A Modular Approach to the Teaching of Mathematical Content at Technical Universities

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Abstract: A new approach to dividing the mathematical content into partial modules is presented. This allows to compose subjects with mathematical content from such partial modules and flexibly adapt these subjects to the needs of specific study programs at technical universities. The consistent and systematic implementation of this approach in a typical learning management system is described in detail. This approach means significant changes in the massive (or bulk) delivery of knowledge using available information technologies. The main benefits of the presented system consist in the increase the resulting level of knowledge of students along with their satisfaction with the results and the form of their study. The most important changes arising from our approach are the following. First, the study process became distributed in space and in time. Second, it can be piecewise continuous in time, and, since all students can study at their own pace, it runs in multiple individual time scales. The most important change, however, is the shift of the paradigm the educational process from transmissive “teach–learn” to active “study”.

Keywords: mathematics education; modular approach; educational paradigm

MSC: 97D40; 97D60; 97U50; 97U80

1. Introduction

For many decades, if not centuries, we were used to the classical way of teaching: the lectures took place in a transmissive way, where the teacher, during the entire process of teaching, passes on the finalized knowledge to students, who, in the best case, just listen, write, and memorize. Exercises took place in auditoriums or smaller classrooms, where students often solved assigned problems at the teacher’s direct guidance and interventions.

With the arrival of a new generation of students and the development of IT technologies, a massive development of new teaching methods based on active learning emerged. This development was accompanied by efforts to eliminate transmissive teaching and to shift the focus to the constructive creation of students’ knowledge and forming their active approach to acquiring their knowledge, abilities, and skills. Emphasis is placed on motivating students to create their own knowledge.

Another motivation or impulse for changes in the educational processes comes also from the requirements of the industry/business for the preparation of the university students. Increasingly, emphasis is placed by industrial companies and businesses on the development of soft skills of students, not only hard skills.

In 2020, due to the COVID-19 pandemic, the entire world locked down for a relatively long period of time—about two years. In the case of higher education, that happened even for a longer period than in the case of industry or other sectors. This fact created a demand and an opportunity for the rapid development of the distance forms and methods of education, which until then did not receive appropriate attention. Thanks to the information technology hardware and software, it was, and is, possible to work relatively efficiently with students while moving a large part of the educational process to the online environment.
Another question arose: not whether online education is a suitable alternative to direct transmissive teaching, but how to prepare and implement it so that it takes its meaningful place in the process of education. The discussion about the necessity of e-learning has stopped—in fact, it simply disappeared. The pandemic and health protection measures have shown that e-learning has significant advantages for all participants of the process of education (teachers and students), and often is the only usable solution.

The task of education at a technical university is not to simply provide students with theoretical mathematical education. The primary task is to ensure appropriate level of understanding of the mathematical content, which is essential for correct understanding of subsequent and related technical disciplines. Therefore, mathematics belongs to the core subjects at all technical universities (universities of technology). However, students do not come to technical universities for studying mathematics, physics, or chemistry as such, and this is a huge difference compared to classical universities with faculties of mathematics or natural sciences. Mathematical subjects at technical universities usually have generic names (like Mathematics 1 or Mathematics 2) and comprise the parts (like linear algebra, univariate differential calculus, analytical geometry, etc.) that, say, at faculties of mathematics are studied separately during one or two semesters each.

In the context of the present paper, we should first explain our view on the term “module”, which is used with different meaning in different context. The term “module” can be found in the literature in several basic meanings.

One of them means that some study program is divided into courses, and those are called modules, e.g., Endocrinology module or Emergency module for some medical study program [1]. In other context, the term “module” is often understood as a subject that is organized and preserved using digital technologies [2,3]. In some countries and universities, a different terminology is used, where a course is a study program and individual course subjects are referred to as modules [4,5].

In some other cases, a study program is divided into groups of related subjects, and those groups of subjects are called “modules”—for example, Technology-based module, Chemistry-based module, Soft skills, Fundamental module, Practical skill development, Technology laboratories, etc. [6]. The Fundamental modules includes, among others, one course of Mathematics (12 months, in the first year) comprising Algebra, Analytical Geometry, and Univariate Calculus.

Still, in other cases, the term “module” means more a support tool for subjects than some comprehensive theoretical and practical portion of knowledge [7].

