Energy Dissipation of Rock with Different Parallel Flaw Inclinations under Dynamic and Static Combined Loading

Bing Dai 1, Guifeng Zhao 1, Lei Zhang 1, Yong Liu 1,2, Zhijun Zhang 1, Xinyao Luo 1,3 and Ying Chen 1,*

1 School of Resource Environment and Safety Engineering, University of South China, Hengyang 421001, China
2 School of Physics and Optoelectronic Engineering, Shenzhen University, Shenzhen 518060, China
3 School of Resources and Safety Engineering, Central South University, Changsha 410000, China
*
Correspondence: usc_chen@usc.edu.cn; Tel.: +86-18569069231

Abstract: Deep surrounding rocks are highly statically stressed before mining (excavating) and will inevitably experience disturbances from unloading, mining, stress adjustment or their combinations during mechanical or blasting excavation, which actually suffer from a typical coupled static-dynamic stress. A split Hopkinson pressure bar was used to carry out dynamic-static loading test on rock specimens with different fracture angles. The results show that the change law of energy utilization efficiency is similar to the energy absorption rate that they increase first and then decrease with the increasing of axial pressure. The elastic energy of specimens would also increase first and then decrease with the increasing of axial pressure, while the plastic energy generally decrease overall. Both the energy utilization efficiency and energy absorption rate increase with the growth of dynamic compressive strength under impact loading, which indicate that the energy dissipation exhibits a positive with the dynamic strength. The energy absorption density and energy utilization efficiency gradually increase linearly with the increasing of the average strain rate, while the relationship between energy utilization efficiency and incident energy basically follows the exponential function increasing law. The rock burst of pre-flawed rock is related to the static load level under dynamic-static loading, it occurs obviously under the action of medium energy when the axial pressure is high. Based on the energy dissipation theory, the damage variable model was further established, the damage variable can reasonably describe the damage evolution of crack granite under dynamic-static loading.

Keywords: SHPB tests; double-flaw rocks; dynamic and static combination; fractured rock; energy dissipation; damage variable

MSC: 00A69

1. Introduction

In the fields of resource extraction, transportation, and underground protection engineering, the construction of deep rock mass engineering is becoming more and more frequent, and the research on deep rock mass mechanics has received extensive attention [1–3]. In addition to the high static load of self-weight stress and tectonic stress, deep fractured rock mass is often affected by engineering disturbances such as blasting vibration, mechanical excavation, drilling, and rock caving [4]. Body fissures and engineering disturbances are both important factors that affect the mechanical properties of deep high-stress rock mass, especially deep hard rock has significant brittleness characteristics, which are more likely to cause engineering disasters such as rock burst and rock mass engineering instability and damage. Therefore, it is of great significance to further ensure the safety and stability of rock mass engineering by carrying out research on the energy dissipation law of fractured rock mass under combined dynamic and static loading.

Up to now, a lot of work has been carried out to study the impact of mining on the rock mass and to recommend measures for reducing strain changes of rock mass and
minimising the stress on the surface. Since mining production has a significant effect on the stress-strain behaviour of rock mass, the issues of reducing this influence are very relevant and scientists around the world are trying to minimize it. For example, the stress-strain behavior control in rock mass were studied by using different-strength backfill [5], and the proposed approach to the stress-strain analysis of rock mass in rock-back-fill contact zones is recommended for the applied geotechnical assessment in the conditions of mining with manmade support of mined-out area. Through analysing the impact of underground mining on the undermined mass, this impact is investigated using FLAC 3D modelling software. Mining operations cause destruction of the mass above, bringing this disturbance to the surface and producing subsidence and sinkholes. Ways to minimise the impact of underground mining on the surface are suggested [6]. Some researcher proposed to minimize the change in the stress-strain behavior of rock mass using a different-strength backfill. The assessment of the stress-strain behavior of the undermined mass allows to set the stress values boundaries depending on the formation of an artificial mass [7]. A methodology of selecting a mineral deposit development technology and a rational backfill material includes fuzzy models and algorithms, and it can provide processing of large amounts of information and form the significance of environmental factors [8]. Affected by long-term geological tectonic movement and human activities, various types of defects such as cracks, holes and joints are often formed in the rock mass. As an important rock mass structural plane, cracks are widely distributed in deep rock mass engineering.

