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Exponential Stability of Switched Neural Networks with Partial State Reset and Time-Varying Delays †

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Abstract: This paper mainly investigates the exponential stability of switched neural networks (SNNs) with partial state reset and time-varying delays, in which partial state reset means that only a fraction of the states can be reset at each switching instant. Moreover, both stable and unstable subsystems are also taken into account and therefore, switched systems under consideration can take several switched systems as special cases. The comparison principle, the Halanay-like inequality, and the time-dependent switched Lyapunov function approach are used to obtain sufficient conditions to ensure that the considered SNNs with delays and partial state reset are exponentially stable. Numerical examples are provided to demonstrate the reliability of the developed results.

Keywords: switched systems; Lyapunov function; partial state reset; impulsive systems

MSC: 93Cxx; 93Dxx

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1. Introduction

Over the past few decades, neural networks (NNs) have been extensively investigated [1–6] and successfully applied to image recovery, genetic regulatory networks and intermittent control [4,7,8]. These applications rely on the basic stability theory for the equilibrium point of neural networks [9–12]. Thus, it is of great importance to investigate the stability problem of neural networks. Nevertheless, when applying neural networks to practical scenarios, the interactions between collaterals and neurons are generally asynchronous, and time delay effects are inevitable. The presence of time delay will prevent the disturbance of the system from being detected early and the control effect from being effective in time [13–15]. For such reasons, the research on the delayed neural networks has always been a problem of great concern in control science.

In addition, there has been increasing attention to the stability analysis of switched systems since switching effects widely exist in our social life. The phenomenon that systems undergo transitions between various modes occurs in diverse areas of applications, such as the dynamic control of a vehicle with manual gearbox [16] and various biological networks model, including gene regulatory networks [8,17,18]. In [16], under each mode (each gear), the continuous dynamic evolution of speed and position can be described by some ordinary differential equations. The driver's shift behavior triggers the switching between modes. In [18], each gene expression process can be roughly defined by a continuous dynamic behavior in a gene regulatory network, which is made up of a set of interacting genes, but when the protein concentration exceeds a certain threshold, the regulation kinetics will change abruptly. Furthermore, stable and unstable subsystems usually coexist in complex networks [19–21] since some subsystems in a switched system may be unstable due to disturbances, highly nonlinear dynamics, or possible failures [19,22]. As a result, considering switched neural networks (SNNs) with only stable or unstable subsystems are impractical. However, the existence of stable and unstable subsystems may bring more uncertainty in the stability analysis of switched systems. For instance, although each subsystem is stable, improper switching may lead to the instability of the whole system, and even if each subsystem is unstable, the whole system can be stabilized by choosing the proper switching signal [23,24]. Therefore, it is exciting and challenging to study the stability of switched systems with both stable and unstable subsystems. In order to investigate the stability of switched systems, there are two popular ways to characterize the switching signal: one is the dwell time approach, i.e., all switching intervals have common upper and lower bounds, and the other is the average dwell time approach [3].

In some existing results on the stability of SNNs, it has been implicitly assumed that the system state is unchanged at the switching instant [21,25,26]. However, in practice, when the system switches from one subsystem to another, the system may experience discontinuous state jumps (state reset) at the switching instant, such as in complex dynamic networks and electronic networks [27,28]. Moreover, in some cases, the reset of some of these components may be prohibited. Thus, modeling switched systems with partial state reset is more realistic [29], where only a fraction of the states of the system can be reset at some instants. In mathematics, total state reset is typically described by invertible reset matrices. However, partial state reset cannot be described by invertible reset matrices since there are some components of the state vector that remain unchanged. Like most cases, the partial state reset is defined by letting some state components be 1 in the reset matrix [30–32]. Generally, the multiple Lyapunov function method is popular for studying the stability of switched systems [21,23,24]. To guarantee that the multiple Lyapunov function drops at the switching moment, the spectral radius of the reset matrix is usually assumed to be less than 1 [24]. However, when partial state reset is considered, one cannot derive that the multiple Lyapunov function drops at the switching moment since the partial reset matrix's spectral radius is 1. To our knowledge, few models simultaneously cover these factors, including switching effects between stable and unstable subsystems, time delay effects, and partial state reset. These factors can make our current research more applicable in real-world settings.

Note that there are some results on the stability of switched systems with impulsive effects, where the system state was reset at impulsive instants [24,33–36]. The switched system with state reset is usually modeled as an impulsive differential or impulsive functional differential system [30,37,38]. The earliest work on impulsive functional differential systems was reported in 1989, which can be found in [39,40]. Since then, many scholars have devoted themselves to the improvement of the impulsive differential system [41–43]. A useful method to study the stability of impulsive functional differential systems is the famous Halanay differential inequality [44]. For example, based on the generalized Halanay inequality, the global exponential stability was investigated for nonlinear non-autonomous time-delayed systems in [45]. In [46], the dissipativity results of Volterra functional differential equations are obtained by generalizing Halanay inequality. Motivated by the above results, the Halanay-like inequality will be utilized in deriving the main results of this paper.

Note that some results on the stability of SNNs with partial state reset have been reported [29,32,47], but the majority of the above results were focused on linear systems, and the main goal is to find a common Lyapunov function [29,32]. However, it is hard to find a proper common Lyapunov function for nonlinear switched systems with partial state reset. According to what the author knows, the exponential stability of SNNs with partial state reset is always an open but complicated issue, and the main aim of this paper is to bridge this gap. Thus, it is of great significance to model SNNs with time delays and partial state reset. In this paper, the time-dependent switched Lyapunov function will be proposed to handle partial state reset effects. The contributions of this paper are set out below.

(1) SNNs with partial state reset is proposed in this paper, where the partial state reset is more realistic than pinning impulses and time-varying impulses considered in the existing literature [24,35,48,49]. What is more, stable subsystems, unstable subsystems, and time-delay effects are also taken into account, which renders our SNNs model more practical.

(2) A time-dependent switched Lyapunov function method is proposed to handle the stability of SNNs with partial state reset. Moreover, the comparison principle and the generalized Halanay-like inequality are used to derive the stability criteria of the SNNs under consideration.

Notations: This paper employs the following regular notations. I_n and $\mathbb{R}^n$ represent a n-dimensional identity matrix and the Euclidean space of dimension n. $\mathbb{N}$ and $\mathbb{N}^+$ denote the sets of non-negative and positive integers, respectively. x^T denotes the transpose for vector x, and its norm refers to the Euclidean norm. For matrix $A \in \mathbb{R}^{n\times n}$, $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$ represents its spectral norm, where $\lambda_{\max}(\cdot)$ stands for the largest eigenvalue of a square matrix. For any given $\tau > 0$, $PC([-\tau, 0]; \mathbb{R}^n)$ denotes the class of piecewise right continuous functions from $[-\tau, 0]$ to $\mathbb{R}^n$. ψ: $[-\tau, 0] \to \mathbb{R}^n$ with $\tau > 0$ is a vector-valued function and the norm $\|\psi(t)\|_\tau$ is defined as $\sup_{0\le s\le\tau} \psi(t-s)$. $D^+y(t) = \lim\sup_{h\to 0^+} (y(t+h) - y(t))/h$ denotes the upper-right Dini derivative of the function $y(t)$. $\#S$ stands for the number of elements of a finite set S.

2. Problem Formulation

This section presents the SNNs model and some preliminary information; among them, the basic definition, assumptions, and lemmas that are crucial for deriving the main results are offered.

The following SNNs with time delay are considered in this paper:

$$\dot{x}(t) = -A_{\alpha(t)}x(t) + B_{\alpha(t)}\tilde{f}(x(t)) + C_{\alpha(t)}\tilde{f}(x(t-\tau(t))), \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is the state vector connected with the neurons, $A_{\alpha(t)}, B_{\alpha(t)}, C_{\alpha(t)} \in \mathbb{R}^{n\times n}$ are system weight matrices, and $\tilde{f}(x(t)) = (\tilde{f}_1(x(t)), \tilde{f}_2(x(t)), \ldots, \tilde{f}_n(x(t)))^T \in \mathbb{R}^n$ represent the activation functions of the neurons. $\tau(t)$ is the time-varying delay that satisfies $0 < \tau(t) \le \tau$, and $\alpha : [0, +\infty) \to S$ is a piecewise right continuous constant function with $S = \{1, 2, \ldots, N\}$ being a finite set, where $N > 0$ is the number of subsystems. For simplicity, let $\alpha(t) = l_k = r, \forall t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$ representing the switching index of the active subsystem on $[t_k, t_{k+1})$, where $\{t_k\}_{k\in\mathbb{N}}$ is a sequence satisfying $0 \le t_0 < t_1 < t_2 < \cdots < t_k < \ldots, \lim_{k\to\infty} t_k = \infty$. Naturally, $\alpha(t)$ is called the switching signal of the SNNs (1). Without loss of generality, we assume that there are no switching effects or state reset at the initial time t_0. Consequently, system (1) can be described as

$$\dot{x}(t) = -A_r x(t) + B_r\tilde{f}(x(t)) + C_r\tilde{f}(x(t-\tau(t))), r \in S, \#S = N. \tag{2}$$

Remark 1. *For the convenience of the subsequent proof, we use $r = l_k, k \in \mathbb{N}$ to denote the index of active subsystem. Correspondingly, the time-varying parameters matrix $A_{\alpha(t)}$ is replaced by certain constant matrices $A_r = A_{l_k}, k \in \mathbb{N}$ at time interval $[t_k, t_{k+1})$, where each r belongs to the index set S. Thus, SNNs (1) composed of N subsystems can be rewritten in the N time-invariant systems (2).*

Assumption 1. *Assume that the nonlinearity $\tilde{f}_i(x(t))$ meets the following Lipschitz condition:*

$$\|\tilde{f}_i(x) - \tilde{f}_i(y)\| \le l\|x - y\|, \tag{3}$$

$\forall x, y \in \mathbb{R}^n$, and the positive constant l is called the Lipschitz constant.

