

Editorial

# Preface to the Special Issue on “Advances in Differential Dynamical Systems with Applications to Economics and Biology”

Eva Kaslik <sup>1,2,\*</sup>, Mihaela Neamtu <sup>2,3,\*</sup> and Anca Rădulescu <sup>4,\*</sup>

<sup>1</sup> Department of Mathematics and Computer Science, West University of Timișoara, 300223 Timișoara, Romania

<sup>2</sup> Institute for Advanced Environmental Research, West University of Timișoara, 300223 Timișoara, Romania

<sup>3</sup> Department of Economics and Business Administration, West University of Timișoara, 300223 Timișoara, Romania

<sup>4</sup> Department of Mathematics, State University of New York at New Paltz, New Paltz, NY 12561, USA

\* Correspondence: eva.kaslik@e-uvt.ro (E.K.); mihaela.neamtu@e-uvt.ro (M.N.); radulesa@newpaltz.edu (A.R.)

† These authors contributed equally to this work.

In recent research on natural processes, mathematical modeling has become a very useful tool. It is often the case that in fields such as economics and biology, a temporal lag between cause and effect must often be taken into consideration. In modeling, a natural and practical implementation of this phenomenon is through the use of distributed delays. This is because they illustrate the situation where temporal lags arise in certain ranges of values for certain related probability distributions, taking into account the variables' entire history of behavior. Another mathematical tool that allows for the memory and inherited properties of systems to be encompassed in a model is the replacement of integer-order derivatives with fractional derivatives. To address realistic conditions, stochastic perturbation framed by a stochastic differential delay system can be used to explain the ambiguity about the context in which the system operates.

This Special Issue comprises 16 scientific contributions and focuses on the dynamical analysis of mathematical models arising from economy and biology and innovative developments in mathematical techniques for their applications.

Musaev et al. [1] explored the evolutionary self-organization of control techniques using the example of speculative trading in a non-stationary immersion market environment. Because of the extreme volatility and non-stationarity of the observation series, it is particularly challenging to employ adaptive computational algorithms. The authors suggest a strategy based on evolutionary modeling that provides a control model with structural and parametric self-organization.

In a second paper [2], the same authors take into account the short-term forecasting of a process that is an output signal of a nonlinear system seen against an additive noise background. The authors show that it is fundamentally possible to make profit, even in fields with complicated dynamics and sudden changes in the process under consideration, suggesting updated channel strategies and outlining key methods for boosting their efficiency.

In the paper by Najafi et al. [3], the dynamics of CD4<sup>+</sup> T-cells under the influence of HIV-1 infection are studied for the first time in the context of a generalized fractal-fractional structure by using a new mathematical model. Analytical and numerical results reveal stability properties and a lack of a discernible order in the early stages of the illness' dynamics.

In a more theoretical study, Elshenhab et al. [4] studied the finite-time stability of nonhomogeneous systems of second-order linear delay differential equations. The results are applicable to all singular, non-singular and arbitrary matrices.

Using multidimensional statistical analysis, Musaev et al. [5] explored the problem of evaluating the current value of financial instruments, examining various methods for



**Citation:** Kaslik, E.; Neamtu, M.; Rădulescu, A. Preface to the Special Issue on “Advances in Differential Dynamical Systems with Applications to Economics and Biology”. *Mathematics* **2022**, *10*, 3561. <https://doi.org/10.3390/math10193561>

Received: 23 September 2022

Accepted: 26 September 2022

Published: 29 September 2022

**Publisher's Note:** MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



**Copyright:** © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

building computational schemes for regression. The chaotic nature of the observation series, which is caused by the instability of the starting data's probabilistic structure, is a key aspect of this issue; hence, the primary approach for examining the effectiveness of forecasting and analytical algorithms is numerical experimentation.