Some important achievements have been made in the research on the mechanical properties of rock with cracks. Many scholars have studied the strength properties, deformation properties, energy transfer laws, etc. of rock under quasi-static or dynamic loading, and they also found that there are large differences in the energy dissipation laws under quasi-static or dynamic loading [9–11]. These results show that the mechanical properties and crack propagation mode of crack rocks are different under static or dynamic loads, but both of them are affected by the number and angle of cracks. Some scholars have studied the influence of crack angle and strain rate on dynamic cracking behaviours and energy evolution of multi-crack rock under dynamic loading, they found that the dynamic strength increases with the increase of strain rate, decreases first and then increases with the increase of crack angle, showing evident rate-dependence [12]. To get insight into rock unloading failure, some researchers have used the discretized virtual internal bond model in conjunction with element partition method to study the dynamic fracturing process. First of all, the fracturing pattern exhibits the tensile-shear transition with the stress level increasing, and shear cracks are more likely to occur when the flaw angle is closer to 45°. Then, the results shows that the inclination angle of the parallel flaws has a large effect on the unloading failure of rock by simulating the unloading failure process of rock with multiple flaws [13,14]. Subsequently, the DIC was used to investigate crack initiation and propagation of rock with different angles, some scholars have presented a simple calibration method, which involves adjusting the size and spacing of subsets of pixels within images of a specimen’s surface, and the crack opening displacements and displacement vectors can be determined at all stages of loading. Besides, different ranges of load contact configurations are also considered, for both the flat platen and small wooden cushion, they observed that the cracks initiates far from the disc centres for 0° and 90° anisotropy angles, the development of the fracture process zone is also identified after the major crack initiation [15]. Some scholars have studied the compression behavior of Kuru Gray granite at a wide range of confining pressures by using the servo-controlled hydraulic testing machine, and the tests were divided into low strain rate compression test and dynamic test. The results show that the rock strength increases significantly with the increase of strain rate and confining pressure. At confinements higher than 20 MPa, the strength of the material increases faster at the lower strain rate while at confinements lower than this, the effect of confining pressure is weaker at the higher strain rate [16]. Additionally, to study the fragmentation characteristic and burst behavior of coal under impact load, some researchers found that coal samples have high peak stress, pulverized fragmentation, and
intensive burst energy under impact load. And the fragments induced by the impact load have a relatively consistent distribution mode, which can be characterized by a fractal model [17,18]. Since rocks in underground engineering constructions are likely to be confronted with static stress and dynamic disturbance simultaneously, it is of great scientific significance and engineering application value to study the excavation damage caused by blasting, mechanical drilling and other impact dynamic loads of crack rock mass in the stratum or the effect of dynamic natural disasters such as seismic wave and rock burst on the stability of surrounding rock in deep rock engineering [19,20], the coupled static-dynamic loading on flawed rocks has also been performed [21,22]. Due to the different failure mode caused by static and dynamic loads, some studies have been carried out on engineering materials under coupled static and dynamic loading [23,24]. Under uniaxial compression, brittle solids are frequently observed to fail by splitting parallel to the direction of loading. Although this failure has long been recognized, considerable confusion regarding how such splitting mechanisms develops still remains [25], based on this, some researchers used the Brazilian disk method and high-speed camera to study the dynamic tensile mechanical properties of granite rock under dynamic and static combined loading, they have found that a slowing rising stress wave would be created in the dynamic and static combined loading test through wave analyses, stress measurement and crack photography, which allows the stress in the sample gradually accumulated, whilst the loads at both ends of the sample remained in a balanced state. The tests results showed that the tensile strength of the granite decreases with the increase of pre-stress under the dynamic and static loads, which might lead to the major repair of the blasting design or support design in deep underground projects. In addition to it, the failure patterns of specimens under dynamic and static loads have been investigated [26,27]. Based on the Brazilian disk method to study the dynamic strength under coupled dynamic and static loads, an experimental method is used to study the dynamic failure process of pre-stressed rock specimen with a circular hole. The researchers have found that the rock debris ejected at the surrounding circular hole because of the high static pre-stress coupled with dynamic loading, while the rock failure can not be induced by the lower static pre-stress coupled dynamic loading. Besides, the dynamic stress concentration occurred when the half-sine wave was generated around the circular hole [28]. Cyclic impact load tests of rock with a single hole were carried out by using Split Hopkinson Pressure Bar device under different impact loads and impact methods. The results showed based on one-dimensional stress wave theory and interface continuity conditions, an improved damage calculation formula is obtained suitable for rock specimen. Damage accumulation of granite increases in a power function with the increase of strain rate. Moreover, cumulative specific energy absorption value increase gradually with cyclic impact. [29] The split Hopkinson pressure bar(SHPB) was used to carry out the crack development and damage patterns under dynamic and static loads [30–32], and DIC is a non-contact monitoring device, some scholars have used it to studied the strain and displacemengt field, failure mode and the crack trend of double-fractured specimens, the results showed that the fractures had a promotional effect on final failure of specimens with different angles [33]. The damage to the specimens was caused firstly by large principal and shear strains near the fissure tips, which illustrated that the parallel double fractures was different from the single crack rock. In addition to it, there were two main types of cracks discovered under the dynamic and static loading: tensile cracks and shear cracks in the ultimate failure mode. In addition to it, some experts and scholars have conducted research on the energy dissipation of single-fracture rock mass under combined dynamic and static loading [34,35]. For example, some researchers have studied the rockburst characteristics and tendency indicators under the combined dynamic and static loading conditions, and reached a meaningful conclusion: the rockburst tendency was not only related to the characteristics of its sample, but related to the static load level and dynamic load energy density under different axial pressures and impact numbers [36,37]. In order to study the mechanical properties and failure laws of intact primatic sandstone specimens and specimens with a prefabricated internal circular
cavity under coupled static and dynamic loads, coupled static and dynamic loads tests were carried out with a modified split Hopkinson pressure bar (SHPB) apparatus, and the high-speed camera was also applied to record and analyze the fracturing and damage evolution of specimens. The tests results showed that the strength of the fractured specimen is obviously lower than that of the intact specimen, and that, with increasing of the axial pre-stress, the dynamic strength and dynamic elastic modulus generally increased first and then decreased, the combined strength generally increased while the dynamic strain generally decreased [38]. To reveal the mechanical characteristics and failure modes of rock samples with a mini-tunnel under combined static-dynamic loads, some scholars have used the digital image correlation technique to monitor the fracturing in real time, fracture evolution and energy consumption characteristics were further summarized. The results showed that: the pre-stress level decided on the dynamic crack initiation stress, failure mode and dynamic crack velocity of the specimen under otherwise similar dynamic and static disturbance conditions [39].

