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Editorial

Preface to the Special Issue on "Quantum Computing Algorithms and Computational Complexity"

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1. Call for Papers

In 1982, Richard Feynman stated that in order to simulate quantum systems, we would rather go for a sort of brand-new powered quantum processor instead of a classical one. Since then, Quantum Computation has been growing in terms of both architectural issues associated with such quantum computers and the algorithms that can be run with them. All this has attracted much interest from the computer science community.

Just to mention some facts, it is obvious that the intrinsic parallelism that comes with the superposition of quantum states together with interference features provides us with a very good perspective to deal with heavy computational problems, such as encrypting/decrypting tasks or studying quantum issues of matter.

Quantum computing is a hot field of research at the intersection of mathematics, computer science, and physics that promises to significantly revolutionise many aspects of the technology industry such as medicine, machine learning, artificial intelligence, cryptography, and operations research to name a few. Investors and governments from all over the world promote its development considering that it is crucial and of strategic importance for countries, companies and, therefore, society as a whole. The huge investments in resources to develop quantum computing by countries such as China, India, the United States, Russia, and so on only confirms this reality.

This Special Issue was mainly concerned with quantum algorithms, the mathematics underlying them, and those complexity issues arising from them.

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2. Published Papers

This is a Special Issue of *Mathematics* belonging to the section "Mathematics and Computer Science", which was closed on 30 June 2022.

A total of 13 papers were submitted to it, of which 7 have been accepted. This represents an acceptance ratio of 53.8%. The average time for accepted papers to be published is 43.4 days.

The main contributions of these seven papers are the following:

Two of these seven papers are focused on improving the performance by means of quantum algorithms over the best instances of classical ones:

- Yan Li, Dapeng Hao, Yang Xu, and Kinkeung Lai, in their paper "A Fast Quantum Image Component Labeling Algorithm" [1], improve the performance of one of the most time-consuming tasks within digital image processing. They propose a fast quantum image component labelling algorithm that improves the efficiency of its classical computing counterpart. The time and spatial complexities are $O(n^2)$ and $O(n)$, respectively.
- Kamil Khadiev, Artem Ilikaev, and Jevgenijs Vihrovs, in their paper "Quantum Algorithms for Some Strings Problems Based on Quantum String Comparator" [2], improve the performance of three classical problems over strings: "sorting of n strings of length

k", "the most frequent string search problem", and "searching intersection of two sequences of strings". Based on the quantum procedure for comparing two strings of length k in $O(\sqrt{k})$ queries, they are able to reduce time complexities, thus moving the factor k to $\sqrt{k}$ in all its instances as parameters.

Another four papers deal with former quantum algorithms (or part of them) for which some improvements or different perspectives have been addressed:

- Daniil Rabinovich, Richik Sengupta, Ernesto Campos, Vishwanathan Akshay, and Jacob Biamonte, in their paper "Progress towards Analytically Optimal Angles in Quantum Approximate Optimisation": [3], present proof that the optimal quantum approximate optimisation algorithm's (QAOA) parameters for a single layer reduce to one free variable and that optimal angles can be recovered in the thermodynamic limit. They also demonstrate that conditions for vanishing gradients of the overlap function are so similar that reveals a linear relationship between both parameters regardless the number of qubits.
- Tieyu Zhao, Tianyu Yang, and Yingying Chi, in their paper "Quantum Weighted Fractional Fourier Transform" [4], present a reformulation of the weighted fractional Fourier transform (WFRFT) and prove its unitarity, thereby proposing a quantum weighted fractional Fourier transform (QWFRFT) which seems to be very usable for signal processing.
- Mauro Mezzini, Jose J. Paulet, Fernando Cuartero, Hernan I. Cruz, and Fernando L. Pelayo, in their paper "On the Amplitude Amplification of Quantum States Corresponding to the Solutions of the Partition Problem" [5], present a quantum computing piece of code that increases the amplitude of the states corresponding to the solutions of the partition problem by a factor of almost two. Unfortunately, this algorithm cannot be iterated in contrast to the amplitude amplification part of Grover's algorithm.
- Serena Di Giorgio and Paulo Mateus, in their paper "On the Complexity of Finding the Maximum Entropy Compatible Quantum State" [6], follow Jaynes' principle in order to characterize a compatible density operator with maximum entropy. They first stated that comparing the entropy of compatible density operators is complete for the quantum computational complexity class QSZK, even for the simplest case of three chains. They show that for the case of quantum Markov chains and trees, there exists a procedure which is polynomial in the number of subsystems that constructs the maximum entropy compatible density operator. An extension of the Chow–Liu algorithm to the same subclass of quantum states is also provided.

Finally, there is a paper that researches a classical Operational Research problem by means of quantum annealing:

- Saul Gonzalez-Bermejo, Guillermo Alonso-Linaje, and Parfait Atchade-Adelomou, in their paper "GPS: A New TSP Formulation for Its Generalizations Type QUBO" [7], propose a new Quadratic Unconstrained Binary Optimization (QUBO) formulation of the Travelling Salesman Problem (TSP) with a smaller number of necessary variables, together with a thorough study of the constraints and their management. This study includes a practical test over D-wave quantum annealers platform.

As Guest Editors of this Special Issue, we would like to thank all the authors who make contributions on these quite conceptually similar fields of research.

We also would like to thank all the reviewers for their big effort in developing so constructive reports that contribute to improve the quality and quantity of the results provided within this Special Issue on "Quantum Computing Algorithms and Computational Complexity".

We hope that the research papers published in this Special Issue promote more extensive research and lend further support to quantum computing. We are believers of the wide and crucial effect that quantum computing can have in our society, from the domain of energy-efficient computing, through to high-performance computing, up to the management of many of the most challenging problems which still remain open.

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