In this paper, we base on the following: a study program consists of subjects (like Mathematics, Physics, Electrical Circuits, Signal Processing, etc.), and each subject can eventually be split into modules. We further use the term “module” in this particular meaning, and this is important for the presentation of our approach.

Typically, the content of the subjects that are generically denoted “Mathematics” is divided into several consecutive semesters and at the same time forms prerequisites and the foundation for the subjects that form the professional content of the study programs at technical universities (machine engineering, civil engineering, industrial logistics, etc.). Clearly, different specializations require different combinations of the parts of mathematics. Here the following challenge arises: how to efficiently deliver customized mathematical content to students of different specialized study programs within a limited time, which depending on those study programs varies from one semester of “Mathematics” to two, or exceptionally three semesters.

The above-mentioned circumstances and current post-COVID conditions require significant changes in the view of the education process and its implementation. For the teaching of mathematics and subjects with mathematical content (like control theory, signal processing, etc.), we implemented the re-design of those subjects based on smaller modules. We have also created complex teaching materials (text and multimedia) for all individual modules. These materials were used during implementation using the LMS (learning management system) Moodle [8], in which we introduced the necessary
elements of automation of the education process and continuous testing of knowledge level acquired by students. This led us to changing our view on the real meaning of different forms of study (e.g., in-person teaching versus online teaching), on continuous testing and educational feedback, as well on other aspects, which are presented below. Ultimately, that means changing the educational paradigm from “teach–learn” to “study”. The system that we have created is of the “open system” type and allows its continuous expansion by adding further (sub) modules that can be used as building blocks not only for generic mathematical courses, but mainly for specialized subjects needing particular advanced mathematical chapters. For example, most study programs would not need the Fourier transform, the Laplace transform, and the Z-transform, so there is no justification for including these advanced topics into generic “Mathematics” courses for all study programs; but the course on signal processing needs these transforms as a part.

Our system proved its usability and efficiency. It was met with extremely positive feedback from students and led to an increase in the average level of students’ mathematical knowledge and competencies.

Below we present the re-design of the courses with mathematical content at the Technical University of Kosice, Slovakia (further denoted as TUKE), its implementation, and its resulting effect on the education process and its outcomes.

2. Redesign (Re-Engineering) of Courses with Mathematical Content

The mathematical content of basic mathematics courses is practically the same at all technical universities, and is usually taught in the first semester or two. This basic content is followed by additional (advanced) mathematical chapters that are embedded in parts into professional subjects where they are needed, depending on the specific field of study. For these reasons, we re-designed the subjects with mathematical content by splitting them (“atomizing”) into modules, from which the content and scope of a specific subject are subsequently created (Figure 1).

![Figure 1. Modular approach to splitting mathematical content.](image-url)
Such modules are our basic building blocks, and the content and scope of the subject are created by combining several modules in a meaningful sequence. The sequence of completion of modules and their mutual dependences are determined by the prerequisites for the individual modules.

One module belongs to one area of mathematics and contains the following components:

- Videos from lectures;
- Slides from lectures;
- Workbooks for exercises and independent work of students [9,10];
- Tutorials for solving tasks using modern engineering software (MATLAB);
- Solutions with explanations to examples from the workbooks [9,10]:
  - in the form of visualizations (web pages with JavaScript, interactive PDF documents);
  - in the form of videos with solutions elaborated and presented by students;
  - in the form of an online collection of solved examples (Blogger).

Based on the analysis of the current curricula of engineering study programs at our university and other technical universities in Slovakia, we created the following structure of modules:

**Basic level modules (Figure 1, inner circle):**

- Linear Algebra;
- Differential Calculus I (Functions of one real variable and their differential calculus);
- Integral Calculus I (Integral calculus of a function of one real variable);
- Analytical Geometry;
- Differential Calculus I (Functions of several real variables and their differential calculus);
- Integral Calculus II (Integral calculus of functions of several variables);
- Ordinary Differential Equations (Ordinary differential equations and their systems);
- Infinite Series;
- Applications of Mathematics.

**Advanced level modules (Figure 1, outer ring):**

- Probability and Statistics;
- Graph Theory;
- Discrete Mathematics;
- Game Theory;
- Integral Transforms;
- Partial Differential Equations;
- Linear Optimization;
- Nonlinear Optimization;
- Numerical Methods.