The research on mechanical properties of rock materials with defects is mainly focused on static loads, dynamic loads test and numerical simulation. At present, only some scholars have carried out the research on mechanical properties of rock mass with defects under dynamic-static combined loading. In addition to it, the combined static-dynamic SHPB tests are mainly limited to the mechanical responses and fracturing characteristics of pre-flawed rocks and the energy dissipation of the intact rocks. Since the energy dissipation and damage variable of flawed rocks significantly differ from that of intact rocks, it is essential to acquire the energy dissipation and damage variable of flawed rocks under combined static-dynamic loading.

In this paper, double-flawed granite specimens were tested under dynamic-static loading with an modified SHPB system, and the influences of the crack angle and axial pressure on the energy dissipation and damage variable of pre-flawed rocks were revealed. The structure of this paper is: Section 2 presents sample preparation, testing machine and data processing. Section 3 illustrates the experimental results, including the influences of axial pressure, compressive strength, average strain rate and incident energy on energy absorption density and energy utilization efficiency, as well as the influences of axial pressure on elastic energy, plastic energy and rock-burst. Section 4 discusses the new damage variable model and the influences of absorbed energy and pre-stress on damage degree. Sections 5 and 6 concludes the discussions and the whole study.

2. Test System and Specimen Preparation

2.1. Sample Preparation

By virtue of its isotropy in mechanics and homogeneous in texture, the test material is granite with relatively good integrity and uniformity, as shown in Figure 1 (made by author). In order to ensure the accuracy of experimental results, all test specimens were manufactured according to the International Society for Mechanics and Rock Engineering [40]. First, intact rocks were cut with a nominal size of $45 \times 45 \times 20 \text{ mm}^3$. Then, diamond saw is used to cut the flaws with different different inclination angles $\alpha$ (the angle is between horizontal direction and flaw orientation as shown in Figure 1), which were divided into $0^\circ$, $45^\circ$, $90^\circ$. Finally, we polished the parallel-flawed rocks until the surface roughness was less than 0.02 mm, the depth of the cracks is about 1 mm, the crack length is 10 mm and the width is 1 mm. The average density of the rock samples is 2580 kg/m$^3$, the average elastic modulus is about 9.86 GPa. Before dynamic loading test, it is essential to use the INSTRO 1346 test system to conduct uniaxial compression test on rock specimens. Three specimens are tested for the specimen configuration and the average compressive strength is 139.66 MPa. This experiment included two kinds of specimens: intact specimens and pre-flawed specimens with different angles, a total of 66 samples were required. Among them, there were 6 intact specimens and 18 samples for other 3 kinds of crack angles.
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Figure 1. Geometry of the double-flawed rock specimens.

2.2. Testing Machine

In the test, a modified Split Hopkinson Pressure Bar (SHPB) loading system was used to perform the dynamic-static loading test. The system was mainly composed of an incident bar, transmitted bar, buffer bar, spindle shaped punch, pressure device and data acquisition system, the axial pressure was provided by the axial pressure system at end of the transmitted rod. Strain signals were collected by strain gauges pasted on the incident and transmitted bar, and relevant test data were processed by oscilloscope. The diameter of incident and transmitted bar was 50 mm. The anisotropic punch was used to achieve constant strain loading, as shown in Figure 2. Before the test, a layer of butter was evenly applied at both ends of the rock specimen to ensure good contact between the two ends of the sample and incident and transmitted bar, it would also reduce the interface friction between the rock specimen and elastic rod. The sample was sandwiched between the incident and transmitted bar, before the dynamic and static loading, the axial pressure was slowly applied to the set value first and then the axial pressure loading pump valve closed. The oscilloscope was adjusted to receive the signal transmitted through the strain gauge. After all the preparation was ready, the nitrogen switch was started. The impact bar first struck the pulse shaper, and then impacted the incident bar to generate an incident bar when the incident wave reached the rock surface, a part of incident wave would be transmitted to the transmitted bar through the rock, the others formed the reflected wave and reflected back to the surface of rock specimen. At this time, the strain gauge recorded the signals of three stress wave at the same time.
2.3. Data Processing

The half-sinusoidal stress loading wave generated by the spindle-shaped punch can enable the constant strain rate loading. At the same time, the system is equipped with ultra-dynamic strain gauge, oscilloscope and data processing device, which can realize the functions of stress wave signal acquisition, recording and data processing. Based on the one-dimensional stress wave theory, the mechanical parameters such as the average dynamic stress \( \sigma(t) \), strain \( \varepsilon(t) \) and strain rate \( \varepsilon'(t) \) of the specimen can be obtained by the following equation [41]:

\[
\sigma(t) = \frac{A_e}{2A_s} [\sigma_I(t) - \sigma_R(t) + \sigma_T(t)] 
\]  

\[
\varepsilon(t) = \frac{1}{\rho_e C_e L_s} \int_0^t [\sigma_I(t) + \sigma_R(t) - \sigma_T(t)] \, dt 
\]  

\[
\varepsilon'(t) = \frac{1}{\rho_e C_e L_s} [\sigma_I(t) + \sigma_R(t) - \sigma_T(t)] 
\]  

In the above equations, \( \sigma_I(t) \) is the incident wave strain; \( \sigma_R(t) \) is the reflection wave strain; and \( \sigma_T(t) \) is the transmitted wave strain; \( A_e \) is the sectional area of the elastic rod; \( C_e \) is the velocity acoustic wave; \( \rho_e \) is the density of the rod; \( A_s \) is the sectional area of the sample; and \( L_s \) is the length of the sample.