Let $s(t) = (s_1(t), s_2(t), \ldots, s_n(t))^T$ be an equilibrium solution of system (2). By letting $\xi(t) \triangleq x(t) - s(t)$, we can shift the equilibrium point of the system (2) to the origin by denoting $f(\xi(t)) \triangleq \tilde{f}(x(t)) - \tilde{f}(s(t))$ and supposing $f(0) = 0$, we have

$$\dot{\xi}(t) = -A_r\xi(t) + B_r f(\xi(t)) + C_r f(\xi(t-\tau(t))), r \in S. \tag{4}$$

Now, we take state reset into account and assume that state reset only occurs at switching instants t_k, then the following dynamical system can be obtained:

$$\begin{cases} \dot{\xi}(t) = -A_r\xi(t) + B_r f(\xi(t)) + C_r f(\xi(t-\tau(t))), t \neq t_k, r \in S, \\ \xi(t_k) = \xi(t_k^+) = \Re_k \xi(t_k^-), t = t_k, k \in \mathbb{N}, \end{cases} \quad (5)$$

where $\Re_k \in \mathbb{R}^{n\times n}$ is the diagonal matrix representing state reset matrix at switching instant t_k. In what follows, we will consider two kinds of state reset matrices: one is the partial state reset matrix $\hat{\Re}_k = \mathrm{diag}\{I_p, \mu_k I_{n-p}\}$ with $\mathrm{rank}(I_n - \hat{\Re}_k) < n$, and the other is the total state reset matrix $\tilde{\Re}_k = \mu_k I_n$, $\mathrm{rank}(I_n - \tilde{\Re}_k) = n$.

From the perspective of the influence on the stability of the neural networks, in the existing results, there are time-varying impulses [49] and pinning impulses [6,48,50]. In [48], the pinning impulsive control problem was investigated for complex networks, where only a fraction of the states of the system are subject to stabilizing impulsive effects. It is clear that the partial state reset proposed in this paper can take pinning impulses as a special case. Thus, the partial state reset is more tough and realistic than the impulses considered in [3,24,35,48,49].

It should be mentioned that if $p = 0$ is considered in $\hat{\Re}_k = \mathrm{diag}\{I_p, \mu_k I_{n-p}\}$, then SNNs (5) are subject to the total state reset, which is analogous to the time-varying impulses in [49]. In this case, if the impulsive strength $|\mu_k| > 1$, these impulses are known as destabilizing impulses since the state's absolute value increases at the impulse instants, and when the impulsive strength $|\mu_k| < 1$, these impulses are called stabilizing impulses since the state's absolute value is decreased at the impulse instants [49]. Unless otherwise specified, the strengths of the stabilizing and destabilizing impulses are denoted uniformly as μ_k.

Remark 2. *When partial state reset effects are considered, we have* $\mathrm{rank}(I_n - \hat{\Re}_k) = n - p < n$, *where p is a positive integer. In practice, it is likely that p-dimension elements of neuron state are not subject to the impulses, and the other $n - p$-dimension elements are subject to the impulsive effects, which can be either stabilizing or destabilizing.*

In this paper, the limits from the left and the right at instant t_k are denoted by $x(t_k^-)$ and $x(t_k^+)$, respectively. For simplicity, we assume that $x(t_k) = x(t_k^+)$, $t_0 \geq 0$, and $\xi(t) = \phi(t) \geq 0$, $\forall t_0 - \tau \leq t_0 - \tau(t) \leq t \leq t_0$ is the initial state, where $\phi(t) \in PC([-\tau, 0]; \mathbb{R}^n)$.

The following basic definition, assumption, and lemmas are required to derive the exponential stability criteria for SNNs with partial state reset effects.

Definition 1. *The SNNs (5) are said to be exponentially stable about the origin, if there exist $m > 0, M_0 > 0$ such that*

$$\|\xi(t)\| = \|x(t) - s(t)\| \leq M_0 e^{-m(t-t_0)}$$

holds for all $t \geq t_0$.

Assumption 2. *Discontinuity only occurs at switching time t_k. That is to say, the trajectory of system (5) is piecewise continuous.*

Lemma 1 ([51])**.** *Let Q be a positive definite matrix, then the following inequality holds:*

$$2x^T y \leq x^T Q x + y^T Q^{-1} y, \ x, y \in \mathbb{R}^n. \quad (6)$$

Lemma 2 (Comparison Principle [24]). *Let* $0 \leq \tau_i(t) \leq \tau$, $F(t, x, \bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m) : \mathbb{R}^+ \times \overbrace{\mathbb{R} \times \cdots \times \mathbb{R}}^{m+1} \to \mathbb{R}$ *be nondecreasing in* $\bar{x}_i$ *for each fixed* $(t, x, \bar{x}_1, \ldots, \bar{x}_{i-1}, \bar{x}_{i+1}, \ldots, \bar{x}_m)$, $i = 1, 2, \ldots, m$, *and* $I_k(x) : \mathbb{R} \to \mathbb{R}$ *be nondecreasing in* x. *Suppose that*

$$\begin{cases} D^+ x(t) \leq F(t, x(t), x(t-\tau_1(t)), \ldots, x(t-\tau_m(t))), \\ x(t_k^+) \leq I_k(x(t_k^-)), k \in \mathbb{N}^+, \end{cases}$$

and

$$\begin{cases} D^+ y(t) > F(t, y(t), y(t-\tau_1(t)), \ldots, y(t-\tau_m(t))), \\ y(t_k^+) \geq I_k(y(t_k^-)), k \in \mathbb{N}^+. \end{cases}$$

If $x(t) \leq y(t)$, *for* $t_0 - \tau \leq t \leq t_0$, *then* $x(t) \leq y(t)$ *for all* $t \geq t_0$.

Lemma 3 ([52]). *Assume that* Δ, Ψ_1 *and* Ψ_2 *are appropriate-sized constant matrices,* $0 \leq \rho(t) \leq 1$, *then*

$$\begin{cases} \Delta + \Psi_1 < 0 \\ \Delta + \Psi_2 < 0 \end{cases}$$

is equivalent to

$$\Delta + (1 - \rho(t))\Psi_1 + \rho(t)\Psi_2 < 0.$$

3. Main Results

This section focuses on investigating the exponential stability of the SNNs with both stable and unstable subsystems in (5). Using the time-dependent switched Lyapunov function technique, the comparison principle and the Halanay-like inequality, sufficient conditions of exponential stability are developed for SNNs with partial state reset and total state reset, respectively.

Firstly, we consider the SNNs (5) with partial state reset, i.e., only a part of the states of system (5) are subject to the state reset. The exponential stability of system (5) with partial state reset matrix $\hat{\mathfrak{R}}_k$ is investigated by making the following assumption.

Assumption 3. *Suppose that the switching sequence* $\{t_k\}_{k \in \mathbb{N}}$ *satisfies* $\sigma_1 \leq t_{k+1} - t_k \leq \sigma_2$, *where* σ_1, σ_2 *are two positive constants, and the notation* $S(\sigma_1, \sigma_2)$ *denotes the class of switching sequence satisfying the above condition.*

Remark 3. *The upper and lower bounds are utilized to avoid the activation duration of the stable subsystems being too short and the activation duration of unstable subsystems being too long. In view of Assumption 3, we can get* $\frac{t-s}{\sigma_2} - 1 \leq N(t, s) \leq \frac{t-s}{\sigma_1} + 1$, *where* $N(t, s)$ *is the switching numbers during the time interval* $[s, t)$.

In this paper, let T^- be the index set of stable subsystems and denote T^+ as the index set of unstable subsystems. At the same time, we let $T^-(t, t_0)$ (respectively, $T^+(t, t_0)$) denote the total activation duration of stable subsystems (respectively, unstable subsystems) during $[t_0, t)$.