Various mathematical subjects (such as Mathematics 1, Mathematics 2, Mathematics 3, etc.) and other mathematically oriented subjects (such as Modeling processes and systems, etc.) are (can be) subsequently created by a suitable combination of modules. Basic level modules are used for constructing basic courses, like “Mathematics 1” and similar. Advanced level modules are those parts (or chapters) of mathematics that are embedded into specialized engineering subjects (like, for example, Integral Transforms are a part of Signal and Image Processing, etc.)

Our approach to re-design of teaching mathematics is based on two main pillars. The first pillar is that the mathematical content is divided into parts that we call **modules**. Modules in our approach are not courses or units; we use them as self-contained parts of mathematics that can be used to joint or connect them in order to create one-semester subjects (or courses).

The main reason for dividing the mathematical content into modules is the possibility of delivering individual configurations of mathematical content for specific faculties or study programs. The use of modules allows us to create the content of specific subjects (basic—Mathematics 1, Mathematics 2, Mathematics 3, etc., as well as study-program specific subjects with mathematical content) according to the needs and the focus of individual
faculties or study programs, which, in principle, do not need the same mathematics. This can be used at the level of study groups, and can even be used for configuring (or customizing) the study of individual students. Thanks to this approach, each student can create his own configuration of the mathematical content (methods, tools, skills, etc.) necessary for his further technical studies. The study credit system, which is implemented at most universities, is also based on this idea of adjusting the subjects and their content to the needs of students under the condition of fulfilling the requirements of the study program and related administrative restrictions.

The second pillar of our re-design is based on the fact that, because of the COVID-19 pandemic, we have come to a form of study that we call a hybrid (or combined) form of teaching, where online teaching and face-to-face teaching are suitably combined. Although the method of combining online and face-to-face teaching has been known for a long time and applied in various forms (such as flipped learning, blended learning, and active learning), the current situation takes it to a much higher level by necessarily bringing forward the current information technologies, devices, and software.

Due to the COVID-19 pandemic, we have developed and revised our existing online and multimedia resources and materials for teaching using IT tools and technologies. This is what gives us the possibility to create different configurations according to the programs or according to the individual needs of the students. It also became possible to automate the teaching process to a large extent. The student studies independently and prepares for a test after each topic, and after delivering his test answers and solutions he automatically receives feedback from our system. If the test is completed to a specified number of percent (threshold), the student continues to the next topic within the module. If the test is not completed, the student is returned to the related study materials (a specific part of the lecture video and lecture slides, a specific part of the practical exercises video and handouts, specific tasks from the workbook, etc.). After completing these additional activities, which are monitored automatically, the student will take the test again on the same topic. Tests are randomly generated from a large bank of problems, which allows for a high variety of tests of the same level (difficulty). Each student receives an individual test. After successful completion of the module, he continues his studies with the next module while observing the prerequisites of the modules.

Each student studies using the provided complex teaching materials independently, that is, without the physical presence of the teacher, which is a huge difference from the so-called standard (traditional, classical) form of education.

In the standard (traditional, classic) form of education, it is exactly the presence of the teacher and students in the same room that plays a key role. In the classic form of education, the student answers the questions of the teacher, who manages the entire educational process and takes responsibility for it.

In our approach, education takes place with the follow-up assistance of a teacher after the phase of the independent study of the student with the help of our system. The basic element of this phase is the student–teacher contact, in such a way that the student (students) asks the teacher specific questions about the given topic. The role of the teacher is to provide necessary additional explanations to students in order to complement their previous work within the online system. This establishes the rules for in-person teaching, which takes place when students have questions about the issue. The student–teacher contact itself can take place directly in the classroom (if the situation allows) or remotely online using video-conferencing applications (e.g., Cisco Webex Meetings, MS Teams, etc.). This represents a significant change in the view on in-person (face-to-face) teaching in modern conditions: it can be physical (in the same real classroom) or online (in the same virtual room). Here, the real and the online reality naturally interlace and complement each other. Altogether this means a change of view on in-person teaching. It is directed by the student with his questions—and not by the questions posed by the teacher like in classical teaching. This phase lasts as long as the student has questions about the topic being studied. It can take place directly in the classroom or online.
During such a generalized in-person teaching phase, the teacher adapts to the current understanding of the topic by the students. Since the student actually controls the communication with the teacher by his questions, the attention and focus of the teaching shifts to the student’s understanding of the topic. Hence, this changes the whole philosophy of higher education, and also modifies the role of teacher and student. The main paradigm changes from “teach–learn” to “study”.

