Article

Optimal Location and Operation of PV Sources in DC Grids to Reduce Annual Operating Costs While Considering Variable Power Demand and Generation

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Abstract: Due to the need to include renewable energy resources in electrical grids as well as the development and high implementation of PV generation and DC grids worldwide, it is necessary to propose effective optimization methodologies that guarantee that PV generators are located and sized on the DC electrical network. This will reduce the operation costs and cover the investment and maintenance cost related to the new technologies (PV distributed generators), thus satisfying all technical and operative constraints of the distribution grid. It is important to propose solution methodologies that require short processing times, with the aim of exploring a large number of scenarios while planning energy projects that are to be presented in public and private contracts, as well as offering solutions to technical problems of electrical distribution companies within short periods of time. Based on these needs, this paper proposes the implementation of a Discrete–Continuous Parallel version of the Particle Swarm Optimization algorithm (DCPPSO) to solve the problem regarding the integration of photovoltaic (PV) distributed generators (DGs) in Direct Current (DC) grids, with the purpose of reducing the annual costs related to energy purchasing as well as the investment and maintenance cost associated with PV sources in a scenario of variable power demand and generation. In order to evaluate the effectiveness, repeatability, and robustness of the proposed methodology, four comparison methods were employed, i.e., a commercial software and three discrete–continuous methodologies, as well as two test systems of 33 and 69 buses. In analyzing the results obtained in terms of solution quality, it was possible to identify that the DCPPSO proposed obtained the best performance in relation to the comparison methods used, with excellent results in relation to the processing times and standard deviation. The main contribution of the proposed methodology is the implementation of a discrete–continuous codification with a parallel processing tool for the evaluation of the fitness function. The results obtained and the reports in the literature for alternating current networks demonstrate that the DCPPSO is the optimization methodology with the best performance in solving the problem of the optimal integration of PV sources in economic terms and for any kind of electrical system and size.

Keywords: DC networks; discrete–continuous metaheuristic; parallel processing tool; photovoltaic generation; variable power demand; variable renewable generation

1. Introduction

Nowadays, DC grids are widely used around the world given their different advantages in comparison with their alternate current (AC) counterparts, i.e., their easy
implementation, low investment and maintenance costs, and low operating complexity [1–4]. For these reasons, several electrical industry manufacturers and researchers have put their efforts and money into developing efficient methodologies for the operation and planning of this kind of grid, with the optimal integration of distributed energy resources currently being the topic of interest [5–8]. The optimal integration of distributed energy resources aims to locate and operate distributed generators (DGs) and energy storage systems within electrical systems in order to improve the different technical, economic, and environmental indices of DC grids [9–11]. This problem has been highly studied in alternating current networks by multiple authors, who obtained excellent results in terms of solution and processing times [12–14] and have taken advantage of different methodologies for solving problems with similar characteristics proposed in the literature [15,16], with particular adaptations to different electrical issues. However, the performance of the methodologies for AC networks reported in the literature must be evaluated in DC grids through the proposal of very new methodologies; in each of these cases, the mathematical formulations are different due to the absence of reactive or frequency components in the DC networks, which generates a completely different problem. Consequently, the optimal integration of PV sources into DC grids has become a widely studied topic in recent years [17–19]. In the specialized literature, many studies have been reported with regard to solving the problem of the optimal integration of PV sources into DC grids in order to improve technical conditions (power loss, voltage profiles, system chargeability, etc.) and reduce the environmental impact associated with fossil fuel-based generation as well as with operating and investment costs [20–22].

As for the methodologies used to improve the technical characteristics of the grids, multiple works can be found in the literature. One example is [23], which proposed the use of mixed-integer quadratic programming and the General Algebraic Modeling System (GAMS) software as a solution. In this work, the authors demonstrated the effectiveness of GAMS solvers in terms of standard deviation and processing times. However, this kind of software is often stuck in the local optima and increases the costs and complexity of the solution methodology. Other works have used integer non-linear programming methods to represent the mathematical model that could describe the optimal integration of DGs into DC grids for the reduction of power losses [24], which also requires specialized software, thus increasing the acquisition costs and the complexity of the solution. In order to minimize the implementation of specialized software, different authors have used optimization methods that were based on sequential programming and developed in free software, aiming to reduce power losses via algorithms such as the particle swarm optimization method [12] and the vortex search algorithm [25], among others [26]. The solution impact, standard deviation, and processing time from these works’ simulation results were then evaluated in order to demonstrate the effectiveness and robustness of the proposed solution methodologies.