Theorem 1. *Suppose that Assumptions 2 and 3 hold. If there exist positive definite matrices $P_r^1, P_r^2 \in \mathbb{R}^{n\times n}$, positive scalars $\eta_r^-, \eta_r^+, \zeta_r$, $r \in S$ and $0 < \mu < 1$ satisfy the following inequalities:*

$$c_1 I_n \preceq P_r^m \preceq c_2 I_n, \tag{7}$$

$$-2P_r^m A_r + 2c_2 l\|B_r\| I_n + c_2 l\|C_r\| I_n - \eta_r^+ P_r^m + \frac{P_r^1 - P_r^2}{\sigma_h} \prec 0, r \in T^+, \tag{8}$$

$$-2P_r^m A_r + 2c_2 l\|B_r\| I_n + c_2 l\|C_r\| I_n + \eta_r^- P_r^m + \frac{P_r^1 - P_r^2}{\sigma_h} \prec 0, r \in T^-, \tag{9}$$

$$c_2 l\|C_r\| I_n - \zeta_r P_r^m \prec 0, \tag{10}$$

$$\eta^- - \eta\zeta > 0, \tag{11}$$

$$\sigma_1 > \frac{(\eta^- + \eta^+)\sigma_2}{\eta^- - \eta\zeta - \ln(\mu)/\sigma_2}, \tag{12}$$

$$\begin{pmatrix} -\mu P_{\bar{r}}^1 & \hat{\Re}_k^T P_r^2 \\ P_r^2 \hat{\Re}_k & -P_r^2 \end{pmatrix} \prec 0, \forall r, \bar{r} \in S, \tag{13}$$

$$m, h = 1, 2, \tag{14}$$

where $\eta^- = \min_{r\in T^-}\{\eta_r^-\}$, $\eta^+ = \min_{r\in T^+}\{\eta_r^+\}$, $\zeta = \max_{r\in S}\{\zeta_r\}$, $\eta \triangleq \frac{1}{\mu} e^{2(\eta^+ + \eta^-)\sigma_2}$, $\lambda_1 \triangleq \eta^- - \frac{(\eta^+ + \eta^-)\sigma_2}{\sigma_1} - \frac{\ln\mu}{\sigma_2}$, $c_1 = \min\{\lambda_{\min}(P_r^1), \lambda_{\min}(P_r^2)\}$, $c_2 = \max\{\lambda_{\max}(P_r^1), \lambda_{\max}(P_r^2)\}$, $\varsigma = \eta c_2 \|\phi(t)\|^2$. *Then the SNNs (5) with partial state reset is exponentially stable.*

Proof. We select the following time-dependent switched Lyapunov function:

$$\begin{aligned} V_1(t, \xi(t), r) &= \xi^T(t) P_r(t) \xi(t) \\ &= \xi^T(t)((1-\rho(t))P_r^1 + \rho(t)P_r^2)\xi(t), \end{aligned} \tag{15}$$

where $P_r^1, P_r^2 \in \mathbb{R}^{n\times n}$ are positive definite matrices and $P_r^1 \neq P_r^2, r \in S$.
Letting $\rho(t) \triangleq \frac{t_{k+1}-t}{t_{k+1}-t_k}$, $\forall t \in [t_k, t_{k+1}), k \in \mathbb{N}$, then it is easy to notice $0 < \rho(t) \leq 1$. For convenience, we write down $\xi(t-\tau(t))$ as $\xi_\tau(t)$ in the following content. Along the trajectory of (5), the derivative of $V_1(t)$ can be obtained for $\forall t \in [t_k, t_{k+1})$,

$$\begin{aligned} D^+V_1(t) &= 2\xi^T(t)P_r(t)\dot{\xi}(t) + \xi^T(t)\dot{P}_r(t)\xi(t) \\ &= 2\xi^T(t)P_r(t)[-A_r\xi(t) + B_r f(\xi(t)) + C_r f(\xi(t-\tau(t)))] + \xi^T(t)\dot{P}_r(t)\xi(t). \end{aligned} \tag{16}$$

Combined with the Cauchy–Schwarz inequality, (7) and Lemma 1, it follows that

$$\begin{aligned} 2\xi^T(t)P_r(t)B_r f(\xi(t)) &\leq 2c_2\|\xi(t)\|\sqrt{\|B_r f(\xi(t))\|^2} \\ &\leq 2c_2 l\|B_r\|\xi^T(t)\xi(t), \\ 2\xi^T(t)P_r(t)C_r f(\xi_\tau(t)) &\leq 2c_2 l\|C_r\|\sqrt{\|\xi(t)\|^2}\sqrt{\|\xi_\tau(t)\|^2} \\ &\leq c_2 l\|C_r\|\xi^T(t)\xi(t) + c_2 l\|C_r\|\xi_\tau^T(t)\xi_\tau(t). \end{aligned} \tag{17}$$

From (16) and (17), we have for $t \in [t_k, t_{k+1}), r \in T^+$,

$$\begin{aligned} D^+V_1(t) \leq& \xi^T(t)\left[-2P_r(t)A_r + 2c_2 l\|B_r\| I_n + c_2 l\|C_r\| I_n - \eta_r^+ P_r(t) + \dot{P}_r(t)\right]\xi(t) \\ &+ \xi_\tau^T(t)[c_2 l\|C_r\| I_n - \zeta_r P_r(t-\tau(t))]\xi_\tau(t) + \eta_r^+ V_1(t) + \zeta_r V_1(t-\tau(t)). \end{aligned} \tag{18}$$

We can deduce from the expression of $\rho(t)$ that

$$\begin{aligned}\dot{P}_r(t) &= -\dot{\rho}(t)P_r^1 + \dot{\rho}(t)P_r^2 \\ &= (P_r^1 - P_r^2)/(t_{k+1} - t_k).\end{aligned} \tag{19}$$

From Assumption 3, one has $\frac{1}{\sigma_2} \leq \frac{1}{t_{k+1}-t_k} \leq \frac{1}{\sigma_1}$, thus there exists a mapping $\sigma(t) : [0, +\infty) \to [0, 1]$ such that

$$\frac{1}{t_{k+1} - t_k} = (1 - \sigma(t))\frac{1}{\sigma_1} + \sigma(t)\frac{1}{\sigma_2}. \tag{20}$$

From (18)–(20), we have

$$\begin{aligned}&-2P_r(t)A_r + 2c_2 l\|B_r\|I_n + c_2 l\|C_r\|I_n - \eta_r^+ P_r(t) + \dot{P}_r(t) \\ =&(1-\rho(t))(-2P_r^1 A_r - \eta_r^+ P_r^1) + \rho(t)(-2P_r^2 A_r - \eta_r^+ P_r^2) + 2c_2 l\|B_r\|I_n + c_2 l\|C_r\|I_n \\ &+ (1-\sigma(t))\frac{P_r^1 - P_r^2}{\sigma_1} + \sigma(t)\frac{P_r^1 - P_r^2}{\sigma_2}.\end{aligned} \tag{21}$$

According to Lemma 3 and and $\rho(t) \in (0, 1]$, we obtain that (8) and (10) are respectively equivalent to

$$\begin{aligned}&-2\Big[(1-\rho(t))P_r^1 + \rho(t)P_r^2\Big]A_r + 2c_2 l\|B_r\|I_n + c_2 l\|C_r\|I_n \\ &-\eta_r^+\Big[(1-\rho(t))P_r^1 + \rho(t)P_r^2\Big] + \frac{P_r^1 - P_r^2}{\sigma_h} \prec 0,\end{aligned} \tag{22}$$

and

$$c_2 l\|C_r\|I_n - \zeta_r\Big[(1-\rho(t-\tau(t)))P_r^1 + \rho(t-\tau(t))P_r^2\Big] \prec 0. \tag{23}$$

According to Lemma 3 and $\sigma(t) \in [0, 1]$, we further have that (22) is equivalent to

$$\begin{aligned}&-2\Big[(1-\rho(t))P_r^1 + \rho(t)P_r^2\Big]A_r + 2c_2 l\|B_r\|I_n + c_2 l\|C_r\|I_n - \eta_r^+\Big[(1-\rho(t))P_r^1 + \rho(t)P_r^2\Big] \\ &+ (1-\sigma(t))\frac{P_r^1 - P_r^2}{\sigma_1} + \sigma(t)\frac{P_r^1 - P_r^2}{\sigma_2} \prec 0.\end{aligned} \tag{24}$$

We can obtain from (18) and (21)–(24) that

$$\begin{aligned}D^+V_1(t) &\leq \eta_r^+ V_1(t) + \zeta_r V_1(t-\tau(t)) \\ &\leq \eta^+ V_1(t) + \zeta V_1(t-\tau(t)), t \in [t_k, t_{k+1}), r \in T^+.\end{aligned} \tag{25}$$

By following the similar proofs as those in deriving (25), we obtain

$$\begin{aligned}D^+V_1(t) &\leq -\eta_r^- V_1(t) + \zeta_r V_1(t-\tau(t)) \\ &\leq -\eta^- V_1(t) + \zeta V_1(t-\tau(t)), t \in [t_k, t_{k+1}), r \in T^-.\end{aligned} \tag{26}$$