According to the loading principle of SHPB and the law of conservation of energy, the curves of incident wave \( \sigma_I(t) \), reflected wave \( \sigma_R(t) \) and transmitted wave \( \sigma_T(t) \) obtained
The energy absorbed by the rock specimen \( E_A \), the energy utilization efficiency \( E_d \) defined in this study, and the energy absorption density \( N_d \) of the rock specimen can be determined as follows [41]:

\[
E_A = E_I - E_R - E_T \quad (7)
\]

\[
E_d = \frac{E_A}{V_S} \quad (8)
\]

\[
N_d = \frac{E_A}{E_I} \quad (9)
\]

where \( V_S \) is the volume of the rock specimen. The absorbed energy can be mainly divided into: (1) For crack initiation, propagation, penetration and final failure of the sample. (2) Elastic energy stored inside the rock. (3) The kinetic energy and other forms of energy required for flying fragments. The energy absorption rate and energy absorption density can reflect the tensile damage and deformation failure of the specimens during the dynamic and static loading.

3. Results and Discussion

3.1. Dynamic Stress Equilibrium Check

In order to ensure the accuracy of SHPB test results, both ends of the specimen must reach dynamic stress equilibrium before rock failure under dynamic loading. The curve of dynamic stress wave at end of the specimen \( S_0\text{-flaw0}\text{-}2 \) is shown in Figure 3. It can be seen that the transmitted stress wave \( \sigma_T(t) \) is basically coincident with the superposition wave of incident and reflected stress wave, which indicates that the balance condition of dynamic stress can be achieved and maintained during the dynamic-static loading, and it also verifies the validity of the test results.

Figure 3b depicts the typical evolution curve of the strain rate and stress of \( S_0\text{-flaw0}\text{-}2 \). When the stress reaches the peak stress, there is a gentle stage in the curve of strain rate-time for a period time, and the strain rate hardly changes with the time, which indicates that the deformation rate of rock is constant at this stage. In this study, the average strain rate is defined at the constant deformation rate stage.

3.2. Dynamic Deformation Characteristics

The SHPB device was used to perform the impact tests on the rock samples under different axial (0, 14, 27.9, 41.9, 69.8 and 83.8 MPa), respectively corresponding to 0%, 10%, 20%, 30%, 50%, 60% of the uniaxial compressive strength of the standard sample. Besides on Equations (1)–(9), the incident energy, reflected energy, transmitted energy, energy absorbed by the samples, energy absorption rate and the energy absorption density. The results of the dynamic and static combined loading test were listed in Table 1. In the test, the specimen with cracks failed under all loading conditions, and the dynamic stress-strain curve was shown in Figure 4.
3.3. The Relationship between Energy Utilization Efficiency and Energy Absorption Density and Axial Pressure

Due to space limitations, Figure 5 shows the energy evolution curve of $0^\circ$ fracture under different axial pressures, the four kinds of energy increase first and then remain unchanged with the increase of time. At the beginning of the energy change, the reflected energy is almost equal to the absorbed energy, and the transmitted energy is nearly zero. This is because the signal of incident wave can only be received by the strain collection system after multiple reflections and transmissions. Therefore, the arrival time of the incident wave was delayed but then subsequently increased. At the same time, the reflected and transmitted wave continued to increase at a faster speed. During the dynamic and...
static combined test, the reflected energy was consistently the largest, followed by the absorbed energy, and the transmitted energy accounted for the minimum proportion. It can be seen from the Figure 4, the energy evolution curve is obviously affected by the axial static pressure. As the axial pressure increased, the absorbed energy gradually decreased, and it accounted for the largest proportion at 10%UCS. When the axial pressure was 50%UCS and 60%UCS, the absorbed energy was clearly lower than the reflected energy. When the axial pressure is 20%UCS and 30%UCS, the absorbed and reflected energy of different crack specimens were almost the same level. The evolution curves can be divided into three stages: (1) compaction stage. The micro-cracks are compacted under the action of axial pressure, which is similar to the stress-strain curve in the dynamic compression test; (2) Linear stage. The consumption of energy increases linearly, and the crack further initiates and expends; (3) Stable stage. The absorbed energy remains nearly constant because the accumulative strain energy is sufficient to cause the specimen to collapse.

![Figure 4. Stress-strain curves of specimens with an artificial flaw under coupled static and dynamic loads: (a) 0° flaw; (b) 45° flaw; (c) 90° flaw.](image)

Using Equations (4)–(7), the energy partitions of the confined double-flawed specimens under different axial pressure conditions are determined. Figure 6 presents the incident energy of confined double-flawed specimens under different axial pressure in each crack angle group. The abscissa at the bar grasp bottom represents different crack angle groups, and the axial pressure increases successively from left to right in each crack angle group. Under the same incident energy and impact velocity, the axial pressure makes rocks deform more difficulty. As the axial pressure increases, specimen require more incident energy to obtain a similar strain rate. The reflected energy, transmitted energy, and dissipated energy are normalized by the incident energy, which are shown in Figure 5 in the form of a percentage. As the axial pressure increased, the proportion of reflected energy decreases gradually, which changes from 55% to 44%, and the transmitted energy decreases from 21% to 14%, while the dissipated energy first increases and then decreases, the maximum value is reached when the axial pressure is 0.1 MPa. The higher axial pressure improves the bearing capacity of the specimen, and more energy is used to fracture the rock. When the axial pressure is small, the micro-cracks inside the rock are closed, the dynamic strength increases, and the energy used to destroy the rock will increase. As the axial pressure continues to increase, the micro-cracks inside the sample grow and expand, the energy storage inside the rock gradually increases, and the energy required for rock failure also decreases. Therefore, the proportion of transmitted energy is negatively correlated with the axial static pressure. Under a similar axial pressure, the proportion of reflected energy first decreases and then increases with the increasing of crack angle, while the proportion of transmitted energy and absorbed energy increases first and then decreases, which indicates...
that there is no obvious relationship between crack angle and energy utilization efficiency of rock under combined dynamic and static loading.