In the last decade, several works have been reported whose objective function is to reduce CO₂ emissions in DC grids by optimally siting and sizing PV sources. These works employed optimization methods based on sequential programming and aimed to improve the quality of the solution as well as reduce processing times by avoiding the implementation of specialized software [27,28]. However, most of the works published in recent years have focused on improving economical indices such as the reduction of energy purchasing/production, investment, and maintenance costs associated with DG as these indices directly affect both users and electrical operators. In this vein, PV sources are the most developed and installed technology around the world. An example of this is the work presented in [29], which used the BONMIN solver of GAMS to solve the mathematical model that represents the problem of optimal integration of DGs in DC grids in order to reduce annual costs. This methodology remained stuck in the local optima, with short processing times and a standard deviation of zero. Similarly, in [30], a two-stage optimization process was proposed for the integration of PV and wind generators in a DC grid, with the aim of reducing production and investment costs. The results demonstrated
the effectiveness of the proposed methodology in terms of its objective function. However, the authors did not analyze the method’s repeatability and processing times. In addition, this work employed a master–slave methodology that is traditionally used in the literature to solve for the optimal integration of DGs in both DC and AC grids [31]. This methodology employs two kinds of optimization methods, i.e., discrete and continuous, thus requiring more time and increasing the complexity of the solution.

Seeking to improve the effectiveness and robustness of the solution methodologies for the problem under study, the literature has proposed modified optimization algorithms that employ discrete–continuous methodologies to solve the problem of PV sources’ optimal integration into DC grids. These techniques involve continuous optimization methods that force some variables within the solution to be discrete, thus offering a solution to the location problem while keeping the rest of the variables continuous in order to solve the DG sizing problem. An example of this is [29], where a discrete–continuous version of the vortex search algorithm for integrating PV sources in DC grids was proposed. Its aim was to reduce the annual costs associated with energy purchasing, investment, and maintenance. In this paper, the authors compared the results obtained by the proposed methodology to a discrete–continuous version of the Chu and Beasley genetic algorithm and the BONMIN solver of GAMS, which are used to solve the same problem in AC grids [32]. This work compared the average results obtained by the solution methodologies, demonstrating the effectiveness of the proposed solution, but it did not analyze the effects of all solution methods on aspects such as standard deviation and processing times. Furthermore, within the mathematical model and its validation, the voltage and branch current limits associated with the test systems were not analyzed. By using a discrete–continuous codification, ref. [33] proposed a modified version of the generalized normal distribution optimizer to solve the studied problem. In their manuscript, the authors compared the results obtained with a discrete–continuous version of the vortex search algorithm, a genetic algorithm, and the BONMIN solver of GAMS. The results obtained demonstrated the effectiveness of the proposed methodology, and all constraints related to the operation of the microgrids were satisfied. However, the proposed methodology reported longer times in comparison with other methods. Moreover, the authors did not include the analysis of standard deviation and the impact of processing times.

Based on the aforementioned works, it is possible to conclude that it is currently necessary to propose new methodologies for solving the problem of integrating PV sources into DC grids, ones that reduce complexity by implementing discrete–continuous codifications and by reducing the processing times. These must also guarantee excellent performance in terms of the objective function and standard deviation, with the aim of obtaining a solution of good quality each time that the algorithm is executed. Another objective should be to achieve shorter processing times in order to explore a large number of scenarios while planning the energy projects that are to be presented in public and private contracts, as well as to offer a solution to the technical problems in distribution electrical companies within a short period of time [34].

In light of the issues mentioned above, this study implemented a discrete–continuous parallel version of the particle swarm optimization (PSO) algorithm to solve the problem regarding the integration of PV sources into DC grids. This solution methodology employs a parallel processing tool that takes advantage of all the functions of the computer, with the purpose of reducing processing times. This methodology has been used in the literature to solve the problem of the optimal integration of PV sources in AC grids [13]; however, its performance has not been validated in DC grids. The reduction of annual costs associated with energy purchasing, investment, and maintenance in PV sources installed in DC grids was thus used as an objective function by implementing a fitness function to ensure compliance with the technical and operative constraints that represent the operation of the DC in an environment of PV sources. Furthermore, to demonstrate the effectiveness of the proposed methodology, two test systems of 33 and 69 buses were used, and we focused on the methodologies that consider the integration of distributed energy resources into
DC grids. In addition, four methods were employed for comparison (most of which were discrete–continuous, as identified in the state of the art), including the BONMIN solver of GAMS. This paper makes the following contributions:

i. A new approach for the discrete–continuous version of the particle swarm optimization algorithm;
ii. The implementation of parallel processing to solve the problem concerning the integration of PV sources into DC grids in order to reduce processing times and improve the exploration of the algorithm;
iii. The identification of the most efficient methodology to date via simulation results, which could solve the problem of the optimal integration of PV sources into AC and DC grids for annual cost reduction. This considers the results reported in [13] for AC networks.

This paper is structured as follows: Section 2 presents the mathematical model used to solve the problem of the optimal integration of PV sources into DC grids for annual cost reduction; Section 3 describes the proposed solution methodology; Section 4 describes the 33- and 69-node test systems as well as the generation and demand curves used and the considerations made in obtaining the simulation results; Section 5 analyzes the results obtained by the methodologies in terms of the solution, processing times, and repeatability; and Section 6 presents the conclusions and future works derived from this research.

2. Mathematical Formulation

This section presents the mathematical formulation of the problem concerning the integration of PV sources into DC grids for the reduction of energy production/purchasing costs associated with conventional generators (electrical grid, DIESEL generators, among others) as well as the initial investment and maintenance costs related to PV generators. Furthermore, this mathematical model includes all constraints related to the technical and operating constraints of DC grids in the context of PV sources.