From the definition of $\rho(t)$, it is easily obtained that $\rho(t_k) = \rho(t_k^+) = 1$ and $\rho(t_k^-) = 0$, and then $V_1(t_k^+) = V_1(t_k) = \xi^T(t_k^+)P_r^2\xi(t_k^+)$, $V_1(t_k^-) = \xi^T(t_k^-)P_{\bar{r}}^1\xi(t_k^-)$. Left multiplying and right multiplying (13) by diag$\{\xi^T(t_k^-), I_n\}$ and its transpose, respectively, one has

$$\begin{pmatrix}\xi^T(t_k^-) & \\ & I_n\end{pmatrix}\begin{pmatrix}-\mu P_{\bar{r}}^1 & \Re_k^T P_r^2 \\ P_r^2\Re_k & -P_r^2\end{pmatrix}\begin{pmatrix}\xi(t_k^-) & \\ & I_n\end{pmatrix} = \begin{pmatrix}-\mu\xi^T(t_k^-)P_{\bar{r}}^1\xi(t_k^-) & \xi^T(t_k^-)\Re_k^T P_r^2 \\ P_r^2\Re_k\xi(t_k^-) & -P_r^2\end{pmatrix} \prec 0. \tag{27}$$

Using the Schur complement Lemma, we know from (27) that

$$-\mu V_1(t_k^-) + \xi^T(t_k^-)\Re_k^T P_r^2\Re_k\xi(t_k^-) < 0. \tag{28}$$

Since $\xi(t_k) = \xi(t_k^+) = \hat{\Re}_k \xi(t_k^-)$, one has

$$V_1(t_k) = \xi^T(t_k)P_r^2\xi(t_k) = \xi^T(t_k^-)\hat{\Re}_k^T P_r^2 \hat{\Re}_k \xi(t_k^-). \tag{29}$$

In view of (28) and (29), the following inequality can be easily obtained:

$$V_1(t_k) < \mu V_1(t_k^-). \tag{30}$$

Thus, we obtain

$$\begin{cases} D^+V_1(t) \le \pi(t)V_1(t) + \zeta V_1(t-\tau(t)), t \neq t_k, t \ge t_0, \\ V_1(t_k^+) \le \mu V_1(t_k^-), k \in \mathbb{N}, \\ V_1(t) \le c_2\|\phi(t)\|^2, t_0 - \tau \le t \le t_0, \end{cases} \tag{31}$$

where $\pi(t) = \begin{cases} -\eta^-, r \in T^-, \\ \eta^+, r \in T^+. \end{cases}$

Let $\gamma(t)$ be a unique solution of the following impulsive delay system for any $\varepsilon > 0$:

$$\begin{cases} \dot{\gamma}(t) = \pi(t)\gamma(t) + \zeta\gamma(t-\tau(t)), t \neq t_k, \\ \gamma(t_k^+) = \mu\gamma(t_k^-), \\ \gamma(t) = V_1(t), t_0 - \tau \le t \le t_0. \end{cases} \tag{32}$$

According to Lemma 2 and (30)–(32), we have

$$V_1(t) \le \gamma(t), t \ge t_0. \tag{33}$$

By the formula for the variation of parameters, it follows from (32) that

$$\gamma(t) = W(t,t_0)\gamma(t_0) + \int_{t_0}^{t} W(t,s)[\zeta\gamma(s-\tau(s)]ds, \tag{34}$$

where $W(t,s)$, $t,s \ge t_0$ is the Cauchy matrix of following linear system corresponding to system (32):

$$\begin{cases} \dot{y}(t) = \pi(t)y(t), t \neq t_k, \\ y(t_k^+) = \mu y(t_k^-), t = t_k, k \in \mathbb{N}. \end{cases} \tag{35}$$

Next, we aim to prove that $\gamma(t)$ is exponentially decreasing. From the properties of Cauchy matrix, we know

$$W(t,s) = e^{\int_s^t \pi(u)du}\mu^{N(t,s)}. \tag{36}$$

Note that $0 < \mu < 1$. In view of Assumption 3 and (36), we know that $\frac{t-s}{\sigma_2} - 1 \le N(t,s) \le \frac{t-s}{\sigma_1} + 1$, and the following estimation can be easily obtained:

$$\begin{aligned} W(t,s) &\le e^{-\eta^- T^-(t,s) + \eta^+ T^+(t,s)}\mu^{N(t,s)} \\ &= e^{-\eta^-(t-s) + (\eta^- + \eta^+)T^+(t,s)}\mu^{N(t,s)} \\ &\le e^{-\eta^-(t-s)}e^{(\eta^- + \eta^+)\sigma_2(1+N(t,s))}\mu^{N(t,s)} \\ &\le e^{-\eta^-(t-s)}e^{(\eta^+ + \eta^-)\sigma_2}e^{(\eta^+ + \eta^-)\sigma_2(\frac{t-s}{\sigma_1}+1)}e^{(\frac{t-s}{\sigma_2}-1)\ln\mu} \\ &= \frac{1}{\mu}e^{2(\eta^+ + \eta^-)\sigma_2}e^{-\eta^-(t-s)}e^{(\eta^+ + \eta^-)\sigma_2\frac{(t-s)}{\sigma_1}}e^{\frac{(t-s)}{\sigma_2}\ln\mu} \\ &= \eta e^{-\lambda_1(t-s)}, \end{aligned} \tag{37}$$

where $\eta \triangleq \frac{1}{\mu} e^{2(\eta^+ + \eta^-)\sigma_2}$, $\lambda_1 \triangleq \eta^- - \frac{(\eta^+ + \eta^-)\sigma_2}{\sigma_1} - \frac{\ln \mu}{\sigma_2}$. Let $\varsigma = \eta c_2 \|\phi(t)\|^2$, then it can be verified from (34) and (37) that

$$\forall t, \gamma(t) \leq \varsigma e^{-\lambda_1(t-t_0)} + \int_{t_0}^{t} \eta e^{-\lambda_1(t-s)} [\zeta\gamma(s-\tau(s)] ds. \tag{38}$$

Noting the third formula of (32), we can conclude from (38) that

$$\gamma(t) = V_1(t) \leq c_2 \|\phi(t)\|^2 = \frac{1}{\eta}\varsigma < \varsigma, t \in [t_0 - \tau, t_0]. \tag{39}$$

Define $h(s) \triangleq s - \lambda_1 + \eta\zeta e^{\tau s}$. It can be obtained from (12) that $h(0) \triangleq -\lambda_1 + \eta\zeta < 0$. Owing to $h(+\infty) = +\infty$ and $\dot{h}(s) > 0$, accordingly, there exists a unique $\lambda > 0$ such that $\lambda - \lambda_1 + \eta\zeta e^{\lambda\tau} = 0$. In addition, (12) implies $\lambda_1 - \eta\zeta > 0$, and from (39), we get

$$\gamma(t) < \varsigma < \varsigma e^{-\lambda(t-t_0)} + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta}, \forall t \in [t_0 - \tau, t_0]. \tag{40}$$

In what follows,

$$\gamma(t) < \varsigma e^{-\lambda(t-t_0)} + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta} \tag{41}$$

will be proved via the contradiction method. In this way, as $\varepsilon \to 0$, we can easily obtain that $\gamma(t) \leq \varsigma e^{-\lambda(t-t_0)}, \forall t \in [t_0 - \tau, +\infty)$. Suppose that (41) is not true, then there exist some $t^* > t_0$ such that

$$\gamma(t^*) \geq \varsigma e^{-\lambda(t^*-t_0)} + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta}. \tag{42}$$

Assume that $t^* = \inf\left\{t \in (t_0, +\infty) : \gamma(t) \geq \varsigma e^{-\lambda(t-t_0)} + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta}\right\}$. In view of (38) and (41), it follows that,

$$\begin{aligned}
\gamma(t^*) &\leq \varsigma e^{-\lambda_1(t^*-t_0)} + \int_{t_0}^{t^*} \eta e^{-\lambda_1(t^*-s)} [\zeta\gamma(s-\tau(s))] ds \\
&< e^{-\lambda_1(t^*-t_0)} \left\{ \varsigma + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta} + \int_{t_0}^{t^*} \eta e^{\lambda_1(s-t_0)} (\zeta\varsigma e^{-\lambda(s-\tau(s)-t_0)} + \frac{\eta\zeta\varepsilon}{\lambda_1 - \eta\zeta}) ds \right\} \\
&\leq e^{-\lambda_1(t^*-t_0)} \left\{ \varsigma + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta} + \int_{t_0}^{t^*} \eta e^{\lambda_1(s-t_0)} (\zeta\varsigma e^{\lambda\tau} e^{-\lambda(s-t_0)} + \frac{\lambda_1\varepsilon}{\lambda_1 - \eta\zeta}) ds \right\} \\
&= e^{-\lambda_1(t^*-t_0)} \left\{ \varsigma + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta} + \int_{0}^{t^*-t_0} \left[\eta\zeta\varsigma e^{\lambda\tau} e^{(\lambda_1-\lambda)s} + \frac{\eta\lambda_1\varepsilon}{\lambda_1 - \eta\zeta} e^{\lambda_1 s} \right] ds \right\} \\
&= e^{-\lambda_1(t^*-t_0)} \left\{ \varsigma + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta} + \varsigma e^{(\lambda_1-\lambda)(t^*-t_0)} - \varsigma + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta} e^{\lambda_1(t^*-t_0)} - \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta} \right\} \\
&= \varsigma e^{-\lambda(t^*-t_0)} + \frac{\eta\varepsilon}{\lambda_1 - \eta\zeta},
\end{aligned} \tag{43}$$

where the first inequality is derived from (38), the second inequality is obtained from $\frac{\eta\varepsilon}{\lambda_1 - \eta\zeta} > 0$ and $t - \tau(t) < t^*$ for $t \in (t_0, t^*)$, the third inequality follows from $\lambda_1 - \eta\zeta > 0$. It is easy to see that (43) contradicts (42), and therefore (41) holds for $t \in [t_0 - \tau, +\infty)$, and thus $\gamma(t) \leq \varsigma e^{-\lambda(t-t_0)}, \forall t \in [t_0 - \tau, +\infty)$. Noting that $V_1(t) \leq \gamma(t)$, we have

$$V_1(t) \leq \gamma(t) \leq \varsigma e^{-\lambda(t-t_0)}, \forall t \in [t_0 - \tau, +\infty). \tag{44}$$