The teacher’s role in this process is twofold: the teacher is the author of the teaching materials, and the teacher guides the students’ process of study by answering their questions and filling the gaps in their understanding and/or skills.

The responsibility for studying is, therefore, shifted to the student, which makes sense even from the viewpoint of linguistics. Similarly, the time management of educational activities is also transferred to the student.

3. Implementation: The Tools

In our conditions, the presented approach has been applied to teaching several mathematics subjects. For illustration and clarity, below we focus on the subject “Mathematics 1” for the undergraduate level, which is the basic mathematics subject for all students of our faculty in the first semester of their university studies. We chose the subject Mathematics 1 also because the students are just getting to know such a teaching system, since they are freshmen in the first semester of university studies, and it is necessary to pay considerable attention to the explanation of the study rules as well as the modularity of the subject content itself.

This subject is currently taught in accordance with the described modular approach by four full-time teachers to more than 300 students of the first year (all undergraduate study programs) and comprises the following modules: Linear algebra, Calculus I (Function of one real variable and its differential calculus), and Integral Calculus I (Integral calculus of a function of one real variable). These modules follow each other in the listed order.

The original weekly time load for the subject “Mathematics 1” in case of standard teaching in a classroom is two academic teaching hours of lectures and three teaching hours of seminars (each academic hour is 45 min). In the process of classical teaching, we cannot control or estimate the amount of time devoted by students to the subject outside the classroom.

3.1. Moodle LMS as a Suitable Platform

Our re-design, however, is uses the Moodle system (Figures 2 and 3), which allows monitoring of students’ activities and their progress on daily basis. Moodle (Modular Object-Oriented Dynamic Learning Environment) [8] is an online open-source learning management system (LMS) used for the preparation of online courses for various types of schools (from primary and secondary schools to universities). It consists of various packages that provide many educational features and collaborative tools such as quizzes, wikis, chats, forums, etc.

As a typical LMS, Moodle provides a set of expected features for managing the content and the roles:

1. Roles for system administration (site administrator, manager, and course creator), roles for teachers (teacher and non-editing teacher), for students (student), and guests (quest). A teacher has the right to manage his courses, add content to them, and assign and manage students in the course. After integration with the academic information system, it enables protected access to the university Moodle for all students and teachers.

2. It supports a large number of different types of study materials that the teacher can use in the course or subject, such as multimedia, text documents in various formats, code snippets, URL references to external materials, and so forth. Basic course activities include assignments, quizzes, and tests, that are used for assessment, as well as chat,
forum, a glossary that are used as helper tools, and also survey, wiki, questionary, that are used for general feedback from the participants.

3. The teacher manages own courses, adds content to them, depending on the type of the course (subject). The teacher also creates quizzes and tests that are used to evaluate students activity, and creates the banks of problems that are used for generating quizzes and tests (from simple tests to self-assessment tasks with detailed feedback).

4. It allows a student to display all available resources and own activities, follow and fulfill tasks, follow tasks deadlines, view his grades, and track his progress. If the bank of problems is properly configured, the system can automatically provide suitable feedback to students.

5. It provides the teacher with monitoring of course activities as a whole, and also at the level of individual students—not only their progress, but also their errors. As a result, the teacher gains a certain control over the activities of students during their self-study.

![Representation of the created (split) modules in Moodle LMS.](image1)

**Figure 2.** Representation of the created (split) modules in Moodle LMS.

![An example of one of sets of textual and multimedia study materials from Module A.](image2)

**Figure 3.** An example of one of sets of textual and multimedia study materials from Module A.

### 3.2. LaTeX for Interactive PDFs

LaTeX [11] is a typographic system for the preparation and creation of documents created by Leslie Lamport in 1985 as a set of macros for Donald E. Knuth’s TeX. Similar to TeX, it can be extended by further set of macros. One of the LaTeX extension packages is Animate [12]—the package for creating animations. These animations are created from sets of vector graphics or raster image files or from inline graphics.
The animation of the examples can be controlled using the controls buttons: first frame, step backwards, play backwards/stop, play forwards/stop, step forwards, last frame, slower, faster, and default speed.