2.1. Objective Function

The objective function is presented in Equation (1). This equation considers the minimization of the annual costs associated with energy purchasing in relation to conventional generators \( f_1 \) as well as to investment and maintenance with regard to the installation of PV sources \( f_2 \).

\[
    OF = \min\text{Annual costs} = \min(f_1 + f_2)
\]

To calculate the annualized energy purchasing costs while considering the lifetime of the PV generators and the increase in power demand, Equation (2) was used, where \( C_{kWh} \) corresponds to the cost of each kWh, \( T \) represents the number of days in a regular year (365), and \( F_a \) is the factor that annualizes the total energy purchasing/production costs by conventional generators installed in the DC grid. In Equation (3), \( t_a \) represents the fixed return rate for the investment made in the integration of the PV generators, \( N_t \) corresponds to the number of years contained inside the useful life of the PV sources, and \( F_c \) corresponds to the annual increase in power demand within the planning horizon. This factor is described in Equation (4), where \( t_e \) represents the increase in the energy purchasing cost within the analyzed time (expressed as a percentage); \( p_{i,h}^{cg} \) represents the power supplied by the conventional generator located at node \( i \) in the period of time \( h \), and \( \Delta h \) is the duration of the said power supply. Finally, \( \Omega_N \), \( \Omega_H \), and \( \Omega_T \) represent the set of nodes that make up the DC grid, the total periods of time considered for a day of operation, and the useful life of the PV generators (years).

\[
    f_1 = C_{kWh}TF_aF_c \left( \sum_{i \in \Omega_H} \sum_{h \in \Omega_N} p_{i,h}^{cg} \Delta h \right)
\]
\begin{align*}
F_a &= \left( \frac{t_a}{1 - (1 + t_a)^{-N_t}} \right) \\
F_c &= \left( \sum_{i \in T} \left( \frac{1 + t_e}{1 + t_a} \right)^{t_e} \right) \tag{3}
\end{align*}

Equation (5) calculates the investment and maintenance costs associated with the integration of PV sources into the DC grid, where \( C_{pv} \) corresponds to the cost per kW of the PV sources, \( p_{pv}^i \) represents the total PV power installed at bus \( i \), \( C_{pv}^{O&M} \) denotes the maintenance cost of the PV generators by kW generated, and \( p_{i,h}^{pv} \) is the power supply of the PV sources located at bus \( i \) for the period of time \( h \). In this equation, \( \Omega_{pv} \) denotes the set of buses that contain PV sources.

\begin{align*}
F_2 &= C_{pv} F_a \left( \sum_{i \in \Omega_{pv}} p_{pv}^i \right) + T \left( \sum_{i \in \Omega_i} \sum_{i \in \Omega_{pv}} C_{pv}^{O&M} p_{i,h}^{pv} \Delta h \right) \tag{5}
\end{align*}

2.2. Constraints

The set of constraints that represent the problem of the optimal integration of PV sources into DC grids while considering variable power demand and generation is presented in Equations (6)–(11).

\begin{align*}
p_{i,h}^{cg} + p_{i}^{pv} C_{h}^{pv} - p_{d}^i &= v_{i,h} \sum_{j \in \Omega} G_{ij} v_{j,h} \tag{6} \\
p_{i}^{cg,\text{min}} &\leq p_{i}^{cg} \leq p_{i}^{cg,\text{max}} \tag{7} \\
x_i p_{i}^{pv,\text{min}} &\leq p_{i}^{pv} \leq x_i p_{i}^{pv,\text{max}} \tag{8} \\
\sum_{i \in \Omega_{pv}} x_i &\leq N_{pv}^{dev} \tag{9} \\
v_{i,\text{\min}} &\leq v_{i,h} \leq v_{i,\text{\max}} \tag{10} \\
I_{i,j,h} &\leq I_{ij}^{\text{max}} \tag{11}
\end{align*}

Equation (6) represents the power balance of the electrical grid. Here, \( p_{i}^{cg} \) denotes the active power demanded at node \( i \) in the period of time \( h \), \( C_{h}^{pv} \) is the factor that determines (from 0 to 100%) the power production of PV sources in relation to the radiance potential of the region where the electrical system is located, \( v_{i,h} \) and \( v_{j,h} \) represent the voltage profiles at buses \( i \) and \( j \) in period of time \( h \), and \( G_{ij} \) denotes the conductance value related to the branch that connects buses \( i \) and \( j \). Equation (7) formulates the constraint that ensures that the minimum \( (p_{i}^{cg,\text{min}}) \) and maximum \( (p_{i}^{cg,\text{max}}) \) power are supplied by the conventional generator located at node \( i \). Equation (8) establishes the minimum \( (p_{i}^{pv,\text{min}}) \) and maximum \( (p_{i}^{pv,\text{max}}) \) power bounds for the PV sources located at node \( i \), where \( x_i \) corresponds to a binary variable that takes a value of 1 when a PV source is located at node \( i \) and a value of 0 when it is not. In this way, Equation (8) limits the maximum number of PV sources \( (N_{pv}^{dev}) \) to be located in the DC grid. Equation (10) describes the constraints related to the voltage profile limits, where \( V_{i,\text{\min}} \) and \( V_{i,\text{\max}} \) correspond to the minimum and maximum voltage profiles allowed at node \( i \). Finally, Equation (11) represents the maximum branch limit, where \( I_{i,j,h} \) and \( I_{ij}^{\text{max}} \) correspond to the branch and maximum current allowed for the branch that connects buses \( i \) and \( j \).