Paşca et al. [6] propose the least-squares homotopy perturbation as a straightforward and accurate method for obtaining approximate analytical solutions for systems of ordinary differential equations. The technique is used to resolve a problem involving the laminar flow of a viscous fluid in a semi-porous channel, which may be used to simulate blood flow through a blood vessel while taking the effects of a magnetic field into account.

In the paper by Popescu et al. [7], the Pontryagin Maximum Principle and Lie geometric methods are employed to study two optimal control problems at the level of the Lie algebroid. It is demonstrated that the cotangent bundle is not the best framework for finding the best solutions to some driftless control affine systems with holonomic distributions. In this context, a financial application is also presented.

Badralexi et al. [8] analyzed the processes of erythropoiesis and leukopoiesis in the context of maintenance therapy for acute lymphoblastic leukemia by considering two mathematical models expressed by delay differential equations. The stability of every equilibrium point is examined either analytically or numerically. The mathematical results are interpreted from a biological point of view.

Another study by Musaev and Grivoriev [9] examined the applicability of conventional statistical management decision-making methods under stochastic chaos. Compositional algorithm variations are proposed, aimed at adjusting statistical methods to the non-deterministic circumstances brought on by the peculiarities of chaotic processes.

Alonso-Quesada et al. [10] investigate an SIRS epidemic model involving the immunization of susceptible individuals and the treatment of contagious individuals, which are both governed by a designed control system for which its inputs are the subpopulations of the epidemic model. Additionally, newborn vaccinations are also taken into account and control strategies are proposed to eradicate the infectious disease.

In the paper of Kaslik et al. [11], a five-dimensional mathematical model for analyzing the labor market was proposed, with a particular emphasis on the number of open positions, migration, fixed-term contractors, full-time employment and unemployment. The rate of change of open positions, which depends on historical regular employment levels, takes the distributed time delay into account.

By incorporating age structures and overall infection rates into a cholera model, Jiang [12] examined the model's global dynamics, the existence and point dissipativeness of the orbit and asymptotical smoothness. Next, they focus on the existence and local stability of equilibria and also discuss uniform persistence, followed by numerical simulations.

Aletti et al. [13] present an epidemiological SEIR population-based model with many groups, which may represent a geographically limited population or a social subpopulation with similar tendencies while also taking into account the heterogeneity in the weighting of contacts between two groups. In order to minimize the sum of the economic and social costs, they suggest a straightforward control algorithm in which connection weights are optimized.

The aim of the paper of Mousa et al. [14] was to examine the dynamics of a fractional-order susceptible-infectious-recovered (SIR) model that simulates epidemiological diseases. An efficient numerical method based on the Grunwald–Letnikov fractional derivative was proposed and a qualitatively stability analysis was also carried out.

Abia et al. [15] investigate the dynamics of a particular consumer-resource model for *Daphnia magna* from a numerical perspective. Malthusian, chemostatic, and Gompertz growth laws are taken into account in this study for the evolution of the resource population, and the ensuing global dynamics of the model are contrasted as various model parameters vary. The numerical simulations demonstrate that the biological effect of resource scarcity is a real reduction in the size of the consumer population's members.

The paper of Lăzureanu [16] considers systems of three autonomous first-order differential equations such that the sum of the three variables is constant in time. Hamilton–Poisson formulations and integrable deformations are presented, and the case of Kolmogorov systems is also analyzed. As an application, the author examines the three-dimensional Lotka–Volterra system with constant populations from the perspective of Poisson geometry.

As Guest Editors, we are grateful to all authors who contributed to the success of this Special Issue, and to all reviewers for their constructive comments that helped improve initial submissions.

This Special Issue’s objective was to draw original contributions in the area of “Advances in Differential Dynamical Systems with Applications to Economics and Biology”. We expect that the international scientific community will find this collection of research papers influential and that they will spur additional investigations on diverse applications with respect to dynamical systems in all scientific areas.