Figure 5. Energy evolution curves of tested specimens.

Figure 6. Incident, reflected, transmitted and dissipated energy variation of the single-flawed specimens under different axial pressures and crack angles.

Figure 7. Change of energy utilization efficiency with axial pressure.

Figure 8. Energy absorption density of samples under different axial static pressures.
Figure 7 shows the relationship between the energy utilization efficiency of four types of specimens and different axial static pressures. It can be seen that with the increasing of the axial pressure, the energy utilization efficiency of specimen first increased slightly and then it shows a downward trend, and the energy utilization efficiency reaches the maximum when the axial pressure is 14 MPa. The energy utilization efficiency of sample is always positive during the whole loading process, it is indicated that during the loading process, the rock continues to absorb energy for the initiation and propagation of internal cracks. The initial loading causes the micro-cracks inside the sample to become dense and hard, and the energy required for rock failure will increase, so the energy utilization efficiency rises briefly; As the axial pressure continues to increase, the micro-cracks in the sample initiate and gradually increase, the micro-cracks in the sample initiate and gradually expand, the elastic strain energy stored in the rock gradually increase, the energy required for rock failure decreases, and the energy utilization efficiency also decreases.

\[ \varepsilon = \varepsilon^* + \varepsilon^u \]  

(Figure 7. Change of energy utilization efficiency with axial pressure.)

Figure 8 shows the relationship between energy absorption density of four types of specimens and different axial static pressures. The energy absorption density of samples first increases and then decreases with the increasing of the axial pressure. When the axial pressure is 14 MPa, the energy absorption rate reaches the largest, its characteristic law is similar to Figure 6, the three kinds of fracture samples continuously absorb energy from the outside during the dynamic and static combined loading test process, but do not release energy.

\[ E_u = E_E + E_P \]  

(Figure 8. Change of energy absorption density with axial pressure.)
3.4. Energy Ratio

Figure 9 presents the relationship between the various energy ratios and the axial pressure, the energy ratios of three crack samples are similar, and the energy absorption rate increases with axial pressure, at the same time, energy reflectance has a rise. However, there is no significant change in energy transmittance. Take 0° crack as an example (similar to other fracture angles), as the axial pressure increases, the reflective energy is used significantly, and the energy absorption rate is gradually reduced.

3.5. The Relationship between Elastic Energy and Plastic Energy and Axial Pressure

According to the law of thermodynamics, the elastic energy density $u_k$ and plastic energy density $u_p$ can be obtained according to the dynamic stress-strain curve. In the following equations, $\sigma^*$ is the dynamic peak stress, $\varepsilon^*$ is the dynamic peak strain, $E_d$ is the dynamic elastic modulus, $E_K$ and $E_P$ are the elastic energy and plastic energy in the loading process, as shown in Figure 10.

$$ u = u_k + u_p \tag{10} $$

$$ u = \int_0^{\varepsilon^*} \sigma d\varepsilon, \quad u_k = \frac{(\sigma^*)^2}{2E_d} \tag{11} $$

$$ u_p = \int_0^{\varepsilon^*} \sigma d\varepsilon - u_k = \int_0^{\varepsilon^*} \sigma d\varepsilon - \frac{(\sigma^*)^2}{2E_d} \tag{12} $$

$$ E_K = u_k \cdot V = \frac{(\sigma^*)^2}{2E_d} \cdot V \tag{13} $$

$$ E_P = u_p \cdot V = \int_0^{\varepsilon^*} \sigma d\varepsilon - \frac{(\sigma^*)^2}{2E_d} \tag{14} $$

As shown in Figure 11a, the elastic energy stored by the rock under the combined dynamic and static loading is generated by the axial pressure and the impact load. When the impact load is constant, the elastic energy stored in the rock first increases and then decreases with the increasing of the axial pressure, this is because the impact loading time is very short and the rock sample instantly completes the two stages of energy storage and energy release. When the axial pressure reaches a certain value, the damage of the rock sample increases sharply, and the internal micro-cracks develop rapidly. The damage degree of the rock sample is small in the initial stage of the impact, and the ability to resist the impact load is not obviously, resulting in elastic energy of the rock sample increases rapidly, it indicates that the rock sample has a tendency to transform from brittleness to ductility. When the axial pressure is more than 41.9 MPa, the elastic energy decreases with the increasing of the axial pressure, it proves that the high axial pressure aggravates the internal damage, reduces its strength, and weakens the ability of energy storage. In
addition to the change of elastic energy, the rock often has a change of plastic energy during dynamic and static combined loading. As shown in Figure 11b, the plasticity energy generally decreases with the increasing of the axial pressure, it is indicated that irreversible structural changes have occurred inside the rock sample when the rock sample is irreversibly deformed under impact loading, and studying the change law of plastic energy can reveal the evolution characteristics of the internal structure of sample. When the axial pressure is relatively low, the original cracks or formed due to the pre-axial pressure are not fully closed, and they continue to close and consume more energy under impact loading. The crack of rock closes more sufficient with an increasing of the axial pressure, and the energy consumed for micro-crack closing is correspondingly reduced. The new cracks caused by the impact load continue to expand and the energy consumed at this time increases slightly with an increasing of axial pressure.