\begin{align*}
FF &= \min (f_1 + \beta PF) \tag{12}
\end{align*}
\begin{equation}
PF = \begin{cases} 
\max \left\{ \sum_{i \in N} \left( p_i^{PV} - p_i^{PV,max} x_i^{DG} \right) \right\} \\
+ \min \left\{ \sum_{i \in N} \left( p_i^{PV} - p_i^{PV,min} x_i^{DG} \right) \right\} \\
+ \max \left\{ \sum_{i \in N} (v_i - V_i^{max}) \right\} \\
+ \min \left\{ \sum_{i \in N} (v_i - V_i^{min}) \right\} \\
+ \max \left\{ \sum_{i \in N} \sum_{j \in N} (I_{ij} - I_{ij}^{max}) \right\} \\
+ \max \left\{ \sum_{i \in N} x_i - N_{dev}^{PV} \right\} 
\end{cases}
\end{equation}

In order to guarantee all the previously described constraints and improve the exploration of the solution space, this paper used the fitness function presented in (12), which allows the penalization of the objective function when some constraints are violated by permitting the solution methodology to explore infeasible regions. This helps to improve the solution and reduce processing times [35]. In FF, \( \beta \) is entrusted with normalizing the values calculated by the penalty Factor (PF) described in Equation (13). For this manuscript, \( \beta = 1000 \), which is obtained through a heuristic process.

3. Proposed Solution Methodology

This paper used a parallel discrete–continuous version of the PSO method [31] to solve the problem regarding the optimal integration of PV sources into DC grids. This solution methodology resolves the discrete–continuous codification that describes the problem. It avoids the use of the traditional master–slave strategy used in the specialized literature for siting and sizing DGs in electrical networks [34,36], which requires two solution methods: a binary/discrete optimization method to solve the location problem and a continuous optimization method to solve the sizing problem. This offers a solution to the problem, but it results in increased complexity and processing times.

The discrete–continuous codification used here is illustrated in Figure 1. It can be seen that the location and power values (sizing) of the different PV sources in the DC grid are included in the same vector of size \( 1 \times 2N_{dev}^{PV} \). The discrete variables are associated with the location problem, and the continuous variables are related to the sizing of the DGs. Note that in the same figure, the DGs located at node 4 have a nominal power of 0.19 Kw, while the PV sources installed at node 41 have a nominal power of 2.4 kW.

![Half of vector size](image)

<table>
<thead>
<tr>
<th>Node location of PV DGs</th>
<th>4</th>
<th>10</th>
<th>41</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sizing of PV DGs</td>
<td>0.19</td>
<td>1.25</td>
<td>2.4</td>
</tr>
</tbody>
</table>

**Figure 1.** Codification used for the optimal integration of PV sources.

To solve the problem concerning the optimal integration of PV sources into DC grids via the aforementioned codification, this paper used the DCPPSO. This modified version of PSO discretizes the variables related to the location for each particle by using the number of candidate buses in the electrical system, and it allows for the variables associated with PV source sizing to remain continuous. Furthermore, the modified PSO uses parallel processing to evaluate the FF of each particle, which enables the reduction of processing times. It is important to highlight that in order to evaluate the FF, it is necessary to use an hourly power flow (HPF) that allows for the inclusion of variable power demand and generation related to PV sources, which is caused by the variation in the solar radiance in
the region where the electrical system is located [34]. The pseudo-code that describes the DCPPSO/HPF methodology is presented below:

It can be observed that the first step of Algorithm 1 consists of reading all the DC grid data and the parameters associated with DCPPSO. A maximum number of iterations (iter\(_{max}\)) is set as a stopping criterion. Then, the first iteration generates the initial particle swarm via the codification presented in Figure 1 while considering random values between the number of buses and the power limits assigned to the DGs. Afterwards, the FF of each particle is calculated using Equation (12), for which an hourly power flow based on the successive approximations method is used [35]. Every hour, this method updates the power demand and PV generation during an average operation day in order to calculate the objective function described in Equation (1), thus taking variable demand and generation into account.