As $c_1 \|\xi(t)\|^2 \leq \xi^T(t) P_r(t) \xi(t) = V_1(t) \leq \varsigma e^{-\lambda(t-t_0)}$, it is easy to know that

$$\|\xi(t)\| \leq \sqrt{\frac{\varsigma}{c_1}} e^{-\frac{\lambda}{2}(t-t_0)}. \tag{45}$$

Then the SNNs in (5) with partial state reset and time-varying delay are exponentially stable. The proof is therefore completed. □

Remark 4. *In [6,48,50], the pinning impulse was considered, in which only a section of the system states was subjected to the same strength of stabilizing impulses. In this paper, our main focus is to investigate the exponential stability of SNNs (5) with the partial state reset, where the impulse effects can be either stabilizing or destabilizing. In [49], the exponential stability was addressed for delayed neural networks with time-varying impulses. One distinguishing trait of time-varying impulses is that both stabilizing and destabilizing impulses are taken into account simultaneously. Different from the impulses in [49], the partial state reset considered in this paper can not only be time varying, but also only a part of neurons can be reset. More specifically, if we let* $p = 0$ *in* $\hat{\Re}_k = diag\{I_p, \mu_k I_{n-p}\}$ *and neglect the switching effects, the model in this paper is reduced to the model in [49]. As a result, compared with the pinning impulses in [48] and the time-varying impulses in [49], the partial state reset in this paper is more general. Since some states of dynamical networks are prohibited, unmeasurable, or too expensive to measure, it is of great significance to consider SNN with partial state reset. For example, in order to achieve coordinated control of vehicles and reduce costs at the same time, we prefer to measure only the states of position or velocities. From a theoretical and realistic factor of view, it is desirable to study the stability of SNNs with partial state reset, that is, only a portion of the states are reset at some switching instants.*

Remark 5. *It is worth mentioning that the time-dependent switched Lyapunov function* $V_1(t)$ *is crucial for deducing (30). Previous literature [47] has proved that a sufficient and necessary condition for exponential stability of linear switched systems is that all stable subsystems have a common quadratic Lyapunov function (CQLF)* $V(t) = \xi^T(t)P\xi(t)$. *However, if the system state is subject to partial state reset and time-varying delay, owing to the fact that some reset matrix components are 1, we can hardly find a common positive definite solution P that satisfies the conclusion (30) by using the LMI tool (the reason can be seen from the following illustrative example). Thus, the time-dependent switched Lyapunov function* $V_1(t)$ *in Theorem 1 is very useful for handling the delay and partial state reset.*

If we want to obtain that $V(t_k) < \mu V(t_k^-), 0 < \mu < 1$ holds at each switching instant t_k, we need to show $\hat{\Re}_k^T P \Re_k \prec \mu P$ holds. That is

$$V(t_k) < \mu V(t_k^-) \Leftrightarrow \xi^T(t_k^-)\hat{\Re}_k^T P \Re_k \xi(t_k^-) < \mu \xi^T(t_k^-) P \xi(t_k^-) \Leftrightarrow \hat{\Re}_k^T P \Re_k \prec \mu P.$$

The partial state reset matrix and positive definite matrix P are assumed as follows,

$$\hat{\Re}_k = \begin{pmatrix} 1 & 0 \\ 0 & 0.2 \end{pmatrix}, P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix},$$

then we can get $\hat{\Re}_k^T P \hat{\Re}_k = \begin{pmatrix} P_{11} & 0.2P_{12} \\ 0.2P_{21} & 0.04P_{22} \end{pmatrix}$. Obviously, the equation $\hat{\Re}_k^T P \hat{\Re}_k \prec \mu P$ can never be proven true since P is positive definite.

Remark 6. *It can be seen that a larger* μ *can reduce the conservativeness of the condition in Theorem 1 from two aspects. Firstly, in view of (37), it will lead to a smaller upper bound of the switching interval, which means that shorter stable subsystems can be enforced (this possibility is presented in Table 1 through a numerical simulation in Example 1). Secondly, from (30), it admits the system state to be decreased at a slower rate.*

Table 1. The relationships between the parameters μ, σ_1, σ_2.

μ	0.6	0.7	0.8	0.9
σ_1	0.044	0.045	0.046	0.046
σ_2	0.052	0.05	0.048	0.047
$\tilde{h}_1$	0.0415	0.2010	0.3206	0.4104
$\tilde{h}_2$	0.0319	0.0293	0.0238	0.0052

Remark 7. *A sufficient condition for the exponential stability of SNNs with partial state reset is obtained. Theorem 1 can be applied to the system (5) with various of partial state reset matrices, which can be represented as* $\hat{\Re}_k = diag\{I_p, \mu_k I_{n-p}\}$ *or* $\hat{\Re}_k = diag\{I_p, \mu_k I_q, \nu_k I_{n-p-q}\}, \mu_k \neq \nu_k$ *are constants. Moreover, it is not only suitable to the case that system (5) with partial state reset but also suitable to the case of system (5) with total state reset.*

In what follows, if $p = 0$ is considered in $\hat{\Re}_k = \mathrm{diag}\{I_p, \mu_k I_{n-p}\}$, i.e., all system states of SNNs (5) are subject to total state reset at switching instants, then the following corollary can be obtained.

Corollary 1. *Suppose that Assumptions 2 and 3 hold and the reset matrices are chosen as* $\tilde{\Re}_k = \mu_k I_n$. *Denote* $\bar{\mu} = \sup_k |\mu_k|$; *if there exist positive definite matrices* $P_r, R_r \in \mathbb{R}^{n\times n}$, *positive scalars* $\eta_r^-, \eta_r^+, \zeta_r, r \in S$, *and* $\nu > 1$ *such that the following inequalities exist:*

$$c_1 I_n \preceq P_r \preceq c_2 I_n, \forall r \in S, \tag{46}$$
$$-2P_r A_r + 2c_2 l\|B_r\| I_n + c_2 l\|C_r\| I_n - \eta_r^+ P_r \prec 0, r \in T^+, \tag{47}$$
$$-2P_r A_r + 2c_2 l\|B_r\| I_n + c_2 l\|C_r\| I_n + \eta_r^- P_r \prec 0, r \in T^-, \tag{48}$$
$$c_2 l\|C_r\| I_n - \zeta_r P_r \prec 0, \tag{49}$$
$$P_r \preceq \nu P_{\bar{r}}, \forall r, \bar{r} \in S, \tag{50}$$
$$\eta^- - \eta\zeta > 0, \tag{51}$$
$$\sigma_1 > \frac{(\eta^- + \eta^+)\sigma_2 + \ln\omega}{\eta^- - \eta\zeta}, \tag{52}$$

where $\eta^- = \min\limits_{r\in T^-}\{\eta_r^-\}$, $\eta^+ = \min\limits_{r\in T^+}\{\eta_r^+\}$, $\zeta = \max\limits_{r\in S}\{\zeta_r\}$, $\omega \triangleq \bar{\mu}^2\nu > 1$, $\eta \triangleq \omega e^{2(\eta^+ + \eta^-)\sigma_2}$, $\lambda_1 \triangleq \eta^- - \frac{(\eta^+ + \eta^-)\sigma_2 + \ln\omega}{\sigma_1}$, $c_1 = \lambda_{\min}(P_r)$, $c_2 = \lambda_{\max}(P_r)$, $\varsigma = \eta c_2\|\phi(t)\|^2$, *then system (5) with total state reset is exponentially stable.*

Proof. Firstly, we select the switched Lyapunov function as follows:

$$V(t, \xi(t), r) = \xi^T(t) P_r \xi(t), r \in S. \tag{53}$$

Taking the derivative of $V(t)$ along the solution curve of system (5), we have for $\forall t \in [t_k, t_{k+1}), k \in \mathbb{N}$,

$$D^+ V(t) = 2\xi^T(t) P_r[-A_r\xi(t) + B_r f(\xi(t)) + C_r f(\xi_\tau(t))]. \tag{54}$$