Each example that we provide as a complement to our workbooks [9,10] is divided into standard parts: problem formulation, step-by-step solution, control buttons, and eventually necessary graphics (Figure 4). The animations created in this way have only one limitation—they must be downloaded to the client computer and run on supported tools like Adobe Reader.

![Figure 4. An example of an interactive PDF document with step-by-step explanations to a sample solution. Color frames denoting separate areas are added for clarity.](image)

3.3. JavaScript

JavaScript [13] is a scripting language widely used in the development of web applications, and it works with many specialized libraries and technologies. We used the WebGL (Web Graphics Library) [14], three.js [15], and MathBox.js [16] libraries for creating mathematical animations. WebGL allows rendering 2D and 3D animated graphics in a web browser with the help of the library three.js. The MathBox.js is a library for rendering mathematical expressions, and it also works under WebGL.

Our JavaScript animations provide step-by-step illustrations of the process of solution of selected sample problems from our workbooks (Figure 5). A user (a student) can move back and forth between the individual steps of the presented solution. During this transition, transitions are triggered, such as adding or removing parts of the graphical representation.

3.4. Blogger

Blogger [17] is a free Google service that allows creation of private or multi-user blogs. For providing the mathematical content (equations, formulas, etc.) we had to integrate Blogger with the MathJax library. We started using this service to create our collections of solved examples from our workbooks [9,10]. Each example contains a problem formulation and its commented sample solution (Figure 6). For easy orientation within the collections, all sample problems have appropriate tags (keywords). Since Bloggers works (is correctly rendered together with MathJax) on all current devices (desktops, laptops, smartphones, and tablets), our tool became very popular and frequently used by students, as we observe from the statistics of accesses.
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Figure 5. An example of an interactive JavaScript web page step-by-step explanations to a sample solution. Color frames denoting separate areas are added for clarity.

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Figure 6. An example of a solution of a sample problem in Blogger [18]. Color frames denoting separate areas are added for clarity.

4. Implementation: The Process

In-person teaching in our conditions takes place according to the schedule for individual study groups. The two-hour block is taught to all students enrolled in this subject, and it follows the preceding study of the material by students. The teacher responds directly to students’ questions, or after reviewing the success in completing individual activities in the Moodle system, opens a guided discussion on the relevant topic. Due to a large number of students at one time, the teacher only deals with problematic typical parts or questions. When the conditions of emergency situation apply, such as the COVID-19 pandemic or anything else of similar restrictions, this two-hour block can take place exclusively online; otherwise, it can have the form of a meeting in a real lecture room, or a hybrid form (some students are present physically, some are present virtually). The students’ participation in this block is not controlled by the teacher and is voluntary (non-mandatory) for students, as it is defined in the TUKE study rules.

During the three-hour blocks for seminars, students are divided into smaller groups, usually according to the study program (up to 50 students in one group). Within one group, the students are further divided into smaller groups (usually five students), where a discussion on the topic takes place within the group, in which the teacher intervenes only at the request of the students.

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5. Results

The main results of the implementation are related to the level of knowledge acquired by students and their understanding and appreciation of the changes in the educational process which we have introduced, and which are described above.

5.1. Positive Changes in the Level of Knowledge of Students

Objective grading is widely accepted as an estimate of the knowledge of students. We used this measure for monitoring and evaluating the impact of our approach on students. The overall assessment of the subject “Mathematics 1” consists of continuous tests during the semester, and the final exam at the end of the semester. All tests during the semester and the final exam are carried out in our online system described above; a student has to enter answers to the problems and to attach a photo of the elaborated solution; each problem contributes to the final result only if the correct answer has been obtained by some of the possible correct procedures or ways of solution. As mentioned earlier, all tests and final exams are generated from our very extensive banks of problems. All exams are graded by a committee of three examiners. This procedure guarantees the objectivity of the assessment and its usability for measuring the progress of students.

At our university (TUKE), the final (overall) grade is expressed by letters A, B, C, D, E, and FX, where FX means failure, and the rest are different levels defined by the total number of points achieved as a sum of the continuous evaluation during the semester (max. 30%) and the final exam (up to 70%); so A corresponds to 91–100%, B to 81–90%, C to 71–80%, D to 61–70%, and E to 51–60%.