**Algorithm 1:** Pseudo-code for the DCPPSO/HPF

```plaintext
Data: Read DC grid data and optimization parameters

for iter = 1 : iter\(_{max}\) do
    if iter == 1 then
        Generate the particle swarm;
        Calculate the FF for each particle by using the HPF;
        Select the FF and position obtained by each particle as the best particle solution and position;
        Select the best solution in the swarm and its position as the incumbent;
    else
        Calculate the velocity vector (VV);
        Update the position of the particle swarm by using the last position, the incumbent, and the VV;
        Calculate the FF for each particle by using the HPF;
        Update the best particle solution and position;
        Update the incumbent;
        if The stopping criterion has been met? then
            Finish the optimization process;
            Print the incumbent;
            Break;
        else
            Continue;
        end
    end
end
```

The base of the Particle Swarm Optimization algorithm (PSO) is the use of a population to explore the solution space in each iteration, taking advantage of the social and cognitive knowledge to converge on a solution of good quality. In each iteration of the algorithm, it is necessary to evaluate the FF of each particle that makes up the population, which requires long processing times. The proposed methodology considered the implementation of a parallel processing tool that uses all Workers (W) of the computer to evaluate as individuals the number of workers that exist, which reduces the processing time [35]. In order to carry out this task, a highly used tool in the literature called “parfor” of Matlab [13] was employed, which made the parallel evaluation of the FF that comprises the population possible by allowing for the reduction of the processing times inside the iterative process. In this parallel processing, the PC employs all workers to evaluate the different particles that make up the swarm in groups with a size equal to W, executing as many processes as necessary to evaluate all particles in the swarm. The time required for this task can be calculated by means of Equations (14) and (15). The former allows obtaining the number of parallel processes (NPP) required for evaluating all particles in the swarm, while the latter
calculates the total processing time required to carry out the process (PPT). Here, MPT represents the maximum processing time required for all particles to be evaluated [35].

\[
NPP = \text{CEIL}(n/W)
\]

\[
PPT = NPP \cdot MTRP
\]

After evaluating the FF of all particles, in the first iteration, the solution and position of each particle are assigned as the best values. Furthermore, the particle with the best solution is selected as the incumbent of the problem (the best solution) by storing its position and FF.

From the second iteration until the end of the process, each iteration calculates the velocity vector (VV) by using random values, as well as the position information of each particle (cognitive knowledge) and the incumbent (social knowledge). Subsequently, the particle movement is generated by using the VV and the current position. Then, the FF of the swarm is calculated, and the best particle position and solution as well as the incumbent are updated. At the end of each iteration, the stopping criterion is analyzed. In this particular case, it is verified whether the maximum number of iterations has been achieved. If this has occurred, the optimization process ends; if not, another iteration is carried out.

4. Test Scenarios and Considerations

4.1. Test Systems

In this paper, the DC versions of the 33- and 69-bus test systems were used, which are illustrated in Figure 2. These are widely used for validating planning strategies in DC grids [33].

Figure 2a illustrates the first test system, which comprises 33 buses and 32 branches. It employs a voltage of 12.66 kV and a power of 100 kW as base values. The parametric information of this test system is described in [37]. To obtain the branch current limits for this test system, the HPF described in the last section was used while considering the power demand and generation curves for Antioquia, Colombia (Figure 3). Consequently, the maximum current allowed for this test system was 310 A, and the electrical conductor was 350 kcmils.

Figure 2b presents the second test system employed in this research. It comprises 69 buses and 68 branches and uses the same base values as the 33-bus grid [37]. To obtain the branch current limits, the same methodology was employed, which provided a maximum allowed current of 335 A for an electrical conductor with a caliber of 400 kcmils.

4.2. Power Generation and Demand

To estimate the average impact on the annual costs of the electrical grid, this study considered the power demand and generation of PV sources in the region of Antioquia,
Colombia for an average day of operation. Figure 3 describes this behavior for a period of 24 h.

Figure 3. Typical power demand and PV generation behavior in Antioquia, Colombia [35].

4.3. Comparison and Considerations

One of the main objectives of this paper is to demonstrate the effectiveness of the proposed methodology in terms of the solution, repeatability, and processing times. To this effect, four comparison methods were selected from the specialized literature, which used specialized software and sequential programming optimization methods to solve the studied problem. The first solution method was the BONMIN solver of GAMS, the second was a discrete–continuous version of the Chu and Beasley genetic algorithm, the third was a discrete–continuous version of the vortex search algorithm (DCVSA), and the fourth was a discrete–continuous version of the generalized normal distribution optimizer (DCGNDO). The selection of these methods was based on the fact that they have all been used to solve the problem regarding the optimal integration of PV sources into DC grids, and that they have been evaluated in the same test systems and conditions as those used in this paper [29,33]. Furthermore, most of these comparison methods take advantage of the discrete–continuous codifications used by the proposed methodology (DCPPSO).