Combined with the Cauchy–Schwarz inequality, (46) and Lemma 1, the following statements hold:

$$\begin{aligned} 2\xi^T(t) P_r B_r f(\xi(t)) &\le 2c_2 l\|B_r\|\xi^T(t)\xi(t), \\ 2\xi^T(t) P_r C_r f(\xi_\tau(t)) &\le c_2 l\|C_r\|\xi^T(t)\xi(t) + c_2 l\|C_r\|\xi_\tau^T(t)\xi_\tau(t). \end{aligned} \tag{55}$$

From (54) and (55), we know

$$D^+V(t) \leq \xi^T(t)\big[-2P_rA_r + 2c_2l\|B_r\|I_n + c_2l\|C_r\|I_n - \eta_r^+P_r\big]\xi(t) + \xi_\tau^T(t)[c_2l\|C_r\|I_n - \zeta_rP_r(t-\tau(t))]\xi_\tau(t) + \eta_r^+V(t) + \zeta_rV(t-\tau(t)). \quad (56)$$

Thus, from (47), (49) and (56), it is easily derived that

$$\begin{aligned} D^+V(t) &\leq -\eta_r^-V(t) + \zeta_rV(t-\tau(t)) \\ &\leq -\eta^-V(t) + \zeta V(t-\tau(t)) \end{aligned} \quad (57)$$

holds for $\alpha(t) = l_k = r \in T^-, t \in [t_k, t_{k+1})$.
Similarly, from (48), (49), we have that

$$\begin{aligned} D^+V(t) &\leq \eta_r^+V(t) + \zeta_rV(t-\tau(t)) \\ &\leq \eta^+V(t) + \zeta V(t-\tau(t)) \end{aligned} \quad (58)$$

holds for $\alpha(t) = l_k = r \in T^+, t \in [t_k, t_{k+1})$.
Secondly, when $t = t_k$, in view of the second formula in (5), it follows from (50) that

$$\begin{aligned} V(t_k) &= \xi^T(t_k)P_r\xi(t_k) \\ &\leq \nu{\mu_k}^2\xi^T(t_k^-)P_{\bar{r}}\xi(t_k^-) \\ &\leq \omega V(t_k^-), \end{aligned} \quad (59)$$

where $\omega = \nu\bar{\mu}^2$. Analogizing the steps from (31)–(36), we can obtain an estimation for $W(t,s)$ that is similar to (37) in Theorem 1,

$$\begin{aligned} W(t,s) &\leq e^{-\eta^-(t-s)}e^{(\eta^-+\eta^+)\sigma_2(1+N(t,s))}\omega^{N(t,s)} \\ &\leq e^{-\eta^-(t-s)}e^{(\eta^++\eta^-)\sigma_2}e^{(\eta^++\eta^-)\sigma_2(\frac{t-s}{\sigma_1}+1)}e^{(\frac{t-s}{\sigma_1}+1)\ln\omega} \\ &= \omega e^{2(\eta^++\eta^-)\sigma_2}e^{-\eta^-(t-s)}e^{(\eta^++\eta^-)\sigma_2\frac{(t-s)}{\sigma_1}}e^{\frac{(t-s)}{\sigma_1}\ln\omega} \\ &= \eta e^{-\lambda_1(t-s)}, \end{aligned} \quad (60)$$

where $\eta = \omega e^{2(\eta^++\eta^-)\sigma_2}$, $\lambda_1 = \eta^- - \frac{(\eta^++\eta^-)\sigma_2+\ln\omega}{\sigma_1}$. In view of (51), (52), it follows that $\lambda_1 > \eta\zeta$. Therefore, the proof is finished. □

Remark 8. *Compared with Theorem 1, in Corollary 1, we only utilize the condition (50) to deduce $V(t_k) < \omega V(t_k^-)$. The switched Lyapunov function is chosen as $V(t,r) = \xi^T(t)P_r\xi(t), r \in S$, which can be regarded as a specific case of the proposed time-dependent switched Lyapunov function ($P_r^1 = P_r^2 = P_r$).*

Remark 9. *Corollary 1 is applied to the case where all system states are subject to destabilizing impulses at switching instants. Moreover, it is also applied to the case where some dimensional node states are subject to stabilizing impulses and others are subject to destabilizing impulses because $\omega > 1$ is satisfied in the above two cases. When all states of the node are subjected to stabilizing impulses, we can discuss the applicability of Corollary 1 in two situations, one is that $\omega > 1$, and the other is $\omega < 1$. The distinction lies in whether the upper bound or the lower bound of the dwell time of switching interval is chosen to estimate $\omega^{N(t,s)}$ in $W(t,s)$. If $\omega > 1$, we utilize the lower bound σ_1 to magnify the term $\omega^{N(t,s)}$, if $\omega < 1$, we use the upper bound σ_2.*

The following two Corollaries show that our results can be applied to two special cases that SNNs only have stable or unstable subsystems. In Case 1, SNNs with only stable subsystems are considered. In Case 2, SNNs with only unstable subsystems are considered.

If there are no unstable subsystems in (5), we have the following.

Corollary 2. *Consider the SNNs (5) with only stable subsystems which are subjected to partial state reset. Suppose that Assumptions 2 and 3 hold and the conditions (7), (9), (10), (13) remain unchanged. If (11) and (12) in Theorem 1 are changed into into* $\eta^- - \frac{1}{\sigma_2}\ln\mu > \frac{1}{\mu}\zeta$, *then the SNNs (5) with partial state reset are exponentially stable.*

If there are no stable subsystems in (5), one has the following:

Corollary 3. *Consider the SNNs (5) with only unstable subsystems. Suppose that Assumptions 2 and 3 hold and the conditions (7), (8), (10), (13) remain unchanged. If (11) and (12) are changed into* $-\eta^+ - \frac{1}{\sigma_2}\ln\mu > \frac{1}{\mu}\zeta$, *then the SNNs (5) with partial state reset are exponentially stable.*

4. Numerical Example

Two examples are listed in this section to affirm the validity of the findings obtained in this paper. In example 1, the partial state reset matrix is in the form of $\hat{\Re}_k = \mathrm{diag}\{I_p, \mu_k I_{n-p}\}$. In the second example, the partial state reset matrix is in the form of $\hat{\Re}_k = \mathrm{diag}\{I_p, \mu_k I_q, \nu_k I_{n-p-q}\}$, where $\mu_k \neq \nu_k$ are constants.

Example 1. *We consider SNNs (5) with two unstable and two stable subsystems subjected to partial state reset in the form of* $\hat{\Re}_k = diag\{1, 1.1\}$. *The switching sequence is* $1 \to 2 \to 3 \to 4 \to \ldots$,

$$A_1 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0.56 & 0.08 \\ 0.064 & 1.04 \end{bmatrix}, C_1 = \begin{bmatrix} -0.2 & 0.024 \\ 0.08 & -0.12 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 3.8 & 0 \\ 0 & 3.8 \end{bmatrix}, B_2 = \begin{bmatrix} 0.56 & 0.08 \\ 0.064 & 1.04 \end{bmatrix}, C_2 = \begin{bmatrix} -0.2 & 0.024 \\ 0.08 & -0.12 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, B_3 = \begin{bmatrix} 0.1 & 0.1 \\ 0.4 & 0.05 \end{bmatrix}, C_3 = \begin{bmatrix} 0.1 & -0.3 \\ -0.1 & 0.4 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 3.8 & 0 \\ 0 & 3.8 \end{bmatrix}, B_4 = \begin{bmatrix} 0.1 & 0.1 \\ 0.4 & 0.05 \end{bmatrix}, C_4 = \begin{bmatrix} 0.1 & -0.3 \\ -0.1 & 0.4 \end{bmatrix},$$
$$P_1^1 = e^{-10}\begin{bmatrix} 0.1926 & 0 \\ 0 & 0.1926 \end{bmatrix}, P_1^2 = e^{-10}\begin{bmatrix} 0.5528 & 0 \\ 0 & 0.4998 \end{bmatrix},$$
$$P_2^1 = e^{-10}\begin{bmatrix} 0.2461 & 0 \\ 0 & 0.2458 \end{bmatrix}, P_2^2 = e^{-10}\begin{bmatrix} 0.5593 & 0 \\ 0 & 0.4982 \end{bmatrix},$$
$$P_3^1 = e^{-10}\begin{bmatrix} 0.2373 & 0 \\ 0 & 0.2317 \end{bmatrix}, P_3^2 = e^{-10}\begin{bmatrix} 0.5452 & 0 \\ 0 & 0.4926 \end{bmatrix},$$
$$P_4^1 = e^{-10}\begin{bmatrix} 0.3044 & 0 \\ 0 & 0.2989 \end{bmatrix}, P_4^2 = e^{-10}\begin{bmatrix} 0.5544 & 0 \\ 0 & 0.4938 \end{bmatrix}.$$