3.6. The Relationship between Energy Utilization Efficiency and between Energy Absorption Density and Compressive Strength

It can be observed from Figure 12, as the dynamic compressive strength of the sample gradually increased, the energy absorption density and energy utilization efficiency also increase. This is because the impact load is very short in the crack rock, the elasticity can
not be done in time, therefore, both of the energy absorption density and energy utilization efficiency increase with the improvement of dynamic compressive strength. When the axial pressure is small, microfitting of the rock closes, the evolution of energy absorption rate makes the intensity of the crack rock deteriorate, eventually result in structural destruction. With an increase in the axial pressure, the higher the compressive strength is, the faster the small crack of the sample expands, the remaining carrying capacity of the moving load gradually decreases, the more energy the sample absorbs before it breaks, the more reflected and transmitted energy the sample coverts during destruction, and the damage of the sample is more serious, thus the energy absorption density and energy utilization efficiency are directly related to the damage and strength deterioration of the fractured granite.

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3.7. The Relationship between Energy Absorption Density and Energy Utilization Efficiency and Average Strain Rate

Figure 13 presents the relationship between energy absorption density, energy utilization efficiency and average strain rate. The average strain rate of the specimen is defined by choosing the moment when the specimen reaches the stress uniformity to failure (or the stress peak value). The energy absorption density and energy utilization efficiency gradually increase linearly with the increasing of the average strain rate under a certain impact load, which shows an obvious rate effect. The correlation coefficient of the linear fit between the two is relatively high, it shows that among the energy absorption density, energy utilization efficiency and average strain rate have a positive correlation. When the average strain rate is approximately the same, the larger the axial compression ratio is, the smaller the energy absorption density and energy utilization efficiency are. This is because the increase in axial pressure means that the small cracks inside the sample increase and expand gradually, the sample reduces its own energy storage limit, the energy absorption density and energy utilization efficiency decrease accordingly.

3.8. Relationship between Energy Absorption Density and Energy Utilization Efficiency and Incident Energy

In order to qualitatively analyze the relationship under combined dynamic and static loading, the energy absorption density, energy utilization efficiency and incident pressures can be fitted during the different axial. Figure 14 and Table 1 show incident energy decreases with an increasing of axial pressure, and the energy absorption density of sample increases first and then decreases with increasing of the axial pressure. It can be seen that energy absorption density and energy utilization efficiency of rock sample basically follow the...
exponential function increasing law with the increasing of incident energy by nonlinear fitting, and the overall trend is on the rise.

![Graph](image_url)

**Figure 13.** Relationship between Average strain and (a) Energy utilization efficiency (b) Energy absorption rate.

![Graph](image_url)

**Figure 14.** Relationship between incident energy and (a) Energy utilization efficiency (b) Energy absorption rate.

### 3.9. Energy Dissipation of Rock under Coupled Static-Dynamic Loads

Under the action of static pre-stress, assuming that the elastic energy stored inside the rock is $E_s$, and the disturbance energy is $E_d$, when it is disturbed by the outside, then the surface energy required for rock failure under $E_s$ conditions is $E_c$, without considering the thermal energy and radiation energy generated in the process of rock failure, $E_c = \gamma S_R$ ($\gamma$ is the surface energy per unit area; $S_R$ is the total area of new crack surfaces generated by rock failure, which is an increasing function of $E_d$), that is, $E_c = \gamma S_R = R_f (E_d)$. When the $E_d$ is larger, the fragmentation of the rock increases, and the consumption of $E_c$ is larger. According to it, we propose a rock-burst occurrence criterion based on dynamic and static energy indicators ($E_s$ and $E_d$) whether there is internal elastic energy storage and release.

Under impact loading, the excited internal elastic energy is greater than the surface energy required for rock fracture, the residual elastic energy is converted into the kinetic...
energy of rock that needs to be broken and it also leads to the rock burst. The rock burst in deep rock is affected by the dual effects “Internal elastic energy and External impact “kinetic energy” according to the energy angle. It can be seen that whether there is a occurrence of rock-burst based on dynamic and static energy indicators (\(E_s\) and \(E_c\)) that released by internal elastic energy storage. And the premise of the rock-burst is that the elastic energy of the rock is partially redundant that is used for the ejection of the rock no matter how large the incident energy outside the world, the stress status and energy storage of rock can be. Therefore, the elastic energy inside rock is greater than the surface energy which is required for the rock fracture under the effect of impact loading, so there is [42],

\[
E_s - E_c > 0, \text{ rock burst occurs} \\
E_c = \gamma f
\]