We monitored the overall assessments of students during the past six years, four of which were before we introduced our system (academic year 2016/2017 to 2019/2020), and two were after its implementation (2020/2021 and 2021/2022). Our goal was to investigate the impact of our new modular approach on students’ results. We observed that (Figure 7):

- The proportion of excellent students (ratings A, B) did not change significantly. This means that for excellent students with good mathematical background from high school and good mathematical abilities the method of delivery of a subject does not affect their achieved level of knowledge.

- In the group of average and weak students we, however, observed a very significant shifts from lower grades to grades that are one level higher. Some portion of students,
who were able to get only D in the past, was able to get C now (approximately 5% compared to the previous 4-year average). Even larger portion of students shifted from E to D (approximately 12%), and—this is probably most important—from FX to E (approximately 18%).

![Figure 7. Distributions of grades for the last six academic years.](image)

Therefore, it can conclude that our new approach increased the proportion of students who successfully completed the course.

5.2. High Level of Student Appreciation

The anonymous electronic self-developed questionnaire was made available to those 186 students, who participated in the education process during the whole semester. The questionnaire was sent only to students who actively communicated with teachers throughout the semester (not to all students enrolled in Mathematics 1). The questionnaire was available to students for one week, and we received 106 complete responses (57% response rate).

As a part of a questionnaire, we asked the students of “Mathematics 1” to express their opinion regarding our modular approach to teaching this subject. The overall level of appreciation exceeded our expectations: 80% of the students replied that they were highly satisfied, 11% of the students did not like this approach, and 9% of the students did not provide a comment to this question (Figure 8). At the same time, 97% of students replied that they were happy with the continuous assessment system, and only 3% did not like the continuous testing during the semester.

In the following lines, we quote some of the verbal feedback from students. The provided quotations are taken from the written questionnaire, and the translations from Slovak to English are made by us.

Student 1:

Original: “Cítil som sa dobre na výučbe Matematiky, ale som nie hned’ pochopil ako sa úlohy riešia. Pozeral som videá k seminárom a tak som to pochopil. Som rád, že som uspel na skúške a teším sa na ďalší semester.”

Translation: “I felt good about studying Mathematics, but I did not immediately understand how the tasks have to be solved. I watched the seminar videos and...
that is how I understood it. I am glad I passed the exam and I am looking forward to the next semester.”

Student 2:

Original: “Krášne Vám d’akújem prednášky a cvičenia boli reorganizovane úplne dobre. A vy ste profesionální.”
Translation: “Thank you very much, the lectures and seminars were reorganized perfectly well. And you are professional.”

Student 3:

Original: “Ja nemám žiadne negatívne komentáre, keďže máme z matematiky nahraté prednášky, prezentácie, Livescripty z MATLABu, ktoré si môžeme kedykoľvek pozriť. Myslím, že pre študentov sa viac ani nedá spraviť, a je to už len na nás, či to využijeme.”
Translation: “I do not have any negative comments, as for Mathematics we have recorded lectures, presentations, and LiveScripts from MATLAB that we can watch at any time. I think that there is nothing more that can be done for students, and it is up to us whether we use it.”

Student 4:

Original: “S priebehom vyučovania som bola spokojná.”
Translation: “I was satisfied with the educational process.”

Student 5:

Original: “Vyhovuje mi skupinová metóda spolupráce pri štúdiu, a tiež riešené úlohy v Moodle a aj v MATLABe.”
Translation: “I like the team method of collaboration in study, as well as solved problems in Moodle and MATLAB.”

6. Discussion

In our approach to the re-design of the mathematical content, a module is the basic building element (“block”, “brick”) with its fixed (or closed) content and scope. This view differs from the views mentioned in the introduction, where usually the whole subject or even a group of subjects is associated with the term “module.”
The system that we have created has significantly changed our view on the process of education at technical universities. It leads to the higher efficiency of the educational process, which is reflected in:

- the lower number of teachers for providing in-person lecturing, seminars, and tutoring;
- the higher number of students, to whom the knowledge is served using our approach;
- an increase in the level of knowledge acquired by students in the case of initially less prepared ("weaker") students;
- does not disqualify the most capable (talented, well prepared) students.