$f(x) = (\tanh(x_1), \tanh(x_2))^T$, $\tau(t) = \frac{e^t}{1+e^t}$, *then we can get* $l = 2$. *Choose* $\mu = 0.7$, $\sigma_1 = 0.045, \sigma_2 = 0.05$. *According to (7)–(13) in Theorem 1, using Matlab LMI toolbox, we can obtain the above feasible solution* $P_i^1, P_i^2, i = 1,2,3,4$, *and* $\eta^- = 1.1, \eta^+ = 1.2, \zeta = 0.5, \sigma_1 - \frac{(\eta^- + \eta^+)\sigma_2}{\eta^- - \eta\zeta - \ln(\mu)/\sigma_2} = 0.0293 > 0$. *The state trajectories of the whole system and each subsystem with partial state reset in (5) are shown in Figures 1 and 2 , respectively. From the state trajectories of* $\xi(t)$, *we can see that the SNNs in (5) with partial state reset can be exponentially stable.*

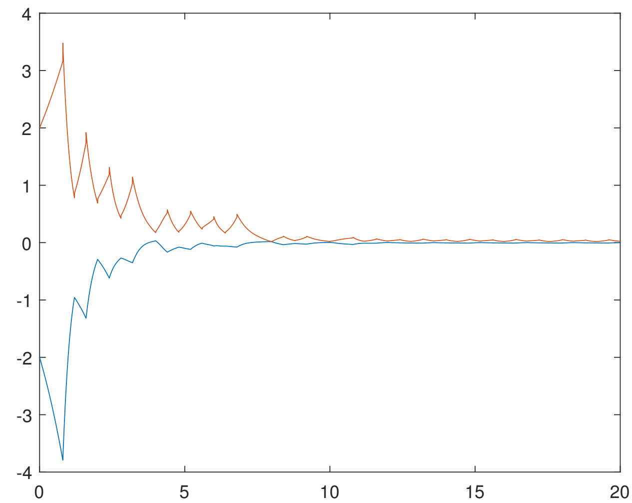
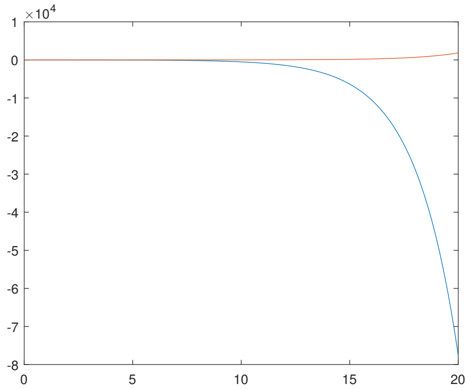
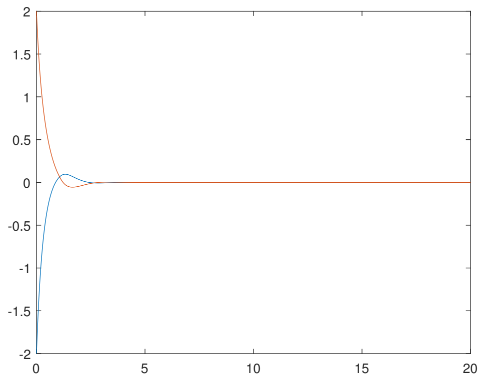


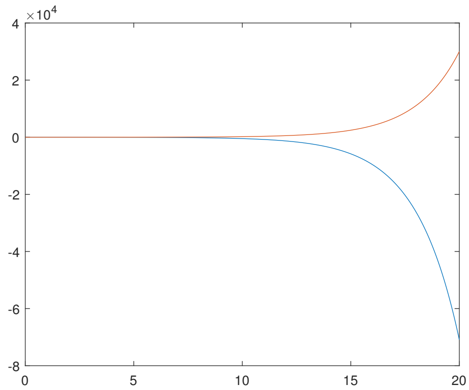
Figure 1. State trajectory of the whole switched system $\xi(t)$.



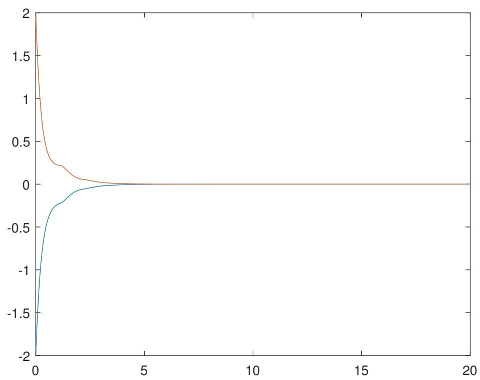
(**a**) State trajectory of the first subsystem $\xi_1(t)$



(**b**) State trajectory of the second subsystem $\xi_2(t)$



(**c**) State trajectory of the third subsystem $\xi_3(t)$



(**d**) State trajectory of the fourth subsystem $\xi_4(t)$

Figure 2. State trajectories of the four 2-dimensional subsystems $\xi_i(t), i = 1, 2, 3, 4$.

$\mu = 0.6$

$$P_1^1 = e^{-10}\begin{bmatrix} 0.2968 & 0 \\ 0 & 0.2640 \end{bmatrix}, P_1^2 = e^{-10}\begin{bmatrix} 0.7915 & 0 \\ 0 & 0.7054 \end{bmatrix},$$
$$P_2^1 = e^{-10}\begin{bmatrix} 0.4004 & 0 \\ 0 & 0.3636 \end{bmatrix}, P_2^2 = e^{-10}\begin{bmatrix} 0.7876 & 0 \\ 0 & 0.6998 \end{bmatrix},$$
$$P_3^1 = e^{-10}\begin{bmatrix} 0.3621 & 0 \\ 0 & 0.3259 \end{bmatrix}, P_3^2 = e^{-10}\begin{bmatrix} 0.7928 & 0 \\ 0 & 0.7071 \end{bmatrix},$$
$$P_4^1 = e^{-10}\begin{bmatrix} 0.4921 & 0 \\ 0 & 0.4529 \end{bmatrix}, P_4^2 = e^{-10}\begin{bmatrix} 0.7874 & 0 \\ 0 & 0.7003 \end{bmatrix},$$

$\mu = 0.8$

$$P_1^1 = e^{-10}\begin{bmatrix} 0.1217 & 0 \\ 0 & 0.2981 \end{bmatrix}, P_1^2 = e^{-10}\begin{bmatrix} 0.3212 & 0 \\ 0 & 0.7054 \end{bmatrix},$$
$$P_2^1 = e^{-10}\begin{bmatrix} 0.1459 & 0 \\ 0 & 0.1597 \end{bmatrix}, P_2^2 = e^{-10}\begin{bmatrix} 0.3292 & 0 \\ 0 & 0.2977 \end{bmatrix},$$
$$P_3^1 = e^{-10}\begin{bmatrix} 0.1451 & 0 \\ 0 & 0.1528 \end{bmatrix}, P_3^2 = e^{-10}\begin{bmatrix} 0.3170 & 0 \\ 0 & 0.2934 \end{bmatrix},$$
$$P_4^1 = e^{-10}\begin{bmatrix} 0.1759 & 0 \\ 0 & 0.1844 \end{bmatrix}, P_4^2 = e^{-10}\begin{bmatrix} 0.3286 & 0 \\ 0 & 0.2969 \end{bmatrix},$$

$\mu = 0.9$

$$P_1^1 = e^{-10}\begin{bmatrix} 0.1977 & 0 \\ 0 & 0.2178 \end{bmatrix}, P_1^2 = e^{-10}\begin{bmatrix} 0.4114 & 0 \\ 0 & 0.3938 \end{bmatrix},$$
$$P_2^1 = e^{-10}\begin{bmatrix} 0.2345 & 0 \\ 0 & 0.2554 \end{bmatrix}, P_2^2 = e^{-10}\begin{bmatrix} 0.4155 & 0 \\ 0 & 0.3875 \end{bmatrix},$$
$$P_3^1 = e^{-10}\begin{bmatrix} 0.2321 & 0 \\ 0 & 0.2438 \end{bmatrix}, P_3^2 = e^{-10}\begin{bmatrix} 0.4115 & 0 \\ 0 & 0.3918 \end{bmatrix},$$
$$P_4^1 = e^{-10}\begin{bmatrix} 0.2781 & 0 \\ 0 & 0.2903 \end{bmatrix}, P_4^2 = e^{-10}\begin{bmatrix} 0.4206 & 0 \\ 0 & 0.3918 \end{bmatrix}.$$

From Table 1, it should be noted that $\tilde{h}_1 = \eta^- - \eta\zeta > 0$, $\tilde{h}_2 = \sigma_1 - \frac{(\eta^- + \eta^+)\sigma_2}{\eta^- - \eta\zeta - \ln(\mu)/\sigma_2} > 0$ implies that conditions (11) and (12) hold in Theorem 1, respectively. The case that $\mu = 0.7$ is detailed in Example 1.