Figure (b, e, h) in Table 2 describe the relationship between incident energy and the unused energy. It can be seen that, the energy partition can be separated into two regions: (1) Region I is linear and very close to 1:1, where almost all incident energy is dissipated in the form of transmitted and reflected energy (unused energy), but hardly any damage is caused to the rock sample; (2) In region II, when the incident energy is close to the critical value, the sum of transmitted energy and reflected energy increases with the increasing of incident energy, and deviates from the 1:1 line. The energy partition becomes larger, denoting that the sample absorbs less energy, eventually the unused energy reaches the upper limit. When the axial compression ratio exceeds 25% of the crack damage threshold (25% of UCS), almost all samples with different angles occur rock-burst under medium and high axial pressure. Figure (a, d and g) illustrate relationship between the absorbed energy and incident energy, it can be observed that the total absorbed energy increases with the increasing of incident energy under medium and low axial compression, while at high axial pressure, the absorbed energy gradually decreases, especially for rock samples with 45° cracks, when the axial compression is 69.8–83.8 MPa, the final absorbed energy of the sample during the experiment is negative (releasing energy), this is because the rock first accumulates a large amount of strain energy under the action of axial pressure, which is mainly used for rock spalling and formation of expansion cracks, and then continues to release the strain energy after dynamic loading, a small fraction of stored strain energy is dissipated for the growth and interaction of cracks, while the remaining part predominately delivers into elastic bars as released energy.

Table 2. Energy relations for specimens under different static axial pressure.

<table>
<thead>
<tr>
<th>Angle/°C</th>
<th>Energy Relationship</th>
<th>(E_s - E_c &gt; 0), rock burst occurs</th>
<th>(E_c = \gamma f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0° flaw</td>
<td>Incident Energy and Unused Energy</td>
<td>(E_s)</td>
<td>(E_c)</td>
</tr>
<tr>
<td></td>
<td>Absorbed Energy and Incident Energy</td>
<td>(E_s)</td>
<td>(E_c)</td>
</tr>
<tr>
<td></td>
<td>Absorbed Energy and Pre-Stress</td>
<td>(E_s)</td>
<td>(E_c)</td>
</tr>
</tbody>
</table>
4. Rock Damage Characteristics

Theoretically, the definition of the damage value of the rock samples based on the dissipated energy of crushing absorbed energy is closer to the nature of rock damage. Therefore, this paper defines a new damage variable form based on the study of granite energy absorption.

\[
D_i = \frac{U_{ih}^b}{U_{ih}^b}
\]  

(17)

In the above formula, \(U_{ih}^b\) is the energy absorption density of fractured rocks with different axial compression under the action of dynamic and static loading, and \(U_{ih}^b\) is the maximum value of energy absorption density.

Figure 15 shows the damage value of the rock samples increases continuously with the progress of the dynamic and static combined loading test, there is an obvious nonlinear relationship between the damage value and axial strain, the damage value increases with the increasing of axial pressure. And the peak damage value generated by sample in high axial pressure is faster, indicating that a certain axial preload can enhance the failure strength of rock. The damage value develops relatively gently in the initial compaction and elastic sections, increases significantly in the crack development stage, decreases slowly at the early stage of pre-peak unstable fracture and gradually approaches to 1. It can be seen that the damage value does not appear to be abnormally reduced or even “negative damage” at each stage. The damage development reflected in different stages is the same as that reflected by the stress-strain curve characteristic analysis, it can be inferred that the damage degree of rock under static and dynamic loading is reasonable.

Table 2. Cont.

<table>
<thead>
<tr>
<th>Angle/(°C)</th>
<th>Energy Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Incident Energy and Unused Energy</td>
</tr>
<tr>
<td>45° flaw</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>90° flaw</td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>
which proves that there is a similar change law between the damage value variable of the pressure. The energy utilization efficiency and energy absorption density increase linearly proportional relationship. From this, it can be concluded that the damage value of the three value increases again. It can be seen from the changing curve during the loading procedure decreases with an increase of the axial pressure ratio, but an abnormal phenomenon occurs that the damage value increases when the axial pressure is 69.8–83.8, this is because the micro-cracks inside the sample expand seriously under high axial pressure, and the damage value increases again. It can be seen from the changing curve during the loading procedure that the damage value of the sample decreases with a decrease of the absorbed energy, which proves that there is a similar change law between the damage value variable of the sample and the absorbed energy during the loading procedure, and both of them are in a proportional relationship. From this, it can be concluded that the damage value of the three different fracture specimens generally decreases with an increase of the axial pressure, the energy absorbed by the specimen during the loading process is used to initiate and expand the cracks inside a sample and gradually damages the sample until it fails.

Figure 15. Damage degree under different strain of: (a) 0° flaw; (b) 45° flaw; (c) 90° flaw.

Figure 16 shows the relationship between the damage value and the absorbed energy of the specimen during the combined dynamic and static loading. The damage value decreases with an increase of the axial pressure ratio, but an abnormal phenomenon occurs that the damage value increases when the axial pressure is 69.8–83.8, this is because the micro-cracks inside the sample expand seriously under high axial pressure, and the damage value increases again. It can be seen from the changing curve during the loading procedure that the damage value of the sample decreases with a decrease of the absorbed energy, which proves that there is a similar change law between the damage value variable of the sample and the absorbed energy during the loading procedure, and both of them are in a proportional relationship. From this, it can be concluded that the damage value of the three different fracture specimens generally decreases with an increase of the axial pressure, the energy absorbed by the specimen during the loading process is used to initiate and expand the cracks inside a sample and gradually damages the sample until it fails.

Figure 16. Relationship between damage degree and absorbed energy.

