical systems with one input. Accordingly, without restrictions, this approach is applicable to MIMO systems with m outputs, in which: (i) the number of inputs is not less than outputs; (ii) the system is representable in the form of m input-output subsystems with one input, in which the matrix before the controls has full rank; (iii) there is no internal dynamics subsystem or its solutions are bounded (i.e., the system is a minimum phase). The more general case of MIMO systems requires additional research.

5. Discussion

The main result of this work is the use of S-shaped smooth sigmoid functions in the feedback loop as fictitious and true controls when unmatched non-smooth disturbances act on the system. The parameters of the nonlinear stabilizing controller are iteratively selected at the stage of synthesis based on inequalities obtained from the worst possible values of the parameters of the control plant and the boundaries of changes in external influences. This approach does not require reconfiguring the controller when internal and external factors change within acceptable limits. Thus, it simplifies the structure of the controller and decreases the formation time of the control signal, since additional identification of unknown parameters, a compilation of models, and the use of an external disturbance observer are not required. In the process of regulation, the sigmoid fictitious and true controls converge to the unknown bounded external signals matched with them in a finite time and repeat their shape with a predetermined accuracy. Thus, a mechanism of suppressing disturbances, including those that are not into the space of true control, is automatically implemented, which ensures the invariance of the output (controlled) variable.

The boundness of sigmoid feedbacks is their undoubted advantage over the traditionally used linear feedbacks with high-gain factors, leading to a large overshoot. In the paper [33], the results of comparative analysis and modeling of systems with linear and nonlinear local feedbacks operating under uncertainty conditions are shown. In [34,35], the results of modeling closed systems with sigmoid local feedbacks as applied to various electromechanical control plants are presented. The disadvantages of the method include a more complex computational implementation compared to a linear control. However, given the constantly increasing power of modern control microprocessors, this is not a serious obstacle to the use of nonlinear functions in automatic control systems of modern and promising technical objects.

Due to the organization of local feedbacks, the state variables of the closed initial system will "track" bounded sigmoid signals, while the maximum deviations of fictitious controls from "reference influences" are bounded (51). This fact is a prerequisite for the creation of analytical methods of the synthesis of invariant systems, taking into account design constraints on the state and control variables. The solution of this problem is the subject of future research by the authors.

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