Example 2. *Compared with Example 1, we reconsider SNNs (5) subjected to the partial state reset in the form of $\hat{\mathfrak{R}}_k = diag\{1, 1.1, 0.4\}$. It implies that the strength of partial state reset can be different from each other. The switching sequence is $1 \to 2 \to 3 \to 4 \to \ldots$,*

$$A_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0.56 & 0.08 & 0 \\ 0.064 & 1.04 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}, C_1 = \begin{bmatrix} -0.2 & 0.024 & 0 \\ 0.08 & -0.12 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 5.4 & 0 & 0 \\ 0 & 5.4 & 0 \\ 0 & 0 & 2.7 \end{bmatrix}, B_2 = \begin{bmatrix} 0.56 & 0.08 & 0 \\ 0.064 & 1.04 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}, C_2 = \begin{bmatrix} -0.2 & 0.024 & 0 \\ 0.08 & -0.12 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}, B_3 = \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0.4 & 0.05 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}, C_3 = \begin{bmatrix} 0.1 & -0.3 & 0 \\ -0.1 & 0.4 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 5.4 & 0 & 0 \\ 0 & 5.4 & 0 \\ 0 & 0 & 2.7 \end{bmatrix}, B_4 = \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0.4 & 0.05 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}, C_4 = \begin{bmatrix} 0.1 & -0.3 & 0 \\ -0.1 & 0.4 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$P_1^1 = e^{-10} \begin{bmatrix} 0.7364 & -0.1851 & -0.3206 \\ -0.1851 & 0.7243 & -0.2697 \\ -0.3206 & -0.2697 & 0.5552 \end{bmatrix}, P_1^2 = e^{-10} \begin{bmatrix} 1.1660 & 0.0820 & 0.1220 \\ 0.0820 & 1.1050 & 0.1520 \\ 0.1220 & 0.1520 & 1.1790 \end{bmatrix},$$
$$P_2^1 = e^{-10} \begin{bmatrix} 0.9059 & -0.2285 & -0.3618 \\ -0.2285 & 0.9160 & -0.3270 \\ -0.3618 & -0.3270 & 0.5279 \end{bmatrix}, P_2^2 = e^{-10} \begin{bmatrix} 1.0180 & 0.1480 & 0.1880 \\ 0.1480 & 0.9590 & 0.2060 \\ 0.1880 & 0.2060 & 1.1020 \end{bmatrix},$$
$$P_3^1 = e^{-10} \begin{bmatrix} 0.8139 & -0.1980 & -0.2331 \\ -0.1980 & 0.7819 & -0.2057 \\ -0.2331 & -0.2057 & 0.8040 \end{bmatrix}, P_3^2 = e^{-10} \begin{bmatrix} 1.1710 & 0.0440 & 0.1170 \\ 0.0440 & 1.0980 & 0.1400 \\ 0.1170 & 0.1400 & 1.2260 \end{bmatrix},$$
$$P_4^1 = e^{-10} \begin{bmatrix} 1.0290 & -0.2050 & -0.2330 \\ -0.2050 & 1.0230 & -0.2250 \\ -0.2330 & -0.2250 & 0.9040 \end{bmatrix}, P_4^2 = e^{-10} \begin{bmatrix} 1.0430 & 0.1230 & 0.1640 \\ 0.1230 & 0.9790 & 0.1720 \\ 0.1640 & 0.1720 & 1.1800 \end{bmatrix}.$$

$f(x) = (\tanh(x_1), \tanh(x_2), \tanh(x_3))^T$, choosing $\tau(t) = \frac{e^t}{1+e^t}$, then we can obtain $l = 2$. Let $\mu = 0.9$, $\sigma_1 = 0.045, \sigma_2 = 0.05$. According to (7)–(13) in Theorem 1, we know that $\eta^- = 1.2, \eta^+ = 0.9, \zeta = 0.6, \sigma_1 - \frac{(\eta^- + \eta^+)\sigma_2}{\eta^- - \eta\zeta - \ln(\mu)/\sigma_2} = 0.0027 > 0$. The state trajectories of the whole system and each subsystem with partial state reset in (5) are depicted in Figures 3 and 4 , respectively. From the state trajectories of $\xi(t)$, we can see that the SNNs in (5) with more general partial state reset effects can also be exponentially stable.

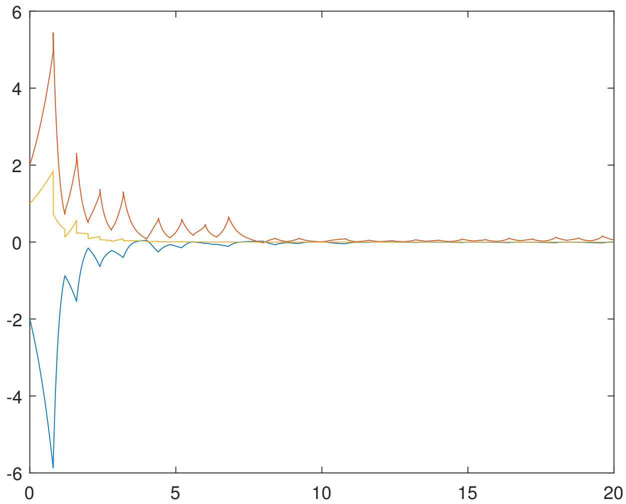
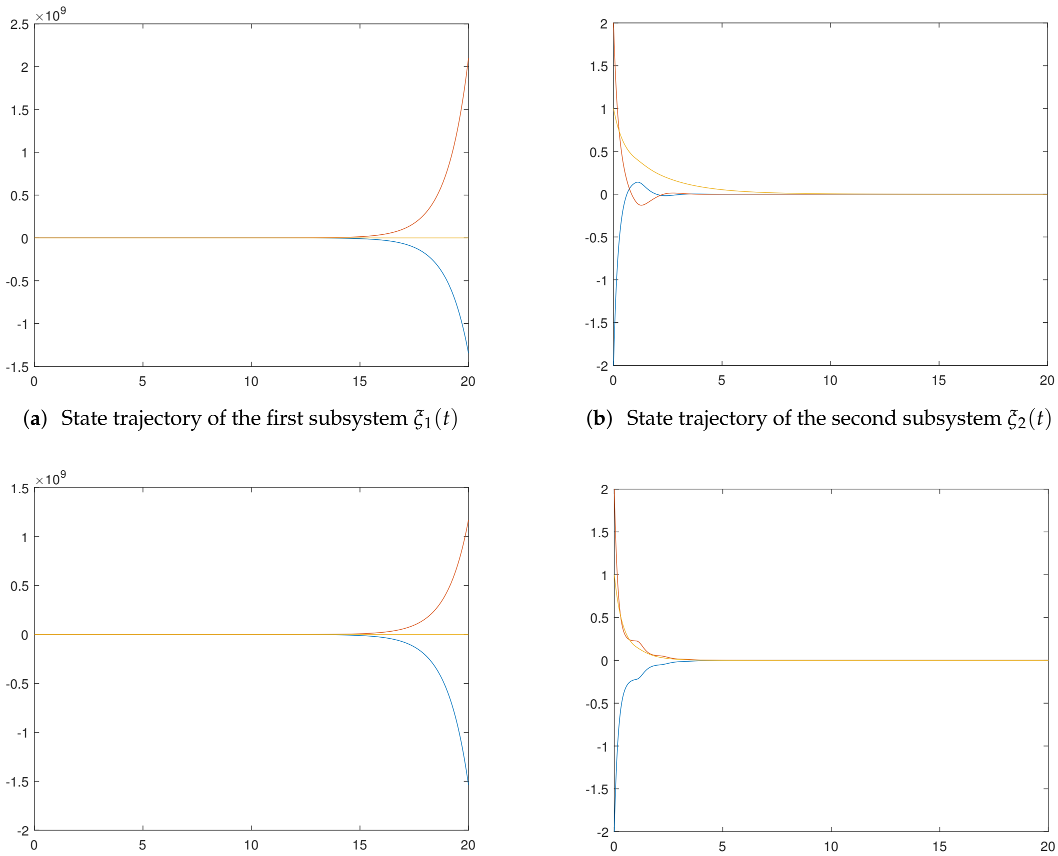


Figure 3. State trajectory of the whole switched system $\xi(t)$.



(**a**) State trajectory of the first subsystem $\xi_1(t)$

(**b**) State trajectory of the second subsystem $\xi_2(t)$

(**c**) State trajectory of the third subsystem $\xi_3(t)$

(**d**) State trajectory of the fourth subsystem $\xi_4(t)$

Figure 4. State trajectories of the four 3-dimensional subsystems $\xi_i(t), i = 1, 2, 3, 4$.

5. Conclusions

In this paper, the exponential stability problem is investigated for delayed SNNs with both stable and unstable subsystems, where only a part of the system states can be reset. By means of the time-dependent switched Lyapunov function, the Halanay-like differential inequalities, and the comparison principle, sufficient criteria are obtained to guarantee that the proposed delayed SNNs with stable and unstable subsystems can be exponentially stable. Finally, the effectiveness of the theoretical result is confirmed by two examples. For future works, it would be practical to consider external inputs, such as noise and disturbance in SNNs (5).

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