The main considerations and information used to validate the effectiveness and robustness of the proposed methodology in relation to the comparison methods are described below:

- All parameters used to evaluate the effects of PV source methods are presented in Table 1 [29].
- The maximum number of PV sources considered for installation was 3, and the maximum power capacity was 2.4 p.u. [29].
- The maximum allowed oscillation for the voltage profiles was \(+/-10\%\) of each test system’s nominal voltage.
- Both test systems considered non-telescopic grids, for which the maximum current in all branches corresponds to the maximum current allowed: 310 and 350 A for 33- and 69-bus test systems, respectively.
- The optimization parameters of the comparison methods were taken from the original paper [29,33], while the parameters of the DCPPSO are reported in Table 2.
- To evaluate the average processing time and the repeatability of the proposed methodology, each technique was executed 100 times while also analyzing the standard deviation obtained.
The simulations were carried out in a Dell Precision T7600 Workstation with an Intel(R) Xeon(R) CPU ES-2670 @2.50 GHz and 32 GB of RAM while running the Matlab software, version 2022a.

### Table 1. Parameters for the optimal integration of PV sources into electrical grids.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Value</th>
<th>Unit</th>
<th>Param.</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{dWh}$</td>
<td>0.1390</td>
<td>USD/kWh</td>
<td>$T$</td>
<td>365</td>
<td>days</td>
</tr>
<tr>
<td>$f_a$</td>
<td>10</td>
<td>%</td>
<td>$N_l$</td>
<td>20</td>
<td>years</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>2</td>
<td>%</td>
<td>$C_{r,bom}$</td>
<td>0.0019</td>
<td>USD/kWh</td>
</tr>
<tr>
<td>$N_{pv}$</td>
<td>3</td>
<td>-</td>
<td>$\Delta V$</td>
<td>$\pm 10$</td>
<td>%</td>
</tr>
<tr>
<td>$s_{pv, min}$</td>
<td>0</td>
<td>kW</td>
<td>$s_{pv, max}$</td>
<td>2400</td>
<td>kW</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>$100 \times 10^4$</td>
<td>USD/V</td>
<td>$\alpha_2$</td>
<td>$100 \times 10^4$</td>
<td>USD/V</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>$100 \times 10^4$</td>
<td>USD/W</td>
<td>$\alpha_4$</td>
<td>$100 \times 10^4$</td>
<td>USD/A</td>
</tr>
</tbody>
</table>

### Table 2. DCPPSO parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>33-Bus Test System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of particles ($N_i$)</td>
<td>100</td>
</tr>
<tr>
<td>Maximum iterations ($t_{max}$)</td>
<td>1000</td>
</tr>
<tr>
<td>Maximum inertia</td>
<td>0.4133</td>
</tr>
<tr>
<td>Cognitive component</td>
<td>1.96236</td>
</tr>
<tr>
<td>Social component</td>
<td>2</td>
</tr>
</tbody>
</table>

### 5. Simulation Results

Tables 3 and 4 present and analyze the simulation results obtained with the proposed methodology and the comparison methods in the 33- and 69-bus test systems. This table shows, from left to right, the following information: the methodology used, the PV source location and capacity, the total annual costs, the reduction achieved with respect to the baseline case (the scenario without PV sources), the average processing time, the standard deviation, the worst voltages, and the maximum current.

### Table 3. Simulation results obtained via different methodologies in the 33-bus test system in both AC and DC grids.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Bus/Power (MVar)</th>
<th>$A_{cost}$ (USD/year)/Reduction (%)</th>
<th>Time (s)</th>
<th>STD (%)</th>
<th>$V_{worst}$ (p.u)</th>
<th>$I_{max}$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline case</td>
<td>[0–2.4]</td>
<td>3,644,043.01</td>
<td>- - -</td>
<td>- - -</td>
<td>[0.9–1.1]</td>
<td>310</td>
</tr>
<tr>
<td>BONMIN</td>
<td>18/1.4301 32/2.0611 33/1.7155</td>
<td>2,664,089.12/26.8919</td>
<td>0.77</td>
<td>0</td>
<td>0.93</td>
<td>304</td>
</tr>
<tr>
<td>DCCBGA</td>
<td>11/1.1629 14/0.9434 31/1.4827</td>
<td>2,662,724.82/26.9293</td>
<td>2.43</td>
<td>0.0557</td>
<td>0.93</td>
<td>304</td>
</tr>
<tr>
<td>DCVSA</td>
<td>9/0.5803 15/1.2913 31/1.7155</td>
<td>2,662,425.32/26.9375</td>
<td>76.86</td>
<td>0.0620</td>
<td>0.93</td>
<td>304</td>
</tr>
<tr>
<td>DCGNDO</td>
<td>10/0.9742 16/0.9202 31/1.6925</td>
<td>2,662,371.59/26.9390</td>
<td>166.15</td>
<td>0.0601</td>
<td>0.93</td>
<td>304</td>
</tr>
<tr>
<td>DCPPSO</td>
<td>10/0.9680 16/0.9189 31/1.6999</td>
<td>2,662,371.59/26.9390</td>
<td>8.52</td>
<td>0.0398</td>
<td>0.93</td>
<td>304</td>
</tr>
</tbody>
</table>