5. Discussions

Energy consumption characteristics and damage degree are the key components to study rock samples with different fractures under combined dynamic and static loading at a certain incident energy. In this study, the fracture angle and axial pressure of the rock sample have a certain influence on the energy dissipation and damage degree, which is similar to the analytical and experimental results [23,39], they showed that energy utilization efficiency and energy absorption density both decrease with the increasing of the axial pressure. The energy utilization efficiency and energy absorption density increase linearly
with the increasing of mean strain rate and increase exponentially with the increasing of incident energy [43], this is because fractured rocks have relatively with good homogeneity, and rocks can absorb more energy for crack propagation and failure. In addition, we define the sum of the reflected energy and the transmitted energy as the unused energy of the rock. Regardless of the axial pressure, the unused energy in the dynamic and static combined loading process is divided into two stages with the change of the incident energy, while the absorption energy of rock first increases and then decreases with the increasing of axial pressure. We find that rock burst occurs when the sample is under medium and high axial pressure and exceeds 25% of the crack damage stress, this is because part of the elastic strain energy will be stored in the sample under high preload, under the action of a certain impact, the rock absorbs another part of the energy, resulting in a rock burst phenomenon.

The damage variable is an important index to define the damage of rock with the change of energy during the loading process. In this paper, we define a new damage degree form, that is, the ratio of the energy absorption density to the maximum energy absorption density of the rock during the combined dynamic and static loading process. We find that the newly defined damage variable exhibits an obvious nonlinear relationship with the axial strain, and the damage degree increases with the increasing of the axial strain. Figure 17 shows the coupled effects of axial pressure and energy absorption density on rock damage. As shown in Figure 16b, for a certain crack angle, with the increasing of axial pressure, the rock damage degree shows an obvious decreasing trend, which verifies that the axial pressure has a certain inhibitory effect on the final rock damage degree. Figure 16c shows that the damage degree increases with the increasing of energy absorption density, which indicates that the energy absorbed under impact loading is further converted into elastic strain energy required for rock failure.

This study may contribute to some rock engineering practices, underground protective structure design, rock blasting and rock excavation. For deep rock structures, the failure process is controlled by the combined effects of strain energy stored in rock mass (in situ stress) and the external dynamic disturbance, in which the strain energy storage dominates the growth and interaction of micro-cracks within the rock, while the dynamic disturbance only participates as triggering source for rock dynamic response and energy supply for rock fragmentation. Based on the analysis of energy feature, the failure of rock in underground engineering can be categorized as follows: (1) The rocks without pre-stress need to absorb external energy to break them. (2) When the pre-stress is moderate and the crack density is very low, the elastic strain energy stored under the impact loading is suddenly released, resulting in the occurrence of rock-burst. (3) When the pre-stress exceeds the crack damage...
threshold, the rock experiences large deformation and has considerable strain energy storage capacity.

6. Conclusions

In this study, we performed combined dynamic and static SHPB tests on granites with different fractures to investigate the coupling effects of fracture angle and axial pressure on rock energy dissipation and damage characteristics. Under certain incident energy and impact velocity, six groups of granites with axial pressure in the range of 0% UCS, 10% UCS, 20% UCS, 30% UCS, 50% UCS and 60% UCS were tested. Main conclusions can be drawn as follows:

(1) It was found that the energy absorption density and energy utilization efficiency of the sample first increase and then decrease with the increase of axial pressure, and both of them increase first decrease and then increase with increase of the flaw angle.

(2) As a result of this study, the energy absorption density and energy utilization efficiency increase linearly with an increase of the average strain rate, which belongs to the deterioration effect of rock dynamic mechanical properties during the dynamic and static combined loading process.

(3) The energy absorption density and energy utilization efficiency increase with the increasing of the incident energy, it can also be seen from the curve fitting that both of them increase exponentially with the incident energy.

(4) The dependence of plastic and elastic energy and axial pressure was obtained, the elastic energy first increases and then decreases with the increase of axial pressure, while the plastic energy decreases with the increase of axial pressure.

(5) When the axial compression ratio exceeds 25% of the crack damage stress (accounting for 25% of the UCS), the dynamic strength of rock decreases because of the impact loading, and a small part of strain energy inside the rock is released. Overall, the fractured rock generally absorbs energy in rock bursts.

(6) As the absorbed energy decreases, the damage variable of the specimen decreases overall. The damage variable defined based on the energy absorption density can reasonably describe the damage evolution process of fractured rock under static and dynamic loading. The damage variable is relatively gentle in the initial compaction and elastic deformation stages under different axial pressures, it slightly increases in the crack development stage. After that, it decreases and gradually becomes stable.

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Data Availability Statement: Data sharing not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

\( \sigma, \varepsilon, \dot{\varepsilon} \)  
dynamic compressive stress, strain, strain rate; MPa, s\(^{-1}\);

\( \sigma^*, \varepsilon^* \)  
dynamic peak stress, dynamic peak strain MPa;

\( A_e, A_s, L_S \)  
Sectional area of the elastic rod, sectional area of the sample, length of the sample; m\(^2\), m;

\( C_e, \rho_e \)  
velocity acoustic wave, density of the rod; m/s, Kg/m\(^3\);
\( E_i, E_R \) incident, reflected energy, J;
\( E_T, E_A \) transmitted, absorbed energy, J;
\( E_k, E_p \) elastic energy, plastic energy, J;
\( E_d, N_d \) energy utilization efficiency, energy absorption density, J/cm³;
\( u_c, u_p \) elastic energy density, plastic energy density, J/cm³;
\( D_j, U^b_j, U^p_j \) damage variable, energy absorption density of fractured rocks, maximum value of energy absorption density

UCS uniaxial compressive strength, MPa;

SHPB split Hopkinson pressure bar；

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