The system that we created is an open system, which is in line with the current trends in the world of information technology and education. Our modular system enables the use of our electronic teaching materials not only at all TUKE faculties, but also in the whole of Slovakia. This system allows adding additional users to the "teacher" role as well as to the "student" role. After granting the necessary administration rights, teachers can add additional modules or levels to the system, as each module can have several levels of difficulty. (For example, in the case of the "Linear Algebra" module for technical universities, "Level 1" is sufficient, and "Level 2" would also be necessary for faculties of natural sciences.)

We continuously expand our system, which, thus, becomes more and more usable for all technical universities. It is also suitable for non-mathematical study programs (e.g., biology, chemistry, economics, etc.) at classical universities, where mathematics is also a tool and not the main objective of study. With the approach that we have implemented, it is necessary to ensure only in-person teaching during seminars at individual faculties or universities, because the in-person lectures can be eliminated, and students’ own studies are ensured and managed automatically by our system.

For specialized (professional) subjects within various study programs, it is also possible to use a suitable combination of basic modules and advanced modules (see Figure 1), which makes the entire process of education more efficient compared to the typical current transmissive way of teaching.

With the proper use of the proposed approach, it is possible to use teachers’ time dedicated to in-person (face-to-face) teaching more efficiently. Our modular approach also allows more efficient teaching of subsequent professional subjects, since it is not necessary to repeat parts of the basic modules and it is sufficient to require completion of the necessary prerequisites.

Our system enables natural integration and symbiosis with other programs and systems for online study. One of them is MATLAB, which we use in teaching from day one. TUKE has a license of TAH type (Total Academic Headcount), which allows all teachers and students access not only to all components and toolboxes for MATLAB, but also to online courses provided by the MathWorks, Inc.—from the basic MATLAB OnRamp, through "Linear Algebra", "Ordinary Differential Equations", etc. The courses offered by MathWorks are suitable complements to our modules.

7. Conclusions

The presented solution, which we created and currently use at TUKE, significantly changes the approach to the educational process and has significant implications. This solution generally changes the view of knowledge transfer and de facto stands for something that we can call the **bulk delivery of knowledge**.

The use of the mentioned information technologies and systems means consistent digitization of teaching. With this, teaching changes its form from concentrated to distributed. Teaching is **distributed in space**, as students can be in different places. Teaching is also **distributed in time**, because students can access the provided text and multimedia, programs, and study materials independently at different times. The system created by us allows students to use those study materials at a pace that suits the needs and abilities of individual students, because each student can choose his speed of playback of multimedia materials, the time devoted to individual examples and tasks, etc.
It should also be noted that the study of individual materials does not have to take place in continuous time intervals, but may consist of several time periods; therefore, it can be **piecewise continuous**. In other words, each student can study in such a **time scale** and at such a pace that suits him according to his individual skill needs. It follows from the above that the differences between internal (full-time) students and external (distant) students disappear.

The modularity of our approach enables flexible creation (configuration) of mathematics curriculum and subsequent subjects with mathematical content according to the needs or requirements of those study programs and even individual students. This aspect is very important for technical universities, because different study programs require different sets of mathematical tools, which can be achieved by combining the modules prepared by us or eventually by other contributors. The unifying element for all modules is not only the language of mathematics, but also the language of MATLAB—a standard for all technical universities and industry in the world. Both these languages—mathematics and MATLAB—are essential for communication between students and teachers at technical universities, and for the future professional life of graduates.

Continuous monitoring of the level of knowledge through regular tests helps students maintain adequate motivation and ensures regular feedback during the process of education. The feedback is bi-directional: a student gets an idea of his academic standing and progress, while a teacher obtains a statistical overview of the overall academic progress of all students involved. This process naturally develops the ability for self-organization and a proactive approach of students to their studies. In other words, these are elements of the agile study [19].

The most important impact of the system, that we have developed and implemented, is the change of educational paradigm from “teach–learn” (teacher–pupil) to “study” (student). This differentiates not only the level of study (primary, middle, and high school versus university), but also the method of study and access to it (a pupil versus a student).

It is the overcoming of an important barrier, because the main result of university studies, in contrast to lower-level schools, is the developed ability to acquire, process, and apply information and knowledge. In the future, we plan to follow up the progress and level of knowledge of the students who studied using our modular approach, and the sustainability of their acquired mathematical knowledge and skills. We will extend modular approach to other subjects.

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