Table 4. Simulation results obtained by solution methods in the 69-bus test system.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Bus/Power (MVAR)</th>
<th>$A_{\text{cost}}$ (USD/year)/Reduction (%)</th>
<th>Time (s)</th>
<th>STD (%)</th>
<th>$V_{\text{worst}}$ (p.u)</th>
<th>$I_{\text{max}}$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>[0–2.4]</td>
<td>3,817,420.38</td>
<td>- - -</td>
<td>- - -</td>
<td>[0.9–1.1]</td>
<td>335</td>
</tr>
<tr>
<td>BONMIN</td>
<td>21/0.4971, 61/2.3999, 65/0.8530</td>
<td>2,785,208.63/27.0395</td>
<td>2.02</td>
<td>0</td>
<td>0.93</td>
<td>304</td>
</tr>
<tr>
<td>DCCBGA</td>
<td>19/0.7908, 61/1.7890, 64/1.1474</td>
<td>2,785,598.84/27.0292</td>
<td>7.74</td>
<td>0.1289</td>
<td>0.93</td>
<td>319</td>
</tr>
<tr>
<td>DCVSA</td>
<td>23/0.7720, 62/2.3402, 63/0.6185</td>
<td>2,785,538.58/27.0308</td>
<td>269.22</td>
<td>0.0974</td>
<td>0.93</td>
<td>319</td>
</tr>
<tr>
<td>DCGNDO</td>
<td>19/0.4969, 61/2.3999, 64/0.8470</td>
<td>2,785,011.53/27.0446</td>
<td>376.88</td>
<td>0.2384</td>
<td>0.93</td>
<td>319</td>
</tr>
<tr>
<td>DCPPSO</td>
<td>22/0.5310, 61/2.4, 64/0.8105</td>
<td>2,784,987.68/27.0452</td>
<td>28.24</td>
<td>0.0226</td>
<td>0.93</td>
<td>319</td>
</tr>
</tbody>
</table>

5.1. 33 Bus Test System

In the results presented in Table 3, it can be observed that all solution methodologies reduce the annual costs by 26.92% on average in comparison with the baseline case. However, the DCPPSO obtained the best results. Figure 4 illustrates the improvement achieved with this methodology, which obtained reductions of $6.44 \times 10^{-2}\%$, $1.32 \times 10^{-2}\%$, $2 \times 10^{-3}\%$, and 0 when compared to the BONMIN, DCCBGA, DCVSA, and DCGNDO techniques, respectively. Thus, the average annual cost reduction was $2.62 \times 10^{-2}\%$. It is possible to conclude that DCGNDO achieved the same results as DCPPSO in this test system.

![Figure 4. Improvements regarding annual cost reduction achieved by the DCPPSO with respect to the comparison methods in the 33-bus test system.](image)

As for the processing times, the fastest method was the BONMIN solver, followed by the DCCBGA (Figure 5). These methods required 95.41% and 85.54% less time than the DCPPSO. In the 33-bus test system, the DCPPSO was in third place, with a reduction

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**Table 4**. Simulation results obtained by solution methods in the 69-bus test system.
of 78.12% and 89.88% in processing times when compared to the DCVSA and DCGNDO. Despite these results, DCPSO had the best performance in terms of the solution. Figure 5 shows that the proposed methodology obtained the best results in terms of repeatability when compared to the solution methods based on sequential programming; it achieved an average standard deviation (STD) reduction of 48.80%. It is important to highlight that the BONMIN solver showed an STD of 0% since it belongs to the family of specialized software that guarantees the same solution each time that the algorithm is executed, although it has disadvantages in terms of purchase costs and implementation complexity [35,38].

Finally, columns 6 and 7 of Table 3 show that all solution methods satisfy the voltages and branch current bounds.

Figure 5. Improvements obtained by the DCPPSO with respect to the comparison methods in the 33-bus test system in terms of processing time and standard deviation.

5.2. 69-Bus Test System

The results obtained for the 69-node test system are shown in Table 4. Figure 6 illustrates the improvements achieved by the DCPPSO with respect to the comparison methods. Figure 6 shows that the proposed methodology obtained the best results in terms of annual cost reduction, with an average reduction of $1.24 \times 10^{-2}$%.

As for the processing times (Figure 7), DCPSO was in third place, being outperformed by BONMIN and DCCBGA by 92.81% and 72.59%, respectively. However, DCPSO was superior to the DCVSA and DCGNDO, reducing the processing times by 89.51% and 92.50%. When comparing these results with those obtained in the 33-bus test system, it can be observed that the differences with respect to the faster methods (BONMIN and DCCBGA) were reduced, while the quality of the solution was also the best. The proposed methodology achieved STD reductions of 82.46, 82.46, and 82.46% with respect to the DCCBGA, the DCVSA, and DCGNDO, respectively, (i.e., 83.23% on average). In this test system, the BONMIN solver reported an STD equal to 0, given its aforementioned characteristics. Finally, columns 6 and 7 of Table 4 demonstrate that all methodologies satisfied the voltage and current branch bounds established for the 69-bus test system.
Figure 6. Improvements in the reduction of annual costs as achieved by DCPPSO with respect to the comparison methods in the 69-bus test system.

Figure 7. Improvements obtained by the DCPPSO with respect to the comparison methods in the 69-bus test system in terms of processing time and standard deviation.