**Funding:** This research received no external funding.

**Conflicts of Interest:** The authors declare no conflict of interest.

## References

1. Musaev, A.; Makshanov, A.; Grigoriev, D. Evolutionary Optimization of Control Strategies for Non-Stationary Immersion Environments. *Mathematics* **2022**, *10*, 1797. [[CrossRef](#)]
2. Musaev, A.; Makshanov, A.; Grigoriev, D. Numerical studies of channel management strategies for nonstationary immersion environments: EURUSD case study. *Mathematics* **2022**, *10*, 1408. [[CrossRef](#)]
3. Najafi, H.; Etemad, S.; Patanarapeelert, N.; Asamoah, J.K.K.; Rezapour, S.; Sitthiwirattam, T. A study on dynamics of CD4+ T-cells under the effect of HIV-1 infection based on a mathematical fractal-fractional model via the Adams-Bashforth scheme and Newton polynomials. *Mathematics* **2022**, *10*, 1366. [[CrossRef](#)]
4. Elshenhab, A.M.; Wang, X.; Bazighifan, O.; Awrejcewicz, J. Finite-time stability analysis of linear differential systems with pure delay. *Mathematics* **2022**, *10*, 1359. [[CrossRef](#)]
5. Musaeu, A.; Makshanov, A.; Grigoriev, D. Statistical Analysis of Current Financial Instrument Quotes in the Conditions of Market Chaos. *Mathematics* **2022**, *10*, 587. [[CrossRef](#)]
6. Pașca, M.S.; Bundău, O.; Juratoni, A.; Căruntu, B. The Least Squares Homotopy Perturbation Method for Systems of Differential Equations with Application to a Blood Flow Model. *Mathematics* **2022**, *10*, 546. [[CrossRef](#)]
7. Popescu, L.; Militaru, D.; Tică, G. Lie Geometric Methods in the Study of Driftless Control Affine Systems with Holonomic Distribution and Economic Applications. *Mathematics* **2022**, *10*, 545. [[CrossRef](#)]
8. Badralexi, I.; Halanay, A.D.; Mghames, R. Stability Analysis of Equilibria for a Model of Maintenance Therapy in Acute Lymphoblastic Leukemia. *Mathematics* **2022**, *10*, 313. [[CrossRef](#)]
9. Musaeu, A.; Grigoriev, D. Numerical studies of statistical management decisions in conditions of stochastic chaos. *Mathematics* **2022**, *10*, 226. [[CrossRef](#)]
10. Alonso-Quesada, S.; De la Sen, M.; Nistal, R. An SIRS Epidemic Model Supervised by a Control System for Vaccination and Treatment Actions Which Involve First-Order Dynamics and Vaccination of Newborns. *Mathematics* **2021**, *10*, 36. [[CrossRef](#)]
11. Kaslik, E.; Neamțu, M.; Vesa, L.F. Global Stability Analysis of a Five-Dimensional Unemployment Model with Distributed Delay. *Mathematics* **2021**, *9*, 3037. [[CrossRef](#)]
12. Jiang, X. Global Dynamics for an Age-Structured Cholera Infection Model with General Infection Rates. *Mathematics* **2021**, *9*, 2993. [[CrossRef](#)]
13. Aletti, G.; Benfenati, A.; Naldi, G. Graph, spectra, control and epidemics: An example with a SEIR model. *Mathematics* **2021**, *9*, 2987. [[CrossRef](#)]
14. Mousa, M.M.; Alsharari, F. A Comparative Numerical Study and Stability Analysis for a Fractional-Order SIR Model of Childhood Diseases. *Mathematics* **2021**, *9*, 2847. [[CrossRef](#)]
15. Abia, L.M.; Angulo, Ó.; López-Marcos, J.C.; López-Marcos, M.Á. Computational Study on the Dynamics of a Consumer-Resource Model: The Influence of the Growth Law in the Resource. *Mathematics* **2021**, *9*, 2746. [[CrossRef](#)]
16. Lăzureanu, C. Integrable Deformations and Dynamical Properties of Systems with Constant Population. *Mathematics* **2021**, *9*, 1378. [[CrossRef](#)]