6. Conclusions and Future Work

This paper proposed a discrete–continuous version of the particle swarm optimization algorithm to solve the problem regarding the optimal integration of PV sources into DC grids, with the purpose of reducing the energy purchasing/production costs associated with conventional generators as well as the investment and maintenance costs related to
distributed generation. To evaluate the effectiveness, repeatability, and robustness of this solution methodology, four comparison methods and two test systems (33 and 69 buses) were used, which executed each methodology 100 times.

The advantages that the DCPPSO have are related to the implementation of a discrete–continuous codification that allows for the resolution of discrete and continuous problems at the time of its execution, which considers the discrete variables related to the location problem in this particular case as well as the continuous variables associated with the sizing problem of PV distributed generators on the DC grid. Furthermore, this methodology takes advantage of the PSO, a metaheuristic optimization technique that has been highly used in the literature to solve non-linear problems [12,35] due to its excellent effectiveness and robustness in solving non-linear and non-convex problems, such as those addressed here. Finally, inside the DCPPSO proposed, a parallel processing tool for evaluating the objective function of the population in each iteration was used, which allowed for the reduction of the processing time required for the solution methodology; thus, it was possible to present a solution of excellent quality with short processing times.

As for the results obtained in the 33-bus test system, even though DCPPSO was not the fastest method, it yielded excellent results in terms of processing times, with an average of 16.81 seconds. This method also achieved the best result in terms of the quality of the solution (annual cost reduction), with an average reduction of 0.0199% in comparison with the other methods. Furthermore, the proposed methodology was in second place in terms of the standard deviation, with a reduction of 48.80% when compared to the other discrete–continuous methodologies; it was only surpassed by the BONMIN solver of GAMS, a commercial software that increases the complexity and cost of solving the studied problem. Based on these results, the DCPPSO is considered to be an excellent methodology for the optimal integration of PV sources into small DC grids, with excellent results in terms of processing times and standard deviation.

For the 69-bus test system, the simulation results showed a similar behavior. Regarding the solution impact, DCPPSO obtained the best results, with an average reduction of 0.0126% with respect to the other methods. It also obtained the best results in terms of standard deviation; it was only surpassed by the BONMIN solver. As for the processing times, the DCPPSO was also in third place, taking longer than the BONMIN solver and the DCCBGA. This was due to the fact that BONMIN is a commercial solver and that the DCCBGA does not operate similarly to the other discrete–continuous solution methods; it does not work with population in its iterative process, which reduces the processing times and negatively impacts the quality of the solution. With respect to the other discrete–continuous methods, DCPPSO obtained an average processing time reduction of 91%. These methods were the most efficient methods after DCPPSO with regard to the quality of the solution. Based on the last results, the DCPPSO is the solution methodology with the best outcome in terms of the solution (annual cost reduction), as it showed excellent performance in terms of standard deviation and processing times for DC grids of any size. Furthermore, the results reported in [13] concluded that the DCPPSO is the most effective methodology for solving the problem of the optimal integration of PV sources in AC networks of different sizes. It can thus be concluded that to date, the DCPPSO is the solution methodology with the best trade-off in economic terms to solve the integration problem of PV sources in any kind of electrical network, with reduced processing times. The above-mentioned conclusions are based on the state-of-the-art knowledge acquired by the authors.

The proposed methodology works in other DC networks that consider the main generator, distribution system load, and PV distributed generators due to the mathematical model used, which considered all technical and operative constraints that represent the DC grids in an environment of PV distributed generations. However, in the particular case when other distributed energy resources are considered, e.g., energy storage systems, all equations that model this kind of electric devices must be included in the mathematical
formulation with the aim of guaranteeing an adequate representation of the electrical networks used.

Future work could consider the implementation of new solution methodologies and other parallel processing tools that allow for the improvement of the results in terms of the quality of the solution and processing times. In addition, the mathematical model could include other technical, economic, and environmental indices related to the operation of the DC grid, such as the power losses produced in the transport of energy, voltage stability, load chargeability, reductions in fossil fuel-based energy production and CO₂ emissions, among others. By including the technical, economic, and environmental aspects of the grid, it is possible to consider a multi-objective optimization algorithm in solving the problem of the optimal integration of PV sources into the DC grid. Finally, DCPPSO could be used to solve the problem concerning the optimal integration of energy storage systems into DC grids by taking advantage of the power of PV sources at times without solar production, which would improve the technical, economic, and environmental conditions of electrical networks.

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Conflicts of Interest: The authors of this paper declare no conflicts of interest.

Acronyms

PV Photovoltaic
MINLP Mixed-Integer Non-Linear Programming
GAMS General Algebraic Modeling System
DCCBGA Discrete–Continuous Chu and Beasley Genetic Algorithm
DCNMA Discrete–Continuous Newton Metaheuristic Algorithm
DCVSA Discrete–Continuous Vortex Search Algorithm
DCGNDO Discrete–Continuous Generalized Normal Distribution Optimizer
DCPPSO Discrete–Continuous Parallel Particle Swarm Optimization

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